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EXPLORING THE METHODS OF COINTEGRATION PROCEDURES USING STOCK PRICES

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Abstract: Stationary models are essential class of stochastic models for describing time series data which have received a great attention. In reality, however, business and economic data are non-stationary multivariate time series that are often better understood by cointegration analysis. This study investigates the cointegration testing methods of Engle-Granger two-step estimation technique, Phillip-Ouliaris test and Johansen's multivariate test. The stock prices of selected companies in Nigeria from 2008-2014 are used in the study. Findings revealed that the three techniques produced different results and that the Johansen's method and Engle-Granger two steps procedure exhibits higher efficiencies than Phillips-Ouliaris methods but their efficiency is dependent on the number of variables and correct selection.

Keywords: Augmented Dickey-Fuller test, Cointegration, Engle-Granger method, Stationarity, Johansen's test, Phillip-Ouliaris method, Stock price, Unit root

1. INTRODUCTION

The behaviour of economic time series data have been studied in various ways using different statistical and economic tools. Most business data are collected in series over a period and exhibits trends that are usually non-stationary which makes analysis challenging sometimes as it may results in spurious regressions (Wei, 2006; Utkulu, 1994). Spurious regression occurs when there appears to be a statistically significant relationship between variables but the variables are unrelated. A way of solving this non-stationarity challenge is by transformation of the series by differencing. The concept cointegration in the time-series econometrics was introduced by Granger (1981) and Engle and Granger (1987) and they provided a theoretical frameworks for representing, testing, estimating and modelling of cointegrated non-stationary time-series variables. Ever since, the concept has undergone various developments and transformations from researchers such as Utkulu (1994), Alexander (1999) and Ssekuma (2011). By cointegration analysis a non-stationary data can used such that spurious results are avoided, and also provides effective framework for testing and estimating long-run models from time-series data. Cointegration has evolved into is a time-series modelling methodology and Ssekuma (2011) discussed in details, the three popular techniques of measuring cointegration which are Engle-Granger estimation procedure; the Phillip-Ouliaris residual-based test and Johansen's multivariate technique.

In this article, the three cointegration measuring methodologies were investigated using the Nigeria stock exchange data. The study also seeks to determine the inherent long run

relationship between stock prices of five quoted stocks and the circumstances when it is reasonable to expect that two or more stock prices may be cointegrated. That is, if at least one of the processes is driving the other and if the prices are being driven by the same underlying process. The remainder of this article is arranged thus: methodology, material and data, results, summary and conclusion.

2. mthodology

Before cointegration analysis is carried out, it is imperative to first check the *unit roots* aso as to ascertain whether the variables are I(1) at levels and I(0) at differences Kazi (2008). Unit root is concern with the existence of characteristic root that is equal to one. The simplest model that may contain a unit root is the AR (1)

$$Y_t = \emptyset Y_{t-1} + \varepsilon_t \tag{1}$$

Where, ε_t is the uncorrected white noise error term with mean zero and variance is constant. If \emptyset =1, then (1) leads to a random walk without drift model, making it a non-stationary process. When this happens we are faced with a problem called unit root. However, if \emptyset < 1, the series is said to be stationary. It is important that a series is stationary because correlation may persist in a series that is, non-stationary even if the sample is large and may lead to what is called spurious(nonsense) regression (Yule 1989). The problem of unit root or having a stationary series is solved by differencing and time trend regression (Wei, 2006). The differencing is done by using the Augmented Dickey-Fuller Test (ADF).

The ADF test is a lower tail, if F_{τ} is less than the critical value, then null hypothesis of the unit root is rejected and conclude that the variable of the series does not contain unit root and is stationary.

The testing procedure for the ADF unit root is used to the model;

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \alpha_i \sum_{j=1}^{\rho} \delta_j \Delta y_{t-j} + \varepsilon_{ti}$$
 Where, (2)

 $\Delta y_t = y_t - y_{t-1}$ are the first difference of y_t , y_{t-1} is the lagged value of order of y_t , Δy_{t-1} are the changes in lag values and ε_{ti} is the white noise

Another test for stationarity used in this study is Kwiatkowski- Phillip-Schmidt-shin (KPSS). Checking the stationarity of the series is necessary because if correlation persist in non-stationary time-series data even if the sample is very large it may lead to spurious regression (Yule, 1989). (Wei, 2006) identified that unit root problem can be solved, or stationarity can be achieved by differencing the data set.

