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Engaging Students in Authentic Mathematical Discourse in a High School Mathematics Classroom

Whitney Ann Evans

ENGAGING STUDENTS IN AUTHENTIC MATHEMATICAL DISCOURSE IN A HIGH SCHOOL MATHEMATICS CLASSROOM

A Masters Thesis

Presented to

The Graduate College of

Missouri State University

In Partial Fulfillment

Of the Requirements for the Degree

Master of Science in Education, Secondary Education

By

Whitney Ann Evans

May 2017

ENGAGING STUDENTS IN AUTHENTIC MATHEMATICAL DISCOURSE IN A HIGH SCHOOL MATHEMATICS CLASSROOM

Mathematics

Missouri State University, May 2017

Master of Science in Education

Whitney Ann Evans

ABSTRACT

The purpose of this study is to gain insight into the successes and challenges a teacher faces when implementing authentic mathematical discourse in a high school mathematics classroom. Nine lessons were implemented that emphasized student engagement in Common Core Mathematics Practice Three (MP3) (construct viable arguments and critique the reasoning of others.) This was a qualitative action research study that took place in a Geometry classroom with 16 9th – 11th grade students. Personal journal entries were recorded by the teacher-researcher after each lesson which included detailed summaries of the lesson and reflections on instances of students engaging in MP3 as well as observed challenges that hindered students' engagement in MP3. Observational data from a building-level instructional coach and student artifacts were also collected. Data showed that the following three factors were prominent in leading to student engagement in MP3: 1) transferring responsibility from the teacher to students; 2) investing into lesson plans, time management, class structure, and relationships with students; and 3) establishing a classroom culture of safety, respect, and high expectations. The results of the study indicated that high school mathematics students are capable of engaging in authentic mathematical discourse when teachers intentionally consider these three factors and implement lessons that provide opportunities and encourage students to engage in MP3.

KEYWORDS: authentic mathematical discourse, Common Core, Eight Standards for Mathematical Practice, high school mathematics classroom

This abstract is approved as to form and content

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Chairperson, Advisory Committee
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CHAPTER I: OVERVIEW OF THE STUDY

Teenagers sit in mathematics classrooms around the world learning everything from how to find the area of a circle in Geometry to how to differentiate and integrate in Calculus. How many students truly understand what they are learning and could go beyond completing textbook mathematics problems to discussing the topics they encounter in their mathematics classes?

Paul Lockhart (2009) in “The Mathematician’s Lament” agonizes over the lack of true mathematics that high school students encounter in early 21st century classrooms. He compares mathematics with an art form that requires creativity and imagination. Lockhart (2009) contrasts the beauty of putting a triangle inside a rectangle and trying to figure out how much space the triangle takes up with teachers giving students the area formula for a triangle. He argues that simply providing a formula completely removes any mathematics problem from the lesson. A solution has been given and there is no room or reason for students to use their imaginations to make sense of a simple and elegant mathematical scenario (Lockhart, 2009). Most teachers were taught with this emphasis on formulas and algorithms. As a result, many teachers are continuing to spread this misconception about the nature of mathematics. Lockhart (2009) questions whether traditional mathematics education is optimal or even effective at all for the majority of students.

According to the Common Core State Standards (CCSS), the ultimate goal for any mathematics teacher should be for students to understand the presented material at a conceptual level and to see the value and beauty in what they are learning (National

Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). Mathematics is intrinsically interesting, beautiful in its simplicity, and also necessary for everything from telling time (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010) to modeling particles in quantum mechanics (Missouri State University Physics Department, 2016). Is it possible that most of the population, students and even teachers alike, does not really know what mathematics is? Paul Lockhart (2009) said the following about mathematics:

> Mathematics is the music of reason. To do mathematics is to engage in an act of discovery and conjecture, intuition and inspiration; to be in a state of confusion—not because it makes no sense to you, but because you gave it sense and you still don't understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be alive, damn it. (p. 8)

Many teachers, students, and parents believe they excel at mathematics, when in reality, they may simply be good at following rules and using formulas. Plugging numbers into formulas and following the rules of mathematics without any deeper understanding is not considered "mathematically proficient" by the authors of the CCSS (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). The CCSS has published Eight Standards for Mathematical Practice (App. A) that have been derived from the National Council of Teachers of Mathematics (NCTM) and a report by the National Research Council (NRC) that "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010, para. 1).

Mathematics teachers face the challenge of developing these mathematical practices in their students when they themselves may not feel confident using all eight of the mathematics practices. Significant time may be necessary to teach high school students the required mathematical content as well as teach students to think critically and engage in authentic mathematical discourse (MP3). Preparing to teach mathematics by incorporating the eight Standards for Mathematical Practice may mean a greater workload for mathematics teachers. The eight Standards for Mathematical Practice require that mathematics teachers guide and mentor young mathematicians in mathematical thinking, reasoning, and discourse.

Rationale for the Study

As a high school mathematics teacher, I have experienced students who can find an answer to a mathematics question but lack any knowledge of why it is correct. My colleagues and I have witnessed mathematics students who are able to score A's on tests throughout the semester but are unable to pass a comprehensive final exam. Standardized test scores, inconsistencies between classroom grades and final exam and standardized test exam scores, and my own experiences are indicators that something is missing in the traditional approach to secondary mathematics education.

Students participating in a study by Nogueira de Lima and Tall (2008) showed evidence of the need to move beyond procedural skills to conceptual understanding in high school mathematics classrooms. Nogueira de Lima and Tall (2008) found that students were not making viable arguments or thinking critically about the mathematical concepts. A researcher at a Finland University noticed a similar mathematical understanding gap among their first year mathematics students (Oikkonen, 2009). These studies (Nogueira de Lima & Tall, 2008 & Oikkonen, 2009) showed the need for further research into how to implement mathematical experiences in the classroom that would engage students in conceptual mathematical thinking as well as how those experiences might positively impact student learning and understanding.

Multiple researchers have sought to discern and understand the value of mathematical discourse as a strategy to increase conceptual understanding and academic achievement in students (Oikkonen, 2009; Mercer & Sams, 2006; Kazemi & Stipek, 2008; & Bonotto, 2013). Teacher-researcher Oikkonen (2009) found that increasing the mathematical discussion during the class period seemed to significantly increase the passing rates for students in his university mathematics courses. Mercer and Sams (2006) studied the impact of providing students with modeling and direct instruction regarding how to engage in meaningful mathematical discussion. These students were better able to solve, understand, and make sense of mathematics problems when compared with students in the same classrooms who did not receive the modeling and direct instruction. The instructed students also scored significantly higher on standardized tests than their peers who did not receive the modeling and direct instruction on mathematical discussion (Mercer & Sams, 2006). Kazemi and Stipek (2008) found that with teacher support, when students were pressed to persevere, they were capable of discussing and making sense of challenging mathematics problems. Bonotto (2013) has shown that incorporating contextual scenarios is one way to encourage students to engage in authentic mathematical discourse.

While these researchers all found data to support the positive implications of mathematical discourse in the classroom, little research has explicitly studied the implementation of authentic mathematical discourse through the eyes of a secondary teacher. Heaton (2000) studied her experience implementing The *Professional Standards for Mathematics Teaching* (NCTM, 1991) in an elementary classroom. Her research further showed the importance of allowing students to make mathematical discoveries and engage in various types of mathematical thinking and discussion. Developing "mathematically proficient" students per the CCSS Standards for Mathematical Practice (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010) is a daunting task, and few researchers have studied the successes and challenges of high school teachers as they attempt to implement these practices in their classrooms. The implications of implementing Mathematical Practice Three, just one of the Eight Standards for Mathematical Practice, could be significant for both teachers and students.

Purpose of the Study

The purpose of this study is to gain insight into the successes and challenges a teacher faces when implementing Mathematics Practice Three (MP3) from the CCSS Standards for Mathematical Practice in a high school mathematics classroom. Researcher-journals, self-reflection, and observations by a fellow educator were used to identify successful strategies or challenging obstacles to implementation of MP3 and mathematical discourse in the classroom.

Research Questions

The following questions will guide my research.

1. Which teaching strategies lead to successful student engagement in Common Core Mathematical Practice 3 (MP3: *Construct viable*

arguments and critique the reasoning of others)?

2. What challenges was the teacher confronted with, and how did she react as she attempted to develop MP3 in her students and classroom?

Research Design

This qualitative study used an action research design (Hendricks, 2013). Data was collected between November and December of 2016 in the form of journal entries I recorded based on my own observations and experiences in the classroom, observational notes from my instructional coach, and student artifacts. My primary source of data was observations (recorded in my journal and made by my instructional coach). I recorded as many of my observations as possible throughout the study that showed evidence of my implementation of MP3. I also asked the building instructional coach to observe me teaching to look for evidence of my implementation of MP3 and ways in which my instruction seemed to be successful or unsuccessful at implementation. The instructional coach has experience teaching English but is passionate about secondary mathematics education. I hoped having an observer with experience in a different content area would increase the validity of my study by helping me avoid bias.

The instructional coach and I used an observational protocol to allow for consistency in identifying occurrences of the implementation of MP3 (App. B). This protocol allowed the instructional coach who observed me to better recognize implementation of MP3. I also collected samples of student artifacts which showed evidence of successes and challenges as I developed MP3 in my students. Nine lessons were taught with an emphasis on student engagement in MP3. All occurrences of student engagement in MP3 or of challenges that impeded or hindered student engagement in MP3 were categorized and dated so that themes could be identified.

Significance of the Study

The goal of this study was to gain insight into how a teacher can implement CCSS MP3 (construct viable arguments and critique the reasoning of others) and the successes and challenges along the way. Prior research supports inquiry and authentic discussion in the classroom to improve student understanding and learning (Oikkonen, 2009; Mercer & Sams, 2006; Kazemi & Stipek, 2008; Bonotto, 2013; and Heaton, 2000). Since I was the researcher as well as a participant, this study aimed to investigate how I taught my students to engage in MP3 through authentic mathematical discourse. The results of this study show mathematics teachers what they can do to develop MP3 in their own students. It also prepares educators for the challenges teachers face when trying to teach students to engage in authentic mathematical discourse. I hope the honesty with which I shared my findings, both my successes and challenges, will give other teachers hope, validation, and encouragement.

Also of significance was insight into how students received this type of mathematical instruction. Teacher-student and student-student interactions throughout my journey to implement MP3 showed specific examples of how students increased their mathematical understanding and the benefits from being taught to make and critique viable mathematical arguments (MP3). To successfully teach mathematics to high school students, teachers must continue to learn how they can successfully implement practices into their classrooms that will promote student engagement in the Standards for Mathematical Practices outlined in the CCSS and, in particular, MP3.

Most importantly, my action research was significant because it provided me with learning that I could not gain from any other experience and could immediately apply to my classroom and lesson planning. I was able to see firsthand what it can look like to develop a mathematical approach in my students that relies on mathematical discourse and MP3. My findings allowed me to grow and improve as a teacher. As I continue to feel more comfortable teaching lessons with a focus on students constructing viable arguments (MP3), I can begin to shift my focus to developing some of the other Mathematical Practices in my students as well.

Assumptions

The following is a list of assumptions I made while conducting my study.

1. I assumed students would act in a typical manner throughout this study and not be exceptional from students in any of my other classes. While individual students are unique, I assumed the class as a whole would be representative of most general education high school mathematics classes.

2. I assumed students would not have been previously introduced to mathematics lessons in the past with a focus on constructing viable arguments and critiquing the reasoning of others (MP3). I assumed the majority of students' mathematical experience had been focused primarily on procedures with the teacher as the primary source of logic and reason.

Limitations

The following is a list of limitations I encountered during my study.

1. Time and required curriculum were limitations. Because I was required to teach a list of mathematics topics by the end of December, I was unable to spend as much time as I would have liked developing MP3 in my students. Furthermore, the

duration of my study was a limitation as my data collection took place over the course of two months. More data could have been collected and analyzed if my study had spanned an entire school year. Ideally, many lessons would be conducted with the same group of students over an entire school year (or longer) so that students could get comfortable with the new way of teaching and learning, and long-term data on the teacher's experience could be studied. I also had limited personal time to create the lesson plans that I studied. With more personal time for lesson-planning, I could have created, implemented, and studied more than nine lessons with an emphasis on MP3.

2. Consistency with other teachers was a limitation. Our mathematics department requires that all Geometry teachers teach and assess topics within a few days of each other. Had maintaining this level of consistency not been a factor, I could have spent more time focusing on implementing MP3. I may have chosen to spend more time allowing students to make mathematical discoveries and explore the content in depth through writing and mathematical discourse.

3. The time the class occurred was a limitation. Some students were slow to engage in the mathematical thinking and discourse due to having class so early in the morning (7:25 a.m.).

4. My possible unrecognized bias toward certain students may be a limitation. While I intentionally question, offer assistance, and engage in discussions with all students, it is possible that I spend more time with some students than others. This could be due to students' willingness to ask questions or accept help. This could be because of my prior knowledge of which students have needed extra support during previous class periods. It could also be due to students' response to my encouragement to persevere when they do not see a solution strategy.

Definition of Terms

The following is a list of important definitions.

1. Authentic Mathematical Discourse: Authentic mathematical discourse is defined for the purposes of this study as any instance in which a student is discussing a mathematics problem or pattern, thinking or writing analytically about a mathematics problem or pattern, or solving or posing a mathematics problem or pattern with a peer, teacher, or group.

2. Inquiry: The act of learning mathematics through discovery (as opposed to simply being told facts and procedures).

3. MP3: MP3 is an acronym for "Mathematics Practice 3" and refers to the third Standard for Mathematical Practice as listed in the CCSS (App. A). MP3 states

that mathematically proficient students will “construct viable arguments and critique the reasoning of others.” I will identify student engagement in MP3 with the following list from the CCSS’ description of MP3:

a. Students using stated assumptions, definitions, and previously established results to construct arguments.
b. Students making conjectures.
c. Students building a logical progression of statements to explore the truth of their conjectures.
d. Students breaking a situation into cases for analysis.
e. Students recognizing and/or using counterexamples.
f. Students justifying their conclusions.
g. Students communicating their conclusions and justifications to others.
h. Students responding to the arguments of others.
i. Students reasoning inductively about data, taking context into account.
j. Students comparing arguments and distinguishing between correct and flawed logic.
k. Students asking questions to clarify or improve arguments.

(National Governor’s Association Center for Best Practice, Council of Chief State School Officers, 2010, para. 4)

4. Success: A success, for the purpose of this study, will be evaluated by a student engaging in MP3. I will recognize one of my teaching strategies as successful if it results in any evidence of student engagement in MP3 as determined by students engaging in any one of the eleven parts in the definition of MP3.

Summary

CCSS MP3 states that proficient mathematics students engage in authentic mathematical discourse by making viable arguments and critiquing the reasoning of others (National Governor’s Association Center for Best Practice, Council of Chief State School Officers, 2010). Researchers have studied the possible benefits of incorporating mathematical discourse into mathematics classrooms (Oikkonen, 2009; Mercer & Sams, 2006; Kazemi & Stipek, 2008; & Bonotto, 2013), but few have studied the role of the teacher. Heaton’s (2000) study sought to gain insight into the successes and challenges she faced as she implemented new teaching strategies in an elementary classroom. My study is significant because my findings provide support to high school mathematics teachers that want to implement lessons and teaching strategies that lead to increased student engagement in MP3.

In this study, I used action research to learn from my experience as a secondary mathematics teacher as I implemented MP3 into my classroom. The purpose of the study is to gain insight into successes and challenges that teachers face when implementing authentic mathematical discourse to engage students in MP3. Observation notes from a visiting instructional coach and detailed journals of my own experiences were the main sources of collected data. My findings were significant because they provide mathematics educators with specific strategies to encourage student engagement in MP3 in the classroom. My findings also provided a detailed analysis of the challenges that arose and the teacher’s reaction.

CHAPTER II: REVIEW OF RELATED LITERATURE

Mathematics education in most traditional classrooms does not emphasize inquiry or conceptual understanding. Many students (and possibly even teachers) seem to have a false perception of the nature and significance of mathematics (Lockhart, 2009). Existing research states that conceptual understanding improves for students when teachers focus on understanding rather than procedure (Nogueira de Lima & Tall, 2008). CCSS MP3 and evidence from related literature point to mathematical discourse in the classroom as one way of shifting from a procedural focus to an emphasis on conceptual understanding. Evidence also suggests the importance of pressing students to persevere not only in finding a solution, but also in justifying their thinking (Kazemi & Stipek, 2008). Research will also be discussed which shows some obstacles other teachers have faced when implementing lessons that attempt to engage students in authentic mathematical discourse as well as help students attain deeper conceptual understanding (Heaton, 2000; Dennis & O'Hair, 2010).

Conceptual Understanding is Missing

Nogueira de Lima and Tall (2008) studied the necessity of shifting teaching mathematics from a procedural focus to a conceptual focus. They chose to focus their study on solving linear equations and students' ability to make viable mathematical arguments. They likened many students' solving of equations to "magic" and hypothesized that students did not understand why they could do the various allowed algorithms such as moving terms from one side of an equation to another and changing the sign.

Nogueira de Lima and Tall (2008) conducted interviews after students completed an assessment containing equations of varying difficulty levels. The interview responses were analyzed for students' perceptions and thinking. Not one interview included a student mentioning *why* they performed their steps the way they did. No mathematical reasoning was documented; only explanations of procedures. Evidence of a lack of conceptual understanding was witnessed by students' inability to correctly solve and make sense of problems. Students struggled to use mathematical reasoning when discussing their thinking with the researchers. Students appeared to be manipulating numbers and symbols almost like "magic." Nogueira de Lima and Tall (2008) wrote, "The shift from many actions to a single thinkable concept is an act of compression that is an essential tool in making profound sense of mathematics" (p. 5). They concluded that the students they interviewed did not have a conceptual understanding of the mathematical concept of equality.

Similarly, Marshall et al. (2009) studied the amount of inquiry instruction taking place in mathematics classrooms, which showed the necessity of incorporating strategies in the classroom with the potential to increase students' mathematical proficiency. Marshall et al. (2009) hypothesized that inquiry learning was a key to improving student achievement and conceptual understanding. They compared how much time teachers believed should be spent on inquiry in the classroom with how much time teachers perceived they actually spent incorporating inquiry lessons in their classrooms. The researchers hoped to gain insight into the implementation of inquiry instruction in mathematics classrooms by studying “teachers’ behaviors, beliefs, and motivation regarding inquiry instruction,” (p. 576) to determine if any of these factors were related.

Marshall et al. (2009) found that all mathematics teachers at all grade levels believed they should incorporate more inquiry instruction than they were currently implementing. The amount of time spent using inquiry instruction appeared to be positively correlated with self-efficacy (specifically toward teaching inquiry lessons), grade level taught, and available support. Their data suggests teachers need to be empowered through long-term professional learning with a high level of support to improve their self-efficacy and enable teachers to improve their instructional practice. While their study focused on inquiry, rather than mathematical discourse, as one strategy for improving students’ conceptual understanding in mathematics, their study considered factors that affected the teacher and thus the implementation of non-traditional teaching strategies. According to Bonotto (2013), there is also a relationship between providing students with opportunities to make their own mathematical discoveries and encouraging authentic mathematical discourse in students.

Mathematical Discourse to Address Lack of Conceptual Understanding

Bonotto (2013) studied the use of contextual artifacts as a way to link inquiry, relevance, and mathematical discourse to achieve increased conceptual understanding in students. Bonotto (2013) predicted that presenting students with real artifacts (receipts, brochures, maps, advertisements, etc…), and asking them to pose problems concerning those artifacts would provide relevant opportunities for students to create connections between mathematics and real life.

Bonotto (2013) conducted two qualitative studies with fifth graders to observe the impact of problem-posing and problem-solving activities on creativity and sense-making. In the first study, fifth graders analyzed a grocery store advertisement and wrote word problems involving percentages using the advertisement. Students solved each other's problems and the teacher facilitated a large-group discussion about errors and confusion in the various students' problems. Students were critiquing the reasoning of their peers. The data showed students making sense of the mathematics involved in the ad to create more creative, more complex, and more authentic word problems than typically encountered in a mathematics textbook.

Bonotto (2013) found that all but one of the pairs created solvable, logical word problems. The final group created a problem with very complex calculations necessary for solving, so the class worked together to modify the data and create a story problem that was more straightforward for everyone to solve. In conjunction with collaborating to edit that group's word problem, students were encouraged to think of further questions to pose in all of their peers' word problems, as well. The emphasis was shifted from getting an answer, whether it made sense or not, to engaging in mathematical discourse regarding the complexity, relevance, and reasonableness of the students' generated mathematics problems.

