All Numbers Big and Small – An Infinite Problem

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Introduction

Students tend to think of math as being defined or concrete. However, when considering something fundamental to math, ***the number***, some of the concepts are inherently abstract. *Dictionary.com* provides many definitions for the noun: ***number***. Essentially, this site defines a number as being considered as one or as many. A number can be a sum, a total or a collection. It can mean a word or a symbol as long as it pertains to the action of counting. A number can be assigned to an object. This begs the question: *how are numbers assigned*? Furthermore, how do you determine when a number is bigger than another number? The term itself is rather ambiguous.

However, numbers still seem concrete. 'One' is easy to relate to when counting fingers on a hand, or people in a room. 'One' person is easy to see, to define and seems very real, or concrete. But in that 'one' person is 'many'. In fact, that 'many' (in terms of cells) has been estimated to be about 37.2 trillion (http://www.smithsonianmag.com/smart-news/there-are-372-trillion-cells-in-your-body-4941473/?no-ist).

The human brain is primed to deal with numbers. It likes to catalog and categorize. Memory relies on the ability to group similar items. Numbers make the abstract easier to comprehend. Numbers are necessary when taking measurements in construction, or when used in accounting. Taking inventory of items in a grocery store requires the ability to count – and an understanding for the numerical symbols with appropriate meanings behind them. Counting the amount of roads to take to get home from the mall, the amount of milk to pour into a bowl for a specific recipe, the time you send your kids to sleep… all of this requires some sort of understanding of the meaning behind these symbols we use. Moreover, these meaningful symbols must be similarly understood amongst a large population.

It seems many numbers can be viewed as defined and concrete by the general population. Sometimes the numbers we use, however, are *very very* small or *very very* large. These extremes are hard to relate to and may be perceived abstractly. People don't typically play with the extreme numbers on a daily basis. The unfamiliar can be daunting and then, suddenly, the abstract nature of numbers becomes apparent.

Many fields of study, like science and engineering, require an understanding of these extremes. When trying to teach these concepts and utilize these very small or very large numbers (or the infinite), students may gloss over the significance, thereby, not completely learning or retaining pertinent information. Students may find it hard to understand the concepts being taught. The activities included in this session are geared toward making these extreme numbers more attainable, or more familiar. By doing so, the concepts behind these numbers become more concrete and easy to understand.

Standards

Activity 1: Imagine a Numberless World….

Common Core State Standards - Mathematics

CCSS.MATH.PRACTICE.MP2: Reason abstractly and quantitatively
CCSS.MATH.PRACTICE.MP3: Construct viable arguments and critique the reasoning of others
CCSS.MATH.PRACTICE.MP4: Model with mathematics

Activity 2: Something So Small

Next Generation Science Standards:

NGSS.5-ESS2-2: Describe and graph the amounts and percentages of water and fresh water in various reservoirs to provide evidence about the distribution of water on Earth.
NGSS.MS-ESS3-1: Construct a scientific explanation based on evidence for how the uneven distributions of Earth's mineral, energy, and groundwater resources are the result of past and current geoscience processes.
NGSS.SEP-2: Developing and using models
NGSS.SEP-4: Analyzing and interpreting data

Common Core State Standards - Mathematics:

CCSS.MATH.CONTENT.6.RP.A.3.C: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.MATH.CONTENT.6.RP.A.3.D: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
CCSS.MATH.CONTENT.7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
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Activity 3: Too Much to Handle

Common Core State Standards - Mathematics:

CCSS.MATH.CONTENT.8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
CCSS.MATH.CONTENT.8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
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Activity 4: The Winning Tower of Sticky-Notes

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CCSS.MATH.PRACTICE.MP4: Model with mathematics

Common Core State Standards – ELA:

CCSS.ELA-LITERACY.RST.11-12.9: Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.

Activity 5: The Finish Line

Common Core State Standards - Mathematics:

CCSS.MATH.CONTENT.HSN.CN.B.6: Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
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CCSS.ELA-LITERACY.RST.11-12.3: Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
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Summaries and Objectives

Activity 1: Imagine a Numberless World….