Cointegration analysis is fundamentally multivariate, as a single time series cannot be cointegrated. To test for cointegration between two or more non-stationary time series, there are several methods but the ones considered in this study are namely; Engle-Granger two-step procedure; Phillips-Ouliaris residual-based tests; and Johansen procedure. The Phillips-Ouliaris methods are effected by using two residual-based tests: The variance ratio test and the multivariate ratio test.

The variance ratio statistic $\hat{P_u}$ is defined as

$$\widehat{P}_{u} = \frac{11.2}{T^{-1} \sum_{t=1}^{T} \widehat{u}_{t}}$$
(3)

where \hat{u}_i is the residual of the long-run regression equation

$$y_t = \hat{\beta} x_t + \hat{u}_t \tag{4}$$

$$\widehat{w}_{11,2} = \widehat{w}_{11} - w_{21} \mathbf{z}_{22} \widehat{w}_{21} \tag{5}$$

and

$$\widehat{\Omega} = T^{-1} \sum_{t=1} \widehat{\varepsilon}_t \widehat{\varepsilon}_t + 1 \quad \sum_{s=1} \omega_{sl} 1 \quad \sum_{t=1} (\widehat{\varepsilon}_t \widehat{\varepsilon}_{t-s} + \widehat{\varepsilon}_{t-s} \widehat{\varepsilon}_t)$$
(6)

The variance ratio test is a residual-based to test a null hypothesis of no cointegration. The null hypothesis is formulated in terms of the conditional variance parameter $\omega_{11,2}$ as follows:

$$H_1: \omega_{11,2} = 0 \tag{7}$$

against

$$H_0: \omega_{11,2} \neq 0$$
 (8)

With two-step procedure and Phillips-Ouliaris only a single cointegrating relationship can be estimated. However, if one more than two time series are involved, it is possible that more than one cointegrating relationship will exist, which requires the use of vector cointegration techniques like Johansen's procedure. The multivariate trace statistic, denoted as \hat{P}_z is defined as:

$$\widehat{P}_z = Ttr(\widehat{\Omega}M_{zz}^{-1}) \tag{9}$$

Where T is the number of observations, $M_{zz}^{-1} = t^{-1} \sum_{t=1}^{T} z_t z_t'$, and $\hat{\Omega}$ is estimated.

Engle and Granger steps for identifying whether two integrated variables of the same order cointegrate are done as following:

Pre-test: a pre-test is carried out on each variable so as to determine the order of integration. By definition, cointegration requires that two variables be integrated of the same order. Augmented Dickey-Fuller (ADF) unit root test can be used to know the number of unit roots (if any) in each of the variables under study. The testing procedure for the ADF unit root test is applied as follows;

$$\Delta y_t = \alpha + \beta t + \theta y_{t-1} + \alpha_i \sum_{j=1}^{\rho} \delta_j \Delta y_{t-j} + \varepsilon_{ti}$$
 (10)

Where;

 ρ is the lag order of the autoregressive process,

 $\Delta y_t = y_t - y_{t-1}$ are the first differences of y_t ,

 y_{t-1} is the lagged values of order one of y_t ,

 Δy_{t-j} =changes in lagged values, and

 ε_{ti} =white noise.

Regress the long run equilibrium model: We estimate the long-run equilibrium relationship in the form of an OLS regression line, if the hypothesis of the unit root test is rejected,

$$y_t = \beta_0 + \beta_1 x_1 + \varepsilon_t \tag{11}$$

If the variables cointegrate, the Ordinary Least Square OLS regression in equation (11) implies that there is a strong linear relationship between the variables under study (Ender, 2004). The strong linear relationship can be tested in either of the following ways.