Similarly, Bonotto's (2013) second study also involved fifth graders creating word problems from real artifacts. Four classrooms were given a brochure to an amusement park, and spent two hours discussing and interpreting the information on the brochure before being given 45-60 minutes to independently write as many mathematical word problems involving the brochure information as possible. Four of the word problems were selected by the teacher based on their probability of producing mathematical discussion and given to all students who then had two hours to solve them, write about how they solved them, and critique the problems. Bonotto's (2013) findings showed that after engaging in mathematical discussions surrounding the artifacts, the students independently generated an average of more than two problems per student, and more than 60% of the generated mathematics problems were solvable and required multiple steps. These findings were evidence of conceptual understanding in part due to students' opportunity to make their own mathematical inquiries and engage in discourse about their mathematical ideas. The teacher's impact, teaching practices, and classroom environment were recommended areas for future study (Bonotto, 2013).

Rather than attempt to alter teaching strategies and study the outcome, Sahin, Isiksal, & Koc (2015) conducted a qualitative study with the goal of observing and then painting a vivid picture of the typical dialogue taking place in elementary mathematics classrooms in Turkey. Turkey has followed in America's footsteps and released similar standards for mathematical practice. Sahin et al. (2015) stated the following Turkish standard with similarities to MP3:

> Students can be required to write about the way of their solving a problem or want them to explain what a rule means. To talk and write about/on mathematics will help them to construct the mathematical concepts easier. Teacher should provide appropriate classroom discourse in which students have opportunities to explain their ideas, discuss, and explain by writing. (p. 301)

This Turkish standard wants students to explain mathematical rules and their meanings and use writing and discussion to achieve conceptual understanding. Similarly, MP3 requires that students create their own viable arguments and think critically about the mathematical claims of others. The CCSS document lists various ways that mathematics educators should encourage and aid students in using writing and discussion to achieve increased mathematical proficiency in MP3 (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). Sahin et al. (2015) recognized the necessity of their study because they believed the possible outcomes of effective mathematical discourse could include students learning to find different strategies, students understanding meaning behind procedures (not just being able to replicate the procedures), and students receiving empowerment that comes with greater understanding and the ability to communicate their understanding of mathematics. They also believed teachers might improve at identifying misconceptions and monitoring the effectiveness of their lessons (Sahin et al., 2015).

Sahin et al. (2015) tracked the types of discourse present in different content and learning activities in the classroom as either procedural, conceptual, problem-solving, making connections, active engagement, alternate solution strategies, representations, or justification. The results painted a picture of little authentic mathematical discourse. Procedural thinking was seen in all twenty observed lessons, and active engagement was identified in 50% of the lessons. Every other category was present less than half the time, with conceptual thinking and connections being observed in only three of the twenty lessons and alternate strategies and representations being observed only one time each (Sahin et al., 2015). The majority of lessons had a very low cognitive demand for students. Although justification was identified in nine lessons, the students were doing the justifying in only one of those instances. The other eight were referencing justification given by the teacher (Sahin et al., 2015). Their data showed the pervasiveness of procedural emphasis in the studied classrooms. Sahin et al. (2015) made the point that “another reason for the procedural nature of the mathematics noted in this study can be the characteristics of teacher questions. It was observed that teacher questions were not motivating the students to think in alternative ways and develop different solution strategies” (p. 315). Finally, Sahin et al. (2015) noted that having such a large class size, lack of pedagogical knowledge of how to lead a classroom using more student discourse, and comfort with a teacher-dominant environment may also have been factors in the lack of mathematical discourse, particularly by students.

Role of the Teacher

Sahin et al. (2015) predicted that teachers played a large role in affecting the quality and quantity of discourse and mathematical thinking that takes place in the classroom. Kazemi and Stipek (2008) wanted to describe the role teachers can play in encouraging students to think more deeply and to persevere when faced with challenging mathematics problems. Their findings provided practical strategies to increase conceptual understanding for students.

Teachers were selected for Kazemi and Stipek’s (2008) study who were encouraging, sensitive, enthusiastic, and nonthreatening so that fear, embarrassment, or a negative climate would not interfere with the study. The researchers believed a teacher’s ability to motivate students would be a significant factor in students’ willingness to persevere in engaging in mathematical discourse and problem-solving. Kazemi & Stipek (2008) defined motivation using the following four criteria:

1. How much the teachers focused on understanding and learning.

2. How much the teachers "pressed" students to persevere and work through difficult problems.

3. How much the teachers promoted student autonomy (self-evaluations and choices; personal responsibility)

4. How much the teachers deemphasized performance and getting right answers.

Kazemi and Stipek (2008) observed teacher vs. student roles, from the creation of the lesson plan to the discussions, questions, and engagement in the actual lesson. They also noted participation, frustration, perseverance, and depth of student engagement in both large and small groups. Initially, they believed four socio-mathematical norms needed to be present in the classroom to promote conceptual thinking and learning. The researchers categorized the data by "high press" and "low press" examples. "High press" was defined by Kazemi and Stipek (2008) using the following characteristics:

> (a) an explanation consists of a mathematical argument, not simply a procedural description; (b) mathematical thinking involves understanding relations among multiple strategies; (c) errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies; and (d) collaborative work involves individual accountability and reaching consensus through mathematical argumentation (para. 1)

The researchers determined that various relationships existed between the "high press" classes and the students achieving the four socio-mathematical norms listed below.

1. <u>Students describe their thinking</u>: In "high press" classes, students were expected to explain and justify using mathematical reasoning while "low press" teachers accepted a list of steps and "yes or no" answers. Student-teacher exchanges were longer in the "high press" classes, and answers that did not show conceptual understanding were questioned and critiqued. The "low press" students were often praised for correct answers and for why the answer was correct or for whether a better way might exist that was never discussed. Some students in the "low press" classes never actually engaged in the mathematics they were supposed to be learning. The teacher told one such group of students that doing so would probably be too difficult.

2. <u>Students solve problems in multiple ways and share with the teacher and their peers</u>: In the "high press" classrooms, multiple strategies were compared and

contrasted. Incorrect answers were used for learning rather than being brushed over as in the "low press" classes. In the "low press" classes, the teacher often told students which answer was correct and moved on without giving the students opportunities to make sense on their own or to discuss the various student ideas.

3. Mistakes are okay: All teachers made sure the students felt safe making mistakes. In the "high press" classes, both correct and incorrect answers were discussed. In the "low press" classes, the teacher usually corrected any mistakes or glossed over them until someone got the correct answer.

4. Students solve problems collaboratively: Students in the "high press" classrooms knew all students were accountable for understanding and being able to justify the group's answer so they questioned each other, listened to all ideas, and expected their group mates to give justifications. Students in the "low press" classrooms usually accepted whatever the students said who were perceived to be "most skilled."

All of the evidence in this study suggests that regular social norms and superficial good teaching strategies are not enough to achieve deep conceptual understanding in the classroom. Kazemi and Stipek (2008) believe more authentic mathematical discourse and conceptual understanding can be gained with the use of their four socio-mathematical norms. They also believe more research is needed on how their socio-mathematical norms influence mathematical understanding for students, as well as which other norms may be helpful or necessary to achieve a "high press" classroom.

Conner, Singletary, Smith, Wagner, & Francisco (2014) also studied how a teacher's interaction with students could impact students' conceptual understanding, specifically manifested in the students' ability to make a mathematical argument. Two student teachers were observed teaching a two-week Geometry unit in the same rural high school. Data was collected to analyze three separate aspects: teachers' direct contributions to making mathematical arguments, questions used, and supportive actions used.

The data was analyzed for which teacher contributions, questions, and actions led to mathematical arguments from students. Conner et al. (2014) found that all questions that led to collective classroom dialogue of mathematical arguments fell into one of these five categories in no particular order:

- Requesting Facts
- Requesting Ideas
- Requesting Methods
- Requesting Elaboration
- Requesting Evaluation

Similarly, five actions were found to be the most conducive to classroom mathematical discussions.

- Directing – Refocusing students on the problem, questions, or argument
- Promoting – Making suggestions or encouraging students
- Evaluating – Letting students know something they are doing is correct or incorrect
- Informing – Clarifying or giving more information
- Repeating – Restating student idea or writing ideas on the board

A comparison of Conner et al. (2014) and Kazemi and Stipek (2008) leads to questions concerning interpretation of data and making determinations about what is valuable in promoting conceptual understanding in mathematics education. Conner et al. (2014) state that while a framework of five questions and actions have been identified that contribute to collective classroom argument, the effectiveness and value of the actual arguments that result from their framework have not been studied. Evaluating and letting students know if something they are doing is correct or incorrect was identified by

Conner et al. (2014) as a strategy for promoting collective argumentation, but it sounds more like the “low press” teacher described in Kazemi and Stipek’s (2008) study. It may result in a student making a new claim or questioning something they thought previously, but the student may be missing an opportunity to make sense on their own and find their own misconceptions through being pressed to justify their thinking without the teacher “promoting” or making suggestions as Conner et al. (2014) would seem to suggest. Similarly, requesting factual information from students does not require any justification and is thus a question devoid of any opportunity for evidence of conceptual understanding or the formation of a viable argument from a student.

Achieving Authentic Mathematical Discourse

Mercer and Sams (2006) researched elementary students’ use of language to reason. They questioned if students’ individual understanding of mathematics or their ability to problem-solve and communicate effectively in groups would increase as the students received direct instruction on how to engage in mathematical discourse at an after-school program called *Thinking Together*.

By reviewing data from interviews, observations, and pre- and post-assessments, Mercer and Sams (2006) found an increase in student learning in classrooms with students who had participated in the *Thinking Together* program and even noticed a positive correlation between more mathematically dialogic classrooms and higher mathematical achievement. Video evidence showed that the teacher played a huge role in modeling sense-making with the use of language. Interviews confirmed that the students who participated in the *Thinking Together* program felt better-equipped to discuss mathematics problems (Mercer & Sams, 2006).

Similar findings occurred in a study at an university in Helsinki, Finland. Oikkonen (2009) gave students the opportunity to write in detail about missed exam questions until they understood their previous misconceptions. Students could discuss the mathematics problems with classmates and even teachers, and Oikkonen (2009) observed students gaining deeper understanding through this process. Although his study included no statistical analysis, Oikkonen (2009) believed that the introduction of mathematical writing and discourse led to an increase in the number of students passing introductory university mathematics courses.

Oikkonen (2009) also began incorporating the same discussion techniques in his large classes as he had previously been using in his small discussion sections. He spent more time teaching and discussing central themes with his classes even if it meant certain topics were never covered. His findings showed an increase in the number of students remaining in the university's mathematics program and succeeding academically in their first year mathematics courses.

A study by Page and Clarke (2014) also looked at the potential benefits of giving learners opportunities to engage in mathematical discourse in the form of journaling. Page and Clarke (2014) documented the experiences of third to fifth year elementary teachers as they attempted to implement various mathematics journaling strategies. The study focused on the teachers' experiences and perceptions as they implemented various writing strategies. Some teachers asked their students to write a response to the question, "Why?" Some participating teachers identified that asking this one question allowed them to gain deeper understanding into what students were gaining from the lessons.

Factors to Consider When Implementing Mathematical Discourse

Heaton (2000) studied her successes and challenges as a teacher as she attempted to implement the new mathematics reforms outlined in the *Mathematics Framework for California.* She spent a year periodically teaching mathematics in a fourth grade classroom. The *Professional Standards for Mathematics Teaching* (NCTM, 1991) provided her with the following necessary shifts in her teaching:

- Toward classrooms as mathematical communities
- Toward logic and mathematical evidence as verification
- Toward mathematical reasoning
- Toward conjecturing, inventing, and problem-solving
- Toward connecting mathematics, its ideas, and its applications

while encouraging her to move

- Away from classrooms as simply a collection of individuals
- Away from the teacher as the sole authority for the right answers
- Away from merely memorizing procedures
- Away from an emphasis on mechanistic answer-finding
- Away from treating mathematics as a body of isolated concepts and procedures. (Heaton, 2000, pp. 6-7; NCTM, 1991, p. 3)

Heaton (2000) made many important discoveries about her teaching practices and her mathematical knowledge and understanding. She learned that she needed to accept that she would never understand everything there was to understand about mathematics. She clearly showed evidence of this finding through two classroom examples, one in which she realized too late that she did not understand what was meant by a pattern in mathematics and a second example in which she found herself unable to answer a conceptual question about the addition of fractions. In both situations, she explored the problems with the students, did additional research outside of class, and maintained the role of a guide to help students make their own sense of the mathematics.

Heaton (2000) learned that she needed to be willing to deviate from her teacher's guide and share leadership, and sometimes control, with the students. She gave the details of a situation in which she was skeptical to call on a particular student due to his prior responses. She chose to let him share his question with the class, and it resulted in a rich mathematical discussion that never would have taken place had she not shared the control of the classroom with him. Initially, Heaton (2000) had believed that "telling" students anything was a bad teacher practice in an authentic constructivist mathematics classroom. Her findings, however, were that "telling" was a powerful tool that could be used strategically to improve student understanding in the proper context.

She shared three specific stories that highlighted her findings and the growth and learning that took place from September to June. She gave the details of a situation in which a student was very confused about the difference between addition and multiplication in a specific example. Until this point in the year, she had refrained from "telling" students anything and had expressed frustration that she felt as though she was not actually teaching and sometimes students left class with incorrect mathematical ideas. During this particular lesson she said, "I would like to show you something here," (Heaton, 2000, p. 56). As she explained to the students how to complete the problem, the students expressed understanding. The previously confused student was then able to explain the difference between the addition and multiplication that he had initially been confused about, evidence that sometimes “telling” is an important tool for a teacher, even when trying to implement more authentic mathematics in the classroom. The teacher can act as a model of how to construct reasonable arguments and critique suggested arguments (Heaton, 2000).

Dennis and O’Hair (2010) also studied the impact a mathematics teacher can have on the amount and quality of mathematical thinking and discourse that takes place in the classroom. In contrast to Heaton’s (2000) study, Dennis and O’Hair (2010) interviewed and observed multiple teachers to identify some of the major factors that surface when planning and teaching nontraditional lessons that incorporate “higher order thinking, deep knowledge, substantive conversation, and connections to the world beyond the classroom,” (p. 7). The study found that, while factors such as attendance, lack of funding, and time were all identified obstacles, quality and knowledgeable instructors had a greater impact when attempting to implement conceptually based lessons and mathematical discourse (Dennis & O’Hair, 2010).

Summary

Mathematics teachers and educational researchers agree that students’ opportunities to engage in conceptual understanding via mathematical inquiry and discourse should increase (Marshall et al., 2009; Nogueira de Lima & Tall, 2008). When Noguiera de Lima and Tall (2008) interviewed participating students, students showed a lack of conceptual understanding, leading the researchers to conclude that conceptual understanding was missing and should be emphasized over a focus on procedural knowledge.

Sahin et al. (2015) and Bonotto (2013) believed mathematics students could benefit from opportunities to engage in mathematical discourse in the classroom. Sahin et al. (2015) studied the types of discourse taking place in mathematics classrooms and found it to be primarily procedural with limited student engagement in mathematical discourse. Bonotto (2013) witnessed students' ability to solve and generate complex mathematics problems when given opportunities to engage in mathematical discourse about contextual artifacts.

Studies have been conducted that focused primarily on the role of the teacher in encouraging students to engage in mathematical discourse (Kazemi & Stipek, 2008; Conner et al., 2014; & Heaton, 2000). Conner et al. (2014) identified five questions and five actions that were most likely to result in mathematical discussion and argumentation. Kazemi and Stipek (2008) studied the role of the teacher to motivate students and the impact of teachers that expected all students to work collaboratively, as well as justify and discuss their mathematical thinking.

Multiple researchers have looked for specific ways to achieve authentic mathematical discourse in the classroom (Page & Clarke, 2014; Oikkonen, 2009; & Mercer & Sams, 2006). Journaling (Page & Clarke, 2014), collaborative writing assignments to address mathematical misconceptions (Oikkonen, 2009), teacher modelling (Mercer & Sams, 2006), and direct instruction on effectively communicating with peers about mathematics (Mercer & Sams, 2006) were strategies that have been studied for their impact on students' mathematical discourse and conceptual understanding.

Lastly, research on the challenges teachers face and various factors to consider when implementing teaching strategies that begin to develop mathematical discourse in students showed that teacher quality had a greater impact than school setting on successful implementation of new teaching strategies (Dennis & O'Haire, 2010). Heaton (2000) conducted extensive research on her experiences implementing more mathematical thinking and discourse into an elementary classroom. Her findings identified of the importance of teacher flexibility and willingness to surrender some control so that students can take a more active role in the mathematical inquiry, argumentation, and discussion in the classroom.

CHAPTER III: METHODOLOGY

Limited research has been conducted specifically on the implementation of the CCSS Standards for Mathematical Practice, and in particular the teacher's role in engaging students in MP3 in a secondary classroom. In my study, I observed and recorded specific instances in which students engaged in MP3 as a result of my lesson or instruction. I also noted specific challenges that I experienced in attempting to teach my students to solve and make sense of mathematics problems using authentic mathematical discourse. I made decisions about the methodology to use in my study based on what would allow me to gain insight into the successes and challenges teachers face when implementing mathematical discourse in a high school classroom.

Research Design

My study used a classroom action research design. This type of study is described by Hendricks (2013) as, "… A form of action research that is conducted by teachers in their classrooms with the purpose of improving practice. It values the interpretations that teachers make based on data collected with their students" (p. 12). The goal of action research is to provide deepened understanding which can inspire change and improvement in my individual practice.

I believed that a statistical analysis of student/teacher responses or test scores would not be sufficient to describe the complexities of a human being trying to learn new strategies, teach those new strategies, and analyze her successes simultaneously. To reach the level of depth that I desired in my study, I required flexibility and the ability to collect whatever valuable data arose. I had no way of knowing ahead of time what responses or experiences to expect, so it would have been difficult to quantify the results of my study. I had multiple research questions but intentionally refrained from posing a research hypothesis. I entered my study with the goal of discovery and exploration rather than trying to control specific variables to test an expected result.

Out of the 23 times that I met with my first period Geometry class during November and December, I taught lessons that emphasized students' engaging MP3 and a focus on authentic mathematical discourse on nine occasions. The other fourteen times the class met, students either took an assessment or received instruction without an intentional emphasis on MP3. The nine studied lessons were not consecutive. I took factors such as the content and my personal time available for planning to determine which class meetings to teach lessons with an emphasis on MP3. After each of the nine lessons, I reflected in an electronic journal. My reflections included detailed summaries of the lesson and my observations on students' engagement or lack of engagement in MP3. I also reflected on what aspects of the lesson resulted in students engaging in MP3, as well as what obstacles hindered or stopped students from engaging in MP3. I used an observational protocol that was designed using the MP3 Definition to help me recognize when students were engaging in MP3.

During two of the nine lessons, the building-level instructional coach observed my class, and took detailed notes. I provided her with the observational protocol to ensure consistency in how we identified student engagement in MP3. She used the protocol to identify specific examples of student engagement in MP3 and to list specific challenges that kept students from engaging in MP3 (App. C). The observational notes from the instructional coach, as well as student artifacts, provided data triangulation. Many of the nine lessons included writing components, so I collected a few samples of students' written responses as artifacts showing student engagement inMP3.

Site of the Study

The study took place in a suburban U.S. high school in the Midwest. The high school had 1,324 students enrolled in the 2015-2016 school year. The district has one high school (9^{th}-12^{th} grades), one middle school (7^{th}-8^{th} grades), one intermediate school (5^{th}-6^{th} grades), and five elementary schools (K-4^{th} grades) with a total of approximately 4,572 students. Ninety percent of high school students are White. In 2015, 3.7% of high school students were Black, 3.3% were Hispanic, 0.4% were Asian, and 0.8% were Indian. Also of note is that 35.1% of students received free or reduced lunch.

The school district is one-to-one in technology, with all students from fifth grade through high school having access to a personal school laptop they can use at school and take home with them to use for the entire school year. Students must pay $50 to use the laptop for the entire school year. Students may choose to bring a personal device to school instead of using a school-provided laptop. The high school also has laptops available for daily checkout. All of my participating students had access to a laptop, either their own person computer or one provided by the school.

Participants

Since I conducted classroom action research, I, as the mathematics teacher, was a participant. I used convenience sampling (Gay, Mill, & Airasian, 2012) for my study.

Upon receiving consent, 16 of the 17 students in the class were participants as well. The class consisted of three 9^{th} graders, eleven 10^{th} graders, and three 11^{th} graders. The students ranged from 14 to 17 years of age. The class was composed of eight males and nine females. One student received special education accommodations through an IEP. Another student received English as a Second Language (ESL) services and accommodations. Sixteen students were self-identified as White and one student as Hispanic. At the conclusion of my data collection, seven students were earning course grades of A's, one had a B, five had C's, three had D's, and one student had an F. Four of the students scored "Advanced" on their most recent Algebra 1 EOC (End of Course Exam), twelve students scored "Proficient," and one student did not have a score available for the Algebra 1 EOC. Pseudonyms have been used to keep all participants' identities confidential. I distributed informed consent forms (App. D-E) to my first period Geometry students and their guardians.

I teach Geometry to three classes of students every day, but I chose to use my first period students only for this study. Since my study was qualitative, I anticipated collecting large quantities of data. I thought I could gain depth in my analysis by focusing on just one group of students.

