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Dyscalculia: possible causes, symptoms, and teaching techniques

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DYSCALCULIA:
POSSIBLE CAUSES, SYMPTOMS, AND TEACHING TECHNIQUES

by

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CHAPTER 1

INTRODUCTION

Elementary schools today are faced with the task of preparing children for higher learning by teaching them the basics of reading, language, and mathematics. With the development of these three tools, the student may become a self-sufficient learner for the rest of his formal education, in deed, for the rest of his life. While reading and language skills are considered by some as the most important of the learning and communication skills, mathematic skills are certainly a very basic part of an individual's daily life. Mathematic skills are needed in purchasing commodities, getting to work on time, cooking a meal, paying for services rendered, paying taxes, banking, making investments, and calculating quantities of raw goods needed in making things as in clothing, fences, and gardens.

While most elementary school teachers intellectually realize the importance of developing mathematic skills in children, many teachers feel less secure about teaching math than they do about teaching reading or language. There are many reasons for this insecurity. Some educators feel that the cause for poor teaching skills in mathematics is due to the teacher's own bad experiences in the subject. Other educators feel the reason for this teaching weakness is due to a failure of the colleges to prepare elementary teachers to teach mathematics. Possibly the cause is due to the preciseness of mathematics and, therefore, the lack of redundancy that is apparent in language and reading. In other words, a child is able to comprehend what he has read even if he misses a couple of words in a story. Mathematics is so exacting a procedure that misinterpreting a minus sign for a division sign, or an addition sign for a multiplication sign has serious consequences in solving the problem. The child who has difficulty in mathematics is often more frustrated by his learning problems because his teacher is not sure how to help him; therefore, the child's inability to succeed in mathematics is perpetuated.

Purpose

The purpose of this research paper was to examine the causes of dyscalculia in learning disabled children and to examine the characteristics and symptoms such a child exhibits. In other words, questions such as, "What causes a child to not succeed in mathematics?" and "What are the behaviors a mathematically-disabled child will show in the classroom and at home?" were answered. Then, in the hopes of helping children with dyscalculia and their instructors, multisensory remedial activities have been presented to use as starting points or as guidelines to ameliorate the dyscalulic child.

Scope and Limitations

This review of literature was limited to the publishings available in the metropolitan Milwaukee area. The remedial activities that have been presented are age-appropriate mainly for the elementary school aged child; though in some cases they would be beneficial for children in the thirteen to fifteen year age group.

This author, though well aware of the need for equality of the sexes, finds that writing and reading literature with total dual sexuality cumbersome. For this reason, when referring to one child, the term "he" will be used. This term will mean both male and female children, and is not meant to be a sexist slur to male children.

Definitions

To better understand this paper, certain definitions are herewith presented.

Acalculia: The total failure of all mathematic ability.

Auditory Perception: The ability to interpret and organize sensory data received aurally.

Body Image: An awareness of one's own body and the relationship of the body parts to each other and to the outside environment (Lerner, 1976).

Conceptual Disorder: A disturbance in the thinking process and in cognitive activities (Lerner, 1976).

Developmental Dyscalculia: For the purpose of this paper the accepted definition for developmental dyscalculia is a

> structural disorder of mathematical abilities which has its origin in a genetical or congenital disorder of those parts of the brain that are the direct anatomical-physiological substrate of the maturation of the mathematical abilities adequate to age, without simultaneous disorder of general mental functions. (Kosc, 1974, p. 192)

Further discussion of this definition will be presented later.

Directionality: The ability to tell right from left and top from bottom and to consistently move in a prescribed direction.

Dysgraphia: Extremely poor handwriting or the inability to perform the motor movements required for handwriting.

Figure-ground Perception: The ability to attend to one aspect of the visual field while perceiving it in relation to the rest of the field (Lerner, 1976).

Lateral Confusion: The tendency to perform some acts with a right side preference and others with a left, or the shifting from right to left for certain activities (Lerner, 1976).

Oligocalculia: This condition refers to a relative decrease in all mental faculties and, in particular, the decrease in mathematic abilities to an appropriately equal degree as the loss in other functional areas.

Perception: The process of organizing and/or interpreting the raw data obtained through the senses.

Perceptual-motor: Perceptual-motor is a term describing the interaction of the various channels of perception with motor activity. These learning channels include visual, auditory, tactual, and kinesthetic.

Perseveration: Perseveration is the tendency to continue an activity once it has been started, and to be unable to change or stop the activity even though it is understood that the activity is no longer appropriate.

Position in Space: The perception of the size and movement of an object in relation to the observer.

Postlesional Dyscalculia: Postlesional dyscalculia is different from developmental dyscalculia in that it refers to a regression of previously normal mathematical abilities. This form of dyscalculia is caused by illness or insults to the brain and is mainly acquired as an adult.

Spatial Relationship: The position of two or more objects in relation to each other.

Strephosymbolia: The perception of visual stimuli in reversed or twisted order (Lerner, 1976).

Visual Perception: The identification, organization, and interpretation of sensory data received through the individual's eye.

Summary

Elementary school teachers in general, and specifically teachers of children having learning disabilities, are at a loss when attempting to remediate children with mathematical dysfunctions. The reasons for this weakened teaching ability may be varied, but nevertheless, the child with math disabilities is the one who suffers.

As stated, the purpose of this paper was to examine the causes and symptoms of dyscalculia; and then to discuss teaching techniques for remediating dyscalculia.

The scope and limitations were given to be literature and research available in the Milwaukee area. The scope of the paper dealt primarily with elementary school age children with some material appropriate for children from thirteen to fifteen years of age. Definitions were given to aid the reader in understanding the terminology used in this endeavor.

CHAPTER 2

REVIEW OF LITERATURE AND RESEARCH

Discussion of Definitions

Although one definition of developmental dyscalculia has been given, others are also used by educators. Flinter (1979) calls dyscalculia a disturbance in arithmetic that results from one or more disorders of quantitative thinking. Bakwin (1960) calls it "a difficulty in counting" (Kosc, 1974, p. 165). Cohn (1968) states that dyscalculia is "a failure to recognize numbers or manipulate them in an advanced culture" (Kosc, 1975, p. 165). Gerstman (1957) finds it an "isolated disability to perform simple or complex arithmetic operations and an impairment of orientation in a sequence of numbers and their fractions" (Kosc, 1974, p. 165). Kosc (1974) believes that developmental dyscalculia ought to include "those disorders of mathematical ability which are a consequence of heredity or congenital impairment of the growth dynamics of the brain centers." While Kosc is more specific and, in that sense, more limited, he defends his definition by stating that the other scholars do not take into account several factors. First, they do not take into account the general mental ability of the person's specific mathematical ability. Second, he feels that others ignore the developmental aspect of the disability; he believes that because the term is called "developmental dyscalculia" those dysfunctions which occur in adulthood should not be included. Third, he believes that ignoring the developmental aspect causes confusion when attempting to identify developmental dyscalculia in general and its symptoms in particular. Finally, he feels that others neglect the structural analysis of mental ability in general, and mathematical ability in particular as seen in test results. These justifications are consistent with the criteria used in the definition of learning disabilities as stated in the "Children with Specific Learning Disabilities Act" of 1969, PL 91-230 (Bryan, 1975). Dyscalculia is usually characterized by a retarded growth in mathematical skills while the person demonstrates normal auditory ability and may have an excellence in reading vocabulary and syllabication. Pure dyscalculia is only when there is a disorder in arithmetic functioning without a parallel disability in the person's general mental attitude (Flinter, 1979).

Causes of Dyscalculia

Conceptual Development

The causes of dyscalculia are complicated, as one can see from the definitions which have been presented. Each dyscalculic child may have one disability or a combination of disabilities causing his dysfunction in mathematics. These areas of dysfunction may be in the conceptualization of numeric principles, or they may be caused by physiological problems. Processing or perception may be the main problem; however, in many cases language development and/or poor teaching techniques are the reason for failure to attain mathematical skills. Each of these reasons for failure are now discussed.

Piaget (1953) is a leading authority in the area of mental development in children, particularly in the field of readiness for the acquisition of mathematical concepts. He feels that many mathematical concepts are developed within one's self; and when adults try to impose knowledge before the child is developmentally ready, the responses are merely verbal and have no meaning (Piaget, 1953). He maintains that there are three concepts that each child must develop within himself prior to learning arithmetic skills which are academically presented. The first skill is that of "relations." This term refers to a child's ability to comprehend words such as bigger, smaller, faster, slower, the same as, mine and yours. Also, within "relations" is the ability to understand one-to-one relationships between sets of objects (Herold, 1979). An example of the one-to-one concept would be to have the child arrange blocks and beads in such a way that there is one block for each bead, establishing that there are the same number of each. Then the child rearranges the items into separate groups and in a different placement, realizing that there are still the same number in each set.

The second prenumeric skill is "classification." This step involves the ability to place objects into sets. Examples of the classification concept would be the ability to take a set of several dolls and several trucks and place the dolls in a "doll box" and the trucks in a "truck box." On a higher level of classification, the child should know that when given several squares and circles of different sizes and told to classify the items by shape; large and small circles should be put in the same group. But if the child is to classify by size, small circles and small squares belong to the same set.

The third phase of prenumber development is the "conservation of quantity." This concept refers to the fact that quantity does not change no matter what arrangement it takes. An example of conservation of quantity is to take two balls of clay of the same size, establish that, in deed, they are the same; then, flatten one of the balls. The child should acknowledge that though the shape is different, the quantity of clay in each form remains the same. These three developmental steps are a part of Piaget's concrete operational stage of development which takes place in most children between the ages of seven and eleven (Copeland, 1970; Piaget, 1953).

Psychologists agree that children under six or eight years of age are able to form abstract concepts, as found in arithmetic, only through experiences (Herold, 1979). Further, the ability to coordinate different perspectives in thought does not come until about age nine or ten (Piaget, 1953). An example of the coordination of different perspectives is the ability to look at an arrangement of objects from across a table and duplicate it in front of the observer. In light of the illustrations above, if a child must experience concepts in order to understand them or is not of the mental age to duplicate arrangements in reverse, difficulties in developing arithmetic skills often follow.

Physiological Causes

While mental development is an important possible cause for dyscalculia, findings show that sometimes children are born with certain dispositions toward math. In conjunction with the genetic predisposition, if children receive head injuries within their first year of life, irreparable damage may cause similar effects as those who are genetically imperfect (Kosc, 1974).

Many neurological investigations give proof of the existence of special aptitudes for mathematics. If special centers in the brain are damaged, mathematical disorders result. Barakat (1951) made several such studies into the existence of genetic dispositions. He made an investigation of families and the mathematical abilities among their members. He demonstrated that both gifted and subnormal individuals could exist in the same family.

In another study (Barakat, 1951) the mathematical abilities of identical twins were examined. This study showed a correlation of .76 in the groups of twins from seven to nine years of age and a correlation of .61 in the groups of twins from eleven to thirteen years of age. According to this study these are the highest correlations of any two individuals. Another investigation (Barakat, 1951) showed a difference between male and female participants. In this study, males performed at a higher level than the females. These results indicate a predisposition of male superiority in mathematical abilities. However, because of the fact that the expectations in math are higher for males, this author questions whether the results of this phase of Barakat's studies are valid.

A final reported study by Barakat (1951) was limited to gifted individuals. This investigation showed a surprising amount of mathematical knowledge from a very early age (Kosc, 1974). In light of the information it seems logical that in at least some cases, dyscalculia can be caused by factors of heredity.

Processing Problems

Closely related to genetically founded causes of dyscalculia are problems of processing dysfunction. The concepts that the mathematic pupil must manipulate mentally may be relatively unfamiliar, and/or difficult to visualize mentally. Recall of all of the relevant features and properties of any mathematical concept may be difficult (Muro, 1979).

Statistically a higher number of incidents of learning problems occur in boys than in girls. This is also true in mathematic difficulties (Lansdown, 1978). Lansdown (1978) feels that the reason for this is because of the "casual relationship between math and spatial ability" (p. 183).

Another cause for problems in numeric abilities is poor spatial perception. Spatial relationships are disturbed effect concepts such as before, after, bigger, smaller, more than and less than. They can also cause the child to perceive numbers, letters and shapes incorrectly. For instance, these concepts may be perceived inversely. Spatial problems may also cause difficulties in the ability to tell which is closer, e.g. "which is closer to five, three or six?" (Herold, 1979; Myklebust & Johnson, 1967).

Poor visual perception causes difficulties in learning the process signs. A child with this problem may confuse an equals sign with a subtraction sign. Visual perception problems may cause a child to be confused by the number of problems or numbers on a page. This difficulty is often the cause for a child to lose his place in working a problem. Finally, visual perception problems may cause difficulties in discriminating shapes. The child may wonder if the shape he is to identify is a square or rectangle, an open curve or a closed one.

A poor grasp of visual-spatial relationships may cause difficulties in discriminating different sizes, quantity, and length, and hence is also the reason a child may have trouble with place value. In other words, if the child cannot see that a bundle of ten sticks is different from a single stick, the numeral 43 will have little meaning to him (Herold, 1979; Mykelbust & Johnson, 1967).

Poor directionality will cause the pupil to confuse right and left, and up and down; hence the child will reverse numerals or parts of them. The place value of digits is confused and the pupil often cannot remember where to start working a problem (Herold, 1979).

Poor memory, whether it be visual memory, auditory memory, or a combination of the two, causes great difficulty in learning the basic facts. A disability in this area of processing hampers the child who understands the concepts of the different operations, but cannot work the problems because he does not have the mathematical "tools" readily at his disposal (Herold, 1979).

