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Original Article

On r-Fuzzy λ-Closed sets and smooth Fuzzy λ-Continuous functions

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Abstract

This paper deals with the interrelations of r-fuzzy λ-closed sets with other types of closed sets with the suitable counter examples. Also, the interrelations of smooth fuzzy λ-continuous functions with other types of smooth fuzzy continuous functions are established with the necessary counter examples. Finally, the properties and the characterizations of smooth fuzzy λ-compact spaces are discussed.

Keywords:
r-fuzzy λ-semi-closed sets, r-fuzzy λ-b-closed sets, smooth fuzzy λ-continuous functions, smooth fuzzy λ-semi-continuous functions, smooth fuzzy λ-b-continuous functions and smooth fuzzy λ-compact spaces
2000 Mathematics Subject Classification: 54A40, 03E72.

1. Introduction and Preliminaries

The concept of fuzzy set was introduced by Zadeh [18] in his classical paper. Fuzzy sets have applications in many fields such as information [13] and control [17]. Chang [7] introduced the notion of afuzzy topology. Later Lowen [10] redefined what is now known as stratified fuzzy topology. Sostak [14] introduced the notion of fuzzy topology as an extension of Chang and Lowen's fuzzy topology. Later on he has developed the theory of fuzzy topological spaces in [15] and [16]. In 1992, he introduced and studied the smooth fuzzy topology. Rajarajeswari and Bagyalakshmi [12] studied the concept of intuitionistic fuzzy λ-closed sets.

Throughout this dissertation, let X be a nonempty set, $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$.

A fuzzy set in X is an element of the set I^X of all functions from X to I.[7]

Definition 1.1 [14] A function $T: I^X \to I$ is called a **smooth fuzzy topology** on X if it satisfies the following conditions:

(1) $T(\overline{0}) = T(\overline{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$.

(3) $T(\vee_{j \in \Gamma} \mu_j) \geq \wedge_{j \in \Gamma} T(\mu_j)$ for any $\{\mu_j\}_{j \in \Gamma} \in I^X$.

The pair (X, T) is called a **smooth fuzzy topological space.**

Definition 1.2 [1] Let (X, T) be a smooth fuzzy topological space. For any $\lambda \in I^X$ and $r \in I_0$,

(1) λ is said to be an r-fuzzy semi-open set iff $\lambda \leq C_T(I_T(\lambda, r), r)$.

(2) λ is said to be an r-fuzzy semi-closed set iff $\overline{1} - \lambda$ is an r-fuzzy semi-open set.

Definition 1.3 [3] Let (X, T) be a smooth fuzzy topological space. For any $\lambda \in I^X$ and $r \in I_0$,

(1) λ is said to be an r-fuzzy b-open set iff $\lambda \leq C_T(I_T(\lambda, r), r) \vee I_T(C_T(\lambda, r), r)$.

(2) λ is said to be an r-fuzzy b-closed set iff $\overline{1} - \lambda$ is an r-fuzzy b-open set.

Definition 1.4 [12] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an

(1) intuitionistic fuzzy λ-closed set (IF λ-CS) if $A \supseteq cl(U)$ whenever $A \supseteq U$ and U is an intuitionistic fuzzy open set in X.

(2) intuitionistic fuzzy λ-open set (IF λ-OS) if the complement A^C is an intuitionistic fuzzy λ-closed set A.

Definition 1.5 [4] A smooth $\tilde{g}$-P-fuzzy topological space (X, $\tilde{g}$-P(T)) is called $\tilde{g}$-P-fuzzy almost compact if for any $\tilde{g}$-P(T) covering of (X, $\tilde{g}$-P(T)), there exists a finite subset J_0 of J such that $\vee_{j \in J_0} \tilde{g}\text{-}C_{R(T)}(\lambda_i, r) = \overline{1}$, $r \in I_0$.

DEFINITION 1.6 [4] A smooth $\tilde{g}$-P-fuzzy topological space (X, $\tilde{g}$-P(T)) is called $\tilde{g}$-P-fuzzy nearly compact if for any $\tilde{g}$-P(T) covering of (X, $\tilde{g}$-P(T)), there exists a finite subset J_0 of J such that $\vee_{j \in J_0} \tilde{g}\text{-}I_{P(T)}(\tilde{g}\text{-}C_{R(T)}(\lambda_i, r), r) = \overline{1}$, $r \in I_0$.

Definition 1.7 [5] A smooth fuzzy topological space (X, T) is called **fuzzy $\tilde{g}$-$T_{1/2}$** if every r-gf$\tilde{g}$-closed setis r-fuzzy $\tilde{g}$-closed.

2. On r-Fuzzy λ-Closed sets

This section deals with the interrelations of r-fuzzy λ-closed sets, r-fuzzy λ-semi-closed sets and r-fuzzy λ-b-closed sets with necessary counter examples.

Definition 2.1 Let (X, T) be a smooth fuzzy topological space. For any $\mu \in I^X$ and $r \in I_0$, μ is said to be an **r-fuzzy λ-closed set** (briefly, rfλ-cls) if $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is an r-fuzzy open set. Its complement is said to be an **r-fuzzy λ-open set.**

Definition 2.2 Let (X, T) be a smooth fuzzy topological space. For any $\mu \in I^X$ and $r \in I_0$, μ is said to be an **r-fuzzy λ-semi-closed set** (briefly, rfλ-s-cls) if $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is an r-fuzzy semi-open set. Its complement is said to be an **r-fuzzy λ-semi-open set.**

Definition 2.3 Let (X, T) be a smooth fuzzy topological space. For any $\mu \in I^X$ and $r \in I_0$, μ is said to be an **r-fuzzy λ-b-closed set** (briefly, rfλ-b-cls) if $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is an r-fuzzy b-open set. Its complement is said to be an **r-fuzzy λ-b-open set.**

Proposition 2.1
Every r-fuzzy λ-closed set is r-fuzzy λ-semi-closed.