Ethical Considerations

Prior to conducting my study, I received approval from the Missouri State University Institutional Review Board (IRB) (App. F) and school principal. Since I worked with students under the age of 18, informed consent from both the students (App. D) and their parents or guardians (App. E) was required. During class time, research-based best practice was used so students who had permission and students who did not have permission to participate received the same instruction and experience in the classroom. The only data used for the purpose of this study, however, was from those students who gave informed consent.

I taught three classes of Geometry students every day, but only studied my first period class. To ensure that all students received the best instruction and educational experience possible, if I believed the lesson plans that emphasized MP3 were best for students in the other sections, they received similar instruction to my first period class. My first priority in all of my Geometry classes was to provide the highest quality mathematics instruction to all students.

Data Collection Procedures

From November 16, 2016 through December 21, 2016, I met with my first period Geometry class 23 times. Each class period began at 7:25 with a 48 minute duration. On nine of those class sessions, I implemented lessons I had created with an emphasis on MP3. The following is a brief description of each of the nine lessons. A detailed summary of Lesson 9, as well as corresponding lesson materials are in the appendices (App. G).

1. Lesson 1: Students made discoveries about perpendicular bisectors in triangles by working in small groups to complete a guided GeoGebra activity. I engaged in mathematical dialogues with small groups to discern their thinking and understanding.

2. Lesson 2: Students used writing to individually make and justify conjectures regarding ratios involving the rate of water dripping from a leaky faucet. Then students discussed their responses in both small and large groups.

3. Lesson 3: Students used writing to individually make and justify conjectures regarding ratios and proportions involving the amount of pizza per person that should be ordered for a party. Students discussed their responses with partners before ideas were shared with the whole class via a large group discussion.

4. Lesson 4: I guided a large group discussion about the meaning of the word *scale factor* before students collaborated to use their understanding of that concept to strategize how to find missing sides in similar triangles. Three different student-generated strategies were shared and discussed with the entire class. Then students were given an opportunity to practice applying what they had learned by solving various types of proportional reasoning problems.

5. Lesson 5: Students used writing to individually make and justify conjectures regarding the sides and angles in similar figures. Students then had an opportunity to complete similar figures problems individually or in small groups. I posed and answered questions to provoke students to think conceptually about the strategies they were applying to the problems.

6. Lesson 6: Students used GeoGebra applets either individually or in small groups to make discoveries about guaranteeing similarity in triangles. Each time I observed all students had completed one of the applets, I had students stop and engage in a large group discussion. Students shared their discoveries and rephrased their peers' comments. Students summarized what they had learned in writing at the end of class, and I collected their responses as student artifacts.

7. Lesson 7: Students spent the entire class period solving problems involving similar figures and determining similarity. Answers and teacher explanations were provided so that immediate feedback was available to students. I rotated between students and small groups and posed questions that required students to justify their thinking and explain why their answers made sense.

8. Lesson 8: Students corrected mistakes from a prior quiz giving them an opportunity to critique their own prior reasoning and misconceptions. Students spent the remainder of the class working in small groups completing an error analysis of common misconceptions. Students were required to identify the mistakes of an imaginary student and write an analysis of the mistake and how it could be corrected.

9. Lesson 9: To review previous concepts to prepare for semester final exams, students discussed various methods of using triangle congruence postulates and theorems before sharing their conclusions and justifications with the class. A contextual distance problem was also discussed. Throughout the lesson, students were encouraged to both ask and respond to questions from both their peers and the teacher.

During the remaining 14 class periods, either students received a more traditional mathematics lesson without specific emphasis given to engaging in MP3, or an assessment was given.

Data on the nine studied lessons were collected in multiple ways. I kept a journal where I reflected on my experiences and observations after each of the lessons. I wrote these journals during my planning period when I did not have a class of students. Approximately two hours passed between the end of the lesson and my planning period when I was able to write my journal entries. I specifically focused on experiences and observations relating to my attempts to develop MP3 in my students. After one of my studied lessons, I collected students' written responses to use as data as well.

I also collected data by being observed twice, during Lesson 3 and Lesson 9, by our high school instructional coach. I chose her to observe me because, while she has years of experience teaching high school English and coaching teachers in all subjects, the majority of her teaching experience is not in mathematics. I chose an observer who was not a mathematics teacher in an attempt to avoid bias. I provided my instructional coach with a list of dates when I would be teaching lessons for my study, and she chose to observe me on two of the dates from that list when she was available. Although we scheduled her observations, the lessons she watched were random. I informed my instructional coach ahead of time about the purpose of my study. I asked her to look for specific instances when students showed evidence of engaging in MP3. I provided the instructional coach with an observational protocol (App. B) which provided her with a list of student behaviors that would indicate engagement in MP3.

Instrumentation. One tool for measurement used in my study was the observational protocol I provided for my instructional coach to identify when my students were engaging in MP3 (App. B). Tarr and Austin (2015) created the observational protocol. I revised it by incorporating the list of eleven student behaviors from the detailed description of MP3 on the CCSS website (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). This protocol provided consistency in what my observer and I were looking for during the lessons. It gave her a structure to record specific times throughout the lesson when I was able to successfully get students to engage in MP3. It also provided a structure for her to list any factors she observed during the lesson that prevented students from engaging in MP3 (App. C).

While I occasionally referenced the observational protocol's list of MP3 related student behaviors as I wrote my personal journal entries, I followed my own structure as I reflected on each lesson. I wrote each journal entry using the following structure. First, I recorded a detailed summary of the lesson. Next, I wrote a descriptive list of successes and a descriptive list of challenges that I encountered. I defined a success as any instance in which a student engaged in MP3 in some way. I used the CCSS' description of MP3 to consistently recognize if students were engaging in MP3. I identified a challenge as any obstacle that resulted in decreased student engagement in MP3. Lastly, I wrote a list of implications for future mathematics lessons based on my reflection of the challenges and successes I encountered (App. H).

Role of the researcher. My role as the researcher was first to teach, and then to record, analyze, and tell the story of the data next. As the regular classroom teacher conducting action research in my classroom, I had a unique relationship with each student in my class that a non-teacher researcher would not have had. My goal was to share the topic of the study with the students and allow them not to solely be participants, but co-researchers as well. I reminded the students often throughout the study that I was researching the implementation of lessons that would give us opportunities to engage in MP3.

It was important for me to remain aware of my biases as I analyzed the data so that I maintained accuracy at all times. It was also crucial for me to be honest and vulnerable, even in the face of my shortcomings, so that my findings can be of use to myself and to others. Being observed by my instructional coach was one way that I safeguarded against bias. I informed the instructional coach of her role as an observer and provided her with the observational protocol. I wanted to be observed more than once throughout the study by the coach, and because of conflicting schedules, I determined that two observations would be sufficient. Although we scheduled her observations, the lessons she observed were random to avoid bias.

Data Analysis

To analyze my data, I reread all of my personal journal entries and the observational notes of my instructional coach. I also looked at available student artifacts during and after data collection. I tracked all instances when I attempted to get students to engage in MP3 as well as the students' responses to my attempts. I tracked evidence of successes and challenges separately. When I noted a piece of data that showed such evidence of a success or challenge, I looked for additional data from the same moment in the class to provide data triangulation.

I expected to have evidence of both successes and challenges throughout my study; therefore, I considered specific lessons or experiences in the classroom that highlighted the challenges as well as some contrasting lessons or experiences that showed evidence of success. I then analyzed the challenges and successes and determined how to respond or adjust in the challenging situations and also how to recreate the positive experiences. To ensure that what I had learned was not simply my opinion, but based on analysis, I considered multiple data types and gave the most specific, detailed, and descriptive data analysis possible.

Heaton (2000) provided a design for her data analysis that proved useful in my own study. She selected three specific lessons that showed evidence of her challenges, growth, and successes as she attempted to implement the new mathematical reform standards of her time. I selected specific themes that became apparent throughout my data and analyzed them in a similar manner. I analyzed specific occurrences from multiple lessons to determine how each theme provided opportunities for the teacher to cultivate success and how each theme highlighted obstacles that created challenges for the teacher. I then revisited each theme to consider the overall implications for both teachers and students.

To aid me in identifying themes, I used a spreadsheet to track lesson characteristics and teacher strategies that either led to a success or posed a challenge (App. I). As I reread my personal journal entries, I categorized successes or ways in which I observed students engaging in MP3 in one spreadsheet. I categorized challenges that I identified in another. After completing my categorized lists of observed successes and challenges, I read my journal entries again, this time putting the date next to any item on the list with evidence that it was present during that lesson. After inputting the occurrences by date, I highlighted the dates with corroborated data identified by the observation notes of my instructional coach or by student artifacts. This spreadsheet provided me with a visual representation of my data, which made themes more apparent.

Summary

My classroom action research study took place during November and December of 2016. My participants were 16 out of the 17 students in my first period Geometry class. All participating students ranged from 9th to 11th grade at a suburban high school in the Midwest. Nine lessons were taught with an emphasis on engaging in MP3 and authentic mathematical discourse. After each lesson, I recorded detailed observations in the form of personal journal entries. The high school instructional coach observed two of the lessons at random and provided observation notes as well. Student artifacts showing written mathematical discourse were collected. The journal entries and observation notes were reread and analyzed for themes that provided insight into both recreating the successes and addressing the challenges. An in depth analysis of the evidence from various lessons that supported the main themes provided both broad and specific implications for mathematics teachers.

CHAPTER IV: FINDINGS

The purpose of this study is to gain insight into the challenges and successes a teacher experienced when implementing one of the Common Core's eight Standards for Mathematical Practice, MP3. The goal was to discover what teaching strategies were successful and what challenges exist in fostering MP3 in high school mathematics students. Between November, 2016 and December, 2016, nine out of the 23 lessons taught to my first period Geometry class were intentionally focused on providing opportunities for student engagement in MP3. Using my personal experience as both a participant and a researcher, detailed reflective journals, student artifacts, and observations and notes from an instructional coach, I categorized all successes and challenges throughout the study into major themes. I recorded the dates that each specific theme was represented in my data with evidence from a lesson. Three main themes emerged which provided insight into increasing student engagement in MP3.

1. Transfer of Responsibility
2. Hopeful Investment
3. Establishing Classroom Culture

Transfer of Responsibility

The teaching strategies that most often led to students engaging in MP3 all included lesson components that transferred the responsibility of understanding a concept and making sense of the mathematics from the teacher to the student. Rather than always relying on direct instruction, I focused on allowing and guiding students to be the mathematical authority in the classroom. Whenever possible, I tried to use small group and large group discussion. This allowed students to generate and share multiple strategies to solve problems and multiple ways to explain and justify their thinking. I discovered the importance of individual accountability. I also discovered that the use of writing, error analysis, and adequate time to make sense of mathematics problems were ideal ways of holding students accountable for showing evidence of their levels of understanding. Giving the students more control and responsibility, while successful, did lead to challenges.

Successes as students became the mathematical authority. One particular instance of students taking on more responsibility for sense-making can be evidenced in Lesson 3 on the topic of proportional reasoning. Students taking on the role of the mathematical authority in the classroom was a large goal of this lesson. Students were given the following writing prompt.

> *Last year there were 26 students on the academic team and for their End-of the-Year party, the coach ordered 13 pizzas for them to share. This year, there are only 22 students on the academic team. The new coach has decided that since there are 4 less people on the team this year, she should order 4 less pizzas. She is planning on ordering 9 pizzas for this year's End-of-the-Year party. Critique the reasoning of the Academic Team's new coach. Do you agree with her logic? Why or why not? Write an explanation of your thinking. You may also include diagrams, equations, etc…, but make sure to include a written explanation.*

Rather than giving notes over proportional reasoning, I chose to place the responsibility for making sense of an authentic mathematics problem within a real-world context on the students. All students wrote their individual conjectures in their notebooks. After five minutes had passed, students shared their conjectures with their partners. I rotated throughout the room and engaged in discussions with pairs of students. Izzy shared her strategy with her partner as I listened. She had figured out that since 13 pizzas were purchased for 26 students, “that meant two students per pizza.” She went on to explain that since there were four less students this year, they should buy two less pizzas. She also volunteered a justification that for the previous year’s academic team, every two students had gotten a pizza. Izzy had clearly made sense of this problem and found student-friendly language that was meaningful to her to justify her thinking. Since she was sharing her strategy with her neighbor, she was seen as having the mathematical authority rather than the teacher.

While Izzy was sharing her thinking, the pair of students next to her was listening as well. I wanted to encourage sharing of different strategies, and I was hopeful that different students would make sense of the problem in different ways. I asked the two students sitting next to Izzy and her partner if they had done something similar or if they had a different way of thinking about the problem. Nicole shared her strategy, “... she had noticed that 13 pizzas for 26 students meant they ordered half as many pizzas as there were students. This told her they should order half as many pizzas as there are students this year, too. Since there are 22 students, they should order 11 pizzas.” Nicole’s partner, Rhiannon, changed her answer and was able to self-critique her initial thinking when she heard Nicole’s answer and reasoning. As Rhiannon critiqued Nicole and Izzy’s reasoning and compared it with her own, she was able to come to the conclusion that the coach was not using correct reasoning. She never asked me if she was correct to change her answer, and she never asked me for an explanation of what the correct answer was and why. Similarly, Izzy and Nicole shared their arguments with me without expecting validation of whether or not they were correct. The students viewed themselves as the mathematical authority.

Not only were the students the mathematical authority, but this lesson also showed students searching for multiple ways to represent the same problem or solution (MP3 Definition, parts b, f, g, h, i, and j). Nicole considered the relationship between the total number of students and the total number of pizzas and used that to create her argument about how many pizzas the coach should buy for the new team. Izzy, on the other hand, considered the smaller ratio of students to pizzas and used that to figure out how many less pizzas the coach should buy for the new team. The same question had multiple entry points and strategies for reaching the same justifiable conclusion.

After already witnessing successful engagement in MP3 with two pairs of students, I wanted to give all students a chance to hear the thinking of their peers through a large group discussion. Nicole volunteered to share what she had told her partner and me earlier during small group discussion. As soon as she finished, I randomly chose Aaron and said, "What did she just say?" He was successfully able to rephrase Nicole's answer and explanation. I noted in my observations that this question made Aaron appear very uncomfortable. It felt very uncomfortable for me as well, I think due to the control this question takes from the teacher. Nicole's and Aaron's responses would both be critiqued by everyone listening. If Aaron was unable to give a logical or clear response rephrasing Nicole, then I would feel pressure to maintain both students' sense of value. Without knowing what Aaron would say ahead of time, it was impossible to prepare to respond. Despite the awkwardness, Aaron's response was evidence that he understood Nicole's thinking because he could rephrase it in his own words. I noted in multiple other lesson journals that I often tend to rephrase what a student says in *my* own words. Rather than sending the message that I can say something in a better way than the students can, in this lesson I held the students accountable for engaging in MP3 and experienced success.

I had already witnessed a success in that Aaron was able to rephrase the reasoning of one of his peers. This was an example of students communicating their conjectures and justifications to others (MP3 Definition, part g). A second success occurred when he shared his strategy with the class. He was making his own conjecture (MP3 Definition, part b). I did not know what to expect because I had not had an opportunity to check in with his small group before moving to the whole class discussion. He explained that "… he used the ratios like we had done yesterday instead. He explained that he put 26/13 as one ratio and 22/9 as the other. The first ratio simplified to 2 students per pizza while the second ratio came out to approximately 2 students per 0.81 pizzas." The resulting ratios were different which informed Aaron that the coach was not using correct logic. This was evidence of a third different viable argument to address the logic of the academic team coach. Students were comparing arguments and distinguishing between correct and flawed logic (MP3 Definition, part j).

When given the time and opportunity, students were able and willing to generate multiple strategies to solve the same problem. By valuing their various reasoning strategies, students were treated as the mathematical authority. Throughout the remainder of the lesson, I never once had to establish what the correct answer was. I wrote as a success in my journal for this particular day, "Students were the mathematical authority for the first fifteen minutes of class. They weren't looking to me for the right answer. By the time they had written their answers and discussed, I didn't even have to ask or clarify if the coach in the question was using good logic or not because they all agreed unquestioningly that she wasn't based on their own arguments and the arguments of others." I identified the discussed components of this lesson as success because of the student behaviors from my definition of MP3 that were evident. Students made and justified conjectures, they explored the truth of their own conjectures and those of their peers, they took context into account, compared arguments, and responded to the arguments of others (MP3 Definition, parts b, c, f, g, h, i, and j).

During Lesson 9, reviewing congruent triangle theorems and postulates, I experienced another example of sharing mathematical authority, which also showed the shift to students being more responsible for the sense-making and critiquing. Groups were assigned a Congruence Theorem or Postulate and given a diagram containing potentially congruent triangles (App. G).

They were instructed to discuss different pieces of information that could be given to allow them to use their assigned theorem or postulate. After some small group discussion, each group took a turn coming to the white board and sharing with the whole class. After the first group, I asked a few questions and allowed the presenting group and the rest of the class to work together to figure out how to address my questions. For the groups that followed, I said, "I have a question for them, but I want to hear what questions you guys have for them first." My hope was that this would set the expectation that there were questions to be asked and that I knew the students were capable of coming up with them without just waiting to hear my thoughts. It worked, because the students successfully came up with their own questions for the last three groups (MP3 Definition, part k). I used my authority as the teacher to make it clear there were questions to be asked and critiquing to be done. While the authority for the class structure remained with me, the mathematical authority was placed on the students to generate meaningful questions and scenarios to consider (MP3 Definition, part d and k).

A specific example of a student-generated question came after one particular group had just presented what information could be given to use the ASA Congruence Theorem. Two girls had shown a picture accurately portraying ASA, but when they described what would need to be given to produce one of the pairs of congruent angles, the students kept saying that the shared side, HP, was bisecting both of the angles causing them to be congruent. The students' use of the word *bisect* did not make sense. I waited to see if any of the students would generate a question to address this issue. I had to wait for what felt like a long time, but was probably only a few seconds, before anyone said anything. This indicates that a long enough wait time may be crucial to transferring authority to students. If I had not been willing to accept a few extra minutes of silence while waiting on the students, they would not have had the opportunity to remain the mathematical authority, as I would have asked the question instead.

Finally a few students asked what was being bisected. The two presenting students, Rhiannon and Becky, were unable to answer the question. Multiple students from other groups said they did not think it was possible to get those two angles to be congruent as the result of bisecting something. I decided to share the authority and asked a few leading questions to help the class find a way to resolve the presenting girls' correct diagram with their incorrect description. This scenario shown success as students asked questions to clarify and improve arguments, made, justified, and communicated their arguments to others, and distinguished between correct and flawed logic (MP3 Definition, parts b, f, g, j, and k).

At this point in the discussion, another success occurred when one student independently took charge of the discussion and asked if we could bisect the top angle (MP3 Definition, part k). The two girls that were presenting, marked the picture appropriately to show the result of bisecting the top angle. Students from other groups took on the mathematical authority and said that the resulting congruence theorem would be AAS rather than ASA. I could see that Becky and Rhiannon were struggling to see how some of their classmates were deciding on AAS. There were multiple markings on the picture at this point. I used my authority over the class structure to have a volunteer join them at the board and point out where AAS was coming from. Hannah volunteered, and this allowed students to remain the mathematical authority in the classroom. This interaction shows evidence of students communicating their conjectures and justifications as well as responding to the arguments of others (MP3 Definition, parts b, f, h, and j).

The building-level instructional coach was observing this particular lesson. She also identified successes in this lesson in her following reference to my MP3 definition (part g, h, j, and k). She noted the shift to students as the mathematical authority when she said in her observation notes, “Students asked many questions either challenging logic respectfully or offering alternative explanations.” This is further evidence of the shift from looking immediately to the teacher for answers and confirmation to looking instead to peers as capable of generating and justifying reasonable arguments. It may not have happened without the structures I put in place and the way I guided the discussion. I discovered that I had the power to manipulate the shift in responsibility and the result was greater responsibility on the students and engagement in MP3.

While giving up control to the students was sometimes a challenge for me, during Lesson 6, I experienced success as I addressed this struggle of giving up mathematical authority by using a writing strategy. During Lesson 6, I taught a lesson using GeoGebra in which students manipulated sides and angles in the computer software to make discoveries about similar triangles. All students were working at different paces making large group discussion difficult. I interrupted the students periodically to ask them questions about their discoveries, but it felt disjointed, especially since I had had similar conversations already with some small groups but not with others.

A success came at the end of the lesson when students were asked to write a summary of what they had learned. When reading their summaries, I recognized some of their written statements as things that other students had said during our short large group discussions. Giving an individual writing assignment forced all students to make independent statements about what they had learned. Having both small and large group discussions throughout the class period allowed students to learn from their peers and find ways of describing what they were witnessing in GeoGebra in a student-friendly way that made more sense to them. This was James' summary: "If triangles have 2 proportionate sides the triangles can be anything. If they have 3 proportionate sides, they have to be similar. If they have 2 proportionate sides and an included angle, they will be similar." James' summary was memorable because Connie had said during the lesson that the angle that was between the two sides was called an included angle. James' use of that word shows that he was listening to Connie and was able to improve on his understanding of similar triangles because of what she said.

