Three-Act Math Tasks
A Pedagogical Approach

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Note: All materials can be downloaded at *http://mr-stadel.blogspot.com/2012/04/file-cabinet.html*

Logistics: 6-8 Grades *(Instructional approach is appropriate for all K-12 Grades)*

Materials for *File Cabinet:*

- Act 1 *File Cabinet Video* File
- Act 2 Photos
  - File Cabinet Dimensions Estimate
  - File Cabinet Height
  - File Cabinet Width
  - File Cabinet Depth
  - Sticky Note Dimensions Estimate
  - Sticky Note Dimensions
- Act 3 *File Cabinet Video* File
- Student Pages

Time: 30-90 Minutes *(varies depending on the activity)*

Location: Classroom Setting

Objectives:

- Students will generate questions, make estimates, identify information needed to solve math problems, and engage in meaningful mathematical discourse.
- Students will apply their understanding of geometric properties in a real-world situation.

Standards:

- CCSS.MATH.CONTENT.6.G.A.4 *Solve real-world and mathematical problems involving area, surface area, and volume.*
- CCSS.MATH.CONTENT.7.G.A.1 *Draw, construct, and describe geometrical figures and describe the relationships between them.*
- CCSS.MATH.PRACTICE.MP1 *Make sense of problems and persevere in solving them.*
- CCSS.MATH.PRACTICE.MP2 *Reason abstractly and quantitatively.*
- CCSS.MATH.PRACTICE.MP3 *Construct viable arguments and critique the reasoning of others.*
- CCSS.MATH.PRACTICE.MP4 *Model with mathematics.*
- CCSS.MATH.PRACTICE.MP5 *Use appropriate tools strategically.*
- CCSS.ELA-LITERACY.CCRA.SL.1 *Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.*

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Introduction

Proficient mathematics students embody a variety of skills including, but not limited to: the ability to play an active role in discourse, awareness of quantities, establishing a need for appropriate tools and techniques, and applying known skills in a systematic way to understand a real-world scenario. As instructors, our role is to develop these students. We do this by building our "teaching toolbox," hand-selecting instructional methods and strategies appropriate for building student mastery.

In recent years, instructional jargon such as "real-world", "inquiry-based" and "student-centered" has gained popular attention. Students are now being encouraged to approach open-ended problems that promote curiosity and interest. Dan Meyer, an innovative and highly regarded mathematics educator, developed an instructional approach that redefines "real-world" mathematics and clarifies the role that a student plays in the learning process.

Three-Act Tasks pose a mathematical problem through active storytelling. Dan Meyer explains, "Storytelling gives us a framework for certain mathematical tasks that is both prescriptive enough to be *useful* and flexible enough to be *useable*." (source: *The Three Acts of a Mathematical Story*) This instructional design begins with students viewing an image or short video clip, and ends with students piecing together identified mathematical concepts and skills to arrive at a solution. Throughout the process, they collaborate with peers, ask purposeful questions, seek needed information, and evaluate their answer for reasonableness and sources of error. This process exemplifies one hallmark of a Three-Act Task: student-centered learning.

A second hallmark of Three-Act Tasks lies in the seemingly unconventional real-world portrayal of rich problems. While many students experience "real-world" scenarios as those depicted in textbooks, Dan Meyer suggests otherwise. Identifying this idea as a relative concept, he proposes:

> If you can ask questions about it, it's in your real world. If you can guess about it, it's in your real world. If you are able to argue about it, it is in your real world.
>
> *Source: Beyond Relevance & Real World: Stronger Strategies for Student Engagement*

Venturing from traditional instruction, Three-Act Tasks present real-world, real-time opportunities to gain mathematical proficiency.

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Inquiry Overview

The following activity allows students to investigate the application of properties of geometrical figures through a Three-Act Task. First, students will view a short video clip intended to introduce the "central conflict" of a mathematical problem. Students will then pose a question(s) in response to the video. This series of student-generated questions will set the foundation for the activity. As learners continue, they will work collaboratively to estimate a solution to the mathematical problem and ask purposeful questions to obtain information needed to solve. Students will then seek new math skills and vocabulary to assist in solving, and merge these ideas with previously mastered concepts. Throughout the process, students will continuously evaluate their solution for accuracy, reasonableness, and error. Finally, students will determine the actual answer and reflect on the relationships between the theoretical math and practical outcome.

Throughout this document, guided questioning and pedagogical insights are provided for the instructor.

File Cabinet Activity *(Author: A.Stadel)*

ACT 1: (15-20 minutes)

During this activity, students will work individually while occasionally collaborating with a neighbor. Provide all students with the appropriate student pages.

Inform the students that they will watch a short video. At this time, play the Act 1 *File Cabinet Video* at least once, or multiple times if requested by the students.

