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Original Article

Intuitionistic λ-Regular closed sets

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Abstract

In this paper, the interrelations among intuitionistic open set, intuitionistic regular open set, intuitionistic β open set and intuitionistic semi open set are studied with the necessary counter examples. Also a new class of intuitionistic closed sets namely intuitionistic λ-closed set, intuitionitic λ-regular closed set, intuitionistic λ-β closed set and intuitionistic λ-semi closed set are introduced in intuitionistic topological spaces and investigated their properties.

Keywords:

Intuitionistic regular open set, intuitionistic β open set, intuitionistic semi open set, intuitionistic λ-closed set, intuitionitic λ-regular closed set, intuitionistic λ-β closed set and intuitionistic λ-semi closed set

2000 Mathematics Subject Classification 54A40, 03E72

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [10] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Krassimir T. Atanassov [1] and many works by the same author and his colleagues appeared in the literature [2, 3, 4]. Later topological structures in fuzzy topological spaces [5] is generalized to "intuitionistic fuzzy topological spaces" by coker in [6], and then the concept "intuitionistic set" is introduced in Coker [7]. This is a discrete form of intuitionistic fuzzy set, where all the sets are entirely the crisp sets. Still it has membership and non-membership degrees, so this concept gives us more flexible approaches to representing vagueness in mathematical objects including engineering fields with classical set logic. In 2000, Coker [8] also introduced the concept of intuitionistic topological spaces with intuitionistic sets, and investigated basic properties of continuous functions and compactness. λ-Closed sets in intuitionistic fuzzy topological spaces was introduced by Rajarajeshwari and Bagyalakshmi[9]. In this paper, a new class of intuitionistic closed sets namely intuitionistic λ-closed set, intuitionitic λ-regular closed set, intuitionistic λ-β closed set and intuitionistic λ-semi closed set are introduced in intuitionistic topological spaces and investigated their properties.

2. Preliminaries

Definition 2.1[8] Let X be a nonempty fixed set. An intuitionistic set (IS for short) A is an object having the form A = ⟨ X, A^1, A^2 ⟩, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of members of A, while A^2 is called the set of non-members of A.

Definition 2.2[8] Let X be a nonempty set, A = ⟨ X, A^1, A^2 ⟩ and B = ⟨ X, B^1, B^2 ⟩ be intuitionistic sets on X, and let { A_i : i ϵ J } be an arbitrary family of intuitionistic sets in X, where A^i = ⟨ X, A_i^1, A_i^2 ⟩.

(1) A ⊆ B iff $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$;
(2) A = B iff A ⊆ B and B ⊆ A;
(3) $\bar{A}$ = ⟨ X, A^2, A^1 ⟩;
(4) $\cup A_i$ = ⟨ X, $\cup A_i^1$, $\cap A_i^2$ ⟩;
(5) $\cap A_i$ = ⟨ X, $\cap A_i^1$, $\cup A_i^2$ ⟩;
(6) [ ]A = ⟨ X, A^1, $(A^1)^c$ ⟩;
(7) < >A = ⟨ X, $(A^2)^c$, A^2 ⟩;
(8) $\emptyset_\sim$ = ⟨ X, ∅, X ⟩;
(9) $X_\sim$ = ⟨ X, X, ∅ ⟩.

Corollary 2.1 [8] Let A, B, C and A_i be intuitionistic sets in X (i ϵ J). Then,

(1) $A_i \subseteq$ B for each i ϵ J ⇒ $\cup A_i \subseteq$ B;
(2) B ⊆ A_i for each i ϵ J ⇒ B ⊆ $\cap A_i$;
(3) $\overline{\cup A_i} = \overline{\cap A_i}$, $\overline{\cap A_i} = \overline{\cup A_i}$;
(4) A ⊆ B ⟺ $\bar{B} \subseteq \bar{A}$;
(5) $\overline{\overline{(A)}}$ = A;
(6) $\overline{\emptyset_\sim X_\sim}$, $\overline{X_\sim}$ = $\emptyset_\sim$.

Definition 2.4 [8] An intuitionistic topology (IT for short) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(T_1) $\emptyset_\sim = X_\sim \in \tau$;
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic topological space (ITS for short) and any IS in τ is known as an intuitionistic open set (IOS for short) in X.

