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Teaching Arithmetic Sequences Using Situated Problem Solving Tasks

Thomas A. Fisher

State University of New York College at Brockport, tfish1@brockport.edu

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Teaching Arithmetic Sequences
Using Situated Problem Solving Tasks
Thomas A. Fisher
The College at Brockport, SUNY

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Abstract

This curriculum project was developed to help teachers educate students about arithmetic sequences using an inquiry-based situative approach. The curriculum incorporates meaningful contexts within problem solving tasks to help students make connections between linear functions and arithmetic sequences. The materials are aligned to the Algebra I standards within the Common Core Learning Standards (CCLS).

Chapter 1: Introduction

Prior to the Common Core implementation, sequences were a part of the Algebra 2/Trigonometry standards (Algebra 2 and Trigonometry, 2005). The previous state standards, most often the National Council of Teachers of Mathematics (NCTM) *Standards*, in Algebra 2/Trigonometry, referred to as Algebra 2/Trig, stated that students were expected to identify sequences, determine the common difference, write a formula for the n th term, and determine the value of a specific term (Algebra 2 and Trigonometry, 2005). In Algebra I with the Common Core Learning Standards (CCLS), students are expected to “write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms” (New York State P-12 Common Core Learning Standards for Mathematics, n. d., P. 59). With these new standards, teachers now teach all students sequences since everyone in New York State is required to pass the Algebra I course in order to graduate high school. Algebra I teachers need materials which can support the teaching of arithmetic sequences in a way that fosters problem solving behaviors in contexts that are relatable to Algebra I students.

Purpose

The purpose of this curriculum is to provide a unit of materials for teaching arithmetic sequences using situated problem solving tasks. These situated tasks are defined as problems which are placed within relatable contexts since how the content is learned and “the situation in which a person learns, are a fundamental part of what is learned” (Peressini et al., 2004, p. 68). These lessons are written to support students in making connections between linear relationships and real life situations involving arithmetic sequences. Teaching arithmetic sequences for conceptual understanding is a

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complex task and it requires explicit work on the part of the teacher. A complex task is defined as “a category of learning tasks that confronts learners with situations where concept definitions within or between domains are unclear” (Van Merriënboer & Sweller, 2002). The contexts used will be meaningful to students because they incorporate authentic situations that students could experience in their daily life. So teachers who use this curriculum will be able to use it with confidence that the real life applications are more than just minor connections to arbitrary situations.

Arithmetic sequences are often taught disjointed from linear relationships, which creates challenges for students in understanding their relevance and their connection to previously learned content. Also, sequences require learning new and different mathematical syntax and symbolism. As with other mathematics topics, weakness in arithmetic sequences is “often due in part to the obstacles [students] face in focusing on these symbols as they attempt to read the ‘language of mathematics’” (Adams, 2003, p. 786). Similar to learning English, “mathematics is a language that people use to communicate, to solve problems, to engage in recreation, and to create works of art and mechanical tools” (Adams, 2003, p. 786). The tasks in the provided lessons build upon student knowledge of linear relationships while introducing students to a variety of ways to represent the sequences. Teachers will be able to use this curriculum to teach recursive and explicit definitions of arithmetic sequences using learning tasks that demonstrate flexibility in symbolism while maintaining the connection to a linear relationship. The situative perspective recontextualizes the concept of linearity to apply it to new problems so that the content is more applicable to students instead of arbitrary symbols and procedures (Peressini et al., 2004).

Chapter 2: Literature Review

Situative Perspective

The situative perspective is focused on collaboration within situations so that the content learned is meaningful to students (Peressini et al., 2004). This approach is vastly different from the cognitive perspective that “knowledge is an entity that is acquired in one setting and then transported to other settings” (Peressini et al., 2004, p. 69). Thus, the cognitive perspective does not value the context of where the content was learned or where it is applied. The cognitive method of teaching is how students traditionally learn mathematics by transferring knowledge between settings. Rather than relying on this transportation of information, the situative perspective instead recontextualizes the material and previous discourses to apply them to new problems (Peressini et al., 2004). This situative process makes the content more real to students because they are able to make connections back to the situation in which the material was learned.

What separates a situative perspective from its cognitive counterpart is the idea that mathematics should be taught using worthwhile tasks (Peressini et al., 2004). If these tasks “are rich with mathematical possibility and opportunity” (Peressini et al., 2004, p. 77) students will be hooked into the lesson and will be able to contribute using their own ideas. In this perspective, the goal of the teacher is to facilitate learning by presenting students with tasks which require them to build off of previously encountered situations and discuss their ideas collaboratively. Students are encouraged to come up with multiple methods to solve these problems and usually are required to justify and interpret the discussed ideas (Peressini et al., 2004). This discourse is essential because it allows students to “gain access to the phenomena of mathematics” (Peressini et al., 2004, p. 78) because they are

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active participants and all ideas are encouraged. Mathematics becomes something much more than rote practice; it instead is a negotiated discussion between the teacher and students about concepts and ideas.

Many who enter into mathematics education programs believe, and often relish, the notion that “mathematics means finding correct answers, quickly, using the (one, correct) standard procedure” (Peressini et al., 2004, p. 73). Their understanding is that to be good at mathematics is to be good at mastering these procedures, without any regard to their conceptual basis (Peressini et al., 2004). For example, given a question on functions most future math educators will reply with some version of “a mapping from x to a corresponding y ” (Peressini et al., 2004, p. 75). Although correct, it is lacking the conceptual depth which ties the idea to all of its representations (e.g., equations, graphs, and tables). Without this conceptual understanding, students see functions as three separate ideas: functions as equations, functions as graphs, and functions as tables. Instead, Peressini et al. (2004) have stated that functions are better understood as a development from “‘action’, to ‘process’, and finally to ‘object’” (as cited in Confrey & Doerr, 1996, p. 75). By teaching the concept of a function in context, students can move easily through its various representations and are prepared for more rigorous concepts in later courses, which also have multiple forms.

Engagement

Situated problems are intended to get students more involved in learning and provide them with deeper understandings of the material learned. Beswick (2010) analyzed how effective these real world problems are at engaging students and educating them about mathematical concepts. She argued that if the literature suggests that

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mathematics education should involve authentic tasks, the research should also support this claim. To this end, Beswick (2010) reviewed the findings of various studies regarding contextual problems in mathematics to determine how they enhanced participation, engagement, and achievement.

The major concern over lack of student engagement stems from the lack of participation in mathematics courses and in mathematics related careers (Beswick, 2010). The connection is made between declining participation in mathematics courses beyond what is required and decreasing interest in fields, which require a strong mathematics background (Beswick, 2010). This lack of interest in these fields is often attributed to the lack of interest in mathematics at an early age; if students are not interested in mathematics in school, there is little chance they will choose a mathematical career (Beswick, 2010). In the modern age of technology, there is a growing demand for careers in mathematics, science, and engineering so this decline in interest is even more troublesome. In an effort to increase engagement in mathematics, teachers are attempting to use more innovative tasks, which involve appealing contexts to students (Beswick, 2010). These contextual mathematics problems are used by educators to “enhance achievement, engagement, and participation in mathematics education” (Beswick, 2010, p. 368) to fix the aforementioned issue by getting students excited about mathematics.

Although the contextual problems are intended to provide richer mathematics instruction, this cannot simply be assumed to be true. In her efforts to determine their effectiveness, Beswick (2010) first defined the different types of contextual problems students are presented with. Problems typically defined as ‘word problems’ are considered “not to be authentic because their simplicity means that algorithms are readily available”

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(Beswick, 2010, p. 369). Conversely, situated problems simulate situations that people face in their daily lives because they are “meaningful, purposeful, and goal-directed” (as cited in Jurdak, 2006, p. 248). So a typical ‘word problem’ may be something as simple as ‘If I spend \$5.00 on a shirt and the tax is 8%, how much is my total?’ whereas an authentic problem is much more meaningful. Word problems take previously taught concepts and translate them into simple real life situations whereas authentic problems are more open-ended and cannot be solved by using an already mastered algorithm. A word problem such as the one previously stated does not provide students with a better understanding of the content, since it is nothing more than a simple application of a learned procedure. These types of ‘real-world’ problems are typically characterized by students as “monotonous, meaningless, individualized work” (Beswick, 2010, p. 370) because they do not allow students to make connections and fully engage with the concepts. To teach this same content using an authentic task, the problem would involve much more context and will provide less support. For example, the same content could be represented as:

‘Carol is going clothes shopping with her friends. She brought \$20 and is planning on purchasing new shirts. The original price of a shirt was \$7.99, but they are on sale for 20% off. If there is 8% sales tax, how many shirts would she be able to purchase with the money she brought?’

This problem covers the same content of finding percentages, but incorporates a much more authentic and meaningful example.

The real goal of authentic problems is to develop “flexible mathematical concepts able to be used in whatever context when and as required” (Beswick, 2010, p. 377). If students are given enough freedom with a task, they will be better able to draw upon

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previous knowledge to formulate a solution. The evidence from Freudenthal (1968), Boaler (1993), and Ainley et al. (2006) suggests that traditional teaching of procedures followed by some applications later is ineffective (Beswick, 2010). Instead, the evidence shows that “using mathematical ideas in contexts can facilitate the development of understanding of them” (Beswick, 2010, p. 379). Boaler (1993) examined 100 eighth grade students from two schools who solved the same problem presented in differing contexts and without context to determine if the context supported the conceptual understanding. The results were inconclusive. Beswick (2010) reported that either the contexts made it “more difficult for some students to demonstrate their understanding when the relevant mathematics was already familiar” or the contexts enhanced meaning but did not allow the students to “connect the meanings derived” to their prior knowledge (p. 372). This does not suggest that contexts are ineffective at enhancing instruction, but rather the contexts need to be appropriate for the students due to the “complex and individual nature of learning” (Beswick, 2010, p. 373). Using authentic problems alone is not the solution, students must also “be given a degree of autonomy in determining their approach to tasks” so that they can “render it personally meaningful” (Beswick, 2010, p. 373). In order to be effective learning tools in mathematics instruction, authentic tasks must do more than simply make connections between algorithms and real life, they must require students to develop flexible skills which can be applied to any context (Beswick, 2010).