Proof: Let (X, T) be a smooth fuzzy topological space. For $\mu \in I^X$ and $r \in I_0$, let μ be an r-fuzzy λ-closed set. Then, $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is r-fuzzy open. By Proposition 2.1, every r-fuzzy open set is r-fuzzy semi-open. Thus for any r-fuzzy semi-open setγ, μ is r-fuzzy λ-semi-closed. Therefore, **every r-fuzzy λ-closed set is r-fuzzy λ-semi-closed.**

Remark 2.1

The converse of the Proposition 2.1 need not be true.

Example 2.1

Every r-fuzzy λ-semi-closed set need not be r-fuzzy λ-closed.

Let X = { a, b } and let $\lambda_1, \lambda_2 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.2, \lambda_1(b) = 0.4;$

$\lambda_2(a) = 0.8, \lambda_2(b) = 0.5.$

Define the smooth fuzzy topology T: $I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases}$$

Let r = 0.1 and $\gamma, \mu \in I^X$ be defined as follows:

$\gamma(a) = 0.2, \gamma(b) = 0.45;$

$\mu(a) = 0.8, \mu(b) = 0.55.$

Then, $C_T(\gamma, 0.1) = (0.2, 0.5) < \mu$ whenever $\gamma < \mu$ and γ is 0.1-fuzzy semi-open.

Hence, μ is 0.1-fuzzy λ-semi-closed.

But for any $T(\lambda_2) \geq 0.1$,

$C_T(\lambda_2, 0.1) = (0.8, 0.6) \nless \mu$ whenever $\lambda_2 < \mu$.

Hence, μ is not 0.1-fuzzy λ-closed.

Therefore, **every r-fuzzy λ-semi-closed set need not be r-fuzzy λ-closed.**

Proposition 2.2

Every r-fuzzy λ-closed set is r-fuzzy λ-b-closed.

Proof: Let (X, T) be a smooth fuzzy topological space. For $\mu \in I^X$ and $r \in I_0$, let μ be an r-fuzzy λ-closed set. Then, $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is r-fuzzy open. By Proposition 2.2, every r-fuzzy open set is r-fuzzy b-open. Thus for any r-fuzzy b-open set γ, μ is r-fuzzy λ-b-closed. Therefore, **every r-fuzzy λ-closed set is r-fuzzy λ-b-closed.**

Remark 2.2

The converse of the Proposition 2.2 need not be true.

Example 2.2

Every r-fuzzy λ-b-closed set need not be r-fuzzy λ-closed.

Let X = {a, b} and let $\lambda_1, \lambda_2 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.2, \lambda_1(b) = 0.4;$

$\lambda_2(a) = 0.8, \lambda_2(b) = 0.5.$

Define the smooth fuzzy topology T: $I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases}$$

Let r = 0.1 and $\gamma, \mu \in I^X$ be defined as follows:

$\gamma(a) = 0.2, \gamma(b) = 0.45;$

$\mu(a) = 0.8, \mu(b) = 0.55.$

Then, $C_T(\gamma, 0.1) = (0.2, 0.5) < \mu$ whenever $\gamma < \mu$ and γ is 0.1-fuzzy b-open.

Hence, μ is 0.1-fuzzy λ-b-closed.

But for any $T(\lambda_2) \geq 0.1$,

$C_T(\lambda_2, r) = (0.8, 0.6) \nless \mu$ whenever $\lambda_2 < \mu$.

Hence, μ is not 0.1-fuzzy λ-closed.

Therefore, **every r-fuzzy λ-b-closed set need not be r-fuzzy λ-closed.**

Proposition 2.3

Every r-fuzzy λ-semi-closed set is r-fuzzy λ-b-closed.

Proof: Let (X, T) be a smooth fuzzy topological space. For $\mu \in I^X$ and $r \in I_0$, let μ be an r-fuzzy λ-semi-closed set. Then, $C_T(\gamma, r) \leq \mu$ whenever $\gamma \leq \mu$ and γ is r-fuzzy semi-open. By Proposition 2.3, every r-fuzzy semi-open set is r-fuzzy b-open. Thus for any r-fuzzy b-open set γ, μ is r-fuzzy λ-b-closed. Therefore, **every r-fuzzy λ-semi-closed set is r-fuzzy λ-b-closed.**

Remark 2.3

The converse of the Proposition 2.3 need not be true.

EXAMPLE 2.3

Every r-fuzzy λ-b-closed set need not be r-fuzzy λ-semi-closed.

Let X = {a, b} and let $\lambda_1, \lambda_2 \in I^X$ be defined as follows:

$\lambda_1(a) = 0.3, \lambda_1(b) = 0.2;$

$\lambda_2(a) = 0.7, \lambda_2(b) = 0.3.$

Define the smooth fuzzy topology T: $I^X \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases}$$

Let r = 0.1 and let $\gamma_1, \mu \in I^X$ be defined as follows:

$\gamma_1(a) = 0.3, \gamma_1(b) = 0.6;$

$\mu(a) = 0.7, \mu(b) = 0.7.$

Then, $C_T(\gamma_1, 0.1) = (0.3, 0.7) < \mu$ whenever $\gamma_1 < \mu$ and γ_1 is 0.1-fuzzy b-open. Hence, μ is 0.1-fuzzy λ-b-closed.

