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Conservation of number

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THE CONSERVATION OF NUMBER

by

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CHAPTER I

INTRODUCTION

Early childhood education is designed to promote the development of young children. Curricula frequently attempt to fulfill this objective through enrichment programs with an experience or activity center approach to learning. Such enrichment programs usually try to stimulate a rather broad range of different abilities at any given time.

Although children benefit from enrichment programs, precision teaching should generate more learning because it focuses on the specific ability level of the child in a particular area. Precision teaching is the continual evaluation and integration of each child's level of development along three continua. First, what are the various levels of development within each concept or content area, and how do they interrelate? Secondly, what materials and methods affect the child's ability to decode or internally represent stimuli? Finally, how does the child use body movements or words to encode or respond? Given this information, the teacher should know: (1) the precise concept and level within the concept to develop, (2) the method of presentation and the materials to use, and (3) the type of response to request from the child.

Precision teaching should offer much satisfaction and learning to children with special learning needs. Although he has a deficit or weakness in one or more areas, a learning disabled child still learns most effectively according to developmental sequences. Due to its totally developmental nature, precision teaching maximizes each child's abilities while simultaneously improving his weakness(es). The National Advisory Committee on Handicapped Children of the U.S. Office of Education describes children with learning disabilities:

> Children with special (specific) learning disabilities exhibit a disorder in one or more of the basic psychological processes involved in understanding or in using spoken or written language. These may be manifested in disorder of listening, thinking, talking, reading, writing, spelling, or arithmetic.[1]

The learning disabled child should profit from precision teaching in early mathematics. The importance of mathematics is not only due to its inclusion in the standard school curriculum but also to its relationship to cognitive development. Piaget (1965) hypothesized,

> . . . The construction of number goes hand in hand with the development of logic, and that a pre-numerical period corresponds to the pre-logical level. . . . logical and arithmetical operations therefore constitute a single system . . . the second resulting from generalization and fusion of the first.[2]

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[1]National Advisory Committee on Handicapped Children, Special Education for Handicapped Children, Final Report (Washington, D.C.: U.S. Department of Health, Education and Welfare, January 31, 1968), p. 4.

[2]Jean Piaget, The Child's Conception of Number (New York: W. W. Norton, 1965), p. viii.

Commenting on Piaget's hypothesis, Sawada (1972) states,

> Piaget's inference that intellectual structures are isomorphic with certain mathematical structures implies that the teaching of mathematics has a direct and intrinsic role to play in the development of intelligence. . . . A cogent and empirically based argument can now be given to view the study of mathematics as a legitimate way of developing the intelligence. It is important to note that Piaget's theory can be used to justify such a role for mathematics if and only if mathematics is taught in such a way that the child's adaptation to the world of mathematics takes place through encounters with mathematics in which deeper and deeper levels of cognitive equilibrium are reached through internal reorganization by a process Piaget calls equilibration.[3]

An important concept in early mathematics and cognitive development is the conservation of number--the ability to correctly judge two sets the same in numeric value regardless of the physical arrangement of the sets. The child's ability to conserve number is significant because it heralds the child's entry into operational or logical thought.

Much psychological research is available on number conservation. Unfortunately, very little research relates number conservation to learning. The studies that do exist are primarily devoted to training procedures for number conservation in a research setting but don't provide any suggestions for implementation in a school setting.

Besides no transference of research on number conservation to curriculum, none of the available research relates

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[3]Daiyo Sawada, "Piaget and Pedagogy: Fundamental Relationships," Arithmetic Teacher 19 (April 1972): 297.

any of Piaget's theory to learning disabilities. However, children with other special needs have been examined in a Piagetian context (Woodward 1961; Hood 1962; Inhelder 1968; Brown, Bellamy, and Gadberry 1971; Wilson and Boersma 1974; Kahn and Reid 1975). The research indicates that children with special needs appear to follow the same sequence of stages in cognitive development as normal children. Therefore, a review of the available research on number conservation should provide pertinent information about number conservation in all children, including those with learning disabilities.

The purpose and outline of this paper were:

1. to review Piaget's theory of cognitive development and the characteristics of preoperational thought.
2. to review Piaget's research on number conservation, including the stages in development.
3. to evaluate studies, which indicate that conservation of number is innate and not stage-related.
4. to describe the routine test for conservation.
5. to examine the thought processes involved in the standard conservation problem.
6. to review the research on the non-instructional variables which are related to number conservation.
7. to analyze various factors in the presentation of number conservation problems.
8. to examine response factors in number conservation problems.
9. to discuss learning, particularly the research on training procedures for number conservation.

Five terms are pertinent to a discussion of number conservation and are defined as:

1. Cardinal number: the number of elements of a set.

2. Equivalent sets: the elements of two sets can be placed in one-to-one correspondence.

3. Number: a mathematical entity.

4. One-to-one correspondence: occurs "if the elements of two sets can be paired in some way so that each element of each set is associated with a single element of the other. . . . Each member of each set is paired with one and only one member of the other."[4]

5. Set: A collection of distinct, separate objects that are recognized as belonging to the specific collection.

Summary

Precision teaching is the sequential presentation of concepts. Each presentation is determined by the on-going evaluation and integration of each child's level of development along three continua: conceptualization, decoding, and encoding. An area for precision teaching is mathematics for young children with learning disabilities. The general aim of this paper was to explore one mathematical concept--the conservation of number--and to present training procedures based on the research findings.

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[4]National Council of Teachers of Mathematics, Topics in Mathematics, no. 1: Sets (Washington, D.C.: National Council of Teachers of Mathematics, 1964), p. 11.

CHAPTER II

PIAGET AND THE FACTORS INVOLVED IN NUMBER CONSERVATION

The concept of number conservation originated with the developmental psychology of Jean Piaget. A summary of his theory and the characteristics of the preoperational child are presented in order to provide background information for the reader who is not familiar with Piaget's work. Next, Piaget's research on number conservation is presented and followed by a review of research which conflicts with Piaget's. Finally, after a description of the routine test for conservation, a review of research examines the factors which are involved in number conservation, namely, the non-instructional variables, such as sex and age; conceptualization, decoding, and encoding.

The Developmental Psychology of Jean Piaget

The developmental psychology of Jean Piaget focuses on the ontogenetic development of intelligence. According to Piaget, intelligence evolves from biological structures through a person's active interaction with his environment. All structures have two interdependent, functional properties in common: organization and adaptation. As invariant and fundamental characteristics of intellectual activity, organization and adaptation are the essence of intelligence.

Cognitive organization is two-fold. First, each cognitive structure is internally organized into a "schema." Flavell defines a schema as:

> . . . a cognitive structure which has reference to a class of similar action sequences, these sequences of necessity being strong, bounded totalities in which the constituent behavioral elements are tightly interrelated. . . . A schema is a kind of concept, category, or underlying strategy which subsumes a whole collection of distinct but similar action sequences.[5]

Secondly, the individual schemas are closely integrated with each other into a stable, coherent whole. As the organization increases, the schemas simultaneously become more and more interrelated yet differentiated from each other. For example, a red block could be placed in many schemas, such as those for block, red, square, cube, wood, hard, and so on. But as Flavell stresses,

> All intellectual organizations can be conceived of as <u>totalities</u>, systems of <u>relationships</u> among elements. . . . An act of intelligence, be it a crude motor movement in infancy or a complex and abstract judgment in adulthood, is always related to a system or totality of such acts of which it is a part.[6]

The schemas are fluid and subjected to change through adaptation to the environment. Adaptation involves two complementary and simultaneous processes: accommodation and assimilation. Accommodation requires the individual

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[5] John H. Flavell, <u>The Developmental Psychology of Jean Piaget</u> (New York: Van Nostrand Reinhold, 1963), pp. 53-54.

[6] Ibid., p. 47.

to modify his structure(s) to fit the environment, such as in imitation. Accommodation is the coming to grips with the special properties of the thing apprehended. On the other hand, assimilation occurs when the person changes reality to suit his structure(s), such as in play. In assimilation, the individual interprets or assigns meaning to something in external reality according to his current structures. Flavell emphasized,

> The cognitive incorporation of reality always implies both an assimilation to structure and an accommodation of structure. To assimilate an event it is necessary at the same time to accommodate to it and vice versa. . . .
>
> . . . Changes in assimilatory structure direct new accommodations, and new accommodatory attempts stimulate structural reorganization.[7]

Major reorganization of structures are denoted by stages which have qualitative similarities and differences. The stages continuously develop in an invariant sequence, earlier stages being incorporated and transformed into the present stage. In the transition from one stage to the next, the structure's properties, which will define the coming stage, are being formed and organized. Temporary instability and disorganization result during the transition period. But gradually the new overall structure emerges as a unified, integrated whole in stable equilibrium--the balance between assimilation and accommodation. Equilibrium is the main goal and the fundamental process of mature

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[7]Ibid., pp. 48-50.

thought.

The Stages in Cognitive Development

The developmental sequence consists of three major stages or periods: (1) sensorimotor, (2) concrete operations, and (3) formal operations.

Sensorimotor Period (0-2 years)

To the neonate, nothing is differentiated. The infant doesn't even primitively perceive himself as something separate from his environment. The world is an unknown mass which temporarily exists when it is within the child's immediate perception.

The neonate initially is centered about his body. He first responds to the environment and his needs through his reflexes but gradually develops new action-responses. Through his increasingly complex interactions and the accompanying structural development, the infant decenters from the self and slowly learns to differentiate his self and other objects from the whole. At the end of this period, he has developed the concept of object permanence, which is vital to future cognitive development.

Concrete Operations Period

Preoperational subperiod (2-7 years)

At the beginning of this subperiod, the child is developing primitive inner representations of his world. The representations are still closely related to concrete objects and actions but become increasingly abstract or symbolic.

The primary characteristic of this subperiod is the child's mental disequilibrium--the inability to balance accommodation and assimilation. Lack of equilibrium results in an unstable, disorganized, present-oriented, discontinuous cognitive structure.

Another principal characteristic of this subperiod is egocentrism, which is similar to the <u>action</u> centering of the neonate. So too, the preschooler <u>thinks</u> the world centers about him. The child views events from his own perspective and can not perceive another's position. Through arguing and social interaction, the young child gradually decenters his thought.

Concrete operation subperiod (7-11 years)

In contrast to the preschooler who is in transition from action to inner thought, the child's cognitive structures are in a state of equilibrium. His mental system is a coherent, flexible, enduring, integrated organization. The cognitive actions of this structure are operations. An operation is the transformation of reality by means of internalized action, which is characterized by one element--the representations are grouped into coherent, <u>reversible</u> systems. The structure of an operation is the grouping, of which there are nine variations. The grouping is the logical composite of the mathematical concepts of group and lattice. In comprehending the various groupings, Flavell suggests,

> . . . A useful rule of thumb, one Piaget has used, . . . is to say that all the actions implied in common mathematical symbols like +, -, X, =, <, >, etc., belong to, but do not exhaust, the domain of what he terms <u>intellectual operation</u>.[8]

<u>Formal Operations Period (11-15 years)</u>

An adolescent's structure is in the final, highest state of equilibrium. The adolescent can deal with the possible and the hypothetical and is not limited to reality. He explores problems by first considering all <u>possible</u> solutions. Through deductive reasoning and experimentation, the adolescent determines which of the possible relations or solutions are true or real. His reasoning includes combinatorial analysis through which he isolates all the variables in the problem and all of the different combinations of these variables. In addition, the adolescent manipulates propositions which are based on the results of concrete operations on reality data. Prepositional thinking is formal operations and is called "operations to the second-power."

The Characteristics of Preoperational Thought

The child, who is developing number conservation, is in the subperiod of preoperational thought. Preoperational thought is characterized by:

1. <u>Centration</u>: the child sees only one aspect of a thing, e.g., the child sees the height of a container but not the width.

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[8]Ibid., p. 166.

2. Irreversibility: the child can't reverse his thought or return to the beginning of his thought, e.g., the child does not understand that 3+8=11 is related to 11-8=3.

3. Actions: the child replicates but doesn't reconstruct, e.g., the child repeats an operation point for point but can't go immediately from start to finish.

4. States: the child sees things staticly, e.g., the child draws a picture of a pencil falling, which he first shows as being vertical to the table top and then horizontal to the table top without any of the intervening angles of the fall.

5. Concepts: the child fails to find stable identity in contextual change, e.g., the child can't see that table, chair, and tree are all made of wood.

6. Egocentrism: the child sees the world through his own eyes and can't see other persons' points of view, e.g., the child has his own personal language.

7. Disequilibrium: there is no balance between accommodation and assimilation in the child's thought, e.g., the child assimilates by making a box into a plane.

8. Realism: to the child, thought, dreams, and names are real events and objects, e.g., "My dream was in the room and went to bed with me."

9. Animism: to the child all things are alive and conscious, e.g., "The clouds are alive like people."

10. Artificalism: everything was made by and for man and God, e.g., "Daddy, make the cloud stop raining."

The Conservation of Number

As defined earlier, the conservation of number is the equation of two sets, each set having the same number of units, without regard to the physical arrangements of the units in each set. Piaget used two different types of materials and their corresponding methods of presentation to study the development of the conservation of number.

The two forms are provoked one-to-one correspondence and spontaneous correspondence.

Provoked one-to-one correspondence employs objects which are dissimilar in appearance but which are qualitatively complementary or associated to each other, such as by function. The specific association partially determines the degree to which one-to-one correspondence is provoked or stimulated. The three sets of associations in Piaget's original research and their corresponding "provoking" capacities are:

1. Least provoking: a glass placed near a bottle.

2. More provoking: a flower placed in a vase with the possibility of the child's placing more than one flower in a vase.

3. Most provoking: an egg cup which can hold only one egg.

These sets of associated items were presented to each child by first placing one set of objects on a table and then showing the child the associated set on a tray. The child was instructed to match one item in one set to one item in the other set. Piaget provided numerous, verbatim accounts of his research interviews with children:

> . . . "Look at these little bottles. What shall we need if we want to drink?--<u>Glasses</u>.--Well, there they are. Take off the tray just enough glasses, the same number as there are bottles, one for each bottle." The child himself makes the correspondence, putting one glass in front of each bottle. If he takes too many or too few, he is asked: "Do you think they're the same?" until it is clear that he can do no more. . . . Once the correspondence is established, the six glasses are grouped together and the child is again asked: "Are there as many glasses as bottles?" If

> he says "no," he is then asked: "Where are there more?" and "Why are there more there?" The glasses are then rearranged in a row and the bottles grouped together, the questions being repeated each time.[9]

On the other hand, spontaneous correspondence uses similar but not associated sets of objects, such as red and blue counters. A series of figures, which were made with counters, were presented:

> . . . I, "badly-structured" figures, e.g., a collection of counters distributed at random, but neither touching nor overlapping; II, open series, e.g., two parallel rows of counters; III, closed figures, the shape of which did not depend on the number of elements used, e.g., a circle, a house, a right angle; IV, closed figures of which the shaped depended on the number of counters, e.g. a square, a cross, etc. V, more complex closed figures, less familiar to the child, e.g. a rhombus, etc.[10]

After receiving his own set of objects, the child was shown the series of figures and told, "'There is a number of objects: pick out the same number.'"[11] No method was suggested to the child as how to accomplish this. After the child finished this, he was asked if the figures and his counters were the same and why. If the child thought both sets were the same, Piaget would rearrange one set and again question the child.

The children's reactions can be separated into three stages in the development of both provoked and spontaneous correspondence.

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[9] Piaget, The Child's Conception of Number, pp. 42-43.

[10] Ibid., p. 66.

[11] Ibid., p. 65.

Stage 1: Global Comparison

In the first stage, the child totally relies on perception to determine the equivalence of two sets. The child views a set of discrete units as a continuous or single quantity. The length or density (closeness) of the rows are the critical perceptual cues. A child in Stage I usually thinks the longer row has more elements even though it may contain four elements while the shorter row has four or more elements which are more compressed or closer together. Also, the two rows are judged equal if both rows are the same length or have the same density, irregardless of the number of objects in each row. With the series of figures, the children try to reproduce both the configuration and the dimensions of the model, but they aren't concerned with details.

Stage II: Intuitive Correspondence

During the second stage, the child can intuitively or visually establish one-to-one correspondence and equivalence by placing one object near or in the associated object. In the series of figures, the child makes exact copies of the models. But the established visual equivalence is not permanent. If one of the two rows or figures is rearranged, such as by being extended or compressed, the child asserts that one set has more items. In both Stages I and II, the child ultimately depends on the overall appearance of the set rather than the number of units and consequently makes simple quantitative relationships, such as "big," "long," and "narrow."

Stage III: Operational Correspondence

The child in Stage III overcomes the intuitive or optical comparison and relies on operational thought. He understands that one-to-one correspondence remains and that, therefore, the number is constant or invariant, irrespective of the sets' configurations or rearrangements.

The permanency of one-to-one correspondence results from the reversibility of operational thought. Piaget (1967) stresses that reversibility does not mean empirical return, e.g., elastic can become longer and then shorter, but is a logico-spatial or operatory concept, such that movement from A to B is nullified by returning B to A. For example, the child establishes visual equivalence, then extends or compresses one set, and finally mentally comprehends that he can return the objects to visual equivalence by the inverse operation. In coordinating these actions, the child basically understands the two displacements or "equilization of differences": (1) that if the elements of a row or figure are spread out, the number per unit of length diminishes; and (2) that if the objects are compressed, the number per unit of length increases.

