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Mathematical concept formation for the primary school-aged learning disabled student

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MATHEMATICAL CONCEPT FORMATION
FOR THE PRIMARY SCHOOL-AGED
LEARNING DISABLED STUDENT

by

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To my wife Jan,

A dedication is not unique . . . nor is love
For many things, for many reasons . . . Amo Te!

To Sr. Joanne Marie Kliebhan O.S.F. Ph. D.,

For all those, "May I see you for just a minute?",
For all those inconveniences, I thank you!

This research paper has been approved for the Graduate Committee of the Cardinal Stritch College by

Sister Joanne Marie Kliebhan
(Advisor)

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CHAPTER I

Introduction

The Learning Disabled (LD) student often encounters difficulty in mathematics and other related areas of cognition where quantitative reasoning is required. This paper attempts to define the nature of the problem, as well as the developmental sequences and processes that lead to the remediation of those difficulties.

The researcher, as a learning supervisor of LD students, has often encountered difficulty in teaching mathematics to those students and has hoped for a more precise technique(s) for doing so.

The researcher feels that when a student fails to understand a basic mathematical statement, such as, 1+1=2, that the problem lies far deeper into the mental processes of mathematical psycholinguistics, rather than the mere failure of producing the correct verbal or written response, through the arithmetical manipulation of numbers.

Rationale

As previously alluded to, the reason for writing this paper was to provide insight into helping the LD student to learn mathematics.

It has been observed by the researcher, that mathematics curricula, that have been devised for the normal student are not sufficient for the LD student, even though, the pace of presentation of the material has been "slowed to fit the needs of the individual".

It is the opinion of the researcher that mathematics should be developed as a psycholinguistic skill to optimize the learning environment for the exceptional learning person. Rather than assuming that the student has the necessary readiness skills in mathematics, the researcher asserts that these concepts can be more fully integrated in a linguistic environment.

Scope

The areas examined are: definition of mathematical ability, formation of mathematical concepts, psycholinguistics of mathematics, number concept, and finally, theory of transference.

It is hoped that these areas will provide the necessary framework for the reader to understand the developmental processes of learning and teaching mathematics. It may be noticed by the reader that some areas examined for research were not as extensively covered as other areas, and this seems to indicate to the researcher that some areas of exceptionality are being presently researched by the scientific community and the relative newness of the LD student as a topic of research by that community. Due to this fact, the amount of research that is readily available, is somewhat limited at this time.

Limitations

The relative newness of the study of the LD child in itself limits the paper. Although there is no clear consensus for the meaning of the term "learning disabilities" among the educational communities, the 1962 definition of Kirk and Bateman and that of the National Advisory Committee on Handicapped Children, represents the most widely accepted consensus statement.

For purposes of clarity, the researcher has included those definitions at this point. The Kirk and Bateman definition is as follows:

> A Learning Disability refers to a retardation, disorder, or delayed development in one or more of the processes of speech, language, reading, writing, arithmetic, or other school subjects resulting from a psychological handicap caused by a possible cerebral dysfunction and/or emotional or behavioral disturbances. It is not the result of mental retardation, sensory deprivation, or cultural or instructional factors.[1]

The National Advisory Committee on Handicapped Children states:

> A Learning Disability refers to one or more significant deficits in essential learning processes requiring special educational techniques for its remediation.
>
> Children with learning disability generally demonstrate a discrepancy between expected and actual achievement in one or more areas, such as spoken, read, or written language, mathematics, and spatial orientation.
>
> The disability referred to is not primarily the result of sensory, motor, intellectual, or emotional handicap, or lack of opportunity to learn.
>
> Essential learning processes are those currently referred to in behavioral science as perception, integration, and expression, either verbal or nonverbal.

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[1]Samuel A. Kirk and Barbara Bateman, "Diagnosis and Remediation of Learning Disabilities," Exceptional Children, XXXIX, No. 2, (October, 1962), 73.

> Special education techniques for remediation require educational planning based on the diagnositic procedures and findings.[2]

The reader will observe that both statements contain the subject of mathematics as an area of difficulty for the LD student. As previously stated, this paper addresses itself to that area of difficulty.

This paper is also limited in that it will be primarily concerned with the primary school-aged learning disabled student and not with the whole continuum of mathematics. It is concerned with those concepts that are presently deemed necessary for mathematical understanding by the pedagogic world.

Some of those concepts discussed are: more, less, equal to, more than, greater than, less than, set, number, and the appropriate language in introducing those concepts to the student.

Summary

This paper is concerned with mathematics and the Learning Disabled student. In this chapter, the rationale, the scope, and the limitations of the study were stated, as well as the definitions of terms to be employed in the research review.

The population to be discussed is the primary school-aged Learning Disabled student. The concepts to be examined

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[2]National Advisory Committee on Handicapped Children, Special Education for Handicapped Children, First Annual Report, (Washington D.C.: Department of Health, Education, and Welfare, Office of Education, January 31, 1968), p. 34.

are: more, less, equal to, more than, greater than, less than, number, set, and the appropriate language for each concept.

CHAPTER II

Definition of Mathematical Ability

There have been many researchers attempting to define the term mathematical ability. If mathematical ability can be isolated as a separate mental function, is there a working definition? Werdelin derived this definition:

> . . . the ability to understand the nature of mathematics (and) similar problems, symbols, the methods and proofs, to learn them, to combine them with other problems, symbols, methods, and proofs, and to use them when solving mathematical and (similar) tasks.[3]

Werdelin, in trying to factor-analyze mathematical ability as a separate mental function, has encountered opposition because the existence of a broad group factor of mathematical ability is a matter of present controversy.

British factor-analysts, Blackwell, Barakat, and Wrigley,[4] have stated that a group factor of mathematical

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[3]I. Werdelin, "The Mathematical Ability: Experimental and Factorial Studies," (Lund, Sweden: C. W. K. Gleerup), cited by Lewis Aiken, Jr., Review of Educational Research, XLIII, (Fall, 1973), p. 405.

[4]A. M. Blackwell, "A Comparative Investigation into the Factors Involved in Mathematical Ability of Boys and Girls, Parts I and II," British Journal of Educational Psychology, 1940, X, pp. 143-153, pp. 212-222. M. K. Barakat, "Factors Underlying the Mathematical Abilities of Grammar School Pupils," British Journal of Educational Psychology, 1951, XXI, pp. 239-240. J. Wrigley, "Factorial Nature of Ability in Elementary Mathematics," British Journal of Educational Psychology, 1958, XXVIII, 61-78.

ability does exist, defined as quantitative reasoning or problem solving ability.

American psychologists, Guilford, Hoepfner and Petersen, and Hills[5] have questioned the idea of a group factor and have assessed mathematical ability as a composite of mathematical ability as shown in TABLE I.