Thinking of numbers as something that is abstract may be unfamiliar for students. Through forcing the students to devise a system of counting items without the use of numbers, students are introduced to the abstract nature of numbers. They learn to categorize and define a numerical system using different terms or symbols – ultimately determining that they are still using that abstract term – ***numbers***.

Objectives and Standards:

- Use a modeling activity to think abstractly using numbers
- Define what is a number

Activity 2: Something So Small

This activity was originally developed to provide a concrete understanding to how little potable freshwater is currently available, or easy to attain. The percentage of easily attainable, potable freshwater is approximately 0.07% of all the water on the planet. This number, 0.0007, is very small and difficult to comprehend. By providing concrete items to count, and magnifying the number, the figure becomes more familiar and easier to understand.

Objectives:

- Model the distribution of salt to fresh water, and the availability of potable water on the planet.
- Work with data to determine the ratios and percentages of available potable water to the total amount of water available on Earth.

Activity 3: Too Much to Handle

This activity was developed to provide a hands-on method to introducing the concept and mechanics behind scientific notation. Scientific notation is a method of writing numbers in terms of a number no smaller than 1, but less than 10, multiplied by powers of 10. It is a way to make really large or really small numbers easy to working with or manipulate.

Objectives:

- Determine ways to simplify or create a short-hand system for very large numbers.
- Model the steps of converting a large number to scientific notation.
- Understand that scientific notation uses powers of ten and exponents to shorten the written expression of a very large (or very small number).

Activity 4: The Winning Tower of Sticky-Notes

This activity was developed to introduce the concept of Infinity. Specifically, this activity deals with the expanding nature of infinity (where numbers grow larger and larger, without end.

Objective:

- Model the expanding nature of infinity.

Activity 5: The Finish Line

This activity was developed to provide a hands-on understanding of how the concept of infinity can be applied to decreasing numbers. Through an examination of the first of Zeno's paradoxes, students discover idea that space can be infinitely divided. Therefore, the infinite isn't merely large; it can also refer to smaller and smaller numbers.

Objective:

- Gain an understanding that infinity doesn't just refer to ever increasing numbers, but also includes infinitely smaller quantities.

Activity 1: *Imagine a Numberless World…*

Thinking of numbers as something that is abstract may be unfamiliar for students. Through forcing the students to devise a system of counting items without the use of numbers, students are introduced to the abstract nature of numbers. They learn to categorize and define a numerical system using different terms or symbols – ultimately determining that they are still using that abstract term – ***numbers***.

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- Use a modeling activity to think abstractly using numbers
- Define what is a number

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Materials:

for each team of 4:

- 1 bag of various small items, like farm animals or shapes
- 1 Index Card
- 1 pencil or pen

Estimated Time:

- **5 minutes – Introductory Discussion**
- **10 minutes – Activity**
- **5 minutes – Debrief**

Suggested Inquiry Approach:

Introduce the activity by informing the students that they are being challenged. Their challenge will be to sort and count items without using numbers. Ask the students to define the term: number. Ask the students for examples. Once students agree upon a definition, move on with the activity.

Group the students into teams of 4 and distribute the bags of items, the index cards and pencil or pen. Instruct the students to begin devising a way to sort and count these items without the use of numbers.

Once they have completed that task, instruct the students to determine the system they developed to complete the task and to record a description of this system on an index card.

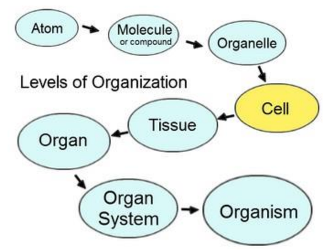
When all the groups have completed this task, collect the cards and redistribute the cards amongst the groups. Be sure that each group does not receive their own card.

Activity 1: *Imagine a Numberless World…*

Instruct the students to examine and discuss the system described on the card. They must compare this system to the definition the class derived for the term: number. Ask each group if the system successfully sorted and counted these items without the use of numbers.