Counting

Additional research revealed that counting each set of objects did not influence the child's judgment of equivalence. Some children were able to correctly count the objects in a set but did not assign the last number to the set. For an example, some children counted a set of five pennies, "One, two, three, four, five." When asked how many pennies there were, many children either repeated the series of numbers or replied, "Four." The children could not determine the cardinal number for a set.

Research on Stage Development

Dodwell (1960), Elkind (1961), Beard (1963), and Almy, Chittenden, and Miller (1966) confirmed Piaget's delineation of three stages for the development of number conservation. Initially categorizing the subjects' responses according to Piaget's three stage sequence, Little (1972) suggested classifying the responses into ten groups rather than the clear stage sequence:

Category	Main Characteristics of Responses
"A	Random actions, seems to lack comprehension of basic concepts of 'more' or 'same.'
"B	Understands basic concepts but makes global undifferentiated responses.
"C	Negative, 'silly,' and tangential.
"D	Perceptual attribute and unable to explain choice.
"E	Perceptual attribute and explains reason for choice.
"F	Makes comparisons using fingers, pencils, etc., to 'measure.'

"G Changed answer when queried, unable to explain why.

"H Changed answer on query and can give reason.

"I 'Knew' correct answer on first question and unable to give reason.

"J 'Knew' correct answer on first question and could give explanation.

"Note.--level 1 = categories A, B, C; level 2 = categories D, E, F; level 3 = categories G, H, I, J."[12]

Number Conservation in Very Young Children

Before proceeding with the review of research of the factors involved in conservation, research which seriously differs from Piaget's research on number conservation is first examined. Estes (1956), Mehler and Bever (1967), and Bever, Mehler, and Epstein (1968) found that very young children are capable of conserving number. This contradicts Piaget's and others' findings, which place the acquisition of number conservation at approximately 6 or 7 years, and questions the validity of stage-theory.

First, Estes (1956) employed 52 children, who ranged in age from 4 to 6 years, to study number conservation and obtained no evidence to support Piaget's theory as to the development of stages in the acquisition of mathematical concepts.

> . . . This study found (a) that if children could count, they counted correctly whatever the arrangement of objects; (b) they did not confuse extension of line

[12]Audrey Little, "Longitudinal Study of Cognitive Development in Young Children," Child Development 43 (September 1972): 1028.

> with increase in number; (c) they did not mistake an apparent increase with a true increase in number . . .[13]

Wohlwill (1960) criticized Estes' use of the cross-sectional method, the relatively small number of subjects tested, and inadequate methodology.

In another attempt to determine the age of acquisition of number conservation, Mehler and Bever (1967) tested seven age groups of children, who ranged in age from 2-4 to 4-7, in order to study a form of quantity conservation. Each subject participated in two number experiments. The format for both experiments was:

> . . . One of the experimental sequences for each child had clay pellets while the other had M&M candies. . . . In each experimental sequence the child was first presented with adjacent rows of four, as in 1a, and he was asked if they were the "same." The experimenter then modified the arrays into a situation like 1b, in which a short row of six is adjacent to a longer row of four. In the experiment with clay pellets he was then asked which row had "more." In the experiment with M&M the responses to situation 1b were nonverbal: instead of asking the child to state a quantity judgment, the experimenter asked him to "take the row you want to eat, and eat all the M&M's in that row."[14]

Mehler and Bever concluded from the results that "under 3 years 2 months (3-2), children exhibit a form of quantity conservation; they lose it as they get older and do not exhibit it again until they are about 4 years 6 months (4-6)."[15]

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[13]Betsy W. Estes, "Some Mathematical and Logical Concepts in Children," Journal of Genetic Psychology 88 (June 1956): 221.

[14]J. Mehler and T. Bever, "Cognitive Capacity of Very Young Children," Science 158 (October 1967): 141.

[15]Ibid., p. 141.

The temporary inability to conserve between 3-2 and 4-6 appears to result from an overdependence on perceptual stategies and expectancies of length, which develop from experiences in which the longer row usually has more elements. Through additional experience, the child becomes more sophisticated at integrating logical operations with his perceptual rules, which allow the subject to count the items in each set and discount or ignore his perceptual expectancies. In summary, conservation is an innate structure which eventually surfaces. Bever, Mehler, and Epstein (1968), Calhoun (1971), and Bryant (1972) also subscribed to the idea that children can retain quantity judgments right from the start.

In a somewhat similar experiment, Gelman (1972) used three groups of 32 children, whose age ranges were 3-8 to 3-11, 4-0 to 4-11, and 5-0 to 6-5 years, to study children's reactions to unexpected subtractions, additions, and displacements. The experiment involved three phrases: (1) examiner played with each subject individually in order to establish rapport; (2) expectancies about two arrays of mice were established in an identification task; and (3) the children's reactions to surreptitious subtractions, additions, and displacements in the sets were assessed.

Gelman concluded:

> . . . The experimental paradigm employed above yields clear evidence that, for small numbers, children as young as 3 years old possess a concept of number that is independent of the irrelevant dimensions of length and density. Furthermore, they possess a logic that treats the cardinal number of a set as invariant under spatial displacement of its elements. The logic

requires that subtraction or addition operations intervene if the cardinal number of a set decreases or increases and appears to recognize that addition operations reverse subtraction operations and vice versa.[16]

Mehler and Bever (1967) have received much criticism from many sources (Piaget 1968, Rothenberg and Courtney 1968, Achenbach 1969, and Siegel and Goldstein 1969). Mehler and Bever (1968) and Bever, Mehler, and Epstein (1968) have attempted to answer these criticisms. Other studies (Beilin 1968a, Beilin 1968b, Rothenberg and Courtney 1969, Siegel and Goldstein 1969, Willoughby and Trachy 1971, Gelman 1972, Pufall and Shaw 1972, Rose 1973, and Winer 1974) have failed to support Mehler and Bever's results, that children under 3-2 years conserve number. Some of the same critical comments can be applied to the other studies.

One such critic is Piaget (1968). Piaget first explained that "perception" of length, which supposedly interfers with number conservation from ages 3-2 to 4-6, is already based on cognition structures, and, therefore, is not a process to be later integrated with logical operations. Instead, Piaget proposed that very young children base number judgment on a topological relation of "crowding," which refers to density, or on the relative lengths of the two rows. The child depends on the particular configurations to select either density or length as the basis for his

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[16]Rochel Gelman, "Logical Capacity of Very Young Children: Number Invariance Rules," Child Development 43 (March 1972): 86-87.

judgment. Piaget posited that the ratio of the lengths of the two rows perhaps determines the selection of density or length. For an example, as the lengths' ratio approaches 1, the probability of using relative density to determine judgment increases.

Second, in addition to other research which is noted later in this chapter, Piaget found that the subjects in his experiment considered the row, which the experimenter manipulated in any way, as having "more" or "a lot" and the undisturbed row as having "a little." This is important because Mehler and Bever's experiment always manipulated the most numerous row.

Third, Piaget asserted that Mehler and Bever's experiment had nothing to do with number conservation. According to Piaget, Mehler and Bever dealt with conservation of inequality, which does not prove or disprove conservation of equality. In this regards, Piaget insists that conservation be defined as:

> . . . the invariance of a characteristic despite transformations of the object or of a collection of objects possessing this characteristic. Concerning number, a collection of objects "conserves" its number when the shape or disposition of the collection is modified, or when it is partitioned into subsets.[17]

Fourth, Mehler and Bever apparently thought that Piaget disagreed with them on the natural tendency of young children to conserve. On the contrary, Lunzer (1972) and Piaget agreed that young children conserve as long as

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[17] Jean Piaget, "Quantification, Conservation, and Nativism," Science 1968 (November 1968): 978.

they are not confronted by facts, which they don't expect. The confrontation places the child in disequilibrium, and the facade or pseudoconservation becomes apparent.

> In general, children expect conservation, but since they cannot know beforehand what will be conserved and what will not be conserved, they have to construct new means of quantification in every new sector of experience. The inadequacy of the means of quantification explains nonconservation, and it is worth noting that nonconservation therefore indicates an effort to analyze and to dissociate variables; very young children and severely mentally retarded subjects pay no attention to these variables, whereas the older, normal children pass through a stage of nonconservation as they reorganize relations which they cannot yet grasp in full.[18]

Fifth, Beilin (1968a) notes that Mehler and Bever "both added objects to their numerical arrays and relocated them in a single operation. Thus it is not possible to know whether a child's response was due to addition or relocation, or both."[19]

Sixth, Rothenberg and Courtney (1968) note that Mehler and Bever based their conclusions on a single biased question in both experiments, which is much easier than the standard conservation question.

Finally, in reading the remaining sections of this chapter, the reader will note additional grounds for criticism, such as the experimenter did not request the subject to explain his judgment, the earlier appearance of conservation of inequality than conservation of equality,

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[18] Ibid., p. 978.

[19] Harry Beilin, "Cognitive Capacities of Young Children: A Replication," Science 162 (November 1968): 920.

and the earlier understanding of "more" than "same."

Piaget's Crowding Theory

As presented in Figure 1, Pufall and Shaw (1972) explained that preoperational thought employs Rules 1, 2, and 3 to partially coordinate length, density, and number in order to correctly judge the numeric relations between linear sets. Only operational thought generates Rule 4 to completely coordinate length and density, which are inversely related, and number.

As discussed earlier, Mehler and Bever (1967) and Bever, Mehler, and Epstein (1968) indicated that 2-year-olds can make correct judgments of numeric equality, as in Configuration 5, and of numeric inequality, as in Configuration 6, even when the subject was not shown the first phase of the standard conversation problem, Row A equals Row B. Piaget suggested "crowding" to explain these findings. If the child does use crowding as the basis of his number judgment, a very young or older child should judge that the more dense rows in Configurations 3 and 4 and the longer rows in Configurations 2, 5, and 7 have more. Piaget's theory would predict that the children would only be successful on the number judgment tasks, which meet the criteria for "crowding."

In order to test the validity of Piaget's theory, Pufall and Shaw presented 163 children, who ranged in age from 3 to 6 years, 7 number problems, which were identical

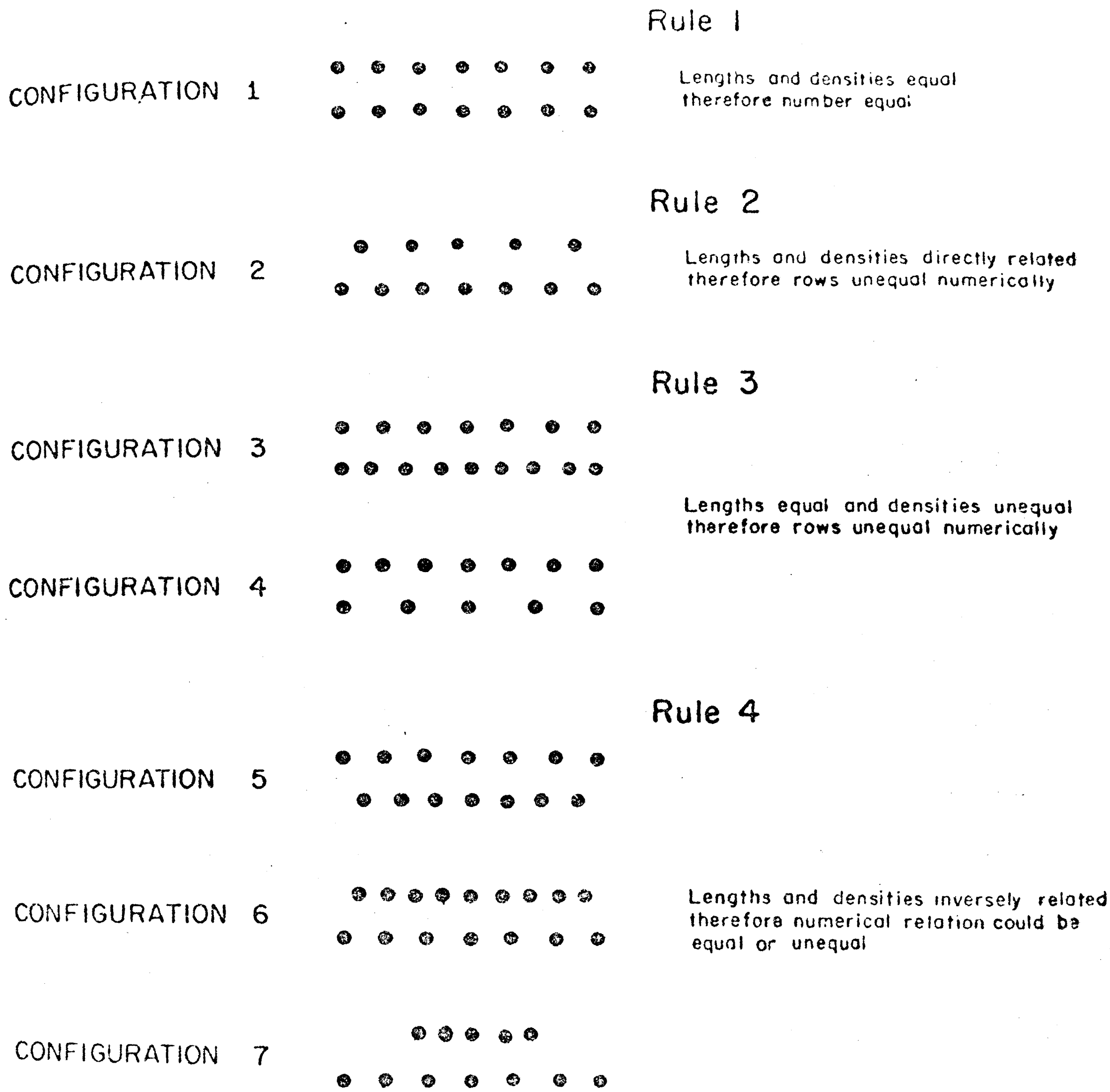

FIG. 1. Configuration and rules relating their spatial properties to number judgments.[20]

to the configurations in Figure 1. Each problem was presented in the static form so that the children did not see a transformation in order to "maximize the child's dependence on perceptual rules and eliminate judgments based on

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[20]Peter B. Pufall and Robert E. Shaw, "Precocious Thoughts on Number: The Long and the Short of It," Developmental Psychology 7 (July 1972): 63.

transformational relations."[21]

The results for Configuration 1 show that 42% of the 3-year-olds made correct judgments in comparison to all of the older children. On the other hand, there were no significant age differences across ages, which indicated that more 3- and 6-year-olds than 4- and 5-year-olds made correct judgments. Based on the percentage of correct answers, Configurations 1, 2, and 7 were the easiest; Configurations 3, 4, and 6 were more difficult. The most difficult was Configuration 5.

A further analysis of performance determined that 4% of the children made the correct judgment on all seven problems. No child consistently used the relative density hypothesis, but 22% of 96 4- and 5-year-olds appeared to use the relative length hypothesis on all seven tasks. On Configurations 3, 4, and 6, the youngest and the oldest children made significantly fewer length judgments than the middle-aged groups did.

Pufall and Shaw concluded that the youngest children did not conserve equivalence, thus failing to support Bever, Mehler, and Epstein's (1968) findings. The youngest children's use of either "crowding" or relative length as the basis for their number judgments was consistent with Piaget's (1968) hypothesis. However, the results suggested a different order

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[21] Ibid., p. 65.

of difficulty for the configurations within Rule 3 and Rule 4 than that generated by Piaget's theory. In order to explain these findings, three developmentally ordered models were proposed. Overall though, Pufall and Shaw (1972) and Pufall, Shaw and Syrdal-Lasky (1973) found the configurations, which were generated by Rules 1, 2, and 3, were solved prior to the configurations, which were generated by Rule 4.

Routine Test for Number Conservation

Most of the subsequent research on number conservation was modeled after Piaget's initial studies. Gelman (1969) described the routine procedure for the test for number conservation:

> In general, a test for a child's ability to conserve quantity involves the following sequence of events: (1) An S is shown two identical objects or set of objects; (2) he is then asked to judge whether the two objects are quantitatively equal; (3) if S says that they are equal, E alters some perceptual but no quantitative properties of one of the stimuli; (4) S is asked once more if the two objects (changed versus altered) are still equal with respect to amount; (5) and finally S is asked to explain his judgment. If S says the stimuli still have equal amounts and is able to explain his answer logically (Piaget, 1952), he is judged a "conserver." Alternatively, if he fails to indicate that the amounts are equal or gives a nonlogical explanation, he is judged a "nonconserver."[22]

Elkind (1967) symbolized this entire procedure as:

$$S \overset{t_0}{=} V \qquad V \overset{t_1}{\rightarrow} V' \qquad S \overset{t_2}{?} V'$$ [23]

"S" represents the standard stimulus, "V" for the variable

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[22]Rochel Gelman, "Conservation Acquisition: A Problem of Learning to Attend to Relevant Attributes," Journal of Experimental Child Psychology 7 (April 1969): 167.

[23]David Elkind, "Piaget's Conservation Problem," Child Development 38 (March 1967): 16.

stimulus, and "V'" for the transformed variable stimulus. As previously noted, the transformation (V → V') of the variable only alters the perceptual but not the quantitative equivalence between the variable and the standard. The standard question about equivalence is usually referred to as "Q" and frequently worded, "Does one of the rows have more checkers, or do they both have the same amount?"

Conceptualization

Identity and Equivalence Conservation

Piaget (1953, 1965) initially divided his research on the conservation of number into provoked correspondence and spontaneous correspondence. Although the distinction between provoked and spontaneous correspondence is important, Elkind (1967) and Piaget (1967) noted a more basic theoretical distinction within the standard conservation task format, regardless of the content. Elkind posited that the subject's judgment of the equality or the inequality of S ? V' involves two different forms of conservation: identity and equivalence.

