Context is Critical

K-5th Grade Three-Act Math Tasks

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Note: All materials can be downloaded at *https://gfletchy.com/arraybow-of-colors/*

Logistics: 4th Grade *(Instructional approach is appropriate for all K-12 Grades)*

Materials for *Array-bow of Colors:*

- Act 1 *Array-bow of Colors* File
- Act 2 Photos
  - Estimate the number of Skittles packages used
  - The actual number of Skittles packages used
  - How many Skittles are in an individual package?
- Act 3 *Array-bow of Colors* File
- Student Page(s)

Time Estimate: 30 – 90 Minutes *(varies depending on the activity and grade level)*

Location: Classroom Setting

Objectives:

- Students will generate questions, make estimates, identify information needed to solve math problems, and engage in meaningful mathematical discourse.
- Students will multiply whole numbers using strategies based on place value and the properties of operations.
- Students will illustrate their understanding of a real-world mathematical situation using equations, rectangular arrays and/or area models.

Standards:

- CCSS.MATH.CONTENT.4.NBT.5 *Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using questions, rectangular arrays, and/or area models.*
- CCSS.MATH.PRACTICE.MP1 *Make sense of problems and persevere in solving them.*
- CCSS.MATH.PRACTICE.MP2 *Reason abstractly and quantitatively.*
- CCSS.MATH.PRACTICE.MP3 *Construct viable arguments and critique the reasoning of others.*
- CCSS.MATH.PRACTICE.MP4 *Model with mathematics.*
- CCSS.MATH.PRACTICE.MP5 *Use appropriate tools strategically.*
- CCSS.MATH.PRACTICE.MP6 *Attend to precision.*
- CCSS.ELA-LITERACY.CCRA.SL.1 *Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.*

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Introduction

Proficient mathematics students embody a variety of skills including, but not limited to: the ability to play an active role in discourse, awareness of quantities, establishing a need for appropriate tools and techniques, and applying known skills in a systematic way to understand a real-world scenario. As instructors, our role is to develop these students. We do this by building our "teaching toolbox," hand-selecting instructional methods and strategies appropriate for building student mastery.

In recent years, instructional jargon such as "real-world", "inquiry-based" and "student-centered" has gained popular attention. Students are now being encouraged to approach open-ended problems that promote curiosity and interest. Dan Meyer, an innovative and highly regarded mathematics educator, developed an instructional approach that redefines "real-world" mathematics and clarifies the role that a student plays in the learning process.

Three-Act Tasks pose a mathematical problem through active storytelling. Dan Meyer explains, "Storytelling gives us a framework for certain mathematical tasks that is both prescriptive enough to be *useful* and flexible enough to be *useable*." (Source: *The Three Acts of a Mathematical Story*) This instructional design begins with students viewing an image or short video clip, and ends with students piecing together identified mathematical concepts and skills to arrive at a solution. Throughout the process, they collaborate with peers, ask purposeful questions, seek needed information, and evaluate their answer for reasonableness and sources of error. This process exemplifies one hallmark of a Three-Act Task: student-centered learning.

A second hallmark of Three-Act Tasks lies in the seemingly unconventional real-world portrayal of rich problems. While many students experience "real-world" scenarios as those depicted in textbooks, Dan Meyer suggests otherwise. Identifying this idea as a relative concept, he proposes:

> If you can ask questions about it, it's in your real world. If you can guess about it, it's in your real world. If you are able to argue about it, it is in your real world.
>
> *Source: Beyond Relevance & Real World: Stronger Strategies for Student Engagement*

Venturing from traditional instruction, Three-Act Tasks present real-world, real-time opportunities to gain mathematical proficiency.

Inquiry Overview

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The following activity allows students to investigate the application of properties of operations (multiplication) and strategies used to explain them through a Three-Act Task. First, students will view a short video clip intended to introduce the "central conflict" of a mathematical problem. During this time, students will simply record what they **notice** and **wonder.** They will then pose a question(s) in response to the video. This series of student-generated questions will set the foundation for the activity. As learners continue, they will work collaboratively to estimate a solution to the mathematical problem and ask purposeful questions to obtain information needed to solve. Students will then seek new math skills and vocabulary to assist in solving. Throughout the process, students will continuously evaluate their solution for accuracy, reasonableness, and error. Finally, students will determine the actual answer and reflect on the relationships between the theoretical math and practical outcome.