But for any 0.1-fuzzy semi-open set $\gamma_2 \in I^X$ be defined by

$\gamma_2(a) = 0.7, \gamma_2(b) = 0.4,$

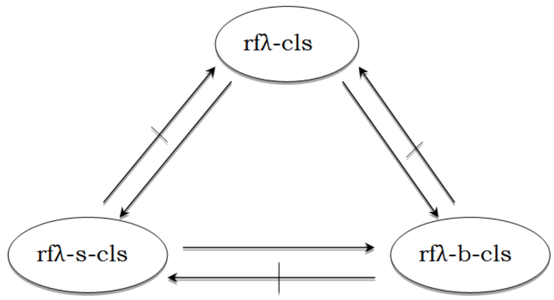
$C_T(\gamma_2, 0.1) = (0.7, 0.8) \nless \mu$ whenever $\gamma_2 < \mu$.

Hence, μ is not 0.1-fuzzy λ-semi-closed.

Therefore, **every r-fuzzy λ-b-closed set need not be r-fuzzy λ-semi-closed.**

Remark 2.4

From the above discussions the following implications hold.



3. On smooth Fuzzy λ-continuous functions

In this section the interrelations of smooth fuzzy λ-continuous functions with other types of smooth fuzzy continuous functions are established with the necessary counter examples.

Definition 3.1 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function $f: (X, T) \to (Y, S)$ is said to be a **smooth fuzzy λ-continuous function** (briefly, sfλ-cf) if for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-closed.

Definition 3.2 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function $f: (X, T) \to (Y, S)$ is said to be a **smooth fuzzy λ-semi-continuous function** (briefly, sfλ-s-cf) if for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-semi-closed.

Definition 3.3 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function $f: (X, T) \to (Y, S)$ is said to be a **smooth fuzzy λ-b-continuous function** (briefly, sfλ-b-cf) if for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-b-closed.

Proposition 3.1 Every smooth fuzzy λ-continuous function is smooth fuzzyλ-semi-continuous.

Proof: Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let any function f: (X, T) → (Y, S) be a smooth fuzzy λ-continuous function. Then for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-closed. By Proposition 2.1, every r-fuzzy λ-closed set is r-fuzzy λ-semi-closed. Therefore, $f^{-1}(\lambda)$ is r-fuzzy λ-semi-closed for every $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$.

Thus, **every smooth fuzzy λ-continuous function is smooth fuzzy λ-semi-continuous.**

Remark 3.1

The converse of the Proposition 3.2 need not be true.

Example 3.1

Every smooth fuzzy λ-semi-continuous function need not be smooth fuzzy λ-continuous.

Let X = {a, b} = Y and let $\lambda_1, \lambda_2 \in I^X$ and $\mu \in I^Y$ be defined as follows:

$$\lambda_1(a) = 0.2,\ \lambda_1(b) = 0.4;$$
$$\lambda_2(a) = 0.8,\ \lambda_2(b) = 0.5 \text{ and}$$
$$\mu(a) = 0.2,\ \mu(b) = 0.45.$$

Define the smooth fuzzy topologies $T: I^X \to I$ and $S: I^Y \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases} \quad \text{and}$$

$$S(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \mu \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, (X, T) and (Y, S) are two smooth fuzzy topological spaces.
Let f : (X, T) → (Y, S) be an identity function.

Let r = 0.1 and let $\gamma \in I^X$ be defined as follows:

$$\gamma(a) = 0.2,\ \gamma(b) = 0.46.$$

Then, γ is 0.1-fuzzy semi-open.
For $S(\bar{1} - \mu) \geq 0.1$, $f^{-1}(\bar{1} - \mu) = \bar{1} - \mu$.
Then for any 0.1-fuzzy semi-open set γ,
$C_T(\gamma, 0.1) = (0.2, 0.5) < \bar{1} - \mu$ whenever $\gamma < \bar{1} - \mu$.
Hence, $\bar{1} - \mu$ is 0.1-fuzzy λ-semi-closed.
Therefore, f is smooth fuzzy λ-semi-continuous.
But for any $T(\lambda_2) \geq 0.1$,
$C_T(\lambda_2, 0.1) = (0.8, 0.6) \not< \bar{1} - \mu$ whenever $\lambda_2 < \bar{1} - \mu$.
Hence, $\bar{1} - \mu$ is not 0.1-fuzzy λ-closed.
Therefore, f is not smooth fuzzy λ-continuous.

Thus, **every smooth fuzzy λ-semi-continuous function need not be smooth fuzzy λ-continuous.**

Proposition 3.2 Every smooth fuzzy λ-continuous function is smooth fuzzy λ-b-continuous.

Proof: Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let any function f : (X, T) → (Y, S) be a smooth fuzzy λ-continuous function. Then for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-closed. But, every r-fuzzy λ-closed set is r-fuzzy λ-b-closed. Therefore, $f^{-1}(\lambda)$ is r-fuzzy λ-b-closed for every $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$.

Thus, **every smooth fuzzy λ-continuous function is smooth fuzzy λ-b-continuous.**

Remark 3.2

The converse of the Proposition 3.2 need not be true.

Example 3.2

Every smooth fuzzy λ-b-continuous function need not be smooth fuzzy λ-continuous.

Let X = {a, b} and let $\lambda_1, \lambda_2 \in I^X$ be defined as follows:

$$\lambda_1(a) = 0.2,\ \lambda_1(b) = 0.4;$$
$$\lambda_2(a) = 0.8,\ \lambda_2(b) = 0.5 \text{ and}$$
$$\mu(a) = 0.2,\ \mu(b) = 0.45.$$

Define the smooth fuzzy topologies $T: I^X \to I$ and $S: I^Y \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases}$$

$$S(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \mu \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, (X, T) and (Y, S) are two smooth fuzzy topological spaces.
Let f : (X, T) → (Y, S) be an identity function.