TABLE I

Representative Mathematical Ability Factors Obtained in Various Factor-Analytic Investigations

Deductive (general) reasoning - (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin 1958, 1966)

Inductive reasoning - (Werdelin, 1958)

Numerical ability - (Kline, 1960; Very, 1967; Werdelin 1958, 1966; Wooldridge, 1964)

Spatial-perceptual ability - (Blackwell, 1940; Very, 1967; Werdelin, 1966)

Verbal Comprehension - (Blackwell, 1940; Kline, 1960; Very, 1967; Werdelin, 1958, 1966; Woolridge, 1964)

*Lewis Aiken, Jr., "Ability and Creativity in Mathematics," Review of Educational Research, XLIII, (Fall, 1973), 406.

[5]J. P. Guilford, R. Hoepfner and H. Petersen, "Predicting Achievement in Gifted Children," School Review, cited by Lewis Aiken, Jr., XLV, pp. 388-414. J. R. Hills, "Factor-analyzed Abilities and Success in College Mathematics," Educational and Psychological Measurement, 1957, IV, pp. 615-622, cited by Lewis Aiken, Jr., Review of Educational Research, XLIII, (Fall, 1973), p. 406.

The term deductive reasoning, in TABLE I is similar to the group factor espoused by the British psychologists.

Krutetskii, a Russian, in a related study, has also addressed himself to the question of mathematical ability and seems to favor the American position as he has listed "components" of mathematical ability as follows:

(a) Formalized perception of mathematical ability
(b) Generalization of mathematical material
(c) Curtailment of thought
(d) Flexibility of thought
(e) Striving for economy of mental forces
(f) Mathematical memory
(g) Spatial concepts[6]

In a related area of research on creative mathematics, Poincaire and Hodamard, claim that much of the work that goes on in mathematics is unconscious or intuitive, rather than logical, formal, and conscious. They have refined their research thus:

(a) Preparation, concentration and deep involvement into a problem
(b) Incubation, during which time the problem is set aside for awhile but work on the solution continues at the unconscious level. If the prolonged work of the second stage is successful, a third stage;
(c) Illumination, or insight into a solution, then;
(d) Verification, elaboration and refinement of the insightful solution takes place[7]

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[6]V. A. Krutetskii, "An Experimental Analysis of Pupils Mathematical Abilities," in Soviet Studies in Psychology of Learning and Teaching Mathematical Ability, ed. by J. Kilpatrick and I. Wirszup, (Stanford, California: School Mathematics Study Group, 1969), pp. 105-111, cited by Lewis Aiken, Jr., Review of Educational Research, XLIII, (Fall, 1973), p. 406.

[7]H. Poincaire, "Mathematical Creation," in The Creative Process, ed. by B. Ghiselin (New York: New American Library Press, 1952), pp. 32-42, cited by Lewis Aiken, Jr., Review of Educational Research, XLIII, (Fall, 1973), p. 406.

If mathematical ability can be defined, can there be quantitative differences in that ability? Aiken goes on to say that there must be further research in the area of mathematical ability, specifically, research in individual differences in psychological and behavioral responses during mathematical problem solving and in perception. Learning and retention should also be assessed in the individual before the issue of what mathematical ability really is can be resolved.[8]

Krutetskii, also acknowledges that there is a need for more research in the psychology of learning, as he indicates that there are people who are more sensitive to relations, symbols, and numbers, and that associations involving them are formed more easily and with less effort and greater retention.[9]

In summary, the reader can see that there is no concensus concerning the definition of what constitutes mathematical ability or even if mathematical ability, as a mental entity, does exist.

There appear to be two schools of thought on the

J. Hodamard, The Psychology of Invention in the Mathematical Field (New York: Dover Press), 1954, cited by Lewis Aiken, Jr., Review of Educational Research, XLIII, (Fall, 1973), p. 406.

[8]Aiken, Jr., Ability and Creativity, p. 407.

[9]Krutetskii, An Experimental Analysis, p. 408.

subject. The British camp expounds that there is a mental function termed quantitative reasoning or problem solving ability. The American school rejects that claim and sees mathematical ability as a composite of mathematical abilities.

Formation of Mathematical Concepts

In beginning to discuss the formation of mathematical concepts, Lazarus[10] has some observations for the teacher. He has coined the term mathophobia indicating a learned repulsion by the student, for the subject of mathematics.

Mathophobia occurs, it seems, as a learned response to the teacher's ineptitude in the hierarchical development of mathematics, and puts the student into unknown areas of a subject where no experiential or psychological readiness has been ascertained as being present in the student.

The end result of this unfortunate situation, is the student opting for non-mathematical courses, or if that is not possible, an unconscious dislike for "those courses" and a fatalistic attitude develops when encountering them.

Many researchers have addressed themselves to the question of when to introduce mathematics to the neophyte learner within the confines of hierarchical development.

The notion of labeling the mathematical mind as logical, formal, intuitive, fast, or slow, has been rejected as an oversimplification of the issue of trying to label

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[10]Mitchell Lazarus, "Mathophobia: Some Personal Speculation," National Elementary Principal, LIII, (January, 1974), 16-22.

the mathematical mind.[11]

Lovell,[12] in directing research on the introduction of mathematics to the LD student, has wondered if the student should wait until he is at the formal operational stage (in the Piagetian sense) before being introduced to mathematics, in order to assimilate ideas fully and formally as mathematics would have them assimilate those concepts ideally.

In continuing his investigation, Lovell goes on to say that it is impossible to say with precision, at present, when children are ready for introduction to mathematical thought.

In further research into the psychology of learning, as applied to mathematics, Lovell asserts that some ideas of comparable mathematical level, structure, and abstraction, are understood better than others, and that there are individual differences within and among individual people, which makes the pinpointing of "now" for mathematical introduction imprecise.

The more familiar students are with concept content, the more readily formal operational thought can occur. Topics can be introduced in different depths, so that the teacher may start with the understanding that the pupil's understanding may be limited at that point in time. Aware

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[11]Aiken, Jr., Ability and Creativity, P. 406.

[12]Kenneth Lovell, "Intellectual Growth and Understanding Mathematics," Arithmetic Teacher, XIX, (April, 1972), 277-282.

of this, the teacher can build the necessary readiness skills, can make the student comfortable with some basic mathematical concepts and terms. Therefore when the teacher returns to the topic at a later time, with a different emphasis of involvement, the subject matter can be reorganized and seen in a different light, i.e. knowledge reinforced with experiential readiness.

Knowledge of the subject matter and of his pupil's abilities allows the teacher to distinguish the level of thinking of the child in relation to the particular topic and the teacher will not attempt to force an understanding not yet available to the student.

Post,[13] in similar research, declares that although there are differences in various learning theorists, all seem to agree that human beings learn from the simple to the complex, and that concept development (at least in its initial stages) is contingent upon readiness skills with sensory media.