Throughout this document, guided questioning and pedagogical insights are provided for the instructor.

Array-bow of Colors Activity *(Author: J.McGavin)*

ACT 1: (10-20 minutes)

During this activity, students will work individually while occasionally collaborating with a neighbor. Provide all students with the appropriate student page(s).

Inform the students that they will watch a short video.

Then, ask students:

- **Would you please write down anything that you notice and wonder? I'm curious to know what you see and what you are thinking about.**

At this time, play the Act 1 *Array-bow of Colors Video* at least once, or multiple times if requested by the students.

Once students have had enough time to review the video (students may ask for the video to be played more than once), allow time for the students to share their "noticings". Record these observations on the whiteboard or chart paper. Then, ask students to turn to their neighbor and share their

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Note: If students are struggling to pose mathematical-questions, you may ask them, "What questions do you think a mathematician may ask about this?"

"wonderings" or any questions they had about what was occurring in the video. Once all students have verbally exchanged their questions, ask for student volunteers to read their question aloud to the whole class. Record these questions on the whiteboard or chart paper for students to refer to throughout the activity. It is important to note that some "wonderings" and "noticings" will be mathematical in nature while some will not. Every idea should be recorded for students to evaluate which questions may be answered using mathematics.

Also, after each question, the instructor may wish to survey the class to determine how many students find that question interesting. This value may be recorded next to its corresponding question.

Once all questions have been recorded, explain to the students:

- We have a lot of great questions here! Let's think about this one: *choose selected question.* Please record this question on your student page.
- I hope we will get around to answering everyone's questions.

In the *Array-bow of Colors* problem, students may generate questions such as:

- *How many Skittles are in the jar?*
- *Why is he putting Skittles in a jar?*
- *How long did it take him to fill the jar with Skittles?*
- *How many packages of Skittles did he use to fill the jar?*
- *How big is the jar?*
- *What is he going to do with all of the Skittles?*

While the goal of this instructional lesson will be to eventually address most of the student-generated questions, the initial focus of this activity will be to determine how many Skittles are in the jar.

Next, inform students:

- On your student page, please record an estimate that you feel is too low (there couldn't possibly be that few Skittles in the jar) and an estimate you think is too high (there couldn't be that many Skittles in the jar).
- Please also write down your best estimate of how many Skittles are in the jar.

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Once all students have determined their series of estimates, ask students to share their estimates with a neighbor. Then, request student volunteers to share their estimates and take a class-wide poll. Record the highest possible estimate and lowest possible estimate on the whiteboard or chart paper along with the names of the corresponding students. You may also choose to list ranges of estimates and poll how many students placed their "exact" estimate within each range. Inform students that they will return to these estimates later in the investigation.

Pedagogical Focus:

"Give yourself one photo or one minute of a video to tell a mathematical story so perplexing that all of your students will want to know the ending, without you saying a word or lifting a finger." - Dan Meyer

ACT 1 serves as the presentation of your mathematical story. It is used to introduce the central conflict, engage the audience, and orient students to the context of a question. In this portion of the activity, a photo or video clip is presented to the students. The image is clear and focused, but contains little to no words. Most importantly, the media ignites student curiosity. The intent of the video is for students to focus on "noticing and wondering" as they watch the clip.

During **ACT 1**, students are given the task of translating their curiosity into questions. Instead of answering prescriptive, and often mundane, questions from a recorded scenario, learners are encouraged to pose questions that are of self-interest. This places little demand on the student and provides all learners with a way to access the content. It leaves the students wondering, "*What happens next?*" While the intent of the lesson may focus on one or several of these generated questions, students will have the opportunity to revisit their list in **ACT 3.**

Finally, students are eased into the mathematical approach by engaging in "Goldilocks guessing". This strategy helps to create a non-threatening, level playing field for students and encourages input from all participants. By sharing estimates with a partner, students are given an opportunity to explain their thinking and listen to the ideas of others. More importantly, it sets the stage for controversy and mathematical discourse. Controversy ignites student interest, engagement and buy-in. Students now want to "prove" they are right.