Problem Solving

The problem solving perspective of mathematics education is closely aligned to that of the situative perspective. Both agree that the notion of traditional education practices which are “based on mastering a corpus of mathematical facts and procedures is severely

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impoverished” (Schoenfeld, 1992, p. 3). Instead, learning mathematics “is an inherently social activity” (Schoenfeld, 1992, p. 3) in which members must participate in active discussions about content to truly grasp its full understanding. This is because “mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us” (as cited in National Research Council, 1989, p. 84). Mathematics must involve “seeking solutions”, “exploring patterns”, and “formulating conjectures” (as cited in National Research Council, 1989, p. 84) since it would be impractical to suggest that all real life problems can be solved using a memorized procedure or formula.

Throughout history, “problems have occupied a central place in the school mathematics curriculum since antiquity, but problem solving has not” (as cited in Stanic and Kilpatrick, 1989, p. 1). Meaning that throughout history students have completed problems in their mathematics classes, but very few have developed the ability to truly problem solve. This is because problems themselves can be two completely different entities: routine exercises or complex situations that are require intense thought and investigation (Schoenfeld, 1992). Routine exercises are organized to “provide practice on a particular mathematical technique that, typically, has just been demonstrated to the student” (Schoenfeld, 1992, p. 11). Such problems limit creative thinking and illustrates to students that problem solving is simply “working the tasks that have been set before you” (Schoenfeld, 1992, p. 13). The argument which aligns with the problem solving perspective is that “real problem solving (that is, working problems of the ‘perplexing’ kind) is the heart of mathematics” (Schoenfeld, 1992, p. 14). This is because “what mathematics *really* consists of is problems and solutions” (as cited in Halmos, 1980, p. 519). By preparing

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students to tackle challenging problems through engagement in “real’ problem solving” (Schoenfeld, 1992, p. 16) teachers can better prepare students for success in academics and in their life experiences.

One of the key aspects of the problem solving perspective is helping students “develop *mathematical power*” (Schoenfeld, 1992, p. 33). To help them develop this power, “classrooms must be communities in which mathematical sense-making, of the kind we hope to have students develop, is practiced” (Schoenfeld, 1992, p. 33). Other goals should include giving students an idea of what mathematics entails and then allowing them to use their knowledge to develop their own conceptual understanding of the content (Schoenfeld, 1992). Rather than present students with a procedure and a list of examples, they should instead be given the opportunity to “explore a broad range of problems” (as cited in Committee on the Teaching of Undergraduate Mathematics of the Mathematical Association of America, forthcoming, p. 2) and develop their own solutions. The instruction should also help students “present their analyses in clear and coherent arguments” and communicate with each other “using the language of mathematics” (as cited in Committee on the Teaching of Undergraduate Mathematics of the Mathematical Association of America, forthcoming, p. 2). If implemented, these goals have the ability to transform a traditional mathematics classroom into a collaborative community of problem solvers who apply their knowledge to find solutions to perplexing problems.

Dating back to Plato was the idea that “those who are good at mathematics tend to be good thinkers” (Schoenfeld, 1992, p. 35) and that students can be trained so that they are good thinkers. The connection is further developed to imply that “problem solving programs ... produced problem solving *behavior*” (Schoenfeld, 1992, p. 37). This view

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contrasts directly with the common interpretation that the “bottom line’ for most mathematics educators is to have students develop an understanding of the procedures and their meanings” (Schoenfeld, 1992, p. 40). There is an apparent discrepancy between how students are educated in mathematics classrooms and how mathematicians “employ a wide range of strategies when confronted with problems beyond the routine” (Schoenfeld, 1992, p. 51). A reliance on rote practice of procedures “may produce surface manifestations of competent behavior” but if it is not grounded in conceptual understanding, the performance may “be error-prone and easily forgotten” (Schoenfeld, 1992, p. 51). Mathematics teachers may inaccurately believe that their students have mastered the content just because they are able to complete numerous examples shortly after memorizing the procedure. However, “virtually everything humans see, hear, or even think about is critically dependent on information stored in long-term memory” (Van Merriënboer & Sweller, 2005, p. 153). And the only way that “humans alter the contents of long-term memory [is] by learning” (Van Merriënboer & Sweller, 2005, p. 154). Learning has been achieved when a child is able to recall the content months after the lesson from their long-term memory.

Although most mathematics educators would agree that problem solving is essential in teaching and learning, there is still widespread misunderstanding of what mathematical problem solving entails. Many textbooks include problem solving as additional or supportive activities that are “sprinkled through the text as rewards or recreations” (Schoenfeld, 1992, p. 56). These problems are interpreted by students and educators as separate from the “real business of doing mathematics” (Schoenfeld, 1992, p. 56) and are just fun little breaks from the numerous practice problems. In other textbooks these

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“problem solving” sections are just lists of even more skill-based questions to drill students on the recently learned material (Schoenfeld, 1992). Neither of these examples align with how Schoenfeld (1992) referred to as true problem solving which he defined as, “learning to grapple with new and unfamiliar tasks, when the relevant solution methods... are not known” (p. 56).

Teaching problem solving is harder for the teacher than traditional process-driven instruction (Schoenfeld, 1992). Mathematically speaking, a teacher has to consider many more approaches and solutions in a problem solving lesson than in a traditional sense. They have to be able to interpret a wide array of suggestions from students and know how to provide feedback when necessary to further their group discussions. And possibly the biggest challenge for teachers is sometimes not knowing all of the answers to the questions students pose in their discussions (Schoenfeld, 1992). Many educators are much more comfortable teaching in the traditional sense largely because in traditional problems, there is typically only a single solution. Although teaching mathematics through true problem solving is more demanding on the teacher, it can also be “far more rewarding, when achieved, than the pale imitations” (Schoenfeld, 1992, p. 57) used in traditional settings.

Teaching mathematics using nontraditional methods that focus on developing problem solvers is a worthwhile but challenging task. Using true problem solving to teach mathematical concepts in contextual situations has been shown to improve student understanding and deepen connections between related concepts. But, it is not always easy to create the situated environments for each mathematical topic. However, in spite of these challenges, true mathematical problem solving in context is worth the extra effort because of how much it can help all students think critically in real life.

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In alignment with both the situative perspective and real problem solving, as defined by Schoenfeld (1992 & 2013) the purpose of this curriculum is to answer the question: *How can real-life contextual tasks be utilized to teach arithmetic sequences so that teachers are better prepared to teach the Common Core curriculum?*

Chapter 3: Methods

This chapter consists of the Algebra I curriculum documents used to teach arithmetic sequences. The design of the lessons is based on the formatting of those from the Connected Mathematics Project 3 (CMP3) (Lappan, 2014). The Connected Mathematics Project is a “problem-centered curriculum promoting an inquiry-based teaching-learning classroom environment” (Connected Mathematics Project, n. d.). Each lesson follows the structure of launch, explore, and summarize to teach the content using an inquiry-based collaborative learning process (Lappan, 2014).

Lesson Design

The first main part of each lesson is called the launch phase. During this phase, the teacher introduces the problem and ties in the students’ background knowledge. This is sometimes started using a video or brief activity to get the students engaged in the topic. The teacher then discusses the opening of the problem with the class to help them understand the context of the situation. If necessary, the teacher defines new vocabulary words or phrases that will be needed in the lesson. Students are also often asked broad questions at this stage in order to engage them in the problem solving process.

After the problem has been introduced, students begin the exploration stage of the lesson. During this phase, students must work with their group members to strategize possible methods of solving the problem. Typically, each problem consists of many

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questions for the students to answer to help scaffold the lesson and guide them to learning the content. During this stage, it is typical for the teacher to circulate among small groups and facilitate discussions through the use of questioning. The teacher guides students by asking them to compare their ideas within their group and provide justification for all strategies. The teacher does not provide students with any immediate solutions or procedures during this portion of the lesson. Students must instead rely on their group members in order to determine the solutions to the problems.

The summary phase of the lesson occurs immediately after the exploration stage so that students can compare their ideas as a whole class. This stage may occur at the end of a lesson or it may alternate between the exploration and summary if a lesson requires more breaks between sections. During this stage the teacher relies on students to provide solutions to the most important parts of the exploration so that the class can formulate conclusions based on their strategies. Students are often encouraged to present their methods in front of the class so that all methods are encouraged and are able to be evaluated. Teachers facilitate this stage by asking for justifications from students in order to form conclusions as a class. Also, they may present other strategies in order to help show more concise methods that students could use or common errors that need to be corrected.

Proposed Curriculum

This curriculum is written using five lessons and a summative assessment. Formative assessments will be used within each lesson to determine student progress as well. Each lesson is intended to be used in one sixty-minute class period, but may be

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stretched across multiple class periods if the teacher needs more time to develop the concepts. The curriculum is written as follows:

1. Lesson 1: Connection to Linearity
2. Lesson 2: Explicit Formulas
3. Lesson 3: Recursive Formulas
4. Lesson 4: Explicit and Recursive Using Various Syntax
5. Lesson 5: Applying the Various Syntax
6. Summative Assessment

Answer keys to these materials are provided in the Appendix.

