# AN IMPROVED LINEAR PROJECTION APPROACH TO ESTIMATE DAILY REAL YIELDS AND EXPECTED INFLATIONS IN A LATENT MULTIFACTOR INTEREST MODEL

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## Abstract

The study improves upon the linear projection approach to estimate daily real yields and expected inflations in a latent multifactor interest rate model. It estimates the projection coefficients for inflation factor exclusively from monthly inflation data, rather than from both inflation and nominal yield data, in order to lessen biasedness. Because these coefficients are the same as those in the daily model, the study uses them with daily nominal yield data to estimate the remaining parameters. Using Thailand's data from March 1, 2001 to August 30, 2013, the study finds that the improved model can fit the nominal yields well. The term structure estimate of real yields has a normal shape, while that of expected inflations is flat. The inflation premiums are significant statistically and economically. They are ten times the ones reported in the past. Inflation premiums cannot be ignored in economic analyses for Thailand.

**Keywords:** Daily Real Yields, Affine Mutifactor Interest Rate Model, Daily Real-Yield Estimation

## 1. INTRODUCTION

Alternative techniques for estimating real yields and expected inflations have been proposed in the literature-among which multifactor affine interest rate models are popular due to their flexibility to explain time-varying risk premiums (Eraker, 2008). For the U.S. market, for example, Ang, Bekeart, and Wei (2008) estimated these term structures using a regime-switching factor model with inflation and nominal yield data. Chen, Liu, and Cheng (2010) proposed a multifactor, modified quadratic term structure model and estimated the term structures using nominal- and TIPS-yield data. Recently, Ho, Huang, and Yildirim (2014) estimated the term structures, using an affine multifactor model and nominal-yield and infla-

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tion-derivatives-price data. For the U.K. market, Evans (2003) employed a regime-switching affine model and nominal- and TIPS-yield data in the estimation. Joyce, Lildholdt, and Sorensen (2010) developed an essentially affine term structure model to estimate the structures using nominal- and real-yield data together with inflation and analyst-forecast inflation data. For Spain, Gimero and Marques (2012) applied an affine model to estimate the structures using the data on nominal yields, inflation and Diebold-Li beta shape factors. For Thailand, Khanthavit (2010) estimated the real curve using a two-latent-factor affine model with the nominal yield and inflation data, while Apaitan and Rungcharoenkitkul (2011) estimated the real curve using a four-macro-factor affine model with nominal yield, inflation and observed macro-variable data.

The daily estimates of real yield and expected inflations are useful and important. They support more active trading of the securities--especially inflation-linked bonds, and closer monitoring of the economy. Noticing that the previous studies could give only monthly or bi-weekly estimates, Khanthavit (2014) proposed a linear projection approach to estimate real yields and expected inflations on a daily basis. The approach employs monthly inflation and daily nominal yield data. It is useful particularly for emerging markets because, in general, these two series are their only available datasets.

In this study, I improve upon the Khanthavit (2014) approach by estimating the projection coefficients for inflation from monthly inflation data, instead of from both inflation and nominal yield data, in order to lessen biasedness. Because these coefficients are the same as those in the daily model, I use these coefficients with the daily nominal yield data to estimate the remaining parameters so that the model captures the motion of daily yield movement better. Using Thailand's data from March 1, 2001 to August 30, 2013, I find that the improved approach can fit the nominal yields well. The term structure estimate of real yields has a normal shape, while that of expected inflations is flat. The inflation premiums are significant statistically and economically. They are ten times the ones reported in the past. Inflation premiums cannot be ignored in economic analyses for Thailand.

## 2. THE MODEL

Khanthavit (2014) adopts the model of Joyce et al. (2010) to describe nominal and real yields in Thailand. The model is an essentially affine term structure model which relates the nominal and real yields with a set of latent factors linearly under a no-arbitrage condition in the real world. It is flexible for it allows time-varying risk premiums and real short rate. The number of latent factors can be raised to capture complex behavior of the yields. Moreover, a latent factor model is found in previous studies to fit yields better than a macro factor model.

### 2.1 The Pricing of Real and Nominal Bonds

In a no-arbitrage environment, the time-t price $P_t^{n,R}$ of a zero-coupon real bond with an n-period maturity must be given by Cochrane (2005)

$$P_t^{n,R} = E_t\{M_{t+1} M_{t+2} \ldots M_{t+n}\}, \quad (1)$$

where $M_{t+j}$ is the real pricing kernel in j periods hence and $E_t\{.\}$ is the conditional expectation operator in the real world. The price $P_t^{n,N}$ of a zero-coupon nominal bond is given in a similar way but with the nominal pricing kernel $M^*_{t+j} = M_{t+j}\frac{I_{t+j-1}}{I_{t+j}}$ being substituted for $M_{t+j}$. $I_{t+j}$ is the consumer price index at time t+j.

$$P_t^{n,N} = E_t\{M^*_{t+1} M^*_{t+2} \ldots M^*_{t+n}\}. \quad (2)$$

## 2.2 Real Yields, Nominal Yields and Their Compositions

From eqs. (1) and (2), because the real yield $y_t^{n,R}$ and nominal yield $y_t^{n,N}$ are $-\frac{1}{n} Ln\{P_t^{n,R}\}$ and $-\frac{1}{n}Ln\{P_t^{n,N}\}$, up to a second order approximation the yields must equal

$$y_t^{n,R} = -\frac{1}{n}\{E_t(\Sigma_{j=1}^n m_{t+j}) + \frac{1}{2}V_t(\Sigma_{j=1}^n m_{t+j})\} \quad (3.1)$$

$$y_t^{n,N} = -\frac{1}{n}\{E_t(\Sigma_{j=1}^n (m_{t+j} - \pi_{t+j})) + \frac{1}{2}V_t(\Sigma_{j=1}^n (m_{t+j} - \pi_{t+j}))\} \quad (3.2)$$

where $m_{t+j} = Ln\{M_{t+j}\}$. $\pi_{t+j} = Ln\{\frac{I_{t+j-1}}{I_{t+j}}\}$ is logged inflation. $V_t(.)$ is the variance operator conditioned on the information at time t.

From eq. (3.1), the 1-period real yield $y_t^{1,R}$ is $-\{E_t(m_{t+1}) + \frac{1}{2}V_t(m_{t+1})\}$. Using this relationship, the real yield $y_t^{n,R}$ can be decomposed into

$$y_t^{n,R} = \frac{1}{n}\{E_t(\Sigma_{j=1}^n y_{t+j-1}^{1,R}) - \Sigma_{j=2}^n Cov_t(\Sigma_{s=1}^{j-1} m_{t+s}, m_{t+j})\}. \quad (4)$$

$Cov_t(.)$ is the conditional covariance operator. The term $\frac{1}{n}E_t(\Sigma_{j=1}^n y_{t+j-1}^{1,R})$ is the average expected 1-period real yield. In the risk neutral world, $y_t^{n,R} = \frac{1}{n}E_t(\Sigma_{j=1}^n y_{t+j-1}^{1,R})$. So, the term $-\frac{1}{n}\Sigma_{j=2}^n Cov_t(\Sigma_{s=1}^{j-1} m_{t+s}, m_{t+j}) = y_t^{n,R} - \frac{1}{n}E_t(\Sigma_{j=1}^n y_{t+j-1}^{1,R})$ can be interpreted as being real term premium.

