A Next-day Earthquake Forecasting Model

庄 建倉　　モデリング研究系　　准教授

【Abstract】

This presemtation gives the technical solutions of implementing the space-time epidemic-type aftershock sequence (ETAS) model for short-term (1-day) earthquake forecasts for the all-Japan region in the Collaboratory for the Study of Earthquake Predictability (CSEP) project in Japan. The optimal model parameters used for the forecasts are estimated by fitting the model to the observation records up to the starting time of the forecasting period, and the probabilities of earthquake occurrences are obtained through simulations. To tackle the difficulty of heavy computations in fitting a complicated point-process to a huge dataset, an "off-line optimization" and "online forecasting" scheme is proposed to keep both the estimates of model parameters and forecasts updated according to the most recent observations. The results show that the forecasts have captured the spatial distribution and temporal evolution of the features of future seismicity. These forecasts are tested against the reference Poisson model that is stationary in time but spatially inhomogeneous.

【Model Formulation】

Space-Time Epidemic-Type Aftershock Sequence (ETAS) model

- Time varying seismicity rate = "background" + "Triggered seismicity":

$$\lambda(t,x,y) = \mu(x,y) + \sum_{i:t_i<t} \kappa(m_i)\, g(t-t_i)\, f(x-x_i, y-y_i; m_i)$$

- Magnitude distribution: the G-R law

$$s(m) = \beta e^{-\beta(m-m_c)}, \quad m \ge m_c$$

- Time distribution: the Omori-Utsu law

$$g(t) = \frac{p-1}{c}\left(1+\frac{t}{c}\right)^{-p}, \quad t>0$$

- Spatial location distribution of children:

$$f(x,y;m) = \frac{q-1}{\pi D e^{\gamma(m-m_c)}}\left(1+\frac{x^2+y^2}{D e^{\gamma(m-m_c)}}\right)^{-q}, \quad q>1$$

- Productivity: mean number of children

$$\kappa(m) = A e^{\alpha(m-m_c)}, \quad m \ge m_c$$

t: time (x,y): spatial location m: magnitude

【Model Estimation Procedures】

- Parameters

$$A,\ \alpha,\ c,\ p,\ D,\ q,\ \gamma,\ \beta$$

- Maximum likelihood estimates

$$\log L = \sum_{i:(t_i,x_i,y_i)\in S} \log \lambda(t_i,x_i,y_i) - \iiint_S (t,x,y)\,dxdydt$$

S: study space-time range

【Estimation of inhomogeneous background rate】

- Background probability: the probability that an event is a background event

$$\varphi_j = \frac{\mu(x_j,y_j)}{\lambda(t_j,x_j,y_j)}$$

- Triggering probability: the probability that event j is directly triggered by i.

$$\rho_{ij} = \frac{\kappa(m_i)\, g(t_j-t_i)\, f(x_j-x_i, y_j-y_i; m_i)}{\lambda(t_j,x_j,y_j)}$$

Once the background probabilities are obtained the background rate can be estimated by the variable kernel functions.

$$\hat{\mu} = \frac{1}{T}\sum_i \varphi_i Z_{h_i}(x-x_i, y-y_i)$$

h_i = distance to the n_pth closest event ($n_p = 3 \sim 7$).

An iterative algorithm is carried out to obtain the model parameters and the background stimuneously.

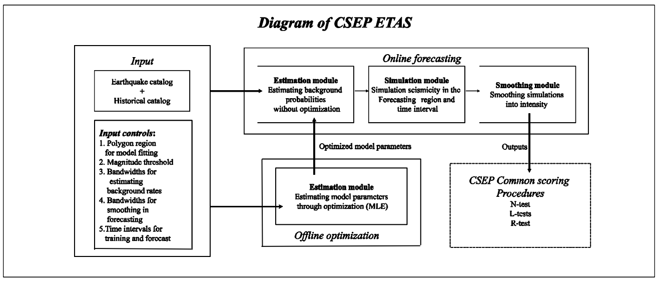
【Simulation algorithm】

1. Generate the background catalog with the estimated background rate $\hat{\mu}$, recorded as Generation 0, according to the simulation procedure for a nonhomogeneous stationary Poisson process.

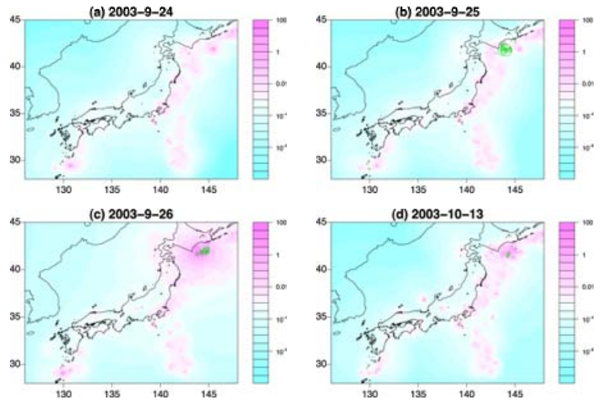
2. For each event, say (t_i, x_i, y_i, m_i), in last simulated genenation, generate its $N^{(i)}$ children, with their occurrence times, locations, and magnitudes from the p.d.f.s $g(t-t_i)$, $f(x-x_i, y-y_i)$ and $s(m)$, respectively, where $N^{(i)}$ is a Poisson random variable with a mean of $\kappa(m_i)$.

3. Repeat last step until no new event is generated. Return with all the events in all generations.

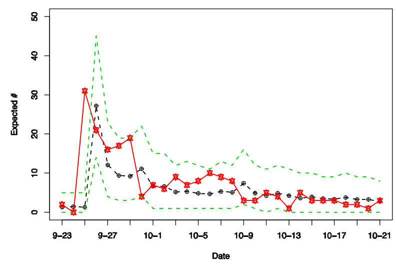
【Illustration of the "off-line optimization and online forecasting" scheme for the implementation of the ETAS model in CSEP】



【Forecasting Example】



Examples of forecasted images of expected numbers of earthquakes on (a) 24 September 2003, (b) 25 September 2003, (c) 26 September 2003 and (d) 13 October 2003. (unit: events/day/deg^2). The green circles mark the locations of earthquakes ($M_J \ge 4.0$) occurring in forecasting time period.



Temporal variation of forecasted daily numbers of earthquakes (sun crosses) in the region (N126°~148°, E28°~45°) during 24 September 2003 to 22 October 2003. The hexagrams mark observed daily numbers of earthquakes.

Main references:

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(3) Zhuang J., Ogata Y., Vere-Jones D. (2004). *Analyzing earthquake clustering features by using stochastic reconstruction.* **J. Geophys. Res.**, 109, No. B5, B05301, doi:10.1029/2003JB002879.
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(5) Zhuang J., (2011) , *Next-day earthquake forecasts for the Japan region generated by the ETAS model.* **Earth Planets Space**, 63, 207-216. doi:10.5047/eps.2010.12.010.