Hooper (1969) defined identity conservation as "the realization that the single stimulus transformation B into C [the same as V→V'] does not alter a fundamental property of the quantity in question."[24] In the standard conservation task, the subject never directly judges the equality or inequality of V and V' after the transformation. The transformation

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[24]Frank H. Hooper, "Piaget's Conservation Tasks: The Logical and Developmental Priority of Identity Conservation," Journal of Experimental Child Psychology 8 (October 1969): 235.

occurs, and the subject is questioned about S ? V'. Based on the subject's response to S ? V', the experimenter infers $V \neq V'$ or $V = V'$, which is identity conservation. Elkind (1967) represented this inference of identity conservation as:

Conservation of Identity	Nonconservation of Identity
S judges S = V	S judges S = V
S judges S = V'	S judges S ≠ V'
E infers V = V'	E infers V ≠ V'[25]

On the other hand, Elkind defined equivalence conservation as "the invariance of a quantitative relation (of equality, inequality, etc.) across a transformation of one of the elements of the relations."[26] Equivalence conservation is directly tested in the standard conservation problem and presupposes identity conservation as is demonstrated in the following paradigms:

Conservation of Equivalence	Nonconservation of Equivalence
S judges S = V	S judges S = V
(Covertly) S judges V = V'	(Covertly) S judges V ≠ V'
S judges S = V'	S judges S ≠ V'[27]

Rewritten with Elkind's symbols, Northman and Gruen's (1970) sequence of the steps, which are involved in the equivalence conservation task, and the corresponding mental operations, which the subject must perform at each step, are:

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[25] David Elkind, "Piaget's Conservation Problem," Child Development 38 (March 1967): 16.

[26] Ibid., p. 17.

[27] Ibid., p. 17.

Before transformation

"Step 1. S = V. Required operation: Given.

After transformation (V → V')

"Step 2. V = V'. Required operation: Identity of substance and conservation of identity of amount, or compensation, or reversibility.

"Step 3. S = V. Required operation: Recall.

"Step 4. S = V'. Required operation: Transitivity or deduction.[28]

Piaget's writing on conservation can be misleading or confusing. Although he used an equivalence conservation format, Piaget's primary interest was the basic mechanism, which was used by the subject to judge the identity or equality between V and V'.

Piaget used the subject's explanation of how he arrived at his S ? V' judgment to study this basic mechanism. Instead of an explanation for equivalence conservation, the subjects' replies usually related to identity conservation. The responses were really *post hoc* rationalizations instead of accurate reflections on how the subjects arrived at their judgments. The subjects felt that conservation was logically necessary and, therefore, needed justification.

The three types of verbal explanation were:

1. Addition-Subtraction Schemas or Identity: nothing has been added or taken away so it

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[28]John E. Northman and Gerald E. Gruen, "The Relationship Between Identity and Equivalence Conservation," *Developmental Psychology* 2 (March 1970): 311.

is the same.

2. Reversibility: if you make it like it was before it will be the same.

3. Equation of Differences or Compensation of Relations: what is lost in one way is gained in another.

The third type of explanation, "equation of differences" or compensation," was the basic mechanism to arrive at identity conservation. As was discussed earlier, Piaget posited that

> . . . the child gradually comes to see that for any given object a change in one dimension is exactly compensated by an equal and inverse change in a second dimension. This discovery--that when the dimension of a given quantity are altered the dimensional differences compensate one another--underlies the child's insight that transformations are reversible and that they leave the object (property or quantity) invariant.
> . . . the child comes to employ a calculus of discontinuous equations or differences so as to arrive at the notion of a continuous or reversible transformation.[29]

The equation of differences does not adequately explain the judgment of equivalence between S and V'. Elkind (1967) reviewed research which has demonstrated that a subject never arrives at equivalence conservation when only presented with S and V' (Northman and Gruen's Step 4) without exposure to S = V and V → V'. The presentation of S and V' in isolation confronts the subject with an illusion which is extremely difficult to overcome, such as in the Müller Lyer.

In addition to identity conservation, equivalence con-

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[29]David Elkind, "Piaget's Conservation Problem," Child Development 38 (March 1967): 18-19.

servation requires the logical deduction of transitivity: if S = V and V = V', then S = V'. Since equivalence conservation requires the additional sequence of transitivity, Elkind further argued that the conservation of identity appears earlier than the conservation of equivalence. On the other hand, Piaget assumed,

> . . . identity and equivalence conservation are simultaneous in time, and that the age of equivalence conservation is also the age of identity conservation, so that it is legitimate to infer the age of the latter from the age at which the former is attained.[30]

Hooper (1969) used two conditions for equivalence conservation. Although similar to the standard paradigms for equivalence conservation, Hooper's equivalence conservation I matched the perceptual features of the identity condition by placing S behind an opaque screen immediately prior to the transformation of V→V', thus requiring the subject to "remember" the appearance of S. Equivalence conservation II followed the conventional paired-stimulus format. The subjects were males and females from kindergarten, first, and second grade classrooms in predominately white, middle-class neighborhoods.

Almost the same percentage of subjects conserved in equivalence conservation I as in equivalence conservation II, which verifies that both conditions actually assessed equival-

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[30]David Elkind, "Piaget's Conservation Problem," Child Development 38 (March 1967): 23.

ence conservation. The percentages of subjects passing identity conservation were 50, 75, and 75 as compared to 9.1, 54.2, and 66.7 for equivalence conservation for the kindergarten, first, and second grade subsamples, respectively. Hooper concluded that identity conservation is developmentally prior to equivalence conservation.

The children's explanations for their judgments were categorized and also indicated a distinction between identity and equivalence conservation.

> . . . the predominant explanation categories for the identity and equivalence cases were noticeably different. Aproximately 50% of the identity explanations focused upon addition-subtraction schemas, e.g., "no seeds were added or taken away." This response has generally been considered an explicit, logically consistent justification and unequivocal evidence of successful conservation performance . . . Its differential appearance in the present Ss' identity explanations adds further support to the developmental priority of identity over equivalence conservation.
>
> The equivalence conservation case, in contrast, is usually "solved" by a reference to the previous state of equality between standard containers A and B [in our case, sets S and V]. Acknowledging the dangers of an uncritical acceptance of young children's introspective rationales, . . . it is noteworthy how often the present equivalence subjects offered reasons closely approximately a logical deduction sequence.[31]

The research on the identity-equivalence problem is far from conclusive. In addition to Hooper's (1969) evidence, Schwartz and Scholnick (1970), Papalia and Hooper (1971), Siegel (1971), and Elkind and Schoenfeld (1972) supported Elkind's hypothesis. On the other hand, several studies have been unable to demonstrate the distinction,

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[31]Frank H. Hooper, "Piaget's Conservation Tasks: The Logical and Developmental Priority of Identity Conservation," Journal of Experimental Child Psychology 8 (October 1969): 245.

including Northman and Gruen (1970), Murray (1970), Moynahan and Glick (1972), and Koshinsky and Hall (1973). The subjects' ages and/or the differences in the content areas analyzed may account for the different results. Additional research is needed to resolve these differences.

Mechanism for Transition

Piaget focused his attention on children in Stage II, the transitional stage between Stage I and Stage III, to examine the mechanism which is responsible for cognitive development. Various theories have attempted to define this mechanism or process.

Gelman (1969) offers a discrimination inhibition theory, which attributes cognitive growth to an inhibition of attention to the previously utilized set of cues and a reorientation or shift in the child's attention to a new set of cues. Jeffrey (1968) proposed that the shift in attention may be due to a process of adaptation to previously relevant cues thus freeing the child to focus on new cues. Extending these discrimination theories, Melnick's (1973) theory is based on the inhibition of stimulus intensity as a mechanism of cognitive development.

> According to discrimination-inhibition theory, the intensity with which primary-concrete stimuli such as form, color, and brightness are experienced by young children captures and holds the orienting response, thereby preventing the children from attending to the relevant stimuli. This is in accord with Titchener's (1966) suggestion that intense stimuli have a binding hold on perception. As the child grows older, the intensity of these stimuli is reduced by both learned and maturational processes. The reduction in intensity frees the orienting response for voluntary control, so

> that the child can refocus on other dimensions, including higher-order invariant abstract relational cues in the stimulus flux. Once the orienting response is freed, factors such as the proportion of relevant and irrelevant stimuli become important, and the child becomes capable of shifting and reversing his orientation to the stimulus array.[32]

In order to test his discrimination-inhibition theory, Melnick used 48 normal and 37 educable mentally retarded students as subjects in a within-subject design and chose longer and longer increases in the length of row V (V → V') as the stimulus distortion. Melnick reported,

> The results support the hypothesis that normal and EMR children who are transitional in respect to conservation of number tend to give conservation responses at small (but noticeable) degrees of stimulus distortion, but fail to give conservation responses at larger degrees of stimulus distortion.[33]

In addition, the study proposed additional research to determine the effects of the intensity of other cues, such as types and number of stimuli and intensity of distortion, on various developmental tasks.

Conservation Extinction

Piaget (1970) stated that structural transformations, such as occurs in the transition from preoperational thought to concrete operations, involve a qualitative reorganization of the mental structures to more adaptative structures. Strauss and Liberman (1974) projected from Piaget's position that a child with the new and qualitatively different

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[32]Gerald Melnick, "Mechanism for Transition of Concrete to Abstract Cognitive Processes," Child Development 44 (September 1973): 599.

[33]Ibid., p. 604.

structures would have difficulty retrieving the old mental structures, and, therefore, should not regress to the previous structure.

In order to test this hypothesis, subjects, who were pretested as conservers and thus had concrete operational thought, were assigned to either a control group or one of two conditions: (1) the subject received various number conservation tasks during which time the experimenter surreptitiously added a bead to the longer row when $V \rightarrow V'$ and (2) the same as the first condition except the experimenter surreptitiously added a bead to the shorter row during the transformation.

The first condition was predicted to be the most likely condition to entice a subject to switch from a conserving to a nonconserving judgment. When confronted with the additional bead in a row after expecting equivalence between the two rows, practically all of the subjects rejected both types of experiences. In addition, the more implausible an empirical violation of a conservation law, as in the second condition, the more likely it was to be suspect.

Group Theory

Although not directly connected with number conservation, Gyr, Willey, Gordon, and Kubo (1974) suggested that the notion of the group of transformations be applied to perception. As a part of Structuralism, a group is:

> . . . any system consisting of a set of elements together with a rule of combination, e.g. a rule of transformation. One basic property of a group is that the transformations applied to elements of the system never lead beyond the system but always engender elements that belong to it and preserve its laws. To introduce the notion of a group of transformations into a discipline means to unify that discipline and to move it to a more advanced theoretical level.[34]

It was positied that successful application of group theory to perception and other psychological areas, such as visual-sensory processes, motor behavior, and cognition, might not only help to unify individual areas but also facilitate unification across areas. However, Piaget (1969) argued that preceptual processes don't possess mathematical group structures since they inhibit the development of cognitive structures.

This author proposes that both cognitive and perceptual development are governed by the same mechanism and are, therefore, mutually interdependent. Next, if the areas are interrelated, the distinction between cognitive stages and the mechanism for transition from one stage to the next might be more thoroughly examined with information from the other areas or from the overall structure. Finally, although Piaget is a Structuralist, he has only related mathematical group structures to operational thought, i.e., the groupings. A small attempt has been made to apply mathematical group structures to preoperational thought, but much additional research is needed for this period of development as well

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[34]John W. Gyr, Richmond Willey, David Gordon, and Richard H. Kubo, "Do Mathematical Group Invariants Characterize the Perceptual Schema of Younger and Older Children?" Human Development 17, n. 3 (1974): 176.

as the sensory-motor stage.

The Neurological Framework Of Mathematical Development

Piaget's cognitive framework for mathematical development is influenced by maturation of the central nervous system. Farnham-Diggory (1968) explored the relationship of the central nervous system and mathematical ability by first noting two basic principles:

> "1. Different areas of the brain are dominantly concerned with special functions like seeing, hearing, and touching.
>
> "2. 'Constellations' of cells from these special areas may work together in carrying out a higher mental activity."[35]

Neurologically correct instruction in mathematics promotes new and strengthens old connections between the vision and motor areas. The notational systems in mathematics must be processed visually and then related to action if comprehension is to occur.

Number Conservation and the Illinois Test Of Psycholinguistic Abilities

Yom, Wakefield, and Doughtie (1975) investigated the relationships between the Illinois Test of Psycholinguistic Abilities (ITPA) (Kirk, McCarthy, and Kirk 1968) and the Concept Assessment Kit - Conservation (CAK) (Goldschmid and Bentler 1968). The ITPA is composed of

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[35]Sylvia Farnham-Diggory, "On Readiness and Remedy in Mathematics Instruction," Arithmatic Teacher 15 (November 1968): 614.

12 subtests to assess children's specific language abilities along three dimensions: (1) two channels which are auditory-vocal and visual-motor; (2) three processes which are reception, organization, and expression; and (3) two levels which are automatic and representational. The CAK consists of six tasks dealing with the conservation of two-dimensional space, number, substance, continuous quantity, weight, and discontinuous quantity.

Fifty-two kindergartners received both the ITPA and the CAK. The product moment correlations between the CAK task for Number Conservation and the ITPA subtests are: (1) Auditory Reception .43 ($p < .01$), (2) Auditory Association .34 ($p < .05$), and (3) Visual Closure .30 ($p < .05$), and (4) Grammatic Closure .33 ($p < .05$).

From the correlations between the 12 subtests on the ITPA and the six tasks on the CAK, the researchers found:

> . . . two subtests were found to be related to all the instances of conservation included in the CAK. These are the Auditory Association and Grammatic Closure subtests. Alston and Wakefield have suggested that these two subtests measure the same process, the organization process, at their respective levels of the clinical model of the ITPA. Wakefield and Carlson (in press) have shown that these two subtests are highly related to Verbal Intelligence, as measured by the Wechsler Intelligence Scale for Children. They suggested that the organization process in the auditory-vocal channel of the ITPA and Verbal Intelligence are similar constructs.[36]

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[36] B. Lee Yom, James A. Wakefield, Jr., and Eugene B. Doughtie, "Psycholinguistic and Conservation Abilities of Five-Year-old Children," Psychology in the School 12 (April 1975): 152.

Non-Instructional Variables

Age

Elkind (1961), Goldschmid (1967), Kahn and Garrison (1973), Pufall, Shaw, and Syrdal-Lasky (1973), Rose (1973), and Nelson (1974) found chronological age to be significantly related to number conservation--the older the child, the greater the number of conserving responses. Miller (1973) found the expected age difference when the easiest conservation task (A) was presented first and the hardest problem (G) last, but not in the G to A condition. Rothenberg (1969) reported significance of age for lower class subjects but not for the middle class subjects. However, Pace (1968), D'Mello and Willemsen (1969), and Roll (1970) did not find any significant difference between chronological age and the three stage placements.

Mental Age and I.Q.

Dodwell (1960), Elkind (1961), Hood (1962), Goldschmid (1967), Pace (1968), Rothenberg (1969), and Klauss and Green (1972) have found intelligence to be positively related to number conservation ability.

Reliability

Rothenberg and Courtney (1969) and Pufall, Shaw, and Syrdal-Lasky (1973) reported a positive linear relation between age and consistence or reliability of judgment. Rothenberg (1969) noted,

When a variety of transformations were presented,

> most 4- and 5-year-old children who gave conserving responses were not consistent in giving such responses for each item. Therefore, the true conservation status of a child appears not to be reliably determined on the basis of one or even two types of transformations.[37]

Dodwell (1960), Wallach and Sprott (1964), Gruen (1965), Peters (1967), and Peters and Rubin (1969) calculated high coefficients of stability in retesting for conservation of number.

Sex

Dodwell (1961), Goldschmid (1967), Pace (1968), D'Mello and Willemsen (1969), Rothenberg (1969), Rothenberg and Courtney (1969), Roll (1970), Klauss and Green (1972), and Nelson (1974) found no significant sex difference on the subjects' performance. However, Pace (1968) reported a significant sex difference in stage placement when the nonconservers, i.e., children at Stages I and II, were grouped together.

Socioeconomic Class

Rothenberg (1969) indicated that the lower-class children had fewer conserving responses; but Dodwell (1961) found no significant difference, although there was a tendency for group differences to favor the higher socioeconomic group.

With educable mentally retarded subjects, Kahn and Reid (1975) reported a significant difference, with

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[37] Barbara B. Rothenberg, "Conservation of Number Among Four- and Five-Year-Old Children: Some Methodological Considerations," Child Development 40 (June 1969): 399.

the low socioeconomic group demonstrating more conserving responses than the middle socioeconomic group when candies were used as the stimuli. No significant difference occurred when paper clips were employed as the stimuli.

In addition, Rothenberg and Courtney (1969) reported,

> . . . The lower SES subjects tended to choose rows to a great extent on the basis of closeness [of the row to the subject] and sometimes also manipulation, while the middle SES subjects tended to choose the longer.[38]

Variance

Miller (1973) presented each subject with seven conservation of number tasks, which differed in order of difficulty. In agreement with Zimiles (1966), Miller sumarized,

> . . .The majority of the children tested (especially those in the younger age groups) were conservers under some conditions and nonconservers under other conditions. Most children had mastered certain aspects of invariance but not others. As suggested earlier, it may be fruitful to think of conservation as a multifaceted concept composed of several levels which are acquired over the course of several months or years. . . . The present study . . . postulates a more extended transitional period than has typically been assumed.[39]

Variables in the Decoding Process

A review of research indicates that numerous

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[38]Barbara B. Rothenberg and Rosalea G. Courtney, "Developmental Study of Nonconservation Choices in Young Children," Merrill-Palmer Quarterly 15 (October 1969): 371.