Auditory memory problems can cause more difficulties than visual memory problems. If a child has a readuitorization problem, it prevents him from recalling numbers. He recognizes numerals when he hears them but cannot always say the number he wants to say. On the other side of the problem, the child may not be able to listen to story problems presented orally and/or assimilate the facts given; therefore he is unable to work the problem (Muro, 1979).

Poor sequencing causes a child to have difficulty in remembering the order of the steps in operations, patterns, and basic facts. A pupil with this disability would have trouble completing a pattern of numbers that began 1, 2, 4, 8 . . . . He also may not be able to compute

$$\begin{array}{r} 43. \\ \times\ \underline{64} \end{array}$$

A child who has poor visual-motor coordination would have difficulty in writing numbers and geometric forms. He may also be penalized for messy work (Herold, 1979; Kosc, 1974; Myklebust & Johnson, 1962).

In a longitudinal study made by Cohn (1968), inconclusive data indicated that dyscoordination motorically usually meant the patient had some form of mathematical deficiency. He further stated that, because of these children's clumsiness, "structural lesions need to be the cause of the particular difficulty encountered" (p. 667). Heretofore, statistical evidence has not borne this out.

Poor auditory perception prevents the child from understanding directions and the new information given orally. The student may know how to add 10 + 6 but when given the new concept of regrouping, he fails to add 12 + 9 correctly (Herold, 1979).

Perseveration causes difficulty in shifting from one act to another. The child with this problem may do poorly when assigned a page of problems where half the work is addition and the other half subtraction (Herold, 1979).

Poor memory is also an important cause for mathematic failure. In a study done by Webster (1979) on the significance of short-term memory in the learning of new mathematic concepts, it was shown that short-term memory abilities played a highly significant role in the learning of new math concepts, as well as previously documented reading abilities. "The results of the presented study support the hypothesis that atypical learners fail to use the same coding mechanisms as efficiently as adequate learners. This inefficiency appears to result in lower levels of achievement and learning gains" (p. 281).

Wilson (1979) made a study using the Gibson et al Test to study the multidimensional nature of perceptual discrimination. In the past this test had been discredited because of the relatively low intercorrelation of its items and reading performance. It had shown that the more complicated the figure was to discriminate, fewer errors were made. However, Wilson's study did not limit correlations to just reading. He tested children before they entered kindergarten and compared the results with their acquisition of sight vocabulary and achievement in arithmetic at the end of third grade. The correlations of this aspect of his study were again low, with correlations ranging from .06 to .31 in reading, and from .01 to .39 in arithmetic. Wilson, however, took these results one step farther. He graphed the reading and arithmetic achievement made among the nonachievers and their performance on the test. It was found that many individual transformations were reliably related to achievement. It was also discovered that patterns of relations were remarkably alike in reading and mathematics. The plotted charts looked very similar and had a correlation of .78. These secondary results of the study done by Wilson indicate that the degree of relationship between perceptual discrimination and achievement in nonachievers is significantly high (Wilson, 1979).

Language Readiness

The child who fails to perform up to the required standards in mathematics because of developmental causes may also exhibit delayed or poor language development. Performance in many areas of human functioning is clearly related to language skills. Low achievement in mathematics, may be in part related to the use of inadequate skills in various areas of language performance (Muro, 1979).

Before a child can comprehend any mathematical statement presented visually or auditorily it is necessary that the child recode it within his own language structure. Many approaches ignore the need for allowing children to understand and use mathematical concepts and relations within their own language. Rather, they stress the use of mathematic symbolism and grammar without having designed experiences in which the child is initially allowed to develop an adequate understanding of the meaning of the elements from within his own language structure. One aspect of determining language readiness, and therefore a readiness for acquiring a particular skill or concept in mathematics may be in considering the words the child will need to understand for that particular skill or concept.

Language Processing

Inadequate language processing abilities in various areas may be related to poor mathematic performance. An example of one such deficit is when the child is given the following problem $\frac{x}{4} = 3$, he answers $x = 3/4$. If a child can relate the idea that 1/4 of a certain number equals three, "What is that number," the probability of error is reduced (Muro, 1979).

A language delayed child often has a problem decoding mathematically related information into ordinary language. Muro (1979) feels the reason for this is the lack of redundancy in verbally expressed arithmetic statements. When mathematic ideas are translated into everyday language, the child may use several different experiences he has had to comprehend the ideas of the verbal arithmetic statements. This decoding represents highly developed and specialized linguistic skills. The child with auditory receptive language disorders (that is, decoding problems) is not necessarily deficient in understanding quantitative relationships (Myklebust & Johnson, 1967). Words are unusually difficult for the receptive aphasic child because he has trouble shifting the meanings for such words as "set," or "times." Numerical symbols are constant in arithmetic, but the spoken words for what they represent vary in meaning (Myklebust & Johnson, 1967).

Muro (1979) concurs with Myklebust and Johnson by defining the three different kinds of words found in mathematics. The first type is words which are defined in normal usuage and preserve their meaning in a mathematical context. An example of such words is "add." The second kind of word is one which occurs exclusively in math's context and is defined in terms of mathematics. One such term is "square root." Finally, there are words which occur in both everyday language and in mathematics but have a different meaning in math. An example of this type of word is "factor." These are the kind of words which the dyscalculic child finds most confusing.

To understand the intended meanings of verbal statements, a child must understand each word in context. Words of normal use in math and ordinary language usually present little trouble to the nonachiever. The words which are exclusively used in mathematics and in which the meanings do not change in context are generally explained and described by teachers. They often give formal definitions, list attributes and characteristics of the word, simulate examples by drawing diagrams, or recall real life examples. However, where words have more than one meaning, one in a mathematical sense and one in a nonmathematical sense, there is likely to be confusion in the mind of the learner.

Many children understand the word meanings used in everyday usuage, but do not understand the mathematical context and have difficulty distinguishing between the two senses in which the words are used (Muro, 1979).

Another kind of problem in comprehension may be seen when there is syntactic difference between verbal arithmetic and ordinary language. A child derives the meaning of a verbal statement by processing it syntactically and semantically. Several common words and phrases have a different meaning in math than in daily meanings. An example would be two, to, and too. These different meanings may be confusing (Muro, 1979).

Language Comprehension

Another language related learning problem in arithmetic involves verbal mathematic statements which code multiple operations. "The ability to comprehend and use verbal statements in which two mathematical elements are related temporally, spatially, or comparatively, is often demanded by children acquiring mathematical skills" (Muro, 1979, p. 904). An example of this problem is seen when the teacher directs a child to do the following: "Before you multiply by three, subtract two from the number" (Muro, 1979, p. 904). The temporal conjunction "before" may cause a problem in doing a serial order of events. This can usually be seen in daily language also.

Math problems coded in normal language can cause children to experience difficulty in translating the "ordinary language" problem into corresponding mathematical forms. One important reason is the inability to relate numerical elements in the way they are meant to be. That is, in some problems terms and/or processes are not presented in an unfamiliar way, e. g. 4 + __ = 7. In other problems the child must arrange the elements relative to each other as well as selecting a process, e. g. (4 x 5) + 2 =___. These problems become more prevalent in word problems (Muro, 1979).

> Many children lack at least some of the cognitive and linguistic abilities demanded for the adequate solution of the verbal arithmetic problems. Many need to be trained to abstract the various pieces of numerical information, to relate them using the syntatic information, and to represent the problem before they begin to perform the arithmetic processes. The problem may be illustrated by either concrete or perceptual means. This kind of representation is essential for some children to select and identify the appropriate process. (Muro, 1979, p. 907)

In a study of the relationship of language deficiency and mathematic skills, Campbell et al (1978) took two groups of educably mentally retarded students matched by mental age as well as I.Q. and tested their mathematic abilities. Then one group was instructed using the Peabody Language Development Kit. The other group was given no such instruction but was taught traditional math. The post-test results showed a significant difference between the two groups. The F-score of 7.09 was significant beyond .05 but less than .01 (point .01 = 7.56). Though the control group was given instruction in math, the mean dropped from 6.5 to 6.25 which was probably due to test reliability. The experimental group's mean rose from 8.9 to 12.1. The conclusions Campbell made indicated that language was an important factor in mathematics but should not be taught in lieu of it.

Teaching As It Affects Dyscalculia

Dyscalculic children often have both a pseudo-disability and actual dyscalculia. In these cases, they could often reach a much higher level of achievement in operational skills than they do; if instruction could be properly organized (Kosc, 1974). Many teachers today are frustrated over what to do with a child with inabilities in math.

> The sensitive teacher today is only too aware of the child with learning disabilities. She may tell herself that his (the child's) distress is the consequence

> of "learning disabilities" and "developmental lags" but this doesn't help her cope with the negative behavior nor even remember that it come from his feelings of inadequacy and anxiety. (Stern, 1977)

Most teachers do not understand how children form number concepts or develop computational skills. The teacher teaches reading with innumerable materials, ideas on phonics, and many attractive easy-to-read books. But in mathematics the teacher is lost (Stern, 1977).

In teaching math most teachers feel more relaxed using concepts they understand. This leads to using counting approaches, where dozens of small objects are counted. Most teachers do not know how to foster sensori-motor skills and activities that are the basis of concept formation. Too often there is an inordinate demand for the act of counting. For instance, when a child cannot memorize combinations he is taught counting as the only way to cope with arithmetic (Stern, 1977). Counting does not lead to mastery in addition and subtraction. Because counting is so often used in these operations, arithmetic becomes a matter of drilling children in hundreds of combinations.

Counting does not lead to an understanding of the relationship of numbers. A child bogs down at the counting stage and the teacher cannot raise the child above this level to acquire any other concepts (Stern, 1977).

More training of teachers would enable them to know how to help those children who are intelligent but are slowed in their progress by learning disabilities. Such training would develop sensitivity and skills that they need in teaching children mathematics. Stern (1977) feels that these skills include the following abilities:

1. The teacher should be able to spot an unusual response from a child.
2. The teacher should think about such a response.
3. The teacher should hypothesize why such a response was made.
4. The teacher should test his hunch by simplifying the task and restructuring it.
5. The teacher should realize the importance of eliciting language from the child. In other words he should not do all the talking. (Perhaps the child was not attending; or perhaps the child was not correctly processing the language.)(p. 173)

It is better for the teacher to observe the kind of mistakes pupils are making and then think of ways the activities or lessons can be restructured. Teachers should understand that mathematics is a mental activity, writing it down is merely an aid (Lovell, 1966). Teaching can promote the acquisition of skills, but when a disposition is impaired, the child usually is not able to fully acquire certain skills and knowledge despite intensive systematic training (Kosc, 1974). However, the teacher needs to develop his full potential in teaching, in order to bring each child to his full potential.

Other Possible Causes

Finally, there are other less documented causes for failure in mathematics. First, emotional factors, that is anxiety can come into play in a student's performance. Lansdown (1978) found studies made by Lynn (1957) and Feldhusen and Klausmeir (1962) which offer experimental evidence that children who display anxiety will often read better than they perform mathematically. It has been shown that anxiety lowers the level of all learning, both behaviorally and academically. Thus, it seems logical that anxiety would have a detrimental effect on students' learning math concepts and skills.

Lansdown (1978) also found evidence (investigated by Barakat, 1951) that schools in poor economic areas produce students with poor mathematic abilities. It also seems logical that children with a mental preoccupation on survival would have trouble learning arithmetic. Therefore, socio-economic factors may be a cause of problems in learning mathematics.

Herold (1979) discussed three more reasons for the failure to learn mathematics. He stated that sometimes parents and/or the teacher put too much pressure on children. This supports the findings of Lansdown (1978). Anxiety which is at too high a level slows learning. However, Herold also feels that sometimes too little pressure to learn is placed on children. For a long time it has been socially acceptable to "have trouble in mathematics." Many people say that they "can't even balance their check book," while their friends listen, laugh, and agree that they have trouble too. Few admit to being a poor reader. Lastly, Herold says that some students fail because they have too little confidence in themselves and their abilities. Group I.Q. tests given by elementary schools often bear out this theory. It is not unusual for a child to score in the low average range of ability on these tests, yet perform well within the average range or above average range; and unfortunately the reverse can also be true. Every child needs to believe that he can do the work before he actually can do it.

Symptoms of Dyscalculia

As stated earlier, the causes of dyscalculia range from genetic to self-imposed factors. The symptoms which children with dyscalculia exhibit are as varied as the causes. A number of people have compiled lists of behaviors dyscalculic children may demonstrate. Cohn (1968) finds dyscalculia manifested by the following behaviors:

1. The patient draws malformed or large number symbols.
2. The patient exhibits strephosymbolia.
3. The patient exhibits an inability to sum single integers.
4. The patient exhibits an inability to recognize operator signs and use linear separators.

5. The patient fails to discriminate separate order characteristics of multi-digit numbers.

6. The patient exhibits an inability to remember and use multiplication tables.

7. The patient exhibits an inability to carry numbers.

8. The patient exhibits inappropriate ordering of numbers in multiplication and division. (p. 666)

Myklebust and Johnson (1967) find the dyscalculic child to manifest these symptoms.