Step 1. The hypothesis is set as follows:

$$H_0$$
: $a_1 = 0$ (no cointegration)

 $H_1: a_1 < 0$ (12)

Step 2. Determine the test statistic using

$$F_{\hat{\epsilon}_t} = \frac{\hat{a}_1}{SE(\hat{a}_1)} \tag{13}$$

Where,

 $SE(\hat{a}_1)$ is the standard error of \hat{a}_1 , the estimate of a_1

According to Johansen (1995), pre-testing the variables so as to know the order of integration is not so important. He mentioned that if one of the variable is I(0) instead of I(1), such a variable will be revealed through a cointegrating vector whose space is covered by the only stationary variable in the mode. Johansen's method takes as a starting point the Vector Auto Regression (VAR) of order p given by:

$$X_{t} = \Pi_{1} X_{t-1} + \Pi_{2} X_{t-2} + \dots + \Pi_{p} X_{t-p} + \underline{u}_{t}$$
(14)

where:

 X_t is an $n \times 1$ vector of variables that are integrated of order one, that is, I(1)

 \underline{u}_t is an $n \times 1$ vector of innovations while Π_1 through Π_p are $m \times m$ coefficient matrices.

If X_{t-1} is subtracted from both sides of equation 16 it becomes;

$$\Delta X_{t} = \Gamma_{1} \Delta X_{t-1} + \Gamma_{2} \Delta X_{t-2} + \dots + \Gamma_{p-1} \Delta X_{t-p+1} - \Pi X_{t-p} + \underline{u}_{t}$$
(15)

Where;

$$\Gamma_1 = \Pi_1 - 1$$
, $\Gamma_2 = \Pi_2 - \Gamma_1$, $\Gamma_3 = \Pi_3 - \Gamma_2$ and $\Pi = 1 - \Pi_1 - \Pi_2 - \dots - \Pi_p$

Equation (15) can be written again in this form:

$$\Delta X_{1t} = \gamma'_{11} \Delta X_{t-1} + \gamma'_{12} \Delta X_{t-2} + \dots + \gamma'_{1p-1\Delta X_{t-p+1}} + \Pi' X_{t-p} + u_{1t}$$
 (16) Where

 γ'_{ij} is the first row of Γ_j , j = 1, 2, ..., p-1 and Π'_1 is the first row of Π

If ΔX_{1t} is stationary, then j = 1, 2, ..., p-1 is also stationary. It is also assumed that u_t is also stationary. So to have a meaningful equation, $\Pi'X_{t-p}$ must be I(0). If none of the components of X_t are cointegrated, they must be zero, but if they are cointegrated, all the rows of Π must be cointegrated but not necessarily dissimilar as (Harris, 1995) pointed out that the number of distinct cointegrating vectors depends on the row rank of Π .

To detect the number of cointegrating vectors, the below likelihood ratio test was proposed by Johansen.

i. The test statistic is given by

$$ltrace = -T\sum_{i=r+1}^{n} \ln\left(1 - \hat{\lambda}_i\right)$$
(17)

 H_0 : r = 0 (no cointegrating vector)

Against

 $H_1: r \ge n$ (at most n cointegrating vector)

ii. The maximum eigenvalue test, tests the null hypothesis of r cointegrating vectors against the alternative hypothesis of (r+1) cointegrating vectors. Its test statistic is given by:

$$lmax = -T \left(1 - \hat{\lambda}_{r+1}\right) \tag{18}$$

 H_0 : r = 0 (no cointegrating vector)

Against $H_1: r+n$ cointegrating vector

2. MATERIAL AND DATA

Opening Stock prices of five quoted stocks of May and Barker, Berger paint, Julius Berger, Livestock Feed PLC and Presco Plc are used in this study. The data represents opening prices from February 2, 2008 to May 20th, 2014 and are sourced from the Nigerian Stock Exchange (NSE). Exploratory data analysis, normality tests and plots were carried out with Gnu Regression, Econometrics and Time-series (gretl) software as presented in Tables 1-2 and in figures 1-5. The implementation was done using the urca package in R by Pfaff, B. (2008).