I do not know if he would have remembered the word *included* if I had been the one to remind students of that particular vocabulary word rather than hearing it from a fellow student. Regardless, the message was sent that other students had enough mathematical authority to remember and use the word *included*, which places responsibility on all students to make use of previously learned vocabulary words and to learn from the reasoning of their peers. Students used written mathematical discourse to write independent mathematical arguments and used prior knowledge in their conjectures. Students' participation in these activities was evidence of engagement in MP3 (MP3 Definition, parts a and b) by all students since everyone was required to write a response.

Challenges as students became the mathematical authority. While I learned to use writing and increased wait time to improve at transferring mathematical authority to students, giving up control was a constant struggle for me as a teacher. After every lesson, I considered specific ways I could have improved since every lesson had its challenges. I recognized a personal struggle to allow students to rephrase each other instead of doing so myself. Even more often, I recognized the challenge of getting *all* students to take on this role of responsibility for their own learning. While Lesson 9 shows students, rather than the teacher, taking the role of mathematical authority, it is important to note that, upon further reflection, not all students were accepting my transfer of responsibility to them. A number of students let their peers accept all responsibility. I noted in general that, "Felicia never said a single thing." Transferring the responsibility for mathematical thinking and discourse from one person, the teacher, to multiple students allowed some students to remain bystanders and refrain from showing any evidence of engaging in MP3. The definition as described in the CCSS documentation requires some type of mathematical discourse as evidence, and I had no evidence from Felicia and a few of the other students.

I encountered another challenge during Lesson 2. I was teaching a lesson about ratios using questions about the speed of water dripping from a leaky faucet. After discussing the questions about the faucet that had a context, I gave the students the following proportion:

$$\frac{5}{10} = \frac{x}{16}$$

Many students immediately said that x would equal 8. When I asked them "why," one student gave a reasonable argument that since 5 is half of 10 and 8 is half of 16, x must equal 8. After this, Hannah asked, "Could we just cross multiply?" I told her that we could cross multiply, and I proceeded to show the students how to complete solving the proportion using cross multiplication. I accepted a question from a student, and then gave all mathematical authority to myself as I worked the problem for them from beginning to end. I wrote about my frustrations later in my journal. "Multiple times throughout the lesson, students read or explained their own arguments as to why they thought the answer was what it was. I constantly rephrased what they said in my own words. … It's like I don't trust the students to be able to listen to their peers and understand each other. How much more powerful would our discussions be if students were rephrasing each other and making sense of each others' thoughts? I think that is the whole point of MP3." Upon reflection, I recognized lost opportunities to allow students to be the mathematical authority. My inability to relinquish some control and provide longer response time made it impossible for students to engage in any of the behaviors listed in my definition of MP3. I think time influenced some of my decisions, and I believe time is an ever present dilemma that I will discuss when I address the theme of "Hopeful Investment."

Later in Lesson 2, I encountered another challenge related to transferring responsibility to students. I presented students with a question about the ratio of the sides in a rectangle and the rectangle's perimeter. I asked the students to try to come up with a reasonable solution within their small groups, but they came up with nothing productive. I was not prepared for what to do when the students were unable to generate any ideas so I took back the mathematical authority and showed them how to complete the problem.

One specific implication that I took from that experience was to share the authority and give quite a bit of help on problems like the perimeter problem that were too different or difficult for students to come up with strategies for solving them on their own. I noted in my journal that, "I changed my approach in later classes and solved the perimeter problem with the help of the whole group. Then I had them try to apply what they had just seen to help them solve Example 2, which was similar but used triangle angles instead of the sides of a rectangle. This made the time they spent trying to figure out a strategy for Example 2 more worthwhile because they had ideas about what to try based on how I had done the first one with them."

After expressing frustrations from Lesson 2 over not giving students enough mathematical authority, I continued to struggle with this same challenge when I was teaching Lesson 3 the next day. This was the lesson concerning how many pizzas to buy for the new academic team that I referenced previously. I still struggled with the balance of mathematical authority. I forced myself to ask Paul at one point, “What did he just say?” after hearing an explanation of Kyle’s thinking. Paul was able to show his understanding that every two people got a pizza, but he was unable to restate, in Kyle’s words or his own, how Kyle used that relationship in his explanation.

Paul had not been paying close attention when Kyle had explained his thinking, so I did not know if Paul’s inability to answer was because he had not been paying attention or because he did not understand the problem. I became uncomfortable because I was unsure of how to proceed, and made the split-second decision not to make Kyle repeat himself. I had requested his explanation originally. He had not volunteered it. For this reason, it felt like a punishment to require Kyle to repeat himself when Paul may have simply been off task and that was the reason he was unable to rephrase Kyle’s idea.

When this attempt to get students to rephrase each other failed, I still tried to maintain students as the mathematical authorities by having Izzy share her thinking with the class. I knew she had used good logic that was very similar to Kyle’s since I had been part of her small group’s initial discussion. Izzy had not yet shared anything with the large group, so it was an opportunity for another student’s thinking to be heard by the whole class. This would also allow Paul one more opportunity to hear the strategy that he had missed from Kyle. Unfortunately, after Izzy’s explanation, I second-guessed how to handle the situation and never held Paul responsible for eventually being able to explain the logic behind Kyle’s and Izzy’s strategy. One indication that a student is engaging in MP3 is communicating and responding to the arguments and justifications of others (MP3 Definition, parts g and h), and I did not press Paul to do either. I fear I sent the message that, while some students in our class are capable of being the mathematical authority in the classroom, others are not expected to reach the same level.

Multiple strategies encouraged student authority. I was constantly impressed with the students' use of prior knowledge and vocabulary. In seven of the nine lesson I studied, students found multiple ways to say or write an explanation using mathematical vocabulary from past lessons. Students also found multiple ways to represent the same scenario or multiple ways to represent possible solutions to mathematics problems. In many of my experiences prior to this study, students often looked to the teacher to show them the "best" or "fastest" or "right" way that a problem "should be" solved. In an attempt to get my students constructing viable arguments and critiquing the reasoning of others (MP3), I tried to intentionally highlight any alternative solutions, representations, or ideas that arose from my students. I hoped this would lead to students comparing arguments, recognizing or using counterexamples, and exploring the truth of conjectures (MP3 Definition, parts c, e, and j).

I mentioned an instance previously when students' use of multiple strategies took place during Lesson 3 on proportional reasoning. Three different students found three different ways to determine that the academic coach was using flawed reasoning when she purchased four fewer pizzas for four fewer students. One student even made a visual representation of the ratio of students to pizzas that included circles and meaningful shading. After observing Lesson 3, my instructional coach made the following comments that support students' use of multiple strategies, explanations, and vocabulary words (MP3 Definition, part a).

- "It allowed for natural recall of vocabulary."

- “It was a non-threatening way to share ideas and compare. I heard often, ‘I said the same thing, but in a different way.’”
- “Sharing out allowed students to see how many ways one could approach this problem successfully.”

Multiple strategies were generated by the students took place during Lesson 4, as well. The students had already learned about scale factors. The lesson’s purpose was for students to make their own viable arguments about how to find missing side lengths in various similar figures using a scale factor. I started by drawing two similar triangles and a similarity statement on the board and having students copy it into their notebooks.

After discussing which angles were congruent and why based on the similarity statement, I asked students to discuss with their small groups what they remembered about the definition of a scale factor. Two main responses were shared with the whole class. Connie remembered our use of scale factor from two days prior to check polygons to determine if they were similar. This influenced her memory that scale factor was “when we made the ratios of all the sides and they all equaled the same thing.” Gordon volunteered that he thought scale factor was “what you had to multiply by to get the sides in the new triangle.” My initial internal response to Gordon’s description of scale factor was that it could be potentially misleading. I considered the usefulness of his definition if the scale factor was 3:1, but was unsure of how that definition could be applied to find missing sides when two similar figures had a scale factor such as 3:5. I hope my uncertainty did not show on my face or in my response. I once again struggled with not wanting to give up control, and I responded to his definition with that very example of two shapes that had a scale factor of 3:1. I rambled on for a few seconds about how that would mean we could multiply all the sides of the small figure by 3 and that would indeed give us the sides in the new triangle. I put aside my concern for any confusion that might arise from trying to use Gordon's definition because I knew that the class had heard Connie's definition, as well, which was what I had initially imagined all students would learn and use.

We moved on as a large group to determine which sides could be put together to generate a scale factor. I was still struggling to give up mathematical authority, but I did ask questions that the students were able to answer. For example, I asked the class which sides must be corresponding based on the similarity statement and how to simplify the scale factor. These questions were not conceptual and did not require problem-solving, but they did keep many of the students engaged. Finally, I gave small groups the opportunity to try and come up with a strategy for finding the length of the two missing sides, x and y. I was surprised when I rotated from group to group and observed almost every group using a different strategy. I asked three specific students to put their work on the board. Rhiannon and Delaney used the following two proportions.

Rhiannon: $\frac{9}{11} = \frac{12}{x}$

Delaney: $\frac{18}{22} = \frac{12}{x}$

Both showed their work and final solutions. I asked the class to identify similarities between the two methods shown. Responses were, "They got the same answer," "They both cross multiplied," and, "They both solved for x." When I asked what was different, they stated that Rhiannon had used the scale factor and Delaney had used the actual side lengths. This shows students comparing arguments (MP3 Definition, part j). I asked the students if 9/11 and 18/22 were different fractions. The majority of students nodded their heads up and down, making it clear they recognized the fractions as equivalent fractions, so I moved on to Eric's third strategy.

Eric had simply taken 12 divided by the scale factor. We could see that this strategy yielded the same answer as Rhiannon's and Delaney's methods. I asked Eric how he knew to divide, and he very honestly said he did not know. He just knew that was how he got the answer that made sense. I showed him that we could always multiply if we considered whether or not the new side we were solving for needed to be bigger or smaller. I considered Heaton's (2000) research and decided "telling" would be appropriate in this situation. I did my best to use Gordon's definition of scale factor, in conjunction with common sense about multiplying by numbers that are greater than 1 or less than 1, to show how Eric's method could be tweaked slightly and allow him to use multiplication and also have a strategy for knowing which version of the scale factor to use. Eric showed how he would use multiplication to solve for y using his slightly altered method on the board. In my journals I observed, "As he did future practice problems, I observed him figuring out which version of the scale factor he had to multiply by, and he was successful in getting the correct answers on every problem." Eric was taking context into account as he determined which version of the scale factor he needed to multiply by (MP3 Definition, part i).

Gordon told me that he planned on using Eric's method in the future because he "preferred thinking of scale factor in that way." I recognized after the lesson that if I had not provided an opportunity for students to make their own sense of the problem and share their reasoning with the class, I would have never reconsidered Gordon's meaning of scale factor. The method of choice that made the most sense to Eric and Gordon may never have been discovered by them at all, and it certainly would not have been presented since I had not considered showing that method to the students.

The students were presented with one more problem. We engaged in a large group discussion of how to apply the three different strategies the students had just generated to solve this new problem. Students spent the remainder of the class period completing practice problems using whichever method they preferred. Fourteen of the sixteen students ended up using Delaney's or Rhiannon's methods, but I noted in my journal that I never had to give direct instruction on how to use their methods. Students found at least one method that made sense to them which was either self-generated or peer-generated. This was a success as students were taking on the responsibility of generating strategies for solving similar figure problems as well as taking responsibility for deciding which method made the most sense. Through the process of generating or choosing a method, all students were either forming their own viable arguments or critiquing the arguments presented to them, thus engaging in MP3 (MP3 Definition, parts b, g, and j).

Similar to the students' use of various methods, students were also using words such as *ratio*, *scale factor*, and *proportion* to discuss the two large group problems as well as their practice problems in Lesson 4. This was yet another indication of students finding multiply ways of representing and discussing their thinking.

Successes as students were held individually accountable. During Lesson 8, I was able to use writing and error analysis to hold all students individually accountable for making sense of various mathematics problems and for critiquing the reasoning of others and themselves. First, I returned a quiz from the previous day to the students and instructed them to make corrections. For some students, this was valuable for self-analysis. For other students, making corrections was a valuable activity because they had done very well, and they had an opportunity to help their peers. Teaching something to someone else was an excellent way to improve their individual learning and understanding of the topic. Students were able to work at their own pace and many students got help from their classmates rather than from me, allowing students to, once again, be the mathematical authority in the classroom. This was also a large contrast to the traditional method of the teacher going through commonly missed problems on the board for the entire class. Students were held accountable for correcting their mistakes on their own, asking a classmate for help, or asking the teacher for help. In the common traditional scenario, a student could pretend to listen to the teacher and potentially not learn anything because the teacher was doing all the work and the students' only responsibility in that situation was to appear to be listening. This different strategy successfully held all students accountable for correctly critiquing their own errors (MP3 Definition, part j).

Upon completion of their quiz corrections, students were given a worksheet with six problems that had all been worked out incorrectly. Students had to work individually or in small groups to find the errors, describe the errors in writing, and make viable arguments to explain or show how to correct the mistakes. This activity was a success because it engaged all students in MP3 by requiring justifications, new conjectures, studying a logical projection of statements to find truth, distinguishing between flawed and correct logic, and responding and asking questions for clarification (MP3 Definition, parts b, c, f, h, j, and k). At one point, Rhiannon said, "That is exactly what I would do. I don't think there is anything wrong with it." In the past, her misconception may have gone unnoticed, but requiring her to make sense of a very common mistake made by a fake student provided her with a safe way to critique her own reasoning, the reasoning of the fake student, and the reasoning of her peers as they worked together to understand the error. Students could work in small groups; however, because writing was required, all students were accountable for being able to describe the flaw in each problem. While some students may have gotten answers from their classmates, since a written explanation was required, students were not able to mindlessly copy an answer from their neighboring classmate.

With individual accountability and the writing requirement, I observed an increase in the use of mathematical vocabulary and peers questioning each other for clarification during Lesson 8. I heard the following statements from students collaborating in small groups:

"That's how I would do it."

"Did she just forget to multiply the 4 with the x?"

"Why did she set up a proportion?"

"Is her mistake that she got the ratio flipped?"

"Aren't the angles just congruent?"

"I would have distributed. I can't see what she did, but she definitely didn't distribute."

These phrases show students engaging in MP3 by questioning one another and making hypotheses, using prior knowledge and vocabulary such as *ratio*, *proportion*, and *distribute*, and distinguishing between correct and flawed logic (MP3 Definition, parts a, b, and j). By requiring students to write about the errors, all students were able to engage in MP3 rather than only the few who would have contributed to a large group conversation.

Challenges as students were held individually accountable. Even with all its positive aspects, Lesson 8, the error analysis lesson, had challenges, and I identified methods that I would change for next time. First, although students corrected their quiz mistakes, there was no activity requiring students to show evidence of deeper understanding of why their solutions were incorrect in the first place or why the correct strategy worked. In the future, writing could be used to have students reflect on their quiz mistakes and hopefully show evidence of reaching a greater level of conceptual understanding (MP3 Definition, part f). While I felt there was a level of individual accountability on this lesson, multiple ways of thinking about problems may have been lost since a large group discussion did not seem appropriate due to the various paces of individual students.

There were both successful aspects and challenges to Lesson 4, as well. While multiple strategies were presented during that lesson on scale factors, I recognized that students seeing multiple strategies was not necessarily an indication that all students had found a representation that was meaningful to them. In this and various other lessons, one of the biggest challenges I identified in my notes was including components of the lesson to hold all individuals accountable for showing their level of understanding. Only the students making viable arguments and thinking critically about the arguments of others are actually engaging in MP3. While all students showed they were able to use one of the three student-generated methods to solve for missing sides in similar figures, only a few students were held accountable for showing a deeper level of understanding. Some students may have recognized the patterns that Rhiannon or Delaney used in setting up their proportions and then copied those patterns without making any deeper connections as to why they worked.

In my journal for that lesson, I reflected that, "There was no follow up. While this activity was very powerful for many of the students, some just wrote down what they saw me write on the board and were more concerned with being able to use a method regardless of if they really understood why it worked or not." I was only prepared to make the emphasis on being able to successfully use a method. If I could go back to that lesson, I would place a greater emphasis on expecting all students to show evidence of understanding why the various methods work. The reality was that most students never had to justify their arguments or those of their peers (MP3 Definition, parts f and h). They simply had to be able to follow an algorithm.

In previous lessons, I identified writing as a successful strategy for holding students individually accountable with resulting engagement in MP3. Lesson 4 on scale factors did not include a writing component, but I noted in my journal that "Writing needs to be much more present in the classroom. My evidence of this is that it causes ALL students to make arguments or do critiquing." This was just one lesson where I determined that writing would have been a very practical and powerful way to engage all students in being individually accountable for showing me evidence of their mathematical understanding and thus engaging in MP3 (MP3 Definition, parts b and f).

Summary of transfer of responsibility. The eleven student behaviors that identify engagement in MP3 (MP3 Definition) all begin with the word *students*. In order for students to engage in authentic mathematical discourse, they, not the teacher, must have opportunities to be responsible for making sense of mathematics problems, to communicating their thinking, and to respond to and question others' mathematical ideas. This required a shift that Heaton (2000) described as a shift "away from the teacher as the sole authority for the right answer" (p. 7). I found that intentionally asking questions, using appropriate wait time, and giving assignments that required students to be the mathematical authority encouraged students to accept more of the mathematical responsibility for their learning. This was evident in multiple lessons. One example was when students used written discourse and engaged in discussion to critique the flawed logic of a coach as she tried to solve a contextual problem that required proportional reasoning. Another example of students accepting the role of mathematical authority occurred when I paused long enough for a student to generate a thought-provoking question that allowed the whole class to consider multiple cases, respond to various mathematical arguments, and deepen their understanding of previous theorems and vocabulary.

When I expected students to engage in MP3 and planned questions and activities that would require students to be the mathematical authority, I experienced successes. I recognized, however, that giving up control did not come naturally. Time influenced my decisions, and I often shortened my wait time, rephrased students' thinking in my own words, and accepted when only some of the students accepted the role of mathematical authority. I recognized that I sometimes sent the message that some students, not all, were capable of taking on the role of mathematical authority.

My data contains specific examples of the teacher valuing, modeling, and encouraging the use of multiple strategies for both problems and their potential solutions. One example occurred when I asked three students to share three unique strategies for finding missing sides in similar figures. This led to success as students were communicating their conjectures and justifications, the class was critiquing the strategies of their peers, and students had to compare arguments, respond to their peers, and determine which strategies they could use in the future (MP3 Definition, parts a, b, f, g, and j).

The evidence made it clear that when I found ways such as writing and the use of error analysis to hold students individually accountable, I experienced success, and students engaged in MP3. The related challenge was finding ways to hold students individually accountable during every lesson. Some lessons still allowed students to use algorithms without understanding or explaining why they worked. I needed to intentionally plan a way to hold students individually accountable. Otherwise, some students would not accept the responsibility for mathematical sense-making, but only for getting the correct answer.

Hopeful Investment

The second theme that became evident in my data was the value of "hopeful investment." As I considered my mental state while planning lessons which implemented MP3, I saw an exhaustive amount of decision-making in many cases. Two of my studied lessons required minimal planning. During those lessons, however, the same astronomical amount of decision-making was still required, only during class with less time to make the decisions. The mental and emotional states of my students went into every decision that I made. This required a significant relational investment. Because I believe it is best for all students and because it allowed me to make more informed predictions and decisions about my lesson plans, I have been investing energy and time into learning about my students as human beings and as mathematics students all year. While investing time, energy, thought, and passion can be difficult and draining at times, an investment implies a pay-off. My data shows evidence that making investments will result in future benefits, which is why I have referred to the second theme of my data as "hopeful investment." I am hopeful that continued investments in this area will lead to continued pay-off in students' ability to more independently create viable arguments and critique the reasoning of others.

Challenges as I invested time into lessons and their structure. Every teacher knows an investment of time and planning is necessary in our profession. My time investments during my study were slightly different than they are when a focus on the implementation of MP3 is not emphasized. When I first began collecting data for my study, I was assuming that students did not have much experience making viable arguments and critiquing others. As students became more accustomed to my expectations for justifications, risk-taking, and discussions, the challenges I faced relating to time and lesson planning changed.

The first lesson that I taught with an emphasis on MP3 required students to follow directions independently, use GeoGebra, and draw conclusions where more than one answer was sometimes possible. Some students struggled to follow the directions. One student used the generic "point" feature rather than the "midpoint" feature as the directions called for. The lesson's goal was to observe patterns and create hypotheses about perpendicular bisectors, so without correct midpoints, this student was getting very strange results. As soon as I noticed what had happened, the student had to completely start over, but he had already drawn some incorrect conclusions by this point. This was a clear obstacle to his ability to engage in MP3 since his time had to be spent recreating his figure in GeoGebra rather than spending time making sense of mathematical patterns or engaging in mathematical discourse. I noted in my journals that, "Any arguments or claims he tried to make won't make sense now, but he couldn't know that without knowing what he *should* get and that would defeat the point of students making their own viable arguments."