Let r = 0.1 and let $\gamma \in I^X$ be defined as follows:

$$\gamma(a) = 0.2,\ \gamma(b) = 0.46.$$

Then, γ is 0.1-fuzzy b-open.
For $S(\bar{1} - \mu) \geq 0.1$, $f^{-1}(\bar{1} - \mu) = \bar{1} - \mu$.
Then for any 0.1-fuzzy b-open set γ,
$C_T(\gamma, 0.1) = (0.2, 0.5) < \bar{1} - \mu$ whenever $\gamma < \bar{1} - \mu$.
Hence, $\bar{1} - \mu$ is 0.1-fuzzy λ-b-closed.
Therefore, f is smooth fuzzy λ-b-continuous.
But for any $T(\lambda_2) \geq 0.1$,
$C_T(\lambda_2, 0.1) = (0.8, 0.6) \not< \bar{1} - \mu$ whenever $\lambda_2 < \bar{1} - \mu$.
Hence, $\bar{1} - \mu$ is not 0.1-fuzzy λ-closed.
Therefore, f is not smooth fuzzy λ-continuous.

Thus, **every smooth-fuzzy λ-b-continuous function needs not be smooth fuzzy λ-continuous.**

Proposition 3.3 Every smooth fuzzy λ-semi-continuous function is smooth fuzzyλ-b-continuous.

Proof: Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let any function f: (X, T) → (Y, S) be a smooth fuzzy λ-semi-continuous function. Then for each $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-semi-closed. But, every r-fuzzy λ-semi-closed set is r-fuzzy λ-b-closed. Therefore, $f^{-1}(\lambda)$ is r-fuzzy λ-b-closed for every r-fuzzy closed $\lambda \in I^Y$ with $S(\bar{1} - \lambda) \geq r$.

Thus, **every smooth fuzzy λ-semi-continuous function is smooth fuzzy λ-b-continuous.**

Remark 3.3

The converse of the Proposition 3.3 need not be true.

Example 3.3

Every smooth fuzzyλ-b-continuous function need not be an smooth fuzzy λ-semi-continuous.

Let X = {a, b} and let $\lambda_1, \lambda_2 \in I^X$ be defined as follows:

$$\lambda_1(a) = 0.3,\ \lambda_1(b) = 0.2;$$
$$\lambda_2(a) = 0.7,\ \lambda_2(b) = 0.3 \text{ and}$$
$$\mu(a) = 0.3,\ \mu(b) = 0.3.$$

Define the smooth fuzzy topologies $T: I^X \to I$ and $S: I^Y \to I$ as follows:

$$T(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \lambda_1 \\ 0.6 & \lambda = \lambda_2 \\ 0 & \text{otherwise.} \end{cases} \quad \text{and}$$

$$S(\lambda) = \begin{cases} 1 & \lambda = \bar{0} \text{ or } \bar{1} \\ 0.1 & \lambda = \mu \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, (X, T) and (Y, S) are two smooth fuzzy topological spaces.
Let f: (X, T) → (Y, S) be an identity function.

Let r = 0.1 and let $\gamma_1 \in I^X$ be defined as follows:

$$\gamma_1(a) = 0.3,\ \gamma_1(b) = 0.6.$$

Then, γ_1 is 0.1-fuzzy b-open.
For $S(\bar{1} - \mu) \geq 0.1$, $f^{-1}(\bar{1} - \mu) = \bar{1} - \mu$.
Then for any 0.1-fuzzy b-open set γ_1,
$C_T(\gamma_1, 0.1) = (0.3, 0.7) < \bar{1} - \mu$ whenever $\gamma_1 < \bar{1} - \mu$.
Hence, $\bar{1} - \mu$ is 0.1-fuzzy λ-b-closed.
Therefore, f is smooth fuzzy λ-b-continuous.

Let $\gamma_2 \in I^X$ be defined by

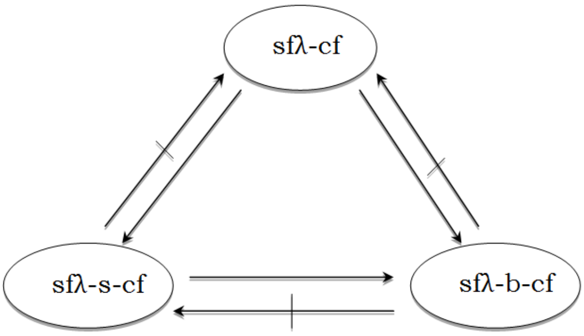
$$\gamma_2(a) = 0.7,\ \gamma_2(b) = 0.4,$$

Then, γ_2 is 0.1-fuzzy semi-open.
Thus, $C_T(\gamma_2, 0.1) = (0.7, 0.8) \not< \bar{1} - \mu$ whenever $\gamma_2 < \bar{1} - \mu$ and γ_2 is 0.1-fuzzy semi-open.
Hence, $\bar{1} - \mu$ is not 0.1-fuzzy λ-semi-closed.
Therefore, f is not smooth fuzzy λ-semi-continuous.

Thus, **every smooth fuzzy λ-b-continuous function need not be smooth fuzzy λ-semi-continuous.**

Remark 3.4

From the above discussions the following implications hold.



Definition 3.4 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function f: (X, T) → (Y, S) is said to be a **smooth fuzzy λ-irresolute function** if for each r-fuzzy λ-closed $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-closed.

Proposition 3.4 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f: (X, T) → (Y, S) be a function, $r \in I_0$. Then the following statements are equivalent:

(a) f is a smooth fuzzy λ-irresolute function.

(b) $f(SF\lambda\text{-}C_T(\lambda, r)) \leq SF\lambda\text{-}C_S(f(\lambda), r)$, for every $\lambda \in I^X$.

(c) $SF\lambda\text{-}C_T(f^{-1}(\mu), r) \leq f^{-1}(SF\lambda\text{-}C_S(\mu, r))$, for every $\mu \in I^Y$.

Proof: (a) ⟹ (b). Let f be a smooth fuzzy λ-irresolute function. Let $\lambda \in I^X$. Then $SF\lambda\text{-}C_S(f(\lambda), r) \in I^Y$ is an r-fuzzy λ-closed set. By (a),

$f^{-1}(SF\lambda\text{-}C_S(f(\lambda), r)) \in I^X$ is r-fuzzy λ-closed. Now, $\lambda \leq f^{-1}(f(\lambda))$.