4. RESULTS

Table 1: Descriptive Statistics

Company	Mean	STDV	SKEW	KURT
May & Baker	4.4458	3.52698	1.953	3.811
Berger Paint	8.7882	3.02150	0.968	3.584
Julius Berger	54.7197	25.09633	1.079	1.607
Life Stock Feed	2.3278	1.88994	1.109	0.782
PRESCO PLC	16.9033	12.43540	0.839	-0.798

Source: Nigeria Stock Exchange (NSE) 2015

Table 2: Test for Normality (H_0 : Normal)

Company	Doornik-Hansen test	Shapiro-Wilk W	Lilliefors test	Jarque-Bera test
May & Baker	2445.68	0.769551	0.183527	2055.93
	(0)	(2.4242e-043)	(0)	(0)
Berger Paint	211.78	0.903291	0.136981	1143.56
	(1.02956e-046)	(3.29551e-031)	(0)	(4.76973e-249)
Julius Berger	377.846	0.911679	0.0906891	499.522
	(8.95141e-083)	(4.63208e-030)	(0)	(3.38906e-109)
Life Stock Feed	712.19	0.863626	0.179326	382.631
	(2.23819e-155)	(9.19768e-036)	(0)	(8.18129e-084)
PRESCO PLC	1098.11	0.828956	0.187454	239.209
	(3.53035e-239)	(9.19768e-036)	(0)	(1.13886e-052)

Note: first entries are the test statistics while the second entries in the same row and column are p-values respectively. In the test: null hypothesis H_0 stands for normality, we reject H_0 in all cases which implies that the data set are not normally distributed.

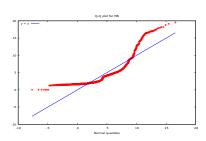


Fig 1: Q-Q Normal Plot for May & Baker

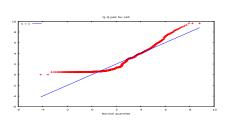


Fig 4: Q-Q Normal Plot for Life Stock Feed Plc

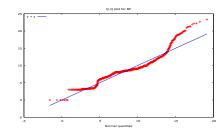


Fig 2: Q-Q Normal Plot for Berger Paint

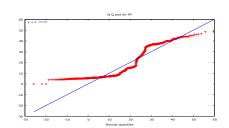


Fig 5: Q-Q Normal Plot for PRESCO

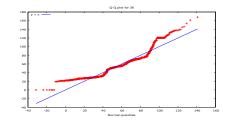


Fig 3: Q-Q Normal Plot for Julius Berger

4.1 Testing for Stationarity

The results of the Augmented Dicker Fuller (ADF) using AIC unit root test procedure are presented in Table 3 while the KPSS tests are in Table 4. Generally, the null hypotheses (H_0) , that variables under observation are non-stationary is accepted if the absolute value of the calculated statistics for any of the variable is lower than the absolute critical value at the stated significant level

Table 5. Augmented Diekel Fullet (ADF) Chit Root Test						
Variable	ADF test Statistics	Decision at 10%	Conclusion at 10%			
May & Baker (MB)	-1.92288	Reject H_0	Stationary			
Berger Paint (PP)	-2.96446	Reject H_0	Stationary			
Julius Berger (JB)	-2.56106	Reject H_0	Stationary			
Life Stock Feed LSF)	-1.68012	Reject H_0	Stationary			
PRESCO PLC (PP)	-1.8395	Reject H_0	Stationary			

Table 3: Augmented Dicker Fuller (ADF) Unit Root Test

The ADF critical value at the 1%, 2.5%, 5% and 10% level of significance are -2.58, -2.23, -1.95 and -1.61. Based on the ADF test, the first difference variables are stationary, which suggests that variables are integrated of order one, I(1). Unit root exist in all at 1% level of significance except Berger Paint, Unit root exist in all at 2.5% level of significance except Berger Paint and Julius Berger, Unit root exist in May and Baker, Live Stock Feed plc, and PRESCO plc at 5% level of significance except Berger Paint.