Debrief:

As a whole class, have students discuss their answers to the following questions:



- ***Did any team successful not use numbers to sort and count? Is it possible to do so?***
- ***Would you change your definition of a number?***
- ***Do you think a number is concrete, real or easy to define?***
- ***Even if numerical symbols changed, is the understanding of what these symbols mean still easy to identify?***
- ***What are some examples in science, history or literacy where items are categorized and sorted? Is this done using a numerical system or some aspect of quantities? Explain.***

Activity 2: *Something So Small*

This activity was originally developed to provide a concrete understanding to how little potable freshwater is currently available, or easy to attain. The percentage of easily attainable, potable freshwater is approximately 0.07% of all the water on the planet. This number, 0.0007, is very small and difficult to comprehend. By providing concrete items to count, and magnifying the number, the figure becomes more familiar and easier to understand. *One note: the mathematics included in this lesson is approximation – the final figure students may calculate will be around 0.05% potable freshwater, and not the actual figure of 0.07%.*

Objectives:

- Model the distribution of salt to fresh water, and the availability of potable water on the planet.
- Work with data to determine the ratios and percentages of available potable water to the total amount of water available on Earth.

Standards:

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Estimated Time:

- **10 minutes – Introductory Discussion**
- **20 minutes – Activity**
- **10 minutes – Debrief**

Activity 2: *Something So Small*

Materials:
for each team of 4:
- 1 bag of 400 chips
- 1 piece of masking tape
- 1 set of ***printed*** cards

Suggested Inquiry Approach:

Begin this activity by asking the students:

- **Which do you think there is more of on Earth: Salt water or Freshwater?**
- **What percentages of the water on Earth do you believe to be Salt water and Freshwater?**
- **What are some of the sources for Freshwater?**
- **Are there any problems with the supply of Freshwater available to drink?**

Allowing the students to first predict the percentages of Salt and Freshwater on the planet before diving into the activity is imperative. One suggestion is to the students create circle graphs to show their predictions.

Regarding the discussion of freshwater sources and problems associated, responses may center on pollution. The topic of amount or availability of freshwater need not be the focus at this point. Once this introductory discussion is complete, place the students into groups of four and distribute the ***printed*** cards, the bags of blue chips, and the pieces of masking tape to the student groups.

Briefly discuss the activity with the students. Instruct the students to work through these cards as a group. Essentially, the activity requires students to work through directions on the cards, asking students to convert the percentages provided to numbers of blue chips (out of 400). As the students are working, circulate amongst the groups, offering mathematical hints when needed.

Once all groups have completed the activity and answered the questions on the cards, reconvene as a large group to debrief.

Debrief:

As a whole class, have students discuss their answers to the following questions:

- ***What surprised you most about the amount of fresh water that is easily acquired for human use and consumption?***
- ***Do you think the availability of fresh water is a problem for the general world population? Explain.***
- ***How would you solve the problem of obtaining more drinkable (potable) water? Which water source is most plentiful? What are the difficulties associated with obtaining water from this source?***

Activity 3: *Too Much To Handle*

This activity was developed to provide a hands-on method to introducing the concept and mechanics behind scientific notation. Scientific notation is a method of writing numbers in terms of a number no smaller than 1, but less than 10, multiplied by powers of 10. It is a way to make really large or really small numbers easy to working with or manipulate.

Objectives:

- Determine ways to simplify or create a short-hand system for very large numbers.
- Model the steps of converting a large number to scientific notation.
- Understand that scientific notation uses powers of ten and exponents to shorten the written expression of a very large (or very small number).

Standards:

CCSS.MATH.CONTENT.8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
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Estimated Time:

- **10 minutes – Introductory Discussion**
- **20 minutes – Activity**
- **10 minutes – Debrief**

Materials:
for each pair:

- 1 bag of number tiles, 0-9
- 1 bag of red beads
- 1 bag of black beads
- Student pages
- ***Too Much To Handle*** Template
- Pencil

Suggested Inquiry Approach:

Begin this activity by asking the students to give examples of some very large number. Explain to the students that people in a variety of fields (like space scientists, engineers, and data analysts) often work with very large numbers.