1. An inability to establish one-to-one correspondence.

2. An inability to count meaningfully.

3. An inability to associate auditory with visual symbols.

4. An inability to learn cardinal and ordinal systems of counting.

5. An inability to visualize clusters of objects within a large group.

6. An inability to grasp the concept of conservation of quantity.

7. An inability to perform arithmetic operations.

8. An inability to understand the meanings of the process signs.

9. An inability to understand the arrangement of numbers on a page.

10. An inability to remember the sequence of steps in various mathematic operations.

11. An inability to understand the principles of measurement.

12. An inability to read graphs and maps.

13. An inability to choose principles in solving problems in mathematic reasoning. (p. 248)

Kaliski (1962) outlines the symptoms of dyscalculia in terms of processing deficits. She states that the dyscalculic child has:

1. A fluctuating attention span.
2. Perceptual disturbances in any or all of the learning channels (visual, auditory and kinesthetic).
3. Figure ground confusion.
4. Distorted body image.
5. Poor visual-motor coordination.
6. Disturbed spatial, size, sequence, and/or temporal relationships.
7. Motor disinhibitedness.
8. Mixed lateral dominance.
9. Perseverative behavior.
10. Language disabilities.
11. Deficient conceptualization.
12. A need for concreteness. (p. 245)

If we examine the various symptoms of these authors' lists there are some areas common to them all. These commonalities are now discussed further.

Visual Problems

Many exceptional children have imperceptions, either visual or auditory, that hamper their success in arithmetic. The research and clinical work of Frostig (1972) documents the high incidence of perceptual dysfunctions among children with severe learning disabilities. Visual perception has been seen as a major probable cause for failure in mathematics (Thorton, 1979). Children with normal vision, but with visual perception problems have trouble interpreting what they see. Visual stimuli are misinterpreted by the brain. Kosc (1974) calls the inability to read mathematic symbols, e. g., digits, numerals, operational signs, and written mathematical operations, lexical dyscalculia. Some children may confuse numbers such as 6 and 9; they often misname 3, 5, and 8; and multi-digited numbers are confused, i.e. 36 may be interpreted as 63. These children may misinterpret fractions for numbers such as 1/4 = 114. Visual perception affects sequencing numbers, reading and interpreting graphs, judging space and time, and aligning and spacing numbers for computation. It may also cause children to lose their place on the page; or it may cause problems in orientating their own bodies and space around them. Consequently, they find it difficult to follow directions such as "up, down, beside, after, and before" (Thorton, 1979).

Visual-spatial disturbances may cause the inability to manipulate objects in the child's imagination. It may also cause different sizes, shapes, amounts, and lengths to become nondistinguishable. Estimations of distances and making judgments related to size may be inaccurate.

Another difficulty which is caused by visual perceptual problems is the inability to perceive operational signs and confuse arithmetic operations. Kosc (1974) calls this problem operational dyscalculia. Children with this dysfunction may confuse a + sign with a x sign, or a - sign with a ÷ sign. They may not understand the meaning of the process. A typical error in this area is seen when the child computes 5 x 2 = 10 correctly but when the problem is given as 5 __ 2 = 10 the child puts a + sign in the blank. Some cases of operational dyscalculia are difficult to spot. An example of such a problem is $\begin{array}{r} 86 \\ \underline{-4} \end{array}$. The student answered 81 but when asked how he got the answer he said 4 + 6 = 10 and 10 + 8 equals 18, but he wrote 81.

Understanding Relations in Mathematics

Another symptom of dyscalculia is discussed by Muro (1979) and Kosc (1974). Muro writes, "Statements coding relationships between math elements, whether they be spatial, temporal, size, or conditional relationships, etc., may be difficult for some children to comprehend" (p. 906). Kosc, (1974) calls this type of dysfunction practognostic dyscalculia. He defines it as a "disturbance of mathematic manipulation with real or pictured objects" (p. 167). Further, he finds mathematic manipulation includes the enumeration of the things and the comparison of estimates of quantity without their addition. In other words, a child with this kind of dysfunction may not be able to set out sticks or cubes in order of magnitude. He therefore would find terms such as bigger, smaller, shorter, longer, more and less as meaningless words. If a child is unable to see the differences in numbers or objects, educators should not expect him to use abstract number concepts or perceptual sequences such as first, next, and last.

Ideognostic dyscalculia is a disability primarily in understanding ideas and relationships and in doing mental calculations (Kosc, 1974). In serious cases the student cannot mentally calculate simple sums, though he may be able to read and write the number and know that "4" is the same as " . . . . " but is unable to understand that "4" is half of 8 or one less than "5."

Children may experience difficulties solving arithmetic problems because of a lack of selective attention to appropriate cues and may require remedial instruction directed at training the children to comprehend and use particular cues and features. For example, a child with attention deficits may not see that in 3 +_ = 7, the __ implies a whole number. Also, children with attention problems may not be able to quickly identify the number of objects in a group. Consequently, when adding 3 + 4 they must count all the objects in order to determine the total number.

Directionality

Poor directionality, or left-right confusion, is frequently an essential factor underlying disorientation in regard to number sequences (Kaliski, 1962). This kind of processing problem is seen in reading as well as arithmetic. Svien and Sherlock (1979) give and discuss many of the typical errors a child with poor directionality may exhibit in mathematics and relate them to similar errors in reading.

	Numerical Errors	Alphabetic Errors
1.	Reads 360, or 402 for 306, 042	Reads brun for burn. This is a transposition.
2.	Working on a written problem with 2 x, midway shifts to x^2.	A transposition
3.	Reads 39 as 93.	Reads was for saw. A transposition.
4.	Writes 3/5 but intends 5/3.	Same as number 3.
5.	Writes 72 but intends 27.	Same as number 3.
6.	Given a dictation of four thousand thirty-two, writes 4,000,32.	Given dictation of the word strain, writes strane.
7.	Writes 4,23. for dictation of "four thousand twenty-three".	Writes "cak" for dictation of "cake." Expected one-to-one correspondence between auditory message and encoding of numerals and letters.
8.	Reads 16 as 61	Reads "u" as "n"
9.	In computing addition, subtraction and multiplication, begin working from the right.	Reads "on" for "no." (p. 274)

Place Value Deficits

The problems which children incur in working with place value are also usually connected to poor directionality. One illustration of this condition is when the child reads 5,102 as "five, one hundred and two hundred." Here, he does not recognize the underlying pattern of hundreds, tens, ones and/or the commas as signals for words that change according to the number of commas, i. e., thousands, millions, etc. (Svien & Sherlock, p. 274). When place value errors occur usually the classroom teacher resorts to sticks and bundles to reinforce this concept. If, however, the problem persists, and place value can be done in isolation, then the problem may be perceptual.

Other symptoms which involve place value deficits and therefore perceptual deficits are seen in computation of problems involving carrying. Usually simple one digit addition, or multiplication does not prove to be a problem. However, with the introduction of two or three digit numbers, difficulties often do occur, especially when the problems are written horizontally. Two such examples are:

$$\begin{array}{r} 15 \\ +\ \underline{18} \\ 213. \end{array}$$

or 15 + 18 = 1518. Uncertainty of direction in which to work is compounded by a total loss of orientation. "Disturbances in loss of direction have been found in counting numbers, which denotes a loss of forward and backward concepts" (Flinter, 1979, p. 45).

Perseveration

Perseveration is noticed at many levels of arithmetic operations, e. g. addition, subtraction, multiplication and division. If one process alone is taught for too long a period of time and practiced for too long, the transfer to another process is extremely difficult and constitutes a real hardship to the perseveric child (Kaliski, 1962). The same problem occurs if numbers are to be counted in one direction and written in another. The two signs which indicate division also may cause a problem; the child may know the process, but not recognize the sign. Traditionally the child who perseverates has a very difficult time counting coins of more than one denomination.

Language Symptoms

Verbal dyscalculia, which is a "disturbed ability to designate verbally mathematic terms and relations" (Kosc, 1974, p. 167) manifests itself through the individual's inability to name amounts, and the number of things, digits and numerals, as well as symbols and mathematic performances. In these cases, individuals who are brain damaged cannot identify numbers which are dictated to them in the form of numerals. However, they are able to read and write the respective number or to count an amount of things. On the other hand, they may not be able to name the amount of things or the value of written numbers, although they are able to read and write the dictated number.

McLeod and Crump (1978) made a study to determine if certain factors among visuospatial skills, verbal abilities, and learning disabilities account for a significant amount of variance among a set of variables. They also wanted to investigate the extent of the relationships among visuospatial skills and verbal abilities and mathematic achievement in content, operations, and applications. Prior to the study it was demonstrated that the substance of math is its symbols and language. Therefore, verbal ability is integral to many facets of mathematics, including concepts of quantity and classification, that is, many of the mathematic concepts are language based (Crump, McLeod, 1978). The results of the study showed a significant relationship among mathematic achievement and visuospatial skills. This suggests that the previous belief that learning disabilities in mathematics is mainly due to visuospatial skills may be an over simplification of the issue.

> Although the degree of difference was small, this difference indicated that verbal activity variables were related more closely to mathematics achievement than visuospatial skills in the sample youngsters with learning disabilities in mathematics. (McLeod & Crump, 1978, p. 240)

Memory Problems

Memory problems are easily seen in dyscalculic children. These deficiencies manifest themselves through object and finger counting. Typically, the child with poor memory demonstrates knowledge of the four major processes but does them very slowly because he must consistently take time to count, or illustrate the problem. Some memory problems seen in dyscalculia are similar to those seen in language (Svien & Sherlock, 1979). For instance, some children cannot recall multiplication facts either visually or auditorily. This condition is similar to dyslexic children who can remember plots and personalities in stories but cannot remember the names of the characters (Svien & Sherlock, 1979).

Dysgraphia

Graphical dyscalculia is a disability in manipulating mathematical symbols in writing (Kosc, 1974). This form of dyscalculia is also called dysgraphia and is an apraxic condition (Myklebust & Johnson, 1967). Children with this motor problem cannot learn the motor patterns for writing letters or numbers. These students may not be able to even count by pointing and saying the numbers in an accurate manner. They may not be able to write dictated numbers or number words; and in severe cases, they may not even be able to copy them. In the classroom, a child with this disability is frustrated because he may understand the concepts and skills presented, but he cannot express this knowledge in a written manner. To make things worse the dysgraphic child is usually penalized for messy work.

Generalizations

The symptoms which have been presented may be found in isolation or in combination. Svien and Sherlock (1979) feel that there are two components which are evident in dyslexia which are also exhibited in dyscalculia. These components are an information-processing deficit and the prerequisite to use a written system in accurate decoding and encoding the symbols used in that system. When looking at the information-processing errors as "constants" they are easily identifiable in many subject areas.

Many of the examples given show that sometimes learning problems which are seen in mathematics in terms of specific categories are due to processing problems rather than concept formation. In working with students who have problems in mathematics,

> it is important to analyze errors. Are they conceptual, perceptual, or both? Is faulty information processing eroding the student's understanding of a concept? Task analysis should include analyses of possible visual or other confusions. With prior task analysis the tutor can anticipate possibilities of errors and adapt teaching strategies to forstall them. Some students can be helped to avoid confusions with symbols

> such as "x" and "+" through verbal mediation, either through subvocalization or "internal" language, or through actual speech. (Svien & Sherlock, 1979, p. 275)

It is the teacher's responsibility to do such analyzing in order to select the most effective activities to ameloriate the child's deficit.

Remedial Teaching Techniques

Knowing the causes and some typical symptoms of dyscalulia can help teachers become better informed about children with mathematic deficits, but if remediating techniques are not apparent many teachers are at a loss as to how to help the children.

Why Teach Math

Mathematics can be used to aid in unlocking the education problems of the learning disabled child. One of the greatest problems for such students is organization. When a child can organize thoughts and activities in a logical way he can go on to master the abstract demands of language development (Swett, 1978). Therefore, the main goal of mathematics is the same as other areas of education and is consistent with most learning disability programs. This goal is ". . . to help any student become an efficient, well organized, and logical problem solver" (Swett, 1978, p. 6).

Math can also be a motivator. It is a break from reading and phonics. The inherent characteristics of mathematics are different than those of language. Its language is concise, and symbols are few. Math is always dependable; there are no exceptions to its rules. Its patterns give more structure to children who yearn for it. The basic skills have "fun tricks" which make them interesting to learn. The basic concepts can be experienced visually and concretely. Finally, mathematics makes sense and can be logically proven, giving the child immediate success (Swett, 1978).

> Math offers a variety of visual and concrete techniques for training students to analyze, to zero in on essentials, to look for patterns, and to classify and organize information in order to solve specific problems. (Swett, 1978, p. 6)

Learning disabled children have trouble doing their school work because they cannot organize their perceptual input meaningfully. Mathematics will help them solve this problem. Academically, the goals in teaching arithmetic are to enable each child to deal with all number situations they are confronted with, and to develop mathematic thinking and logical reasoning (Stern, 1949).

Benefits of Multisensory Teaching

Swett (1978) feels that math is of great benefit in process teaching. Specifically he states,

> Math as a multisensory experience offers great potential in strengthening such vital nonverbal, prereading skills as visual discrimination, pattern-matching, discernment of likes and differences, and the classification of sequencing of visual and tactile data. Whatever Piagetian stage of conceptual development a particular child is at--we can predict that our learning disabled child is lagging, processing slower through these stages than his classmates because of her perceptual difficulties. There are mathematically sound ways to help speed up the child's progress. Whether in the "preoperational stage" or the "period of concrete operations" any child needs concrete manipulative experiences in measuring, comparing and eventually understanding quantitative as well as qualitative relationships. (p. 10)

Swett (1978) goes on to say that the learning disabled child needs to want to take an active role in learning. Games and manipulative activities often give positive and meaningful learning experiences with peers or an understanding adult. They also help develop social skills. Further, the use of manipulatives develop eye-hand coordination. "Organization games that give multisensory experiences with emphasis on sharpening visual-perceptual skills within the solid framework of mathematics, are valuable tools . . . ." (Swett, 1978, p. 11). It has been long known that when a child merely watches an action he does not understand it in the same way as if he actually performs the task. The perceptually handicapped child cannot rely on visual and auditory memory. It is very important that they "do" to learn new concepts no matter what the area of academic learning. The learning disabled child learns best when he touches, feels, and uses all senses together. In multisensory mathematics, slowly and thoroughly, one step at a time, each child experiences and talks about numerical relationships. Later he learns to record his experiences with figures that are meaningful (Stern, 1977).

