SOME APPLICATION OF ORDER STATISTICS TO THE EDUCATIONAL STATISTICS

BY HIROJIRO AOYAMA
INSTITUTE OF STATISTICAL MATHEMATICS

In this paper we treat two applications of order statistics clarifying the meaning of the educational evaluation. The first problem is that of evaluation of pupils and the second is that of estimation of the variances in sub-populations.

1. Recently the so-called 5 point system is adopted in the educational evaluation. The principle of this method lies in that the distribution of the score of pupils is the normal one and that the domain of the score is divided by four points $\bar{x}-1.5\sigma$, $\bar{x}-0.5\sigma$, $\bar{x}+0.5\sigma$ and $\bar{x}+1.5\sigma$, the mean being $\bar{x}$, and the standard deviation, . These sub-domains are called 1,2,3,4 and 5 respectively. This method has some questionable points. We have shown the meaning of the evaluation of the pupils' ability and one method of treating.

When we evaluate the ability of n pupils in a class, let X_i be the ability of the ith pupil, which is measured with the variance σ_i^2. Suppose that we test everyone s times independently. Let x_{ik} $(k=1,2,\dots\dots,s)$ be the score of the ith pupil in the kth trial. Then the estimate of X_i is

$$\xi_i=\frac{1}{s}\sum_{k=1}^{s}x_{ik} \tag{1}$$

And the estimated variance σ^2 is

$$\frac{1}{n}\sum_{i=1}^{n}(\xi_i-\bar{\xi})^2-\frac{n-1}{n^2s(s-1)}\sum_{i=1}^{n}\sum_{k=1}^{s}(x_{ik}-\xi_i)^2 \tag{2}$$

So we can divide the domain of the score by the estimated σ with the center $\bar{\xi}$. If the size of population is N and the sample size is n, we can put, instead of (2),

$$\frac{N-1}{N(n-1)}\sum_{i=1}^{n}(\xi_i-\bar{\xi})^2-\frac{N-1}{Nns(s-1)}\sum_{i=1}^{n}\sum_{k=1}^{s}(x_{ik}-\xi_i)^2 \tag{3}$$

If the ~~distribution of the score is the normal~~ one, and the second term of (3) can be neglected, we can easily estimate σ by means of the range.

Let the maximum score be M, and the minimum score, L, then the range is

$$R_n = M - L$$

Let the mean of the range be $\overline{R_n}$, and sample size, 50, then

$$\overline{R_n} \doteqdot 4.5\sigma \tag{4}$$

So we can divide the domain by four points $\bar{x}-3D, \bar{x}-D, \bar{x}+D$ and $\bar{x}+3D$, with

$$D = \frac{\sigma}{2} = \frac{\overline{R_n}}{9} \doteqdot \frac{M-L}{9} \tag{5}$$

When the distribution of the score is ~~not normal~~ and we can assume the triangular distribution in interval which is represented as

$$f(x) = \frac{h}{c}x, \text{ for } 0 \leqq x \leqq c$$
$$= \frac{h}{b-c}(b-x), \text{ for } c < x \leqq b \tag{6}$$

with $\int_0^b f(x)\,dx = 1$, we get, putting $\frac{hc}{2} = \sin^2\theta_0$,

$$\overline{R_n} = \sqrt{\frac{2c}{h}}\left(\sin\theta_0 - \frac{\sin^{2n+1}\theta_0}{2n+1} - \int_0^{\theta_0}\cos^{2n+1}\theta\, d\theta\right) + \sqrt{\frac{2(b-c)}{h}}\cos\theta_0\Big\{\Big(1-\frac{1}{2n+1}\Big)$$
$$- \sum_{k=0}^{n-1}\frac{2n(2n-2)\cdots(2n-2k)\sin^{2k+2}\theta_0}{(2n+1)(2n-1)(2n-3)\cdots(2n-2k-1)} - \frac{2n(2n-2)\cdots2}{(2n+1)(2n-1)\cdots3}\cos^{2n}\theta_0 \cdots (7)$$