More specific to Lovell's research, is the commonality between the two, that abstraction as in mathematics, is not innate but rather a cognitive ability which develops with time and experience and appropriate concrete experiences. The primary grades are the time for the development of appropriate, major mathematical concepts, a time for the child to explore avenues of interests and appeal, a time to

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[13]Thomas Post, "New Look at the Role of Computation in the Primary Grades," <u>School Science and Mathematics</u>, LXXI, (June, 1971), 527-530.

involve oneself in the understanding of inductive thinking, emanating from those sensory experiences supplied with concrete media.

O'Brien,[14] in trying to characterize the thinking of young children, states that one characteristic of young children's thinking is "atomism" - a view that the things, events, and ideas of one's experiences are unrelated to one another: i.e., they exist as if they were isolated atoms. As an example, he offers, (23 + 41 = 37), where the sum is less than one of the addends. Each addend being seen, by the child, as unrelated to the sum.

Also indicative of children's thinking is "backwards-atomism", whereby children attribute relations without sufficient examination. Backwards-atomism involves an inability to ascertain what <u>must</u> be from those things that <u>may</u> be, as in the numerical example given above.

O'Brien goes on to state that relational thinking is thought that appropriately connects together the "atoms" of a person's experience and that tests the necessity of relationships potentially interweaving them.

Scott, in related research to that of Lovell, Post, and O'Brien, has given the researcher insights into the total concept of mathematical development. Scott has accomplished this task by developing the process of mathematical

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[14]Thomas O'Brien, "Perceptual Goals for Mathematical Education," <u>Childhood Education</u>, L, (February, 1974), 214-216.

conceptualization as follows:

1. The structure of mathematics should be stressed at all levels. Topics and relationships of endurance should be given concentrated attention.
2. Children are capable of learning more abstract and more complex concepts when the relationship between concepts is stressed.
3. Existing elementary arithmetic programs may be severely condensed because children are capable of learning concepts at much earlier stages than thought.
4. Any concept may be taught a child of any age in some intellectually honest manner, if one is able to find the proper language for expressing the concept.
5. The inductive approach of the discovery method is logically productive and should enhance learning and retention.
6. The major objective of a program is the development of independent and creative thinking processes.
7. Human learning seems to pass through the stages of preoperations, concrete operations, and formal operations.
8. Growth of understanding is dependent upon concept exploration through challenging apparatus and concrete materials and cannot be restricted to mere symbolic manipulations.
9. Teaching mathematical skills is regarded as a tidying-up of concepts developed through discovery, rather than as a step-by-step process for memorization.
10. Practical application of isolated concepts or systems of concepts, particularly those applications drawn from the natural sciences, are valuable to reinforcement and retention.[15]

Scott's development of mathematical concepts as contained above, seem to the researcher to be in agreement with Gagne, as his "assumptions" are contained below:

1. Any human task may be analyzed into a set of component parts which are quite distinct from each other in terms of the experiential operations needed to produce them.
2. These tasks components are mediators of the final task performance, that is, their presence insures positive transfer to a final performance, and their absence reduces such transfer to near zero.
3. The basic principle of training design consists

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[15]Lloyd Scott, Trends in Elementary School Mathematics (Chicago: Rand McNally and Co., 1966), pp. 15-16.

> of: a) identifying the component tasks of a final performance; b) insuring that each of these component tasks is fully achieved; and c) arranging the total learning situation in sequence which will insure optimal mediational effects from one component to another.[16]

The researcher included "Gagne's assumptions" because of the emphasis he places on "experiential operations" which places him in some commonality with the other researchers quoted in this section.

In summary, all of the researchers seem to agree on the need for early cognitive, mathematical, stimulation to insure the human organism passing through "hierarchies" or "stages" of development. While the hierarchies or stages are not pure unto themselves, the researchers seem to state and emphasize a "lock-step" system of psychological movement, i.e., one stage must be successfully "passed" before the next can be "entered."

Certainly the researcher of this paper would be amiss if he did not mention the renown Jean Piaget. While this paper is not a paper on Piaget, the researcher does not neglect him. Much of the work quoted is based upon Piaget's research and Piaget, himself, will be quoted where necessary and appropriate. The researcher invites the reader to peruse the works of Piaget as needed on the subject of mathematical concept formation.

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[16]Robert Gagne, "Learning Hierarchies," Educational Psychology, VI, No. 1: 3-6, 1968.

Psycholinguistics of Mathematics

The researcher has stated that the need for revision of mathematical curricula for the LD student exists. As the researcher examined the writings of others, Aiken states the need for a deeper understanding and examination of language, as it pertains to mathematics. Aiken states, "It is generally recognized that not only do linguistic abilities affect performance, but that mathematics itself is a specialized language."[17]

Linville,[18] in acknowledging that mathematics is a specialized language, believes that syntax and difficult vocabulary continue to interfere with effective problem solving.

In trying to ascertain the validity of his assumption that syntax and vocabulary interfere with effective learning, Linville performed research to validate those assumptions. Four arithmetic word problem tests, each consisting of the same problems but varying in difficulty of syntax and vocabulary were prepared: a) Easy Syntax, Easy Vocabulary; b) Easy Syntax, Difficult Vocabulary; c) Difficult Syntax, Easy Vocabulary; d) Difficult Syntax, Difficult

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[17] Lewis Aiken, Jr., "Language Factors in Learning Mathematics," Review of Educational Research, XLII, (Summer, 1972), 359.

[18] W. J. Linville, "The Effects of Syntax and Vocabulary Upon the Difficulty of Verbal Arithmetic Problem Solving with Fourth Grade Students," (Doctoral Dissertation, Indiana University) Ann Arbor, Michigan, University Microfilm, 1970. No. 70-7957. (DA 30A: 4310; April, 1970), cited by Lewis Aiken, Jr., Review of Educational Research, XLII, (Summer, 1972), p. 364.

Vocabulary. These problems were given to 408 fourth graders in twelve schools. Linville, in analyzing the data concluded that significant main effects showed favor toward Easy Syntax and Easy Vocabulary. Secondary findings indicated that pupils who possessed higher general ability and/or higher reading ability made significant higher scores on the arithmetic problems than pupils of lower ability.

In reviewing other studies pertaining to mathematics and linguistics, the researcher has found other examiners who support Aiken's assumptions. Dresher and Johnson[19] found gains in problem solving ability when pupils were given specific training in mathematical vocabulary. Lyda and Duncan[20] found that direct study of quantitative vocabulary contributed to growth in reading, arithmetical computation, and arithmetical reasoning. Lyda's and Duncan's study pertained to a population of twenty-five second graders but the researcher feels that the study is pertinent since it tends to support research by others that claim direct study of terms, as they are applied to mathematics, is a prerequisite to appropriate mathematical understanding.