I also stated that we would have less time for discussing the students' arguments later in the week because the GeoGebra aspect of the lesson took a lot more time than anticipated. I wanted to follow up this lesson with a discussion and a writing prompt to see what the students truly learned and understood from their experimentation with the Geometry software. Unfortunately, time was indeed an obstacle, and because of the number of remaining days before Thanksgiving Break, I felt pressured to move on to a new topic the following day so we never returned to this activity to engage in large group mathematical discourse.

The same pressure to move quickly through content was evident in my second lesson as well. During Lesson 2, I taught the lesson on ratios which started with a successful writing activity and discussion about the rate of water dripping from a leaky faucet. Many aspects of the beginning of the lesson were successful. Students were writing conjectures, regarding the rates of the dripping water, communicating their mathematical ideas and justifications with the class, and comparing arguments (MP3 Definition, parts b, f, g, j). I faced the challenge midway through the lesson when I gave students a mathematics problem that was so difficult that students did not know where to begin. I had to adjust my initial plan and work through that problem with students rather than spending more time trying to give appropriate hints to scaffold the problem so they could realistically solve it on their own.

By giving the students time to attempt the problem and by going through it from beginning to end with the entire class, I spent more time on that particular problem than I had anticipated. When students began the second related example, more time was needed than I had planned because they had not yet surpassed a procedural understanding. The students had not yet struggled through that type of problem on their own, so they still had to go through the sense-making process. Most students were able to figure out the second problem I posed, but I had expected to give the students time for more practice rather than ending the lesson where I did. I commented in my journal, "Ideally, we would have had more time for Example 1 and 2. I think students could have figured it out with more time and hints. My challenge was not feeling like I had enough time. With a strict three weeks remaining until final exams, I feel pressure to get through a certain amount of material. Student confusion makes me think they still don't really understand the concept of a ratio in every context."

If I had been using a typical traditional teaching approach, I would have given notes and given the students a worksheet or a book assignment. I would not have felt the same concern over time that I experienced during these first two lessons as a direct result of my attempt to promote student discovery and mathematical discourse. Predicting the number of minutes required for seventeen unique students to discover a significant pattern or make sense of a new mathematics concept was impossible. Although I invested significant time in planning ahead, I discovered that more time, energy, and planning was still necessary in order to make changes to my lesson plans.

During Lesson 8 students corrected their quizzes and then found the incorrect mathematics in my error analysis activity. While I felt there was adequate class time for students to correct their quizzes and make significant progress on discovering, describing, and correcting the errors in the error analysis activity, I did not feel that time lent itself to both small and large group discussions for the same reasons as discussed for Lesson 7. A comment I wrote about in my journal later was, "Since all students were working at different paces, it wasn't realistic (at least I didn't feel comfortable trying to make it fit appropriately) to have a large group discussion about the error analysis worksheet. I wonder if some students would have had a better understanding if we had been able to have a large group discussion."

Another planning challenge I faced in this particular lesson was the lack of planning time. One change I would make for this lesson in the future would be to plan a follow-up activity for students who make quiz corrections that incorporates writing. Rather than simply making a correction to their quiz and moving on, I would require students to show evidence in writing of their understanding of their mistake, why it was incorrect, and why the correct strategy does work. While simply correcting a problem can often unintentionally result in student engagement in MP3, requiring students to critique their own work in a critical manner and provide justifications will ensure student engagement in MP3 (MP3 Definition, parts b and j). My time was spent creating the error analysis activity and grading students' quizzes, and so I was not prepared with a follow-up activity.

My relationship with time improved from not feeling like I had enough of it and struggling to plan an appropriate amount of time for individual lessons, to more accurately predicting the necessary amount of time and making small changes to lessons to better manage the time I had to work with. One of my only challenges with time in my final lesson, Lesson 9, was the fact that it is irreplaceable. Izzy spent the entire period in the hall making up a test because she had to miss school for an extracurricular activity earlier in the week. The discussion that she missed cannot be replicated or substituted.

Successes provide hope regarding managing time. By the time I reached my final three lessons, challenges related to time had morphed. I noticed that I could accommodate either small group time or large group time, but not always both. Great successes were noted with how I made use of the type of discussion that was present, but both scenarios came with a sacrifice. To prepare for Lesson 7, I was still investing a significant amount of time into my planning. For this particular lesson, I decided to put a greater focus on making sure I provided feedback to students by creating a slideshow that students could open on their laptops. Each slide had a question. The answer, and occasionally a short explanation, could be found on the following slide. This allowed students to work at their own pace and receive immediate feedback on the accuracy of their answers. Students were able to make excellent and productive use of their time. I was also able to make excellent use of my time and focus on the students that were struggling. Although I had not made time to prepare a lesson more intentionally focused on MP3, I did invest significant energy, time, and thought during class into the questions that I asked students as I rotated from group to group. Fifteen of the sixteen participating students chose to work together. If two students got different answers, I heard them justifying their reasoning and critiquing the arguments of their partner or groupmates (MP3 Definition, parts b, e, f, j, and k). Students had opportunities for self-critique. If a student did not understand how I got an answer, I was able to have a very natural, meaningful, and individualized discussion to help the student reach conceptual understanding (MP3 Definition, parts h and k).

My structure and allotment of time were not perfect because if a student mistakenly thought they understood how I arrived at a particular solution, I was not physically able to monitor every group and every students' thinking. If I had been able to make time for a large group discussion as well, perhaps some of the valuable arguments made and justified within small groups could have been shared with the entire class. This would have allowed other students to critique their reasoning, or that of the other groups, and arrive at a deeper level of understanding.

Regardless, my use of time and structure in this lesson gave all students the opportunity to engage in MP3 even if some students avoided accepting the transfer of responsibility for mathematical understanding. By spending time with each student and small group, I was able to press all students to engage in MP3 and explain their thinking when they were discussing the problems with me (MP3 Definition, parts f and h). I also made it clear that I valued mathematical discourse and conceptual over procedural understanding so that hopefully students would continue to engage in MP3 when I was not checking in with their small group.

I had one small group conversation with Lee, Aaron, Kyle, and James about the question, "Congruent triangles are _________________ similar." Students could fill in the blank with *sometimes, always,* or *never.* The boys had already established that congruent triangles are always similar, but I did not know if that was because they read the answer on the next slide or because they had a deeper understanding of the relationship between congruence and similarity. The boys happened to be finishing this question when I approached their group, so I engaged in a small group discussion with them about the reason that all congruent triangles are similar. The boys were forced to make their own arguments, but in an informal and nonobtrusive way.

By continually asking different students questions that required justification, I led many small groups and individuals to make viable arguments they never would have generated if they were doing the slide show activity without my periodic questions and discussion. Similarly, if we had done the whole activity as a large group, many students would never have had the opportunity for individual discussions with myself and a smaller group of people. When a question is posed to 17 students and one person answers at a time, 16 people are left with no voice. Talking to just 2-4 students at a time allowed many more students to share their reasoning and critique different possible strategies. I reflected that, "Small group discussions often yield the greatest value for the most individuals. There is a cost in that the whole class doesn't get to benefit."

By the final lesson of my study, Lesson 9, I was better able to face the issue of time because I had fewer expectations for my ability to predict how much was necessary. For this lesson, I planned three slides with thought-provoking and open-ended questions. I acknowledged ahead of time that planning for the impending discussion was difficult.

While the time required to create my three prepared questions was minimal, when I started the class period, I had to accept a heightened level of flexibility because I had no idea what direction the discussion might take. I did not know if we would discuss all three problems at a sufficient level of depth within 15 minutes or if we would spend the whole period discussing just one or two. While I allowed a few minutes throughout for small group discussion, most of the class period was spent having a large group discussion in which students exhibited many of the behaviors that indicate engagement in MP3. My structure of the class time allowed students to communicate their conjectures and justifications and question and respond to the mathematical ideas of others (MP3 Definition, parts b, f, g, h, and k).

I have already addressed some of the successes from this lesson on congruent triangles and distance related to the transfer of responsibility to students. One success relating to time was my ability to be flexible. I was able to make the decision to skip the second question because we had reached so much depth while discussing the first slide, that a follow-up question on congruent triangles seemed unnecessary. My decision allowed the class to have a successful analysis and discussion over distance in the third problem and gave students an opportunity to make and discuss mathematical conjectures on a new topic (MP3 Definition, parts b, f, and g). For this lesson, we were reviewing for our first semester finals. I did not have new material to cover. This allowed me to better use my time to plan discussions and questions that would focus on students' greatest misconceptions and misunderstandings. Students benefitted from an unrushed opportunity to engage in mathematical discourse about identified topics of difficulty and better allowed me to value the time spent fostering MP3 in my students.

The reward that comes with an investment of time, planning, and lesson structure is that students quickly adapt to teacher expectations. This allows for future lessons with an emphasis on MP3 to run more smoothly as seen by the increased depth, justification, and critical questioning that my students engaged in during our final studied discussion (MP3 Definition, parts a, f, j, and k). As the teacher becomes more comfortable with the required flexibility when implementing MP3, it becomes easier to plan for and accept change, even at the last minute, as evidenced by the freedom I was able to provide myself in the final lesson of my study.

Challenges as teachers pose questions and problems. I learned by experience that significant consideration needed to go into the questions I posed to my students if I wanted them to successfully engage in MP3. The mathematics problems and questions posed must be challenging enough to be interesting but easy enough to be realistic for most of the students in the class.

My first two lessons included problems that were too difficult, and students were unable to make or critique arguments (MP3 Definition, parts b, h, and j) because no one could come up with anything. My second and third lessons included questions that were too easy. There was a clear correct answer, so discussion, argument, and critique were hindered by too much agreement (MP3 Definition, parts h and j). It was not until Lesson 8, the second to the last lesson of my study, that I finally felt I had posed a question that was the perfect difficulty level.

Lesson 1 did not go as smoothly as I had hoped because students were learning a brand new topic. Students could not use prior knowledge to predict what sort of patterns they should be discovering, so some students made completely illogical arguments, but had no idea because the question was disconnected from prior learning and therefore difficult to make meaningful sense of (MP3 Definition, part a). Lesson 2 asked questions about the rate of water dripping from a leaky faucet. The first question had a clear correct answer. This led to a few successes. First, all students engaged in writing their own viable argument with a justification (MP3 Definition, parts b, f, and g), but they were all the same so there was no need for arguments or critiquing logic (MP3 Definition, parts h and j). Second, we expanded on the simplicity of the questions by using diagrams to represent equivalent fractions. We discussed the difference between doubling a fraction and multiplying a fraction by one as a result of multiplying the numerator and denominator by the same value. I think students were encouraged to make arguments because they felt the problem was something they were capable of making sense of. I made this statement, however, in my journal, “While problems were very doable, I think they were almost too easy. I’m not sure if I could have made them a little more difficult and still had 100% of students try to answer the questions and feel confident that solving the problems was within their reach.”

I felt the problems during Lesson 3 regarding the number of pizzas the academic team should buy may also have been too easy. My instructional coach commented, “This particular activity perhaps wasn’t so complex that students needed to clarify understanding. This will be a natural progression as these problems increase in complexity.” Asking clarifying questions is evidence of engagement in MP3 (MP3 Definition, part k) so this is evidence that the simple nature of the question was an obstacle to students engaging in mathematical discourse. As students become comfortable making arguments and critiquing the reasoning of their peers with simpler problems, I hope that investment will pay off as students become more willing to try increasingly more difficult problems.

Successes as teachers pose questions and problems. During Lesson 8, I gave students the error analysis activity which contained six incorrectly solved problems. Had I given all students problems to solve, some students would have unknowingly made the same mistakes that I intentionally highlighted on the error analysis activity. Being forced to find the flawed logic (MP3 Definition, part j), however, gave students what seemed to be the perfect scaffold to allow the problems to be challenging, yet doable. Aaron and James are both students that generally finish assignments quickly without much effort. In my journal I commented, “The difficulty level was just right. Students that could generally fly through an assignment had to think more critically and in a more conceptual and descriptive way… Some students were able to identify where the flaw was even if they weren’t sure how to correct it. That is a great intermediate step to gaining complete understanding of the concept.” As I had hoped, finding errors was more difficult than getting an answer, and both boys asked more questions of each other and of me throughout the activity (MP3 Definition, part k).

My final studied lesson (Lesson 9) in which students had to make viable arguments about congruent triangles and distance on a graph (MP3 Definition, part b) did not require much planning prior to the lesson. Significant effort, however, went into asking questions once the lesson started. I purposely asked questions that could be simplified or expanded on for more difficulty depending on the student engagement and direction of each individual class discussion. This gave me the freedom to address either simple or complex questions related to congruent triangles as appropriate for each class. I had to balance the student and teacher roles to try and achieve a situation where I was merely a guide. The ultimate goal was for students to be doing all of the justifying, arguing, critiquing, and questioning (MP3 Definition, parts j, g, h, and k). This was evident in the presentations each small group gave as well as the questions students asked of each other.

The challenge of investing in teacher-student relationships. In an attempt to guide our discussion during Lesson 9 in a meaningful and conceptual direction, I had to invest significant mental energy into the consideration of multiple factors simultaneously. I devoted much thought into considering each individual student's self-worth, ability, confidence or lack thereof, mood, emotional state, previous contributions to the discussion and class, prior public mathematical mistakes, level of alertness, level of indicated interest, body language, and many other factors that inform my decisions. Before questioning a student, I must consider the individual student and quickly decide on wording, tone, and whether or not I should begin with a praise, make a joke, or find a way to shift the focus to a slightly different aspect of the question or to a different person altogether. High school students respond to respect, encouragement, and the belief that they are genuinely needed and known.

After Lesson 9, I felt mentally exhausted when the bell rang at the end of the period. In fact, I noted in my journal that I actually changed my lesson plan slightly for the subsequent third period Geometry class to include a little bit less discussion as a result of my mental exhaustion from trying to so expertly steer the conversation. It was extremely challenging to keep all students feeling safe and willing to continue making conjectures, critiquing their peers' logic, and listening critically to generate meaningful questions (MP3 Definition, parts b, h, and k). Later class periods received fewer opportunities that day to engage in mathematical discourse as a result.

Successes as a result of investing in teacher-student relationships. Despite the mental exhaustion, Lesson 9 had a multitude of successes. Every single student stood in front of the entire class and shared a conjecture and justification (MP3 Definition, parts f and g). Students, not I, generated questions to clarify or improve arguments (MP3 Definition, part k). Students used prior knowledge to create conjectures as well as consider multiple cases (MP3 Definition, parts a and d). The lesson was only possible because students felt safe, and I was able to make appropriate decisions during the discussion because of the time I had invested relationally to get to know them.

A specific example of how I maintained a discussion based on my relational investment was evident when Becky and Rhiannon were presenting ASA congruence to the class. Becky and Rhiannon had just been asked a question by another student, and they were unsure of the answer. When it was clear they did not know the answer, I immediately said, "Can we all agree that they have a perfectly marked example of ASA?" My purpose was to give them confidence. I wanted to make sure Becky and Rhiannon knew, along with the whole class, that they are intelligent and that their ideas have value. I did not want either one of them to disengage in the discussion because of the experience of not knowing something in front of others.

A final quote from my journal was, "It was difficult to word questions and statements in such a way that I didn't give away answers and encouraged students to not only share what their individual thoughts were, but to also respectfully tell another group that they might be doing something incorrect. I was constantly trying to be aware of the atmosphere of the classroom." The students showed clear evidence of engagement in MP3 throughout this lesson. It came with the cost of being mentally tiresome, but I believe that cost would be alleviated with continued emphasis on this type of mathematical discourse.

I did not address the impact of relational investment in my journal entries as explicitly as some of my other themes, because investing in my relationships with students as people and as mathematicians and thinkers is something that I would be doing regardless of my attempt to implement MP3. Upon reflection, however, my instructional coach and I did find evidence of a payoff in students' engagement in MP3 coinciding with a relational investment in students.

Summary of hopeful investment. At the conclusion of Lesson 9, I identified my belief that students need to continue engaging in similar authentic mathematical discourse. When I taught the first lesson of my study, the main question students were asking was some version of, "Can you check this?" One month later, a student was asking his peers, "What is being bisected?" "Is the scale the same?" and "Why did you say HP was a perpendicular bisector if you don't need the tick marks?" (MP3 Definition, part k). After witnessing the increase in successes that occurred in just one month, it would seem that continued investment in developing mathematical discourse in students will lessen the cost on the teacher, providing hope for mathematics educators.

Initially in my study, managing time and the structure of my lessons was a challenge since I was unsure of how much time to provide for various activities. I did not always have time outside of the class period to plan every desired component of the lessons, and sometimes I had to teach new concepts sooner than preferred to stay on schedule with the other Geometry teachers and our school's curriculum map. Time for students to make sense of the mathematics, time for students to discuss their thinking, and even the time I spent waiting for students to pose or answer questions was sometimes sacrificed as a result. This resulted in fewer opportunities for students to engage in mathematical discourse, so it was clearly an obstacle to students' engagement in MP3.

By the end of my study, I had learned how time and class structure could be managed to achieve more opportunities for students to engage in MP3. I made fewer plans for each lesson but was strategic about what I planned so that students could take responsibility for the direction and time spent on mathematical discussions. I also saw successes by balancing small and large group discussion opportunities so students could gain the benefits from both. Balancing the difficulty level of the presented problems and questions led to successes as well. When questions were too easy, students made conjectures, but the problem-solving process felt irrelevant since everyone's conjectures were similar. When questions were too difficult, students disengaged entirely and time was lost which was an even greater obstacle to students engaging in MP3. The error analysis activity was one example of relatively simple problems students believed they could make sense of which led to mathematical discourse because of the error identification aspect.

Perfect questions, ideal lesson structure, and all the time in the world would not lead to student engagement in MP3 without an investment of energy, time, and passion into teacher-student relationships. This hopeful investment was mentally exhausting at times, but when the teacher is invested relationally, she is able to make decisions during lessons to encourage students to engage in MP3. During my study, students shared conjectures, explained their thinking, and asked questions (MP3 Definition, parts b, f, g, and k) without fear of judgment or embarrassment as a result of my relational investment.

Establishing Classroom Culture

For students to "make viable arguments and critique the reasoning of others," (MP3) they must feel safe, respected, and valued. Investing relationally in students was just one factor that allowed me to create a classroom culture where students could be the mathematical authority without causing chaos. My classroom culture allowed students to ask questions and make mistakes in front of their peers (MP3 Definition, part k). While several aspects of the culture I have created in my classroom have yielded successful student engagement in MP3, other aspects have allowed certain students to assume minimal roles in discussions and activities, decreasing their engagement in MP3.

Successes due to classroom culture. During Lesson 3, when students were asked to critique the reasoning of the academic team coach and her decision about how many pizzas to order, I witnessed the importance of a classroom culture that allows risk-taking and mistakes in allowing engagement in MP3 to be possible. Rhiannon had just finished listening to the thinking of her partner, Nicole, as well as the logic of the group next to her. In my journal I noted, "… without me even asking, she [Rhiannon] volunteered that she had originally agreed with the coach's logic, but she changed her mind after hearing Nicole's reasoning." Rhiannon could have remained quiet and her earlier misconception would have remained completely unknown except from her partner. Instead, she openly shared her mistake in such a way that students around her and I were all made aware of her transition from faulty thinking to correct understanding.

She valued understanding and was unafraid of ridicule by her peers or her teacher for not understanding immediately or for making a conjecture that turned out to be false. She engaged in MP3 initially by making her own viable (if incorrect) argument (MP3 Definition, part b). By critiquing the reasoning of various classmates, she was able to independently reach a different and accurate conclusion (MP3 Definition, parts g and j).

During this same lesson, my instructional coach observed multiple times throughout the class period that students were "sharing arguments and discussing, sharing out, and demonstrating on the board." She was referencing Nicole volunteering to share her argument and reasoning with the whole class. Nicole's argument was followed by that of Aaron, Kyle, and Izzy who all shared their arguments and their reasoning in front of everyone (MP3 Definition, part g). While Aaron, Kyle, and Izzy were selected by me, they all gave thoughtful answers, and none of them tried to avoid answering the question or asked for a pass. A culture of valuing discussion and setting high expectations for all students had already been established, and students knew I valued their willingness to respond thoughtfully more than I valued a "right" answer.

In the majority of my journals, I noted various instances of students' willingness to share their arguments, to receive help from peers, to openly admit to a lack of understanding, and to ask questions that require vulnerability. I did not note a single instance when a student was unwilling to share his or her thinking. Several students admitted they did not know the answer, but no one declined to share their argument or reasoning.

During Lesson 1 of my study, students were making discoveries through an activity on their computers, and then applying what they observed to various statements that could be answered with, “sometimes, always, or never.” I overheard a girl ask her groupmate, Connie, “The last one would be ‘always’ wouldn’t it?” This student was comfortable communicating her mathematical thinking (MP3 Definition, part g) despite being potentially wrong. She was also willing to accept other possible answers from her groupmates (MP3 Definition, part j).

