

# Reverted Mean Log Price for Auction Market and Nationwide Average Data

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## 1 Introduction

Since an expectation of future trend and volatility of log price affects the management decision under stochastic environments, it is important to capture the dynamics through a “reasonable” stochastic model for the purpose of forecasting log price over time. The objective of this paper is to investigate the future trend of two different types of log price dynamics through several variants of the mean-reverting process. One type is the market-based log price data in Fukuoka prefecture, and the other is the nationwide average data. Most analyses on log price changes were based on the nationwide average data, while local forest owners face to their nearby auction market price. Since the aggregated data has less information on price changes than the market data, it is necessary to reveal their difference. In the analysis, we focus on the reverted mean of the price dynamics by four stochastic models with the mean-reverting property.

## 2 Method

We assume the state dependent volatility process with the mean-reverting property for log price dynamics (denoted as SDVP-MR hereafter). Letting  $x_t$  be a log price at time  $t$ , SDVP-MR is expressed by,

$$(2.1) \quad \text{Case 1: } dx_t = (\alpha - \beta x_t)dt + x_t^\gamma \sigma dB_t$$

where  $B_t$  is a standard Brownian motion with the following characteristics. 1.  $B_0 = 0$ , 2.  $\{B_t, t \geq 0\}$  has stationary and independent change. 3. for all  $t (> 0)$ ,  $B_t$  follows the normal distribution with a variance of  $t$  and a mean of 0. The set of parameters,  $(\alpha, \beta, \sigma)$  are positive coefficients in order to strictly constrain equation (2.1) to be mean-reverting. Changing a value of  $\gamma$  to 1, 1/2 and 0, three other mean-reverting models are considered for the analysis here.

$$(2.2) \quad \text{Case 2: } dx_t = (\alpha - \beta x_t)dt + x_t \sigma dB_t$$

$$(2.3) \quad \text{Case 3: } dx_t = (\alpha - \beta x_t)dt + \sqrt{x_t} \sigma dB_t$$

$$(2.4) \quad \text{Case 4: } dx_t = (\alpha - \beta x_t)dt + \sigma dB_t$$

Except the model of equation (2.4), parameter estimation for the other three mean-reverting models is carried out by the pseudo-likelihood approach based on discretization by the local linearization method (Shoji and Ozaki, 1997; Shoji, 1998). In order to strictly constrain parameters  $(\alpha, \beta, \sigma)$  to be positive, the exponential transformation,  $\alpha = e^{\theta_1}$ ,  $\beta = e^{\theta_2}$ ,  $\sigma = e^{\theta_3}$  is applied where  $(\theta_1, \theta_2, \theta_3)$  are unrestricted in the range of  $(-\infty, \infty)$ .

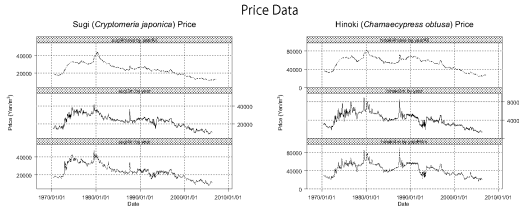


Fig.1. Price dynamics

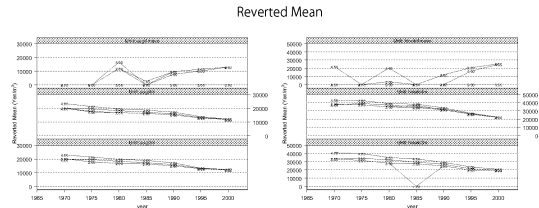


Fig.2. The reverted mean

### 3 Analysis and Results

Analysis was carried out for two kinds of the time series data, i.e. nationwide average and market-based log price on two types of log, i.e. sugi (*Cryptomeria japonica*) and hinoki (*Chamaecypress obtusa*). As for the nationwide price data, monthly time series data of sugi (*Cryptomeria japonica*) and hinoki (*Chamaecypress obtusa*) log price from January 1975 to September 2006 were used along with annual data from 1970 to 1975 with 392 data points to cover the duration from 1970 to 2006. Figure 1 shows price dynamics of these data sets from 1970 to 2006. For the first decade, an increasing trend was observed, and then a decreasing trend took place up to now with a striking pike in the late 1980s.

With some conversion, we can have the following constant volatility process.

$$dy_t = \{\phi'(x_t)(\alpha - \beta x_t) + \frac{1}{2}\phi''g(x_t)^2\sigma^2\}dt + \sigma dB_t$$

Setting the drift term equal to zero, we estimate the reverted mean from the following equation.

$$\phi'(x_t)(\alpha - \beta x_t) + \frac{1}{2}\phi''g(x_t)^2\sigma^2 = 0$$

Figure 2 depicts the reverted mean derived for different log products and different models over different durations for estimation. The nationwide average log data tended to result in the lower reverted mean than the others. As for change in the value over the different starting dates of the time series data for estimation, since 1990 most showed an increasing trend. This could be most likely due to the small degree of concavity of price change. On the other hand, in the case of the market-based price, except one point for hinoki3m with the Case 1 model, all showed a decreasing trend. All in all, a decreasing trend of price change was crucial for the current time series data set with the use of the market-based price, while it was not so for the nationwide average price.

### References

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