Table 4. It widthowski Timilp Seminat Sim (Ki SS) test					
Variable	KPSS test Statistics	Decision	Conclusion		
May & Baker (MB)	11.7438	Reject H_0	Stationary		
Berger Paint (PP)	0.93065	Reject H_0	Stationary		
Julius Berger (JB)	2.25878	Reject H_0	Stationary		
Life Stock Feed LSF)	3.22735	Reject H_0	Stationary		
PRESCO PLC (PP)	11.7026	Reject H_0	Stationary		
Critical value	10%=0.348	5%=0.462	1%=0.743		

Table 4: Kwiatkowski- Phillip-Schmidt-shin (KPSS) test

From the above analysis, it is observed that the test statistics is more than the critical value at 1%, 5% and 10% level of significance respectively so we reject the null hypothesis which implies that the variables are stationary at all levels.

4.2. Phillips-Ouliaris methods

Table 5: Variance ratio test Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.330158 0.129817
                              33.36
                                      <2e-16 ***
                                       <2e-16 ***
z[, -1]BP -0.362231
                    0.013405 -27.02
z[, -1]JB 0.089182 0.002158
                              41.32
                                       <2e-16 ***
z[, -1]LSF 1.114179 0.032503
                               34.28
                                       <2e-16 ***
z[, -1]PP -0.246970 0.003544 -69.69
                                       <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Table 6: The variance ratio test

	10%	5%	1%
Level of Significance			
Critical Value	45.3308	53.2502	71.5214
Decision	Accept H ₀	Accept H ₀	Accept H ₀

The test statistic for the variance ratio test (\hat{P}_u) is 41.0838. From table 6, we observe that the calculated test statistic is lower than the critical value at the 1%, 5% and 10% level of Significance. We therefore reject the null hypothesis of no cointegration against the alternative of the presence of cointegrating variables. We shall examine the Engle-Granger two-step estimation procedure, to examine if the same result is obtained.

Residual standard error: 1.443 on 1660 degrees of freedom, Multiple R-squared: 0.8329, Adjusted R-squared: 0.8325, F-statistic: 2069 on 4 and 1660 DF, p-value: < 2.2e-16

Table 7: Summaries the results of the long-run relationship using the Phillips-Ouliaris test

Level of Significance	10%	5%	1%
Critical Value of $(\hat{P_u})$	45.3308	53.2502	71.5214
Critical value of (\hat{P}_z)	143.0775	155.8019	180.4845

For (\hat{P}_z) : Residual standard error: 1.553 on 1659 degrees of freedom, Multiple R-squared: 0.9945, Adjusted R-squared: 0.9945, F-statistic: 6.042e+04 on 5 and 1659 DF, p-value: < 2.2e-16

The calculated test statistics of the variance ratio test (\hat{P}_u) and the multivariate trace (\hat{P}_z) are 41.0838 and 495.5506 respectively. Since both tests are upper-tailed tests, the null hypothesis is rejected if the test statistic is greater than the critical value. This implies that the null hypothesis is accepted at 1%, 5% and 10% significance level with the variance ratio test, but rejected at the 1%, 5% and 10% significance level with the multivariate trace statistic. Multivariate trace test identifies that there is a relationship between May & Baker opening Stock prices and other four companies while variance ratio test does not.

Table 7: Engle-Graner Cointgration Test

	Table	e 7: Engl	le-Grai	<u>ner Co</u>	ıntgratıon Tes	t	
	coeffic	cient	std.	erro	t-ratio	p-value	
const	4.3301	 L6	0.12	 9817	33.36	3.80e-187 ⁻	***
BP	-0.3622	231	0.01	34046	-27.02	1.36e-133 ³	t * *
JB	0.0891	L818	0.00	215850	41.32	3.24e-257	***
LSF	1.1141	L8	0.03	25026	34.28	3.48e-195 ⁹	+ * *
PP	-0.2469	970	0.00	354409	-69.68	0.0000	+ * *
Mean depend	ent var	4.445	760	S.D.	dependent	var 3.526977	7
Sum squared	resid	3458.	921	S.E.	of regress:	ion 1.443498	3
R-squared		0.832	898	Adjus	sted R-squa:	red 0.832495	5
Log-likelih	ood	-2971.	200	Akail	ke criterio	n 5952.399)
Schwarz cri	terion	5979.	487	Hanna	an-Quinn	5962.438	3
rho		0.869	253	Durb	in-Watson	0.264743	3
	p-value autocorre	$\frac{\overline{2}.699e}{2lation}$	-006 coef	f. for	e: -0.003	0]	

From the test statistics in Table 7, we reject the null hypothesis and conclude that there is cointegrating relationship among the opening stock prices of the companies considered based on F-test.