Lesson 1: Connection to Linearity

60 minutes

Goals	
Focus Question: How is an arithmetic sequence similar to a linear relationship? How is it different?	
Common Core Standard(s): <ul style="list-style-type: none"> ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. 	
Objectives: <i>Students will be able to:</i> <ul style="list-style-type: none"> ▪ Work with, create, and make connections between various forms of a linear relationship (tables, graphs, equations) 	<i>Students will understand that:</i> <ul style="list-style-type: none"> ▪ An arithmetic sequence is a linear relationship with a restricted domain
Assessment: <ul style="list-style-type: none"> ▪ Group discussions about the task ▪ Extensions/Homework section 	
Design	
Launch: <ul style="list-style-type: none"> ▪ Engage students in a discussion about their previous knowledge of linear relationships <ul style="list-style-type: none"> ○ $y = mx + b$ ○ Constant rate of change ○ Graphs of lines with a y-intercept and a constant slope 	
Explore: <ul style="list-style-type: none"> ▪ Allow them to work in their collaborative groups to complete this lesson ▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions: <ul style="list-style-type: none"> ○ <i>How does George's pay relationship relate to something you have seen before?</i> ○ <i>What are the key parts of George and Michelle's pay?</i> ○ <i>How can we see these key features of their pay in the various representations?</i> ○ <i>How are George and Michelle's pay relationships similar? Different?</i> 	
Summarize: <ul style="list-style-type: none"> ▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task. ▪ Discuss their thoughts on part C as a class. Tie this section back to the similarities and differences between the two pay relationships shown in previous parts of the problem. 	

Lesson 1: Connection to Linearity

- A.** George works at an ice cream stand during the summer. He gets paid an initial amount each day and then an hourly rate. The table below represents his daily pay, G , after h hours of work.



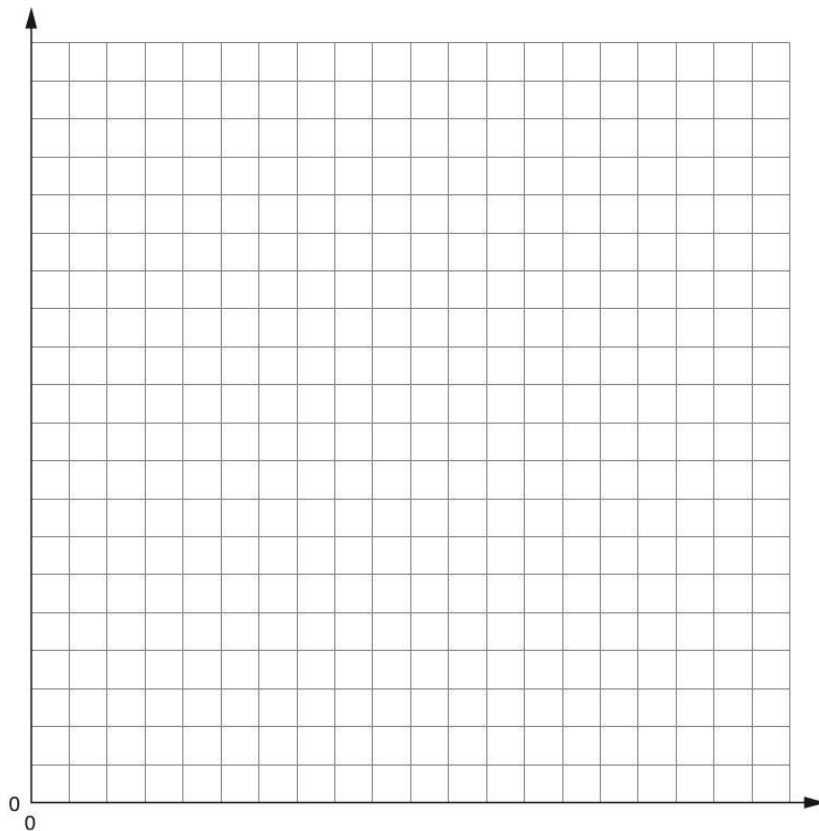
Hours of work	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	23	31	39		55			

- Complete the table above to continue the pattern of (h, G) coordinates.
- Describe the relationship between the number of hours George works, h , and the amount of money he earns each day, G . Include the values for the initial pay and hourly pay in your response.
- Write an equation to represent George's daily pay, G , after h hours of work.
 - Explain how your equation in **(a)** represents the relationship you described in question 2.
 - Determine the amount of money George would earn if he worked a 12-hour shift.

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4.

a. Graph George's daily income relationship on the grid below.



b. Explain how your graph in represents the relationship you described in question 2.

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

d. Describe what the coordinate point $(1.5, 27)$ means in this situation.

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- B.** Michelle works at a lemonade stand by the beach. She is paid an initial amount each day and then earns commission from each drink that she sells. The table below represents her daily pay, M , after n lemonades are sold.



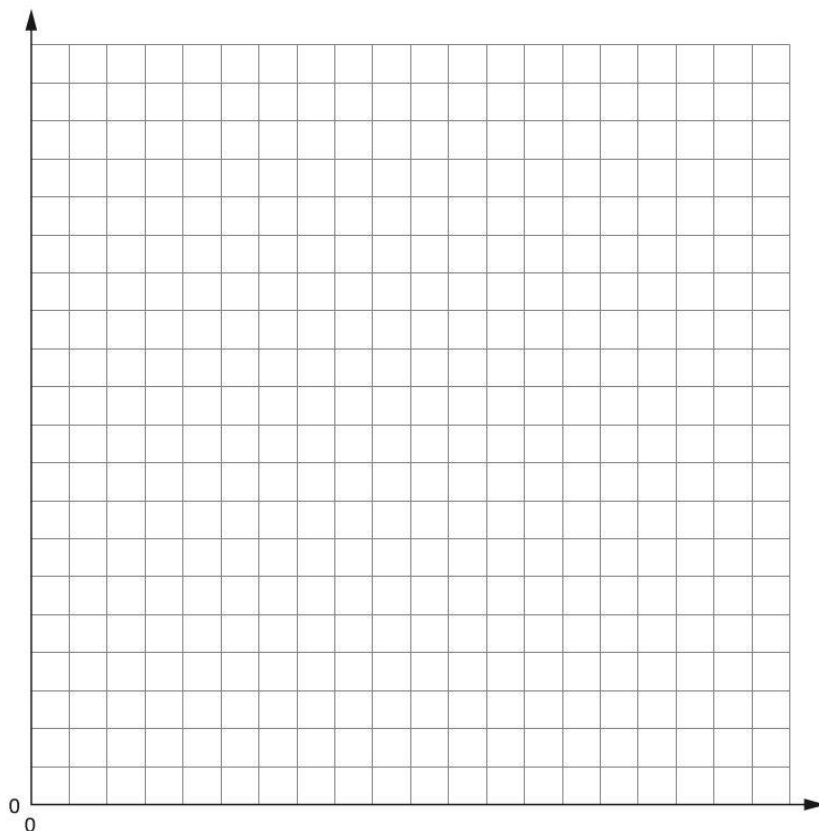
Lemonades Sold	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	15.5	18	20.5	23				

- Complete the table above to continue the pattern of (n, M) coordinates.
- Describe the relationship between the number of lemonades Michelle sells, n , and the amount of money she earns each day, M . Include the values for the initial pay and amount earned per lemonade sold in your response.
 - Write an equation to represent Michelle's daily pay, M , after n lemonades are sold.
 - Explain how your equation in **(a)** represents the relationship you described in question **2**.
 - Compare Michelle's daily earnings to George's. What similarities exist and how are the two situations different?

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4.

a. Graph Michelle's daily income relationship on the grid below.



b. Explain how your graph in represents the relationship you described in question 2.

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

d. Describe what the coordinate point $(1.5, 16.75)$ would mean in this situation. Is this realistic?

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- C.** Both George's and Michelle's pay are examples of linear relationships. Michelle's is also an example of an arithmetic sequence. Based on your work in parts **A** and **B**, what do you think makes a linear situation an arithmetic sequence? Why is George's pay not an example of an arithmetic sequence?

Arithmetic Sequences**Extensions/Homework**

1. Jane works at a nail salon and is paid \$12 an hour. Would her pay represent an arithmetic sequence? Explain.
2. Write an equation for the relationship in problem 1. Compare this relationship to George's pay relationship in part **A**.
3. Would the relationship shown in the table below represent an arithmetic sequence?