Ask the students:

- **What are some difficulties that may arise when manipulating or doing calculations with very large numbers?**

Inform the students that they will be working with some very large numbers in this activity. However, they will be exploring ways to make these numbers easier to handle.

Activity 3: *Too Much To Handle*

Place students into pairs and distribute the bags of number tiles, and red beads, along with the first two student pages. Allow the students time to work through these pages. Once the pairs have completed the activity, regroup for a brief discussion.

Have the students explain their methods for simplifying the numbers provided on the student pages. Ask them to explain how they came up with these methods.

Inform the students that scientists have devised a specific method that works as a sort of "short-hand" to make these numbers easier to manipulate. This method is called: ***Scientific Notation.***

Provide the rules for scientific notation and inform the students that they will be exploring this form of expressing large numbers in the following activity.

Hand out the rest of the student pages to each pair, along with a bag of black beads. Instruct the students to work through the rest of these pages with their partner. Circulate around the room and offer help or hints as needed. Make sure the students discuss the questions on the last page with their partner before reconvening as a large group.

Debrief:

As a whole class, have students discuss the questions on the last page of the student page packet.

- ✓ ***Be sure to emphasize the point that the red beads in the Powers of Ten box represent powers of ten.***
- ✓ ***Ask the students how one could use Scientific Notation on very small numbers, and to demonstrate their ideas (on the board). Have a lengthy discussion around this point.***

These activities offer an introduction to the concept of Scientific Notation. It is good to follow these activities with discussions on how to add, subtract, multiply and divide numbers in scientific notation. This is where students will discover, first hand, how much simpler it is to use this form of numerical expression when dealing with large (or small) numbers.

Activity 4: *The Winning Tower of Sticky-Notes*

This activity was developed to introduce the concept of Infinity. Specifically, this activity deals with the expanding nature of infinity (where numbers grow larger and larger, without end.

Objective:

- Model the expanding nature of infinity.

Standards:

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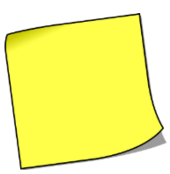
Estimated Time:

- **15 minutes – Activity**
- **10 minutes – Debrief**

Suggested Inquiry Approach:

Materials:
for each team:
- 1 set of sticky notes per turn



Begin this activity by placing students into teams of two to four. Explain that they will be competing against other teams to see which team can build the biggest tower out of sticky-notes.

Be sure to ***NOT*** define the term: bigger.

Inform the student groups of the rules to the challenge:

- Each team is provided a set of sticky notes from which to build.
- For the first round, each team must build their towers. Once they have completed the task, they must step away from the tower.
- The towers from each pair of teams will be compared, and a winning tower will be determined.

Activity 4: *The Winning Tower of Sticky-Notes*

Once the first round is completed, and a winning team from each pairing is determined, ask the winning teams:

- **How can you make the losing teams' tower bigger?**
- **What materials do you need to do so?**

Once they state that they would need more sticky-notes, provide the losing teams more sticky notes, and give them time to make their towers bigger.

Ask the prior winners:

- **They now have the winning tower? How can we make yours bigger?**

When they determine they need more sticky-notes, provide them with sticky notes to make their tower bigger.

After one or two more turns, ask the students how long this process could go on if you could always provide the materials (this should lead them to the concept of infinity).

Reconvene as a large group to discuss the activity.

Debrief:

As a whole class, have students discuss their answers to the following questions:

- ***Which team won the challenge? Explain why.***
- ***What does this tell you about the concept of infinity?***
- ***Based on this activity, how would you define infinity?***

Activity 5: *The Finish Line*

This activity was developed to provide a hands-on understanding of how the concept of infinity can be applied to decreasing numbers. Through an examination of the first of Zeno's paradoxes, students discover idea that space can be infinitely divided. Therefore, the infinite isn't merely large; it can also refer to smaller and smaller numbers.

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Estimated Time:

- **20 minutes – Activity**
- **10 minutes – Debrief**

Suggested Inquiry Approach:

Inform the students that they will be solving a mystery. A turtle had set out to run a race. The turtle firmly asserts that he reached the finish line and completed the race. However, a competitor argues against the possibility that the turtle could even reach the midpoint of the race, let alone, cross the finish line! To solve the mystery, the students must measure the distance between the turtle and the start line at different points of the race.