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[19]R. Dresher, "Training in Mathematics Vocabulary," Educational Research Bulletin, XIII, 1934, pp. 201-204, cited by Lewis Aiken, Jr., Review of Educational Research, XLII, (Summer, 1972), p. 365. H. C. Johnson, "The Effects of Instruction in Mathematical Vocabulary Upon Problem Solving Ability," Journal of Educational Research, XXXVIII, 1934, p. 365.

[20]W. J. Lyda and F. M. Duncan, "Quantitative Vocabulary and Problem Solving," Arithmetic Teacher, XIV, 1967, pp. 289-291, cited by Lewis Aiken, Jr., Review of Educational Research, XXXVIII, 1934, p. 365.

Also, Wilmon[21] revealed that children are introduced to 500 technical terms by the time they reach fourth grade. Wilmon summarized his study by saying that teachers need to reinforce textbooks by concentrating on specialized reading vocabulary in the first three grades.

Harter and Kane[22] in attempting to analyze the comprehensibility of mathematical English passages designed for grades seven through twelve stated that the Cloze method of reading analyzation tends to be a highly reliable and valid predictor of mathematical comprehensibility as contained in textbooks for those grades. Whatever reading formula is employed, Earp[23] noted that the vocabulary of arithmetic texts is frequently at a higher readability level than the performance of the students in grades in which the texts are used.

Smith[24] also reviewed readability levels and

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[21] B. Wilmon, "Reading in the Content Area: A Specialized New Math Terminology List for Primary Grades," Elementary English, XLVIII, 1971, pp. 463-471, cited by Lewis Aiken, Jr., Review of Educational Research, (Summer, 1972), p. 365.

[22] M. A. Harter and R. B. Kane, "The Cloze Procedure as a Measure of the Reading Comprehensibility and Difficulty of Mathematics," English, EDRS Acc. No. ED040881, 1970, cited by Lewis Aiken, Jr., Review of Educational Research, (Summer, 1972), p. 366.

[23] N. W. Earp, "Reading in Mathematics," EDRS Acc. No. ED036397, May, 1969, cited by Lewis Aiken, Jr., (Summer, 1972), p. 366.

[24] F. Smith, "The Readability of Sixth Grade Words," School Science and Mathematics, LXXI, 1971, pp. 559-562, cited by Lewis Aiken, Jr., Review of Educational Research, (Summer, 1971), p. 366.

concluded that readability need not be the primary cause of low scores in mathematics.

Spencer and Russell[25] in following-up research on the reasons for mathematical reading difficulty, found that: a) names of certain numerals are confusing; b) number languages which are patterned differently from the decimal system are used; c) the language expressing fractions and ratios is complicated; d) charts and other diagrams are frequently confusing; e) the reading of computational procedures requires specialized skills.

Gilmary[26] asserted through scientific examination, that training in reading and arithmetic is superior to just instruction in arithmetic alone.

In attempting to offer remediation for difficulties in mathematics, Earp[27] declares that reading adjustment is needed: a) adjustment to a slower rate than that for narrative material; b) varied eye movements, including some regressions, c) reading with an attitude of aggressiveness and thoroughness.

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[25]P. L. Spencer and D. H. Russell, "Reading in Arithmetic," National Council of Teachers of Mathematics Yearbook, XXV, 1960, pp. 202-223, cited by Lewis Aiken, Jr., Review of Educational Research, (Summer, 1972), p. 367.

[26]Sister Gilmary, "Transfer Effects on Reading Remediation to Arithmetic Computation When Intelligence is Controlled and All Other School Factors are Eliminated," Arithmetic Teacher, XIV, 1967, 17-20, cited by Lewis Aiken, Jr., Review of Educational Research, (Summer, 1972), p. 367.

[27]Earp, Reading in Mathematics, p. 369.

In reviewing research on the effects of verbalization versus nonverbalization of mathematical concepts, Stern delineated that requiring students to say the concept aloud was no more effective in improving problem solving ability than not requiring children to do so. Stern, in related research on the same topic, also found that much covert verbalization of talking to oneself occurs during the study of an arithmetic problem. Stern goes on to say that this covert verbalization may be enough to facilitate problem solving.

Stern's points of discussion are thus:

1) Engagement in multi-sensory problem situations.
2) Acquisition of oral language to represent in complete sentence form, the quantitative relations in problem situations.
3) Introduction of written, arithmetical symbols as a shorthand way of writing already known spoken words.
4) Acquisition of meaning of written or spoken arithmetical symbols by representing something in experience.
5) After learning to read them, the writing of numbers, number combinations, algorisms, and so on.
6) Acquisition of computational processes by manipulation and discovery, not by memorization and applying math rules.
7) Teaching rules, principles, and generalizations by the inductive-deductive method.
8) Continuous interrelations between first-hand quantitative experience in life expression of these in oral and written symbolism, and increasing consciousness and knowledge of the nature of arithmetic.[28]

Ausubel and Robinson[29] in a related, cogent study believe that: a) mathematics deals with concepts, the meanings

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[28] C. Stern, "Acquisition of Problem-solving Strategies in Young Children and Its Relation to Verbalization," Journal of Educational Psychology, LVIII, 1967, pp. 245-252, cited by Lewis Aiken, Jr., Review of Educational Research, XLII, (Summer, 1972), p. 371.

[29] D. P. Ausubel and F. G. Robinson, School Learning: An Introduction to Educational Psychology (New York: Holt, Rinehart and Winston, 1969) cited by Lewis Aiken, Jr., Review of Educational Research, XLII, (Summer, 1972), p. 376.

of which are conveyed by simple, explicit images; b) mathematical learning has explicit, dynamic, or kinesthetic images obtained from the child's experiences; c) children must understand the system of propositions.

Gaskill[30] in related research to the child's mathematical development and his experiences also gives the researcher insights into the formation of these mathematical concepts by noting that names, counting, and number concepts, are secondary in development because the child begins his own system of sorting and organizing objects in terms of size and order, first. As the child plays, watches, and listens, he forms his own vague notions about size; big, little, more, many, few, long, short, and tall. He does the same kind of learning about order in terms of up and down, near and far, first and last, before and after, and beginning and end. Words are matched to these ideas gradually as a child uses isolated words to express a whole concept (holophrastic). The child is not only communicating, but also checking out his understanding of a concept and linking this to his understanding of his language. If he gets positive feedback on both of these, the child then "knows" this step in learning and continues to use this technique to confirm his "knowing". Gaskill goes on to say that if the child is allowed to go through the process of trial and error, he elaborates an

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[30]Eleanor, Gaskill, "How Old is Big?" Academic Therapy, VIII, (Fall, 1972), 97-101.

idea until he reaches a generalization or transfers the concept to another situation.