By definition, the break-even inflation rate is $y_t^{n,N} - y_t^{n,R}$. Its structure is given by

$$y_t^{n,N} - y_t^{n,R} = \frac{1}{n}\{E_t(\Sigma_{j=1}^n \pi_{t+j}) - \frac{1}{2}V_t(\Sigma_{j=1}^n \pi_{t+j}) + Cov_t(\Sigma_{j=1}^n m_{t+j}, \Sigma_{j=1}^n \pi_{t+j})\}. \quad (5)$$

The term $\frac{1}{n}E_t(\Sigma_{j=1}^n \pi_{t+j})$ is the expected inflation for the next n periods. The terms $-\frac{1}{n}\frac{1}{2}V_t(\Sigma_{j=1}^n m_{t+j})$ and $\frac{1}{n}Cov_t(\Sigma_{j=1}^n m_{t+j}, \Sigma_{j=1}^n \pi_{t+j})$ are the Jensen's effect (or inflation convexity) and the covariance effect (Ho et al., 2014). Their sum is the inflation premium. Under the Fisher hypothesis, $y_t^{n,N} = y_t^{n,R} + \frac{1}{n}E_t(\Sigma_{j=1}^n \pi_{t+j})$ and the inflation premium is zero.

## 2.3 Stochastic Behavior of Pricing Kernels

The logged, real pricing kernel $m_{t+1}$ takes on the form as in eq. (6).

$$m_{t+1} = -(\bar{r} + \gamma^T z_t) - \frac{\Lambda_t^{'}\Omega\Lambda_t}{2} - \Lambda_t^{'}\Omega^{\frac{1}{2}}\varepsilon_{t+1} \quad (6)$$

The term $(\bar{r} + \gamma' z_t)$ is the real short rate. It can vary over time with a set of K latent factors $z_t^{'} = [z_{1,t}, \ldots, z_{K,t}]$. The real short rate is constant if $\gamma' = [\gamma_1, \ldots, \gamma_K]$ is a zero vector. Vector $\Lambda_t^{'}\Omega^{\frac{1}{2}}$ is time-varying risk premiums.

$$\Lambda_t = \lambda + \beta z_t. \quad (7)$$

Vector $\lambda' = [\lambda_1, \ldots, \lambda_K]$ and matrix $\beta = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1K} \\ \vdots & \ddots & \vdots \\ \beta_{K1} & \cdots & \beta_{KK} \end{bmatrix}$. The risk premium for factor k is constant if vector $[\beta_{k1}, \ldots, \beta_{kK}]$ is zero. $\varepsilon_{t+1}^{'} = [\varepsilon_{1,t+1}, \ldots, \varepsilon_{K,t+1}]$ are Gaussian shocks of factors $z_{t+1}$. Their mean vector is zero and their covariance matrix is

$\Omega = \begin{bmatrix} \sigma_1^2 & 0 \ldots & 0 \\ 0 & \ddots & \vdots \\ \vdots & & 0 \\ 0 & \ldots 0 & \sigma_K^2 \end{bmatrix}$. Factors $z_{t+1}$ follow a VAR(1) process in eq. (8).

$$z_{t+1} = \varphi z_t + \varepsilon_{t+1}. \quad (8)$$

Coefficient matrix $\varphi = \begin{bmatrix} \varphi_{11} & 0 \ldots & & 0 \\ \varphi_{21} & \varphi_{22} & 0 \ldots & \vdots \\ \vdots & & \ddots & 0 \\ \varphi_{K1} & \varphi_{K2} & \cdots & \varphi_{22} \end{bmatrix}$ is a lower triangular matrix.

Because the logged nominal pricing kernel $m*_{t+1}$ is $m_{t+1} - \pi_{t+1}$, from eq. (6), it must equal

$$m*_{t+1} = -(\bar{r} + \gamma' z_t) - \frac{\Lambda'_t \Omega \Lambda_t}{2} - \Lambda'_t \Omega^{\frac{1}{2}} \varepsilon_{t+1} - \pi_{t+1}. \quad (9)$$

## 2.4 The Pricing

Following Duffie and Kan (1996), Joice et al. (2010) derived the solutions for the real and nominal yields as affine functions of latent factors in eqs. (10) and (11).

$$y_t^{n,R} = -\frac{1}{n}\{A_n + \mathbf{B}'_n \mathbf{z}_t\} \quad (10)$$

$$y_t^{n,N} = -\frac{1}{n}\{A^*_n + \mathbf{B}^{*'}_n \mathbf{z}_t\}, \quad (11)$$

where $A_0 = A^*_0 = 0.00$ and $B_0 = B^*_0$ are (Kx1) zero vectors. Coefficients $A_{n>0}$ and $A^*_{n>0}$ and vectors $B_{n>0}$ and $B^*_{n>0}$ are determined sequentially with respect to the systems of equations (12).

$$A_n = -\bar{r} + A_{n-1} - B'_{n-1}\Omega\lambda + \frac{1}{2}B'_{n-1}\Omega B_{n-1} \quad (12.1)$$

$$B'_n = -\gamma' + B'_{n-1}(\varphi - \Omega\beta) \quad (12.2)$$

and

$$A^*_n = -\bar{r} - \mu_\pi + A^*_{n-1} - B^{*'}_n \Omega\lambda^* + \frac{1}{2}B^{*'}_{n-1}\Omega B^*_{n-1} + \sigma_{\frac{1}{2}}^2 + \sigma_1^2 \lambda_1 \quad (12.3)$$

$$B^{*'}_n = -(\gamma' + \varphi_1) + B^{*'}_{n-1}(\varphi - \Omega\beta) + \iota' \Omega\beta, \quad (12.4)$$

where $\varphi_1 = [\varphi_{11}\ 0 \ldots 0]$ and $\iota' = [1\ 0 \ldots\ 0]$. $\mu_\pi$ is the unconditional mean of the inflation. The specifications (12.3) and (12.4) are specific to the perfect correlation assumption of factor $z_{1,t}$ with inflation $\pi_t$. Modification needs be made under a different assumption for $\pi_t$.