Hours worked	1	2	3	4	5
Pay (\$)	2	4	8	16	32

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Lesson 2: Explicit Formulas

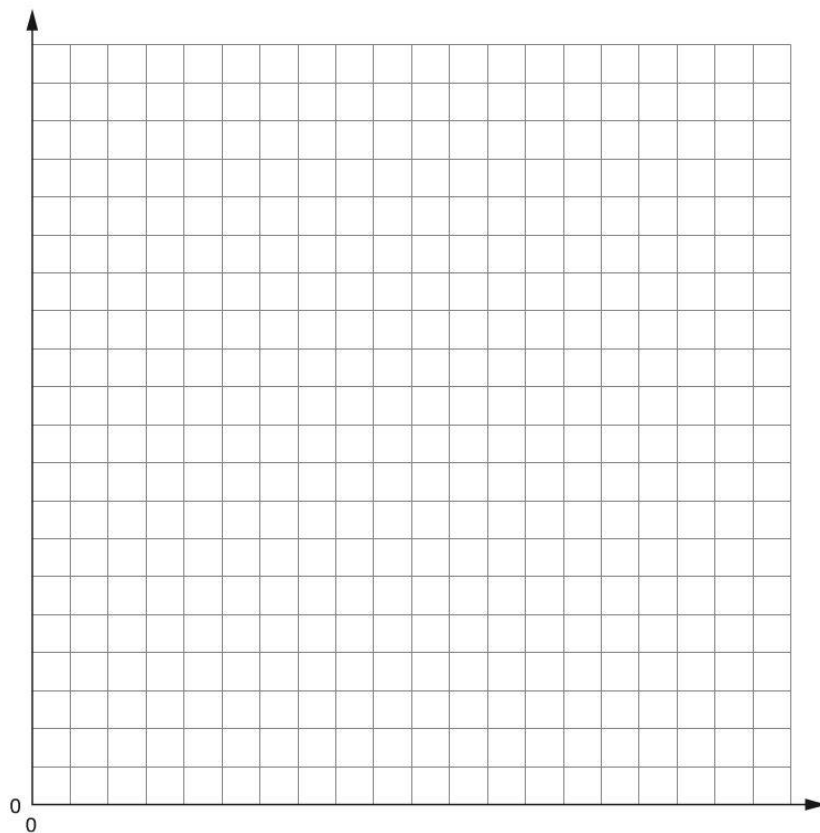
60 minutes

Goals	
Focus Question: How can we apply our knowledge of linearity to write explicit formulas for arithmetic sequences?	
Common Core Standard(s):	
<ul style="list-style-type: none"> ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. 	
Objectives: <i>Students will be able to:</i>	<i>Students will understand that:</i>
<ul style="list-style-type: none"> ▪ Work with, create, and make connections between various forms of a linear relationship (tables, graphs, equations) ▪ Write explicit formulas of the form $y = mx + b$ and $a_n = a_1 + (n - 1)d$ 	<ul style="list-style-type: none"> ▪ An arithmetic sequence is a linear relationship with a restricted domain ▪ An arithmetic sequence can be written using various equivalent forms
Assessment:	
<ul style="list-style-type: none"> ▪ Group discussions about the task ▪ Extensions/Homework section 	
Design	
Launch:	
<ul style="list-style-type: none"> ▪ Engage students in a discussion about the previous lesson’s introduction of arithmetic sequences (Lesson 1 part C). <ul style="list-style-type: none"> ○ <i>How were arithmetic sequences similar to linear relationships? How were they different?</i> ○ <i>How did we see these similarities and differences in the various representations of George and Michelle’s pay?</i> 	
Explore:	
<ul style="list-style-type: none"> ▪ Allow them to work in their collaborative groups to complete this lesson ▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions: <ul style="list-style-type: none"> ○ <i>How does your equation relate to the linear equation $y = mx + b$?</i> ○ <i>What connections can you make between the various equations?</i> ○ <i>Where do we see the y-intercept and slope in each form of the equation?</i> ○ <i>Is the y-intercept shown in each representation? Why or why not?</i> 	
Summarize:	
<ul style="list-style-type: none"> ▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task. ▪ Summarize with C and D as a class to emphasize the connections between the forms. 	

ARITHMETIC SEQUENCES USING SITUATED TASKS

4.

a. Graph Michael's daily income relationship on the grid below.



b. Explain how your graph in represents your equation from part 2a.

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

ARITHMETIC SEQUENCES USING SITUATED TASKS

B. Any arithmetic sequence can be written as an equation in the form

$$a_n = a_1 + (n - 1)d$$

This is often how reference sheets display the generic equation. The variables are defined as follows:

- a_n is the n^{th} term of the sequence. For example, a_6 is the 6th term.
 - n represents the number of the current term. For a_6 , $n = 6$.
 - a_1 is the term when $n = 1$ which is usually the first term of the sequence.
 - d is the common difference. It represents how much the sequence changes from one term to the next.
1. Suppose we apply this new equation format to Michael's daily pay relationship from part **A**. What would a_6 represent in this case?

 2. Michael says that a_1 represents the y -intercept. Whereas Michelle says that a_0 represents the y -intercept. Who do you agree with? Explain.

 3. What is the value of d for Michael's pay situation? What is another name we typically use for this value?

 4. Write an equation using the new format to represent Michael's daily pay.

 5. Simplify your equation from part **4**. How does this equation relate to the one you wrote in part **2a**?

ARITHMETIC SEQUENCES USING SITUATED TASKS

C. Arithmetic sequences can be written as equations using the traditional linear format of $y = mx + b$ or the new format $a_n = a_1 + (n - 1)d$.

1. Which variable in the linear equation corresponds to the a_n variable from the new format?
2. Which variable in the new format corresponds to the m variable from the linear format?
3. Is there a variable in the new format that corresponds to the variable b from the linear format? If not, how could we write it using the new notation?

D. For each sequence shown below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n - 1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

1. y -intercept of 6, slope of -3 .

2. $a_1 = \frac{1}{2}, d = 2\frac{1}{2}$

- 3.

x	1	2	3	4	5
y	14	9	4	-1	-6

E. For each sequence from part **D**, determine the value of a_{10} .

Extensions/Homework

1. Use the equation of an arithmetic sequence show below to answer the following questions.

$$a_n = 6 + (n - 1)(4)$$

- a. What is the first term, a_1 ? What is the common difference, d ?
- b. Simplify the equation completely.
- c. What is the value of the y -intercept of this relationship?
- d. What is the value of a_4 ?

For # below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n - 1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

2. y -intercept of 2, slope of 6.

3. $a_0 = 3, d = -2$

4.

x	1	2	3	4	5
y	7	11	15	19	23

Lesson 3: Recursive Formulas

60 minutes

Goals	
Focus Question: How can we apply our knowledge of linearity to write recursive formulas for arithmetic sequences?	
Common Core Standard(s): <ul style="list-style-type: none"> ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. 	
Objectives: <i>Students will be able to:</i> <ul style="list-style-type: none"> ▪ Use and write <i>NOW – NEXT</i> statements to illustrate a sequence ▪ Write recursive formulas of the form $a_n =$ or $a_{n+1} =$ for a given arithmetic sequence ▪ Use recursive formulas to find values of a sequence 	<i>Students will understand that:</i> <ul style="list-style-type: none"> ▪ A <i>NOW – NEXT</i> statement shows the pattern of change between two consecutive terms of a sequence ▪ Recursive formulas can be written using various syntax but all are of the <i>NOW – NEXT</i> form
Assessment: <ul style="list-style-type: none"> ▪ Group discussions about the task ▪ Extensions/Homework section 	
Design	
Launch: <ul style="list-style-type: none"> ▪ Engage students in a discussion about the previous lesson’s connection between linear formulas $y = mx + b$ and the formula for an arithmetic sequence $a_n = a_1 + (n - 1)d$. <ul style="list-style-type: none"> ○ <i>What connections did we make between the two explicit formulas?</i> ○ <i>How were the two formulas different from each other?</i> 	
Explore: <ul style="list-style-type: none"> ▪ Allow them to work in their collaborative groups to complete this lesson ▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions: <ul style="list-style-type: none"> ○ <i>How would the <i>NOW – NEXT</i> change if Michelle made \$3 per drink?</i> ○ <i>How does the <i>NOW – NEXT</i> form relate to the $a_{n+1} = a_n + d$ form?</i> ○ <i>How can we use recursive formulas to find sequence values?</i> ○ <i>When is a recursive formula more useful? When is an explicit more useful?</i> 	
Summarize: <ul style="list-style-type: none"> ▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task. ▪ Summarize with C and D as a class to emphasize the strengths of each type of formula and more examples of writing recursive formulas. 	

Lesson 3: Recursive Formulas

In the previous two lessons we discussed how both Michelle's and Michael's daily income situations were examples of arithmetic sequences. We used the general linear equation $y = mx + b$ and the arithmetic sequence equation $a_n = a_1 + (n - 1)d$ to represent these relationships.

Michelle's Daily Pay

Lemonades Sold	1	2	3	4
Daily Pay (in dollars)	15.5	18	20.5	23

Michael's Daily Pay

- Initial daily pay of \$20
- \$1 earned for each iced tea sold

- A.** We can use *NOW* – *NEXT* statements to illustrate how to create a sequence of values **recursively**. The *NOW* variable represents the current term and the *NEXT* variable represents the very next term. An example is shown below.

Sequence: 4, 7, 10, 13, ...

$$NEXT = NOW + 3$$

Starting at: 4

This example says to start at a value of 4 and then add 3 to the current term in order to get the next term.

1. Write a *NOW* – *NEXT* statement for both Michelle and Michael's daily pay. Use the *y*-intercept (for 0 drinks sold) as the starting value for each.
2. Which number in each of your equations would be the slope, m , if the equation was in the form $y = mx + b$?
3. Which number in each of your equations would be the *y*-intercept, b , if the equation was in the form $y = mx + b$?

ARITHMETIC SEQUENCES USING SITUATED TASKS

- B.** Another similar method of writing these equations recursively uses different notation to represent the same situation. An example of this notation is shown below.