Definition 2.5 [8] The complement $\bar{A}$ of an IOS A in an ITS (X, T) is called an intuitionistic closed set (ICS for short) in X.

Proposition 2.1 [8] Let (X, τ) be an ITS on X. Then, we can also construct several ITSs on X in the following way:

(1) $\tau_{0,1}$ = { [ ] G : G ϵ τ } ;
(2) $\tau_{0,2}$ = { < > G : G ϵ τ }.

Definition 2.6 [8] Let (X, τ) be an ITS and A = ⟨ X, A^1, A^2 ⟩ be an IS in X. Then the interior and closure of A are defined by

cl(A) = ⋂{ K : K is an ICS in X and A ⊆ K } ;
int(A) = ⋃{ G : G is an IOS in X and G ⊆ A }.

It can be also shown that cl(A) is an ICS and int(A) is an IOS in X, and A is an ICS in X iff cl(A) = A; and A is an IOS in X iff int(A) = A.

Proposition 2.2 [8] For any IFS A in (X, τ), we have cl($\bar{A}$) = $\overline{\text{int(A)}}$, int($\bar{A}$) = $\overline{cl(A)}$.

Proposition 2.3 [8] Let (X, τ) be an ITS and A, B be ISs in X. Then the following properties hold:

(1) int(A) ⊆ A;
(2) A ⊆ B ⇒ int(A) ⊆ int(B);
(3) int(int(A)) = int(A);
(4) int(A⋂B) = int(A)⋂int(B);
(5) int($X_\sim$) = $X_\sim$;
(6) A ⊆ cl(A);
(7) A ⊆ B ⇒ cl(A) ⊆ cl(B);
(8) cl(cl(A)) = cl(A);
(9) cl(A⋃B) = cl(A)⋃cl(B);
(10) cl($\emptyset_\sim$) = $\emptyset_\sim$

3. Interrelations of some types of intuitionistic open sets

In this section, the interrelations among intuitionistic open set, intuitionistic regular open set, intuitionistic β open set and intuitionistic semi open set are studied with the necessary counter examples.

Definition 3.1 Let (X, τ) be an intuitionistic topological space and A ⊂ X. Then,

(1) A is an intuitionistic regular open iff A = int(cl(A)).
(2) A is an intuitionistic regular closed iff A = cl(int(A)).

Definition 3.2 Let (X, τ) be an intuitionistic topological space and A ⊂ X. Then,

(1) A is an intuitionistic open iff A = int(A).
(2) A is an intuitionistic closed iff A = cl(A).

Definition 3.3 Let (X, τ) be an intuitionistic topological space and A ⊂ X. Then,

(1) A is an intuitionistic β open iff A ⊆ cl(int(cl(A))).
(2) A is an intuitionistic β closed iff A ⊇ int(cl(int(A))).

Definition 3.4 Let (X, τ) be an intuitionistic topological space and A ⊂ X. Then,

(1) A is an intuitionistic semi open iff A ⊆ cl(int(A)).
(2) A is an intuitionistic semi closed iff A ⊇ int(cl(A)).

Proposition 3.1 Let (X, T) be an intuitionistic topological space. Then every intuitionistic regular open set is an intuitionistic open.

Remark 3.1 The converse of the above Proposition 3.1 need not be true as shown in the Example 3.1.

Example 3.1

Let X = {a, b, c} and
U_1 = ⟨ X, {c}, {a, b} ⟩;
U_2 = ⟨ X, {c}, {a} ⟩;
U_3 = ⟨ X, {a}, {c} ⟩;
U_4 = ⟨ X, {a, c}, ∅ ⟩;
U_5 = ⟨ X, ∅, {a, c} ⟩.

Then the family T = { $\emptyset_\sim, X_\sim$, U_1, U_2, U_3, U_4, U_5 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.
For an intuitionistic open set U_1 = ⟨ X, {c}, {a, b} ⟩,
Then, int(cl(U_1)) = ⟨ X, {c}, {a} ⟩.

$U_1 \neq$ int(cl(U_1))

Hence, U_1 is not an intuitionistic regular open set.

Therefore, every intuitionistic open set need not be an intuitionistic regular open.

Proposition 3.2 Let (X, T) be an intuitionistic topological space. Then every intuitionistic regular open set is an intuitionistic β open.