In the same lesson, I heard James critique Aaron by saying, “No, you have to go to the vertices. Those should all be the same,” concerning the creation of their GeoGebra figure. I watched Aaron nod his head in understanding, and both boys continued their activity. I later reflected that, although it appeared as though James’ statement was regarding the directions of the activity rather than an independent mathematical claim, “…if these students are comfortable discussing and even correcting each other, then they will hopefully feel safe and more willing to discuss and even correct each other concerning mathematical arguments as well.”

I witnessed these two particular students’ willingness to discuss and even correct each other concerning mathematical arguments throughout my study. One specific example was James’ and Aaron’s willingness to generate questions for other groups during Lesson 9 on congruent triangles. As I reflected on my lesson two hours after the class period ended, four particular students stuck out as being willing to volunteer questions and responses (MP3 Definition, parts g and j). Two of those students were Aaron and James. This was further evidence suggesting that students will engage in MP3 if they are provided with a safe classroom culture where they do not fear being incorrect or lacking a guaranteed correct answer. My instructional coach also noted about Lesson 9 that groups volunteered to share. This supports students' willingness to share despite the possibility of being critiqued, questioned, or potentially incorrect while engaging in MP3.

Various examples of students engaging in MP3 socially in groups of differing sizes have already been identified when discussing the transfer of responsibility to students and the investment into both planning and students. I will not repeat those examples here; however, I must comment that every time a student shared his or her thinking with a peer, a small group, or the entire class, that student was acting in accordance with the safe and respectful atmosphere that I had developed in my classroom. Every instance mentioned previously in which students accepted the transfer of responsibility, they were making the statement that they felt comfortable sharing their thinking whether it was correct or incorrect. Critiquing the reasoning of others requires others to share reasoning. The multitude of examples of students sharing their reasoning highlighted throughout Chapter IV show that my classroom culture both allowed and encouraged students to discuss mathematics. Thus, I was able to see successful examples of students engaging in MP3 as a result of my classroom culture.

Challenges due to classroom culture. While my focus on a safe and encouraging classroom culture allowed students to engage in MP3 by generating their own viable arguments, sharing them with small and large groups, and critiquing the reasoning of their peers (MP3 Definition, parts b, g, and h), I occasionally struggled to hold all students to the same expectations. I found myself asking for volunteers to answer questions or share their logic despite the possibility of misconceptions going unnoticed in the students who chose not to share.

While the classroom culture led to many individuals engaging in MP3 during Lesson 9, I also noted that I never heard Felicia's thinking during the lesson. All students went to the front of the class with their small group at some point during the lesson to share their congruent postulate or theorem and justification (MP3 Definition, parts b, f, and g). Her group members spoke when presenting, so while she may have engaged in MP3 while discussing with her small group, she did not say anything while presenting or during our large group discussion. One of the specific implications I recognized after that lesson was, "Discussion is a valuable tool, but it doesn't always impact all students. Some sort of writing or debriefing assignment or formative assessment would have made the lesson better." When I said the lesson could have been *better*, I was referring to ways that I could have involved more students in further critiquing others' arguments or their own. I recognized the limitations of large group discussions as well as my personal limitations to ensure 100% active participation from all students. It is easy to *intend* for students to engage in MP3; however, it is challenging to maintain total classroom engagement in MP3 from 16 or more students simultaneously.

While my classroom culture allowed for students to share freely and make mistakes without ridicule, I did not always press students to explain their arguments, reconsider others' arguments, or generate questions (MP3 Definition, parts f, h, j, and k) as I believe I could have to maximize their engagement in MP3. The challenge to consistently hold all students accountable for engaging in MP3 simultaneously was constantly present. My perseverance and consideration of the environment, culture, and atmosphere caused some students to engage in MP3 more than if they had not been pushed. Other times, my struggle resulted in missed opportunities. During Lesson 3 regarding how many pizzas to buy the academic team, I called on Paul to rephrase the thinking of Kyle. My decision to call on Paul was based on my observation of his potential disengagement in the large group discussion, so I was attempting to hold him to the same level of responsibility for sense-making as his peers.

When Paul's attempt to rephrase Kyle made it clear he only understood part of what Kyle described, I became flustered as I considered how to address Paul's evident lack of understanding. My consideration of my desired classroom culture made me hesitant to lengthen this interaction for three reasons. First, I did not want to make any decision that would make a student feel as though they were being punished for engaging in MP3. Kyle had clearly explained thinking to the class (MP3 Definition, parts f and g), so I wanted to honor his response and reinforce his willingness to engage in MP3 in front of his peers.

Second, I felt it would be redundant to have Kyle repeat his response since the majority of students had already shown evidence of understanding the problem. Third, I wanted to treat Paul with respect whether he had been paying attention or not because of my desired classroom culture. I hoped he recognized that he needed to remain an active listener when his peers were sharing, and I chose a different student, Izzy, with a response I knew was similar to Kyle's, to share her response with the class instead.

While I understand how my consideration of the classroom culture influenced my decisions, I wonder if I unnecessarily impeded an opportunity for Paul to learn from that experience and have a chance to share his thinking. Despite feeling uncomfortable for both the teacher and students, asking students to rephrase each other encouraged students to listen more carefully and critically. While I did not return to Paul and ask him to rephrase Kyle and Izzy, I did observe him watching Izzy and listening intently to her reasoning. I believe being questioned gave Kyle a clear understanding of my expectation that he be able to explain the thinking of his peers. If I could go back, I would have talked to Kyle when the large group discussion was over to find out his level of understanding without doing so in front of the entire class.

Summary of establishing classroom culture. Throughout my study, I witnessed students asking questions and baring their ignorance to their classmates. I heard students describe their previous misconceptions and how their thinking had changed through the process of mathematical discourse. These students were focusing on learning and understanding the presented mathematics problems. If my classroom culture had not placed value and importance on respect and learning from mistakes, many students would not have volunteered to communicate their thinking in front of the class (MP3 Definition, part g).

While my established classroom culture primarily led to successes, in a few situations, I missed opportunities to press students to persevere in making sense of problems and describing their mathematical thinking because I was concerned about possibly embarrassing students by doing so in front of their classmates.

Summary of All Findings

The purpose of this study is to gain insight into the successes and challenges of a high school mathematics teacher attempting to teach lessons with an emphasis on developing MP3 in her students. Nine lessons were designed and implemented with an emphasis on student engagement in MP3. Through analyzing my personal journal entries, the notes and observations of my instructional coach, and the work of my students, I found three themes that, while they had their challenges, led to increased student engagement in MP3. Transferring responsibility from the teacher to the students, investing time and energy into lesson plans and relationship with students, and establishing a culture that supported learning from mistakes and high expectations for mathematical thinking and discourse, all resulted in student engagement in MP3.

When I retained less control and allowed students to be the mathematical authority in the classroom, many students accepted the increased responsibility. Many students took advantage of the increased opportunities to take part in mathematical discourse as a result. Examples from my study included using wait time to allow student to pose mathematical questions that resulted in student-led mathematical discourse, as well as giving writing assignments that required student to distinguish between correct and flawed logic followed by small and large group discussion. Another identified method of transferring responsibility to students was valuing multiple solution strategies. Students accepted more responsibility as they generated multiple methods to solve problems and had the power to choose which method made the most sense to them. In one example, three students generated and presented three different strategies for finding the missing side in a proportionate triangle and the whole class compared and contrasted the multiple strategies.

Transferring responsibility did not come naturally for me. I recognized that I sometimes held different expectations for different students and challenged myself to send the message that all students are capable of becoming mathematically proficient in MP3. Time occasionally caused me to shorten my wait time and rephrase what students said rather than allowing the students to rephrase the comments of their peers. I found that writing strategies and error analysis were two specific ways for me to increase individual accountability for all students.

Fostering engagement in MP3 required an investment of time, energy, and thoughtfulness into my planning and into my relationships with my students. While working through this process implied a sacrifice, my investment was rewarded, and I witnessed students engaging in authentic mathematical discourse in every studied lesson that I taught. By continuing to emphasize and value MP3 throughout my study, students increased the quality and quantity of mathematical questions posed. I also found an increase in their willingness to consider alternate solutions and engage in authentic mathematical discourse. This evidence gives me hope that continued investment in developing mathematical discourse in students will continue to decrease the cost for teachers.

Similarly, less time investment was required at the end of my study for lesson planning because I had learned how to plan more strategically and make better use of class time by balancing large and small group discussions. I discovered the importance of planning less, but using questions of an appropriate difficulty level that allowed students to take responsibility for the direction and time spent in mathematical discussion. One example was an error analysis activity that contained simple problems that all students believed they could answer. The challenging nature of identifying, describing, and correcting errors required students to question and justify their thinking and the reasoning of their peers. The investment into lesson planning, time management during class, and appropriate questions would not have been as effective if I had not first made an investment into forming relationships with my students. My investment into teacher-student relationships allowed students to engage in MP3 in front the teacher and their peers without fear of judgment or embarrassment.

Finally, I observed the necessity of a safe, encouraging classroom culture with high expectations for all students. My established classroom culture allowed students to engage in MP3 because they knew that mathematical thinking, reasoning, and justifying were more highly valued than correct answers. In one lesson, a student shared her previous misconceptions and how and why she changed her thinking without being prompted. My data includes many similar examples of students explaining their thinking even when they are unsure if their idea is correct because a classroom culture has been established that prioritizes understanding and learning from mistakes.

CHAPTER V: DISCUSSION AND CONCLUSION

The CCSS Standards for Mathematical Practice list eight characteristics of mathematically proficient students. MP3 states that mathematically proficient students "construct viable arguments and critique the reasoning of others," (National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010). The purpose of this study is to gain insight into the successes and challenges a high school mathematics teacher faces when fostering MP3 in her students. I identified which teaching strategies were most successful in getting students to engage in MP3. When I encountered challenges that kept students from engaging in MP3, I identified teaching strategies to address and remove those obstacles so that all mathematics teachers will be better prepared to implement lessons that emphasize the development of mathematical discourse based on my findings and analysis.

Summary of the Study

This study used a classroom action research design. Qualitative data in the form of personal journal entries, notes from the building instructional coach via an observational framework, and student artifacts were collected to determine major themes that arose in response to these two research questions:

1. Which teaching strategies lead to successful student engagement in Common Core Mathematical Practice 3 (MP3: *Construct viable arguments and critique the reasoning of others)*?
2. What challenges was the teacher confronted with, and how did she react as she attempted to develop MP3 in her students and classroom?

I chose to study the implementation of MP3 into mathematics lessons from the perspective of a classroom teacher. A study conducted by Marshall et al. (2009) found that the vast majority of teachers believe student learning would benefit from increased classroom opportunities to engage in mathematical inquiry. Research also supports the importance of teaching for conceptual understanding (Nogueira de Lima & Tall, 2008; Marshall et al., 2009; & Oikkonen, 2009). Nogueira de Lima and Tall (2008) identified the importance of conceptual understanding and showed why teaching procedural knowledge alone in mathematics is a problem. None of these studies, however, provided answers as to how teachers can achieve greater conceptual understanding in their students.

Various researchers experimented with the use of writing, discussion, and contextual problems as possible strategies to address students' lack of conceptual understanding (Bonotto, 2013; Mercer & Sams, 2006; Oikkonen, 2009; Sahin et al., 2015; & Page & Clarke, 2014). Bonotto (2013) presented elementary students with real-world artifacts and observed their ability to pose and solve complex mathematics problems after collaborating and engaging in mathematical discourse. Oikkonen (2009) increased the amount of time his university class devoted to discussion and decreased the covered mathematics content. This shift in his class structure, as well as providing students with assignments that gave them an opportunity to make sense of their misconceptions through written discourse, led to an increase in the number of students that successfully passed these mathematics classes (Oikkonen, 2009).

Mercer and Sams (2006) searched for a relationship between students' quality of mathematical discussion and their ability to problem-solve. They found that elementary students who received direct instruction at an after-school program on how to engage in mathematical discourse, spent more time communicating about mathematics in their regular classroom. These same students outperformed their peers on standardized tests and were better able to solve and make sense of mathematics problems than their peers who did not receive the direct instruction. Furthermore, Mercer and Sams (2006) noted that teacher modeling appeared to improve students' ability to use language to make sense of mathematics.

Other studies also observed the impact of the teacher on students' engagement in mathematical discourse (Conner et al., 2014; Dennis & O'Hair, 2010; Heaton, 2000; and Kazemi & Stipek, 2008). In fact, Dennis and O'Hair (2010) found that the most important indicator of successfully being able to incorporate a new teaching strategy—their study focused on problem solving—into a mathematics classroom is the quality of the teacher. This gave me hope that, although most research focused either on student growth or teacher obstacles, there was evidence that teachers can make progress in overcoming those obstacles.

My study used a similar model to a study conducted by Heaton (2000) in which she also studied her role as the teacher in the implementation of new teaching strategies. Heaton (2000) identified challenges to her implementation, but she discovered ways to overcome her obstacles. I immediately recognized the value of her study because I was able to learn from the strategies that she attempted. I could relate and learn from her vivid descriptions of her lessons, some of the challenges she faced, and some of the successes she experienced. For example, she found evidence that, while students needed time to make their own mathematical discoveries and discuss their mathematical thinking with their peers, occasionally “telling” was the most appropriate strategy. This was significant for me personally because I often feel guilty if students do not learn everything in my mathematics classes via their own logical reasoning and inquiry.

It was evident that my areas of success were closely related to some of my reoccurring challenges. I identified three main themes that related to my challenges and successes. Within each theme, I identified patterns that allowed me to analyze the successes and challenges in greater depth and detail.

1. Transfer of Responsibility

    - Students Became the Mathematical Authority - Students engaged in MP3 when I planned opportunities for them to take on the role of mathematical authority. On the other hand, some students left the role of mathematical authority to their peers and remained bystanders.

    - Multiple Solutions were Encouraged - Valuing and modeling multiple representations led students to accept more responsibility for making sense of mathematics problems and concepts and led to engagement in MP3.

    - Students were Held Individually Accountable - When I implemented strategies that held students individually accountable, they spent more time in mathematical discourse, either verbal or written.

2. Hopeful Investment

    - Investment of Time and Structure - To encourage students to engage in MP3, I recognized the need to invest time and energy into the classroom structure and lesson plans. This investment often led to challenges, especially initially as I learned how best to structure my lessons and use class time.

    - Investment into Posing Appropriate Questions and Problems - The questions and problems I posed needed to be the appropriate difficulty level in order for students to engage in MP3.

    - Investment into Teacher-Student Relationships - I needed to invest time and energy into the well-being of my students. Ensuring students felt safe sharing their ideas in front of classmates was also a challenge, but my data showed that these investments led to success as I observed increased student engagement in MP3.

3. Establishing Classroom Culture

   - Establishing a culture in my classroom where students knew they were expected to treat everyone with respect, share their thinking, and prioritize understanding over answers, led to increased student engagement in MP3.

   - Minimal challenges were encountered as a result of the classroom culture, but at times, I was hesitant to press students to engage in further mathematical discourse in front of the entire class as a result of my consideration for my culture of respect and safety.

The emergence of these three themes has valuable implications for all mathematics educators and for me as I continue to be intentional about creating more opportunities for my students to generate mathematical arguments and critique various logical statements (MP3). My findings imply that using an appropriate wait time is crucial to give students an opportunity accept the role of mathematical authority as well as allow them to formulate and ask questions (MP3 Definition, parts b, h, and k). Finding one acceptable strategy or representation is inferior to analyzing multiple possible strategies and judging their correctness or usefulness (MP3 Definition, parts b, c, g, and j). When I feel mentally exhausted after guiding a conceptual discussion, my findings imply that my students will constantly improving in their ability to accept the transfer of responsibility and take charge of the direction of mathematical discussions so I can anticipate an increase in students' engagement in MP3, as well as a decrease in my investment.

Summary of Results

Transfer of responsibility. When the teacher is doing the work, the teacher is only guaranteeing that she has learned the material. If mathematics educators want to improve students' conceptual understanding and increase students' time spent engaging in authentic mathematical discourse, my findings indicate that teachers achieved success through intentionally transferring responsibility to students for making sense of problems and attempting to find and justify possible solutions. This allowed the students, rather than the teacher, to create arguments, as well as communicate and justify their reasoning (MP3 Definition, parts b, f, and g). To foster this transfer of responsibility, I found it was crucial that students be perceived as the mathematical authority. I encouraged this by having students share their thinking in small and large groups and by having students rephrase and make sense of the logic of their peers (MP3 Definition, parts h, j, and k). I often struggled with the awkwardness and lack of control necessary to allow students to attempt to listen critically to their peers and try to explain someone else's thoughts in their own words. When I expected students to take on the responsibility of listening to each other and finding ways of expressing their own mathematical understanding as well as that of their peers (MP3 Definition, parts g, h, and j), I always witnessed multiple students, if not all, engaging in MP3.

During more traditionally taught lessons, I was often guilty of giving in to time constraints. I would end up teaching students one strategy that I determined would be easiest, fastest, or most likely to be used later in their mathematics careers. During my study, I tried to consistently value when students gave multiple approaches to representing and solving problems. Giving students this opportunity to try different approaches caused them to be more creative. Students came up with solutions I myself had not even considered. This also improved the classroom culture by causing students to think less about finding the "right" answer and more about making sense of mathematical problems and reasoning logically. Students ultimately spent more time engaging in MP3, in particular by considering different cases, comparing various arguments, justifying their thinking, and asking questions to better understand strategies or representations that were different from their own (MP3 Definition, parts d, f, j, and k).

Hopeful investment. Time and how best to manage it within the structure of the class were constant challenges I faced as I attempted to reach 100% classroom engagement in MP3. I identified the importance of including an individual accountability component, such as writing, to a few of my lessons, but this has a larger implication as well. Writing was a powerful way for me to reach my goal of ensuring all students made their own conjectures. My study suggests that students will spend more time engaging in MP3 if mathematics educators incorporate more writing into the classroom. Planning appropriate writing prompts for students presented a challenge as well, because the prompts needed to be easy enough for students to believe they could answer. Questions also needed to be challenging enough or open-ended enough to allow for rich discussion with differing logical arguments and opportunities for critique.

I had to consider the difficulty of the questions I asked and the problems that I posed to students throughout my study. Initially I posed simple questions that gave students confidence and allowed them to become more comfortable writing or discussing their reasoning and then sharing it with their peers and myself. This resulted in success as nearly all students were observed engaging in MP3. Later in my study, I was able to ask more open-ended questions with many possible solutions, and students were still willing to participate.

While investing time and energy into my lesson structure and the questions and problems I posed were factors that affected students' engagement in MP3, my relational investment made it possible for students safely communicating their arguments, justifications, and questions (MP3 Definition, parts g and k). Some of the challenges I encountered in my study showed the reality that students will often not engage in MP3 without significant investment from the teacher in all of these areas. One encouraging result of my study is that I have hope that these investments yields a reward. By Lesson 9, my investment into the lesson structure and my use of time was rewarded by students engaging in MP3 both with small and large groups for the entire class period (MP3 Definition, parts b, f, g, h, j, and k). The investment I made into generating ideal questions for this lesson that would foster mathematical discourse was rewarded when all small groups made attempts to solve their problem and generate questions for other groups (MP3 Definition, parts b and k). Students responded to my relational investment throughout the study by volunteering to share their reasoning and by admitting when they had questions or a lack of understanding (MP3 Definition, parts g and k).

Establishing classroom culture. The first step to encourage students to engage in MP3 is investing time to plan lessons that provide opportunities for mathematical discourse. The second necessary step is ensuring the classroom culture values discussion, input from all individuals, and respect. Students need the opportunity to make sense of mathematics problems. While students may struggle to formulate their own viable arguments, they certainly will not share their thinking with others if they do not also have the safety of an environment where it is acceptable and even valued to make and learn from mistakes. Since the classroom atmosphere is subject to the personalities and temperaments of its students, maintaining this classroom culture is sometimes a challenge. Investing in a culture of safety, respect, and high expectations yielded evidence of some successes in every single lesson that I taught with an emphasis on MP3.

Discussion

Perhaps one of my greatest and most useful discoveries during this study was the value and importance of writing strategies in a secondary mathematics classroom. Page and Clarke (2014) studied the use of journaling in elementary school students. My findings were consistent with theirs that asking students to explain their thinking in writing can give the teacher insight into the students' mathematical understanding. I altered my lesson plans for Lesson 5, as a result of my students' responses to a writing prompt about similar figures. Oikkonen (2009) studied the use of writing in university students, which led me to believe writing could be a useful tool for my study, as well. Just as Oikkonen (2009) attributed increased student success to his implementation of writing assignments, my findings showed that writing led to increased student engagement in MP3.

I found limited research that specifically studied the teacher's role during mathematical discourse. My study showed the complexity of the teacher's role when guiding classroom discussion. Each time I used large or small group discussion during one of my lessons, I commented in my journals on the difficulty of asking questions that were appropriate for the group as a whole or for a specific student. A multitude of factors, ranging from the mood of the student to how the class might interpret particular wording, must be taken into account in a matter of seconds.