# 3. MODEL ESTIMATION

## 3.1 Measurement Equations

Because factors $z_t$ are latent, the econometrician will have to relate them with observed variables. Khanthavit (2014) considers inflation and nominal yields because these variables are observed in most countries. The measurement equations for *day t* are given by

$$\begin{bmatrix} \pi_t \\ -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} = \begin{bmatrix} \mu_\pi \\ A^*_{n_1} \\ \vdots \\ A^*_{n_H} \end{bmatrix} + \begin{bmatrix} \iota' \\ \mathbf{B}^{*'}_{n_1} \\ \vdots \\ \mathbf{B}^{*'}_{n_H} \end{bmatrix} z_t + \begin{bmatrix} 0 \\ \omega_{n_1,t} \\ \vdots \\ \omega_{n_H,t} \end{bmatrix} \quad (13)$$

$y_t^{n_h,N}$) is the daily nominal yield with an $n_h$-day maturity. With respect to Piazzesi (2010), a month of 21 trading days is assumed. So, $n_h$ is 21h and 252h days for h-month and h-year maturities respectively. $\omega_{n_{h,t}}$ is the measurement error due to, for example, bid-ask spreads and zero-curve interpolation. Inflation in eq. (13) ensures its dynamic is consistent with the determining factors of real and nominal yields.[1]

## 3.2 A Linear Projection of Latent Variables

Khanthavit (2014) proposes an approach to estimate the model on a daily basis even though inflation is reported monthly. Latent factors $z_t$ can be projected linearly by a set of $\eta$ observed information variables $q'_t = [q_{0,t} = 1, q_{1,t}, \ldots, q_{\eta-1,t}]$. The projection equation is given by

$$\mathbf{z}_t = \mathbf{b}'\mathbf{q}_t + \mathbf{v}_t, \qquad (14)$$

where $\mathbf{b}' = \begin{bmatrix} b_{1,0}, b_{1,1}, \ldots, b_{1,\eta-1} \\ \vdots \\ b_{K,0}, b_{K,1}, \ldots, b_{K,\eta-1} \end{bmatrix}$ is the matrix of projection coefficients and $v'_t = [v_{1,t}, \ldots, v_{K,t}]$ are projection errors. The linear projection approach follows Mishkin (1981) who estimated unobserved real yields by information variables. When $\mathbf{b}'\mathbf{q}_t + \mathbf{v}_t$ is substituted for $z_t$ in eq. (13), eq. (15.1) is obtained.

$$\begin{bmatrix} [\pi_t] \\ \begin{bmatrix} -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} [\mu_\pi] \\ \begin{bmatrix} A^*_{n_1} \\ \vdots \\ A^*_{n_H} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} [\iota'] \\ \begin{bmatrix} \mathbf{B}^{*\prime}_{n_1} \\ \vdots \\ \mathbf{B}^{*\prime}_{n_H} \end{bmatrix} \end{bmatrix} \mathbf{b}'\mathbf{q}_t + \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + \mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t \\ \vdots \\ \omega_{n_H,t} + \mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t \end{bmatrix} \end{bmatrix} \qquad (15.1)$$

$$= \begin{bmatrix} [b_{1,0} + \mu_\pi & b_{1,1} & \cdots & b_{1,\eta-1}] \\ \begin{bmatrix} A^*_{n_1} + \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_1 & \cdots & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_K \\ & \vdots & & \\ A^*_{n_H} + \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_1 & \cdots & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_K \end{bmatrix} \end{bmatrix} \mathbf{q}_t + \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + \mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t \\ \vdots \\ \omega_{n_H,t} + \mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t \end{bmatrix} \end{bmatrix} \qquad (15.2)$$

$$= \alpha^T q_t + \mu t. \qquad (15.3)$$

$\mathbf{b}'_{q-1}$ is column q of coefficient matrix $\mathbf{b}'$. Eq. (15.2) rearranges the coefficient vectors and matrices in eq. (15.1) by noticing that $q_{0,t} = 1$. $\mathbf{u}_t = \begin{bmatrix} [v_{1,t}] \\ \begin{bmatrix} \omega_{n_1,t} + \mathbf{B}^{*\prime}_{n_1}\mathbf{v}_t \\ \vdots \\ \omega_{n_H,t} + \mathbf{B}^{*\prime}_{n_H}\mathbf{v}_t \end{bmatrix} \end{bmatrix}$ and $\alpha' = \begin{bmatrix} [b_{1,0} + \mu_\pi & b_{1,1} & \cdots & b_{1,\eta-1}] \\ \begin{bmatrix} A^*_{n_1} + \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_1 & \cdots & \mathbf{B}^{*\prime}_{n_1}\mathbf{b}'_K \\ & \vdots & & \\ A^*_{n_H} + \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_0 & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_1 & \cdots & \mathbf{B}^{*\prime}_{n_H}\mathbf{b}'_K \end{bmatrix} \end{bmatrix}$ so that eq. (15.3) is in a familiar regression format.

The regression is linear in information variables $\mathbf{q}_t$. But it is highly nonlinear in the parameters. Eq. (15.3) is important. All the regressors and regressants are observed. Now, the econometrician can use simple regressions for the estimation.

### 3.3 The Proposed Improvement

Eq. (15.3) is the model for the day. Although nominal yields are reported daily, inflation is reported monthly. Khanthavit (2014) adjusts eq. (15.3) to align with the monthly observation of inflation data by summing eq. (15.3) for all day t in month T, totaling $d_T$ days. The sums $\Sigma_{t=1}^{d_T}\pi_t$, $\Sigma_{t=1}^{d_T} y_t^{n_1,N}$, …, $\Sigma_{t=1}^{d_T} y_t^{n_H,N}$ and $\Sigma_{t=1}^{d_T}\mathbf{q}_t$ are observed on a monthly basis. Because $\Sigma_{t=1}^{d_T}\pi_t$ is the sum of daily inflation, by definition it is monthly inflation. The nominal yields and information variables are available daily, so their sums for the month can be computed in a straightforward way. The summation enables the econometrician to estimate the model from monthly inflation and aggregate nominal yields. Because the parameters are the ones from the daily model, daily real yields and expected inflations can be inferred from these estimates and information variables for the days.

Despite the success of Khanthavit's (2014) approach, this study notices that the projection coefficients $[b_{1,0}, b_{1,1}, ..., b_{1,Q-1}]$ for the inflation factor and the expected daily inflation $\mu_\pi$ need not be estimated jointly with the remaining parameters. The coefficients $[b_{1,0,} b_{1,1}, ..., b_{1,Q-1}]$ are the regression coefficients of the monthly demeaned inflation on monthly aggregate information variables, while the expected inflation $\mu_\pi$ can be estimated by the average monthly inflation divided by 21. Estimating these parameters jointly with the remainders may bias the $[b_{1,0}, b_{1,1}, ..., b_{1,Q-1}]$ and $\mu_\pi$ estimates because they must help to explain the motion of nominal yields.