Sequence: 4, 7, 10, 13, ...

$$\begin{aligned}a_{n+1} &= a_n + 3 \text{ for } n \geq 1 \\ a_1 &= 4\end{aligned}$$

1. Match up each part of this new formula with the previous one.
 - a. *NOW* = _____
 - b. *NEXT* = _____
 - c. Starting at = _____
2. Why does this formula need to include the $n \geq 1$ part? Would you be able to create the sequence without this? Explain.
3. Rewrite your *NOW* – *NEXT* statements from **A1** using this new format.
4. What did you label your starting values in part **3** as? Explain your decision.
5. Using your formula for Michelle's daily pay from part **4**, list the steps needed to find the value of a_4 .

ARITHMETIC SEQUENCES USING SITUATED TASKS

C. Using your work from this lesson and previous lessons, answer the following questions.

1. If you had to find the value of a_{20} , which formula would you use? Explain.

Option 1

$$\begin{aligned} a_{n+1} &= a_n + 3 \text{ for } n \geq 1 \\ a_1 &= 5 \end{aligned}$$

Option 2

$$a_n = 3n + 2$$

2. Was your choice in part **1** an explicit or a recursive relationship?
3. When is an explicit formula for an arithmetic sequence (in the form $y = mx + b$ or $a_n = a_1 + (n - 1)d$) more helpful than a recursive one?
4. When is a recursive formula more useful than an explicit one?

D. Write an explicit and a recursive equation for each situation described below.

1. Joan makes \$10 per day plus an additional \$1.30 for each basket she sells.
2. A stack of boxes at a grocery store consists of 22 boxes on first row on the bottom, 18 in the next row up, and so on forming an arithmetic sequence until the top has 2 boxes.

Extensions/Homework

1. At a large grocery store they are having a promotion to give away free samples of pasta sauce. On the first day, they start with 200 cans of sauce. They plan on giving away 12 cans of sauce per day.



- a. Write a recursive formula to represent this situation.

- b. Write an explicit formula to represent this situation.

- c. Find how many cans they will have left after 8 days.

- d. Determine on which day of the promotion they will run out of sauce.

Write an explicit and a recursive equation for each situation described below.

2. To help study for an upcoming test, Julia decides to study each night more than she studied the previous night. She plans on studying for 10 minutes the first night and will increase by 5 minutes each night.

3. James is paying off his student loans every month. In the first month, his loans start at a value of \$30,000. He pays them off by writing checks for \$250 each month.

Lesson 4: Explicit and Recursive Using Various Syntax

60 minutes

Goals	
Focus Question: How can we write explicit and recursive formulas for arithmetic sequences using a variety of syntax?	
Common Core Standard(s):	
<ul style="list-style-type: none"> ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. 	
Objectives: <i>Students will be able to:</i> <ul style="list-style-type: none"> ▪ Write explicit and recursive formulas using a variety of syntax and variables ▪ Use explicit and/or recursive formulas to find values of a sequence ▪ Make connections between different formulas for the same relationship 	<i>Students will understand that:</i> <ul style="list-style-type: none"> ▪ The variables and syntax used in defining arithmetic sequences in interchangeable ▪ Function notation can be used to rewrite sequence equations just like how $f(x)$ is interchangeable with the variable y in $y = mx + b$
Assessment:	
<ul style="list-style-type: none"> ▪ Group discussions about the task ▪ Extensions/Homework section 	
Design	
Launch:	
<ul style="list-style-type: none"> ▪ Engage students in a discussion about how to write recursive and explicit formulas for a given sequence of values. <ul style="list-style-type: none"> ○ <i>How would we write a recursive formula for 3, 8, 13, 18, ...? An explicit one?</i> ○ <i>What are the key features of a sequence needed to write any formula for it?</i> 	
Explore:	
<ul style="list-style-type: none"> ▪ Allow them to work in their collaborative groups to complete this lesson ▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions: <ul style="list-style-type: none"> ○ <i>How can we determine if two formulas represent the same sequence?</i> ○ <i>How does function notation change our previous formulas?</i> ○ <i>Which notation makes sense in each situation? Is there one perfect style for all?</i> 	
Summarize:	
<ul style="list-style-type: none"> ▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task. ▪ Summarize with D as a class to discuss how different syntax can represent the same relationship. 	

Lesson 4: Explicit and Recursive Using Various Syntax

Just like how we can use the variables M and t in place of y and x in $y = mx + b$, we can also use different variables in place of a_n and n in $a_n = a_1 + (n - 1)d$.

- A.** Josh has a job selling tie-dye T-shirts at the beach. His pay is represented by the equation show below:

$$a_n = 8 + (n - 1)(3)$$

1. Simplify this equation completely.
2. Rewrite the equation using different variables that better align with the context. Briefly explain why you chose your variables.
3. Could this equation represent this situation? Explain.



$$J_t = 3t + 5$$

- B.** Function notation is also often used to represent a linear relationship. The form $f(x) = mx + b$ can be used in place of $y = mx + b$.

1. Rewrite your equation from **A2** using function notation.
2. Explain how a_n and $f(n)$ notations are alike.

ARITHMETIC SEQUENCES USING SITUATED TASKS

- C.** Recursive sequences can also be written using various forms. Two recursive formulas for Josh's T-shirt income are shown below.

(a)

$$\begin{aligned} J_{n+1} &= J_n + 3 \text{ for } n \geq 0 \\ J_0 &= 5 \end{aligned}$$

(b)

$$\begin{aligned} J_n &= J_{n-1} + 3 \text{ for } n \geq 1 \\ J_0 &= 5 \end{aligned}$$

1. Compare these formulas. Are they both accurate ways of showing Josh's income relationship?
2. Using both formulas, generate the first five terms of each sequence. Does this support or change your answer from part 1?
3. Rewrite the equations using *NEXT* – *NOW* statements.
4.
 - a. If J_n is the *NOW*, what is the *NEXT*?
 - b. If J_n is the *NEXT*, what is the *NOW*?
5. Rewrite formula **(a)** using function notation like $J(n + 1)$.

ARITHMETIC SEQUENCES USING SITUATED TASKS

- D.** Nicole works for the same T-shirt company as Josh and has to put up posters around town to advertise for an upcoming sale. She starts the day with 240 posters and is able to put up 3 every minute.
1. Write an explicit equation for this sequence using p_m as the number of posters left after m minutes.
 2. Rewrite your equation from part 1 using function notation.
 3. Write a recursive formula for this sequence using p_m as the number of posters left after m minutes.
 4. Rewrite your recursive formula in a different but equivalent way. Use part C as an example.
 5. Rewrite your formula from part 3 using function notation.
 6. Rewrite your formula from part 4 using function notation.
 7. Out of all equations written in parts 1-6, which is the easiest one for you to understand. Justify your choice.

Extensions/Homework

1. Wood is often stacked like the picture shown at right with the bottom layer having the most and the top layer having the least. Suppose the first layer on the bottom has 48 logs. Each additional layer has 4 less logs than the layer below it.



- a. Write an explicit equation for this relationship. Explain why you chose the variables you did.
- b. If your answer in part a is not in function notation, rewrite it so that it is. Otherwise, rewrite it so that the equation is in a_n notation.
- c. Write a recursive formula for this situation.
- d. Write a different, yet equivalent, recursive formula for this situation.
- e. Using any of your formulas, find the number of logs in the first 6 layers.

Lesson 5: Applying the Various Syntax

60 minutes

Goals	
Focus Question: How can we write, interpret, and utilize explicit and recursive formulas for arithmetic sequences using a variety of syntax?	
Common Core Standard(s): <ul style="list-style-type: none"> ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. 	
Objectives: <i>Students will be able to:</i> <ul style="list-style-type: none"> ▪ Rewrite explicit and recursive formulas using a variety of syntax and variables ▪ Interpret given formulas and rewrite them in equivalent forms ▪ Use explicit and/or recursive formulas to find values of a sequence ▪ Make connections between different formulas for the same relationship 	<i>Students will understand that:</i> <ul style="list-style-type: none"> ▪ The variables and syntax used in defining arithmetic sequences in interchangeable ▪ Due to the context, some sequences need to start with a 0th term while others need to start with a 1st term
Assessment: <ul style="list-style-type: none"> ▪ Group discussions about the task ▪ Extensions/Homework section 	
Design	
Launch: <ul style="list-style-type: none"> ▪ Engage students in a discussion about how to rewrite a given sequence formula in as many different ways as possible. <ul style="list-style-type: none"> ○ <i>How many formulas can we write for the sequence 2, –3, –8, ...?</i> ○ <i>Which formula makes the most sense to you? Why?</i> 	
Explore: <ul style="list-style-type: none"> ▪ Allow them to work in their collaborative groups to complete this lesson ▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions: <ul style="list-style-type: none"> ○ <i>How can we think of this recursive relationship as a NOW – NEXT?</i> ○ <i>Should this sequence start with a 0th term or a 1st term? Why does it matter?</i> ○ <i>Is there a better way to write this formula? What makes that way more useful?</i> 	
Summarize: <ul style="list-style-type: none"> ▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task. ▪ Summarize with parts of C and all of D as a class to discuss the different equation forms and the difference between a 0th first term and a 1st first term. 	

Lesson 5: Applying the Various Syntax

- A.** Jeff creates a plan to raise money to clean up the beach. He receives an initial donation and then receives a donation for each T-shirt sold at Josh's T-shirt store. The store offers Jeff a few options to choose from.

For each function below, define what each variable represents. Also, determine how much the initial donation and how much the donation per T-shirt would be.