When encouraging students to generate their own questions, even more decisions must be made as the teacher considers the classroom atmosphere and students being receptive to classmates' questions. When students are not generating their own questions, more decisions need to be made by the teacher. Should the teacher ask her own question? If she does, will that stimulate the discussion and encourage more students to ask questions on their own? Will her question encourage students to critique her reasoning or the reasoning of another classmate, or share a new mathematical argument they have come up with? If the teacher asks a question, will some students view the teacher as the ultimate mathematical authority and leave more of the responsibility for learning in that moment and in the future to the teacher?

Unfortunately, no study can provide an algorithm that guarantees a successful mathematical discussion because students are complex human beings. Conner et al. (2014) attempted to generate such an algorithm by identifying five questions and five actions that were most likely to result in students engaging in mathematical discourse. My study, however, provided sufficient evidence to state with confidence the importance of investing time and energy into knowing and caring about students. This better equips teachers to make decisions that may or may not align with Conner et al.'s (2014) questions and actions, when guiding classroom discussions so that the discourse is appropriate to the individuals that make up the classroom. This evidence is consistent with the research of Dennis and O'Hair (2010) who found that teacher quality was one of the best indicators of successful implementation of a new strategy.

Kazemi and Stipek (2008) also considered the impact of a quality teacher by studying the connection between teachers' ability to motivate students, "press" them to persevere in solving difficult problems, and the students' engagement in four identified socio-mathematical norms they believed were necessary in order to promote conceptual understanding. Three of their four norms—students describe their thinking, students solve problems in multiple ways and share with the teacher and their peers, and students solve problems collaboratively—describe engagement in MP3 (MP3 definition, parts b, f, g, h, and j.) Both their finding and mine showed evidence that intentionally transferring responsibility to students was a specific way to increase mathematical discourse.

The three main themes that emerged from my data also coincided with Kazemi and Stipek's (2008) definition of *motivation*. Kazemi and Stipek (2008) defined motivation using four criteria. Two of the criteria were how much the teacher focused on understanding and learning and how much the teacher deemphasized getting right answers. My theme of establishing classroom culture aligns with both of these criteria. By cultivating a culture in my classroom where justification and logical thinking are more important than getting correct answers, I am also prioritizing understanding and learning. Similarly, I have cultivated a culture where students are encouraged to ask questions and learn from mistakes, thus deemphasizing right answers.

Another one of Kazemi and Stipek's (2008) criteria, how much the teacher "pressed" students to persevere and work through difficult problems, was consistent with my recognition that hopeful investment often resulted in student engagement in MP3. I was able to "press" students to create conjectures and critique their classmates respectfully (MP3) because of the investments I had made relationally with students. I invested time into preparing questions and problems that were the appropriate difficulty level so that students would view them as challenging yet within their ability to solve.

Kazemi and Stipek's (2008) final criterion was how much teachers promoted student autonomy. This directly corresponds to my theme of transferring the responsibility for sense-making from the teacher to the students. While I identified the challenges that arose as I attempted to convince all students of their ability to take on the role of mathematical authority, like Kazemi and Stipek (2008), I also witnessed successes when I found ways to hold students accountable for making independent conjectures and when they began to accept my transfer of responsibility.

My three main themes were consistent with findings from Heaton (2000), as well. After reading Heaton's (2000) research, I hoped to gain similar insight in a secondary-level high school setting. Heaton's study showed the importance of teacher flexibility. In particular, she noted that the teacher must be willing to give up some control and share leadership with students. My study showed that the same is true of high school teachers. One of my main themes was the importance of transferring responsibility to students and the challenges that exist in giving up some teacher control and allowing students to be the mathematical authority and generate their own ideas, arguments, strategies, and representations.

A second important discovery of Heaton (2000) was her revelation that she would never know everything there was to know about mathematics. Perhaps I did not feel this way because of my extensive studies in college mathematics and the fact that I was teaching a relatively basic mathematics course. The hours spent planning my lessons, as well as the previous five years of experience I already had teaching these Geometry concepts, may have contributed to my confidence. My findings validated the importance of teacher quality. I changed my lesson plans, questioning strategies, and discussion techniques in subsequent lessons as I learned from my experiences, which may not have been realistic for a first-year teacher with no experience.

Even though I realize I will never know everything possible about mathematics, this was not something that ever presented itself as either a challenge or cause of success during my study. My study showed, however, that my expertise was often best utilized by creating scenarios in the classroom for students to make their own sense and create their own arguments. Telling students exactly what I wanted them to know and exactly how to solve problems, while occasionally necessary, did not foster student engagement in creating individual mathematical arguments. Students were also not likely to critique my reasoning if I simply told information to them. I discovered the value of using my mathematical knowledge to plan time and opportunities in which students were expected to draw reasonable conclusions and take the time to understand their thinking well enough to be able to discuss and justify (MP3 Definition, parts b, f, and g).

Sahin et al. (2015) also hypothesized when teachers lacked experience or training in preparing for and leading discussions, the result was questions with low cognitive demand and had very low expectations for students' mathematical discourse and justifications. My findings support this hypothesis as I saw evidence of the need to consider multiple factors while leading discussions. My knowledge and experience allowed me to make decisions quickly concerning situations that a first-year teacher may have struggled to handle. For example, when a group of girls was presenting their ideas to the class, and another student asked them a question they did not know how to answer, my relationship with each student and my experience allowed me to move the discussion forward in a way that was mathematically productive and maintain the dignity of all students.

My findings also suggested the importance of investing time in planning questions and lessons that foster student engagement in MP3. Sahin et al.'s (2015) findings suggest that many of the teacher-participants were not making this investment. Since teachers were asking questions that did not require much critical thinking, the students were able to satisfy the teacher's questions with very little justification or further mathematical discussion. I found the inverse to be true. When I planned questions that students viewed as solvable that had multiple solution strategies, mathematical discourse became more prevalent. I invested more time in the questions and problems I posed, but I received an investment in increased student engagement in MP3 as a result.

Recommendations for Future Research

I have evidence that supports transferring responsibility to students, a hopeful investment in students, and establishing a classroom culture of respect and high expectations lead to greater student engagement in MP3. However, I do not know to what extent each theme affects the mathematical discourse in my classroom. I have no quantifiable evidence to determine if one of my themes has a greater impact on students' engagement in MP3 than the others. For this reason, I recommend further research studying the impact on students, perhaps using a quantitative design. If research could determine that one theme had a more significant impact than the others, teachers could prioritize that theme in their practice.

I witnessed students taking on more and more responsibility for mathematical discussion throughout my study. I recommend further study of students' and teachers' roles in mathematical classroom discourse to determine the extent to which students can improve on independently participating in conceptual discussions. If students are continually faced with discussion in mathematics classes where they are expected to be the mathematical authority, to generate questions, to make sense of the logic of their peers, and to critique the ideas and strategies presented (MP3), will students begin to do these things more independently?

Further research is necessary to determine what successes and challenges emerge as teachers begin implementing lesson plans that develop the other seven Standards for Mathematical Practice into their students. Would the emerging success and challenges would be similar to those discovered in my study of MP3? All eight mathematics practices are indicators of mathematical proficiency in students. Further research could include studying the relationships between the standards. As I focused on mathematical discourse and MP3, I also found myself encouraging my students to use some of the other mathematical practices as well. The similarities and differences between the eight Standards for Mathematical Practice, as well as how best to implement each standard into secondary mathematics classrooms, which standards can be easily developed at the same time, and the impact on student understanding and/or perception of and attitude toward mathematics as a result should also be studied.

My findings will help any mathematics educators interested in developing their students' mathematical proficiency to be better prepared to implement lessons that emphasize engagement in MP3, but further research should be done to determine what other factors may also lead to increased student engagement in MP3. I observed students using content-specific vocabulary as I required students to write and discuss. A future study should follow similar implementation of MP3 to determine if there is an increase in students' understanding of content vocabulary. This could have huge implications for educators in every field since every content area teacher faces the challenge of teaching students academic vocabulary. If a future study determined increased understanding in content vocabulary as a result of students engaging in content specific discourse, perhaps the results would be the same for teachers in other fields.

Before and during my study, I informed my students of my emphasis on MP3 and discussed what it means to "create viable arguments and critique the reasoning of others." I did this to provide the students with my expectation that mathematical discourse, respectful and critical listening, and justification were more important to me that finding correct answers. This was part of how I established my classroom culture and encouraged a transfer of responsibility to students. I am unable, however, to compare the behavior of my student-participants with students who were unaware of the purpose of the study and the teacher's desire to emphasize MP3. Further research is necessary to determine the implications of involving students as co-researchers rather than solely participants.

Recommendations for Future Practice

Mathematics educators, administrators, and parents should be encouraged that specific steps can be taken to increase students' engagement in MP3 because when students engage in MP3, they are showing evidence of their mathematical proficiency. I recommend that mathematics educators plan components into their lessons that will transfer the responsibility for making sense of problems and generating multiple strategies for solving them to their students.

The use of writing prompts is one simple yet effective way of doing this. Not only do writing prompts give all students a safe opportunity to generate their own conjectures, thus holding all students individually accountable, but they also give students something tangible to reference later during mathematical discussions. Teachers must not underestimate the difficulty of generating the perfect writing prompts or mathematics problems. Students must be willing to try, but the question cannot have a clear correct answer. If the question is too easy, students will all generate the same argument, leaving no room for critique. Asking the students to identify and describe flawed logic or errors was one way that I found to make simple problems more challenging and more conducive to writing and engaging in mathematical discourse.

In my findings, I identified the value in students generating multiple strategies for solving problems. When presented with various methods, students preferred to use the strategy that made the most sense to them. Students can be empowered to accept my transfer of responsibility when they are presented with choices, especially when some of those choices come from their peers showing the capability of high school mathematics students to be the mathematical authority in the classroom. I recommend that the teacher observe and informally question students during small group discussions so the teacher can be strategic about which students communicate their thinking with the whole class later. This way, the teacher can maintain students as the mathematical authority while still ensuring that multiple strategies or representations are shared.

Teachers should be willing to give up control and allow students to be viewed as the mathematical authority. When students share their thinking, the teacher should not always rephrase the student. The simplest and most powerful way that I discovered to transfer responsibility to the students was by asking the question, “What did (s)he just say?” Responses to this question inform the teacher if students understood the representations or strategies of their peers. This question allows the teacher to hold students individually accountable. It is easy to use because it never requires rewording based on the situation. Another positive result of using this question is that students get to multiple opportunities to hear a particular conjecture and justification, once from the original student and again from the student that the teacher questions. This question also increases the number of students participating in large group discussions, meaning at least double the number of students will have opportunities to engage in MP3.

Although challenges are to be expected when planning the structure, content, and problems of each lesson, I recommend that teachers intentionally invest time in considering all of these factors. My findings lead me to believe that the necessary investment changes over time. As students become more independent learners and accept more responsibility for their learning, teachers can simplify their lesson plans to allow for discussions to be led and guided by student questions, inquiries, and responses.

I also recommend that mathematics teachers discuss the meaning and importance of MP3 with their students and begin to include assessment questions that require written engagement in MP3. These changes would help to shift the classroom culture even farther toward an emphasis on learning, understanding, and justification. These changes could place more value on students' ability to accept responsibility, be the mathematical authority, and engage in authentic mathematical discourse.

I recommend that mathematics teachers prioritize giving students time to make their own arguments whenever possible, but remember that sometimes "telling" the information is necessary. Teachers must be willing to be flexible, from planning their units and working with a curriculum schedule, to the day-to-day and minute-to-minute decisions they constantly make. During my study, I was forced to make certain decisions due to time that were not always in the best interests of providing students' with time to make their own conclusions. Even so, I experienced some examples of successes in every lesson I taught, regardless of circumstances outside of my control.

Administrators, government officials, school board members, and community members can benefit from understanding the time required in allowing students to engage in MP3. All decisions made concerning mathematics curriculum and pacing should take into account my findings about investment and time necessary to implement lessons that focus on engaging in MP3. This is critical, because if students are not engaging in MP3 or the other CCSS Standards for Mathematical Practice, then they are not showing evidence of mathematical proficiency.

Finally, I recommend that teachers invest time in building and maintaining relationships with their students. A classroom culture needs to be cultivated in which students explain their thinking, respect the thinking of their peers, can safely ask questions, and are held to high expectations. In every lesson I studied, I spent time rotating around the room to ensure discussions with as many students as possible. This sent the message to students that I expected them to engage in mathematical discourse and give their best effort to the activity. This also allowed me have a conversation with each individual student which I believe added to their feelings of safety and value in my classroom.

Summary

Implementing MP3 in a secondary mathematics classroom was challenging, but possible. Every challenge that arose was able to be addressed and resulted in witnessing students engaging in MP3 and authentic mathematical discourse. Studying the experience of a teacher is complex. So many factors impact the multitude of decisions teachers make throughout each class period. My experience as a high school mathematics teacher is unique. Due to the diversity of the participants, however, I have gained valuable insight into how best to implement MP3 in any mathematics classroom. I have improved my teaching practice in each of my six mathematics classes as a result of my study.

Lockhart (2009) lamented over the misconception of what mathematics really is and questioned which mathematics practices are beneficial or possibly even harmful. I have seen first-hand that secondary students can think logically and engage in mathematical discourse about complex mathematics problems. High school mathematics teachers can develop MP3 in their students' by transferring responsibility to students, investing in lesson planning, question posing, and students, and intentionally cultivating a classroom culture of high expectations, safety, and respect.

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APPENDICES

Appendix A: Eight Standards for Mathematical Practice

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4 Model with mathematics.

CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6 Attend to precision.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.

(National Governor's Association Center for Best Practice, Council of Chief State School Officers, 2010)

Appendix B: Observational Protocol

Observer:____________________________Date of Observation:________________

As you observe the lesson, record in Column 3 the events of the students as they relate to the following topics:

1. Students using stated assumptions, definitions, and previously established results to construct arguments.
2. Students making conjectures.
3. Students building a logical progression of statements to explore the truth of their conjectures.
4. Students breaking a situation into cases for analysis.
5. Students recognizing and/or using counterexamples.
6. Students justifying their conclusions.
7. Students communicating their conclusions and justifications to others.
8. Students respond to the arguments of others.
9. Students reason inductively about data, taking context into account.
10. Students compare arguments and distinguish between correct and flawed logic.
11. Students ask questions to clarify or improve arguments.

Provide a time stamp in Column 2 to correspond with the events. If you are a certified mathematics educator then fill out the protocol with regards to items 1-7, all others fill out the protocol with regards to items 1-4. After the lesson, assign an appropriate topic number for the events described in Column 3 (e.g., 1, 3, 7) in Column 4. More than one line can be used to discuss any event(s).

Line:	Time:	Event:	Activity Number(s):
1			
2			
3			
4			
5			
6			
7			
…			

Overall, list things you saw that were successful implementations of MP3:

Overall, list things you saw that appeared to be challenges as the teacher tried to implement MP3:

* This observational protocol is adapted from a University of Missouri Project (Tarr & Austin, 2015). The eleven observable behaviors correspond with the MP3 Definition, parts a-k.

Appendix C: Completed Observational Protocol

Observational Protocol
Part 1 – During Observation

Observer: [redacted] Date of Observation: 12/14/16

As you observe the lesson, record in Column 3 the events of the students as they relate to the following topics:

1. Students using stated assumptions, definitions, and previously established results to construct arguments.
2. Students making conjectures.
3. Students building a logical progression of statements to explore the truth of their conjectures.
4. Students breaking a situation into cases for analysis.
5. Students recognizing and/or using counterexamples.
6. Students justifying their conclusions.
7. Students communicating their conclusions and justifications to others.
8. Students respond to the arguments of others.
9. Students reason inductively about data, taking context into account.
10. Students compare arguments and distinguish between correct and flawed logic.
11. Students ask questions to clarify or improve arguments.

Provide a time stamp in Column 2 to correspond with the events. If you are a certified mathematics educator then fill out the protocol with regards to items 1-7, all others fill out the protocol with regards to items 1-4. After the lesson, assign an appropriate topic number for the events described in Column 3 (e.g., 1, 3, 7) in Column 4. More than one line can be used to discuss any event(s).

Line:	Time:	Event:	Activity Number(s):
1	7:25 7:35	Students presented with triangles & asked to discuss congruency based on	1,2,3,7
2		given criteria for each group.	
3		Students discussed approaches in group based on their group info given.	
4	7:35 - 7:45	Student groups presented approach to class.	4,5,7,8
5	7:35- 7:45	students asked (?) and submitted alternatives.	4,5,7,8,9, 10,11
6			
7			
8			
9			
10			

Line:	Time:	Event:	Activity Number(s):
11			
12	7:45	Students presented with points on graph + asked to determine	1, 2, 3
13		distance.	
14	7:45-50	groups discussed + solved problem.	1, 2, 3, 4, 5, 6
15	7:50	volunteer group presented.	6, 7
16	7:55	instructor asked if the same	6, 3, 4, 9,
17		approach could be used to solve	
18		another distance problem on the graph.	
19	7:55-8	students explained why that	7, 8, 9, 10,
20		approach would not work.	11
21			
22	8:00	Common error with info on	1, 2, 4, 5,
23		graph presented. Students	6, 9
24		were asked how they could	
25		adjust answer without	
26		refiguring problem from start	
27		to finish.	
28	8-8:05	Students discussed as a class.	7, 8, 9, 10
29			

Overall, list things you saw that were successful implementations of MP3:

1. The situations presented were linked to current study and prep for upcoming final.
2. Students were deeply engaged in recall ~~[illegible]~~ and Vocabulary had to be used in justifying answers.
   (vectors, right angle, congruent side, bisecting, mid-point, perpindicular, included angle, leg, hypotenuse)
3. Students asked many questions either challenging logic respectfully or offering alterative explanations.
4. Several students clearly had "a-ha" moments when they realized how to approach a problem.
5. Instructor visited with each group to listen to their thinking before they presented, allowing time to offer suggestions and give students a moment to solidify their approach before presenting to class.

Overall, list things you saw that appeared to be challenges as the teacher tried to

Overall, list things you saw that appeared to be challenges as the teacher tried to implement MP3:

Early morning is a beast + students are slow to engage, but she kept a cheery, encouraging disposition. ~~[illegible]~~ students warmed up quickly.

A couple of off-task convos going on, which were quickly + discreetly addressed by the instructor.

Appendix D: Student Informed Consent Form

Dear Student,

I will be conducting a study in our classroom to investigate the challenges and successes involved in teaching more authentic mathematics with a focus on developing students' ability to construct their own reasonable arguments and critique the mathematical reasoning of others. I am writing to ask permission to use the data I collect from you during this process. Participation in this study involves only regular classroom activities. You may ask me questions at any time about this study. The principal, [redacted], has approved this study.

The purpose of this study is to investigate what challenges arise as I use teaching strategies to achieve deeper conceptual understanding rather than using the traditional teaching approach. The study will take place in my 1st period Geometry classroom and will last for the remainder of the school year. I will teach using research-based strategies showing students how to think, argue, and speak mathematically in an attempt to build their authentic mathematical proficiency. I will occasionally have other teachers observe the class and take notes on their observations. I will keep a journal where I will reflect on my own experiences, challenges, and successes. When your student work highlights the challenges or successes that occur throughout the unit, I would like to use your work as evidence.

Benefits of participating in this study include furthering knowledge about the implementation of teaching practices that foster conceptual understanding in students. I will not include your name in any report about this study. You have the right to ask me not to include your data in the study or to tell me later if you no longer want your data included.

If you agree to let me use your data in the study, please print and sign your name below.

I give permission for my data to be used in this study.

____

Student's Printed Name Student's Signature

____

Date

*This letter is adapted from the template in Hendrick's Action Research Book (2013).

Appendix E: Parent/Guardian Informed Consent Form

Dear Parents,

I will be conducting a study in our classroom to investigate the challenges and successes involved in teaching more authentic mathematics with a focus on developing students' ability to construct their own reasonable arguments and critique the mathematical reasoning of others. I am writing to ask permission to use the data I collect from your child during this process. Participation in this study involves only regular classroom activities. You may contact me at any time regarding your child's participation. My phone number is [redacted] ext. [redacted]. The principal, [redacted], has approved this study.

The purpose of this study is to investigate what challenges arise as I use teaching strategies to achieve deeper conceptual understanding rather than using the traditional teaching approach. The study will take place in my 1st period Geometry classroom and will last for one unit of study (approx. 2-3 weeks). I will teach using research-based strategies showing students how to think, argue, and speak mathematically in an attempt to build their authentic mathematical proficiency. I will occasionally have other teachers observe the class and take notes on their observations. I will keep a journal where I will reflect on my own experiences, challenges, and successes. When student work highlights the challenges or successes that occur throughout the unit, I would like to use their work as evidence.

Benefits of participating in this study include furthering knowledge about the implementation of teaching practices that foster conceptual understanding in students. Your child's participation in this study is strictly confidential. Only my three supervising advisors from Missouri State University and myself will have access to data from the study, to your child's identity, and to information that can be associated with your child's identity.