ACT 2: (20-30 minutes)

Students will now determine what information is needed to solve the problem. Ask students to consider the following questions as written on their student pages:

- What information do you need to know to solve the problem?

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Working with a partner, students should appropriately record their thoughts. During this time, it is important for the teacher to refrain from leading students to particular ideas, and instead, encourage discussion between team members. Provide students with the information as requested, and discuss how additional information could be obtained if not readily available.

In the *Array-bow of Colors* problem, students may request the following information:

- *How many Skittles packages were used to fill the jar?*
- *How many Skittles are in an individual package?*
- *How large is the jar?*

Images providing information related to these questions are included in the material list. Note, students may request information that is irrelevant to the intended question. It is suggested that the instructor abstain from dismissing their request, as this presents an important learning opportunity where students will later identify information as unnecessary. Additionally, information may be requested that is not accessible. Discuss this limitation with the students and encourage them to identify how it could be obtained. Reviewing the video to confirm details of the situation may also be useful.

After receiving the desired information, students may also wish to revise their estimate.

Finally, once students have been given access to the necessary tools and pieces of information, allow them time to determine a solution. Encourage students to record all work on their student pages. As students are working, observe how students are approaching the problem and any strategies that are being used to solve. This information will inform the **ACT 3** discussion. For students that finish early, they may begin working on the additional student-generated questions from **ACT 1.**

Note: In the *Array-bow of Colors* problem, students may use a variety of strategies, including using arrays and/or area models to determine a solution.

In the *Array-bow of Colors* problem, students may use the following information and procedure to determine a solution:

- *Exactly 58 packages of Skittles were used to fill the jar.*
- *It is assumed that there are 14 Skittles in each individual package.*
- *There is a total of 812 Skittles in the jar.*

Pedagogical Focus:

"This is the guts of modeling right here…If students aren't grappling with the question, "What's important here and how would I get it?" *they may be doing lots of valuable mathematics, but they aren't modeling." - Dan Meyer*

ACT 2 requires the student to overcome obstacles, look for resources and identify appropriate tools to tackle an evolving mathematical problem. During this exercise, they investigate the constraints and requirements of the problem while searching for the information and skills needed to solve. In the "real world", participants must sift through information, determine what is critical, and devise a strategy on how to use it. For these reasons, students are encouraged to "mess" with the problem and persevere in their ability to determine an answer.

In **ACT 2**, students continue to be active participants in the problem solving process. During this portion of the investigation, the teacher serves as the primary facilitator, thus allowing students to guide the discussion. The collection of ideas reiterates acknowledgement of contributions and each students' role in the learning community. Also, it promotes further dialogue and creative thinking.

ACT 2 also encourages perseverance in problem solving. The National Council of Supervisors of Mathematics reiterates, "Learning is messy and in the process of struggling, students are developing 'grit' as they connect emerging and old ideas in the creation of new learning." During this time, it may be appropriate to teach unfamiliar skills or mathematical concepts that are needed to solve the problem. In this setting, students will be taught new information in a natural context.

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ACT 3: (15+ minutes)

Once all students have determined a solution, redirect students back to the information collected in **ACT 1.** Ask the class to consider:

- Does your answer fall in the range of possible estimates?

Elicit answers from student volunteers. If students are concerned about their calculated answer, encourage them to return to the work completed in **ACT 2.** Revisiting their work provides students with the opportunity to reflect on the models and strategies used to solve.

After students have evaluated the reasonableness of their solution, play the Act 3 *Array-bow of Colors Video.* In this video, the featured presenter begins by arranging the Skittles in an array using the following procedure:

- 10 Skittles in each pile
- 10 piles in each column
- 8 columns with 10 piles in each column are made.

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- 5 extra groups of 10 Skittles are made.
- 3 individual Skittles are placed above the 5 extra groups.

Finally, an over-head view of the Skittle arrangement is shown. This image illustrates the total number of Skittles within each column or group. Students are left with this image and must strategically use this information to arrive at a solution. Using their knowledge of place value, properties of operations and problem solving strategies, students will determine that exactly 853 Skittles were in the jar.

Note: In **ACT 2,** students likely determined that there were approximately 812 Skittles in the jar. However, not all individual Skittle packages contain the same number of candies.