Remark 3.2 The converse of the above Proposition 3.2 need not be true as shown in the Example 3.2.

Example 3.2

Let X = {a, b, c, d, e} and
U_1 = ⟨ X, {a, b, c}, {d, e} ⟩;
U_2 = ⟨ X, {e}, {a, b, c, d} ⟩;
U_3 = ⟨ X, {a, b, c, e}, {d}.

Then the family T = { $\emptyset_\sim, X_\sim$, U_1, U_2, U_3 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let B = ⟨ X, {a, b}, {d, e} ⟩ be an intuitionistic set in X.
Then, cl(int(cl(B))) = ⟨ X, {a, b, c, d}, {e} ⟩.
Therefore, B ⊆ cl(int(cl(B)))
Hence, B is an intuitionistic β open set.
But B ≠ int(cl(B)).
Therefore, B is not an intuitionistic regular open set.

Therefore, every intuitionistic β open set need not be an intuitionistic regular open.

Proposition 3.3 Let (X, T) be an intuitionistic topological space. Then every intuitionistic open set is an intuitionistic β open.

Remark 3.3 The converse of the above Proposition 3.3 need not be true as shown in the Example 3.3.

Example 3.3

Let X = {a, b, c, d} and
U_1 = ⟨ X, {a, b}, {c, d} ⟩;
U_2 = ⟨ X, {c, d}, {a, b} ⟩.

Then the family T = { $\emptyset_\sim, X_\sim, U_1, U_2$ } is an intuitionistic topology on X. clearly the ordered pair (X, T) is an intuitionistic topological space.

Let B = ⟨ X, {a}, {c, d} ⟩ be an intuitionistic set in X.
Then, cl(int(cl(B))) = ⟨ X, {a, b}, {c, d} ⟩.
Therefore, B ⊆ cl(int(cl(B)))

Thus, B is an intuitionistic β open set. But B is not an intuitionistic open set.

Therefore, every intuitionistic β open set need not be an intuitionistic open.

Proposition 3.4 Let (X, T) be an intuitionistic topological space. Then every intuitionistic open set is an intuitionistic semi open.

Remark 3.4 The converse of the above Proposition 3.4 need not be true as shown in the Example 3.4.

Example 3.4

Let X = {a, b, c, d} and
U_1 = ⟨ X, {b}, {a, c} ⟩;
U_2 = ⟨ X, {c}, {a, b} ⟩;
U_3 = ⟨ X, {b, c}, {a} ⟩;
U_4 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim, X_\sim, U_1, U_2, U_3, U_4$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let B = ⟨ X, {a, b}, {c} ⟩ be an intuitionistic set in X.
Then, cl(int(B)) = ⟨ X, {a, b}, {c} ⟩.
Therefore, B ⊆ cl(int(B)).

Thus, B is an intuitionistic semi open set. But B is not an intuitionistic open set.

Therefore, every intuitionistic semi open set need not be an intuitionistic open.

Proposition 3.5 Let (X, T) be an intuitionistic topological space. Then every intuitionistic semi open set is an intuitionistic β open.

Remark 3.5 The converse of the above Proposition 3.5 need not be true as shown in the Example 3.5.

Example 3.5

Let X = {a, b, c} and
U_1 = ⟨ X, {a}, {b} ⟩;
U_2 = ⟨ X, {a, c}, {b} ⟩.

Then the family T = { $\emptyset_\sim, X_\sim, U_1, U_2$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

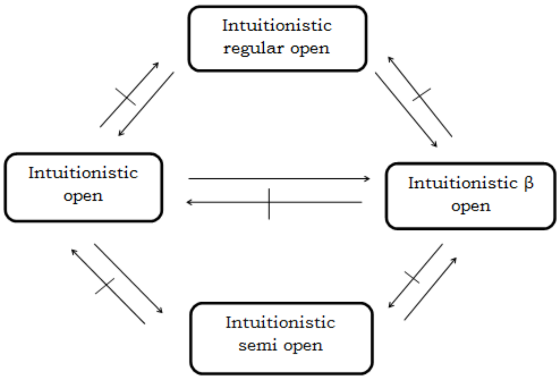
Let B = ⟨ X, {c}, {∅} ⟩ be an intuitionistic set in X.
Then, cl(int(cl(B))) = ⟨ X, X, ∅ ⟩.
Therefore, B ⊆ cl(int(cl(B)))

Thus, B is an intuitionistic β open set. But B is not an intuitionistic semi open set.