I propose to improve the approach by estimating the projection coefficients

$[b_{1,0}, b_{1,1}, ..., b_{1,Q-1}]$ and the expected inflation $\mu_\pi$ from the monthly inflation and monthly aggregate information variable data first. Then I use their estimates $[\hat{b}_{1,0}, \hat{b}_{1,1}, \ldots, \hat{b}_{1,\eta-1}]$ and $\hat{\mu}_\pi$ with daily nominal yield and daily information variable data to estimate the remaining parameters using the daily model for nominal yields in eq. (16).

$$\begin{bmatrix} -n_1 y_t^{n_1,N} \\ \vdots \\ -n_H y_t^{n_H,N} \end{bmatrix} = \begin{bmatrix} A_{n_1}^* \\ \vdots \\ A_{n_H}^* \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{n_1}^{*T} \\ \vdots \\ \mathbf{B}_{n_H}^{*T} \end{bmatrix} \begin{bmatrix} \hat{b}_{1,0}, \hat{b}_{1,1}, \ldots, \hat{b}_{1,\eta-1} \\ \vdots \\ b_{K,0}, b_{K,1}, \ldots, b_{K,\eta-1} \end{bmatrix} \mathbf{q}_t + \begin{bmatrix} \omega_{n_1,t} + \mathbf{B}_{n_1}^{*\prime} \mathbf{v}_t \\ \vdots \\ \omega_{n_H,t} + \mathbf{B}_{n_H}^{*\prime} \mathbf{v}_t \end{bmatrix}. \quad (16)$$

Because the data aggregation can average out valuable information, the use of daily data in eq. (16) should capture the daily motion of the nominal yields better. It is hoped that reduced biasedness in parameter estimates and improved efficiency of information uses will that enhance the model performance.

The two-step estimation being proposed here--in which the first parameter set is estimated in the first step and then is used in the second step to recover the remaining parameters, is similar to that in Joslin, Singleton, and Zhu (2011). Despite the similarity, the objectives of our two-step procedures differ. While Joslin et al. (2011) use a two-step approach to reduce computation complexity, mine uses it to extract information directly and efficiently from daily samples.

### 3.4 The Regressions

I use weighted least squares regression to estimate the projection coefficients for the inflation factor. The weight is $\sqrt{d_T}$ to correct for heteroscedasticity from a $d_T$-day aggregation in month T. I use nonlinear seemingly unrelated regression estimation (SURE) to estimate eq. (16). The nonlinear SURE procedure is two-step. In step 1, I estimate the covariance matrix of the regression errors $\begin{bmatrix} \omega_{n_1,t} + \mathbf{B}_{n_1}^{*\prime} \mathbf{v}_t \\ \vdots \\ \omega_{n_H,t} + \mathbf{B}_{n_H}^{*\prime} \mathbf{v}_t \end{bmatrix}$. Because the model is linear in the information variables, the covariance matrix can be estimated conveniently by linear SURE. In step 2, I use nonlinear SURE to estimate the parameters embedded in eq. (16). I assume the covariance matrix of the errors is the one from the first step.

## 4. THE DATA

### 4.1 Samples and Data Sources

I apply the improved approach to re-estimate the model for Thailand. The sample period is from March 1, 2001 to August 30, 2013. It is the same sample period as in Khanthavit (2014) so that our results can be compared. The nominal yield data are daily for 1-month, 3-month, 6-month and 1-year up to 10-year maturities, with one-year increments, from the Thai Bond Market Association (Thai BMA). The inflation is logged monthly inflation, computed using the headline consumer price index from the Bureau of Trade and Economic Indices, Ministry of Commerce.

Table 1 reports the descriptive statistics of inflation and nominal yields. The average inflation is 2.6804%. This estimate will serve as $\hat{\mu}_\pi$ in the estimation of the remaining parameters. The term structure of average nominal yields has a normal shape, while the volatility structure is inverted. The normal term structure is similar to the ones found for the U.S.A. by Jian and Yan

(2009) and the U.K. by Joyce et al. (2010). But the volatility structures in the U.S.A. and the U.K. are normal. Thailand's inverted volatility term structure is probably because long-termed bonds are less liquid. On a no-trading day, the yields of these bonds are quoted yields from dealers who interpolate today's yields from yesterday's yields.

I test and reject the normality assumption for the inflation and nominal yields. The rejection supports the use of SURE because SURE does not require a normality assumption.

**Table 1: Descriptive Statistics**

| Variables | Average | Max | Min | Std. | Skew. | E. Kurt. | JB Stat. |
|---|---|---|---|---|---|---|---|
| Inflation | 2.6804% | 25.8264% | -36.7878% | 6.6873% | -1.2803 | 9.2715 | 578.2402*** |
| 1M | 2.4260% | 5.0333% | 0.7799% | 1.0915% | 0.6027 | -0.3174 | 198.0985*** |
| 3M | 2.4968% | 5.0536% | 0.7981% | 1.0767% | 0.5817 | -0.3078 | 184.6539*** |
| 6M | 2.5987% | 5.2136% | 0.8633% | 1.0643% | 0.5423 | -0.3619 | 166.6587*** |
| 1Y | 2.7271% | 5.3154% | 0.9314% | 1.0585% | 0.5171 | -0.4169 | 158.5104*** |
| 2Y | 3.0152% | 5.5432% | 1.1781% | 1.0499% | 0.5662 | -0.3152 | 176.1810*** |
| 3Y | 3.2460% | 5.8372% | 1.3491% | 1.0056% | 0.5616 | -0.1891 | 165.3990*** |
| 4Y | 3.4937% | 6.1637% | 1.4515% | 0.9443% | 0.4743 | -0.0377 | 114.9058*** |
| 5Y | 3.7237% | 6.3980% | 1.5680% | 0.9260% | 0.4121 | -0.0992 | 87.8464*** |
| 6Y | 3.9504% | 6.6710% | 1.7383% | 0.8942% | 0.3239 | -0.1741 | 57.3600*** |
| 7Y | 4.1527% | 6.7853% | 1.8978% | 0.8694% | 0.2844 | -0.2531 | 49.4308*** |
| 8Y | 4.3061% | 6.8614% | 2.0604% | 0.8875% | 0.2759 | -0.4829 | 68.5419*** |
| 9Y | 4.4184% | 6.9546% | 2.2364% | 0.9191% | 0.3041 | -0.5455 | 85.0894*** |
| 10Y | 4.5586% | 7.1884% | 2.4839% | 0.9458% | 0.3368 | -0.5930 | 102.6925*** |

**Note:** The statistics for inflation is estimated from monthly data, while those for nominal yields are from daily data. *** = Significance at a 99% confidence level.