Thorton and Reuille (1978) find concreteness important to strengthen processing problems. Visual deficits should be strengthened, using auditory, tactile and color cues.

Auditory deficits should be remediated by using gross discrimination of environmental sounds and then simplified language in teaching. To strengthen memory functions use simple instruction and tapes. To overcome deficits in visual-spatial motor coordination, teach math through manipulatives. In light of the presented information one can see that using multi-sensory learning materials in teaching math may help the child in all areas of academia because they force the child to develop skills in the areas of deficit within himself.

Language Development as a Teaching Technique

Language helps children understand mathematics. By explaining what they have just done, children are forced to relive their mathematic experiences. Stern (1977) feels that it may be that the learning disabled child has not had enough chances to combine actions with mental images and language.

> A curriculum approach in which children are encouraged to express math's statements in ordinary language statements, in concrete forms, or in other symbolic forms that more closely represent the semantic notations symbolized, may be more useful in facilitating the acquisition of math's understanding and provide a firm basis of understanding for the understanding of the conventional math symbolism. (Muro, 1979, p. 902)

Training may involve overt verbalization. This verbalization is helpful to direct the child's attention and to self-instruct the child in operations or concepts (Muro, 1979; Myklebust & Johnson, 1967). To facilitate comprehension in such statements of math education, the child should be given experiences designed to allow him to interpret the findings through either concrete or pictorial situations (Muro, 1979). Because many children cannot process temporal statements and relations, the teacher should make sure that the serial order of the events in the presentation is in the intended order. This preordering can minimize failure due to poor comprehension (Muro, 1979).

Dyscalculic children often need to substitute ordinary language for the exacting language of mathematics. Operational vocabulary should be considered. When referring to addition, other words which mean almost the same thing can and should be substituted. (e. g., adding, plus, sum, total, more than, and greater than) These terms refer to the operation of "putting together." The operation of subtraction should be put in terms of "taking away." Terms which may be substituted for "subtraction" are: less than, taken away from, subtracted from, and minus. Multiplication can be referred to in terms of the addition of groups. Careful consideration should be made, however, not to confuse multiplication with simple addition. For this reason pictorial cues of groups or arrays may be useful. Substitutional terms for "multiplication" may be, multiply, times, product, and multiplier. Division is different from the other three operations in that a literal translation is needed. The child must realize that $9 \div 3$ does not mean the same as $3 \div 9$. This can be related to the fact that $9 - 3$ does not mean the same as $3 - 9$. In translating the division problem given above the child should say that "if 9 is divided into 3 equal groups, there would be 3 objects in each group." However, "if 3 was divided into 9 groups there would not be 3 objects in each group." Substitutional terms for "division" are: divide, into, and quotient (Flinter, 1979). These were some examples given by Flinter; later, a chart will be given presenting other terms to help children with language deficits in mathematics.

Dansforth (1978) gives several other ways to help the language deficient child in math.

1. Give a concise statement of the specific objective for each session.
2. Have the student verbalize in working specific problems.
3. Keep an error-analysis card.
4. Use analogies to emphasize number relations.
5. Present the directions in several different ways.
6. Use concrete materials and real world situations in presenting lessons.
7. Develop aids for avoiding errors.
8. Remove frustration from learning by not giving too many problems at once. (p. 26)

Pre-number Techniques

The dyscalculic child is often lacking the skills that enable him to grasp abstract number concepts. This child should be given activities to develop his readiness for arithmetic. The following are some activities that will help prepare such a child.

1. Use puzzles where only one piece correctly fits each space. This helps develop spatial organization and visual motor integration (Flinter, 1979).

2. Make outlines on the floor. The child should look for the specific form, say its name, and walk along its perimeter (Flinter, 1979).

   "Geometric experiences that focus on organizational and spatial integration skills are particularly helpful to children who have difficulty judging relative distances on a number line or ruler or who are subject to reversal" (Thorton, 1979, p. 25).

3. Trace geometric figures. This is helpful in preparing a child for copying sentences and a row of problems (Thorton, 1979).

4. Take two dimensional figures of different sizes and have the child feel them and describe them.

"When children act on objects physically their mental images become stronger. Carefully planned geometric activities can lead all children, including handicapped learners, toward higher levels of geometric understanding and toward perceptual-spatial, perceptual-motor strengths. (Thorton, 1979, p. 26)

Myklebust and Johnson (1967) relate several activities for developing the concept of one-to-one correspondence.

1. Match rows of pegs.

2. Relate a number of sounds by placing a peg in the pegboard for each drum beat.

3. Match a penny for each finger.

4. Place buttons and button holes on felt and have the child button them.

5. Match hats, etc. on paper dolls.

6. Have the child pass out one piece of paper or pencil to each child.

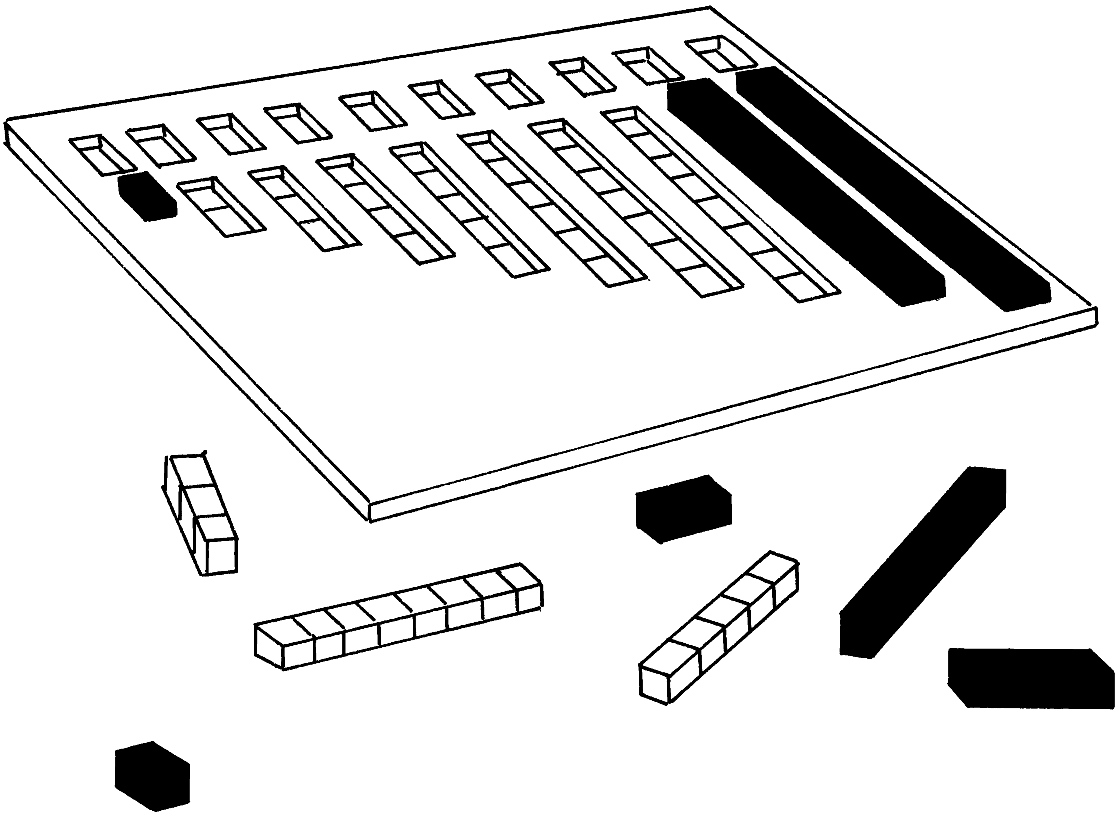
5. Myklebust and Johnson (1967) suggest the use of concrete materials which can be arranged to facilitate numerical thinking in developing concepts such as quantity, order, size, space and distance.

6. Flinter (1979) suggests color coding materials to teach concepts such as "beginning, ending, between, first, last, middle and next row." Manipulatives help a child start to learn concepts by arranging things so as to give them certain physical features to bring out the interrelations between the numbers or concepts (Lovell, 1976, p. 45).

Number Techniques

Basic to all higher mathematical concepts is the idea of the value of each numeral. The following are activities which will help develop value concepts. Many of these activities are used by Catherine Stern (1977) in a mathematical program called Structural Arithmetic. She developed and tested this program for and with dyscalculic children.

The counting board helps children learn to count by counting single cubes aloud in each row. With this activity each child will realize the word refers to the entire group. The next step is to replace the three single blocks with the three block , etc. As he replaces each group of single cubes with the number block of the same size the child "ends up with a concrete representation that shows the cardinal aspect of each number from one to ten" (Stern, 1977, p. 176).



Counting Board[1]

After a few counting activities move to another task. When the blocks in the counting board have been stood on end, they form an upright stairway from one to ten. The child can count this stair with his finger as he recites each counting name. Since there is a meaningful relation between the word he

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[1]Permission for reproduction granted by Houghton Mifflin Company, Boston, Mass. from Structural Arithmetic by Catherine Stern, 1966.

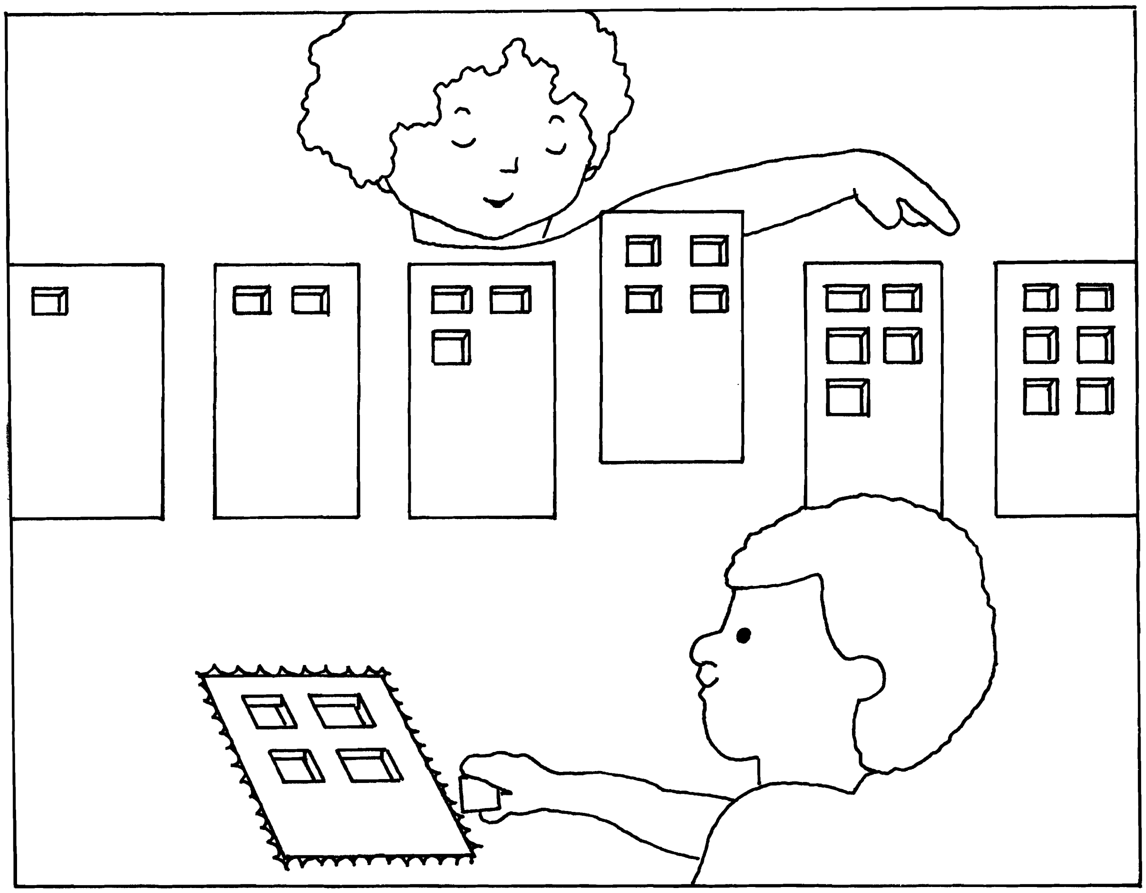
has just said and the block he is touching, the teacher can draw attention to misnamed blocks and the child can coordinate his actions to his words by giving each one its own name. This results in the ability to say each number in one-to-one correspondence with the appropriately counted object.

Other concepts which can be developed with the counting board are language based as well as mathematic. The following are typical questions a teacher may ask:

1. "What block comes <u>next</u>?"
2. "Which block comes <u>after</u> this block?" (between or before)
3. "Which is the <u>biggest</u> block of all?" (tallest, greatest)
4. "Find the <u>smallest</u> block." (littlest, least)
5. "Find the one that is one <u>smaller than</u> this block." (greater than, less than)
6. "Which pattern has one <u>less</u> cube than this one?" (more, greater)

With these activities, vocabulary becomes familiar long before the abstract numerical concepts are presented.

Pattern boards are another of the materials in Stern's <u>Structural Arithmetic</u>. There are ten boards each designed to receive a different number of cubes. In each board the empty blanks are arranged in pairs, which makes the characteristics of eveness and oddness stand out.