[39]Patricia H. Miller, Facilitation of Conservation of Number in Young Children, (Urbana, Ill.: ERIC Clearinghouse on Early Childhood Education) ED 086-319, p. 9.

factors affect the conservation of number. As will be reviewed in the next sections, discrimination learning theory and research have contributed much to an understanding of Piaget's theory, particularly the factors affecting the conservation of number. Bucher and Schneider (1973) described some of the possible assets of discrimination learning theory and research to Piagetian theory and research:

> . . . First, the operant training literature contains analyses of useful training methods such as fading and gradual stimulus change procedures, prompting techniques, methods of shifting reinforcement schedules, etc. Training using such techniques can be more efficient and successful than direct training on some criterion measure. The use of such techniques is essential to reveal the potential value of training for inducing conservation. Second, operant discrimination learning procedures include techniques to control for influences of irrelevant cues and biasing effects. Techniques also exist to analyze responding to complex multidimensional stimuli to determine which aspects control responding. Their use in training studies can provide increased experimental control and sophistication in investigating the variables that contribute to performance in conserving situations. Third, the analytical terminology and procedures of discrimination learning can help clarify empirical grounds for determing the presence or absence of various stages of conceptual development. . . . Carefully controlled experimental contexts are necessary to establish adequate empirical measures of conceptual abilities, to facilitate their systematic exploration.[40]

Attributes of Stimuli

Zimiles (1966) and Rothenberg (1969) reported no significant difference between homogeneous and heterogeneous

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[40]Bradley Bucher and Robert E. Schneider, "Acquisition and Generalization of Conservation by Pre-Schoolers Using Operant Training." Journal of Experimental Child Psychology 16 (October 1973): 202.

materials. Using four sets of testing blocks as described in Table 1, Peters and Rubins (1969) also did not observe any significant differences. The results indicated that variations in the cues, which were provided by the materials, were attended to by some subjects and not by other subjects. In addition, the cues may have facilitated appropriate behavior for some subjects while distracting others from the numeric equality.

TABLE 1

DESCRIPTION OF MATERIALS*[41]

Block Set #	Set Names	Dimensions	Color
1	Neutral	1 3/4" X 1 3/4" X 7/8"	Yellow.
2	Number Accentuated	1 3/4" X 1 3/4" X 7/8"	White blocks with black sequins ordered 1 through 9.
3	Correspondence Accentuated	1 3/4" X 1 3/4" X 7/8"	Red, blue, yellow, dark green, light green, white, silver.
4	Length Accentuated	3 1/2" X 1 3/4" X 7/8"	White block with five evenly spaced black sequins.

* each set included 18 blocks.

Uzgiris (1964) reached similar results:

> . . . Both the analysis of variance and the correlational analysis lead to the conclusion that an individual's position on the conservation sequence is not constant across materials. The variation does not seem to be systematic, in that there was no single

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[41]Donald L. Peters and Kenneth Rubin, "The Effects of Cued Materials and Transformation Variations and Conservation of Number Performance," Alberta Journal of Educational Research 15 (March 1969): 49.

> material on which all Ss were either accelerated or lagging behind. It seems more a matter of individual differences although the discrepancies generally were not large.[42]

Siegel (1973) also found no significant difference between homogeneous and heterogeneous materials for 6- and 7-year-olds but did reach significance for 4- and 5-year-olds, with the heterogeneous condition as the more difficult.

Hood (1962) and Piaget (1965) found that functionally related materials, such as those used for provoked correspondence, tended to facilitate conservation of number more than homogeneous materials, such as those used for unprovoked or spontaneous correspondence.

The attributes of stimuli apparently have a significantly differing affect on young children, especially on those who are in the transitional stage for number conservation. It is hypothesized that, for this population, heterogeneous materials will produce the least number of conservation responses; homogeneous materials, more conservation responses and funtionally related materials, the most conservation responses.

Body Parts

The relation of one's body or body parts to number conservation has received little attention in research. Curcio, Robbins, and Ela (1971) noted that children's counting games, which center upon their fingers and toes,

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[42]Ina C. Uzgiris, "Situational Generality of Conservation," Child Development 35 (September 1964): 840.

may influence the acquisition of number conservation and that children frequently use their own body parts to measure objects before using an independent measuring object.

Five tasks were administered to 167 nursery children, who ranged in age from 42 to 64 months. The third task was number conservation. The experimenter placed five red and five yellow pipe cleaners in horizontal rows with one-to-one correspondence and asked, "'Who has more pipe cleaners, you or I, or do we both have the same number of pipe cleaners?'"[43] Then the experimenter made his row twice the length of the subject's row and repeated Q. The fifth task was number conservation with fingers.

> . . . Subject was asked to hold up his hands, palms outward, fingers slightly spread. Experimenter asked, "Do you have more fingers on this hand (pointing to left hand) or on this hand (pointing to right hand) or do you have the same number of fingers on both hands?" This was the standard question for finger conservation. Then S was asked to spread apart his left hand and leave the fingers on his right hand together. Again Q for fingers was asked. With some minor variations, this procedure was repeated twice.[44]

Fifty-two subjects passed the criteria for conservation with fingers as compared to 13 subjects for conservation with objects. Additional analysis shows 11 passing both tasks, 113 failed both tasks, 41 passed with fingers but failed conservation with objects, and only 2 passed conservation with objects and failed conservation with fingers. The data demonstrated that number conservation with fingers

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[43]Frank Curcio, Owen Robbins, and Susan Ela, "Role of Body Parts and Readiness in Acquisition of Number Conservation," Child Development 42 (November 1971): 1642.

[44]Ibid., p. 1642.

generally precedes conservation with external objects. This conclusion suggests that (1) the horizontal decalage between number conservation with fingers and that with external objects; (2) the possibility that pairing thumbs and so on results in provoked correspondence; and (3) the conservation with fingers is sensory-motor, which precedes those forms, which are more abstract in nature.

Conservation of Equivalence and Difference

As discussed earlier, Piaget's number conservation task requires equivalence conservation. Some researchers, such as Mehler and Bever (1967), have not differentiated between number tasks, which require conservation of equivalence, and those involving conservation of difference.

Gruen (1965), Griffiths, Shantz, and Sigel (1967), Halford (1969), Rothenberg (1969), Halford and Fullerton (1970), and Pufall, Shaw, and Syrdal-Lasky (1973) found a significant difference between the number of correct responses and the two types of conservation and concluded that children acquire conservation of difference or inequivalence before acquiring conservation of equivalence.

In contrast, Zimiles (1966) did not observe a significant difference between the two types of conservation but added,

> . . . It remains possible, of course, that this factor is operative in the conservation behavior of younger children at a developmental period when the equivalence relation has been less firmly established.[45]

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[45]Herbert Zimiles, "The Development of Conservation and Differentation of Number," Monographs of the Society for Research in Child Development 108 (1966): 37.

Desirable and Neutral Stimuli

Using candies and paper clips as stimuli, Kahn and Garrison's (1973) results support Uzgiris (1964) conclusion that desirable stimuli enhance performance on a number conservation task. The results were attributed to the subject's being more attentive when the candies were presented, thereby increasing the likelihood of success. Again employing candies and paper clips, Kahn and Reid (1975) found a significant difference between meaningful and non-meaningful stimuli, particularly when used with educable mentally retarded subjects from a low socioeconomic background. No significant difference was observed for middle socioeconomic children.

Although not finding a significant difference between stimuli (plastic animals and beads) on the overall results, Miller (1973) reported a significant interaction between set effects and stimuli, such that a subject was more likely to succeed with the desirable stimuli, plastic animals, if he had begun with the easiest number conservation task instead of the hardest. The results suggest to this author that desirable stimuli may be more effective with children, who have just acquired number conservation. As Uzgiris (1964) elaborates:

> . . . Although Piaget does not focus on the effects of specific environmental variables on development, he does not deny their importance, as has been sometimes suggested, since he describes the schemata as evolving and differentiating in contact with the environment. . . . It may well be that when a schema is developing, specific contacts with the environment will lead it to accommodate more in certain areas

> than in others, producing situational specificity in terms of specific past experiences of the individual. But after a certain number or a certain variety of encounters, a schema may develop independence and start to be applied universally. This leads to the expectation that schemata would be in a greater state of flux while developing, showing situational specificity, but once they consolidate, the situational variability would be expected to disappear.[46]

Roll (1970) with cinnamon-flavored candies and paper clips and Zimiles (1966) with minature trucks and blocks did not detect any significant difference between these particular stimuli. However, Zimiles questioned the appropriateness of his materials for this purpose.

Feedback

In studying the effect of feedback on number conservation training, Gelman (1969) concluded that

> . . . with feedback, young children quickly learn to use a quantity dimension. In fact, the rapid acquisition . . . strongly suggests that Ss had some preexisting understanding of quantitative relationships. Nevertheless, when irrelevant cues were introduced, they were frequently the basis for responding. This supports the hypothesis that irrelevant nonquantitative cues are salient for the young child and that he is more likely to attend to them. Introducing feedback into the task apparently forces him to eliminate the use of irrelevant cues and to attend to and use relevant quantity cues.[47]

Mode of Presentation

D'Mello and Willemsen (1969) reported that the overall results indicated that conservation of number was

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[46]Ina C. Uzgiris, "Situational Generality of Conservation," Child Development 35 (September 1964): 840.

[47]Rochel Gelman, "Conservation Acquisition: A Problem of Learning to Attend to Relevant Attributes," Journal of Experimental Child Psychology 7 (April 1969): 179.

not influenced by the concrete-abstract stimulus dimension, although training procedures based on this dimension may be effective. Each subject was presented three number conservation problems in four modes of presentation: (1) objects, (2) color photographs, (3) black line drawings, and (4) verbal descriptions.

Additional analysis of the results showed:

> . . . Of the nine subjects who received the modes in this order [1, 2, 3, and 4], there were by the two-thirds criterion 5, 4, 6, and 6 conservers, respectively, with each mode. The numbers are too small to interpret and afford only the loosest speculation, but such speculation should note that in the opposite order were 2, 3, 4, and 4, respectively, for the eight subjects who received the modes in this order, namely 4, 3, 2, 1. Further research with larger samples might make the interpretation of this apparent improvement with two opposite training orders more obvious.[48]

Observation of Transformation

Conflicting research exists on the significance of the subject's observing the transformation during the standard conservation task. With 3- to 5-year-ol subjects, Pufall, Shaw, and Syrdal-Lasky (1973) found that observing or not observing the transformation in a number conservation task did not influence the judgment of a young preoperational child. However, the study continued,

> The fact that the older children did not appear to be influenced by observing the transformation might be due to the fact that they were too young. Halford (1970) reports that children younger than 5 do not organize successive transformations in making quantitative judgments. . . . It is possible, then, that if

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[48]Sydney D'Mello and Eleanor Willemsen, "Development of the Number Concept: A Scalogram Analysis," Child Development 40 (September 1969): 688.

> older nonconservers had been tested, they might have been influenced by observing one transformation, perhaps vacillating between length and density relations.[49]

Zimiles (1966), Fletcher (1970), and Lawson, Baron, and Siegel (1974) did find a significant difference; there were more correct responses with the static arrays than when transformations were observed. Fletcher interprets his findings,

> . . . These results suggest that the observation or awareness of a change--even a change resulting in a new configuration no more perceptually misleading than the original configuration--may offer a stronger misleading cue than the length-oriented perceptual cue. Rather than the change alone, it may be the <u>interaction</u> of the two question procedure with the change that is responsible for the observed phenomenon.[50]

Order of Task Difficulty

Zimiles (1966), Miller (1973), and Siegel (1973) found that children, who began with the easiest trial, gave more conservation responses overall than do children who begin with the more difficult trial.

Proximity of Row to Subject

Rothenberg and Courtney (1969) placed two sets of five objects in one-to-one correspondence. The transformation was the equal subtraction or the removal of the

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[49] Peter B. Pufall, Robert E. Shaw, and Ann Syrdal-Lasky, "Development of Number Conservation: An Examination of Some Predictions from Piaget's Stage Analysis and Equilibrium Model," <u>Child Development</u> 44 (March 1973): 27.

[50] Robert F. Fletcher, "Investigation of the Effect of an Operationally Defined Word on Conservation-of-Number Responses," <u>Arithmetic Teacher</u> 17 (March 1970): 260.

object on the left-hand side of each row. Supporting Rothenberg and Courtney (1968), the results indicated,

> . . . The row on the subject's side was clearly chosen more frequently than the one on the examiner's side. . . . Since both rows were equal in length, density, and manipulation and differed only in proximity to S, these results suggest that all other factors equal, young children more frequently select the closer row as having "more" rather than choosing both rows as often as each other. . . . The choice of a row on the basis of closeness alone was more common among Ss aged 2-5 to 4-2 than among those from 5-3 to 6-2.[51]

It was suggested that the two rows of stimuli be presented perpendicular to the child's front instead of parallel to the child so that both rows are equidistant from the subject.

The Relationship Between Question Structure and Verbal Response

Question structure has greatly varied from study to study with little attempt to systemmatically analyze the effects of different question structures on the subjects' responses. At least three different question structures have been used in number conservation research.

The first and most commonly posed conservation question asks, "Does this row (side or bunch) have more, or does this row have more, or do they both have the same number (amount)?" (Wallach and Sprott 1964 and Zimiles (1966)

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[51] Barbara B. Rothenberg and Rosalea G. Courtney, "Developmental Study of Nonconservation Choices in Young Children," Merrill-Palmer Quarterly 15 (October 1969): 367.

The three parts in this structure are especially difficult for young children to remember and process. As reported by Hood (1962), children frequently repeat the last thing they heard, thus creating a set for a "more" or "same" response. Some studies attempted to avoid this set by alternating the parts of the questions. In either case, it is questionable as to whether the child is actually being evaluated for conservation of number or the ability to process rather complex language structure.

Another structure is the two-part question, such as "Do these two rows have the same number (amount) or does one have more?" (Fleischmann, Gilmore, and Ginsburg 1966 and Wheatley 1968) The two-part question tends to have problems similar to the three-part question.

Finally, Dodwell (1960), Elkind (1961), Wohlwill and Lowe (1962), and Wallach, Wall, and Anderson (1967) asked questions which concerned one event, such as "Are there the same number of eggs and egg cups?" or "Which row has more beads?" Although children remembered better the single phrase in each of these questions, the question's emphasis on "same" and "more" biased the responses.

Rothenberg (1969) studied the biasing effect of two one-part questions:

1. "Does this bunch have the same number of blocks as this bunch?"
2. "Does one bunch have more beads?"

The results showed that there was a much greater percentage correct responses when only the responses to the first question were considered than (1) when correct replies to both questions were required and (2) when only responses to the second question were considered.

In other research on one-part questions, Piaget (1968) and Rose (1973) noted 3- and 4-year-olds' tendency to adopt the set to respond affirmatively to questions in number conservation tasks. Without the aid of justification responses to why S = V', a researcher could easily categorize a very young child as a conserver due to the child's perservation of a "yes" response instead of a true understanding of number conservation.

Three suggestions have been made about how to facilitate questioning. First, in order to detect the set for "yes" without requesting justifications, Rose (1973) suggested the inclusion of inequality items in standard tests of conservation, thus avoiding the need for further questioning of children who may be reluctant or less able to talk, such as the deaf or the child with an expressive language delay. Second, Mood (1962) suggested the use of several forms of the standard single-event question for each problem.

Finally, Fletcher (1970) divided 200 subjects into Groups I (Bimates) and II (Traditional). Each subject was presented with 14 standard number conservation tasks, seven problems with equivalent sets and seven problems with nonequivalent sets. The standard question for Group I was, "Is this <u>bimates</u>?" The rationale and definition for "bimates" were:

> The research of many investigators has clearly established that a child's ability to enumerate collections by counting is no assurance at all of that child's success in conservation-of-number experiments. With what means, then, can a child compare the numerosity of two sets of objects? Fundamental to such a comparison is the physical matching of the members of one set with the members of the other set, or the members of the "smaller" set with the members of a subset of the "larger" set. Lacking a suitable operationally-defined word, our exisiting vocabulary is inadequate for the job of letting a child know exactly what he must do in order to determine whether or not two sets have the same number of members. Consequently, for the purposes of this study, the term bimates was arbitrarily coined by the investigator. It is defined as follows, Given two sets of objects, the answer to "Is this bimates?" (meaning, "Are the sets equivalent?") is "yes" if it is possible to pair the elements of one set with the elements of the other set so that each element is a member of exactly one pair, and is "no" otherwise.[52]

Group II children were asked the traditional standard question, such as, "Do we have the same number of red cards and blue cards?" No rationale was requested for the subject's final answer to Q.

An analysis of results showed that Group I (Bimates) did not perform significantly better than Group II (Traditional) on the conservation of number tasks. A more thorough analysis further revealed that these same results also applied to Group I and II for both equivalent and nonequivalent sets.

Size of the Aggregate

Although Miller's (1973) results were unclear,

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[52]Robert F. Fletcher, "Invetigation of the Effect of an Operationally Defined Word on Conservation-of-Number Responses," Arithmetic Teacher 17 (March 1970): 255-56.

Wohlwill and Lowe (1962), Feigenbaum (1963), Pace (1968), Bucher and Schneider (1973), Lawson, Baron, and Siegel (1974), Smither, Smiley, and Rees (1974), and Winer (1974) found that subjects, particularly younger subjects who may not have completely assimilated the principle of one-to-one correspondence, made more conservation responses with a numerically smaller than larger set of objects. Winer (1974) discussed two interpretations of the data in terms of the developmental relation between conservation of small and larger sets.