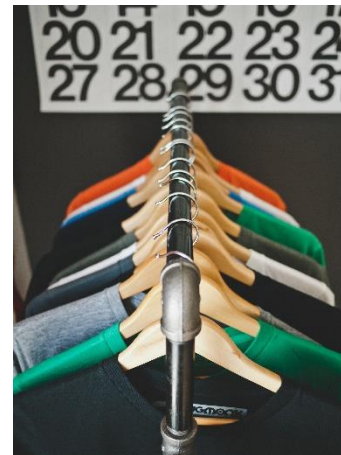
1. $f(0) = 40$
 $f(x + 1) = f(x) + 2$ for $x \geq 0$

2. $d_t = 30 + 2.25t$

3. $x_n = x_{n-1} + 2.75$
 $x_0 = 25$

4. $g(y) = 35 + 2.1y$

5. For this context, is the first term of the sequence the 0th term or the 1st term? Explain.



ARITHMETIC SEQUENCES USING SITUATED TASKS

- B.** Upon hearing about Jeff's fundraiser, the town decides to help with cleanup on the first day. As a result, the group was able to remove 15 barrels of garbage on the first day. After that, the remaining cleaners were able to remove 4 barrels of garbage a day.

1. Complete the table below to represent this situation.

Day	1	2	3	4	5	6
Barrels of Garbage Removed						

2. One of Jeff's workers, Molly, determines that a recursive formula could represent this relationship. Describe what her formula represents.

$$G(d + 1) = G(d) + 4 \text{ for } d \geq 0$$

$$G(0) = 15$$

3. Does her equation accurately depict this situation? If yes, explain. If no, fix her formula so it is accurate.
4. Write an explicit formula using g_d notation for this relationship.
5. Rewrite your formula from part 4 using function, $f(x)$, notation.
6. For this context, is the first term of the sequence the 0th term or the 1st term? Explain.

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C. For each arithmetic sequence defined below, do the following:

- State the first term and the common difference
- Write the first five terms of the sequence
- State whether the formula is explicit or recursive.
- If the formula is explicit, write a recursive formula. If it is recursive, write an explicit equation.
- Write the relationship as a *NEXT – NOW* statement.

1. $f(1) = 12$
 $f(x) = f(x - 1) + 4$ for $x \geq 2$

2. $b_i = 4 + (n - 1)(-2)$

3. $x_{n+1} = x_n - 2$ for $n \geq 1$
 $x_1 = -5$

ARITHMETIC SEQUENCES USING SITUATED TASKS

- D.** Throughout these problems you have seen some sequences that start with the first term as a_1 and others that start with a_0 . Describe the differences between these two types of sequences. Use examples to show how each one may be used.

Extensions/Homework

Jane decides to do her own fundraiser to help with the beach cleanup. Her results from the cleanup form an arithmetic sequence and are shown in the table.

Day	1	2	3	4	5	6
Barrels of Garbage Removed	10	16	22	28	34	40

1. What is the initial value of the sequence? What is the common difference?
2. Write an explicit equation for this relationship. Explain why you chose the variables you did.
3. Josh defines Jane's sequence recursively as shown below. Does his sequence accurately create her results? If yes, explain. If not, what should he fix?

$$j_{n+1} = j_n + 10 \text{ for } n \geq 1$$

$$j_1 = 6$$

Summative Assessment

Name: _____

Multiple Choice #1-5

1. What is the common difference of the sequence 5, 2, -1, -4, ... ?

(1) 3

(2) -3

(3) 5

(4) -5

2. Which formula accurately represents the sequence from question 1?

(1) $a_{n+1} = a_n + 3$ for $n \geq 1$
 $a_1 = 5$

(2) $a_{n+1} = a_n - 3$ for $n \geq 1$
 $a_1 = 5$

(3) $a_{n+1} = a_n + 5$ for $n \geq 1$
 $a_1 = -3$

(4) $a_{n+1} = a_n + 5$ for $n \geq 1$
 $a_1 = 3$

3. What is the fifth term of the sequence described below?

$$f(n) = f(n - 1) + 4 \text{ for } n \geq 2$$

$$f(1) = -3$$

(1) 13

(2) 5

(3) 1

(4) 17

4. Which equation would produce the same sequence as $a_n = 2 + (n - 1)(-3)$?

(1) $y = 2x + 3$

(2) $y = -3x + 2$

(3) $y = -3x - 1$

(4) $y = -3x + 5$

5. Which explicit equation would represent the sequence defined below?

$$x_{n+1} = x_n + 3 \text{ for } n \geq 1$$

$$x_1 = 7$$

(1) $x_n = 3n + 7$

(2) $x_n = 3n + 4$

(3) $x_n = 7n + 3$

(4) $x_n = 7n - 4$

ARITHMETIC SEQUENCES USING SITUATED TASKS

7. Josie is collecting donations for a walkathon. Her donors pledge a certain amount up front and then more for each mile she walks. Her donors provide her with the equations below to represent their pledges. For each equation, determine the initial donation, amount pledged per mile, and first five amounts for walking 1-5 miles.

a. $f(x) = 4x - 2$

Initial Donation	Pledge Per Mile	First Five Terms					
		Mile	1	2	3	4	5
		Donation Total					

b. $b_{n+1} = b_n + 3$ for $n \geq 1$
 $b_1 = 5$

Initial Donation	Pledge Per Mile	First Five Terms					
		Mile	1	2	3	4	5
		Donation Total					

c. $a_n = 4 + (n - 1)(3)$

Initial Donation	Pledge Per Mile	First Five Terms					
		Mile	1	2	3	4	5
		Donation Total					

- d. Are any of the plans from a-c unrealistic? Explain.

ARITHMETIC SEQUENCES USING SITUATED TASKS

8. If Y_n represents the current term of a sequence, what notation would be used to represent the **next** term?
9. If $f(n)$ represents the current term of a sequence, what notation would be used to represent the **previous** term?
10. Robert is planning on running a marathon of about 42 kilometers. To build up his endurance, he plans on running 1.5 km the first day and then running an additional 0.75 km each day.
- Write an explicit formula to represent the distance he runs each day D_n after n days.
 - Will Robert meet his goal of 42 km after a month (30 days)? If not, how close will he be?
 - On what day will Robert reach a distance of 42 km?

Chapter 4: Validity

This unit plan was reviewed by a veteran mathematics teacher who has experience teaching Algebra I. Her knowledge of using inquiry-based methods of instruction validates the curriculum's situated lessons. The feedback was used to modify the lessons so that they could be better implemented to teach arithmetic sequences. The feedback is paraphrased as follows:

- Lesson 1 acts as a way to connect previous knowledge of linear relationships to arithmetic sequences. I like how example #1 was not a sequence and had the students think about why whereas example #2 was a sequence. In part B #4, doesn't d contradict c? If you're saying that you can't order fractional amounts of lemonade, I'm wondering if a whole number of cups should be used for d.
- Lesson 2 – I like how questions 4 and 5 of part B make the connection between the arithmetic sequence form and the slope-intercept form. I loved lesson 2 part D.
- Lesson 4 - The most challenging thing is understanding the math language. Function notation vs sequence notation vs slope intercept form. I do think the best way to learn the math languages is to write a single context in the different ways and then explain what each parts means (which was done in the lesson very nicely). I think it should be emphasized in the teaching of this unit how you substitute in values. For example- a sub n could be substituted in for n to get the sixth term. so we have a sub 6 which then could be replaced entirely with the value of the 6th term.
- Lesson 5 - I liked how you took one context and had a student interpret the math language of the equations. I also thought you differentiated nicely between when we

ARITHMETIC SEQUENCES USING SITUATED TASKS

have a sub 0 vs a sub 1. I also really enjoyed part c as a way to really solidify the understanding of the different syntax and they are actually equivalent.

Overall, the teacher said that the lessons aligned well to the situative perspective and the contexts used were appropriate and meaningful. The primary content goals of this curriculum were to make the connection between linear relationships and arithmetic sequences and to incorporate various syntax to define sequences. These goals were both met by having the students problem solve within contexts. These real life situations allow for students to relate the similarities and differences of linear and arithmetic back to hourly pay and income per lemonade sold. Also, the teacher agreed that the contexts used would help students see the relationships between the different variables and syntax used to write formulas for arithmetic sequences.

In the teacher's feedback she included some suggestions to improve the lessons. One issue she mentioned was regarding Lesson 1 part B #4d. She asked why the question asked about the coordinate point (1.5, 16.75) when in the context, you cannot purchase 1 and a half lemonades. The goal of the question was to highlight this exact issue. If the students connected the points on the graph, hopefully 4d would help them realize that the context only allows for whole number input values. To help with this, the question "Is this realistic?" was added to make sure they consider the situation of the problem. Another recommendation she made was to incorporate more work with substitution of input values into the sequence formulas. This is an important skill that needed to be included throughout the entire unit. To account for this, lessons were adjusted to incorporate more evaluations of formulas at a given domain value. These changes included: adding part E to lesson 2, adding 1d to the extensions in lesson 2, and adding 1c to extensions in lesson 3.

Chapter 5: Conclusion

This curriculum project was designed to help educators teach the challenging topic of arithmetic sequences to all students in Algebra I. Teachers are encouraged to adjust these documents to help meet the needs of their individual students. The author suggests that these materials be presented to students using a collaborative, inquiry-based approach in order to fully develop the connection between linear relationships and arithmetic sequences.