Materials:
for each team of 2:
- 1 ruler
- Masking tape
- 1 toy turtle
- Student Pages
- Set of Activity 5 Cards
- Marker

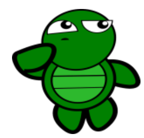
Activity 5: *The Finish Line*

Pair off the students and provide each pair with the necessary materials: one ruler, masking tape, one toy turtle, student pages, the cards and a marker.

Instruct the students to use the masking tape to denote the start line and the finish line for the race. The start line should be placed at one end of the table upon which they are working; whereas, the finish line should be placed at the opposite end.

Be sure the students place the turtle at the finish line before proceeding with the rest of the activity.
Allow time for the students to work through the activity. Students should also read through and answer the questions on the student pages follow the directions on the student pages. Circulate around the room to ensure student conversation.

Once the students have completed the activity, reconvene as a large group to debrief the activity.



Debrief:

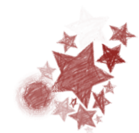
As a whole class, have students discuss their answers to the following questions:

- ***Did the turtle ever cross the finish line? Did it ever leave the start line? Explain.***
- ***Were you limited by the precision of your measuring tool (the ruler)? Explain.***
- ***If the measuring tool were more precise, could you keep measuring smaller and smaller distances?***
- ***What was the mathematical representation of Zeno's paradox that you came up with? Explain.***
- ***Did this change your view of whether or not the turtle made it to the finish line?***
- ***What does this information add to your understanding of the concept of infinity?***

Too Much To Handle

Very large quantities are observed, measured and manipulated throughout science. Below are some examples of very large quantities measured or used in a variety of scientific disciplines:

1. Amount of data generated per day in 2012: 2,500,000,000,000,000,000 bytes
2. Average number observable stars in a galaxy: 100,000,000,000 stars
3. Average number of neurons in a human brain: 100,000,000,000 neurons



Recreate these numbers using the number tiles for the digits 1-9, and the red beads for the zeros. Observe the arrangement of the number tiles and the red beads (representing zeros) for these values.

- What do you notice most about these values, the number tiles and the red beads (zeros)?

- How do you express these numbers with words?

2,500,000,000,000,000,000 = ______________________

100,000,000,000 = ______________________

- In the expressions above, which digits are emphasized (those represented by the number tiles, or those represented by the red beads?) Why?

Too Much To Handle

In the space provided, devise a way to express these values in a simplified manner. Be sure to provide the rules necessary to convert the large numbers to this simplified form.

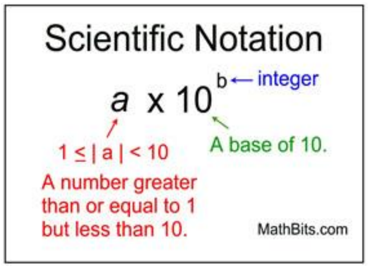
Too Much To Handle

KEY

Red Bead = Power of 10

Black bead = Decimal

Rules for formatting for scientific notation:



- Build the quantity, 23000, using only the number tiles and the red beads as before.

Use the template provided for the following steps:

- Place the quantity denoted by the number tiles in the box labeled: **Significant**.
- Place the red beads in the box labeled: **Powers of 10**.
- Observe the tiles and beads in the template.

  - What is the importance of the quantity in the **Significant** box?

  - What is the importance of the quantity of red beads in the **Powers of 10** box?

  - What is the importance of the **"x"** symbol in between the boxes?

Too Much To Handle

- Is the quantity in the "Significant" box greater than or equal to 1 but less than 10? If not, use a black bead as a decimal to make this number fit the rule. ***Do not proceed to the next step until this number fulfils the rule.***

  - What do the places behind a decimal represent?

- Count the **<u>number of places</u>** behind the decimal. Add this quantity of red beads to the "Powers of Ten" box.

  - What are you doing, *mathematically*, when you place these red beads in the **Powers of Ten** box?