Finally, students should consider:

- **Was the answer closest to the lowest or highest possible estimate?**
- **How close was your answer to your estimate?**
- **If you weren't exactly right, what could account for the error?**

Allow students time to consider possible sources of error in their mathematics and brainstorm any more precise, better models or strategies that could be considered.

At this time, the instructor should formalize the mathematics skills, processes and vocabulary used in the problem. This learning opportunity should involve input from the students. Once students have completed their work, and if time allows, they may continue to work on the additional questions previously generated.

Pedagogical Focus:

"The problem should be deep. It should be rich enough to spend hours, days, weeks, months or years working on variations, generalization and extensions." -Avery Pickford

ACT 3 serves as the resolution of the mathematical story. As stated in *Trying Three-Act Tasks with Primary Students,* "The purpose of revealing the answer is not to ask children to 'check their work,' but to create an opportunity for students to examine how well their models fit the situation and how they could revise their models to improve their accuracy or efficiency." (Lomax, Alfonso, Dietz, Kleyman, &Kazemi 2017).

In this final piece of the investigation, students look for validation by first measuring their calculated solution against their previously generated possible answer list. Again, this practice provides students with a benchmark that encourages them to determine if they "trust" the mathematics. This learning opportunity continues to reiterate that in the problem solving process, students are permitted to make errors, seek out additional information, and search for precise methods of calculation.

This final step in the Three-Act Task also sets up a sequel for the original problem. Determining extensions validates the complexity of the problem.

Resources

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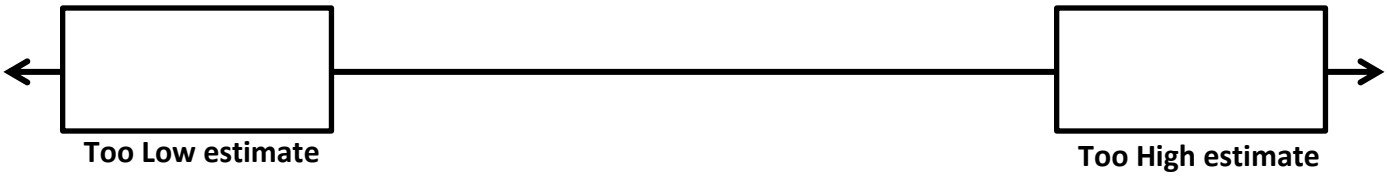
- *Andrew Stadel's blog post highlighting the* File Cabinet *problem, including all materials: http://mr-stadel.blogspot.com/2012/04/file-cabinet.html*
- *Trying Three-Act Tasks with Primary Students: https://www.nctm.org/Publications/Teaching-Children-Mathematics/2017/Vol24/Issue2/Trying-Three-Act-Tasks-with-Primary-Students/*
- *Dan Meyer's blog post explaining the pedagogy and process behind the Three-Act Method: http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/ and http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-one/*
- *Geoff Krall's Common Core Problem Based Curriculum Maps: https://emergentmath.com/my-problem-based-curriculum-maps/*
- *Robert Kaplinsky's Problem-Based Learning Search Engine: http://robertkaplinsky.com/prbl-search-engine/*
- *Robert Kaplinsky's Three-Act Method Gallery: http://robertkaplinsky.com/lessons/*
- *A living document of Three-Act Math Tasks, including standards and practices: https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/htmlview*
- *Greg Fletcher's K-5 Three-Act Tasks with Student Recording Sheets: https://gfletchy.com/3-act-lessons/*
- *When Math Happens: https://whenmathhappens.com/3-act-math/*
- *Kyle Pearce Three-Act Tasks: https://tapintoteenminds.com/3act-math/*
- *A Peek Into My Classroom: https://mrorr-isageek.com/real-world-problems-lessons/*

ACT 1

What did you notice?	What do you wonder?

What is the Main Question that we want answered?

Estimate the solution to the Main Question.



Place your just right estimate along the number line. Be sure to label!

ACT 2

What information would be useful to know to help you solve the problem?

With this information, determine a solution for your question. Show all work.

ACT 3

What is the answer?

Adapted from G. Fletcher's Student Pages and Materials

Estimate

Draw a picture to show your thinking.

Use numbers to show your thinking.

Answer