Although teaching mathematics using nontraditional methods is more challenging, the pursuit of developing problem solving mathematicians is worthwhile. This curriculum allows students to make connections between contexts so that the mathematics learned is easily relatable and better understood. The tasks build off student knowledge of linear functions so that they have a foothold to start with when learning about arithmetic sequences. Also, the contexts used within the lessons deal with mostly monetary situations which students can easily relate to as employees and consumers. Within each lesson students develop and interpret various syntax in formulas to demonstrate equivalence and fluidity between forms. Students are presented with meaningful tasks which allow them to truly problem solve in order to develop “mathematical power” (Schoenfeld, 1992, p. 33). Rather than providing them with the “monotonous, meaningless, individual work” (Beswick, 2010, p. 370) of traditional procedural based lessons, these problems engage students in rich mathematical discussions so that they are able to fully engage with the content. This true mathematical problem solving in context is worth the extra effort because it challenges students appropriately so that they are better prepared to be problem solvers later on in life.

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Appendix

Lesson 1: Connection to Linearity



A. George works at an ice cream stand during the summer. He gets paid an initial amount each day and then an hourly rate. The table below represents his daily pay, G , after h hours of work.

Hours of work	0	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	15	23	31	39	47	55	63	71	79

Handwritten annotations: Green arrows above the table show a constant increase of +1 in hours. Blue arrows below the table show a constant increase of +8 in pay for each hour.

- Complete the table above to continue the pattern of (h, G) coordinates.
- Describe the relationship between the number of hours George works, h , and the amount of money he earns each day, G . Include the values for the initial pay and hourly pay in your response.

George earns \$15 each day initially. He also earns an additional \$8 for each hour he works.

- Write an equation to represent George's daily pay, G , after h hours of work.

$$G = 15 + 8h$$

or

$$G = 8h + 15$$

- Explain how your equation in (a) represents the relationship you described in question 2.

$$G = 15 + 8h$$

Handwritten annotations: A green arrow points to 15 with the label 'money earned'. A purple arrow points to 15 with the label 'initial pay'. A red arrow points to 8 with the label 'hourly pay'. A blue arrow points to h with the label 'number of hours worked'.

- Determine the amount of money George would earn if he worked a 12 hour shift.

$$G = 15 + 8(12)$$

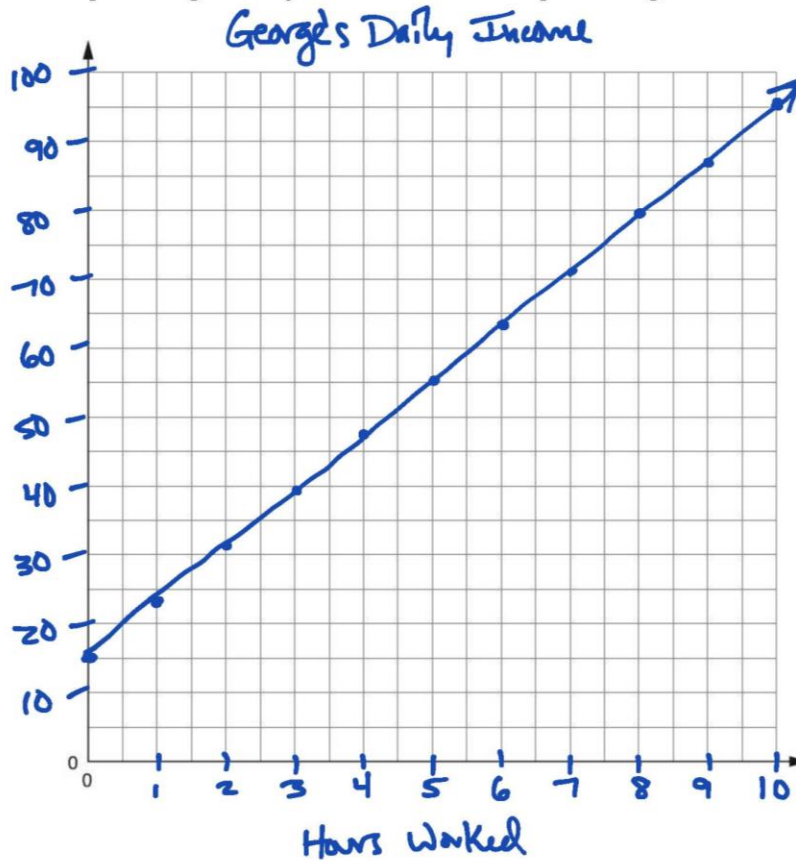
$$G = 111 \qquad \$ 111$$

4.

a. Graph George's daily income relationship on the grid below.

$$G(10) = 15 + 8(10) = 95$$

Pay Earned
(in dollars)



b. Explain how your graph in represents the relationship you described in question 2.

y-int at $(0, 15) \rightarrow$ initial daily pay of \$15.

$(0, 15)$ $(1, 23)$ $\frac{\Delta y}{\Delta x} = \frac{8}{1}$ slope of 8 \rightarrow hourly pay of \$8

$\Delta y = 8$
 $\Delta x = 1$

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

Yes. George can work a fraction of an hour (like 2.5 hrs). So it makes sense to connect the data points to include all fractional hourly values.

d. Describe what the coordinate point $(1.5, 27)$ means in this situation.

for 1.5 hrs of work, George earns \$27.

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- B.** Michelle works at a lemonade stand by the beach. She is paid an initial amount each day and then earns commission from each drink that she sells. The table below represents her daily pay, M , after n lemonades are sold.



Lemonades Sold	0	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	13	15.5	18	20.5	23	25.5	28	30.5	33

- Complete the table above to continue the pattern of (n, M) coordinates.
- Describe the relationship between the number of lemonades Michelle sells, n , and the amount of money she earns each day, M . Include the values for the initial pay and amount earned per lemonade sold in your response.

Michelle initially earns \$13 each day. She also earns \$2.50 for each lemonade she sells.

- Write an equation to represent Michelle's daily pay, M , after n lemonades are sold.

$$M = 13 + 2.5n$$

OR

$$M = 2.5n + 13$$

- Explain how your equation in (a) represents the relationship you described in question 2.

$$M = 13 + 2.5n$$

↑ amount earned
 ↑ initial pay
 ↑ amount earned per lemonade sold
 ← number of lemonades sold

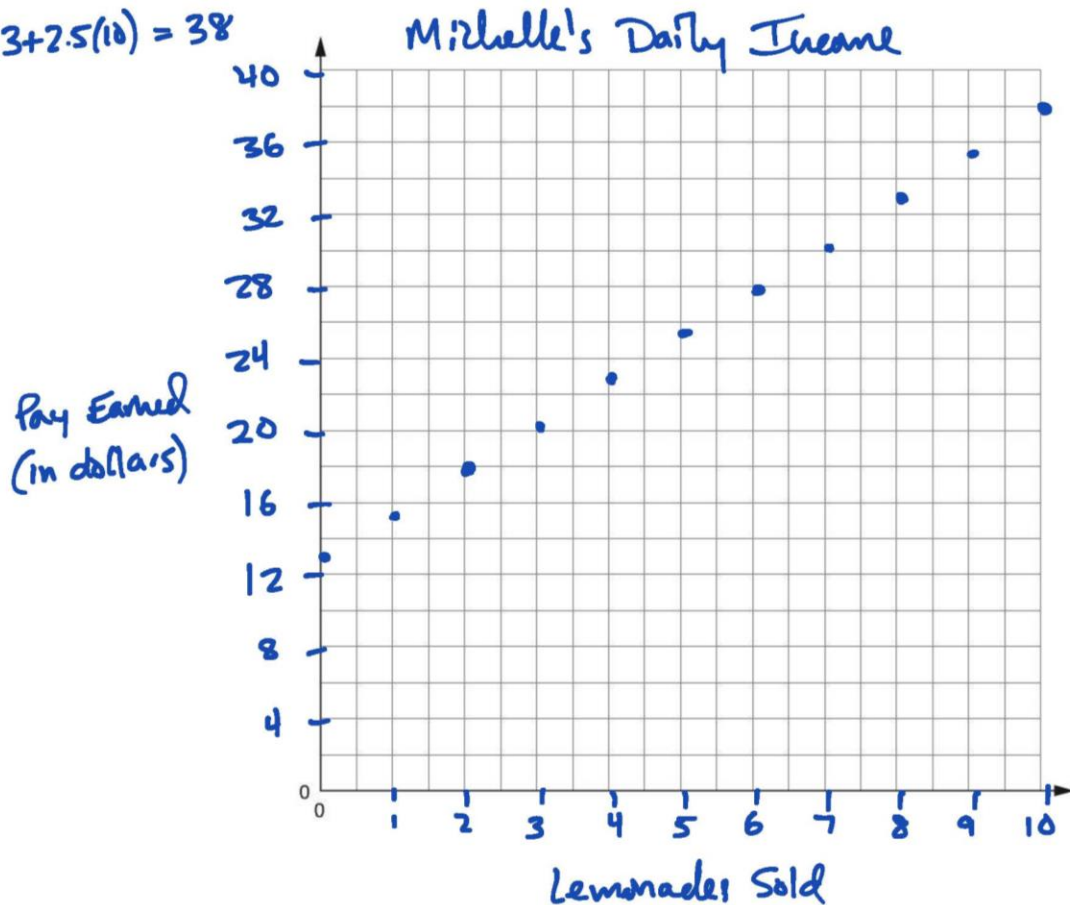
- Compare Michelle's daily earnings to George's. What similarities exist and how are the two situations different?

Both have an initial pay but George's is \$15 while Michelle's is \$13. They both have additional pay but George's is hourly and Michelle's is based on how much lemonade she sells.