**Table 2: Tests for Projection Ability of Information Variables**

| Variables | Constant | Beta F. 1 | Beta F. 2 | Beta F. 3 | Beta F. 4 | $R^2$ |
|---|---|---|---|---|---|---|
| Inflation | 0.0002 | -0.0028 | -0.0042 | 0.0027* | -0.0069* | 0.0223 |
| 1M | 0.0000 | -0.0834*** | -0.0816*** | -0.0023*** | -0.0792*** | 0.9986 |
| 3M | 0.0000 | -0.2496*** | -0.2310*** | -0.0177*** | -0.2144*** | 0.9989 |
| 6M | 0.0000 | -0.4985*** | -0.4253*** | -0.0632*** | -0.3700*** | 0.9979 |
| 1Y | 0.0004*** | -1.0047*** | -0.7669*** | -0.1879*** | -0.6104*** | 0.9980 |
| 2Y | -0.0004*** | -2.0001*** | -1.1898*** | -0.5667*** | -0.7689*** | 0.9967 |
| 3Y | -0.0005*** | -2.9818*** | -1.4066*** | -0.9290*** | -0.7782*** | 0.9962 |
| 4Y | -0.0030*** | -3.9105*** | -1.4856*** | -1.1784*** | -0.7781*** | 0.9955 |
| 5Y | 0.0043*** | -5.0921*** | -1.6612*** | -1.3575*** | -0.9072*** | 0.9931 |
| 6Y | 0.0020*** | -6.036*** | -1.6663*** | -1.4815*** | -0.8786*** | 0.9921 |
| 7Y | -0.0051*** | -6.8659*** | -1.6459*** | -1.5243*** | -0.8437*** | 0.9894 |
| 8Y | -0.0011** | -7.9817*** | -1.8065*** | -1.5333*** | -1.0165*** | 0.9934 |
| 9Y | 0.0020*** | -9.0673*** | -1.8103*** | -1.7124*** | -0.9168*** | 0.9928 |
| 10Y | -0.0039*** | -9.8756*** | -1.5243*** | -1.8261*** | -0.5681*** | 0.9934 |

**Note:** *, ** and *** = Significance at 90%, 95% and 99% confidence levels.

### 4.2 Information Variables

$\eta$ = 5 information variables are considered in the projection. The first is a constant. The remainders are 1-day lagged Bjork-Christensen (1999) beta shape factors. As Khanthavit (2013) reported, these factors could predict Thailand's nominal term structure accurately.

To check for projection ability, I regress daily nominal yields on daily information variables and regress monthly inflation on monthly-aggregate information variables. From eqs. (14) and (15.3) if the information variables are able to project the latent factors, the regression coefficients must be significant. The results are in Table 2. The coefficients for the nominal yields are highly significant. For inflation, the coefficients for beta shape factors 3 and 4 are significant at a 90% confidence level. Based on these results, I conclude that the chosen information variables have projection ability. The regression coefficients for the inflation will serve as $[\hat{b}_{1,0}, \hat{b}_{1,1}, \ldots, \hat{b}_{1,\eta-1}]$ in eq. (16).

It is noted that the $R^2$'s for nominal yields are very high. All are over 99%. The high $R^2$'s and also highly significant coefficients can be explained by Khanthavit's (2013) observation that the nominal yields and beta shape factors were long-memory, near-I(1) variables. So, the results were similar to the ones from co-integration regressions.

## 5. EMPIRICAL RESULTS

### 5.1 Parameter Estimates

I consider a two-factor model because Khanthavit (2014) found that the first two principal components could explain 97.92% of the variation of Thailand's nominal yields. The parameter estimates are reported in Table 3. The estimates are not very close with the ones reported in Khanthavit (2014). These differences are

**Table 3: Parameter Estimates**

| Parameters | Value |
|---|---|
| $\bar{r}$ x 25200 | 0.0339*** |
| $\gamma_1$ | -2.8592*** |
| $\gamma_2$ | 6.9885E-06*** |
| $\lambda_1$ | -19.6616*** |
| $\lambda_2$ | 32.7197*** |
| $\beta_{11}$ | 2976.3640*** |
| $\beta_{12}$ | -28.3074*** |
| $\beta_{21}$ | -364.3557*** |
| $\beta_{22}$ | -12151.5940*** |
| $\varphi_{11}$ | 0.5514*** |
| $\varphi_{21}$ | -1.1895*** |
| $\varphi_{22}$ | 0.6630*** |
| $\sigma_1$ | 0.0016*** |
| $\sigma_2$ | 0.0057*** |
| $\mu_\pi \times \frac{25200}{21}$ | 2.6805*** |
| $b_0^1$ | 5.9730E-05 |
| $b_1^1$ | -0.0027 |
| $b_2^1$ | -0.0042 |
| $b_3^1$ | 0.0027* |
| $b_4^1$ | -0.0068** |
| $b_0^2$ | -0.5674*** |
| $b_1^2$ | -15.0427*** |
| $b_2^2$ | -13.0010*** |
| $b_3^2$ | -3.1100*** |
| $b_4^2$ | -10.0593*** |

**Note:** *, **, and *** = Significance at 90%, 95% and 99% confidence levels, respectively. $\mu_\pi$ is a monthly average divided by 21. $b_0^1, \ldots, b_4^1$ are the regression coefficients of the monthly inflation on the sum of the information variables in the month. The remaining estimates are from the nonlinear SURE model.

expected because our estimates of the projection coefficients for the inflation factor and of the expected inflation are not very close.

The $\varphi_{11}$ estimate in this study equals 0.5514. It is much larger than 0.0179 reported by Khanthavit (2014). This finding is an improvement. A 0.5514 $\varphi_{11}$ implies a 0.0408 autocorrelation for the monthly inflation rate. So, it lies closer to of the AR(1) estimate of 0.3322 from monthly inflation data than does the implied 0.0009 level in Khanthavit (2014).

## 5.2 Performance Comparison

### *5.2.1 Moment Matching*

I follow Ang et al. (2008) to conduct specification tests for the model. If the model fits, the moments of sample and fitted nominal yields should not differ. Comparison of the means, standard deviations, skewnesses and excess kurtoses are in Table 4. The numbers in the first lines are for fitted yields and those in the second lines are their deviations from the sample mo-

**Table 4: Specification Tests**

| **Maturity** | **Descriptive Statistics** | | | |
|---|---|---|---|---|
| | Mean | Std. | Skew. | E. Kurt |
| 1M | 2.4457 | 1.1000 | 0.6874 | -0.1576 |
| | 0.0196 | 0.0085 | 0.0252 | 0.0093 |
| 3M | 2.4836 | 1.1211 | 0.4957 | -0.5612 |
| | -0.0133 | 0.0445 | -0.0682 | -0.1306 |
| 6M | 2.5707 | 1.1089 | 0.4598 | -0.6629 |
| | -0.0280 | 0.0446 | -0.0568 | -0.1087 |
| 1Y | 2.7456 | 1.0642 | 0.4545 | -0.7316 |
| | 0.0185 | 0.0057 | -0.045 | -0.0687 |
| 2Y | 3.0666 | 0.9719 | 0.4680 | -0.8048 |
| | 0.4636 | -1.1636 | -0.3086 | -0.2881 |
| 3Y | 3.3489 | 0.8883 | 0.4788 | -0.8632 |
| | 0.1029 | -0.1173* | -0.0981 | -0.3061 |
| 4Y | 3.5974 | 0.8139 | 0.4828 | -0.9175 |
| | 0.1037 | -0.1303*** | -0.0118 | -0.4586 |
| 5Y | 3.8166 | 0.7480 | 0.4796 | -0.9702 |
| | 0.0929 | -0.1781*** | 0.0498 | -0.3978 |
| 6Y | 4.0104 | 0.6893 | 0.4699 | -1.0221 |
| | 0.0601 | -0.2049*** | 0.1376 | -0.3233 |
| 7Y | 4.1824 | 0.6372 | 0.4540 | -1.0733 |
| | 0.0297 | -0.2322*** | 0.1768 | -0.2446 |
| 8Y | 4.3352 | 0.5907 | 0.4326 | -1.1237 |
| | 0.0291 | -0.2968*** | 0.1852 | -0.0150 |
| 9Y | 4.4715 | 0.5491 | 0.4061 | -1.1730 |
| | 0.0531 | -0.3700*** | 0.1568 | 0.0474 |
| 10Y | 4.5934 | 0.5120 | 0.3750 | -1.2206 |
| | 0.0348 | -0.4339*** | 0.1240 | 0.0948 |