> . . . For one, it seems plausible that what has been labeled conservation of small quantities is based on a primitive and probably perceptual apprehension of numerosity--a notion that does not necessarily seen inconsistent with Piaget's (1952) views regarding the child's judgment of small quantities. When it becomes more difficult to determine numerosity via perception, the child probably then defines quantity in terms of what adults consider irrelevant dimension (e.g. length). . . .
>
> It might also be assumed that young children can employ certain types of operations with particular reference to small quantities (e.g., addition/subtraction, perhaps reversibility) while with larger sets, the distraction from perceptual cues might make the use of these operations more difficult.[53]

On the other hand, Zimiles (1966) and D'Mello and Willemsen (1969) found no difference in conservation responses between problems that differ in the number of objects to be conserved. However, Zimiles (1966) found a very significant interaction between the size of the aggregate and order of task presentation for the kindergarten and some of the less mature first grade subjects:

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[53]Gerald A. Winer, "Conservation of Different Quantities Among Preschool Children," Child Development 45 (September 1974): 841-42.

> . . . The small condition was found to be easier when it appeared first. The smallness of the aggregate facilitated recognition of the conservation principle . . ., as long as this condition constituted the first exposure to the conservation paradigm. If the small condition was preceded by even a single trial involving the large condition, the facilitating effect of smallness was lost.[54]

Transformations

Length cues

Wohlwill and Lowe (1962), Wallach and Sprott (1964), Piaget (1965), Mehler and Bever (1967), Wallach, Wall, and Anderson (1967), Bever, Mehler, and Epstein (1968), Pufall and Shaw (1972), Miller (1973), Pufall, Shaw, and Syrdal-Lasky (1973), Rose (1973), and Lawson, Baron, and Siegel (1974) found a confronting of length and number, particularly around five years of age, in that the children, who gave nonconserving judgments, tended to judge the number of objects in a row by the row's length. According to the child's thought, equal length was judged as having an equal number of items; unequal length was judged as having an unequal number of items. Most children judged that the longer row had more items.

Multiple variations

In order to determine the effects of variations in transformations, Peters and Rubin (1969) presented each subject with six transformations, as pictured in Figure 2,

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[54]Herbert Zimiles, "The Development of Conservation and Differentation of Number," Monographs of the Society for Research in Child Development 108 (1966): 35.

in the following order: two linear transformations, two horizontal figure transformations, and two vertical figure transformations. Although the mean conservation score for the vertical transformations was signficantly higher than those for the other transformations, the significant increase was attributed to test-wiseness from repeated testing.

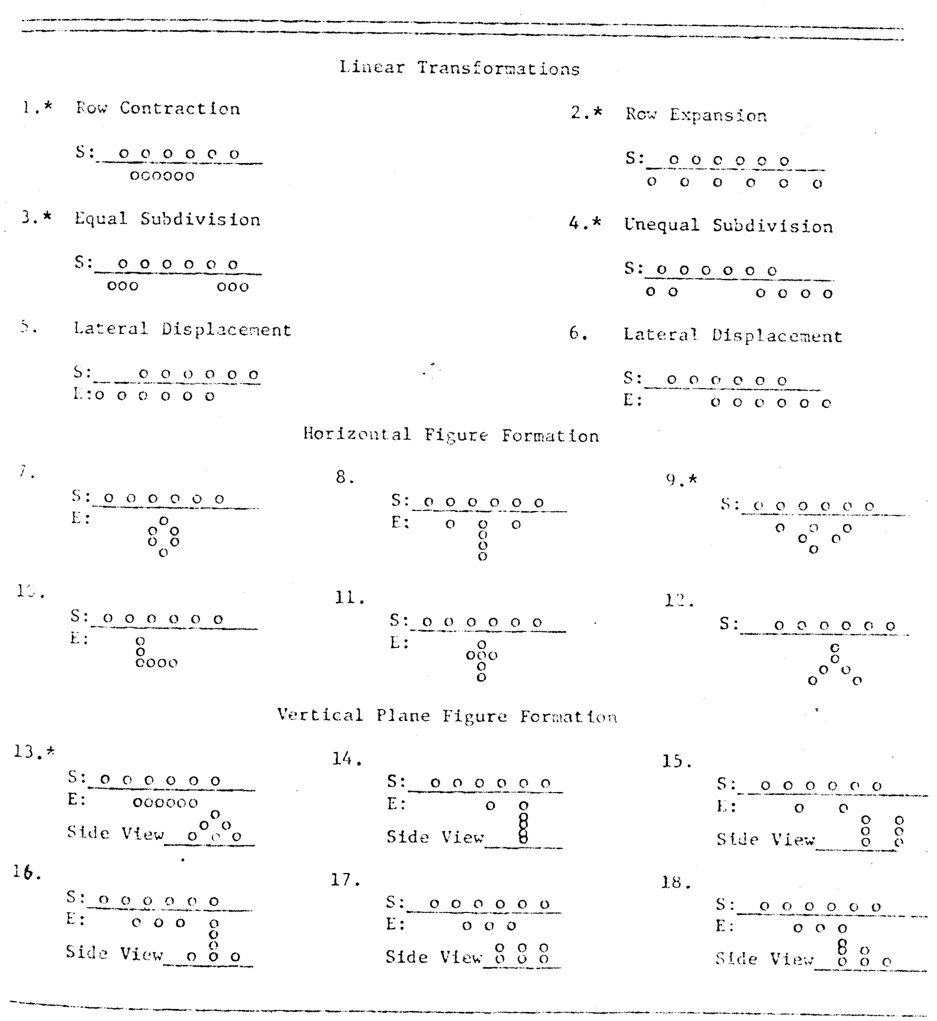

Fig. 2. Conservation of number transformations.[55]

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[55]Donald L. Peters and Kenneth Rubin, "The Effects of Cued Materials and Transformation Variations and Conservation of Number Performance," Alberta Journal of Educational Research 15 (March 1969): 51.

Multiple linear variations

Rothenberg (1969) compared four different linear transformations, as shown in Figure 3, and concluded that all four conservation of equality transformations had approximately the same percentage of conserving responses.

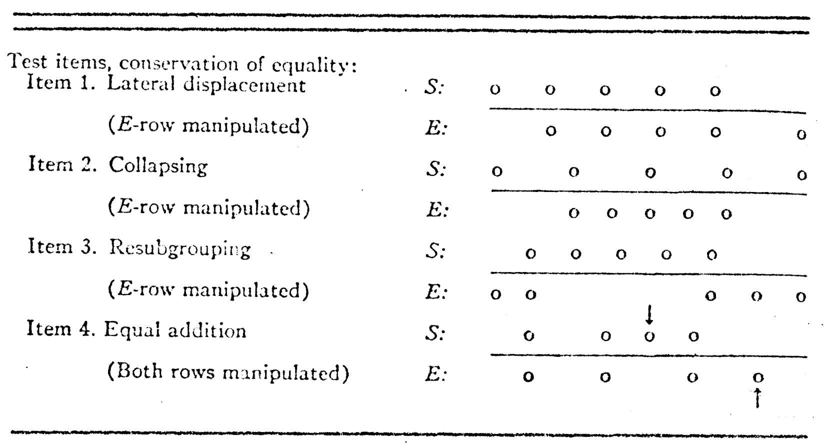

Fig. 3. Conservation of number transformations.[56]

Collapsing

The stimuli for Rothenberg and Courtney (1969) were two sets with five objects in each set in one-to-one correspondence. The transformation consisted of moving in the two end objects in the experimenter's row so as to reduce the row's length, as depicted in Figure 4.

A very high number of nonconserving subjects chose the subject's row, which was closer to the subject and longer, as having more, thus showing the importance of the factors of proximity and length in nonconservation choices.

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[56]Barbara B. Rothenberg, "Conservation of Number Among Four- and Five-Year-Old Children: Some Methodological Considerations," Child Development 40 (June 1969): 390.

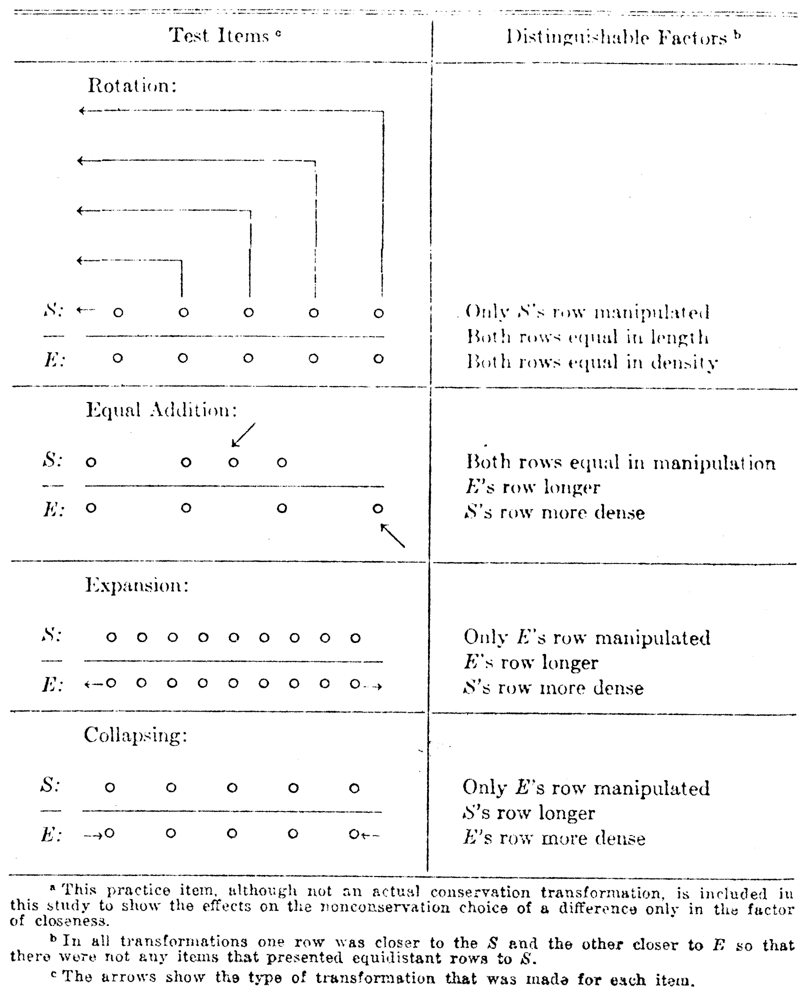

Fig. 4. Conservation of number transformations.[57]

Equal addition

In Rothenberg and Courtney (1969), the stimuli consisted of two sets with three objects per set in one-to-one correspondence. The transformation was the equal addition or adding one item to each row (Figure 4).

The overall results indicated that the longer appearing side, which was the experimenter's, was chosen

[57]Barbara B. Rothenberg and Rosalea G. Courtney, "Developmental Study of Nonconservation Choices in Young Children," Merrill-Palmer Quarterly 15 (October 1969): 366.

more frequently as having "more" by the nonconservers than the more dense and closer row. But the youngest subjects, who were 2-5 to 3-3 years in age, did the oppose of the older subjects by selecting the closer, denser row as having more.

Expansion

Rothenberg and Courtney (1969) used two sets with nine objects per set as the stimuli. The transformation involved an increase in the length of the experimenter's row by moving out the two end objects (Figure 4).

The results were the same as those for Rothenberg and Courtney (1969) in Equal addition, which suggested that "the factor of manipulation when combined with length in contrast to closeness and density does not increase the percentage of total subjects choosing the longer row."[58]

Rotation or lateral displacement

The stimuli for Rothenberg and Courtney (1969) consisted of two sets with five objects in each set in one-to-one correspondence. The transformation was the shifting of the subject's entire row of objects one unit of distance. The density and length of the transformed row remained the same as before the shift (Figure 4).

The subjects ranged in age from 2-5 to 6-2. The nonconservers selected the subject's side more frequently as having more objects than the experimenter's side. The younger

[58]Ibid., p. 370.

the subject, the more likely for him to select the manipulated row as having more than the older subjects, who selected both rows on a nearly equal basis, apparently because both rows were equal in length and density. In addition, the lower class subjects chose the manipulated row more frequently as having more than the middle class subjects.

Variables in the Encoding Process

Eye Movements

Wilton and Boersma (1974) examined 30 nonretarded and 30 mildly retarded subjects to determine if the 15 pretested conservers and the 15 pretested nonconservers for number and liquid would exhibit differential eye-movement patterns in terms of couplings between stimulus elements; number, duration, and position of fixations; number, duration, and position of runs; and examination time on stimulus elements. The mean full-scale I.Q.'s on the Wechsler Intelligence Scale for Children (WISC) (Wechsler 1967) were 116.60 for the nonretarded conservers, 73.87 for the retarded conservers, 109.80 for the nonretarded nonconservers, and 69.07 for the retarded nonconservers.

The procedure involved recording each subject's eye movements with a stand-mounted 16 mm camera while each subject was presented with number, liquid, and continuous quantity conservation tasks on 16 mm black and white movie film. Previous research by O'Bryan and Boersma (1972) indicated that movie and traditional presentations of conservation tasks produced basically the same results.

The results of Wilton and Boersma's research indicated that

> . . . perceptual activity in terms of corneally reflected eye-movement patterns clearly differentiates conservers and nonconservers in both nonretarded and mildly retarded groups. The acquisition of conservation is accompanied by at least two discernible changes in visual perceptual activity. Firstly, conservers engage in considerably more perceptual activity, i.e., their visual exploratory behavior is more active. Secondly, whereas nonconservers tend to concentrate their perceptual activity on the element judged to be greater following stimulus transformation, conservers tend to distribute their perceptual activity more evenly about the stimulus elements, i.e., the perceptual activity of conservers seems more decentered than that of nonconservers.[59]

Justification of Equivalence Judgment

In addition to the subject's answering S = V', Smedslund (1963) required the subject to offer an acceptable explanation of why S = V' before the subject was considered a conserver. Without the verbal explanation, the S = V' reply was a "symptom" response, which Smedslund contended would easily succumb to the examiner's suggestion of nonconservation.

Inhelder, Bovet, Sinclair, and Smock (1966) have raised similar objections:

> . . . The operational structure (as defined by Piaget) underlying the conservation concepts appear to us to be a complex, coordinated system that cannot be properly evaluated by rather summary investigation of answers to preselected questions with no exploration of the child's justification of those answers. Nor can such answers be induced by training the child to direct attention uniquely to those aspects of the

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[59]Keri M. Wilton and Frederic J. Boersma, "Eye Movements and Conservation Development in Mildly Retarded and Nonretarded Children," American Journal of Mental Deficiency 79 (November 1974): 291.

> situation that lead him to a limited (in terms of the criteria for the conservation concept) "correct answer."[60]

In reply to Smedslund, Roll (1970) found that subjects, who correctly responded to S = V' without giving a verbal explanation, resisted the examiner's nonconservation suggestion. However, Gruen (1966), Rothenberg (1969), and Little (1972) determined that merely asking Q resulted in more and probably younger conservers than when an appropriate explanation was also required.

Using 120 3- to 7-year-olds as subjects, Yawkey (1971) supported Gruen's results for 3- to 5-year-old subjects but found that the reply to Q and the justification response were equivalent response measures for the 6- to 7-year-old subjects. Goldschmid and Bentler (1968) found that performance or equivalency judgment and verbal explanation scores on a series of conservation tests correlated .90 or higher; however, performance scores were somewhat higher.

Relational Terms

Griffiths, Shantz, and Sigel (1967) noted that the subject's understanding of the relational terms "more," "same," and "less" may influence the subject's response in a conservation problem.

> . . . If _S_'s knowledge of the terms has not been determined, failure on these classical conservation tasks may indicate that (a) he does not understand the relational terms, (b) he cannot conserve, or

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[60]Bärbel Inhelder, Magali Bovet, Hermine Sinclair, and C. D. Smock, "On Cognitive Development," _American Psychologist_ 21 (February 1966): 16.

(c) both. Most conservation studies have failed to take this factor into account.[61]

As a consequence, the experimenters investigated 64 preschoolers' understanding of these relational terms with three types of materials and the availability of these terms as spontaneous and elicited responses. The three types of materials represented the stimuli used in number, length, and weight conservation problems. In the number tasks, the comparison sets of four, three, and two lollipops were contrasted with the standard set of three lollipops as the stimuli for "more," "same," and "less," respectively.

The results of this study indicated that "same" is a more difficult concept than either "less" or "more." In addition, the standard question's structure determined whether relational terms were used spontaneously and correctly. In conclusion,

> . . . Children may understand the meaning of relational terms but may not use them spontaneously. Thus, it would seem advisable for a researcher to determine whether elicited or spontaneous responses to conservation questions are required, and to pretest the appropriate type.[62]

Summary

According to Piaget, the development of the conservation of number consists of three phases: Stage I--Global Comparisons, Stage II--Intuitive Correspondence (Transition-

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[61]Judith A. Griffiths, Carolyn A. Shantz, and Irving E. Sigel, "Methodological Problem in Conservation Studies: The Use of Relational Terms," Child Development 38 (September 1967): 842.

[62]Ibid., p. 847.

al), Stage III--Operational Correspondence. The review of research indicated that numerous factors are involved in the number conservation task. Subject population, procedural differences, response criteria, and variations of stimuli can significantly affect the conservation judgment.