The regression follows that:

MB = A + 4.33016 - 0.36223BP + 0.089188JB + 1.11418LSF - 0.246970PP there is a negative relationship between May and Baker and (Berger paints and Presco Plc).

Table 8: Johansen's trace test and maximum eigenvalue results

H_o	H_1	Test Statistics	10%	5%	1%	Results
ltrace	-1					
$r \leq 4$	r > 4	3.07	7.52	9.24	12.97	Accept null hypothesis
$r \leq 3$	r > 3	8.90	17.85	19.96	24.60	Accept null hypothesis
$r \leq 2$	r > 2	24.12	32.00	34.91	41.07	Accept null hypothesis
$r \leq 1$	r > 1	56.77	49.65	53.12	60.16	Accept null hypothesis
$r \leq 0$	r > 0	171.27	71.86	76.07	84.45	Accept null hypothesis
<i>l</i> max test						
r=4	r = 5	3.07	7.52	9.24	12.67	Accept null hypothesis
r=3	r=4	5.83	13.75	15.67	20.20	Accept null hypothesis
r=2	r=3	15.22	19.77	22.00	26.81	Accept null hypothesis
r=1	r=2	32.65	25.56	28.14	33.24	Accept null hypothesis
r = 0	r=1	114.49	31.66	34.40	39.79	Accept null hypothesis

Result in table 8 shows that the null hypothesis of no cointegration (r=0) against the alternative of one or more cointegrating vector is rejected at the 10% level of significance in both trace test and maximum eigenvalue techniques. This suggests that there is long-run relationship in the opening stock prices among all the five companies. The null hypothesis $r \le 1$ being rejected at 5% and 10% level of significance shows that there is more than one cointegrating relationship in the opening stock prices of the companies examined.

5. summary and conclusions

The study investigated the methods of testing cointegration by using the Engle-Granger two steps procedure, Johansen method and the Phillips-Ouliaris method. The Engle-Granger two steps procedure requires estimating variables using the ordinary least square and testing the residual of the model for stationary using the Augmented Dickey Fuller ADF test. With the Phillips-Ouliaris methods, two residual-based tests, that is, the variance ratio test and the multivariate trace statistic are employed for testing for cointegration. These tests measure the size of the residual variance from the cointegrating regression of the variables under investigation. The possibility of having more than one cointegrating relationship makes Johansen's procedure quite useful. The tests were examined on the daily opening stock prices of selected companies on the Nigeria stock exchange from 2008 to 2014 so as to know if there is cointegration in the opening stock prices of the companies examined. That is, the study investigated the circumstances when it is reasonable to expect that two or more stock prices may be cointegrated, that is, if at least one of the processes is driving the other and if the prices are being driven by the same underlying process.

The study established that the stock prices in each sector are stationary as confirmed by the Augmented Dickey-Fuller test and are integrated of order one I(1), that is, lag 1; Using the

Engle-Granger two steps procedure, all price combinations cointegrate. Following Alexander (1999), the Engle-Granger two steps procedure show the existence of cointegration if the numbers of k lags are correctly chosen and the number of variables are not more than two; Johansen's method also showed that more than one company's opening stock prices cointegrates. The variance ratio test of Phillips-Ouliaris method does not show the presence of cointegration, but multivariate trace test of Phillips-Ouliaris method showed the presence of cointegration which makes Phillips-Ouliaris method inconsistent. In conclusion, this study has shown that the Phillips-Ouliaris method is contrary to Ssekuma (2011) as his work reveals that both the variance ratio test and the multivariate test to equally exhibit same results. While in the case of Johansen test trace test and maximum eigenvalue exhibits the same results this work agrees with his', this indicates that there is long-run relationship in the opening stock prices among all the five companies. Following Cheung and Lai (1993), the Johansen's method is considered efficient owing to the maximum likelihood and finite sample properties, revealing the existence of cointegration will be detected there is any and the number of cointegrating vectors determined.

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