Therefore, every intuitionistic β open set need not be an intuitionistic semi open.

Remark 3.6

From the above discussions the following implications hold.



4. Intuitionistic λ- Regular closed sets

In this section, the interrelations among intuitionistic λ-regular closed sets with other types of intuitionistic closed sets are discussed with the suitable counter examples.

Definition 4.1 An intuitionistic set A of an intuitionistic topological space (X, T) is called intuitionistic λ-closed if cl(U) ⊆ A whenever U ⊆ A and U is intuitionistic open in X.

Definition 4.2 An intuitionistic set A of an intuitionistic topological space (X, T) is called intuitionistic λ-regular closed if cl(U) ⊆ A whenever U ⊆ A and U is intuitionistic regular open in X.

Definition 4.3 An intuitionistic set A of an intuitionistic topological space (X, T) is called intuitionistic λ-β closed if cl(U) ⊆ A whenever U ⊆ A and U is intuitionistic β open in X.

Definition 4.4 An intuitionistic set A of an intuitionistic topological space (X, T) is called intuitionistic λ-semi closed if cl(U) ⊆ A whenever U ⊆ A and U is intuitionistic semi open in X.

Proposition 4.1 Let (X, T) be an intuitionistic topological space. Then every intuitionistic λ-regular closed set is an intuitionistic λ-closed.

Remark 4.1 The converse of the above Proposition 4.1 need not be true as shown in the Example 4.1.

Example 4.1

Let X = { a, b, c, d, e } and
U_1 = ⟨ X, {b}, {a, c} ⟩;
U_2 = ⟨ X, {c}, {b} ⟩;
U_3 = ⟨ X, {b, c}, ∅ ⟩;
U_4 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1, U_2, U_3, U_4 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {b, c, d}, ∅ ⟩ be an intuitionistic set in X and A is an intuitionistic λ-closed set. But, A is not an intuitionistic λ-regular closed set.

Therefore, every intuitionistic λ-closed set need not be an intuitionistic λ-regular closed.

Proposition 4.2 Let (X, T) be an intuitionistic topological space. Then, every intuitionistic λ-regular closed set is an intuitionistic λ-β closed.

Remark 4.2 The converse of the above Proposition 4.2 need not be true as shown in the Example 4.2.

Example 4.2

Let X = { a, b, c, d } and
U_1 = ⟨ X, {a}, {b, c} ⟩;
U_2 = ⟨ X, {c}, {a} ⟩;
U_3 = ⟨ X, {a, c}, ∅ ⟩;
U_4 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1, U_2, U_3, U_4 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a, c, d}, ∅ ⟩ be an intuitionistic set in X and A is an intuitionistic λ-β closed set. But, A is not an intuitionistic λ-regular closed set.

Therefore, every intuitionistic λ-β closed set need not be an intuitionistic λ-regular closed.

Proposition 4.3 Let (X, T) be an intuitionistic topological space. Then every intuitionistic λ-closed set is an intuitionistic λ-β closed.

Remark 4.3 The converse of the above Proposition 4.3 need not be true as shown in the Example 4.3.

Example 4.3

Let X = { a, b, c, d, e } and
U_1 = ⟨ X, {a}, {b, c} ⟩;
U_2 = ⟨ X, {b}, {a, c} ⟩;
U_3 = ⟨ X, {a, b}, {c} ⟩;
U_4 = ⟨ X, {b, c}, {a} ⟩;
U_5 = ⟨ X, {a, b, c}, ∅ ⟩;
U_6 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1, U_2, U_3, U_4, U_5, U_6 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a, b, d}, {c} ⟩ be an intuitionistic set in X and A is an intuitionistic λ-β closed set. But, A is not an intuitionistic λ-closed set.

Therefore, every intuitionistic λ-β closed set need not be an intuitionistic λ-closed.

Proposition 4.4 Let (X, T) be an intuitionistic topological space. Then every intuitionistic λ- closed set is an intuitionistic λ- semi closed.

Remark 4.4 The converse of the above Proposition 4.4 need not be true as shown in the Example 4.4.