Gaskill continues that concepts are based on nonverbal manipulation, but the ideas have to be expressed in words. Every mathematical relationship is based on these five factors: a) experience, b) addition of language to the experience, c) accumulation of experience with language in order to generalize the idea, d) translation of this process to symbols, and e) repetition and expansion of this learning for improvement of accuracy and insight into patterns and properties of all mathematical processes.

Silverstein[31] in a study related to the exceptional learner found "great gaps in understanding (of) . . . the meaning for the retarded child of indefinite number terms, such as, "a few", "some", and "alot". Closely related, Cohen, Dearnley and Hansely, also found that the above terms proved difficult for the retarded. These authors attributed the difficulty to three possible factors: a) the terms employed, b) the number of objects, and c) the chronological age.[32]

In a replication study into the language of mathematics as applied to the classroom, four terms were employed in the

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[31]A. Silverstein, et al., "The Meaning of Indefinite Number Terms for Mentally Retarded Children", American Journal of Mental Deficiency, LXIX, (November, 1964), 419.

[32]J. Cohen, E. J. Dearnley and C. E. M. Hansel, "A Quantitative Study of Meaning", British Journal of Educational Psychology, XXVIII, pp. 141-148, as cited by A. Silverstein, American Journal of Mental Deficiency, LXVIII, (November, 1964),

study. Kumae and Inoue[33] found that the terms "very many", "many", "a few", and "very few" presented special problems to the exceptional learner.

In analyzing the results, there proved to be no significant distinction between "very many" and "many" but for "few" and "very few" mental retardates took more for a "few" than for "very few".

In summary, the reader sees that mathematics is a special language. Recent research and past research which has now been reviewed, asserts that the child, as a living organism passes through stages of mathematical understanding and that these stages are best "readied" by the teacher (when the child is ready for school enrollment) by understanding the readiness skills of the child which the child will demonstrate. Researchers assert that if the child is not ready for a certain mathematical understanding, failure or confusion, or repulsion for mathematics will follow.

Not all children learn at the same rate nor are ready for the same concepts at the same time; discovery, manipulation, and verbal learning seem to be the best answers and preparation procedures for the normal and exceptional learner.

Number Concepts

In trying to assess the prerequisites for mathematical understandings as these understandings are related to counting,

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[33] J. Cohen, E. J. Dearnley and C. E. M. Hansel, "A Quantitative Study of Meaning", British Journal of Educational Psychology, XXVIII, 1958, 141-148.

measurement, numerical operations, one-to-one correspondence, and teaching methods, Bereiter[34] gives the researcher many useful insights.

As a rationale for writing his book Bereiter states, "Children ought to know what is going on". Bereiter made this statement in reference to the "new math" of the 1960's and that has now been reviewed and in many cases removed from some math curricula. Bereiter goes on to state that the pupil ought to be fully able to understand what the universe of numbers consists of and their relationships to various number systems.

In Bereiter's section on a child's first development of number concepts, number names, as descriptive terms, develop much in the same way as a child begins to recognize "big", "round", "flat". Following this thought to number concepts and naming, the child begins to recognize "oneness", and "twoness" without actual counting being involved. The child may recognize the number concepts, in their concrete form, up to "fiveness" but the child usually needs a formal pedagogic lesson for objects over five.

Bereiter goes on to say that children begin to understand such terms as "more", "less", "many", "few", and similar terms as the child is involved with comparative quantities. A prime learning time for this is when the child is

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[34]Carl Bereiter, Arithmetic and Mathematics (Ontario Institute for Studies in Education, 1968), pp. 3-4.

at the table for meal time, when the child is asked if he would like "more" or "less", or if he has "many" things as compared to a child who has "less". Also related is the learning of "first", "second", "last", etc., the child experientially learns and retains these concepts by being "first", "second", or "last", in a group situation. Throughout the early years of the growing period, the child has the opportunity to experience these situations, and to begin to apply language to these experiences.[35]

When the child enters the classroom, he arrives with many of the above concepts intact but has them reinforced by the teacher. Inconsistencies will occur, as they are normal, but the child must be accepted for what he has to offer.[36]

Of major concern to curriculum developers is the notion of conservation and set. Piaget states, "Our contention is that conservation is a necessary condition for all rational activity . . . A number is only intelligible if it remains identical with itself, whatever the distribution of the units of which it is composed."[37] Conservation is not innate but learned. It is learned through the manipulations of objects in their various shapes of formations and lengths.

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[35]C. Stern, Acquisition of Problem-solving Ability, p. 371.

[36]Bereiter, Arithmetic and Mathematics, pp. 3-4.

[37]Jean Piaget, The Child's Conception of Number (New York: W. W. Norton and Co., 1965), p. 3.

Brown[38] in trying to analyze the data of a study done on conservation with normal, bright, and retarded children, found that the idea of matching the chronological ages with the mental ages of the subject was invalid due to the concept that older retardates had more time or readiness to learn conservation of numbers than did the normal bright child, who would be considered "equal" in ability with just the CA - MA question taken into consideration. The opportunity to learn must be taken into account when matching groups for this type and similar studies.

Keasey and Charles[39] have also stated that the mental age and chronological age must be taken into consideration but it appears that Brown delves deeper into the total implications of the CA - MA question as it specifically applies to the question of conservation of number.

Dodwell and Wheatley[40] have shown that tests of number conservation may be a measure of arithmetic readiness and success. It appears to the researcher that this statement reflects Piaget's statement that conservation of number

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[38]Ann Brown, "Conservation of Number and Continuous Quantity in Normal, Bright and Retarded Children," Child Development, XLIV, (June, 1973), pp. 376-379.

[39]C. T. Keasey and D. C. Charles, "Conservation of Substance in Normal and Mentally Retarded Children," Journal of Genetic Psychology, CXI, 1967, pp. 271-279, cited by Barbara Rothenberg, Child Development, XL, (June, 1969), p. 384.

[40]P. C. Dodwell, "Children's Understanding of Number Concepts: Characteristics of an Individual and of a Group Test," Canadian Journal of Psychology, XV, 1961, pp. 29-36, as cited by Barbara Rothenberg, Child Development, XL, (June, 1969), p. 384.

is the basis of rational thought.

Flavell[41] goes on to state that the lack of understanding the principle of conservation is considered a manifestation of the immature level of functioning of the child's mental processes and of his failure to conform to the operational structures of thought. The understanding of number has been described as lending itself particularly well to investigation of the development of conservation as understood by Flavell.

Calhoun[42] in reviewing research on studies on conservation of number, found that Mehler and Bever[43] examined the development of conservation of number in children of very young age and indicated that children do show the ability to conserve number below the age of three, but appear to "lose" this ability between three and four and one-half years old.

