

# Estimating the maximum earthquake magnitude through short-term earthquake clustering models

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## 【Abstract】

Based on the ETAS (epidemic-type aftershock sequence) model, which is used for describing the features of short term clustering of earthquake occurrence, this poster presents some theories and techniques related to evaluating the probability distribution of the maximum magnitude in a given space-time window, where the Gutenberg-Richter law for earthquake magnitude distribution cannot be applied directly. It is shown that the distribution of the maximum magnitude in a given space-time volume in long term is determined by the background seismicity rate and the magnitude distribution of the largest events in each earthquake cluster. The introduced techniques were applied to the seismicity in the Japan region in the period from 1926 to 2009. It is found that the regions most likely to have great earthquakes are along the Tohoku (Northeastern Japan) Arc and the Kuril Arc, both much higher probabilities than the offshore Nankai and Tokai regions.

## 【Description of methods】

### Space-Time Epidemic-Type Aftershock Sequence (ETAS) model

Time varying seismicity rate = "background" + "Triggered seismicity":

$$\lambda(t, x, y) = \mu(x, y) + \sum_{t_1 < t} \kappa(m_1) g(t - t_1) f(x - x_1, y - y_1; m_1)$$

Magnitude distribution: the G-R law

$$s(m) = \beta e^{-\beta(m-m_c)}, \quad m \geq m_c$$

Time distribution: the Omori-Utsu law

$$g(t) = \frac{p-1}{c} \left(1 + \frac{t}{c}\right)^{-p}, \quad t > 0$$

Spatial location distribution of children:

$$f(x, y; m) = \frac{q-1}{\pi D e^{\gamma(m-m_c)}} \left(1 + \frac{x^2 + y^2}{D e^{\gamma(m-m_c)}}\right)^{-q}, \quad q > 1$$

Productivity: mean number of children

$$\kappa(m) = A e^{\alpha(m-m_c)}, \quad m \geq m_c$$

$t$ : time  $(x, y)$ : spatial location  $m$ : magnitude

### Maximum magnitude in an earthquake cluster

The survivor probability function of the magnitude of the biggest event in a cluster, including the initiating event and all the its descendants, is determined by [Zhuang & Ogata, 2006, Physical Review E, 2006]:

$$F(m) = 1 - \int_{m_c}^m s(m') \exp[-\kappa(m') F(m')] dm', \quad m \geq m_c$$

Given a space-time windows  $V$ , the probability of the maximum magnitude

$$\begin{aligned} & \Pr\{\text{at least 1 event greater than } m \text{ occurs in } V\} \\ &= 1 - \Pr\{\text{all biggest events in each cluster } < m \text{ in } V\} \\ &= 1 - \sum_{n=0}^{\infty} \Pr\{n \text{ clusters occur in total in } V\} \\ & \quad \times \Pr\{\text{all event } n \text{ clusters } < m \mid n \text{ clusters in } V\} \\ &= 1 - \sum_{n=0}^{\infty} [1 - F(m)]^n \frac{\Lambda^n(V)}{n!} e^{-\Lambda(V)} \\ &= 1 - e^{-\Lambda(V) F(m)} \end{aligned}$$

$$\text{where } \Lambda(V) = \iiint_V \mu(x, y) dx dy dt$$

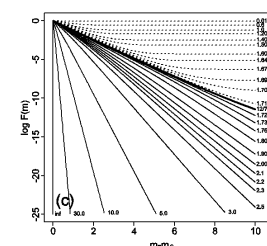
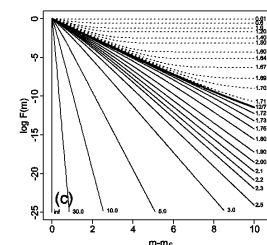
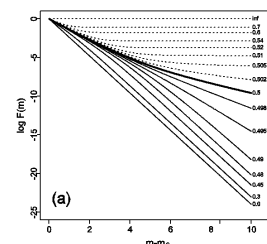


Figure 1. Influence of  $A$ ,  $\alpha$ , and  $\beta$  to function  $F(m)$ . (a) Parameter  $A$  changes,  $\alpha=1.2$  and  $\beta=2.4$  fixed; (b) Parameter  $\alpha$  changes, but  $A=0.3$  and  $\beta=2.4$  fixed; (c) Parameter  $\beta$  changes, but  $A=0.3$  and  $\alpha=1.2$  fixed. The thin solid, thick solid and dashed curves represent the subcritical, critical and supercritical regimes, respectively.