Therefore, $f(SF\lambda\text{-}C_T(\lambda, r)) \leq SF\lambda\text{-}C_T(f^{-1}(f(\lambda), r))$

$\leq SF\lambda\text{-}C_T(f^{-1}(SF\lambda\text{-}C_S(f(\lambda), r)), r)$

$= f^{-1}(SF\lambda\text{-}C_S(f(\lambda), r)$.

Hence, $f(SF\lambda\text{-}C_T(\lambda, r)) \leq SF\lambda\text{-}C_S(f(\lambda), r)$.

(b) ⟹ (c). Let $\mu \in I^Y$ then $f^{-1}(\mu) \in I^X$.

By (b), $f(SF\lambda\text{-}C_T(f^{-1}(\mu), r)) \leq SF\lambda\text{-}C_S(f(f^{-1}(\mu)), r)$

$\leq SF\lambda\text{-}C_S(\mu, r)$.

Thus, $f^{-1}(f(SF\lambda\text{-}C_T(f^{-1}(\mu), r))) \leq f^{-1}(SF\lambda\text{-}C_S(\mu, r))$.

That is, $SF\lambda\text{-}C_T(f^{-1}(\mu), r) \leq f^{-1}(SF\lambda\text{-}C_S(\mu, r))$.

(c) ⟹ (a). Let $\gamma \in I^Y$ be an r-fuzzy λ-closed set.

Then, $SF\lambda\text{-}C_T(\gamma, r) = \gamma$.

By (c), it follows that

$SF\lambda\text{-}C_T(f^{-1}(\gamma), r) \leq f^{-1}(SF\lambda\text{-}C_S(\gamma, r)) = f^{-1}(\gamma)$.

But, $f^{-1}(\gamma) \leq SF\lambda\text{-}C_T(f^{-1}(\gamma), r)$.

Therefore, $f^{-1}(\gamma) = SF\lambda\text{-}C_T(f^{-1}(\gamma), r)$.

Hence, $f^{-1}(\gamma)$ is an r-fuzzy λ-closed set. Thus, f is a smooth fuzzy λ-irresolute function.

Definition 3.5 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function f: (X, T) → (Y, S) is said to be a **smooth fuzzy λ-semi-irresolute function** if for each r-fuzzy λ-semi-closed $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-semi-closed.

Proposition 3.5 Let (X, T), (Y, S) and (Z, R) be any three smooth fuzzy topological spaces. Let any function f : (X, T) → (Y, S) be smooth fuzzy λ-semi-irresolute function and g : (Y, S) → (Z, R) be smooth fuzzy λ-continuous function. Then g∘f: (X, T) → (Z, R) is smooth fuzzy λ-semi-continuous.

Proof: Let $\gamma \in I^Z$, with $R(\bar{1} - \gamma) \geq r$. Since g is smooth fuzzy λ-continuous, $g^{-1}(\gamma) \in I^Y$ is r-fuzzy λ-closed. But, every r-fuzzy λ-closed set is r-fuzzy λ-semi-closed. Therefore $g^{-1}(\gamma) \in I^Y$ is r-fuzzy λ-semi-closed. And since f is a smooth fuzzy λ-semi-irresolute function, $f^{-1}(g^{-1}(\gamma)) \in I^X$ is r-fuzzy λ-semi-closed. Thus $(g \circ f)^{-1}(\gamma) \in I^X$ is r-fuzzy λ-semi-closed. Therefore g ∘ f is smooth fuzzy λ-semi-continuous.

Definition 3.6 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function f: (X, T) → (Y, S) is said to be a **smooth fuzzy λ-b-irresolute function** if for each r-fuzzy λ-b-closed $\lambda \in I^Y$, $f^{-1}(\lambda) \in I^X$ is r-fuzzy λ-b-closed.

Proposition 3.6 Let (X, T), (Y, S) and (Z, R) be any three smooth fuzzy topological spaces. Let any function f : (X, T) → (Y, S) be smooth fuzzy λ-b-irresolute function and g : (Y, S) → (Z, R) be smooth fuzzy λ-continuous function. Then g∘f: (X, T) → (Z, R) is smooth fuzzy λ-b-continuous.

Proof: Let $\gamma \in I^Z$, with $R(\bar{1} - \gamma) \geq r$. Since g is smooth fuzzy λ-continuous, $g^{-1}(\gamma) \in I^Y$ is r-fuzzy λ-closed. But, every r-fuzzy λ-closed set is r-fuzzy λ-b-closed. Therefore $g^{-1}(\gamma) \in I^Y$ is r-fuzzy λ-b-closed. And since f is a smooth fuzzy λ-b-irresolute function, $f^{-1}(g^{-1}(\gamma)) \in I^X$ is r-fuzzy λ-b-closed. Thus $(g \circ f)^{-1}(\gamma) \in I^X$ is r-fuzzy λ-b-closed. Therefore g ∘ f is smooth fuzzy λ-b-continuous.

Definition 3.7 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Any function f: (X, T) → (Y, S) is said to be a **smooth fuzzy λ-closed function** if for each $\lambda \in I^X$ with $T(\bar{1} - \lambda) \geq r$, $f(\lambda) \in I^Y$ is r-fuzzy λ-closed.

Definition 3.8 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. If f: (X, T) → (Y, S) is a smooth fuzzy bijective, smooth fuzzy λ-continuous and smooth fuzzy λ-irresolute function then f is said to be a **smooth fuzzy λ-homeomorphism.**

Definition 3.9A smooth fuzzy topological space (X, T) is said to be **smooth fuzzy λ-$T_{1/2}$ space** if every r-fuzzy λ-closed set$\gamma \in I^X$ is such that $T(\bar{1} - \gamma) \geq r$, $r \in I_0$. Equivalently (X, T) is said to be a smooth fuzzy λ-$T_{1/2}$ space if every r-fuzzy λ-open set$\gamma \in I^X$ is such that $T(\gamma) \geq r$, $r \in I_0$.