Use of data from your child is voluntary. You may contact me at any time if you do not wish to have your child's data included in the study.

Please check the appropriate box below and sign the form:

- ☐ I give permission for my child's data to be used in this study. I understand that I will receive a signed copy of this consent form. I have read this form and understand it.
- ☐ I do not give permission for my child's data to be included in this project.

____

Student's Name Signature of Parent/Guardian

____

Date

*This letter is adapted from the template in Hendrick's Action Research Book (2013).

Appendix F: IRB Approval

Missouri State
UNIVERSITY

To:
Kurt Killion
Mathematics
Gay Ragan

Date: Oct 20, 2016 8:18 AM PDT

RE: Notice of IRB Exemption
Study #: IRB-FY2017-286
Study Title: THE CHALLENGES AND SUCCESSES OF IMPLEMENTING ONE OF THE EIGHT STANDARDS FOR MATHEMATICAL PRACTICE IN A SECONDARY MATHEMATICS CLASSROOM

This submission has been reviewed by the Missouri State University Institutional Review Board (IRB) and was determined to be exempt from further review.

Investigator's Responsibilities:

You are required to obtain IRB approval for any changes to any aspect of this study before they can be implemented. Should any adverse event or unanticipated problem involving risks to subjects or others occur it must be reported immediately to the IRB.

This study was reviewed in accordance with federal regulations governing human subjects research, including those found at 45 CFR 46 (Common Rule), 45 CFR 164 (HIPAA), 21 CFR 50 & 56 (FDA), and 40 CFR 26 (EPA), where applicable.

Researchers Associated with this Project:
PI: Kurt Killion
Co-PI: Gay Ragan
Primary Contact: Whitney Evans
Other Investigators: Whitney Evans

Appendix G: Sample Lesson Plan

Lesson 9: December 14, 2014

Summary of the Lesson:

I made a point ahead of time to remind students to focus on making valid mathematical arguments. Before the first group came to the front of the room, I reminded them that our focus was on making reasonable arguments, but also on critiquing the reasoning of others. I told students to listen respectfully and critically to see if they come up with any questions for the groups. This discussion activity took the entire class period.

I started by making sure all students were in groups of 2, 3, or 4. Many students already choose their seats and were naturally sitting in groups of the appropriate number. I had everyone take out a notebook. I assigned each group 1, 2, 3, 4, or 5 and had each group come up with what information would have to be given to conclude that two triangles were congruent by different congruence theorems of postulates. Each group got either SSS, SAS, ASA, HL, or AAS. After the groups discussed for a little while, each group came to the board and showed what information they believed would need to be given and how it might be worded.

We alternated between groups sharing their conjectures and justifications, the teacher and students posing questions, and the presenters and other students in the class answering and responding to those questions. Multiple times, questions arose about congruence theorems or postulates that were different from the one being presented. We discussed whatever relevant questions arose until students were no longer generating questions or mathematical thoughts even when giving appropriate wait time.

When I determined that the majority of students was confident in their understanding of the five congruent theorems and postulates, I presented one final problem to the students. It was a graph with three points labelled, “Baby Dragon Pit”, “Troll Cave”, and “Scout Camp.” They were all on a coordinate plane but the scale was by 5’s and I only labelled 20, 40, etc… I asked the students to figure out how far apart the baby dragon pit was from the troll cave. After small group discussions, one group presented their strategy of using Pythagorean Theorem to the class. Students asked various questions which were addressed by the presenting group of students.

We also discussed the third point on the graph. A student questioned if it was necessary. I used a “Thumps Up/Thumbs Down” strategy to assess if students thought Pythagorean Theorem would be necessary to determine the distance from the scout camp to the baby dragon pit. We discussed some mathematical vocabulary such as “straight” and “vertical.”

Lastly, I told the students I was going to make a mistake. I set up the Pythagorean Theorem to find the distance between the baby dragon and the troll pit, but I did not use the correct scale. Students were asked to find a way to use my incorrect answer to get the correct answer without reworking out the entire problem.

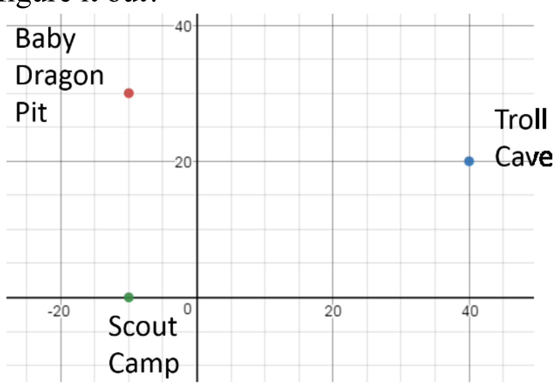
Resources:

<u>Congruent Triangles Diagram</u> (Created by Whitney Evans)
What information would you need to conclude that Triangle HOP and Triangle HEP are congruent by…

[redacted]

SSS? (Group 1)
SAS? (Group 2)
HL? (Group 3)
AAS? (Group 4)
ASA? (Group 5)

<u>Distance Discussion Question</u> (Created by Whitney Evans)
How far apart are the baby dragon pit and the troll cave? What different ways can you figure it out?



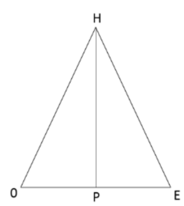
Appendix H: Classroom Journal Sample

12/14/16
Summary of the lesson:

Today I started by making sure all students were in groups of 2, 3, or 4. Many students already choose their seats and were naturally sitting in groups of the appropriate number. I have one girl who prefers to sit by herself. I had her join a group of 2 to make a group of 3. (Felicia with Olivia and Eric) The desks are already placed in groups made for 3 or 4.

I had everyone take out a notebook. I assigned each group 1, 2, 3, 4, or 5 and had each group come up with what information would have to be given to conclude that two triangles were congruent by different congruence theorems of postulates. Each group got either SSS, SAS, ASA, HL, or AAS. The question the students were asked to discuss and prepare to share with the class is shown below.

What information would you need to conclude that Triangle HOP and Triangle HEP are congruent by…



SSS? (Group 1)
SAS? (Group 2)
HL? (Group 3)
AAS? (Group 4)
ASA? (Group 5)

After the groups discussed for a little while, I made each group come to the board and show what information would need to be given and how it might be worded.
(I made a point ahead of time to remind students to focus on making valid mathematical arguments. Before the first group came to the front of the room, I reminded them that our focus was on making reasonable arguments, but also on critiquing the reasoning of others. I told students to listen respectfully and critically to see if they come up with any questions for the groups.)

After the first group shared SSS, I asked them quite a few questions. I alternated between having that group answer and having other students in the room answer.

When the second group came to the front, their answer was incorrect. Thankfully, Hannah was the student doing the talking and writing on the board and she accepted Lee's questioning if she had shown SAS or ASS very well. The group was able to change their answer. I again modeled various questions for them as well as for other groups. I played devil's advocate and asked why it wasn't HL since there was a right angle. Lee very honestly said he actually had no idea. Aaron mentioned that it was because the hypotenuse wasn't marked. I had that group point out the hypotenuses.

The next group that came up was in charge of sharing what information would need to be given to use HL. They successfully gave a way to get HL, but they kept calling the shared side, HP, a perpendicular bisector, but they weren't marking any tick marks. I asked them if the statement that it was a perpendicular bisector was necessary. That group thought it was. Other students in the class also said it was necessary because we needed to have a right angle to use HL. Hannah and Eric were both very adamant about this. Finally, one of the boys in the back group said you didn't need to know it was a bisector. Olivia finally said that if we just knew it was perpendicular that would be good enough. I took it one step farther and asked them what we learned about in Chapter 5 that was the name of a segment in a triangle that went from a vertex and was perpendicular to the opposite side. I had to tell them it started with the letter "A" and Olivia was also able to remember that it could be called an altitude.

Next I had the ASA group go to the front of the room. They showed a picture that was accurately ASA. When they described what information would need to be given to give them their markings, they repeatedly used the word bisect without being able to really tell what was being bisected. They marked the base angles of the triangles congruent (Angle O and Angle E) and they kept saying that the shared side, HP, bisected Angle O and Angle E. I waited to ask questions of my own and let them students ask what was being bisected. The two girls, Rhiannon and Becky, weren't sure how to say what was being bisected. The students said they didn't think it was possible to say something was bisected and get those angles congruent. I asked if they would be congruent if we bisected the entire triangle, and students said they thought so. One student asked if we could bisect the top angle, OHE. We marked it and the students identified that that would give us AAS. Becky and Rhiannon were really struggling to see how it made AAS so I asked for a volunteer to point out the order of angles and sides that would give AAS. Hannah asked to go and showed how she got AAS. Connie remembered that we called sides and angles that are sandwiched between other sides/angles "included" sides so we identified that the shared side, HP, was not included so it couldn't be ASA if we used the right angles, the bisected angles at the top of the triangle, and the shared side. We put Becky and Rhiannon's angles at O and E back and tried to decide if they would help us make ASA. We thought they could if we knew that OE was being bisected or that P was a midpoint.

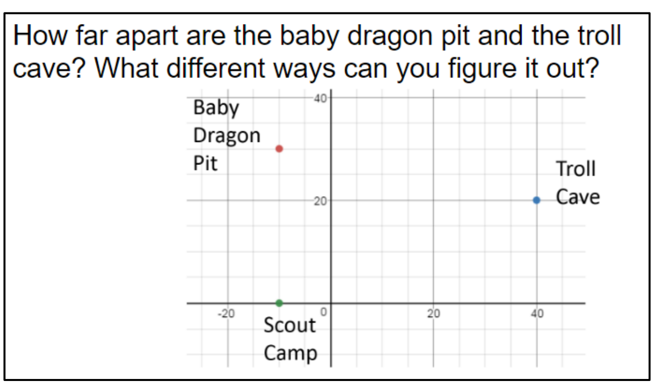
Last, the group of boys in the back went to the front to do AAS. I jokingly minded the class that they had asked a lot of questions so they should try to come up with lots of questions for them too! The atmosphere felt very comfortable because I was able to joke around with them, and they were willing to share their responses without being nervous of what other groups might say. Other students also got into the fun of joking around a little bit, and Hannah made the comment that she would make sure to come up with a question.

After James explained the given information his group had come up with to get AAS, I asked what questions they class had for them. Olivia said she wanted to hear my question, because I had "excellent questions." I told them *they* had excellent questions, and that I wanted to hear their questions first. (I initially forgot to mention that for the last

3 groups, as soon as they finished presenting, I said, “I have a question for them, but I want to hear what questions you guys have for them first.” My hope was that this would set the expectation that there were questions to be asked and that I knew the students were capable of coming up with them without just waiting to hear my thoughts. It worked, because the students did come up with their own questions for all of the last 3 groups.
I was excited when Eric asked the group why they said HP was a perpendicular bisector if they didn’t need tick marks on the bottom sides. I was hoping he would make this connection because the final group had done the same thing Eric and his group had done with HL and called HP a perpendicular bisector when only saying that it was perpendicular to OE was actually necessary. This showed clear evidence that Eric, at least, understood the point of my question and was able to apply what he learned to other students’ arguments. He sort of grinned and Aaron, who was in the group at the front, commented that maybe you didn’t need all that. I asked them what implications Eric’s question had for them. They didn’t respond right away, and finally Aaron said, “What?” So I asked again what implications Eric’s question had for their given information. I also asked if it was necessary to call HP a perpendicular bisector of OE. Olivia, who had been in Eric’s group when this issue came up the first time chimed in and said that you would only have to say that HP was perpendicular to OE. This was evidence that she also understood the point of my earlier question and had learned from being critiqued and was able to apply what she learned to the arguments of other groups.

Next, I skipped a question that was also about Congruent triangles because I felt we had spent enough time messing around with the 5 congruence theorems and postulates to become re-familiarized with them.

I showed the students one final problem. It was a graph with three points labelled, “Baby Dragon Pit”, “Troll Cave”, and “Scout Camp.” They were all on a coordinate plane but the scale was by 5’s and I only labelled 20, 40, etc… I asked the students to figure out how far apart the baby dragon pit was from the troll cave. Below is the actual question.



The back group figured out that they could use Pythagorean Theorem. James and Lee came to the front of the room and showed the whole class how they used Pythagorean Theorem to get the final answer. As they were starting out, they drew in a right triangle and labelled one of the sides “10”. I asked how they knew that distance was 10 rather than 2. Lee pointed to the origin and counted left and right by 5's to show the class that he ended up right at the 20 that was marked. Hannah said, "Isn't the scale different for up and down?" Lee did the same thing. He pointed at the origin and counted up by 5's to clearly show that he landed right at the 20 that was marked when he did so. All the students nodded that they were comfortable labelling that side of the triangle 10 since it covered 2 blocks. They did the same thing and labelled the bottom of the triangle with 50. Hannah said, "Isn't it 55?" I said, "Why don't you count it out and check?" Lee actually used his finger to point and as he got to 50, Hannah agreed that she had simply miscounted. James wrote with a white board marker the steps he took to use Pythagorean Theorem.

Hannah asked why there was a green dot there for the scout camp. I told the students that often in real life there is information we don't actually need so I wanted them to figure out which information was actually useful to solving the question at hand.

I asked them to do a thumb's up for yes and a thumb's down for no for if they thought I would need to use Pythagorean Theorem to find the distance between the scout camp and the baby dragon pit. Every single student put a thumb's down. Most right away, although some may have voted that way because of what they saw the majority of other students doing. I asked a student who said "No" to volunteer to share why. Nicole raised her hand and said it was because the baby dragon pit to the troll cave was slanted, but from the dragon pit to the scout camp was a straight line. I countered with, "but the dragon pit to the troll cave is also a straight line." One student said it was actually that it was diagonal where the dragon pit to scout camp was straight up and down. Paul mentioned the word perpendicular, but I couldn't hear his whole sentence. I asked the students what we call a line that goes straight up and down. They responded with "vertical." At this point I told them I was going to make a mistake and I had one more thing I wanted to show them with it.

I showed them what it would look like if a student didn't notice the different scale and used 2 and 10 for Pythagorean Theorem instead of 10 and 50. I got an answer of 10.189 appx. And asked the students what I could do to get the correct answer if I noticed it wasn't a choice on the final (because all of this is to review for the final) and I didn't want to redo the whole problem. Olivia was the one that noticed that we could just multiply by 5 and it would give us the correct answer. I was tired and struggling to come up with ways of wording a question about why this works, so I told them it was like the proportional reasoning from our last test. If we used 2 and 10, those were 5 times too small so we would have to scale the real length up by 5.

Successes:

1. All students had a chance to come to the front of the room and share an argument that they had come up with in their small groups or partners.
2. All groups were encouraged to ask questions and critique the reasoning of others.
3. James, Lee, Aaron, and Hannah in particular asked quite a few questions. This shows that they were doing the critical thinking and I didn't have to put those questions in their heads, they were starting to come up with them on their own.

4. Eric and Olivia both showed that they learned from the question that I posed to their group about the different between something just being perpendicular vs. being a perpendicular bisector.
5. Various students responded to critiques of their groups' given information. Aaron, Eric, Lee, Becky, and various others all gave responses when asked why they put certain markings on the board.
6. At one point, Becky didn't know how to answer a question. She simply smiled and said, "I don't know." This is a success because she was okay with not knowing the answer and saying so. She later said, "But wouldn't it have to be if…" which showed me that even though she had been unsure about something previously, she was still willing to make statements, share her ideas, and was okay with not always being correct.
7. Hannah went to the board at one point to point out AAS. On her way to the board she said, "I guess I'm not really sure if I know it," but she still showed us what she thought without fear of being wrong.
8. Students were using previous vocabulary to make new arguments.
9. Lee at one asked, "What if HP was a bisector of Angle OHE?" This is evidence that he was manipulating the information around in his head and trying to make sense of it in various ways, not just the way that my instructions required.
10. Here are some of my successes based on how they match with the Common Core's description of MP3:
    a. Students talked about putting tick marks on the shared side, HP, because of the reflexive property. This came up repeatedly which is an example of students using previously established theorems, definitions, or properties to establish their new arguments.
    b. All groups had to make conjectures various times today.
    c. When students put information on the board to support what given information would let them use their congruent theorem or postulate, every single group (there were 5) listed the information in a particular order as they marked their picture. Not one group just put all the markings on and didn't say why in a logical progression.
    d. When Lee asked his "What if" question, we broke the triangle up into cases and he considered what different congruence theorems or postulates would apply to the case his question created. Since he asked the whole class, everyone was able to take part in considering what possibilities would open up with his addition of one more given piece of information.
    e. When groups were asked questions, not a single person said, "pass." The only exception might be the one time when Becky said that she didn't know. She had responded to previous questions already though and just this one topic had her unsure. This tells me that students were justifying their arguments.

f. Students were sharing with the whole class and also discussing in their small groups so they were sharing their arguments and explanations/justifications with many others.
g. Students asked questions and responded to questions from classmates and from me.

Challenges:

1. Izzy was in the hall taking a test she had missed from the day before. When a student misses a discussion day such as this one, it's impossible to replicated.
2. It was difficult to plan for this discussion. I had prepared the slides with the problems, and I knew I wanted to put students in groups, but I didn't know what they would come up with or what direction each problem would take. I felt confident that our discussions today were valuable and resulted in growth and learning. However, at the end of class, I commented to Amber Hainline who came to observe me that I felt exhausted. I actually changed the lesson to include less discussion for 3rd period not because I thought it was best for kids not to spend so much time discussing, but because I was so mentally exhausted from trying to consider all students' understanding, motivating students to share, question, and feel safe opening up about their mathematical ideas. It was difficult to word questions and statements in a way that I didn't give away answers and encouraged students to not only share what their individual thoughts were, but to also respectfully tell another group that they might be doing something incorrect. I was constantly trying to be away of the atmosphere of the classroom. When Becky and Rhiannon were posed a question that they didn't know how to answer, I right away said, "Can we all agree that they have a perfectly marked example of ASA?" to give them confidence and show them and the whole class that they were intelligent and had good ideas even if they didn't know the answer to this one question. Having lessons like this every day would not be sustainable because of the mental strain unless students became so used to this type of classroom experience that they started asking each other questions and doing some of the things that I really had to put forth effort to encourage and motivate them to do all on their own.
3. In looking at my reflection for today, a few students responded over and over, and other students got away with saying little to nothing. For example, Felicia never said a single thing. Paul didn't say very much, but he did interact with me when I discussed how his group was going to share SSS with the large group so I know he was still involved in thinking about the problems. He also took the time to clarify if I wanted the distance straight from the dragon pit to the troll's cave or if I wanted to stay on vertical and horizontal lines.
4. I didn't do any sort of writing component at the end to allow all students to reflect on what they had learned from our large group discussion.
5. I didn't give students any sort of formative assessment to see what they actually took away from today's discussion. I know some students have strong or improved understanding from before, but there are other students that I'm sure

shook their head to make it look like they understood completely, but really didn't.

Implications:

1. Students need to be exposed to this type of discussion situation often enough to start to exhibit the behaviors of good listening, asking questions, critiquing the reasoning of others, and volunteering viable mathematical arguments all on their own. This will mean they are thinking more deeply and more independently and will not be so straining on the teacher since she won't have to focus so much on all these aspects but can focus more on students that are not engaged, students that don't understand, or the more specific misunderstanding of the whole class or small groups.
2. Discussion is a valuable tool, but it doesn't always impact all students. Some sort of writing or debriefing assignment or formative assessment would have made the lesson better.

Appendix I: Data Organization Spreadsheet

The following screenshots were taken from the Excel spreadsheet I used to organize my data into themes by date.

Teacher Strategies that Lead to Successful Student Engagement in MP3					
Transfer of responsibility from teacher to student - during the lesson					
Students generate their own questions/hypothesize about possible answers				1-Dec	2-Dec
Students are viewed as the mathematical authority		28-Nov	29-Nov	1-Dec	
Students shared their own conjectures (in writing or during discussions with small and large groups)	16-Nov	28-Nov	29-Nov	1-Dec	2-Dec
Students questioned each other	16-Nov				2-Dec
Students corrected each other	16-Nov				2-Dec
Students rephrased each other			29-Nov		
Students self-critiqued (rather than the teacher pointing out their mistakes)			29-Nov		
Students generated multiple ways to represent the same thing	16-Nov	28-Nov	29-Nov	1-Dec	2-Dec

Not within zone of proximal development			
Too easy with a clear right answer		28-Nov	29-Nov
Too difficult and students were clueless	16-Nov	28-Nov	

Implications					
Use writing in math!!!					
Write	16-Nov	28-Nov	29-Nov	1-Dec	
error analysis is good			29-Nov		
Classroom culture					
sense of community/safety to share/okay to make mistakes	16-Nov				
make teacher expectations known					2-Dec
Emphasize and value MP3		28-Nov	29-Nov	1-Dec	2-Dec
Intentional transfer of responsibility to students					
make kids do the rephrasing		28-Nov	29-Nov		
Include some individual accountability component	16-Nov		29-Nov	1-Dec	