Pattern Board[2]

Here are some recommended activities to use with the pattern boards:

1. Start by playing games that teach the child to recognize and construct each pattern. They will learn to identify each number later.

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[2]Permission for reproduction granted by Houghton Mifflin Company, Boston, Mass. from Structural Arithmetic by Catherine Stern, 1966.

2. Language may also be developed much in the same way as with the counting boards.

Stern's equipment is of value not because it is manipulative but because it gives certain physical structures which in turn leads to mental structures (Lovell, 1966).

Though Stern's materials are good, here are some other activities of value in developing number concepts.

1. Listen to a drum beat and say the numbers as they are heard (Myklebust & Johnson, 1967).

2. String beads and say numbers (Myklebust & Johnson, 1967).

3. Count dot configurations (Myklebust & Johnson, 1967).

4. Count steps as they are walked (Myklebust & Johnson, 1967).

5. Using a number line slide the finger up and down (Myklebust & Johnson, 1967).

6. Tachistoscope procedures include circling a given number, writing the number of the group that is flashed, and color code the items which are flashed (Myklebust & Johnson, 1967; Flinter, 1979).

7. Use a number chart from 0 - 100. One such chart made by Spitzer has 0 - 9 in the first row so that the tens stand out. It is easy to see, for instance, that 52 has five tens and two more (Flinter, 1979).

8. At the intermediate level graphic representations, that is, linear graphs, bar graphs, and scale drawings make amounts clearer (Flinter, 1979).

9. Slow down the child's counting so that the counting objects are distinct (Flinter, 1979).

10. Large dominoes are a good way of developing recognition of certain numbers in a group (Lovell, 1966).

11. Cuisenaire Apparatus allows the child to play freely with the rods, associate colors with length, compare one rod with two or more rods placed end to end and develop the concepts of "twice as much" or "half of."

It also develops language in much the same way as Stern's equipment (Lovell, 1966).

Place Value Techniques

The concept of place value often gives the dyscalculic child difficulty. The child must understand the reason for writing figures in columns, if there is no understanding it will be done mechanically and this leads to errors and a lack of meaning.

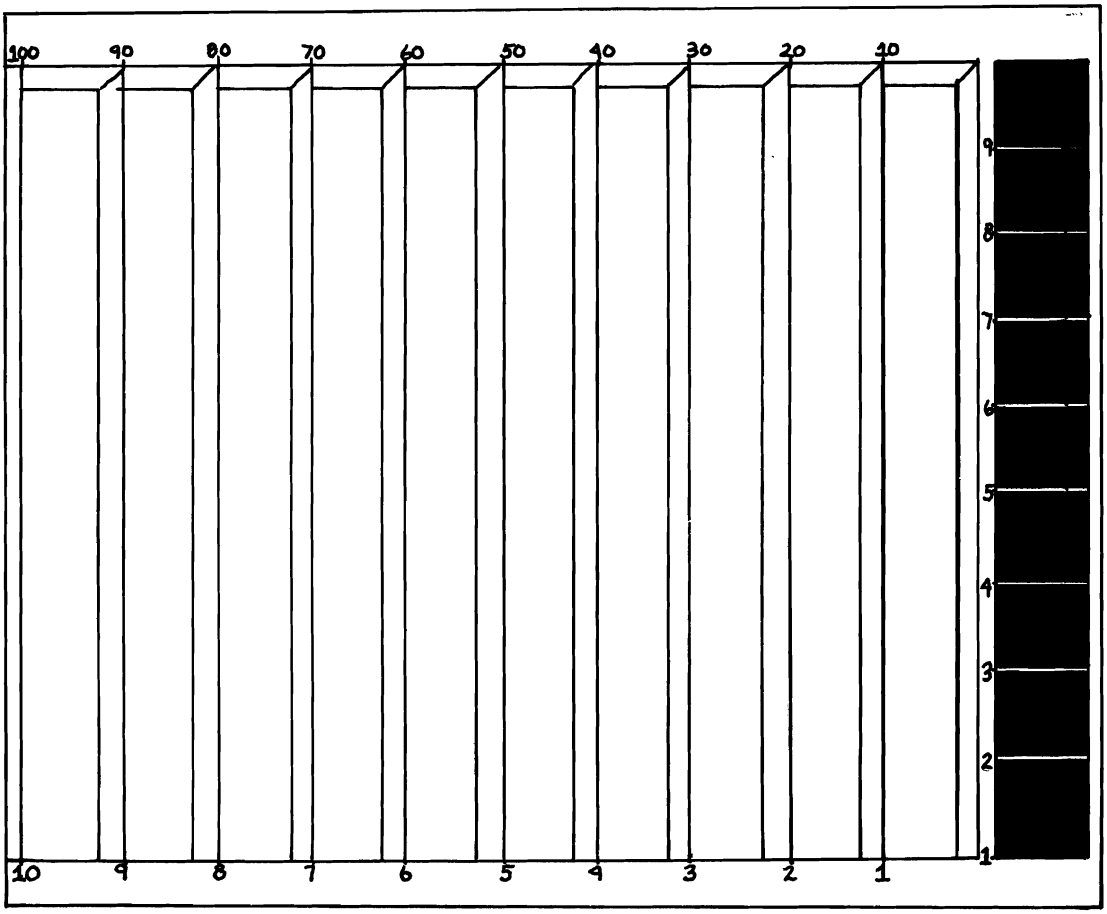
Stern (1977) and Flinter (1979) use a dual board to develop concepts in place value.

It contains ten rows each the length of ten blocks, enough to accommodate up to 100 blocks. There is a separate unit compartment, which is a single row ruled into ten units which are numbered from one through nine. No more than nine cubes are allowed in this separate unit. By deliberately adding one block to this separate unit brings the learner to the point where he must switch to tens. When the amount grows to ten singles, a solid ten block must be placed in the appropriate slot. This technique develops the concept of the value of the tens place. Cuisenaire Powers of Ten and Dunes Multi-Board Arithmetic Blocks also have a similar format and are excellent concept builders (Flinter, 1979).

Other activities recommended are:

1. Write a number and represent it with pictures (Flinter, 1979).

2. Cut out a number and match it with pictorial representations (Flinter, 1979).



Dual Board[3]

3. To train the child to indicate the relative position of digits ask questions as: "In 476, which digit shows the number of tens?" (Flinter, 1979)

4. Given _ _ _, ask the child where to put a six to make it mean six hundreds (Muro, 1979).

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[3]Permission for reproduction granted by Houghton Mifflin Company, Boston, Mass. from <u>Structural Arithmetic</u> by Catherine Stern, 1966.

The Four Operations

Developing skills and concepts in the four main operations, addition, subtraction, multiplication, and division, causes different problems for the dyscalculic child. Here are activities to clarify some common misinterpretations.

Earlier a discussion was made of some uses of Stern's pattern boards. These materials are also of use in developing the concepts of addition and subtraction. The addition and subtraction activity has three parts.

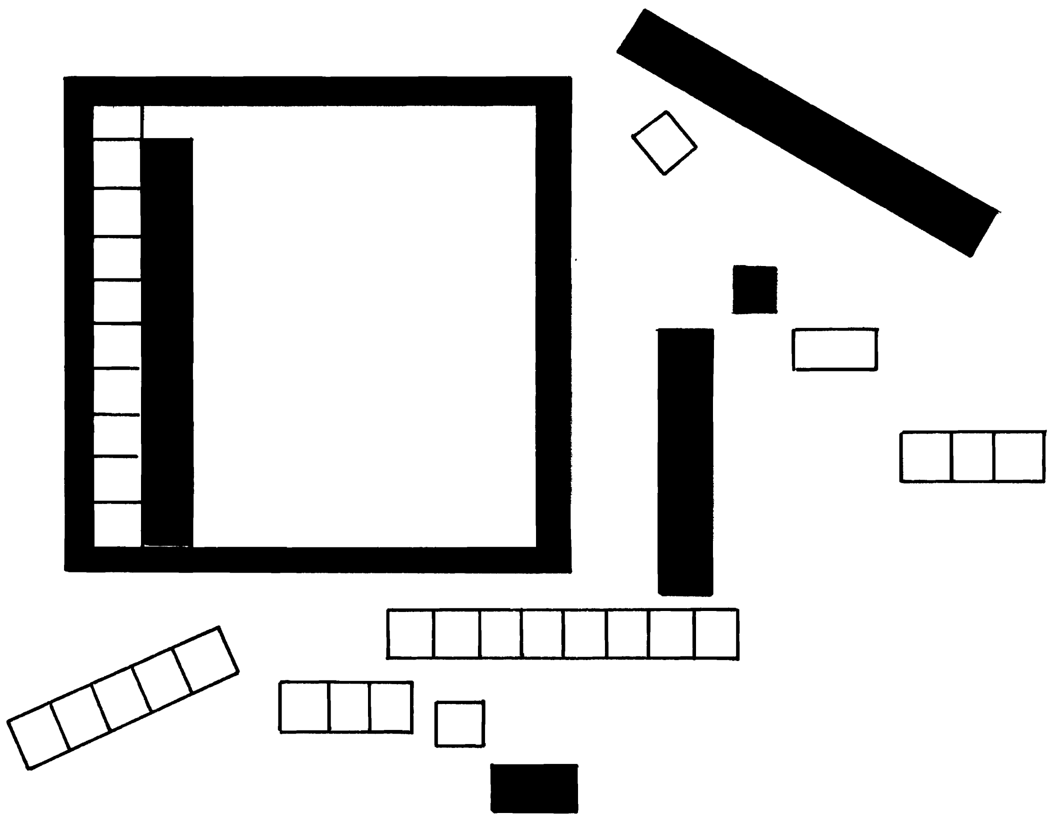
Step A: The teacher places a star by the empty pattern board to indicate which pattern she wants the child to build. The child constructs the matching pattern and names it.

Step B: The teacher points to a goal pattern board and asks how the child can change the pattern that was made to look like the goal. The child compares the two boards, notes the differences, and adds or subtracts the proper amount. The teacher asks the child to explain what he just did.

Step C: The teacher asks the child to name the new pattern number.

In the activity just described, the child is the "doer." He constructed the pattern, studied the desired goal, and met the challenge of altering the original pattern. "These sensorimotor experiences form some of the most important steps in a child's learning" (Stern, 1977, p. 180).

At a higher level of difficulty, Stern (1977) advocates the use of the unit box. In the unit box, pairs of blocks fit together to make combinations totaling ten. This instrument is first used at the sensorimotor level. It has an edge or boundary that helps the child see and feel the size of the total he is building. It develops relations, i. e. nine needs one.



Unit Box[4]

[4]Permission for reproduction granted by Houghton Mifflin Company, Boston, Mass. from Structural Arithmetic by Catherine Stern, 1966.

During the time the blocks are beginning to become familiar in their relations to each other, the teacher can play games to develop other abilities as well (e. g., visual memory, spatial orientation, eye-hand coordination, and mathematics vocabulary) (Stern, 1977). Devices such as number boards, counting boards and pattern boards provide matrices into which block or groups of cubes can be fitted. These kinds of formats set limits for each concept. Much as Montessori's program, the activities are very structured. This structure is good for learning disabled children. These materials also develop Gestalt (that is, a concept of wholeness) and they enable a child to self-correct his work (Stern, 1977).

Learning Math Facts

For the dyscalculic child, learning the basic facts of mathematical operations is usually very difficult. The "successful" learner used the "easy" facts to find the "hard" ones. Here are some activities which Meyers and Thorton (1977) feel will help the mathematically disabled child learn the basic facts. They suggest that addition facts be taught through the combination's relationship with each other. These are the steps they use.

1. Teach one more or one less concept.
2. Teach addition of like numbers.
3. Teach the doubles plus one.
4. Teach the sharing concept, that is in 7 + 5, 7 shares one with 5 making doubles 6 + 6.

5. Stress the zero principle.

If these five steps are followed, there are only ten combinations which must be memorized. They are: 2 + 5, 2 + 6, 2 + 7, 2 + 8, 3 + 6, 3 + 7, 3 + 8, 4 + 7, 4 + 8, and 5 + 8.

Meyers and Thorton (1977) further suggest some game-type activities to reinforce the five teaching steps.

1. Backwards Bingo: The teacher makes a lotto board where doubles plus one (e.g. 15) are randomly placed. The call cards should have two sides; on one side put the doubles plus one as, "7 + 7 and one more"; on the other side, problems as 7 + 8 = 15. The child covers the appropriate box when the problem is called.

2. Balance It: The teacher presents a sharing fact (6 + 8). The student, using Unifix Cubes and a small scale like device balances the problem.

3. Quick Flash: The teacher flashes a problem. Two students of about the same ability say the answer. The one who is first writes the answer in a "quick flash book." At the end of the session, the children total the numbers. The one with the highest score wins the game.

4. Flaps: This activity is designed for overlearning the ten facts which must be memorized. Using a folded piece of construction paper or a file folder, write the problem on the outside and cut a partial rectangle around it. Under the flap that is cut, place the answer. The child looks at the problem, answers it and then checks his answer by lifting the flap.

When the facts are learned, sometimes the dyscalculic child has problems transferring the information to more difficult problems. In this case, the teacher should make the student aware that only known facts are involved. For example, if the problem 38 is given, the teacher should
+ 47

show the pupil that 8 + 7 is solved by the doubles plus one concept and when the one ten is added to 3 + 4, it is the same as having the sharing principle.

Meyers and Thorton (1977) suggest a similar method of teaching multiplication facts to students who have problems in learning mathematics. This method is also done in five steps.