In investigating four and five year olds, Griffiths, Shantz and Siegel[44] found that the verbal responses of "more" and "less" seemed to be spontaneous, in response to questions of a conservational nature. The examiners state that the responses that they received were elicited; therefore, doubt

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[41]J. H. Flavell, The Developmental Psychology of Jean Piaget (Princeton, N. J.: Van Nostrand, 1963), p. 383.

[42]L. G. Calhoun, "Number Conservation in Very Young Children: The Effect of Age and Mode of Responding," Child Development, XLII, (June, 1971), pp. 561-572.

[43]J. Mehler and T. G. Bever, "Cognitive Capacity of Very Young Children," Science, CLVIII, 1967, pp. 141-142, as cited by L. G. Calhoun, Child Development, XLII, (June, 1971), p. 562.

[44]J. A. Griffiths, C. A. Shantz and I. E. Siegel, "A Methodological Problem in Conservation Studies: The Use of Relational Terms," Child Development, XXXVIII, 1967, pp. 841-848, cited by L. G. Calhoun, Child Development, XLII, (June, 1971), p. 563.

upon their validity was cast.

In attempting to delineate the importance of conservation and to impress upon the reader the interrelatedness of conservation with the other sections of this paper, the researcher has found that Cathcart[45] states, "The magnitude of the correlation between achievement in mathematics and conservation seems to be affected by such factors as age, intelligence, social status, and language comprehension". Again the importance of language appears.

In returning to the second consideration of this section on conservation and set, Petersen[46] states that the term set cannot be given a definition in the formal sense since it is a primary idea and cannot be reduced to simpler terms. He does go on to say that "set" is an obvious collection, group, or class of objects. Petersen says that a set is obvious when it is simple for the viewer to determine whether an object is or is not a part of the given set at hand.

Petersen, in his own words will impart his findings to us:

> This concept of set will aid in the precise communication of mathematical knowledge. Teachers have taught through the use of groups of objects for years. It is not so much that the idea of set is a startling

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[45]George Cathcart, "Relationship Between Primary Students' Rationalization of Conservation and Their Mathematical Achievement," Child Development, XLII, (September, 1971), pp. 755-765.

[46]Daniel Petersen, Functional Mathematics for the Mentally Retarded (Columbus, Ohio: Charles E. Merrill Co., 1973), p. 42.

> new idea but rather that the language associated with set enables the learner to deal with the most elementary ideas of mathematics in a way that is clear to him. This will help the child immensely in mastering the skills of correspondence and counting, the concepts of cardinal and ordinal numbers, actions with whole and fractional numbers, measurement, and geometry.[47]

When a child begins to match objects in order, as one cup for each person at the table, he is beginning to develop <u>one-to-one correspondence</u>. At the same time he is reinforcing his inner language for this concept and will begin to express these concepts through spoken language. For the initial period, the child will understand "more than" or "less than" if he finds that he needs more cups or less cups, but soon this becomes inadequate as the child will begin to understand that he will need to know precisely how many more or how many less cups he will need for the people at the table. When the child leaves the "more than", "less than" period he enters the <u>cardinal number</u> period. Here the idea of set is formally realized. The concept of cardinal number is that the number is the last number matched with the last element in the set. Now specifically, the child will begin to ask for "two more" or "ten more" or "one less" or "six less". Gradually the child will see that sets of objects, in themselves, can be "equal to" or "not equal" to each other. The child who now knows how to count and has the appropriate language concepts can begin to express verbally or nonverbally the idea that sets are equal, not equal, and if one set is more or less than the other.[48]

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[47] <u>Ibid</u>., p. 42.

[48] <u>Ibid</u>., p. 42.

Ordinal numbers are numbers that are used to indicate position in a series. One of the major prerequisites is that the child know "left" and "right" so that he arrives at the correct conclusion of which is "first", "second" or "last". The child has an idea of ordinal numbers and of numbers in series but it is the teacher who brings the correct concepts to verbal form for the proper understanding of those number concepts.[49]

Following close to the concept of set is the concept of <u>addition</u>. In this process two separate sets are combined to produce one cardinal number. Actually, addition can have two processes involved. When two sets with no common elements are added, the answer is the sum of the elements, but if the sets have a common element then the answer is the sum without counting the common element.[50]

Subtraction can be difficult for the slow learner or the exceptional learner. It undoes what addition does; therefore, it is called the inverse of addition. In subtraction we are looking for the missing addend. Petersen goes on to say that when children know the basic addition facts they already should know the subtraction facts. This is because the subtraction facts can be taught as the inverse of addition.[51]

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[49] <u>Ibid</u>., p. 103.

[50] <u>Ibid</u>., p. 171.

[51] <u>Ibid</u>., p. 179.

Another major concept to be understood, and actually a primary concept, is the concept of measurement. Bereiter delineates that mathematics is not a collection of separate topics and processes but is a system of thought that may be employed anywhere. Bereiter states, "To view the world in terms of the real number system means to view it in terms of continuous quantities, such as length, weight, time, and temperature."[52]

In summary, the reader will see that a variety of authors agree that conservation and set are basic to the total understanding of the number system. Piaget, to reiterate, says that conservation is the basis for all rational activity. Bereiter concurs by outlining processes that follow along these lines.

With all the concepts presented in this section, language continues to play a major role. The child is not born with mathematical understanding even if he is gifted with normal intelligence. It is more problemmatical for the exceptional learner, who may have a difficult time with the simplest of commands to say nothing of the symbols employed in the classroom for mathematical notation. While development may be slow, the learning disabled student (as defined in Chapter I) can develop these concepts with a modified mode of presentation tailored to his individual needs and states.

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[52]Bereiter, Arithmetic and Mathematics, p. 19.

Transfer of Learning

This section, while it does not pertain directly to mathematics, is included to call to the reader's attention the theory of transference. In many cases teachers are led to believe that once a topic or concept has been taught, it will "automatically" transfer to another situation where similar stimuli are present. If this were true, there would be no "failure" in the classroom and the students would not have to be retained in grades in which they failed.

Teachers recently have been aware of task analysis, whereby simply, the task to be presented to the child is analyzed as to its total components so the teacher and the student, together, know what is to be taught and what is to be learned. This method has been proven to be an improvement over the traditional method whereby the child was presented with the whole concept or topic and expected to arrive at transfer by himself.

The theory of task analysis allows the teacher to reach the level of understanding at which the child presently is, and also allows the creation of a modified curriculum for the child if warranted. The concepts to be learned are still taught in their entirety, but due to task analysis the components of the task are reduced to separate entities for examination by the child and then synthesized back to the whole, for the child.