Although Piaget's concept of "horizontal décalage" has been applied to the developmental sequence of conservation of different content areas, this concept could be extended to explain the difference within a particular content area, in this case, numeric equivalence. "Horizontal décalage" is the

> . . . repetition which takes place within a single period in development. . . . A cognitive structure . . . can first be successfully applied to task X but not to task Y; a year or so later . . . the same organization of operations can now be extended to Y as well as to X.[63]

Lovell (1968) supports this idea and discusses its implications to learning:

> Inhelder and Piaget (1958) pointed out that concrete operations consist of the direct organization of immediately given data and they cannot be generalized to all situations at once. . . . Piaget (1956) also speaks of the notion of "horizontal differentials." This suggests that the same or similar concepts when derived from different materials or situations, develop in staggered sequence rather than simultaneously. . . .
>
> . . . But when the schemas required for the solution to some problem are not too far removed in complexity from those available to the child, the inadequacy of existing schemas will force him to accommodate to the conditions of the problem. Hence the child restructures his own schemas toward greater cognitive adaptation to his environment. Not only does the child solve the problem, but he extends his capacity for further learning. . . . We must always bear in mind that there must not be too great a gap between the schemas available to the child and those demanded by the situation. Yet in spite of the help given by Piaget in assessing a

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[63]Flavell The Developmental Psychology of Jean Piaget, p. 22.

child's level of thinking, what is involved in producing the correct amount of gap between the schemas available to the child and those demanded by the situation remains vague. This is where the intuitive skill of the teacher is called for. It is his task to arrange, or find in the environment, problems which call forth the schemas of the child in new and novel ways.[64]

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[64]Kenneth Lovell, "Developmental Processes in Thought," Journal of Experimental Education 37 (Fall 1968): 18-19.

CHAPTER III

THE ACQUISITION OF NUMBER CONSERVATION

The research on training procedures for the acquisition of number conservation was examined prior to entering a more general discussion on learning. The order of presentation was arranged alphabetically by the researchers' surnames due to the fact that numerous and diversified techniques were frequently studied in a single article.

Research on Training Procedures

Before reviewing the research on training procedures, it should be noted that Piaget (in Duckworth 1964) requires two criteria to be satisfied before training in conservation is considered effective:

1. Generalizability: the concept, which was induced, must transfer to other situations.

2. Durability: the concept, which was induced, should not extinguish over time.

Beilin (1965)

There were three phases to the study: (1) pretests for number, length, and area conservation; (2) training with four experimental procedures on number and length conservation; and (3) posttests for transfer of number, length, and area conservation.

Based on their age and pretest performance, the kindergartners were assigned to either the control group or one of the four training groups: (1) Non-verbal Reinforcement (NVR), (2) Verbal Orientation Reinforcement (VOR), (3) Verbal Rule Instruction (VRI), and (4) Equilibrium (EQ). Each subject in a training group received training in both number and length.

The NVR, VOR, and VRI training procedures were extensions of the pretesting procedure, part of which was a number conservation test.

> . . . There were 2 practice and 12 test trials. Each trial consisted of two parts. In the first part, S was shown the number apparatus with its three parallel columns of corks. One column was equal in number as well as length to the middle (stimulus) column. . . . The other column was unequal in number and also in length to the middle one. . . . The S was instructed to choose the row which was "like" the middle one and to respond by pressing a button at the base of either of the response columns. If correct, he heard a buzzer and was given a token. After S responded, E expanded or contracted the stimulus column so that the first and last corks were aligned with the first and last corks of either the shorter or longer response column [see Figure 5]. No corks were removed or added. All contractions and expansions were made in sight of S. After each change, S was again asked to choose the column that was "like" the middle one, and his correct responses were reinforced in the same manner. . . . On half the trials the incorrect column was shorter than the middle one, and on half it was longer. The number combinations changed in each trial.[65]

The differences between the pretesting procedures and the training procedures were:

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[65] Harry Beilin, "Learning and Operational Convergence in Logical Thought Development," *Journal of Experimental Child Psychology* 2 (December 1965): 321-22.

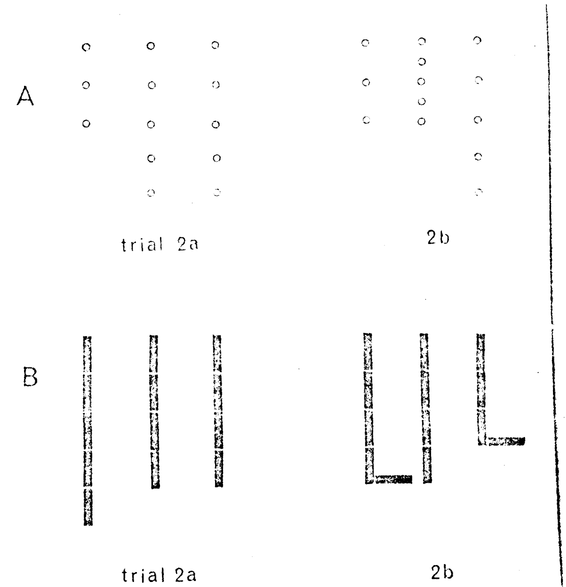

FIG. 5. Sample conservation test trials: A, number; B, length.[66]

1. The NVR training procedure had 36 trials.

2. The VOR training procedure included verbalization of the concept in the instructions and on each trial.

3. The VRI group was given the same starting instructions as the VOR group. After each trial the subject was asked why he choose the column or line that he did. On any trial where the subject responded incorrectly and/or gave an inadequate conservation explanation, the principle of conservation was explained.

The EQ procedure involved transformations in which the objects underwent spatial rearrangements without the addition or subtraction of objects. It was hypothesized that this procedure generated cognitive disequilibrium.

Beilin's results indicated:

> The effect of training in improving performance from pretest to posttest is evident for the tests in

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[66] Ibid., p. 322.

which Ss were trained (i.e., number and length conservation), although not for the test in which Ss were not trained (i.e., the area test). Each treatment group in the study, including the control group, has a significant number of Ss who improved in performance from pretest to posttest. There is only one training group, however, which has significantly more Ss improving than the control group, namely, the VRI group.[67]

Bucher and Schneider (1973)

The study included two major training phases: (1) training to judge the relative numerical sizes of two rows of objects (Number Relations Training) and (2) training in conservation of number, substance, and liquid quantity (Conservation Training). All the subjects first passed two pretests: (1) correctly pointing to black and white blocks after hearing the color named and (2) clear enunciation of "same" and "not the same."

The first phase, Number Relations Training, involved a series of graded steps to train accuracy in judging the numerical equality or inequality of two rows of objects. The subject was presented with numerically equal or unequal rows and required to label the row as "same" or "not the same."

Two training procedures were used, half the subjects in each group, the second procedure being a shorter modification of the first process. In the first training procedure, length and number were cues, so that the larger row was longer and equal rows were the same length. Towards the end of training, length was irrelevant--the two rows

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[67]Ibid., p. 336.

were always unequal in length.

Conservation Training consisted of eight successive steps, which differed in the variety and complexity of the transformations: four for number conservation, two for substance conservation, and two for conservation of liquid quantity. The general trial format followed the standard conservation procedure except for: (1) feedback was given, (2) no explanations for the subject's judgments were required, and (3) a change in the standard question, "Is your row the <u>same</u> as mine or <u>not</u> the same?" Table 2 summarizes the steps.

Two artifacts, which may produce false correct responses in the usual conservation test, were evaluated in the test trials for the first two steps. The first artifact pertained to the effect of a small number of objects in each row. Bucher and Schneider posited that the children may not watch the transformation but merely determine their judgment by counting the few objects. In Step 1, the two rows initially had ten blocks in each row as compared to four blocks per row in Step 2's training.

The second artifact concerned the single use of the conserving operation in other training studies in which the correct reply was always "same" before and after each transformation in every trial. The subject could learn this reply without attending to the transformation. To test this artifact, all the training trials in Step 1 conserved in contrast to Step 1's test trials for inequality. Also, half

TABLE 2[67]

CONSERVATION TRAINING AND TESTING

Step 1.	*Number Conservation. Training.* Ten pairs of blocks. The experimenter's row is lengthened in the transformation. *Test trials.* The experimenter's row is lengthened and one block is removed.
Step 2.	*Training.* Four pairs of blocks. The experimenter's row is lengthened, and on half the trials one block is removed. *Test trials.* Ten pairs of blocks. No change in transformations.
Step 3.	*Training.* Same transformations as in step 2, with 10 pairs of blocks. *Test trials.* The conserving transformations are unchanged. Three non-conserving transformations are used, twice each: stretch the experimenter's row and add one block, stretch the child's row and add one block, stretch the experimenter's row and add one block to the child's row.
Step 4.	*Training.* See step 3 test trials. *Test trials.* See step 5 training trials (conservation of substance).
Step 5.	*Conservation of Substance. Training.* On half the trials the experimenter's ball is rolled into a cigar shape. On the other half, a small portion of the ball is first removed. *Test trials.* Half the trials conserve. Three non-conserving transformations are used: add some material to the experimenter's ball before rolling it out; add to, or subtract from, the child's ball before rolling it out.
Step 6.	*Training.* See step 5 test trials. *Test trials.* See step 7 training trials (conservation of liquid quantity).
Step 7.	*Conservation of Liquid Quantity. Training.* All or 2/3 of the water in the experimenter's standard glass is poured into a broad-bottomed glass. *Test trials.* For conserving transformations all the water in the child's glass is poured into a narrow glass. For the non-conserving transformation 2/3 of the water in the child's glass is poured into the narrow glass (4 trials), or into the broad-bottomed glass (2 trials).
Step 8.	*Training.* See test trials from step 7. *Test trials.* The usual test trials were omitted. There were 20 additional training trials.

the training trials in Step 2 were transformed into a numerical inequality.

In Number Relations Training, a significantly higher percentage of children completed the second shorter procedure and with fewer trial errors. The authors judged the conservation training successful for 25 of 49 children even though (1) no standard pretest was given; (2) the final 20-trial test was not a standard conservation test; and (3) subjects

[67]Bradley Bucher and Robert E. Schneider, "Acquisition and Generalization of Conservation by Pre-schoolers, Using Operant Training," Journal of Experimental Child Psychology 16 (October 1973): 193.

were not asked for verbal explanations for their judgments, even at the end of training.

> First, the present results may be taken to imply that a carefully guided training regime may effectively induce conservation in many preoperational children. . . . The present training program is more lenghty than others that have failed in previous work with preschool children, and its success may be attributed to this fact, and to the use of a successive approximation training procedure. Further, the techniques used to control for use of irrelevant cues, and the use of mixed conserving and non-conserving trials, lend further confidence to the results.[68]

Curio, Robbins, and Ela (1971)

Curio, Robbins, and Ela selected three groups of subjects, 16 per group, who had failed both conservation of external objects and fingers tests but who had passed the other pretests for counting, addition/subtraction, and one-to-one correspondence.

One group received rote-counting (RC) training; another group was assigned addition/subtraction (A/S). In the RC training, the subject saw two rows, with five pipe cleaners (PC) in each row, in one-to-one correspondence. After the subject replied to Q, the experimenter questioned, "How do you know? . Count them." This procedure was repeated seven more times with different transformations, such as expanding and condensing the rows. The A/S training followed the same procedure as RC except that, after the subject replied to Q, one row was lengthened and Q repeated.

When the subject said one row contained more, a

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[68]Ibid., p. 200.

> single PC was added to the other row which, by inference, contained less; Q was then repeated. If S still maintained that the first row contained more, another PC was added to the second row and Q repeated. If S's response implied that it contained less, the previously added PC was removed and Q repeated. This oscillation between adding and subtracting PCs was performed three times or until S asserted the equality of the rows. After the first trial, this procedure, with different transformations, was repeated for four additional trials.[69]

Another 16 subjects, who had passed the conservation with fingers pretest as well as the other three pretests, were assigned to body-part (BP) training. In an attempt to encourage generalization from number conservation with fingers to number conservation with external objects, five yellow and five red PCs were bent into rings and placed on the 10 slightly spread fingers of the subject's raised hands. The BP training procedure continued:

> Experimenter asked, "Do you have more rings on this hand (pointing to right hand) or on this hand (pointing to left hand) or do they both have the same?" Subject was then asked to spread the fingers on his right hand and close those on his left hand. The question was repeated. The rings were then removed from S's fingers and placed on the table so that those on the closed hand were placed close together and those on the spread hand were placed further apart. Then the standard question Q for objects was asked. The rings were then moved into rows for one-to-one correspondence, and Q was repeated. This procedure constituted one trial. The rings were then replaced on S's fingers, and the procedure was repeated twice more with minor variations.[70]

Two identical posttests of number conservation with objects were administered to all training groups, one immediately after training and the other one, one week later. Each

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[69] Frank Curcio, Owen Robbins, and Susan S. Ela, "Role of Body Parts and Readiness in Acquisition of Number Conservation," Child Development 42 (November 1971): 1643.

[70] Ibid., pp. 1643-44.

posttest consisted of three trials. The first and second trials were similar to the pretest trials, which is described under "Body Parts" in Chapter II, except that seven and six PCs were used as stimuli, respectively. The third trial employed six PCs with the subject's row transformed into a starlike configuration.

Posttest results indicated BP training group's superiority to RC and A/S ($p < .01$) for both posttests. The authors noted, however, that this superiority may have been due to the BP group's being initially closer to the conservation of objects as suggested by BP's passing four pretests as compared to three pretests for RC and A/S.

To clarify this issue, two additional training groups, with eight subjects in each, were selected after passing the same four pretests as the BP group. One group received the previously described RC training; the other group took the A/S training. Both groups followed the same format for pretesting, training, and posttests as the BP group.

The RC and A/S posttests were similar and therefore combined for comparison with the BP posttest results. Seven out of 16 subjects in the combined RC and A/S group passed the immediate posttest, whereas 11 out of 16 passed in the BP group. In addition, a significant difference in the delayed posttest results favored the BP training group. The delay posttest results indicated the <u>dual</u> importance of the kind of training experience as well as the closeness of the child to the criterion concept in maintaining the acquisition of number conservation.

Gelman (1969)

Gelman's experiment consisted of three phases: pretesting, training, and posttesting. The pretesting included the random presentation of conservation tests for standard length, number, mass, and liquid amount. For the number conservation tasks, the stimuli were two sets of five black checkers, and the procedure followed the standard format. Twenty kindergartners, who scored as nonconservers on the pretests, were assigned to a control group and each of the three experimental conditions: (1) oddity control (OC) training, (2) learning set (LS) training, and (3) stimulus change (SC) control.

In the OC training, 32 stimulus sets, each of which consisted of two alike and one different toy or vice versa, were used in 32 training problems with six trials per problem. Depending on the task, the subject was requested to point to either (1) the two objects that were the "same" or (2) the two objects that were "different." The subject was informed as to whether his response was wrong or correct, for which he received a prize.

The LS training consisted of 32 six-trial problems: (1) 16 for length and (2) 16 for number. Three stimulus objects or sets were used in each problem. Two contained the same quantities, the third contained a different quantity, such as two rows of five chips versus one row of three chips, or two 6-inch sticks versus one 10-inch stick.

> . . . Ss received extensive training with a large number of different examples of the relevant conservation principles. The choice of number and length concepts derived from examination of the nature of the problems. It has been noted that children often define numerosity in terms of length cues. . . . Alternation between number and length problems here meant that sometimes the length was relevant and sometimes irrelevant. The interchange of number and length tasks was viewed as one way of forcing the child to see that a quantity cue can be either relevant or irrelevant, and that he has to discriminate when a particular cue is, in fact, relevant. To solve all problems, the child would have to learn to separate out the different cue functions of length, as well as, ignore irrelevant cues within a problem.[71]

The two types of problem variations occurred: (1) (1) between problem variations, which were color, size, and chape of stimuli, starting arrangements, and quantity combinations and (2) within problem variations, examples of which are presented in Figure 6.

As in OC training, LS training included feedback to the subjects' responses and randomization. In LS training, the experimenter arranged the stimuli for a particular trial and then said either: (1) "Show me two sticks that are the same (or different) length," or (2) "Show me two rows that have the same (or different) number of things in them."

In the SC control, the stimuli and training procedures were the same as in the LS condition, except that no feedback was given.

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[71]Rochel Gelman, "Conservation Acquisition: A Problem of Learning to Attend to Relevant Attributes," Journal of Experimental Child Psychology 7 (April 1969): 173.

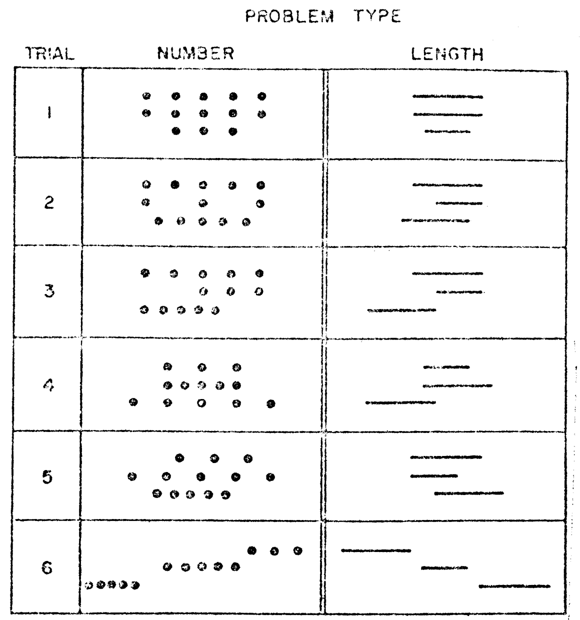

FIG. 6. Schematic representation of intraproblem variations for a length and number problem presented during SC and LS training.[72]

The posttest included all of the pretest items plus additional items in each test. Every subject was tested twice: (1) the day after training and (2) two to three weeks after training.