- Count the number of red beads now contained in the "Powers of 10" box. This number will be the exponent when you write out the quantity in scientific notation ***(represented by the letter b in figure 1 on page 1).***
- The number in the "Significant" box is the quantity represented by the letter b in figure 1 on page 1.
- Place the quantities for "a" and "b" in the appropriate spots to represent this number in scientific notation:

$$\underset{a}{\underline{\hspace{4em}}} \times 10^{\underset{b}{\underline{\hspace{1.5em}}}}$$

Too Much To Handle

Repeat these steps with the following numbers:

a. 250,000,000
b. 704,000,000,000
c. 210

Which numbers were placed in the **Significant** box?

a. ___________

b. ___________

c. ___________

In the **Powers of 10** box?

a. ___________

b. ___________

c. ___________

Write the numbers in Scientific Notation:

a. ________________

b. ________________

c. ________________

Too Much To Handle

Questions

1. Why is using scientific notation beneficial for scientists?

2. Explain why the digits to the left of the decimal (or to the left of the exponent) are considered "significant".

3. Does this system of notation work for very very small values?

4. Does this system of notation work for negative values?

The Finish Line

Harry Hare adapted a paradox thought to be proposed by Zeno of Elea (450 BCE). In this paradox, Achilles and the Tortoise were in a race. The tortoise claimed he would win the race as long as Achilles gave him a head start (10 meters). Of course, Achilles thought the tortoise mistaken but the tortoise claimed he could prove this using mathematics.

The tortoise asked Achilles how fast he could make up the 10 meter head start (essentially, in enough time for the tortoise to move 1 meter more). The tortoise asked if Achilles thinks he could make up that 1 meter pretty quickly. "Why, of course". But in that time, the tortoise would have moved another set distance. By the time Achilles would catch up to where the tortoise had been, the tortoise would have moved ever so slightly ahead – Achilles would always be playing catch-up.

This story is often rephrased as continuously halving the distance from an end-point to a start-point. This "halving of the distances" get infinitely smaller.

The Finish Line

Consider this paradox in mathematical terms. Apply Zeno's paradox to running a 1 mile race. Regard the finish line as equal to 1, and remember you have to run multiple half-distances to reach this finish line. Record the equation in the space below:

Based on this equation, was Harry Hare correct in stating that the tortoise could never move from the start-line? Explain your answer in the space below.

Adapted from http://platonicrealms.com/encyclopedia/zenos-paradox-of-the-tortoise-and-achilles

1

Salt Water Sources

97.5% of the water on Earth is salt water. Most of this is ocean water, but salt lakes and marshes also provide salty habitats. While most of the salt in these waters is similar to table salt (sodium chloride), other dissolved solids also provide a salt component (such as potassium, magnesium, calcium, fluoride, and sulfate).

If the 400 chips in your bag represent all the water on Earth, how many of these chips model salt water?

Answer: __________ chips ***Place these chips on the card and set the card aside.***

2

Freshwater Sources

How much Earth water (chips) do you have left? These chips represent the percentage of freshwater on the planet. Freshwater can be found in lakes, streams, rivers. A lot of the freshwater is "locked up" and hard to access. Examples of these sources are glaciers, ice caps, and deep ground water.

About 90% of the freshwater on Earth is "locked up" in glaciers, ice caps and deep ground water. How many of the remaining chips model this "locked up" freshwater?

Answer: __________ chips ***Place these chips on the card and set the card aside.***

3 Accessible Freshwater

How much Earth water do you have left? The chips that are left represent the percentage of surface water and other freshwater not locked up in glaciers or ice caps. About 80% of ***this*** water is hard to access, largely consisting of ice and permafrost, marshes or swamps, and atmospheric (vapor). We get a lot of our freshwater from the remaining main sources: ***lakes and rivers***.

Determine the percentage of freshwater we have easy access to (can use). Use masking tape to represent this percentage on the remaining chip(s). **Answer: _________ percent**

Place this representation on the card.