1. Teach twos, fives, and nines in that order. These are the easiest. The twos should be explained as meaning the same as the doubles in addition. The fives can be taught in terms of counting by fives. The nines can be reinforced by placing the nines in order on a chart.

   2 x 9 = 18

   3 x 9 = 27

   4 x 9 = 36

   When the child is shown that the sum of the digits of the product always equals nine and that the number in the tens place is always one number less than the lowest multiplier, the child can figure out each product

2. Teach the squares.

3. Teach the zero principle. (Zero times any number equals zero.)

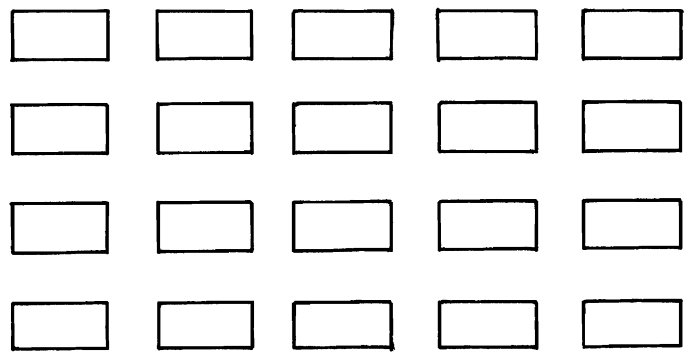
4. Teach the ones principle. (One times any number equals that number.)

5. There are ten facts which must be memorized. These combinations are 3 x 4, 3 x 6, 3 x 7, 3 x 8, 4 x 6, 4 x 7, 4 x 8, 6 x 7, 6 x 8, and 7 x 8.

Flinter (1979) also suggests six different activities for teaching multiplication and division and relating these operations to previously-taught addition and subtraction.

1. Arrange manipulatives into arrays or groups after verbally being given the problem.

   4 x 5 =

   This method enables the child to see the whole. He gets practice with the parts of the whole and becomes more aware of the visual groupings which are possible.

2. Serial order relationships can be worked through number lines.

3. Taking a particular number such as 24, represent it in many different ways, pictorially or concretely.

4. <u>Buzz</u>: The children count in groups and replace a predetermined number, e. g. 4, with the word "buzz." The game would sound like 1, 2, 3 "buzz," 5, 6, 7, "buzz." This also helps develop the concept of grouping.

5. Using graph paper, the teacher can have the child cut out three groups of seven and then seven and three more. This activity will emphasize the differences between addition and multiplication.

6. To illustrate the differences between division and subtraction, take graph paper which has been divided into sections which depict a particular number. Have the child illustrate 24 - 6 by crossing out six squares. Then have the child depict 24 ÷ 6 by making as many groups of 6 as possible by circling them.

These activities enable the child with difficulties in arithmetic to experience the concepts which are used in the four main operations in a multisensory manner. These experiences help the child to integrate the concepts more totally, therefore he learns them better and the ideas stay with him for a longer period of time; consequently, there is less relearning required. When less relearning is needed, the child not only progresses at a faster rate, but also has more successful experiences and fewer unsuccessful ones. Of course, success breeds success and helps the child develop a healthy self image.

Summary

This chapter discussed the possible causes, symptoms, and remedial activities which were found in available literature. The causes of dyscalculia range from genetically presupposed, to process dysfunctions, to poor teaching. The symptoms a child displays may show an inability to conceptualize the ideas being taught. Or the symptoms may be manifested by an inability to count, visualize objects, conserve, perform operations, sequence, or write numbers. Finally, the dyscalculic child may display his problem by showing a lack of attention, distorted body image, perseveration, or language dysfunction. The literature almost unanimously recommends remediation of dyscalculia by multisensory activities to help make mathematics meaningful to children. Finally, multi-faceted teaching techniques were suggested to help the dyscalculic child.

CHAPTER 3

AN EDUCATOR'S VIEW

The activities which have been presented heretofore encompass techniques which other educators, researchers, and clinicians have developed and found successful in ameliorating arithmetic deficits. Having taught mathematics to slow and disabled learners for five years, this educator has found other teaching techniques helpful to dyscalculic children. These techniques are now presented.

In educating, this author has found that it is beneficial to build a strong foundation for the four arithmetic operations by emphasizing place value, inequalities, and numbers and number systems. These three areas of mathematics should be taught in conjunction with each other so that a strong numerical concept is built. By developing these relationships early, the child and teacher can relate to them when teaching or learning more advanced skills in addition, subtraction, multiplication and division.

In order to better understand the different activities, they are presented in the progression used by this educator. Their relationships will be discussed as they are presented.

Numbers and Numbering Systems

The activities presented earlier are those used by this author to develop the concepts of single digit numerals. After the child is comfortable with the numbers from zero through ten, the educator should introduce reading and writing the numbers to one hundred. The progression this educator has found most successful is as follows:

1. Read and write the decades.
2. Read and write all the numbers in the 40s, 60s, 70s, 80s, and 90s.
3. Read and write the 50s, 30s, and 20s, in that order.
4. Read and write the numbers from 13 - 19.
5. Read and write 11 and 12.
6. Read and write numbers to 1000.

These six steps represent a progression which carefully breaks down the objective of learning to read and write numbers to 1000. It can be expanded upon for larger numerals. The following are some suggested successful activities and procedures used by this author.

1. Using a large number of counting sticks or tongue depressors and rubber bands, have the children make many bundles of ten. Emphasize that once the group of sticks gets to ten, a rubber band must be put on it. This procedure reinforces the value of one ten and therefore, the tens place is more meaningful.
2. Using the groups of bundled sticks, have the children count, write, and read the decades to one hundred. The teacher should emphasize that the reason a zero is always in the ones place is because there are no loose sticks which are not bundled.

3. Counting decades: Using a group of from four to ten children, have them each count their own ten fingers. The teacher should point out that each child has one ten. "If three of us use our fingers, how would we write and say the number?" Three tens and no single fingers gives us 30.

4. From the very beginning of writing two digit numbers have the children label the digits "tens and ones" or "T" and "O", e. g. T O
7 2.

5. Using counting sticks or tongue depressors and rubber bands have the children make random numbers in the 40s, 60s, 70s, 80s and 90s. These decades are easiest because the number of tens is cued auditorily.

6. Have the child listen to a dictated number and write it. Only the 40s, 60s, 70s, 80s and 90s should be used and emphasis should be placed on the tens place by the teacher, e. g. FORTY-two, SIXTY-seven.

7. After dictating numbers, have the child place a "T" over the tens place and an "O" over the number in the ones place.

8. Do similar activities as suggested in activities numbered four through seven for the decades 50, 30, and 20. These should not be presented in one day. Rather, devote three separate lessons, emphasizing auditory and verbal use of the words, fifty, thirty, and twenty.

9. Teaching the teens: Because these numbers are auditorily written in reverse from the other decades, children find them difficult to read and write. By explaining that parents often say that their teenagers do things "backwards," the "teen" numbers are written backwards from the way they sound.

10. Have the children make the teen numbers with sticks as before. The teacher should emphasize that the term "teen" comes from "ten."

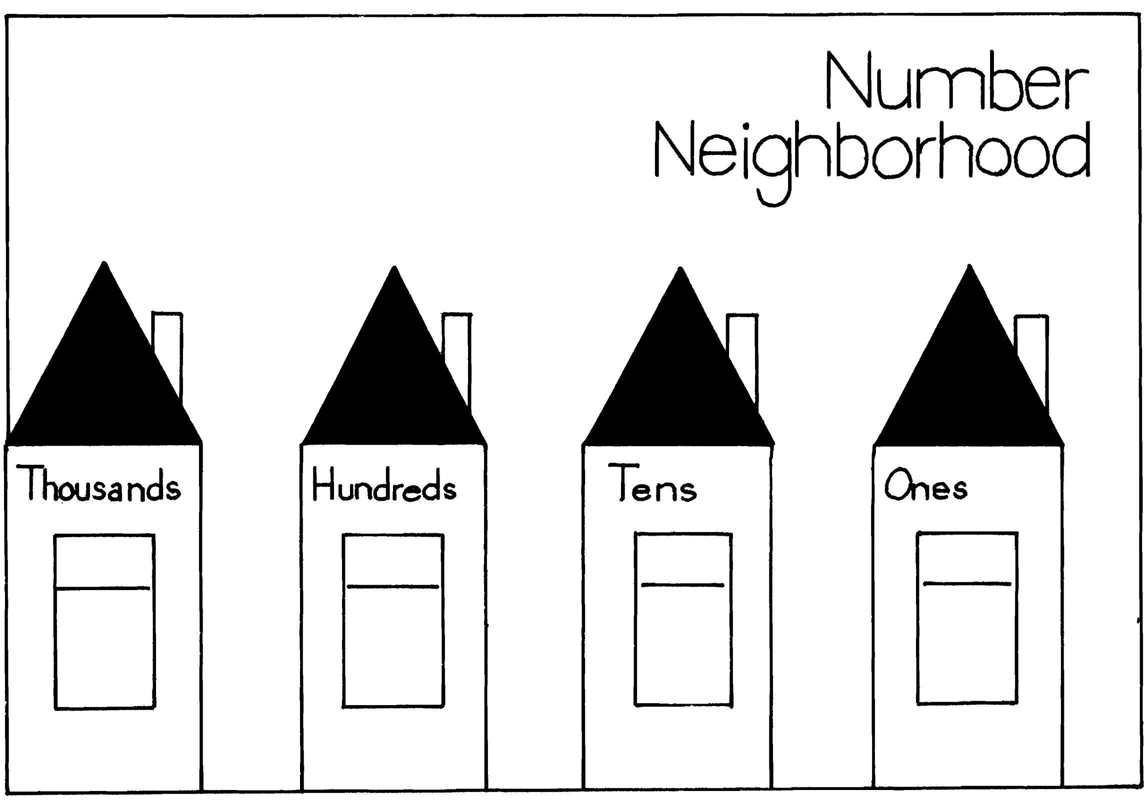
11. The teacher dictates teen numbers and has the children write the numeral and label the tens place "T" and ones places "O".

12. The numbers eleven and twelve usually must be taught in isolation. If the children are comfortable with the concepts given in reading and writing the other numbers to one hundred, writing eleven and twelve comes fairly easily. However, vocabulary drill may be needed in order that the children verbally remember the terms.

13. Reading and writing the numbers from 100 - 999 is done much as the first hundred numbers with one exception. The teacher should introduce the concept of reading and writing all the numerals which do not have a zero as a place holder first.

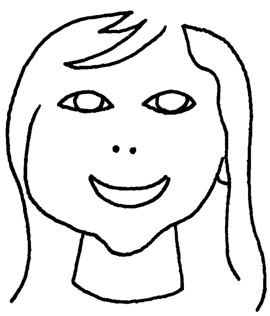
14. Place value houses: This activity is excellent for developing the concept of zero as a place holder. On a piece of poster board, draw three or four houses in a row. Place a library pocket in the center of each house. Label each house according to its place value.

Number Neighborhood

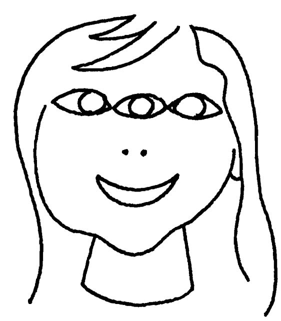


> The teacher should also make cards with single integers which fit in the pockets. By explaining that these houses "belong" to the place that lives there even if no number is "home" much like your house is yours even if no one is home, the child will be able to place integers in the pockets upon receiving a dictated number from the teacher.

The concept of even and odd numbers is often a guessing game on the part of the child who has problems in mathematics. This author introduces the concept by drawing two happy faces on the chalkboard like this:



even



odd

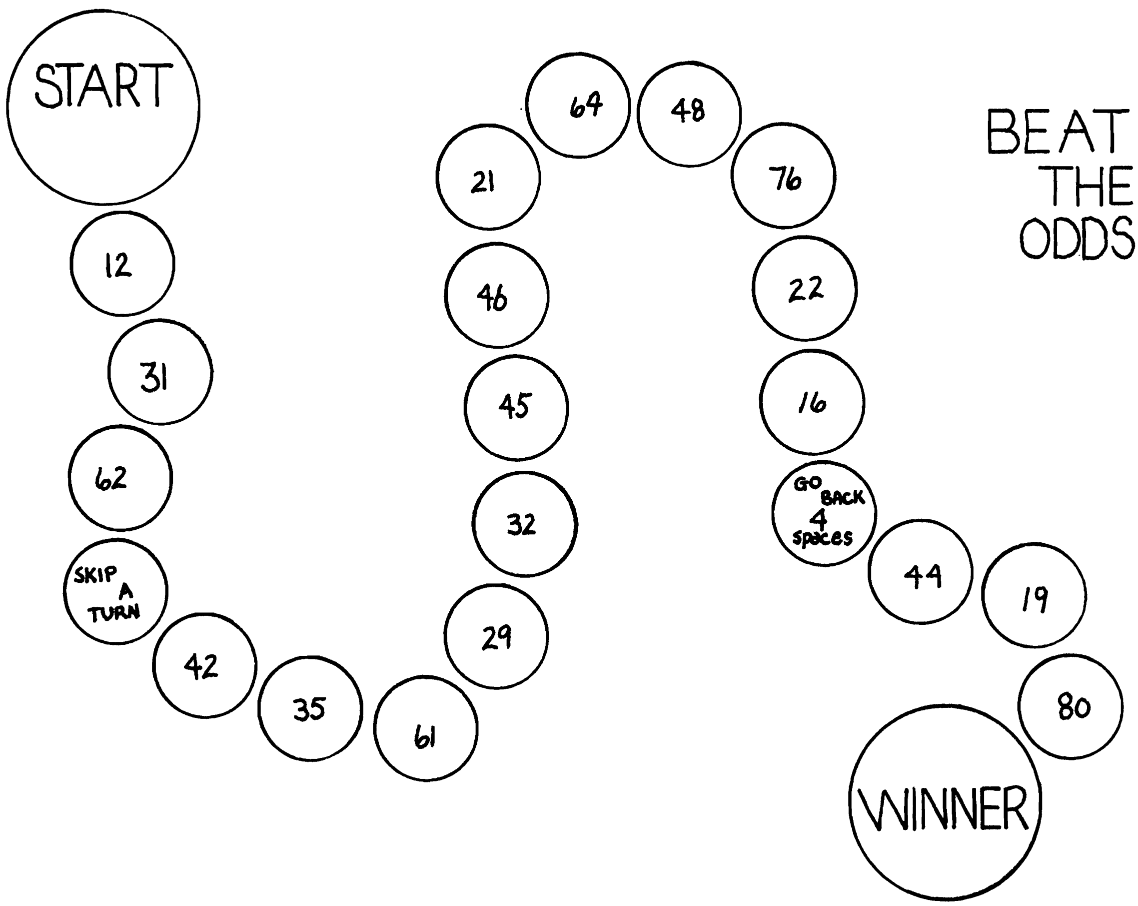
Immediately the children see that something is wrong with "odd." At this point it is easy to explain one definition of "odd" as meaning "strange" or "different." This motivates the children and they want to know more. At this point activities are presented to teach the concepts.