In order for the child to transfer, he must be able to transfer something. Scott[53] writes that there is an urgency

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[53]Herbert Scott, "Transfer in Learning," Education, XC, (April, 1970), p. 299.

about transfer. Transfer requires that you have some skill or knowledge and that you use it in new problems where it is relevant. Scott continues that a teacher has no assurance that a student has "really learned" something until the teacher sees the student use it in a situation where the theory of transference can be observed.[54]

Scott states that there are intellectual processes that we use. These cognitive processes run up a scale from the simplest, namely recall, to the most complex. Each step includes behaviors from the lower step. During this process of going up the scale, something more complex is added. Behavior just below transfer is extrapolation, while application of presented stimuli to new situations is transfer.[55]

Gagne[56] in trying to analyze Piaget's theory of learning as Piaget sees learning in formal operations in the "lock step" method, disagrees with Piaget because Gagne sees Piaget's concepts as teachable and not innate phenomena unto themselves. Gagne prefers to see learning as "cumulative learning" model which is the cumulation of learning organized into the systems of concepts and principles.

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[54] Ibid., p. 299.

[55] Ibid., p. 299.

[56] R. M. Gagne, "Contributions of Learning to Human Development," Psychological Review, CXI, 1968, pp. 177-179, as cited by John Mouw and James Hecht, Journal of Educational Psychology, LXIV, (February, 1973), p. 61.

Bruner, in also addressing himself to the question of transference states that principles and rules are probably best learned through self-discovery and that these principles and rules may also be taught.[57] This statement is also in agreement with those theorists who also advocate the discovery approach to learning which seems to be increasing in popularity.

The researcher at this point, wishes to include the specific suggestions from Gagne, to impress upon the reader the importance of providing for, maintaining, and observing, transfer in the child. Gagne's suggestions are as follows:

> 1. If possible and appropriate, analyze the skill to ascertain the specific psychomotor abilities necessary to perform it, arrange these abilities in order, and help students to master them in sequence.
>
> 2. Provide demonstrations, and as students practice, give verbal guidance to aid mastery of the skill.
>
> a) Demonstrate the entire procedure straight through, then describe the links of the chain in sequence, and finally demonstrate the skill again step by step.
>
> b) Allow ample time for students to practice immediately after the demonstrations. (Remember the importance of activity, repetition, and reinforcement.)
>
> c) As students practice, give guidance verbally or in a way which permits them to perform the skills themselves.
>
> d) Give guidance in a relaxed, noncritical atmosphere and in a positive form.
>
> 3. Be alert to generalization and interference.[58]

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[57] J. S. Bruner, Toward a Theory of Instruction (New York: Norton, 1966), as cited by John Mouw and James Hecht, Journal of Educational Psychology, LXIV, (February, 1973), p. 62.

[58] R. M. Gagne, The Conditions of Learning (New York: Holt, Rinehart and Winston, 1965), as cited by Robert Biehler, Psychology Applied to Teaching (Boston: Houghton Miffin Co., 1971), p. 281.

In summary, the reader will see that transfer of information from one situation to another can be facilitated, and that transfer in itself, can make learning more relevant for the student by the steps as expounded by Gagne.

In mathematics, the researcher feels that transfer can be just as applicable and that the various steps as outlined by Gagne can be utilized. As delineated throughout this paper, a numeral is a symbol which conveys meaning only if the student is able or ready to receive that information. Therefore, it is hoped by the researcher, that task analysis will be utilized in the classroom to a greater extent than it is at the present time.

The reader will also see, that while Piaget enjoys great educational repute, others do not view learning in the same light as he does. In this section, it was presented that Piaget sees learning in "stages" with accompanying concepts, while learning is organized into systems of concepts and principles. Bruner sees many of Piaget's concepts as "teachable" rather than innate at a particular developmental time.

CHAPTER III

Discussion

Mathematics is a specialized language within our various mathematical curricula. It has been established by a myriad of researchers that mathematics, as a specialized language, is best taught by the instructor and best learned by the student, when the various components of mathematics are task-analyzed as to their linguistic and mathematical components.

The issue of whether mathematics is a separate mental function or a composite of mental abilities is not resolved at the present time. Various camps argue in support of their views, but the subject of mathematics continues to be taught and learned while this academic discussion continues.

It has been demonstrated that certain, basic, mathematical concepts are learned by the child without the aid of the teacher, and that gradually, language is matched to these concepts. Internally, the child is processing understandings of a quantitative nature as the human organism develops the ability to receive language in terms of formal concepts and later still, express those concepts in formal expressive language. When the child enters the classroom, he arrives with some vague notions of quantity and also some intact concepts of quantity. The teacher, then, is to understand the readiness skills of the child and accept the child for what he knows and does not know.

Some difficulties in arithmetic class are not of a mathematical nature but rather a linguistic nature. It has been stated that many bright, normal children experience difficulty, not because of the inability to understand the concept being taught, but rather they experience difficulty due to the lack of readiness for the language employed for that particular subject. The same is true for the exceptional learner. Compound his learning problems with inability to understand mathematical concepts and his "failure syndrome" will continue. It has also been demonstrated that when specific training in the language of mathematics is taught, success increases.

Conservation, as expounded by Piaget, is the basis for all rational activity. Piaget expounds that conservation is necessary for the incorporation of mathematical knowledge in the child. While Piaget sees learning in terms of separate stages such that the child cannot pass through a stage until those particular mental processes of that stage are complete, Bruner and Gagne appear to disagree by delineating theories which indicate that learning is cumulative and progressive and not necessarily "lock-step" as viewed by Piaget.

Conclusion

For the primary, school-aged child who is learning disabled, mathematics should be viewed as a specialized language. The child has some intact concepts of a mathematical nature when he enters the classroom but needs them reinforced by the teacher. Modified curricula are necessary for the exceptional learner and this has been demonstrated to be best accomplished through the process of task-analyzation.

Individualized instruction, task-analyzation, low teacher-pupil ratio, and understanding of the total learning process, and the ability to be flexible, according to the needs of the child are best components of the learning milieu for the child.

Much of the above is presently being accomplished at the St. Francis Children's Activity and Achievement Center, located in Glendale, Wisconsin. In this setting, the child enters unlabeled except that he is labeled as a child with learning potential. Who will label the child "stupid" or "dumb"? Who will be content to leave the child in his present condition, when the challenge to change that condition is becoming more and more available to us, as we pass through dark ages of pedagogic knowledge and enter enlightened ages of becoming effective teachers.

In closing, let me end with a quote from Sr. Joanne Marie Kliebhan, director of St. Francis Children's Activity and Achievement Center, "All children can learn, if we can learn how to teach them."

BIBLIOGRAPHY

Aiken, Lewis, Jr., "Ability and Creativity in Mathematics," Review of Educational Research, XLIII, (Fall, 1973), 405-432.

________. "Language Factors in Learning Mathematics," Review of Educational Research, XLII, (Summer, 1972), 359-385.

Ausubel, D. P. and Robinson, F. G. School Learning: An Introduction to Educational Psychology. New York: Holt, Rinehart and Winston, 1969.