After a thorough analysis of the results, Gelman concluded:

> . . . given appropriate training, one can elicit conservation behavior from children who initially fail to conserve on classical conservation tests. Appropriate training seems to involve two factors: (1) an opportunity to interact with many different instances of quantitative equalities and differences and (2) feedback, which presumably tells S what is and what is not relevant to the definition of quantity. This is supported by training and transfer results from both SC and LS Ss. The SC Ss received only changing stimuli, while the LS Ss received both changing stimulus exper-

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[72] Ibid., p. 174.

> ience and feedback. Some of the SC Ss learned to conserve in a limited way. There was a small amount (27%) of specific generalization, but almost no nonspecific transfer. In contrast, with LS training almost perfect specific and considerable nonspecific transfer occurred. In addition, LS Ss were better able to explain their correct answers. Finally, it seems that LS training brought Ss to use a general rule like "it doesn't matter what you do or pay attention to the way it is to start."[73]

Gruen (1965)

Gruen found that neither confronting the child repeatedly with the invariance of numerical values, when irrelevant perceptual cues were present, nor presenting situations, which supposedly induce internal cognitive conflict, was especially effective in inducing number conservation.

However, a substantial number of subjects in both training groups did acquire conservation of number during the experiment, the direction of the results supporting the "equilibration-through-internal-cognitive-conflict" hypothesis.

Hatano and Suga (1969)

Hatano and Suga's most significant result indicated that training, which did not use external reinforcement, did not show any effect upon number conservation. However, the two training programs without reinforcement did provoke a conservation response in 50% of the superior children, who had correctly responded to conflict situations before training. The researchers concluded that it is often

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[73]Ibid., p. 184.

necessary to introduce a training procedure with external reinforcement, especially for those children who appear to be in the initial part of transition.

Pace (1968)

Pace administered the Lorge-Thorndike Intelligence Test, Level 1, Form A, and a pretest on number conservation to 53 kindergartners and 50 first graders. The pretest contained five tasks with two to six objects (checkers) per stimulus set. If the subject passed these five items, three of the five tasks were repeated with 14 checkers per set. If the subject passed these, the three tasks were administered again but with 24 objects per set.

On the basis of the pretest results, the children were categorized into one of the three developmental stages. The 10 subjects in Stage I and the 84 subjects in Stage II were assigned to either an experimental or a control group in each classroom.

The experimental group received 10 to 20 minutes of daily instruction for five weeks by their regular classroom teachers. Organized, concrete experiences with sets attempted to promote the subjects' understanding that:

"1. Two sets X and Y are said to be in one-to-one correspondence when each element of X corresponds to one and only one element of Y, and each element of Y corresponds to one and only one element of X.

"2. Two sets placed in one-to-one correspondence are said to be equivalent. If Set X is equivalent to Set Y, then Set X has <u>as many</u> members <u>as</u> Set Y.

"3. The elements of equivalent sets may be heterogeneous. Similarity in type, size, color, or shape of elements is unimportant in setting up a one-to-one correspondence.

"4. All sets which are equivalent belong to a particular equivalence class and have the same number.

"5. The equivalence of sets is unaffected by the rearrangement of elements within a set."[74]

The control groups continued in the regular mathematics program. The kindergartners participated in activities, which focused on rational counting and the recognition of geometric shapes. The first graders used Modern Arithmetic Through Discovery, Book One, which is published by the Silver Burdett Company.

After the five-week training period, the experimental and the control groups received a posttest, which was identical to the number conservation pretest.

An analysis of the results revealed that 23 out of 45 subjects in the control group advanced at least one stage since the pretest. In the experimental group, 42 out of 47 subjects progressed. Also, 20 control subjects were in Stage III on the posttest as compared to 41 experimental subjects. This same trend between the experimental and the control groups was observed with sets of 14 and 24 objects. Pace's main conclusion was:

> The instructional program was effective in accelerating the attainment of the concept of number as indicated by changes in stage placements from pretest to posttest. Results showed that the experimental group made significant gains at the 1 percent level over the

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[74]Angela Pace, "Effect of Instruction Upon the Development of the Concept of Number," Journal of Educational Research 62 (December 1968): 185-86.

non-instructed group.[75]

Peters (1970)

Peters divided 131 kindergartners, who had a mean age of 67.6 months, into four treatment groups: (1) non-cued discovery (NCD), (2) perceptual cue guided (PCG), (3) verbal didactic instruction (VDI), and (4) no training control (C).

The NCD condition used two nine-block sets of neutral materials. The subject witnessed 12 standard conservation tasks except that the spatial arrangement was returned to the original position after each transformation.

The procedure for the PCG condition was identical to the NCD condition. However, the materials in the PCG condition assisted the subjects in drawing the conservation inference.

The VDI procedure resembled that for NCD except that, after the questioning (S ? V'), a statement of the rule for conservation was given:

> . . . "I have only moved the blocks. They are in another place, but there are just as many as before. See, I can put the whole bunch back the way they were. There are still the same number as before because I did not put in any more blocks or take away any blocks, I only moved them."[76]

In addition, each subject in the three training groups received two training sessions of number conservation

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[75]Ibid., p. 188.

[76]Donald L. Peters, "Task Variations and Individual Differences in Piaget's Conservation of Number," Merrill-Palmer Quarterly 13 (October 1967): 713.

reversibility training, each session presenting three training trials with two transformations in each trial.

Peters' results indicated that

> . . . acceleration of the learning of conservation of numerical correspondence can be brought about through direct training based upon the notion of reversibility. All three trained groups were superior to the nontrained groups at posttest. However, not all the training procedures provided equally durable effects. Only the PCG treatment and VDI treatments were superior to the control after a prolonged period of no training. The superiority of the VDI treatment over the other forms of instruction at posttest replicates Beilin's (1965) findings, but, the lack of significant difference between this training procedure and the PCG indicates it was not the only viable procedure.[77]

Roll (1970)

Roll pretested 87 Colombian kindergartners, who ranged in age from 5-7 to 7-11, and divided the nonconservers into a training and two control groups. The training materials were seven doll beds, which the experimenter arranged in a row, and seven dolls, which the subject placed on the beds. The training continued:

> . . . E asked, "Are there more dolls than beds or are there more beds than dolls or are there just the same?" The E then took the dolls, made a row twice as long as the row of beds (Transformation A) or made two rows of dolls equal in length to the row of beds (Transformation B), and asked the same question. Then E asked what would happen if the dolls were placed back on their beds, "Will there be too many beds or will there be too many dolls or will there be just the right number of dolls and beds?" After that, S was told to put the dolls back on their beds to see if he were right. During a month's time, each S had four trials per day for 11 nonconsecutive days.[78]

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[77]Ibid., p. 717.

[78]Samuel Roll, "Reversibility Training and Stimulus Desirability as Factors in Conservation of Number," Child Development 41 (June 1970): 503.

Control group 1 was allowed to play with the materials, matched session by session with materials used in training. Control group 2 received only the pre- and posttests.

The posttests were identical to the pretests except that the materials were different. On the second set of posttests, the experimenter made a counter-suggestion, which strongly suggested that length and number of rows were crucial factors and that conservation responses were wrong. For an example, the experimenter said, "You mean to tell me that this big, giant row and this tiny row have the same?"

On the basis of their response to S ? V', 11 out of 16 subjects in the training group conserved as contrasted with four out of 28 subjects in both control groups. On the basis of their ability to correctly justify S = V', only four out of the 16 subjects in the training group and three out of the 28 subjects in the control groups conserved. Nine of the 11 posttest conservers from the training group resisted the counter-suggestion; three of the four posttest conservers in both control groups also withstood the non-conservation suggestion.

Wallach and Sprott (1964)

Wallach and Sprott pretested 66 first graders, who had a mean age of 6-11, with two tasks that were very similar to Piaget's provoked correspondence tests. The stimuli were five index cards and five checkers in the first test and six dolls and six beds in the second test. The standard question (Q) was, "Are there the same number of dolls (checkers) as beds (cards)?" If the subject correctly answered Q, the subject was asked, "How do you tell?" The 30 subjects, who did not correctly answer Q on either task, were equally divided into the experimental and the control groups.

The aim of the experimental group was to induce number conservation by demonstrating the reversibility of the transformation back to its initial configuration. The latter test with the dolls and beds was again administered. After incorrectly answering S ? V', the subject was asked to predict, "Do you think we can put a doll in every bed now? Will there be any beds left over? Any dolls left over?"[79] After responding, the subject was requested to put a doll in each bed. A series of similar situations was presented.

> Each situation was repeated till S made the correct prediction and confirmed it. As many situations were presented as were necessary to reach a criterion of correct prediction on the first trial of four situations in succession. Eight situations turned out to be all that were needed, as all Ss reached criterion within this number.[80]

The control subjects played a checker game, which took approximately the same length of time as the training procedure.

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[79]Lise Wallach and Richard L. Sprott, "Inducing Number Conservation in Children," Child Development 35 (December 1964): 1061.

[80]Ibid., p. 1061.

The first posttest was administered to the experimental and the control subjects after the completion of the training procedure and the checker game, respectively. The posttest duplicated the pretest.

The second posttest was given between 14 and 23 days after the first posttest and duplicated the pretest except that an additional test with bowls and spoons was presented before the checkers and cards test.

A nonconservation suggestion was made at the end of the second posttest, if the subject had correctly responded to Q for the dolls and the beds without a doll in front of it, by saying, "But look--here is a bed without a doll in front of it. Aren't there more beds?"[81]

The results of the first posttest indicated that the training procedure strongly influenced conservation. None of the control subjects conserved on either test while 14 out of the 15 experimental subjects conserved for the dolls/beds and 13 out of the 15 transferred conservation to the checkers. Conservation required only a correct reply to Q. Six of the 14 conservers gave acceptable conservation explanations for dolls/beds; eight of the 13 for checkers/cards.

Similar, significant differences between the experimental and the control groups occurred in the second posttest. In addition, 11 of the 13 conserving experimental subjects maintained the equality of dolls/beds after the

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[81] Ibid., p. 1062.

nonconservation suggestion was made.

Wallach and Sprott concluded, "The results clearly indicate that the training procedure was effective in inducing conservation, and thus support the hypothesis that conservation may be acquired by experience with reversibility."[82] In a five page discussion, the authors reasoned that the results can not be attributed to counting or social reinforcement.

Wallach, Wall, and Anderson (1967)

Wallach, Wall, and Anderson administered two pretests to 56 first graders, whose ages ranged from 6-1 to 7-8. First, a doll pretest was given in which the subject was requested to place one doll in each of the six lined-up beds. Then the experimenter asked three questions as to the equality of the beds and dolls. After the subject answered appropriately, the experimenter transformed the stimuli, as in Piaget's provoked correspondence tests, and asked the subject first about the equality of the dolls/beds and then his rationale or which row had more, depending on his response to S ? V'.

The second pretest followed the standard conservation format and questioning with liquid as the stimuli.

The subjects were divided into five groups on the basis of their pretest performance. Three of the five groups continued in the experiment with training and posttests:

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[82]Ibid., p. 1065.

(1) the nonconservation group which had not conserve on either the dolls/beds or the liquid; (2) the partial conservation-liquid group which had conserved with the liquid but not the dolls/beds; and (3) the partial conservation-dolls group which had conserved with dolls/beds but not with the liquid. For the purpose of comparison, each of the first two groups were divided in half. The sequence of procedures for the now five groups is shown in Figure 7.

The doll reversibility training procedure very much resembled Wallach and Sprott's (1964) reversibility training format.

In the doll addition/subtraction training, the subject was requested to put one of the six dolls in each of the six beds. Then a screen was placed between the subject and the beds. Next, the experimenter handed one of the six dolls to the subject to place in a box at the subject's side, asked the subject if he thought there was a bed without a doll and vice versa, and removed the screen to allow the subject to see that there was one bed without a doll. The whole procedure was repeated, first with the return of the sixth doll, and, second, with the addition of a seventh doll. Then the whole cycle was repeated, beginning with the removal of the sixth doll. This cycle continued until the subject correctly described the situation behind the screen four times in succession.

The liquid transfer-series training involved a sequence of 10 somewhat different conservation tasks, all

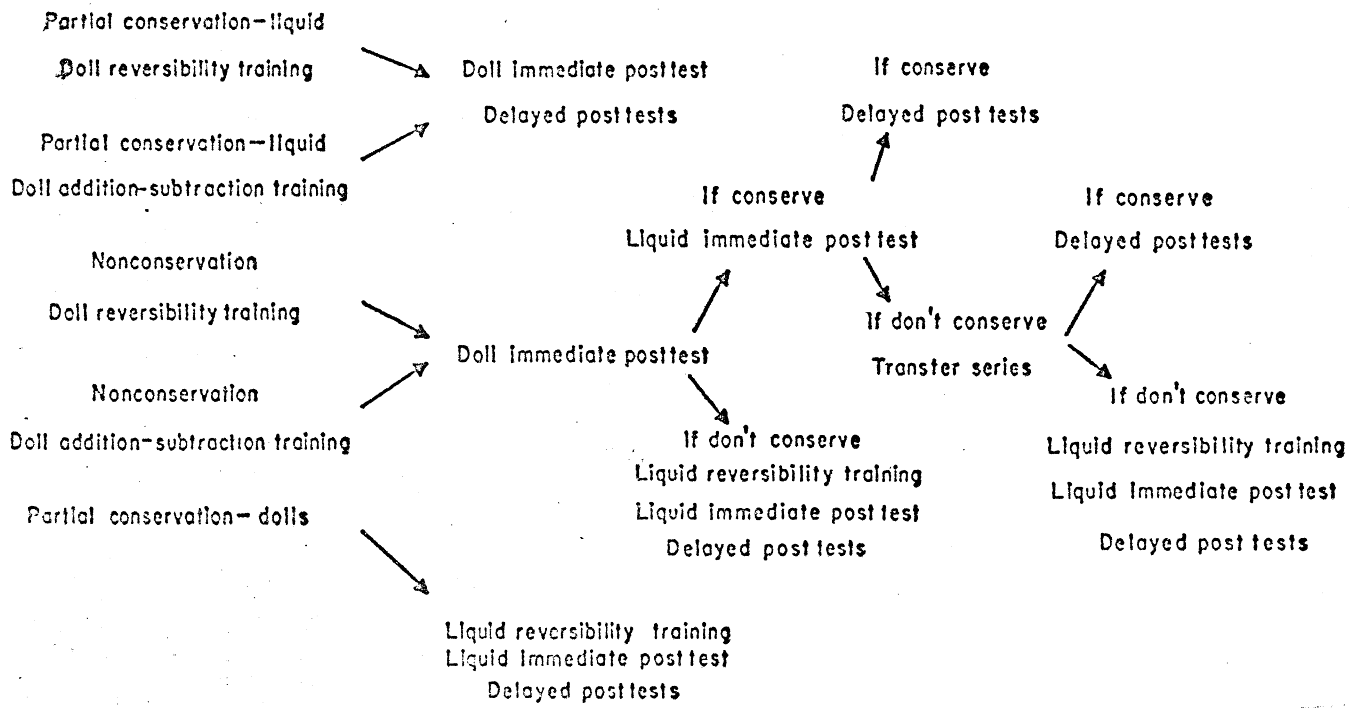

FIG. 7. Sequence of procedures for different groups.[83]

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[83]Lise Wallach, A. Jack Wall, and Lorna Anderson, "Number Conservation: The Roles of Reversibility, Addition-Subtraction, and Misleading Perceptual Cues," <u>Child Development</u> 38 (June 1967): 429.

of which were presented only if the subject had correctly answered each previous question. Each task contained from one to three questions. If at any point an incorrect answer occurred, the transfer-series training was discontinued, and liquid reversibility training was instituted.

In the liquid reversibility training, the experimenter performed the standard conservation task with liquid except for the Q for S ? V'. Instead, the experimenter asked, "If I pour this water back in to this empty glass, will it be filled just like this one?"[84] After the subject had made a prediction, the experimenter poured the water back and asked if the subject's prediction had been correct. This procedure was repeated with three different sets of glasses until the subject had made three correct predictions in succession.

The doll immediate posttest followed the same procedure as the doll/bed pretest. Also, the first part of the liquid immediate posttest was the same as the liquid pretest. However, the posttest was discontinued if the subject had replied S ≠ V'. If the subject conserved, S = V', the experimenter asked the subject his rationale and then repeated the procedure with several variations.

The doll delayed posttest consisted of one standard number conservation task, with two sets of six toy soldiers as the stimuli, and two provoked correspondence tasks, with

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[84]Ibid., p. 433.

five spoons and five bowls as the stimuli in the first task and six dolls and six beds in the second task. If the subject replied S = V' in the dolls/beds task, the experimenter made a nonconservation suggestion, which was the same as that in Wallach and Sprott (1964).

The liquid delayed posttest involved four tasks, which followed the standard conservation task for liquid. The initial stimuli for each task were: (1) two partially but equally filled opaque cups, (2) two completely filled opaque cups, (3) two partially but equally filled test glasses, and (4) two completely filled test glasses. If the subject responded that S = V' in the last task, the experimenter made the nonconservation suggestion, "'But look, in this glass the water goes all the way up to the top, but in this glass it doesn't go nearly so high. Isn't there more water in this glass?'"[85]

The doll immediate posttest indicated that doll reversibility training had a very strong effect on conservation with 12 out of 14 subjects conserving, while two out of 14 subjects conserved in the addition/subtraction training group. The same number conserved throughout the doll delayed posttest except that two of the 12 subjects, who had reversibility training, agreed with the experimenter's nonconservation suggestion. In addition, the doll reversibility training took a median of four trials to reach criterion as compared to a median of six trials to reach

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[85]Ibid., p. 434.

criterion for the doll addition/subtraction training.