1. Using the child's two hands, two ears, two eyes, two legs, etc. explain that things that are in pairs are even, and those that are not in pairs are odd.

2. Giving the children ten items have them experiment and see if certain dictated integers are even or odd. In other words, can they be put into pairs.

3. Using a larger number of manipulatives see if they can, after seeing a dictated two digit number, generalize that they must only look at the ones place to see if a number is even or odd.

4. Beat the Odds: This is a teacher made game to reinforce the concepts of even and odd. On a piece of poster board make a path game.



In each circle write an appropriate numeral for the level of difficulty of the children using it. At the beginning of the game each child should be given five counters. They roll a single die and move to the appropriate circle. The child should read the number, and declare if it is an even or odd number. If it is even, the child receives two more counters from the "bank." If the number is odd, he must "pay" the bank one chip. There are two winners, the one who finishes the board first and the one who has the most chips.

Ordinal numbers usually have a self-motivating factor when the teacher says, "Who wants to be first in line for recess?" etc. However, there are a few gross motor activities which will help to reinforce this numbering system.

1. Role Playing: Playing games such as "Airplane" is fun for the children and helps develop language as well as the ordinal counting system. With a group of students set up a row of chairs, one behind the other. Select a steward or stewardess. The teacher says, "Give some coffee (etc.) to the third passenger." The student takes the coffee to the third passenger and says "Here is your coffee, Mr. Third Passenger. May I do anything else for you?" etc.

2. Using the same idea, "Train" may also be played with a conductor.

3. Ordinal Sitdown: Place ten or more chairs in a row. Have fewer children than the total number of chairs. Each child takes a dictated place and then changes his place as the teacher instructs (i.e., Mary sit in the fourth chair, now sit in the ninth chair).

More About Place Value

While the concept of place value should be first introduced when the child learns to read and write numerals, there are some teaching techniques which this educator uses to reinforce it in isolation.

1. Worksheet activities that have the child circle the integer of a certain place value.

2. Given cards with single integers, have the child construct the largest numeral possible. This also helps reinforce the concept of greatest.

3. Worksheet activities which ask how many tens are in 469.

4. In all activities where a written numeral is used, have the children label the place value with "H" "T" or "O" etc., before beginning to answer the problem further.

Inequalities

The concepts of "not equal," "greater than," "less than," "greatest," and "least" can be as much a language problem as a mathematical one. This educator introduces these concepts by making use of heroes and commercials. The questions which are presented to the class are, "What cereal is grrreat?" and "What boxer says, 'I am the greatest?'" The children immediately know the answers, and a discussion follows as to what the terms "great" and "greatest" mean. The activities which follow are first used to develop the concept of "greater than." When the children are comfortable with this, similar activities are used to teach the concepts of "less than" and "least."

1. Children construct certain sets of numerals and determine which is greater (less).

2. Using two digit numerals, a review of what the integer in the tens place means. Compare two two-digit numbers which have different integers in the tens place.

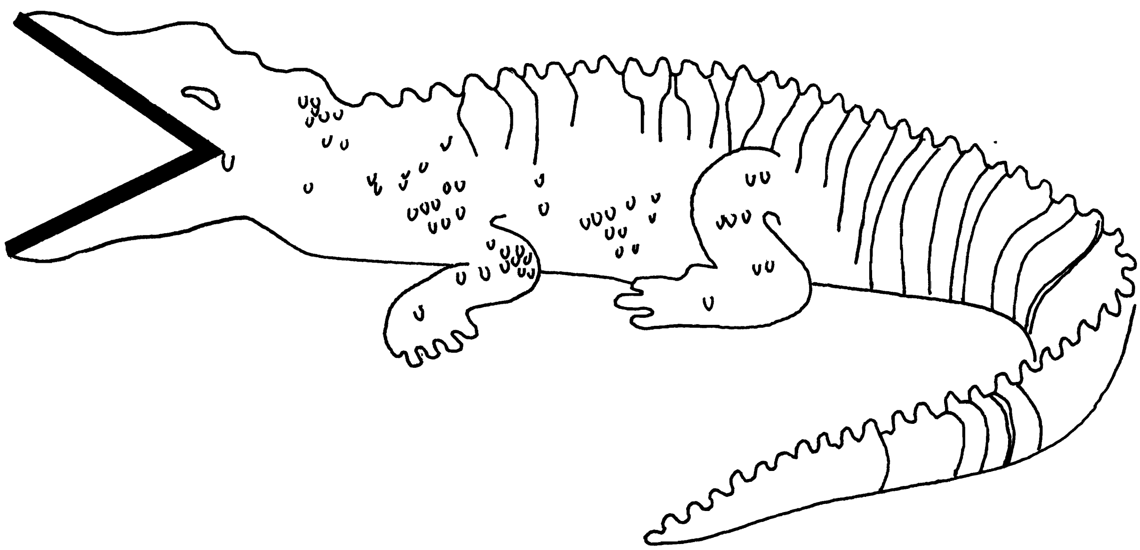
3. Compare two two-digit numerals which have the same integer in the tens place.

4. The child independently compares two numerals and circles the greatest (least).

5. If problems develop in processing the four earlier activities, have the children construct the numerals with like items and place them on a scale to see which is greater or less.

6. Using the scale again, introduce equals by having them construct like numerals and balance them.

The "greater than, less than" signs (<>) almost always are confusing to children. To clarify these this author introduces them to the "greater gator."

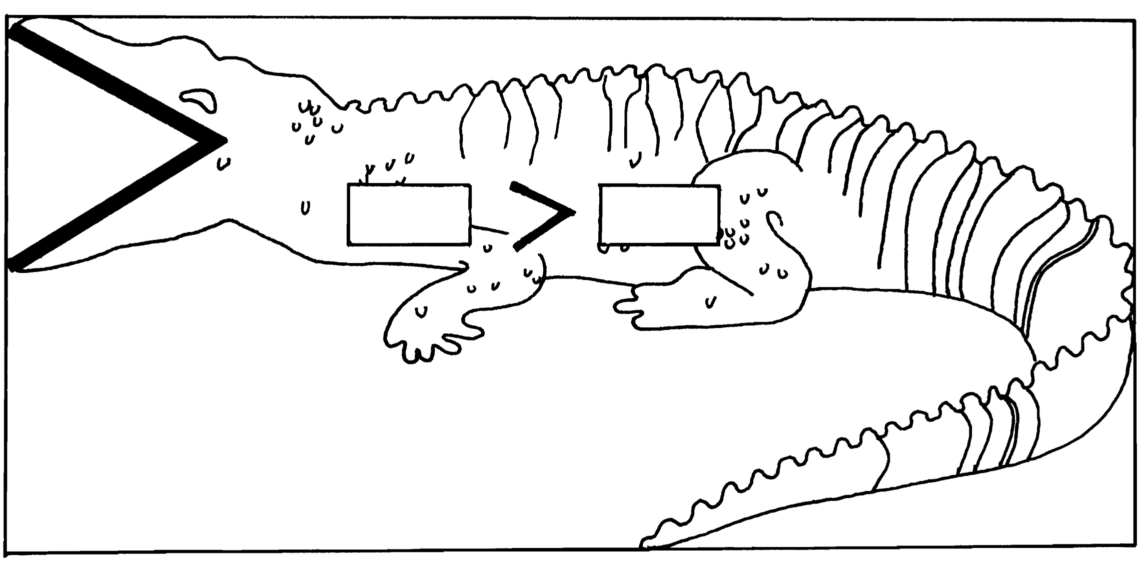


"greater gator"

It is explained that this fellow is <u>very</u> greedy and always wants to eat the greater number. Because he cannot move his mouth, he must always face the greater number so he can eat it.

To reinforce this concept, this educator uses the following activities.

1. The "greater gator" board:



Appropriate task cards are made and put face down in a deck. The first child chooses a card and places it in one of the squares in the center of the greater gator board. Each successive child chooses a card and decides on which side of the "greater than" sign the numeral should be placed to make the mathematical sentence correct. Then the child reads the sentence.

2. Make several gators out of poster board and place adhesive tape on the backs. Make the gators facing different directions. On the chalk board place two numbers across from each other and have the child place the appropriate gator between the numerals. It is at this point that the term "less than" is introduced with the appropriate gator.

3. As much practice in reading mathematical sentences using $>$ $<$ and $=$ should be given. In order that the instructor is sure the children are reading the mathematical sentences in the right direction, the practice in reading them should be done orally.

Addition and Subtraction

Much good literature has been presented about teaching techniques for the instruction of addition, consequently, this operation will not be discussed. However, subtraction has generally been ignored in the available research. There is one especially successful teaching technique which this educator uses to teach the concept of regrouping in subtraction.

When a child is first given a subtraction problem which requires that he regroup the minuend; he traditionally is stumped.

There are three steps which this educator has used to teach the concept of regrouping. These steps have consistently allowed the child to see success. Here are the techniques used:

1. Have the child construct a two digit number between 21 and 98. (Use counting sticks, or tongue depressors and rubber bands.) Tell the child that there is a need to subtract nine ones from the number he has made. Ask if that is possible. When he says yes, tell him to please do it. The child will lift as many ones as there are. (There should be less than nine loose sticks.) At this point. the child usually stops. He is unsure as to what to do. The teacher should guide the child to undo one of the bundles. Ask how many bundles are left. This procedure should be done until the child feels comfortable.

2. In step two, the teacher puts writing with the procedure described above. On the chalk board, the teacher should write a specific problem. As $\begin{array}{r} 32 \\ -\ 9 \\ \hline \end{array}$

   The child, using sticks, should compute the problem but at each step the teacher should stop the child and show how the problem is graphically depicted:

   e. g. $\begin{array}{r} 3\ 2 \\ -\ \ 9 \\ \hline \end{array} \qquad \begin{array}{rr} \overset{2}{\cancel{3}} & \overset{12}{\cancel{2}} \\ - & 9 \\ \hline \end{array}$

3. In the final step, the teacher talks the child through the abstract representation.

The teacher should plan on spending anywhere from one lesson to seven lessons with the child to teach this concept. However, this educator has found that once the child grasps it, there is <u>very</u> little reteaching because the child cannot remember how to regroup.

Multiplication

Teaching multiplication is also a well documented concept. However, few teachers feel that their slow learners can grasp the concepts of the least common multiple and greatest common factor. This educator has found that this is an incorrect judgment. As a matter of fact, the children who have learned these concepts have found division and fractions easier. Here are some suggestions for teaching these concepts.

1. Define each word in the phrases so that the children understand what they are looking for.

2. Least common multiple:

    a. Teach skip counting.
    b. Define multiples as the numbers found when the child skip counts.
    c. Compute two sets of multiples, one under the other.
    d. Circle <u>all</u> the common multiples.
    e. Discuss why they are common.
    f. Color code the least common multiple.
    g. Discuss why it is the least common multiple.
    h. Repeat this procedure until the child can do it independently.

3. Greatest common factor:

    a. Define factor.
    b. Find the factors of several numbers. (Let the child use a matrix chart if he has not committed the factors to memory.)
    c. Choosing two numerals, have the child factor them and choose the greatest common one.

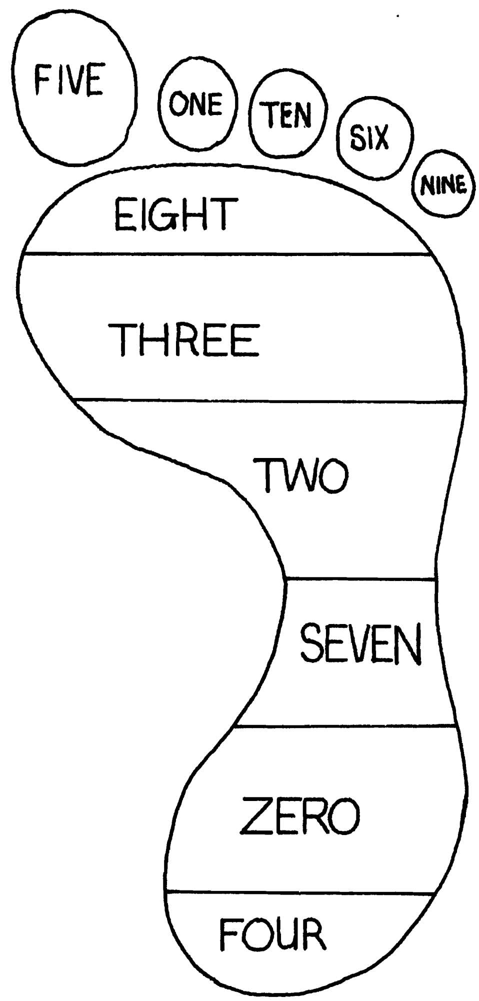
These two procedures will take several weeks. However, the time spent teaching the concept will be very helpful in division.