Proposition 3.7 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be a smooth fuzzy λ-homeomorphism. Then,

(a) If f is a smooth fuzzy λ-closed function and (Y, S) is smooth fuzzy λ-$T_{1/2}$ space then (X, T) is a smooth fuzzy λ-$T_{1/2}$ space.

(b) If f is a smooth fuzzy closed function and (X, T) is smooth fuzzy λ-$T_{1/2}$ space then (Y, S) is a smooth fuzzy λ-$T_{1/2}$ space.

Proof: (a) Let $\lambda \in I^X$ be an r-fuzzy λ-closed set. Since f is a smooth fuzzy λ-closed function, $f(\lambda) \in I^Y$ is an r-fuzzy λ-closed set. Since (Y, S) is smooth fuzzy λ-$T_{1/2}$ space, $S(\bar{1} - f(\lambda)) \geq r$. Now, $\lambda = f^{-1}(f(\lambda))$ is r-fuzzy closed. Hence (X, T) is a smooth fuzzyλ-$T_{1/2}$ space.

(b) Let $\mu \in I^Y$ be an r-fuzzy λ-closed set. Since f is a smooth fuzzy λ-irresolute function, $f^{-1}(\mu) \in I^X$ is r-fuzzy λ-closed. Since (X, T) is smooth fuzzy λ-$T_{1/2}$ space, $T(\bar{1} - f^{-1}(\mu)) \geq r$. Now $\mu = f(f^{-1}(\mu))$ is r-fuzzy closed. Hence (Y, S) is a smooth fuzzy λ-$T_{1/2}$ space.

4. On Smooth Fuzzy λ-Compact Spaces

In this section, the concept of smooth fuzzy λ-compact spaces is studied and some of its properties and the characterizations are also discussed.

Definition 4.1 [1]A smooth fuzzy topological space (X, T) is said to be **smooth fuzzy compact** iff for each family $\{ \gamma_i \in I^X : T(\gamma_i) \geq r, i \in J \}$

with $\underset{i\in J}{\vee}\gamma_i = \bar{1}$, there exists a finite index set J_0 of J such that $\underset{i\in J_0}{\vee}\gamma_i = \bar{1}$.

Proposition 4.1 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be a smooth fuzzy λ-continuous and surjective function. If (X, T) is a smooth fuzzy compact and smooth fuzzy λ-$T_{1/2}$ space then (Y, S) is also smooth fuzzy compact, r ∈ I_0.

Proof: Let S (λ_i) ≥ r, where $\lambda_i \in I^Y$ and let $\underset{i\in J}{\vee}\lambda_i = \bar{1}$. Since f is smooth fuzzy λ-continuous, $f^{-1}(\lambda_i) \in I^X$ is r-fuzzy λ-open. Since (X, T) is a smooth fuzzy λ-$T_{1/2}$ space, T ($f^{-1}(\lambda_i)$) ≥ r. Since (X, T) is smooth fuzzy compact, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee} f^{-1}(\lambda_i) = \bar{1}$. Then, $\bar{1} = f(\bar{1}) = f(\underset{i\in J_0}{\vee} f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee}\lambda_i$.
Hence, (Y, S) is smooth fuzzy compact.

Definition 4.2 Let (X, T) be a smooth fuzzy topological space, r∈ I_0. A **smooth fuzzy λ-open cover** (smooth fuzzy λ-closed cover) of (X, T) is the collection { $\lambda_i \in I^X$: each λ_i is r-fuzzy λ-open (r-fuzzy λ-closed), i ∈ J } such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$.

Definition 4.3 A smooth fuzzy topological space (X, T) is said to be **smooth fuzzy λ-compact** if every smooth fuzzy λ-open cover of (X, T) has a finite subcover.

Proposition 4.2 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be a smooth fuzzy λ-irresolute and surjective function. If (X, T) is a smooth fuzzy λ-compact space then (Y, S) is also smooth fuzzy λ-compact, r ∈ I_0.

Proof Let $\lambda_i \in I^Y$ be r-fuzzy λ-open sets such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$, i ∈ J. Since f is smooth fuzzy λ-irresolute, $f^{-1}(\lambda_i) \in I^X$ is r-fuzzy λ-open. Since (X, T) is smooth fuzzy λ-compact, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee} f^{-1}(\lambda_i) = \bar{1}$. Then, $\bar{1} = f(\bar{1}) = f(\underset{i\in J_0}{\vee} f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee}\lambda_i$. Hence, (Y, S) is smooth fuzzy λ-compact.

Definition 4.4 Let (X, T) be a smooth fuzzy topological space, r ∈ I_0. A **smooth fuzzy regular closed cover** of (X, T) is the collection { $\lambda_i \in I^X$: λ_i is r-fuzzy regular closed } such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$.

Definition 4.5 A smooth fuzzy topological space (X, T) is called

(1) **Smooth fuzzy S-closed** if each smooth fuzzy regular closed cover of (X, T) has a finite subcover.

(2) **Smooth fuzzy S-Lindeolf** if each smooth fuzzy regular closed cover of (X, T) has a countable subcover.

(3) **Smooth fuzzy countable S-closed** if each countable smooth fuzzy regular closed cover of (X, T) has a finite sub cover.

Definition 4.6 A smooth fuzzy topological space (X, T) is called

(1) **Smooth fuzzy λ-S-closed** if each smooth fuzzy λ-closed cover of (X, T) has a finite subcover.

(2) **Smooth fuzzy λ-S-Lindeolf** if each smooth fuzzy λ-closed cover of (X, T) has a countable subcover.