From this formula we get the following table from which we can get the similar results to (4)

	n	2	5	10	50	∞
$\frac{\overline{R_n}}{\sigma}$	Normal	1.128	2.3259	3.0775	4.498	
	($b=100$, $c=50$)	1.2247	3.0740	4.782	4.875	4.899
	($b=100$, $c=81$)	1.2339	3.9468	4.570	4.603	4.612

REPORT OF THE CURRICULUM SURVEY

(PART II. ANALYSIS OF DATE)

H. AOYAMA, S. NISHIHIRA AND N. ZAMA

This report is Part II of the Curriculum Survey which was already reported in No. 3 of this series. We chiefly treat the analysis of data obtained from questionnaires. The parts which are not treated in this report may be referred to in the Reports of the National Institute for Educational Research. Owing to some circumstances, the second survey based on the inspection method was not made in the whole country, but only in Kanto Area. The tests for pupils could not also be carried out. Therefore the discussion of the reliability and validity of this survey made by questionnaires holds true only to some extent.

The report consists of the following paragraphs:

§1. Collection of the questionnaires
§2. Fundamental plan of analysis
§3. On the reliability of this survey
§4. Some remarks of analysis in stratification
§5. Simple and cross tabulations
§6. Influence of the factors on the types of schools and the unit system
§7. Quantification of the types of schools
§8. Effect of the stratification
§9. Others

The collection-rate of the questionnaires reached about 80% and the following-up of the non-responsed schools was done only in Kanto Area. The results showed some heteregeneous features of the non-responsed schools but we recongnized it almost appropriate to handle the responsed schools only.

In this analysis we made 3 grades of unit system, and 3 grades of extra-curricular activities. These grades combined, we made up 5 types of curriculum. On the other hand, we analytically divided the schools into 128 types according to grades of social studies, science, language, mathematics, student-body-activities, club movement and participation in main events of school life.

The above two kinds of classification did not differ from each other on some assumptions. Therefore we analysed the data using the types made by synthetic method. Fundamental plan of analysis of data is shown in § 2.

In § 3., we compared some results with those obtained from the designated statistics issued by the Ministry of Education and from statistics of other sourses. It can safely be said that our data came from the reasonable and representative samples.

In § 4, the relation between the stratification on the basis of the number of pupils and that on the basis of the number of classes was discussed.

In § 5, we showed the tables of some influential factors, the simple tabulated data of unit system and of extra-curricular activities and the relation between these factors.

In § 6, we discussed the influence of the factors upon the types of schools and unit system and found the greatest influential factor was size of schools whose coefficient of correlation was not so large. This results hold true not so much in primary schools as in lower secondary schools.

In § 7, the method of quantification ofthe types of schools based on their size was treated. Making the coefficient of correlation largest, we quantificated their types and sizes, or the former only, and then got the regression plane which made us estimate the types from selected factors.

In § 8, we treated the effect of the stratification in our sampling. Considering the types of schools only, we found the effect was almost from 2 to 11%.

In § 9, we discussed some related items, such as responsibilities of teachers, influence both of the number of teachers and of the number of clerical staff members on the types of schools.

This method is very easy to apply even for unskilled teachers in mathematical calculation.

2. When we use the stratification by means of some characteristics, we must divide the interval with the minimum variance for the estimated value. If we divide with smaller intervals, we will find that the distribution in each stratum is almost uniform. So we calculated the mean range and the variance of the range for this distribution.

Also we calculated those for the linear distribution. These results are applicable to rough estimation or checks of calculation of the variances in each stratum.

This is an issue of the projected series of reports entitled "The Research Report of the I. S. M." "The Research Report of the I. S. M." publishes the reports of researches done in the application of Statistical Mathematics such as initial preparations, study designs, practical procedures and handling of data.

The series aims to be beneficial not only for the theoretical workers, but for research workers who are engaged in the practical problems of surveying, analysis and so on.

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