<u>Game Activities</u>

Games, when used correctly, are a successful way to motivate children and reinforce skills being taught in the classroom. The following suggestions for games are general in nature, but highly motivating. The children love playing them.

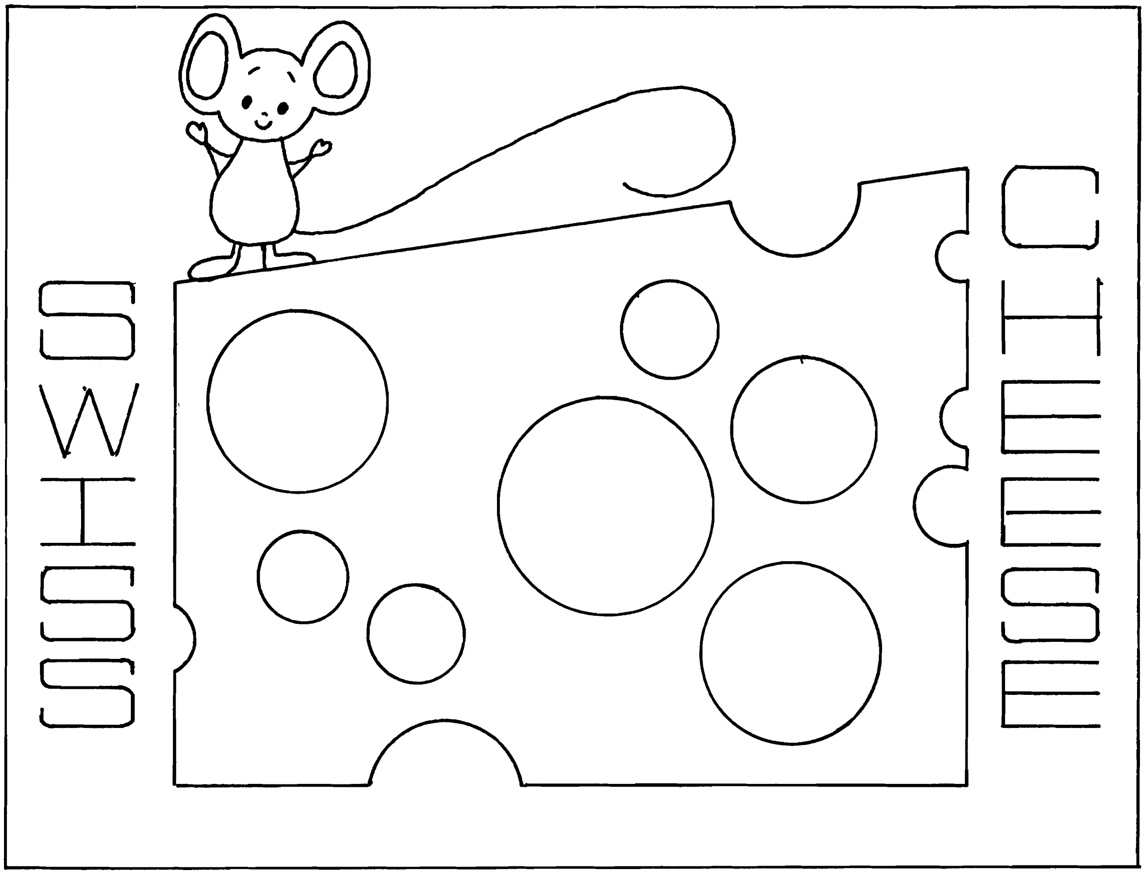
1. Quizmo: This is a commercially made game designed to reinforce math facts.

2. Big Foot: Teacher made game. This is much like Bingo. The shape of the board seems to make the game more fun. All spaces should be covered to win.



3. Swiss Cheese: Teacher made game. The teacher should make circular discs with a certain skill on one side of all the discs but one. On this single disc, a mouse should be drawn.

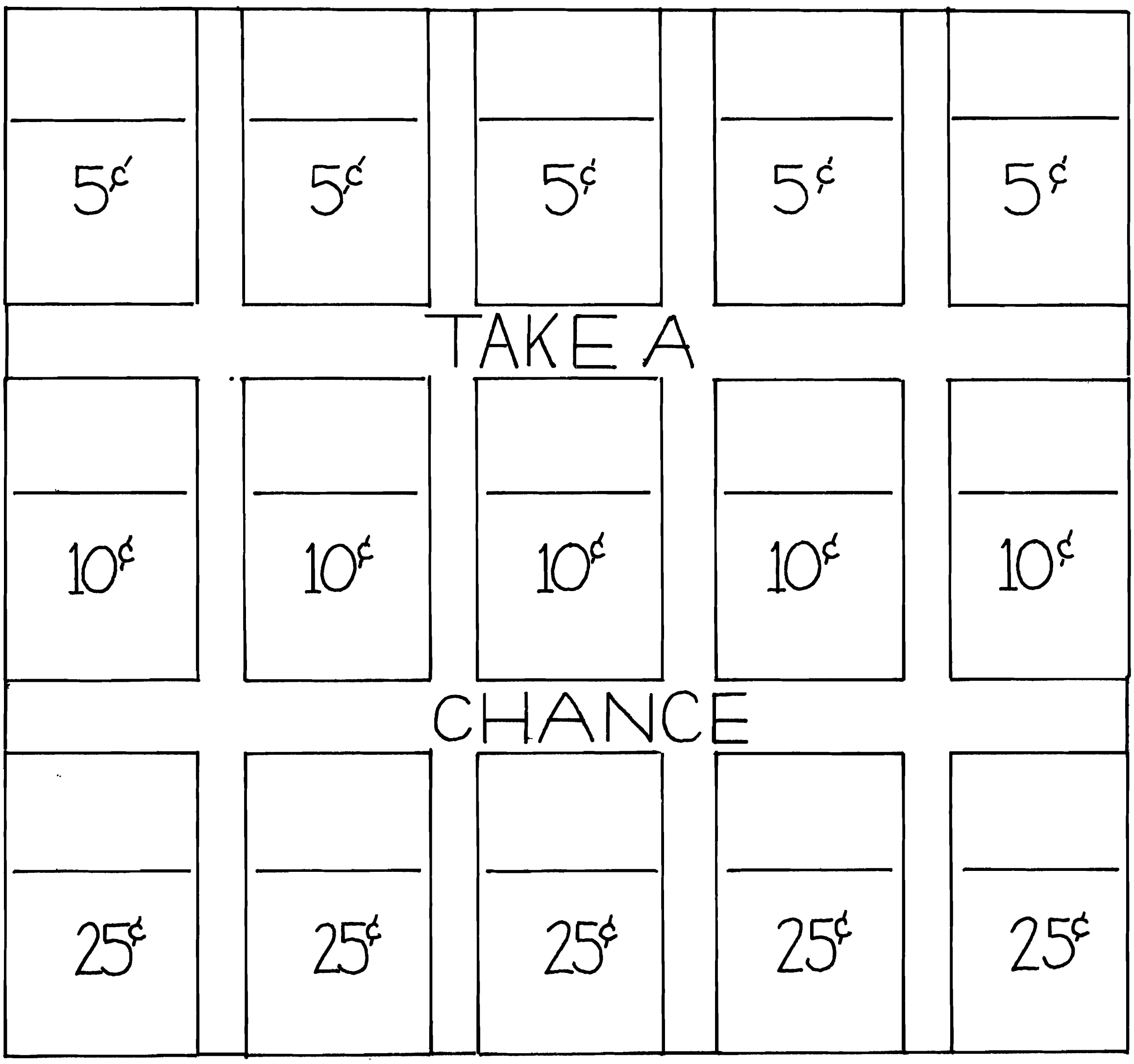
   Rules: In turn, each child draws one of the discs. If he can answer the question, he may keep the disc, if he makes an error he must return the disc to the board. If he draws the mouse, he must return all his accumulated discs. The teacher should mix up the discs that are returned to the board if the mouse disc is drawn. The winner is the child with the most discs when only the mouse card is left.



4. Know the Fact: Teacher made game. This game is played with teams. The teacher should make question cards and place them in a container. After dividing the group into two teams, each team should be given ten points in the form of hash marks on the chalk board. In turn, each team member is allowed to "bet" any number of points his team has. Then he chooses a card. If he answers the question correctly, the team wins that number of points; if he makes a mistake, the team loses that number of points. The winning team has the most points.

5. Take a Chance: Teacher made game. On poster board place rows of library pockets. Each row should be given a value as shown in the diagram. Task cards are made at three levels of difficulty; these cards are placed in the pockets. In turn each child chooses a card and answers it. Harder questions earn more points. The winner has the greatest amount of "money" or points.

TAKE A CHANCE



It is the sincere hope of this educator that these activities will help the reader in teaching more meaningful math lessons.

Conclusions

The findings of this educator in reviewing literature emphasized that dyscalculia has generally not been researched adequately. There was a noticeable absence of research done in this area of dysfunction. What was found was a number of possible causes for dyscalculia. This form of learning disability may be caused by genetic imperfections, neurological processing problems, or physical injuries. Its symptoms are varied and complex. The manifestations may be closely related to dyslexia or language dysfunctions. Finally, the literature has indicated that the best way to help the dyscalculic child is to structure his lessons tightly, and to allow him to experience the concepts in a multisensory manner.

Summary

This author has attempted to examine the possible causes for math dysfunctions as well as discuss their manifestations. Finally, in the hope of being of some aid to those who may read this endeavour, remedial activities of a multisensory nature have been presented. These activities are generally most appropriate for the elementary school-aged child, in deed, most of the literature which has been discussed was most significant for children in the age group from six to twelve years of age.

It seems that, as in other areas of the learning disabilities field, there is no one correct definition for the term dyscalculia. However, it became evident to this researcher that there is one commonality of all the presented definitions of this term. Dyscalculia refers to the inability to grasp mathematical concepts in a functional way, and it is this factor to which this research has been addressed. It was with sincere concern for the mathematically disabled child that this paper has been written.

REFERENCES

REFERENCES

Bryan, J. H. and Bryan, J. H. Understanding learning disabilities. Port Washington, New York: Alfred Publishing Company Inc., 1975.

Campbell, J. The effect of language instruction on math skills of retarded children. Mental Retardation, 1978, 16, 167-169.

Cohn, R. Developmental dyscalculia. The Pediatric Clinic of North America, 1968, 15, 651-659.

Danforth, M. M. Aids for learning mathematics. Arithmetic Teacher, December, 1978, 26, 26-27.

Flinter, P. F. Educational implications of dyscalculia. Arithmetic Teacher, March, 1979, 26, 42-46.

Herold, P. J. Reasons for failure; excerpt from The Math Teaching Handbook. Exceptional Parent, February, 1979, 9, 49-53.

Kaliski, L. Arithmetic and the brain injured child. Arithmetic Teacher, 1962, 9, 245-251.

Kosc, L. Developmental dyscalculia. Journal of Learning Disabilities, 1974, 7, 164-177.

Lansdown, R.. Retardation in mathematics: A consideration of multi-factorial determination. Journal of Child Psychology and Psychiatry, 1978, 19, 181-185.

Lerner, J. W. Children with learning disabilities. Boston: Houghton Mifflin Company, 1976.

Lovell, K. The growth of basic mathematical and scientific concepts in children. London: University of London Press, Ltd., 1966.

McLeod, T. M. and Crump, W. D. Relationships of visuospatial skills and verbal ability to learning disabilities in mathematics. Journal of Learning Disabilities, 1978, 11, 237-241.

Meyers, A. C. and Thorton, C. A. Learning disabled child; learning the basic facts. Arithmetic Teacher, 1977, 25, 46-50.

Muro, J. Language abilities and math performance. Reading Teacher, 1979, 32, 900-915.

Myklebust, H. and Johnson D. Learning disabilities; educational principles and practices. New York: Grune and Stratton, 1967.

Piaget, J. How children form mathematical concepts. Scientific America, 1953, 189, 74-79.

Stern, C. Children discover arithmetic. New York: Harpers, 1949.

Stern, M. B. Structural arithmetic and children with learning disabilities. Bulletin of the Orton Society, 1977, 27, 171-182.

Svien, K. and Sherlock, D. Dyscalculia and dyslexia. Bulletin of the Orton Society, 1979, 29, 269-276.

Swett, S. C. Math and learning disabilities: A new perspective. Academic Therapy, 1978, 14, 5-13.

Thorton, C. A. Rx = geometry; perceptual-motor help for many handicapped learners. Arithmetic Teacher, 1979, 27, 24-26.

Thorton, C. A. and Reuille, R. Classroom teacher, the L.D. child, and math. Academic Therapy, 1978, 14, 15-21.

Webster, R. E. Visual and aural short-term memory capacity deficits in mathematics disabled students. Journal of Education Research, 1979, 72, 277-283.

Wilson, R. Perceptual discrimination as a predictor of achievement in reading and arithmetic. Journal of Educational Psychology, 1979, 71, 220-225.