(3) **Smooth fuzzy countable λ-S-closed** if each countable smooth fuzzy λ-closed cover of (X, T) has a finite subcover.

Definition 4.7 A smooth fuzzy topological space (X, T) is called

(1) **smooth fuzzy strongly S-closed** if for each collection { $\lambda_i \in I^X$, where T ($\bar{1} - \lambda_i$) ≥ r } with $\underset{i\in J}{\vee}\lambda_i = \bar{1}$, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee}\lambda_i = \bar{1}$, r ∈ I_0.

(2) **smooth fuzzy strongly S-Lindelof** if for each collection { $\lambda_i \in I^X$, where T ($\bar{1} - \lambda_i$) ≥ r } with $\underset{i\in J}{\vee}\lambda_i = \bar{1}$, there exists a countable subset J_0 of J such that $\underset{i\in J_0}{\vee}\lambda_i = \bar{1}$, r ∈ I_0.

(3) **smooth fuzzy countable strongly S-closed** if for each countable collection { $\lambda_i \in I^X$, where T ($\bar{1} - \lambda_i$) ≥ r } with $\underset{i\in J}{\vee}\lambda_i = \bar{1}$, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee}\lambda_i = \bar{1}$, r ∈ I_0.

Preposition 4.3 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be smooth fuzzy λ-continuous and surjective. If (X, T) is smooth fuzzy λ-S-closed [resp. smooth fuzzy λ-S-Lindeolf and smooth fuzzy countable λ-S-closed] then (Y, S) is smooth fuzzy strongly S-closed [resp. smooth fuzzy strongl S-Lindeolf and smooth fuzzy countable strongly S-closed], r ∈ I_0.

Proof Let { $\lambda_i \in I^Y$, where S ($\bar{1} - \lambda_i$) ≥ r, i ∈ J } be such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$. From the relation $\bar{1} = f^{-1}(\bar{1}) = f^{-1}(\underset{i\in J}{\vee}\lambda_i)$, it follows that $\bar{1} = \underset{i\in J}{\vee} f^{-1}(\lambda_i)$.

Since f is smooth fuzzy λ-continuous, $f^{-1}(\lambda_i) \in I^X$ is r-fuzzy λ-closed. Hence, { $f^{-1}(\lambda_i) \in I^X$, i ∈ J } forms a smooth fuzzy λ-closed cover of (X, T). Since (X, T) is smooth fuzzy λ-S-closed, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee} f^{-1}(\lambda_i) = \bar{1}$.
Therefore, $\bar{1} = f(\bar{1}) = f(\underset{i\in J_0}{\vee} f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee} f(f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee}\lambda_i$.
Thus, (Y, S) is smooth fuzzy strongly S-closed.
The proof is similar to the respective cases.

Preposition 4.4 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be smooth fuzzy λ-closed and bijective. If (Y, S) is smooth fuzzy λ-S-closed [resp. smooth fuzzy λ-S-Lindeolf and smooth fuzzy countable λ-S-closed] then (X, T) is smooth fuzzy strongly S-closed [resp. smooth fuzzy strongly S-Lindeolf and smooth fuzzy countable strongly S-closed], r ∈ I_0.

Proof Let { $\lambda_i \in I^X$, where T ($\bar{1} - \lambda_i$) ≥ r, i ∈ J } be such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$. From the relation $\bar{1} = f(\bar{1}) = f(\underset{i\in J}{\vee}\lambda_i)$, it follows that $\bar{1} = \underset{i\in J}{\vee} f(\lambda_i)$.

Since f is smooth fuzzy λ-closed, $f(\lambda_i) \in I^Y$ is r-fuzzy λ-closed. Hence { $f(\lambda_i) \in I^Y$, i ∈ J } forms a smooth fuzzy λ-closed cover of (Y, S). Since (Y, S) is smooth fuzzy λ-S-closed, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee} f(\lambda_i) = \bar{1}$.
Therefore, $\bar{1} = f^{-1}(\bar{1}) = f^{-1}(\underset{i\in J_0}{\vee} f(\lambda_i)) = \underset{i\in J_0}{\vee} f^{-1}(f(\lambda_i)) = \underset{i\in J_0}{\vee}\lambda_i$.
Thus, (X, T) is smooth fuzzy strongly S-closed.
The proof is similar to the respective cases.

Preposition 4.5 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f : (X, T) → (Y, S) be smooth fuzzy λ-continuous and surjective. If (X, T) is a smooth fuzzy λ-$T_{1/2}$ space and smooth fuzzy strongly S-closed [resp. smooth fuzzy strongly S-lindelof and smooth fuzzy countable strongly S-closed] then (Y, S) is also smooth fuzzy strongly S-closed [resp. smooth fuzzy strongly S-lindelof and smooth fuzzy countable strongly S-closed], r ∈ I_0.

Proof: Let { $\lambda_i \in I^Y$, where S ($\bar{1} - \lambda_i$) ≥ r, i ∈ J } be such that $\underset{i\in J}{\vee}\lambda_i = \bar{1}$. From the relation $\bar{1} = f^{-1}(\bar{1}) = f^{-1}(\underset{i\in J}{\vee}\lambda_i)$, it follows that $\bar{1} = \underset{i\in J}{\vee} f^{-1}(\lambda_i)$. Since f is smooth fuzzy λ-continuous, $f^{-1}(\lambda_i) \in I^X$ is r-fuzzy λ-closed.
Also, since (X, T) is a smooth fuzzy λ-$T_{1/2}$ space, every r-fuzzy λ-closed set $f^{-1}(\lambda_i) \in I^X$, i ∈ J is such that T ($\bar{1} - f^{-1}(\lambda_i)$) ≥ r.
Moreover, since (X, T) is smooth fuzzy strongly S-closed, there exists a finite subset J_0 of J such that $\underset{i\in J_0}{\vee} f^{-1}(\lambda_i) = \bar{1}$.
Thus, $\bar{1} = f(\bar{1}) = f(\underset{i\in J_0}{\vee} f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee} f(f^{-1}(\lambda_i)) = \underset{i\in J_0}{\vee}\lambda_i$.
Therefore, (Y, S) is smooth fuzzy strongly S-closed.
The proof is similar to the respective cases.