**Note:** * and *** = Significance at 90% and 99% confidence levels, respectively. The statistics on the upper lines are those of the fitted yields and the ones on the lower lines are the deviations from sample statistics.

ments. Significance is based on the White (2000) procedure.

The deviations are small and not significant for all the moments and maturities, except for the standard deviations of 3-year and longer yields. The significance of standard deviations was also reported for most specifications of the Ang et al. (2008) model. With respect to the small number of significant cases and when compared and contrast with the ones reported by the previous study, I conclude that the improved approach satisfactorily fit Thailand's nominal yields. However, it performs slightly less well than the Khanthavit approach in matching the moments of nominal yields. While the improved approach can match the standard deviations of up to 2-year yields, the Khanthavit approach can do up to 4-year yields.

*5.2.2 Estimation Accuracy*

The better approach should give more accurate estimates of the interesting variables. I compare the estimation accuracy of the competing approaches by mean squares errors (MSEs) of the estimates from the sample nominal yields. The results are in Table 5. In the overall accuracy test, the improved approach performs better. Its summed MSE is 4.881 as opposed to 4.9148 of the Khanthavit approach. But the difference is not significant, when it is based on the White (2000) statistics. The improved approach is significantly more accurate for 3-month, 6-month, 8-year, 9-year and 10-year maturities and significantly less accurate for 2-year, 3-year and 4-year maturities. For the remainders, the differences are not significant. The im-

**Table 5: Test for Estimation Accuracy**

| **Maturities** | **Mean Square Errors** | | **Difference** |
|---|---|---|---|
| | Khanthavit (2014) Approach | Improved Approach | |
| Summed MSE | 4.9148*** | 4.8801*** | 0.0347 |
| 1M | 0.0092*** | 0.0083*** | 0.0009 |
| 3M | 0.0153*** | 0.0105*** | 0.0048*** |
| 6M | 0.0173*** | 0.0132*** | 0.0041*** |
| 1Y | 0.0240*** | 0.0247*** | -0.0007 |
| 2Y | 0.1044*** | 0.1177*** | -0.0133*** |
| 3Y | 0.2198*** | 0.2381*** | -0.0183*** |
| 4Y | 0.3349*** | 0.3490*** | -0.0141** |
| 5Y | 0.4562*** | 0.4637*** | -0.0075 |
| 6Y | 0.5556*** | 0.5526*** | 0.0030 |
| 7Y | 0.6449*** | 0.6314*** | 0.0135 |
| 8Y | 0.7467*** | 0.7281*** | 0.0186** |
| 9Y | 0.8349*** | 0.8158*** | 0.0191*** |
| 10Y | 0.9515*** | 0.9271*** | 0.0243*** |

**Note:** ** and *** = Significance at 95% and 99% confidence levels, respectively.

proved approach is more accurate for 5 maturities and less accurate for 3 maturities. Because the improved approach gives smaller MSEs and fits the nominal yields better and significantly better in more cases, I conclude that the improved approach can enhance estimation accuracy over the Khanthavit approach.

### 5.3 Daily Real Yields and Expected Inflations

The estimation of daily real yields and expected inflations is successful. In Panel 6.1 of Table 6, the term structure of Thailand's real yields is time varying. Its average has a normal shape. The averages for 1-month up to 1-year maturities are negative but rising. They turn positive for a 2-year maturity and over. The normal term structure of real yields is similar to the one in Khanthavit (2014). But here the average real yields are much lower of about 60 basis points. The lower average real yields can be explained partly by the higher estimates of the unconditional expected inflation.

In Panel 6.2, the expected inflations are more volatile for short horizons, while those for long horizons do not vary much. The average structure is flat. The results for expected inflations are similar to those in Khanthavit (2014).

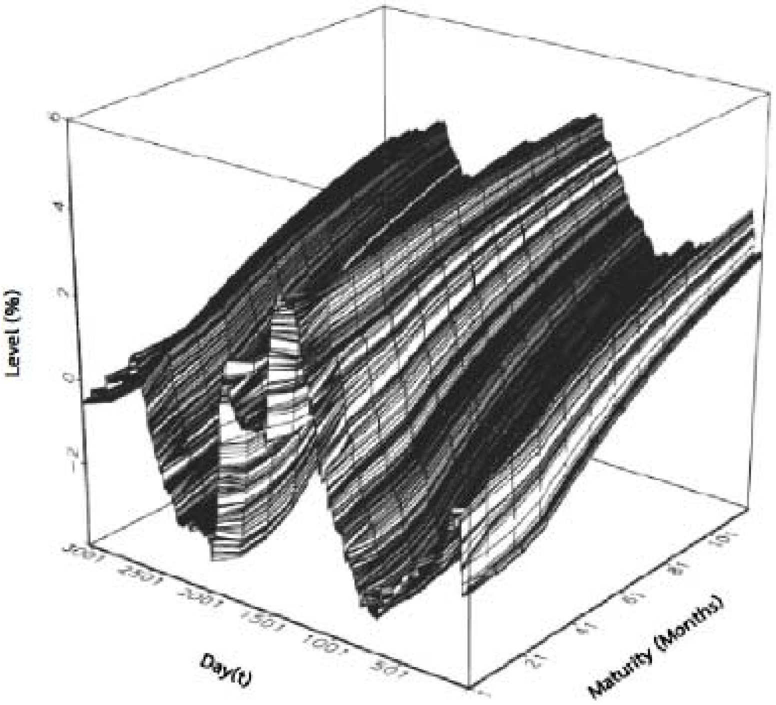
### 5.4 Inflation Premiums

For some reasons, researchers, practitioners and regulators assume zero inflation premiums at times.[2] But inflation premiums need not be zero. In this study, I compute inflation premiums for Thailand by subtracting the real-yield and expected-inflation estimates from the sample nominal yields. The premiums are reported in Panel 6.3. The premiums for short maturities are positive and those for long maturities are negatives. The inverted shape is different from a normal shape in the U.S.A. (Ang et al., 2008) and a humped shape in the U.K. (Joyce et al., 2010).