## 【Results for the Japan region】

JMA catalog: depth 0 - 100 km, Date 1926-01-01 to 2009-12-31,  $M_j \geq 4.0$ .

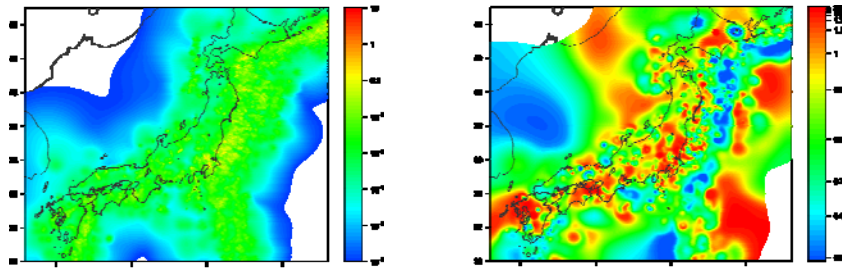


Figure 2. (left panel) Background seismicity rate (occurrence rate of earthquake clusters) [events/(day-deg<sup>2</sup>)] of earthquakes with magnitude  $M_j \geq 4.0$ , obtained based on the space-time ETAS model. (right panel) Spatial variations of  $b$ -values estimated by variable kernel estimates.

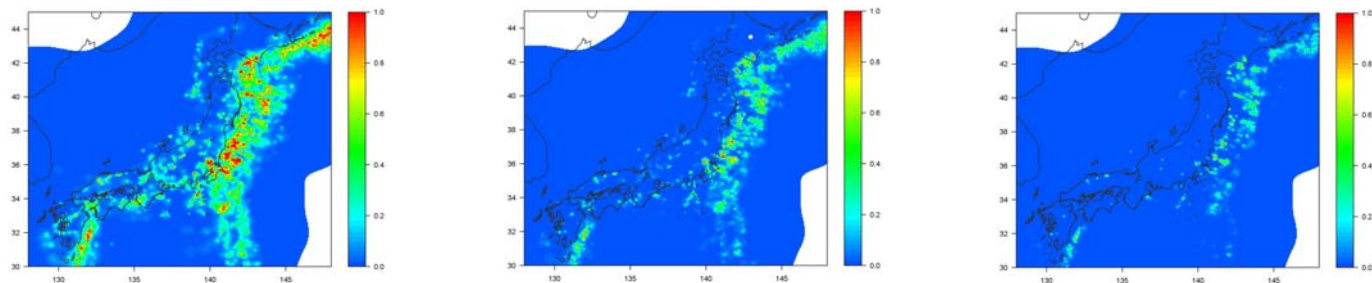


Figure 3 Spatial variations of the occurrence probabilities that at least one earthquake in a  $1 \text{ yr} \cdot \text{deg}^2$  space-time volume is greater than  $M_j \geq 5.0$  (left panel),  $M_j \geq 6.0$  (middle panel) and  $M_j \geq 7.0$  (right panel). Parameters  $A=0.323$  and  $\alpha=1.41$ .

## 【References】

- Zhuang J. and Ogata Y. (2006) Properties of the probability distribution associated with the largest event in an earthquake cluster and their implications to foreshocks. *Physical Review E*, 73, 046134.
- Vere-Jones D. and Zhuang J. (2008) On the distribution of the largest event in the critical ETAS model. *Physical Review E*, 78, 047102.
- Zhuang J. (2012) Long-term earthquake forecasts based on the ETAS model for short-term clustering. *Research in Geophysics*, in press.