Preposition 4.6 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f: (X, T) → (Y, S) be smooth fuzzy λ-closed and bijective. If (Y, S) is a smooth fuzzy λ-$T_{1/2}$ space and smooth fuzzy strongly S-closed [resp. smooth fuzzy strongly S-lindelof and smooth fuzzy countable strongly S-closed] then (X, T) is also smooth fuzzy strongly S-closed [resp. smooth fuzzy strongly S-lindelof and smooth fuzzy countable strongly S-closed], r∈ I_0.

Proof: Let { $\lambda_i \in I^X$, where T ($\bar{1} - \lambda_i$) ≥ r, i ∈ J } be such that $\vee_{i \in J} \lambda_i = \bar{1}$. From the relation $\bar{1}$ = f ($\bar{1}$) = f ($\vee_{i \in J} \lambda_i$), it follows that $\bar{1} = \vee_{i \in J} f(\lambda_i)$.

Since f is smooth fuzzy λ-closed, f (λ_i) ∈ I^Y is r-fuzzy λ-closed.

Since (Y, S) is a smooth fuzzy λ-$T_{1/2}$ space, every r-fuzzy λ-closed set f (λ_i) ∈ I^Y is such that T ($\bar{1} - f(\lambda_i)$) ≥ r. Since (Y, S) is smooth fuzzy strongly S-closed, there exists a finite subset J_0 of J such that $\vee_{i \in J_0}$ f (λ_i) = $\bar{1}$. Therefore, $\bar{1} = f^{-1}(\bar{1}) = f^{-1}(\vee_{i \in J_0} f(\lambda_i)) = \vee_{i \in J_0} f^{-1}(f(\lambda_i)) = \vee_{i \in J_0} \lambda_i$.Thus, (X, T) is also smooth fuzzy strongly S-closed.

The proof is similar to the respective cases.

Definition 4.8 A smooth fuzzy topological space (X, T) is said to be a **smooth fuzzy λ-almost compact** if for any smooth fuzzy λ-open covering of (X, T), there exists a finite subset $J_0$of J such that $\vee_{j \in J_0}$ SFλ-C_T(λ_j, r) = $\bar{1}$, r ∈ I_0.

Definition 4.9 A smooth fuzzy topological space (X, T) is said to be a **smooth fuzzy λ-nearly compact** if for any smooth fuzzy λ-open covering of (X, T), there exists a finite subset J_0 of J such that $\vee_{j \in J_0}$ SFλ-I_T(SFλ-C_T(λ_j, r), r) = $\bar{1}$, r ∈ I_0.

Preposition 4.7 Let (X, T) and (Y, S) be any two smooth fuzzy topological spaces. Let f: (X, T) → (Y, S) be surjective and smooth fuzzy λ-irresolute function. Then the following statements are true:

(a) If (X, T) is smooth fuzzy λ-almost compact, then so is (Y, S).

(b) If (X, T) is smooth fuzzy nearly compact, then (Y, S) is λ-almost compact.

Proof: (a) Let { $\lambda_j \in I^Y$: each λ_j is r-fuzzy λ-open, j ∈ J } be a smooth fuzzy λ-open covering of (Y, S). Since f is a smooth fuzzy λ-irresolute function, $f^{-1}(\lambda_j) \in I^X$ is r-fuzzy λ-open. Since (X, T) is smooth fuzzy λ-almost compact, there exists a finite subset J_0 of J such that $\vee_{j \in J_0}$ λ-C_T($f^{-1}(\lambda_j)$, r) = $\bar{1}$. From the surjectivity of f,

$\bar{1}$= f ($\bar{1}$) = f ($\vee_{j \in J_0}$ λ-C_T($f^{-1}(\lambda_j)$, r)) = $\vee_{j \in J_0}$ f (λ-C_T($f^{-1}(\lambda_j)$, r))

≤ $\vee_{j \in J_0}$ λ-C_S(f ($f^{-1}(\lambda_j)$, r) = $\vee_{j \in J_0}$ λ-C_S(λ_j , r).

Therefore, (Y, S) is smooth fuzzy λ-almost compact.

(b) Let { $\lambda_j \in I^Y$: each λ_j is r-fuzzy λ-open, j ∈ J } be a smooth fuzzy λ-open covering of (Y, S). Since f is a smooth fuzzy λ-irresolute function, $f^{-1}(\lambda_j) \in I^X$ is r-fuzzy λ-open. Since (X, T) is smooth fuzzy λ-nearly compact, there exists a finite subset J_0 of J such that $\vee_{j \in J_0}$ λ-I_T (λ-C_T ($f^{-1}(\lambda_j)$, r), r) = $\bar{1}$.

Hence, $\vee_{j \in J_0}$ λ-C_T($f^{-1}(\lambda_j)$, r) = $\bar{1}$. From the surjectivity of f,

$\bar{1}$= f ($\bar{1}$) = f ($\vee_{j \in J_0}$ λ-C_T($f^{-1}(\lambda_j)$, r))

= $\vee_{j \in J_0}$ f (λ-C_T($f^{-1}(\lambda_j)$, r))

≤ $\vee_{j \in J_0}$ λ-C_S(f ($f^{-1}(\lambda_j)$, r)

= $\vee_{j \in J_0}$ λ-C_S(λ_j , r).

Therefore, (Y, S) is smooth fuzzy λ-almost compact.

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