The average premiums are large from 45 basis points to -88 basis points. They are approximately 10 times those in Khanthavit (2014). Because of the improved information in the estimation of this study, the results here should be more accurate. These levels are significant economically and cannot be ignored. I test for zero inflation premiums and reject the hypotheses for all the maturities. Significant inflation premiums imply that the estimates of Thailand's real yields based on a zero-premium assumption are biased downward for short maturities and biased upward for long maturities. Because the premiums are significant economically and statistically, the role of inflation premiums in economic analyses for Thailand cannot be ignored.

## Table 6: Daily Term Structures

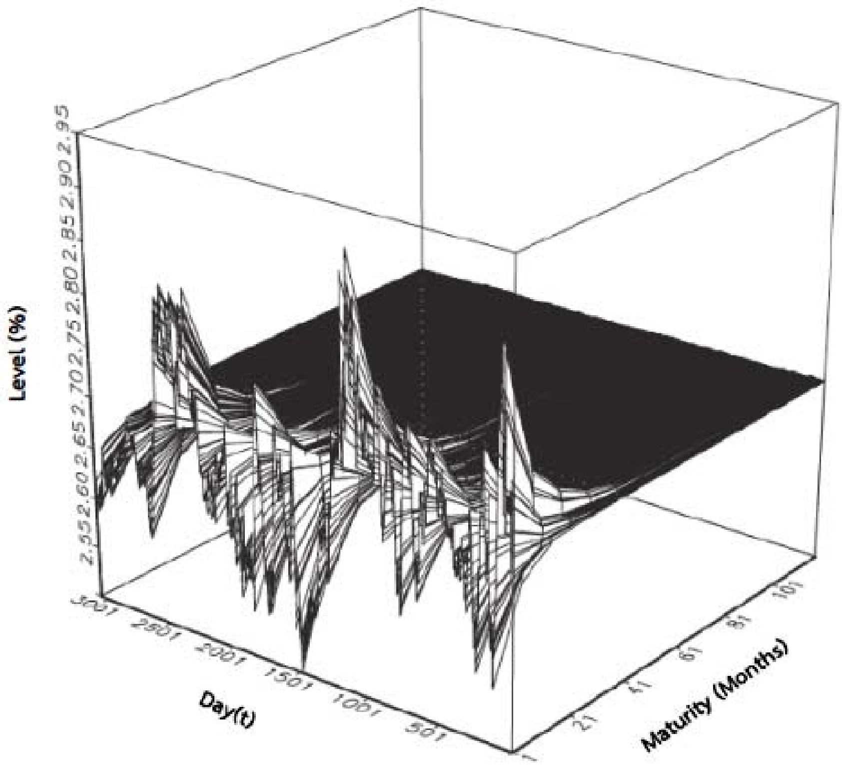
### Panel 6.1: Real Yields




| Maturity | Average | Max | Min | Std. |
|---|---|---|---|---|
| 1M | -0.6557*** | 3.4421 | -3.2531 | 1.7195 |
| 3M | -0.6327*** | 3.5053 | -3.5143 | 1.7492 |
| 6M | -0.5031*** | 3.6038 | -3.3950 | 1.7260 |
| 1Y | -0.2291*** | 3.7160 | -3.0219 | 1.6540 |
| 2Y | 0.2793*** | 3.8828 | -2.2783 | 1.5091 |
| 3Y | 0.7272*** | 4.0202 | -1.6121 | 1.3786 |
| 4Y | 1.1212*** | 4.1381 | -1.0230 | 1.2629 |
| 5Y | 1.4684*** | 4.2406 | -0.5022 | 1.1603 |
| 6Y | 1.7754*** | 4.3303 | -0.0412 | 1.0693 |
| 7Y | 2.0474*** | 4.4092 | 0.3679 | 0.9885 |
| 8Y | 2.2891*** | 4.4790 | 0.7318 | 0.9165 |
| 9Y | 2.5045*** | 4.5408 | 1.0564 | 0.8522 |
| 10Y | 2.6970*** | 4.5958 | 1.3466 | 0.7946 |

**Note:** *** = Significance at a 99% confidence level. Day (t=1) is March 1, 2001 and Day (t=3060) is August 30, 2013.

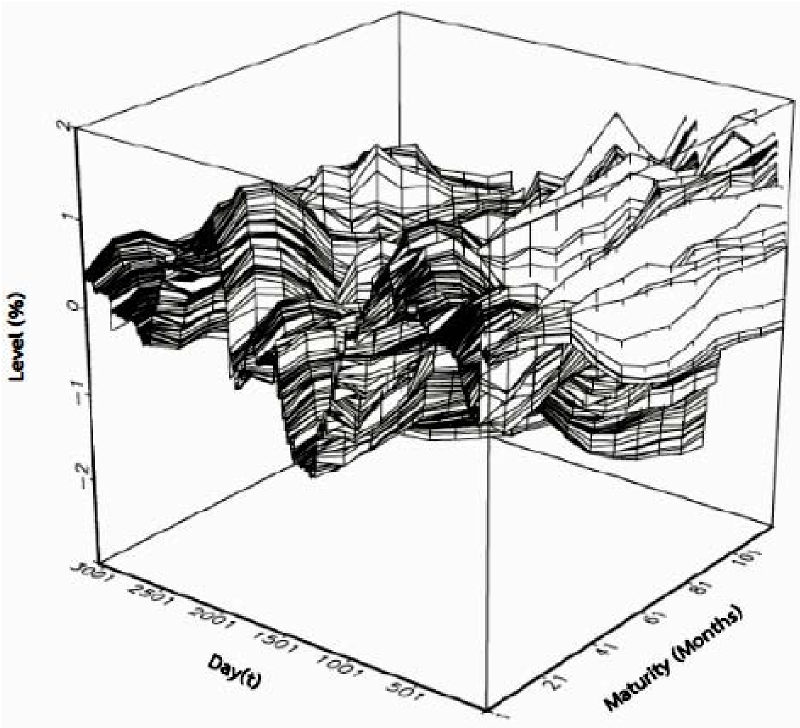
**Panel 6.2: Expected Inflations**




| Maturity | Average | Max | Min | Std. |
|---|---|---|---|---|
| 1M | 2.6804*** | 2.9133 | 2.5018 | 0.0720 |
| 3M | 2.6804*** | 2.7580 | 2.6209 | 0.0240 |
| 6M | 2.6804*** | 2.7192 | 2.6506 | 0.0120 |
| 1Y | 2.6804*** | 2.6998 | 2.6655 | 0.0060 |
| 2Y | 2.6804*** | 2.6901 | 2.6730 | 0.0030 |
| 3Y | 2.6804*** | 2.6869 | 2.6754 | 0.0020 |
| 4Y | 2.6804*** | 2.6853 | 2.6767 | 0.0015 |
| 5Y | 2.6804*** | 2.6843 | 2.6774 | 0.0012 |
| 6Y | 2.6804*** | 2.6836 | 2.6779 | 0.0010 |
| 7Y | 2.6804*** | 2.6832 | 2.6783 | 0.0009 |
| 8Y | 2.6804*** | 2.6828 | 2.6785 | 0.0008 |
| 9Y | 2.6804*** | 2.6826 | 2.6787 | 0.0007 |
| 10Y | 2.6804*** | 2.6823 | 2.6789 | 0.0006 |