4 Swimming in the Numbers

Use the information in this activity to find the percentage of accessible freshwater of ***ALL*** the water on Earth (salt water and all the freshwater sources). ***Hint: it is NOT the answer on card #3.*** In the space below, record your answer, and the steps taken to get the answer.

Too Much To Handle Template

Significant

X

Powers of 10

1

Half Way There

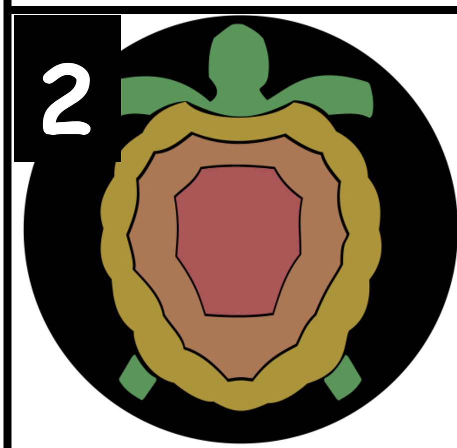
Harry Hare told Timmy Tortoise that there was no way Timmy would beat him to the finish line.

“In fact,” Harry said, “You won’t even be able to move away from the start line – I can prove it using mathematics!”

Timmy Tortoise was not happy but decided to humor Harry. He said “Okay, then, prove it.”

Harry decided to work backwards. Place Timmy Tortoise at the finish line. Measure half way between the start and finish line and mark this spot with a piece of tape. Label the spot with an “A”. Move Timmy to this spot.

2



…And Again

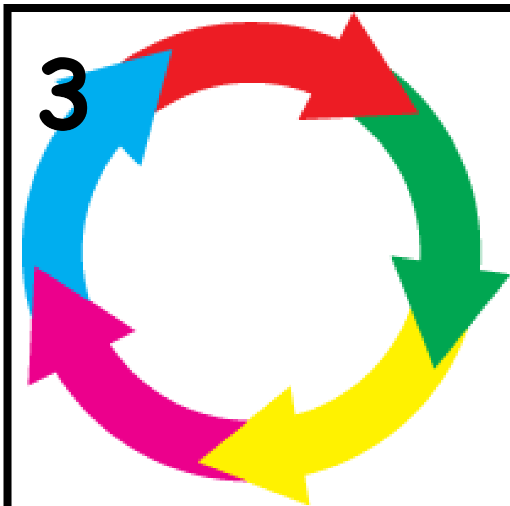
“Harry,” Timmy exclaimed, “I do not understand this explanation at all.”

Harry explained that before he could ever make it to the finish line, he had to make it to the half way point.

He went on to say, “And before you make it there, you need to make it half way between that point and the start.”

Measure the distance half way from the start to point “A”. Mark this spot with tape and label with a “B”. Move Timmy to this spot.

3



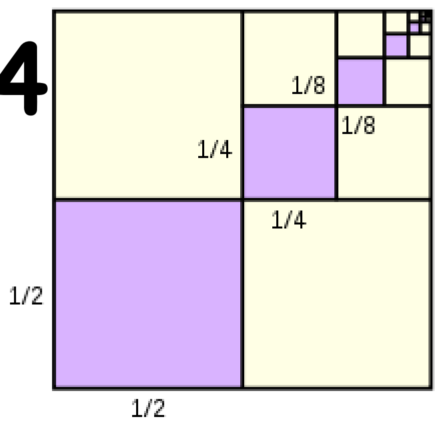
…And Again

"Really, Harry?" said Timmy. "I was just at the Finish line and all you do is just move me backwards, and back, again." Timmy Tortoise was convinced the Hare was tricking him.

"Just go with it, Timmy," said Harry, "I know what I'm doing!"

Measure the distance half way from the start to point "B". Mark this spot with tape and label this spot with a "C". Move Timmy to this spot.

4



Blame it on Zeno

"At this rate, you will send me all the way back to the start line," cried Timmy.

"Since I am working backwards, not quite!" said Harry, "But if we had started at the start line, you would never have left".

Using the pattern of halving the distance, why can Harry say that Timmy would have never been able to leave the start line?

Is Harry correct?