Eight of the 14 subjects, who had conserved on the doll posttests, had been nonconservers on both pretests and, therefore, were given the liquid immediate posttest. Only one subject conserved on this posttest, which basically indicated no direct transfer of either doll training conditions to liquid conservation. The seven nonconservers on the liquid immediate posttest received the liquid transfer-series training. Four out of the seven subjects successfully completed training and maintained liquid conservation in the delayed posttest.

The other three subjects, who did not finish the liquid transfer-series training and the 13 subjects from other groups continued with the liquid reversibility training. Five of these 16 subjects gave conservation answers on both liquid posttests, except that one succumbed to the nonconservation suggestion. However, the authors concluded that this data does not provide sufficient support for the effectiveness of reversibility training on liquid conservation. Figure 8 summarizes the results.

Before presenting a very in-depth and somewhat intricate interpretation of the results, Wallach, Wall, and Anderson summarized,

> Several of the questions which gave rise to this experiment are clearly answered by the results. Our first question was whether, in order to induce number conservation, it was necessary for the reversibility-training procedure which had been found effective in a prior experiment to include, as it had, experience with addition and subtraction. The answer is no; the procedure was as effective here without addition-

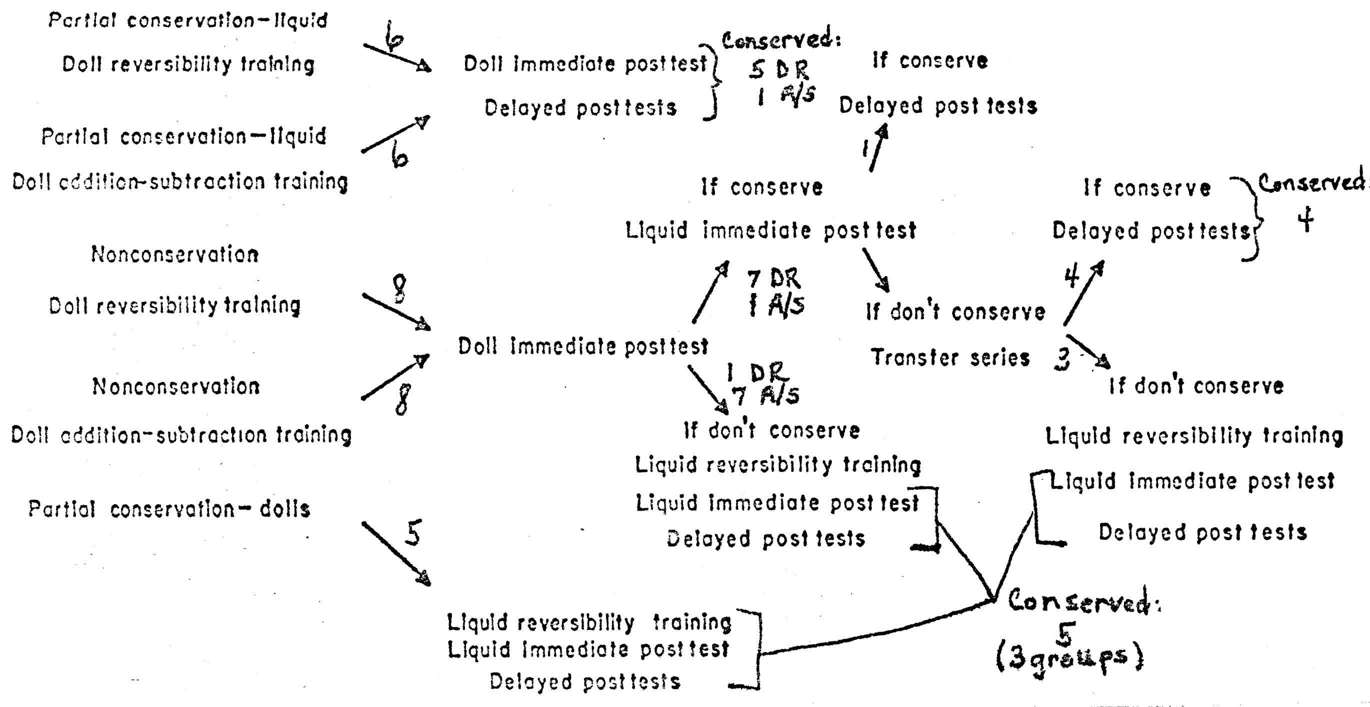

FIG. 8. Number of subjects for each group and summary of the posttests' results.

subtraction experience as it was previously with such experience.

This suggests--but does not necessarily imply--that number conservation is not affected by training in addition and subtraction. In the absence of reversibility training, training with addition and subtraction might still lead to conservation. Our second question was whether this was the case. The answer is no, at least for the particular addition-subtraction training procedure that we used.

Further, the lack of effectiveness of this procedure indicates that the basis for the success of the reversibility training is not that it arouses a number set, as implied by Zimiles' (1963) suggestion. Such training in addition and subtraction ought to be at least as likely, if not much more likely, to arouse a number set as the training in reversibility.

Another question which seems clearly answered is whether the number conservation induced by our reversibility-training procedure transfers directly to such different conservations as that of the amount of liquid: it does not.[86]

Winer (1968)

Winer hypothesized that practice in responding to either additions and subtractions or to changes in length would induce a set in subjects to respond to either of these manipulations. This set would be manifested on conflict trials when changes in length opposed additions and subtractions. Winer further posited that children, who had received addition/subtraction training would acquire conservation.

Winer supported his first hypothesis and, to a limited extent, his second hypothesis.

Winer (1974)

Winer randomly assigned 32 preschooler, who ranged

[86]Ibid., pp. 437-39.

in age from 4-0 to 4-11, to one of two groups: (1) those tested with two and/or three poker chips in a set and (2) those tested with five and/or six poker chips in a set. Based on their pretest results, half of the children from each group were assigned to the corresponding training condition. For the control subjects, an extraneous activity was substituted for the training. All subjects received pretest, training or extraneous activity, posttest, and transfer-test trials.

The training was presented in four blocks of three trials and was designed to encourage the experimental subjects to focus on number and ignore the irrelevant perceptual changes. First, changes in numerical cues were presented by adding or subtracting chips. Later, irrelevant changes in length were made in addition to the addition/subtraction changes.

> . . . On the first trial of each block E showed S two equal rows of chips (Q) and then added a chip to or subtracted a chip from one row (Q). On the next trial an addition/subtraction change was pitted against a change in appearance. Here E showed S two equal rows of chips in one-to-one correspondence (Q), expanded or contracted one row, and then either added a chip to the shorter row or subtracted a chip from the longer row (Q). Thus, on these trials the longer-appearing row ended up with fewer chips. The third trial was similar to the preceding one except that E changed the appearance of a row and added a chip to or subtracted a chip from each row, thus leaving two rows with different appearances but equal quantities.
>
> The first two blocks of trials (sic) always involved the smaller quantities given Ss in a group (e.g., two or five items) and an addition manipulation, while the last two blocks of trials involved the larger quantities (three or six items) and a subtraction change. Also, if S responded correctly on these trials, he was given a token . . ., while, if he responded erroneously, he received no token and was told what the correct response was and why it was

correct.[87]

An analysis of responses, which were made during the training trials, revealed perfect performance for seven of the eight experimental subjects, who were tested with the smaller sets, and less than perfect performance for all eight subjects, who were tested with the larger sets. With the smaller quantities, five experimental and five control subjects had less than perfect pretest performances. On the posttests, all five of these experimental subjects improved as compared to none of the control subjects. With the larger quantities, eight experimental and seven control subjects had less than perfect pretest performance. Two of the eight experimental subjects improved as compared to no control subjects.

Wohlwill and Lowe (1962)

Wohlwill and Lowe presented 72 kindergarten subjects, whose mean age was 5-10, with four diagnostic questions, five tasks in a verbal number conservation pretest with seven objects per stimulus set, and two of the five conservation tasks with twelve items per set. The subjects then participated in pretraining in number matching, which required the subject to count the six to eight stars on the stimuli card and then select the corresponding window from the three windows, which were inscribed with the numerals

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[87]Gerald A. Winer, "Conservation of Different Quantities Among Preschool Children," Child Development (September 1974): 840.

"6." "7," and "8." If he chose the correct window, the subject found a chip behind the window. Otherwise, the subject had to correct any counting errors and received guidance in choosing the correct window when necessary.

The fourth phase of the experiment involved a nonverbal conservation pretest with three two-phase trials.

> . . . Ss were presented with a row of colored stars, either six, seven, or eight in number, mounted on a set of corks which rested on a series of connected scissors-like slats. This apparatus permitted lengthening or shortening the row while preserving the straight-line arrangement. E told S that he was to count the stars in order to find the chip behind the correct window. Following S's initial response, he was made to return the chip to E, who replaced it behind the same window, and then, depending on the trial, either extended or shortened the row of stars. S was allowed to count only on the first phase; he thus had to find the correct window on the second phase on the basis of the knowledge gained in the first and in the face of the perceptual changes in the row of stars.[88]

The subjects were next assigned to either an experimental group or the control group. The three experimental conditions were: (1) reinforced practice, (2) addition and subtraction, and (3) dissociation. All four groups received two sets of nine trials each, which were administered on successive days. Wohlwill and Lowe described each series of trials:

> a. Reinforced Practice (RP). The procedure here was the same as for the preceding conservation trials, with this modification: if S made an incorrect response on the second phase of the trial, he was told to count the stars, so as to find out which window he should have chosen. E then exposed the chip behind that window but

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[88]Joachim F. Wohlwill and Roland C. Lowe, "Experimental Analysis of the Development of the Conservation of Number," Child Development 33 (March 1962): 158.

did not allow S to remove the chip.

b. Addition and Subtraction (A&S). These trials were similar to the conservation trials, except that on two-thirds of thr trials, following the S's initial response after counting, E either added or subtracted a star at the end of the row before changing its length. The remaining third of the series consisted of straight conservation trials which were interspersed with the A&S trials.

c. Dissociation (Diss.). Unlike the above, these were single phase trials, with the length of the row varying from one trial to the next over a range of four times the smallest length. S was urged to count the stars and open the corresponding window; if correct, he received the chip. Over the series of trials each number of stars appeared equally often at each of the different settings of length.[89]

Immediately after the completion of the training, all the subjects took a nonverbal conservation posttest and a verbal number conservation posttest. Both posttests were identical to the pretests except that, if the subject correctly responded on the last trial of the nonverbal conservation posttest, he was asked, "'How did you know where to look for the chip that time?'"[90]

Wohlwill and Lowe found that the greatest amount of improvement from the pretest to the posttest trials took place in the A&S group. Wohlwill and Lowe's results were analized by Zimiles (1963) in a rather comprehensive article.

General Implications to Education

If a person had reviewed the literature on the effectiveness of conservation training procedures a few years ago, the person would have concluded that the concept of conser-

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[89]Ibid., p. 159.

[90]Ibid., p. 159.

vation was resistive to training and perhaps based on a very different type of learning process. However, as evidenced in the first section of this chapter, a number of more recent studies have demonstrated the effectiveness of a variety of training techniques. As noted by Wohlwill (1970),

> . . . The interesting point to emerge from a comparison of the results of these various studies is that the amount of transfer observed in these various studies was roughly in proportion to the breadth and intensity of the training experiences.[91]

How and what activities should the educator provide in order to achieve this necessary "breadth and intensity" of training? Piaget and many of his proponents recommend the discovery approach to learning. This greatly contrasts with the highly structured approaches in the training studies, which were designed to promote a Piagetian concept.

Much controversy exists as to the actual interpretation or implementation of the discovery approach. On the one hand, Piaget and others would present an enriched environment which would provide a child with a variety of experiences to assimilate when and as he chooses. This position reflects Piaget's concern for creativity and for possible harm due to cognitive acceleration.

> . . . The real problem is knowing whether it is advantageous to accelerate development. There are two issues here. Pedagogically, I think it is better for a child to find, and invent his own solutions rather than being taught. Teaching something to a child prevents him from inventing the solution. They will be

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[91]Joachim F. Wohlwill, "The Place of Structured Experience in Early Cognitive Development," Interchange 1, n. 2 (1970): 14.

> more constructive and creative if left to themselves.
> Psychologically, acceleration is possible, but whether this is advantageous to development--we don't know. . . .
> . . . So the real problem is that there is probably an optimal development speed. Future research will determine this. Psychologically, we don't know, so we must be careful.[92]

On the other hand, other researchers and educators (Brearley 1969; Sharp 1969; Lavatelli 1970; Biber, Shapiro, and Wickens 1971; and Weikart, Rogers, Adcock, and McClelland 1971) suggest structured experiences which are not as rigid as the training procedures on number conservation. As Roeper and Sigel (1970) stated,

> This brings up the issue of incidental learning. The young child is most eager for learning. Every experience therefore becomes a learning situation. Early childhood education has realized the child's great potential for learning by himself and it has become an integral part of preschool education. This type of learning, however, is unselective in the case of the child who functions on a preoperational level. He is not yet equipped to differentiate between different categories of facts and therefore to build his judgment on proper relevancy. . . . The young child is deeply motivated toward understanding the world but is not yet mentally equipped for it. The only solution for his dilemma seems to be knowledgeable adult guidance. It is for this reason that we believe the young child should be helped toward proper concept formation through an organized goal-directed approach built on knowledge of his cognitive growth.[93]

Given both positions, how can transferability of intellectual functioning be fostered? Gagné suggested the "approach to generalizability via learning hierarchies":

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[92]"Interview with Jean Piaget," Times (London) Educational Supplement, 18 February 1972, p. 19.

[93]Annemarie Roeper and Irving Sigel, "Finding the Clue to Children's Thought Processes," in Educational Implications of Piaget's Theory, ed. Irene J. Athey and Duane O. Rubadeau (Waltham, Mass.: Ginn-Blaisdell, 1970), pp. 88-89.

> Assuming that transferability can be insured by using the method described, there is a major reason for preferring programmed over unprogrammed basic learning experiences. Simply stated, this reason is the possibility that "gaps" will occur because of accidental variations in the child's early experience, and these gaps will have the effect of making vertical transfer inordinately difficult. Since learning has a cumulative effect, according to this notion, intellectual development will be slowed.
>
> Thus use of pre-planned learning hierarchies in the design of instruction by no means precludes the provision of variety of experiences at each "level" of vertical transfer.[94]

Through his theory on learning hierarchies, Gagné and numerous other authors (Stendler 1962, Coxford 1963, Rosenbloom 1964, Bidwell 1969, Adler 1970, Smart 1970, and Lovell 1971) support the principle of precision teaching, which is a totally developmental arpproach to instruction. Precision teaching not only emphasizes the structure of and materials used in the learning process, but also stresses the correspondence of these learning components to the individual's abilities.

In line with this correspondence, a word frequently associated with the concept of precision teaching is "match":

> . . . there must be some kind of match between the quality of the thinking skills of the child and the complexity of the mathematical ideas to which he is introduced.[95]
>
> The job of the teacher is to use his professional skill and provide learning situations for the child which demand thinking skills just ahead of those which are

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[94]Robert M. Gagné, "Structured Experience and Pre-planned Learning," Interchange 1, n. 3 (1970): 115.

[95]Kenneth Lovell, The Growth of Understanding in Mathematics (Kindergarten through Grade Three) (Chicago: Holt, Rinehart and Winston, 1971), p. 1.

> available to him. It is a question of keeping the carrot just ahead of the donkey's nose.[96]

O'Brien and Shapiro (1969) restate Lovell's (1971) explanation of "match" in Piagetian terms:

> Perhaps it is the situation slightly different from the student's existing cognitive structure which causes him to query the existing structure and change it as necessary to restore equilibrium between the internal and the external world.[97]

Summary

Fourteen studies on training procedures for the acquisition of number conservation have been reviewed. Some of the procedures appear more effective than others. By combining the more effective training procedures with the other factors which affect number conservation, the reader should be able to apply his creativity and psychological and educational knowledge to the development of a program, which is specifically designed to meet the needs and abilities of his individual students.

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[96] Ibid., p. 17.

[97] Thomas C. O'Brien and Bernard J. Shapiro, "Problem Solving and the Development of Cognitive Structure," Arithmetic Teacher 16 (January 1969): 12.

CHAPTER IV

CONCLUSION

The preoperational child would benefit from a combination of structured experience and a variety of open-ended play. The structured experiences in number conservation would be most effective with the child in Stage II (Transitional) and the child in Stage III, who has not generalized the concept of number conservation to all materials and situations.

In structuring the experience, reversibility training appears to be the most successful approach at this time. Although the research is not conclusive at this time, sessions which also utilize the principles of verbal cue, perceptual guidance, and addition/subtraction may reinforce reversibility training and quicken the acquisition of number conservation.

In addition, the learning experiences would be enhanced by the application of discrimination learning theory and research. The most promising format is successive approximations. The first step in this approach would use functionally related items as the stimuli, each set containing two or three items. The stimuli should be considered desirable to the particular child. Initially, the materials should be presented in rows, which are perpendicular to the child's front. Static arrays should first be used, and only one-part questions asked. No justification should be required in the beginning.

Later steps would involve less provoking and more neutral material. Each set gradually increases in size. More difficult tasks would ask two- and three-part questions, include observable transformations, and require justifications.

The fundamental concept of precision teaching is the matching of the learning to the child's present ability level. Through the information provided in the review of literature, the educator should be more capable of providing appropriate learning experiences for number conservation to each student.

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