Barakat, M. K. "Factors Underlying the Mathematical Abilities of Grammar School Pupils," British Journal of Educational Psychology, XXI, 1951, 239-240.

Bereiter, Carl. Arithmetic and Mathematics. Dimensions in Early Learning Series. U.S.A.: Ontario Institute for Studies in Education, 1968.

Blackwell, A. M. "A Comparative Investigation into the Factors Involved in Mathematical Ability of Boys and Girls, Parts I and II, British Journal of Educational Psychology, X, 1940, 143-153.

Brown, Ann. "Conservation of Number and Continuous Quantity in Normal, Bright, and Retarded Children," Child Development, XLIV, (June, 1973), 376-379.

Bruner, J. S. Toward a Theory of Instruction. New York: Norton, 1966.

Calhoun, L. G. "Number Conservation in Very Young Children: The Effect of Age and Mode of Responding," Child Development, XLII, (June, 1971), 561-572.

Cathcart, George. "Relationship Between Primary Students' Rationalization of Conservation and Their Mathematical Achievement," Child Development, XLII, (September, 1971), 755-765.

Cohen, J.; Dearnley, E. J.; and Hansel, C. E. M. "A Quantitative Study of Meaning," British Journal of Educational Psychology, XXVIII, 1958, 141-148.

Dodwell, P. C. "Children's Understanding of Number Concepts," Canadian Journal of Psychology, XV, 1961, 29-36.

Dresher, R. "Training in Mathematics Vocabulary," Educational Research Bulletin, XIII, 1934, 201-204.

Earp, N. W. "Reading in Mathematics," EDRS Acc. No. ED036397, May, 1969.

Flavell, J. H. The Developmental Psychology of Jean Piaget. Princeton, N. J.: Van Nostrand, 1963.

Gagne, R. M. "Contributions of Learning to Human Development," Psychological Review, CXI, 1968, 177-179.

________. "Learning Hierarchies," Educational Psychology, VI, No. 1: 1968, 3-6.

________. The Conditions of Learning. New York: Holt, Rinehart and Winston, 1965.

Gaskill, Eleanor. "How Old is Big?" Academic Therapy, VIII, (Fall, 1972), 97-101.

Gilmary, Sister. "Transfer Effects on Reading Remediation to Arithmetic Computation When Intelligence is Controlled and All Other School Factors are Eliminated," Arithmetic Teacher, XIV, 1967, 17-20.

Griffith, J. A.; Shantz, C. A. and Siegel, I. E. "A Methodological Problem in Conservation Studies: The Use of Relational Terms," Child Development, XXXVIII, 1967, 841-848.

Guilford, J. P.; Hoepfner, R.; and Petersen, H. "Predicting Achievement in Gifted Children," School Review, 1957, XLV, 388-414.

Harter, M. A. and Kane, R. B. "The Cloze Procedure as a Measure of the Reading Comprehensibility and Difficulty of Mathematics," English, EDRS Acc. No. ED040 881, 1970.

Hodamard, J. The Psychology of Invention in the Mathematical Field. New York: Dover Press, 1954.

Hills, J. R. "Factor-analyzed Abilities and Success in College Mathematics," Educational and Psychological Measurement, IV, 1957, 615-622.

Johnson, H. C. "The Effects of Instruction in Mathematics Vocabulary Upon Problem Solving Ability," Journal of Educational Research, XXXVIII, 1934.

Keasey, C. T. and Charles, "Conservation of Substance in Normal and Mentally Retarded Children," Journal of Genectic Psychology, CXI, 1967, 271-279.

Kirk, Samuel, and Bateman, Barbara. "Diagnosis and Remediation of Learning Disabilities," Exceptional Children, XXXIX, No. 2, (October, 1962), 73.

Krutetskii, V. A. "An Experimental Analysis of Pupils' Mathematical Abilities," Soviet Studies in Psychology of Learning and Teaching Mathematical Ability. Edited by J. Kilpatrick and I. Wrszup. Stanford, California: 1969, 105-111.

Mitchell, Lazarus. "Mathophobia," National Elementary Principle, LIII, (January, 1974), 16-22.

Linville, W. J. "The Effects of Syntax and Vocabulary Upon the Difficulty of Verbal Arithmetic Problem Solving With Fourth Grade Students," (Doctoral Dissertation, Indiana University) Ann Arbor, Michigan, University Microfilm, 1970. No. 70-7957. (DA 30A: 4310; April, 1970.

Lovell, Kenneth, "Intellectual Growth and Understanding Mathematics," Arithmetic Teacher, XIX, (April, 1972), 277-282.

Lyda, W. J. and Duncan, F. M. "Quantitative Vocabulary and Problem Solving," Arithmetic Teacher, XIV, 1967, 289-291.

Mehler, J. and Bever, T. G. "Cognitive Capacity of Very Young Children," Science, CLVIII, 1967, 141-142.

National Advisory Committee on Handicapped Children, Special Education for Handicapped Children, First Annual Report. Washington D. C.: Department of Health, Education and Welfare, Office of Education, January 31, 1968.

O'Brien, Thomas, "Perceptual Goals for Mathematical Education," Childhood Education, L, (February, 1974), 214-216.

Petersen, Daniel. Functional Mathematics for the Mentally Retarded. Columbus, Ohio: Charles E. Merrill Co., 1973.

Piaget, Jean. The Child's Conception of Number. New York: W. W. Norton and Co., 1965.

Poincaire, H. "Mathematical Creation," in The Creative Process. Edited by B. Ghiselin. New York: New American Library Press, 1952.

Post, Thomas. "New Look at the Role of Computation in the Primary Grades," School Science and Mathematics, LXXI, (June, 1971), 527-530.

Scott, Lloyd. Trends in Elementary School Mathematics. Chicago: Rand McNally and Co., 1966.

Silverstein, A., et al. "The Meaning of Indefinite Number Terms for Mentally Retarded Children," American Journal of Mental Deficiency, LXIX, (November, 1964).

Smith, F. "The Readability of Sixth Grade Words," School Science and Mathematics, 1971, 559-562.

Spencer, P. L. and Russell, D. H. "Reading in Arithmetic," National Council of Teachers of Mathematics Yearbook, XXV, 1960, 202-223.

Stern, C. "Acquisition of Problem-solving Strategies in Young Children and Its Relation to Verbalization," Journal of Educational Psychology, LVIII, 1967, 245-252.

Werdelin, I. "The Mathematical Ability: Experimental and Factorial Studies," Lund, Sweden: C. W. K. Gleerup, 1958.

Wilmon, B. "Reading In the Content Area: A Specialized New Math Terminology List for Primary Grades," Elementary English, XLVIII, 1971, 463-471.

Wrigley, J. "Factorial Nature of Ability in Elementary Mathematics," British Journal of Educational Psychology, XXVIII, 1958, 61-78.