Example 4.4

Let X = { a, b, c, d, e } and
U_1 = ⟨ X, {b}, {a, c} ⟩;
U_2 = ⟨ X, {c}, {b} ⟩;
U_3 = ⟨ X, {b, c}, ∅ ⟩;
U_4 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1 , U_2 , U_3, U_4 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {b, c, d}, ∅ ⟩ be an intuitionistic set in X and A is an intuitionistic λ-semi closed set. But, A is not an intuitionistic λ-closed set.

Therefore, every intuitionistic λ-semi closed set need not be an intuitionistic λ-closed.

Proposition 4.5 Let (X, T) be an intuitionistic topological space. Then every intuitionistic λ-semi closed set is an intuitionistic λ-β closed.

Remark 4.5 The converse of the above Proposition 4.5 need not be true as shown in the Example 4.5.

Example 4.5

Let X = { a, b, c, d, e } and
U_1 = ⟨ X, {a}, {b, c} ⟩;
U_2 = ⟨ X, {b}, {a, c} ⟩;
U_3 = ⟨ X, {a, b}, {c} ⟩;
U_4 = ⟨ X, {b, c}, {a} ⟩;
U_5 = ⟨ X, {a, b, c}, ∅ ⟩;
U_6 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1, U_2, U_3, U_4, U_5, U_6 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a, b}, ∅ ⟩ be an intuitionistic set in X and A is an intuitionistic λ-β closed set. But, A is not an intuitionistic λ-semi closed set.

Therefore, every intuitionistic λ-β closed set need not be an intuitionistic λ- semi closed.

Proposition 4.6 Let (X, T) be an intuitionistic topological space. Then every intuitionistic closed set is an intuitionistic λ- closed.

Remark 4.6 The converse of the above Proposition 4.6 need not be true as shown in the Example 4.6.

Example 4.6

Let X = { a, b, c} and
U_1 = ⟨ X, {a}, {b, c} ⟩;
U_2 = ⟨ X, {b, c}, {a} ⟩.

Then the family T = { $\emptyset_\sim$, $X_\sim$, U_1, U_2 } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a}, {c} ⟩ be an intuitionistic set in X and A is an intuitionistic λ-closed set. But, A is not an intuitionistic closed set.

Therefore, every intuitionistic closed set need not be an intuitionistic λ-closed.

Proposition 4.7 Let (X, T) be an intuitionistic topological space. Then every intuitionistic closed set is an intuitionistic λ- regular closed.

Remark 4.7 The converse of the above Proposition 4.7 need not be true as shown in the Example 4.7.

Example 4.7

Let X = {a, b, c} and
U_1 = ⟨ X, {a}, {b, c} ⟩;
U_2 = ⟨ X, {b, c}, {a} ⟩.

Then the family T = { $\emptyset_{\sim}, X_{\sim}, U_1, U_2$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a, b}, {c} ⟩ be an intuitionistic set in X and A is an intuitionistic λ-regular closed set. But A is not an intuitionistic closed set.

Therefore, every intuitionistic λ-regular closed set need not be an intuitionistic closed.

Proposition 4.8 Let (X, T) be an intuitionistic topological space. Then every intuitionistic closed set is an intuitionistic λ- semi closed.

Remark 4.8 The converse of the above Proposition 4.8 need not be true as shown in the Example 4.8.

Example 4.8

Let X = { a, b, c, d} and

U_1 = ⟨ X, {a}, {b, c} ⟩;

U_2 = ⟨ X, {c}, {a, b} ⟩;

U_3 = ⟨ X, {a, c}, {b} ⟩;

U_4 = ⟨ X, ∅, {a, b, c} ⟩.

Then the family T = { $\emptyset_{\sim}, X_{\sim}, U_1, U_2, U_3, U_4$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a, b, d}, {c} ⟩ be an intuitionistic set in X and A is an intuitionistic λ-semi closed set. But, A is not an intuitionistic closed set.

Therefore, every intuitionistic closed set need not be an intuitionistic λ- semi closed.

Proposition 4.9 Let (X, T) be an intuitionistic topological space. If A and B are two intuitionistic λ-closed sets in an intuitionistic topological space (X, T), then A ∩ B is an intuitionistic λ-closed set.

Remark 4.9 The converse of the above Proposition 4.9 need not be true as shown in the Example 4.9.