**Note:** *** = Significance at a 99% confidence level. Day (t=1) is March 1, 2001 and Day (t=3060) is August 30, 2013.

**Panel 6.3: Inflation Premiums**




| Maturity | Average | Max | Min | Std. |
|---|---|---|---|---|
| 1M | 0.4013*** | 1.4766 | -1.0912 | 0.6106 |
| 3M | 0.4491*** | 1.6624 | -1.1603 | 0.6841 |
| 6M | 0.4214*** | 1.6441 | -1.1156 | 0.6737 |
| 1Y | 0.2758*** | 1.5209 | -1.1084 | 0.6253 |
| 2Y | 0.0554*** | 1.5013 | -1.3002 | 0.6151 |
| 3Y | -0.1616*** | 1.4534 | -1.5182 | 0.6853 |
| 4Y | -0.3079*** | 1.6377 | -1.6657 | 0.7742 |
| 5Y | -0.4252*** | 1.6667 | -1.9210 | 0.8434 |
| 6Y | -0.5054*** | 1.6741 | -2.1572 | 0.9040 |
| 7Y | -0.5751*** | 1.9308 | -2.2772 | 0.9532 |
| 8Y | -0.6634*** | 1.6361 | -2.4413 | 0.9963 |
| 9Y | -0.7665*** | 1.8412 | -2.6567 | 1.0265 |
| 10Y | -0.8188*** | 1.8013 | -2.6792 | 1.0807 |

**Note:** *** = Significance at a 99% confidence level. Day (t=1) is March 1, 2001 and Day (t=3060) is August 30, 2013

### 5.5 Composition of Nominal Yields

Nominal yields equal real yields plus expected inflations plus inflation premiums, while real yields equal average expected 1-day real yields plus real premiums.[3] It is interesting to examine how much these variables contribute to nominal yields. I estimate the percentage shares of the four variables in the nominal yield's variation using the slope coefficients from linear regressions of the variables on nominal yield. They are reported in Table 7.

The movement of nominal yields is principally driven by real premiums and inflation premiums. For short-termed yields, real premiums contribute the most. Their percentage shares fall when maturities are lengthened. For the 10-year nominal yield, the share of real premium falls to 20% while that of inflation premium rises to 80%. Average expected 1-day real yields and expected inflations contribute little. These results are expected due to the small size and low volatility of the average expected 1-day real yields and the low volatility of expected inflations.

## 6. CONCLUSION

Alternative techniques for estimating real yields and expected inflations have been proposed in the literature. But they could give only monthly or bi-weekly estimates. The daily estimates of real yield and expected inflations are useful and important. They support more active trading of the securities and closer monitoring of the economy. So, recently Khanthavit (2014) proposed a linear projection approach to estimate real yields and expected inflations on a daily basis from monthly inflation and daily nominal yields. The approach is use-

**Table 7: Composition of Nominal Yields**

| **Maturity** | **Average Expected 1-Day Rate** | **Real Premium** | **Expected Inflation** | **Inflation Premium** |
|---|---|---|---|---|
| 1M | -6.3447*** | 163.6477*** | -2.3102*** | -54.9928*** |
| 3M | -1.7891*** | 163.4717*** | -0.6569*** | -61.0256*** |
| 6M | -0.6849*** | 162.0854*** | -0.2554*** | -61.1450*** |
| 1Y | -0.2243*** | 154.8086*** | -0.0864*** | -54.4979*** |
| 2Y | -0.0327*** | 136.1284*** | -0.0153*** | -36.0804*** |
| 3Y | -0.0032*** | 120.7236*** | -0.0036*** | -20.7168*** |
| 4Y | -0.0043 | 105.8029*** | -0.0033 | -5.7953*** |
| 5Y | -0.0040 | 87.0082*** | -0.0027 | 12.9986*** |
| 6Y | 0.0022 | 70.3837*** | -0.0002 | 29.6143*** |
| 7Y | 0.0083 | 54.5010*** | 0.0022 | 45.4885*** |
| 8Y | 0.0085* | 40.2784*** | 0.0025 | 59.7106*** |
| 9Y | 0.0115*** | 30.5865*** | 0.0037*** | 69.3984*** |
| 10Y | 0.0149*** | 19.9905*** | 0.0050*** | 79.9896*** |

**Note:** * and *** = Significance at 90% and 99% confidence levels.

ful particularly for emerging markets because, in general, these two series are their only available datasets.

In this study, I improve upon the Khanthavit approach by estimating the projection coefficients for inflation from monthly inflation data, instead of from both inflation and nominal yield data, in order to lessen biasedness. Because these coefficients are the same as those in the daily model, I use these coefficients with the daily nominal yield data to estimate the remaining parameters so that the model captures the motion of daily yield movement better. Using Thailand's data from March 1, 2001 to August 30, 2013, I find that the improved approach can fit the nominal yields well. The term structure estimate of real yields has a normal shape, while that of expected inflations is flat. The inflation premiums are significant statistically and economically. Hence, inflation premiums cannot be ignored in economic analyses for Thailand. The resulting policy implications include careful applications of the Fisher hypothesis by regulators to manage the economy and by academics to improve and test interesting theories. As for investors, assuming zero inflation premiums will lead them to misprice inflation-linked bonds.

I am aware that the results are based on the model with restrictive assumptions. At least two important assumptions are worth discussing. One, the model assumes a linear relationship between the short rate and the latent factors. And two, I assume two latent factors in the model. As for the first assumption, although nonlinear multifactor models have been developed recently, for example, by Jian and Yan (2009), they are not very popular due to complexity and difficulty to be used in practice. Moreover, a linear model can be thought of being linear approximation of the nonlinear model and the number of latent factors can be raised to ensure that the linear approximation model can fit the data. As for the second assumption, a small number of factors can simplify the model greatly. I tested for the number of latent factors against the data and found that two factors sufficed. In all, despite these restrictive assumptions, the model can fit the data well. In the performance tests, the theoretical nominal yields can match the actual yields up to four moments.

## Endnotes

[1]It is assumed factor $z_{1,t}$ correlates perfectly with inflation in order to simplify the model's structure. The first factor then can be interpreted as being inflation factor. The perfect correlation assumption is not restrictive. The factors are latent. When the first factor is inflation, the remaining factors can be rotated so that the fit of the model remains unchanged.

[2]For example, the Bank of Thailand (2014) assumes the Fisher hypothesis to explain the channel through which the adjustment of policy interest rates can affect the economy.

[3]The results for average expected 1-day real yields and real premiums can be obtained from the author upon request.

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