Then, ask students:

- **Would you please write down the first question that this video brings to mind, if any? If you don't have any questions, that's just fine.**

Next, ask students to turn to their neighbor and share their question. Once all students have verbally exchanged their questions, ask for student volunteers to read their question aloud to the whole class. Record these questions on the whiteboard or chart paper. Also, after each question,

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survey the class to determine how many students find that question interesting. You may choose to record this value next to its corresponding question.

Once all questions have been recorded, explain to the students:

- **We would love to get to all the questions on your list, but because of time, we will start here….***choose selected question.* **Please record this question on your student page.**
- **I hope we will get around to answering everyone's questions.**

In the *File Cabinet* problem, students may generate questions such as:

- *How many sticky notes will cover the file cabinet?*
- *How many packs of sticky notes would you need to cover the file cabinet?*
- *Will he cover the top and bottom of the file cabinet?*
- *How long will it take for him to cover the file cabinet?*
- *Why is he covering a file cabinet with sticky notes?*

While the goal of this instructional lesson will be to eventually address all student-generated questions, the initial focus of this activity will be to determine **how many sticky notes will cover the file cabinet.**

Next, inform students:

- **On your student page, please record a guess that you feel is too low (there couldn't possibly be that few sticky notes) and a guess you think is too high (there couldn't be that many sticky notes).**
- **Please also write down your best guess of how many sticky notes will cover the file cabinet.**

Once all students have determined their series of estimates, ask students to share their guess with a neighbor. Then, request student volunteers to share their guesses and take a class-wide poll. Record the highest possible guess and lowest possible guess on the whiteboard or chart paper along with the names of the corresponding students. Determine what portion of the class believes the answer is closest to the lowest possible guess and how many believe the answer is closest to the highest possible guess. Inform students that they will return to these estimates later in the investigation.

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Pedagogical Focus:

"Give yourself one photo or one minute of a video to tell a mathematical story so perplexing that all of your students will want to know the ending, without you saying a word or lifting a finger." - Dan Meyer

ACT 1 serves as the presentation of your mathematical story. It is used to introduce the central conflict, engage the audience, and orient students to the context of a question. In this portion of the activity, a photo or video clip is presented to the students. The image is clear and focused, but contains little to no words. Most importantly, the media ignites student curiosity.

During **ACT 1**, students are given the task of translating their curiosity into questions. Instead of answering prescriptive, and often mundane, questions from a recorded scenario, learners are encouraged to pose questions that are of self-interest. This places little demand on the student and provides all learners with a way to access the content. It leaves the students wondering, "*What happens next?*" While the intent of the lesson may focus on one or several of these generated questions, students will have the opportunity to revisit their list in **ACT 3.**

Finally, students are eased into the mathematical approach by engaging in "Goldilocks guessing". This strategy helps to create a non-threatening, level playing field for students and encourages input from all participants. By sharing guesses with a partner, students are given an opportunity to explain their thinking and listen to the ideas of others. More importantly, it sets the stage for controversy and mathematical discourse. Controversy ignites student interest, engagement and buy-in. Students now want to "prove" they are right.

ACT 2: (20-45 minutes)

Students will now determine what information is needed to solve the problem. Ask students to consider the following questions as written on their student pages:

- What information would be useful to know to help you solve the problem?
- What tools will you need?
- How can you get this information?

Working with a partner, students should appropriately record their thoughts. During this time, it is important for the teacher to refrain from leading students to particular ideas, and instead, encourage discussion between team members. Provide students with the information as requested, and discuss how additional information could be obtained if not readily available.

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In the *File Cabinet* problem, students may request the following information:

- *What are the height, width and depth of the file cabinet?*
- *What are the dimensions of a sticky note?*
- *How many sides will be covered with sticky notes?*

Images providing information related to these questions are included in the material list. Note, students may request information that is irrelevant to the intended question. It is suggested that the instructor abstain from dismissing their request, as this presents an important learning opportunity where students will later identify information as unnecessary. Additionally, information may be requested that is not accessible. Discuss this limitation with the students and encourage them to identify how it could be obtained.

Finally, once students have been given access to the necessary tools and pieces of information, allow them time to determine a solution. Encourage students to record all work on their student pages, and then take several minutes to summarize how the information was sequenced to arrive at their solution. For students that finish early, they may begin working on the additional student-generated questions from **ACT 1.**

In the *File Cabinet* problem, students may use the following information and procedure to determine a solution:

- *The area of the front and back sides of the file cabinet is 2,592 in^2.*
- *The area of the left and right sides of the file cabinet is 1,296 in^2.*
- *The area of the top of the file cabinet is 648 in^2.*
- *It is assumed that the bottom of the file cabinet will not be covered with sticky notes.*
- *The total area of the file cabinet that will be covered with sticky notes is 9,072 in^2.*
- *The total area of a sticky note is 9 in^2.*
- *The number of sticky notes needed to cover the file cabinet is 936.*

Note: In the *File Cabinet* problem, students exercise their knowledge of the surface area of a rectangular prism, area of a square and measurement units.