Example 4.9

Let X = { a, b, c, d, e } and

U_1 = ⟨ X, {d, c}, ∅ ⟩;

U_2 = ⟨ X, {a, d}, ∅ ⟩;

U_3 = ⟨ X, {a, d, c}, ∅ ⟩;

U_4 = ⟨ X, {d}, ∅ ⟩;

U_6 = ⟨ X, ∅, {d} ⟩.

Then the family T = { $\emptyset_{\sim}, X_{\sim}, U_1, U_2, U_3, U_4, U_5, U_6$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {b, c, d}, ∅ ⟩ and B = ⟨ X, {a, d, e}, ∅ ⟩ be an intuitionistic sets in X. Then A⋂B are intuitionistic λ-closed set. But A and B are not intuitionistic λ-closed sets.

Therefore intersection of two intuitionistic sets is λ-closed set but the two sets may not be an intuitionistic λ-closed.

Remark 4.10 The union of two intuitionistic λ-closed sets in an intuitionistic topological space (X, T) may not be an intuitionistic λ-closed set as seen in the following example.

Example 4.10

Let X = { a, b, c, d, e } and

U_1 = ⟨ X, {a}, {d} ⟩;

U_2 = ⟨ X, {d}, {a} ⟩;

U_3 = ⟨ X, {d, c}, ∅ ⟩;

U_4 = ⟨ X, {a, d}, ∅ ⟩;

U_5 = ⟨ X, ∅, {a, d} ⟩;

U_6 = ⟨ X, ∅, {d} ⟩;

U_7 = ⟨ X, {a, d, c}, ∅ ⟩;

U_8 = ⟨ X, {d}, ∅ ⟩.

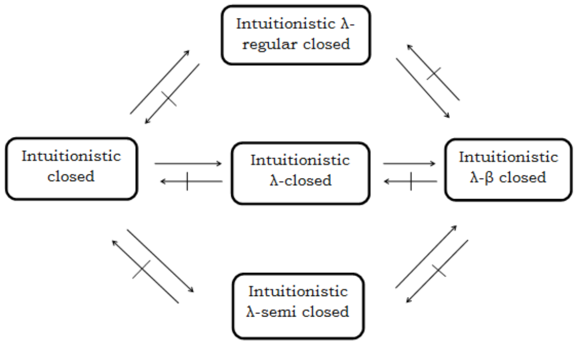
Then the family T = { $\emptyset_{\sim}, X_{\sim}, U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8$ } is an intuitionistic topology on X. Clearly, the ordered pair (X, T) is an intuitionistic topological space.

Let A = ⟨ X, {a}, ∅ ⟩ and B = ⟨ X, {c, d, e}, ∅ ⟩ be an intuitionistic sets in X. Then A and B are intuitionistic λ-closed sets. But A∪B is not an intuitionistic λ-closed set.

Therefore union of two intuitionistic λ-closed sets may not be an intuitionistic λ-closed.

Remark 4.11

From the above discussions the following implications hold.



Proposition 4.10 Let (X, T) be an intuitionistic topological space. A is an intuitionistic λ-open set in X iff A ⊆ int(B) whenever A ⊇ B and B is an intuitionistic closed set in X.

Proof

Assume that A is an intuitionistic λ-open set in X. Then A^c is an intuitionistic λ-closed set in X. An intuitionistic set A is an intuitionistic λ-open set in X if A ⊇ int(B) whenever A⊇B and B is an intuitionistic closed set in X. Let B be an intuitionistic closed set in X such that A⊇B which implies $A^c \supseteq B^c$. Since A^c is an intuitionistic λ-closed and B^c is an intuitionistic open, $A^c \supseteq$ cl(B^c). But cl(B^c) = $[int\,(B)]^c$. And so $A^c \supseteq [int(B)]^c \Rightarrow$ A ⊆ int (B).

Conversely assume that A ⊆ int(B) where A ⊆ B and B is an intuitionistic closed in X. Let C be an intuitionistic open set such that $A^c \supseteq$ C which implies A⊆C^c then, A ⊆ int(C^c). But int(C^c) = $[cl(C)]^c$ Therefore, A⊆cl$(C)^c$. Then $A^c \supseteq$ cl(C). Therefore A^c is an intuitionistic λ-closed set. Therefore A is an intuitionistic λ-open set.

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