Pedagogical Focus:

"This is the guts of modeling right here…If students aren't grappling with the question, "What's important here and how would I get it?" *they may be doing lots of valuable mathematics, but they aren't modeling." - Dan Meyer*

ACT 2 requires the student to overcome obstacles, look for resources and identify appropriate tools to tackle an evolving mathematical problem. During this exercise, they

investigate the constraints and requirements of the problem while searching for the information and skills needed to solve. In the "real world", participants must sift through information, determine what is critical, and devise a strategy on how to use it. For these reasons, students are encouraged to "mess" with the problem and persevere in their ability to determine an answer.

In **ACT 2**, students continue to be active participants in the problem solving process. During this portion of the investigation, the teacher serves as the primary facilitator, thus allowing students to guide the discussion. The collection of ideas reiterates acknowledgement of contributions and each students' role in the learning community. Also, it promotes further dialogue and creative thinking.

ACT 2 also encourages perseverance in problem solving. The National Council of Supervisors of Mathematics reiterates, "Learning is messy and in the process of struggling, students are developing 'grit' as they connect emerging and old ideas in the creation of new learning." During this time, it may be appropriate to teach unfamiliar skills or mathematical concepts that are needed to solve the problem. In this setting, students will be taught new information in a natural context.

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ACT 3:

Once all students have determined a solution, redirect students back to the information collected in **ACT 1.** Ask the class to consider:

- Does your answer fall in the range of possible guesses?
- Should we trust the mathematics?

Elicit answers from student volunteers. If students are concerned about their calculated answer, encourage them to return to the work completed in **ACT 2.**

After students have evaluated the reasonableness of their solution, play the Act 3 *File Cabinet Video.* In this video, the actual answer *(935 sticky notes)* will be determined.

Finally, students should consider:

- Was the answer closest to the lowest or highest possible guess? *(Recognize the student responsible for this guess and show appreciation.)*
- How close was your answer to your guess?
- If you weren't exactly right, what could account for the error?

Allow students time to consider possible sources of error in their mathematics and brainstorm any newer, better models that could be considered. All thoughts should be recorded in their student pages.

Note: In **ACT 2,** students determined that it would take approximately 936 sticky notes to cover the file cabinet. However, they may not have taken the file cabinet handle into consideration, as was demonstrated in **ACT 3.**

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At this time, the instructor should formalize the mathematics skills, processes and vocabulary used in the problem. This learning opportunity should involve input from the students. Once students have completed their work, and if time allows, they may continue to work on the additional questions previously generated. Additionally, they are asked to determine a title for the lesson based on the mathematics used and manner in which it was utilized. This final task reinforces the mathematics applied in context to the real-world situation.

Pedagogical Focus:

"The problem should be deep. It should be rich enough to spend hours, days, weeks, months or years working on variations, generalization and extensions." -Avery Pickford

ACT 3 serves as the resolution of the mathematical story. In this final piece of the investigation, students look for validation by first measuring their calculated solution against their previously generated possible answer list. Again, this practice provides students with a benchmark that encourages them to determine if they "trust" the mathematics. This learning opportunity continues to reiterate that in the problem solving process, students are permitted to make errors, seek out additional information, and search for precise methods of calculation.

This final step in the Three-Act Task also sets up a sequel for the original problem. Determining extensions validates the complexity of the problem.

Resources

- *Andrew Stadel's blog post highlighting the* File Cabinet *problem, including all materials: http://mr-stadel.blogspot.com/2012/04/file-cabinet.html*
- *Dan Meyer's blog post explaining the pedagogy and process behind the Three-Act Method: http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/ and http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-one/*
- *Geoff Krall's Common Core Problem Based Curriculum Maps: https://emergentmath.com/my-problem-based-curriculum-maps/*
- *Robert Kaplinsky's Problem-Based Learning Search Engine: http://robertkaplinsky.com/prbl-search-engine/*
- *Robert Kaplinsky's Three-Act Method Gallery: http://robertkaplinsky.com/lessons/*
- *A living document of Three-Act Math Tasks, including standards and practices: https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFeJimZxnhgwOVsWkTQEsfqouLWNNC6Z4/htmlview*

Three-Act Math Tasks

Student Pages

ACT 1

A guess too LOW: *A guess too HIGH:* *What is your guess?*

ACT 2

What information would be useful to know to help you solve the problem?

What tools will you need?

How can you get this information?

With this information, determine a solution for your question. Show all work.

ACT 3

How close was your answer to your guess?

If you weren't exactly right, what could account for the error?

SEQUEL

What extension question(s) do you want to solve? Show your work.

TITLE

*How should we title this lesson so that it captures **the math we used** and **where we used it?***

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