4.

a. Graph Michelle's daily income relationship on the grid below.

$$13 + 2.5(10) = 38$$



b. Explain how your graph in represents the relationship you described in question 2.

y -int at $(0, 13) \rightarrow$ initial daily pay of \$13
 Slope of 2.5 \rightarrow commission of \$2.50 per lemonade sold
 $\Delta y = 2.5$
 $(0, 13)$ $(1, 15.5)$
 $\Delta x = 1$

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

No. Michelle cannot sell a fraction of a lemonade.

d. Describe what the coordinate point $(1.5, 16.75)$ would mean in this situation. Is this realistic?

One and a half lemonades sold would earn Michelle \$16.75. This is not realistic because you cannot sell half of a lemonade.

ARITHMETIC SEQUENCES USING SITUATED TASKS

- C. Both George's and Michelle's pay are examples of linear relationships. Michelle's is also an example of an arithmetic sequence. Based on your work in parts A and B, what do you think makes a linear situation an arithmetic sequence? Why is George's pay not an example of an arithmetic sequence?

An arithmetic sequence has a domain of positive integers (sometimes including 0) only. Linear situations can have domains of all real numbers. George can work for a non-integer value number of hours, so his relationship is not an arithmetic sequence.

Arithmetic Sequences

→ Linear relationships

→ Positive integer domains

$\{0, 1, 2, 3, 4, \dots\}$
 sometimes included

Extensions/Homework

1. Jane works at a nail salon and is paid \$12 an hour. Would her pay represent an arithmetic sequence? Explain.

No. The domain of her hours worked could include non-integers (like 4.25 hrs).

2. Write an equation for the relationship in problem 1. Compare this relationship to George's pay relationship in part A.

$J = 12h$ Jane earns more than George per hour but does not make any initial amount like he does.

3. Would the relationship shown in the table below represent an arithmetic sequence?

Hours worked	1	2	3	4	5
Pay (\$)	2	4	8	16	32

$\times 2$ $\times 2$ $\times 2$ $\times 2$

No. This relationship is not linear. It is exponential.

Lesson 2: Explicit Formulas

- A. Michael, Michelle's brother, works at an iced tea stand near his sister's stand by the beach. Michael's boss pays him more each day but he earns less for each drink he sells. He earns a daily pay of \$20 and then earns \$1 for each iced tea he sells.



1. Complete the table below to represent how much Michael earns in one day of sales.

Iced Teas Sold	0	1	2	3	4	5	6	7
Daily Pay (in dollars)	20	21	22	23	24	25	26	27

- 2.
- a. Write an equation to represent the amount of money Michael earns, M , in one day after he sells t iced teas.

$$M = 20 + 1t$$

- b. Label how each variable and number in your equation represents this situation.

$$M = 20 + 1t$$

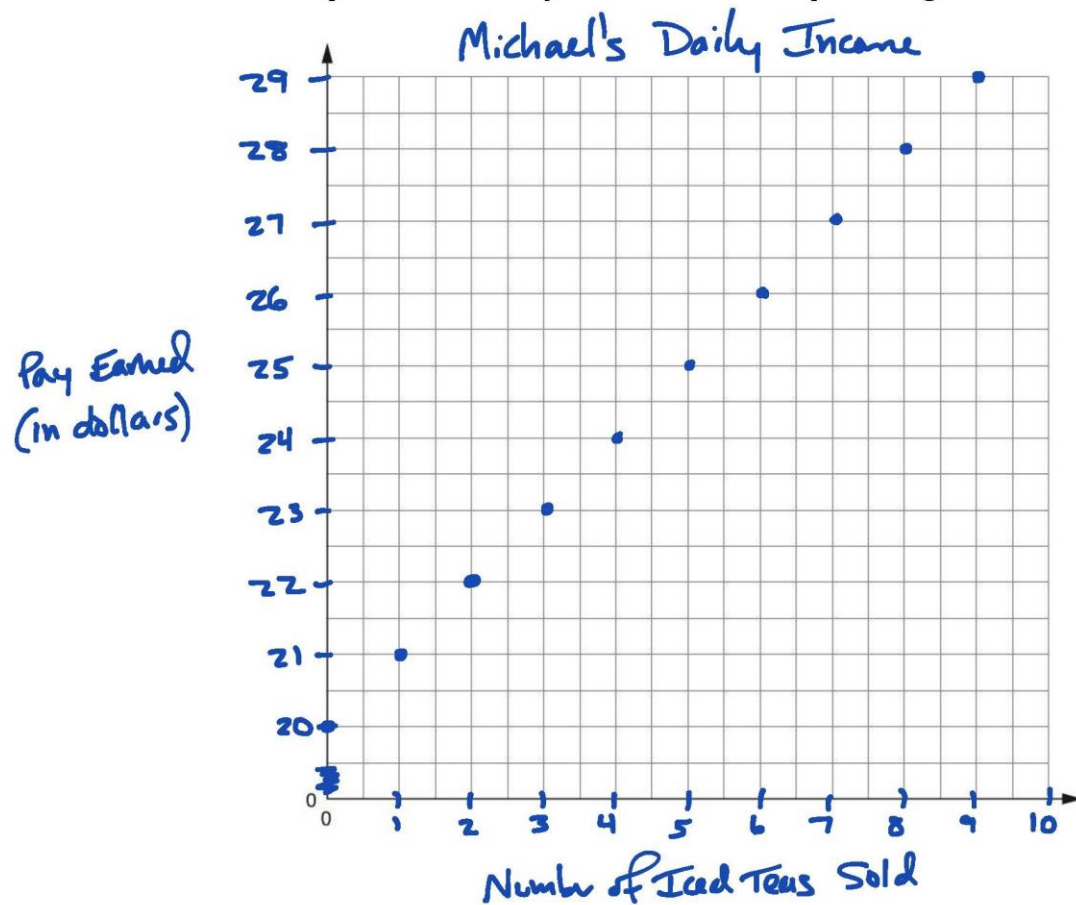
Michael's daily pay (points to 20)
 initial daily pay (points to 20)
 \$1 per iced tea (points to 1)
 number of iced teas (points to t)

3. Does Michael's daily pay relationship represent an arithmetic sequence or is it just a linear relationship? Explain.

Arithmetic sequence. The domain is positive integers since you can't buy part of an iced tea.

4.

a. Graph Michael's daily income relationship on the grid below.



b. Explain how your graph in represents your equation from part 2a.

y -int at $(0, 20) \rightarrow$ initial pay of \$20 = b
 $(0, 20)$ $(1, 21)$ $\frac{\Delta y}{\Delta x} = 1$ Slope = 1 \rightarrow \$1 per iced tea = m
 $\Delta y = 1$
 $\Delta x = 1$

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

No. Michael cannot sell a fraction of an iced tea.

ARITHMETIC SEQUENCES USING SITUATED TASKS

B. Any arithmetic sequence can be written as an equation in the form

$$a_n = a_1 + (n - 1)d$$

This is often how reference sheets display the generic equation. The variables are defined as follows:

- a_n is the n^{th} term of the sequence. For example, a_6 is the 6th term.
 - n represents the number of the current term. For a_6 , $n = 6$.
 - a_1 is the term when $n = 1$ which is usually the first term of the sequence.
 - d is the common difference. It represents how much the sequence changes from one term to the next.
1. Suppose we apply this new equation format to Michael's daily pay relationship from part A. What would a_6 represent in this case?

a_6 would be the pay Michael earns in a day if he sells 6 iced teas.

2. Michael says that a_1 represents the y-intercept. Whereas Michelle says that a_0 represents the y-intercept. Who do you agree with? Explain.

$a_0 \rightarrow$ the y-intercept is when $x=0$ not 1.

3. What is the value of d for Michael's pay situation? What is another name we typically use for this value?

$d = 1 \rightarrow$ slope

4. Write an equation using the new format to represent Michael's daily pay.

$$a_n = 21 + (n-1)(1)$$

5. Simplify your equation from part 4. How does this equation relate to the one you wrote in part 2a?

$$a_n = 21 + n - 1$$

$a_n = 20 + n \rightarrow$ equivalent to $M = 20 + 1t$, just with different variables

ARITHMETIC SEQUENCES USING SITUATED TASKS

C. Arithmetic sequences can be written as equations using the traditional linear format of $y = mx + b$ or the new format $a_n = a_1 + (n - 1)d$.

1. Which variable in the linear equation corresponds to the a_n variable from the new format?

y

2. Which variable in the new format corresponds to the m variable from the linear format?

d

3. Is there a variable in the new format that corresponds to the variable b from the linear format? If not, how could we write it using the new notation?

No. Should be a_0

D. For each sequence shown below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n - 1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

1. y -intercept of 6, slope of -3 .

a_0 $a_1 = 3$

$$y = -3x + 6$$

$$a_n = 3 + (n-1)(-3)$$

$$a_n = 3 - 3n + 3$$

$$a_n = -3n + 6$$

2. $a_1 = \frac{1}{2}$, $d = 2\frac{1}{2}$

$$y = 2\frac{1}{2}x - 2$$

$$a_n = \frac{1}{2} + (n-1)(2\frac{1}{2})$$

$$a_n = \frac{1}{2} + 2\frac{1}{2}n - 2\frac{1}{2}$$

$$a_n = 2\frac{1}{2}n - 2$$

3.

x	0	1	2	3	4	5
y	19	14	9	4	-1	-6

$$y = -5x + 19$$

$$a_n = 14 + (n-1)(-5)$$

$$a_n = 14 - 5n + 5$$

$$a_n = -5n + 19$$

E. For each sequence from part D, determine the value of a_{10} .

1. $a_{10} = -3(10) + 6$
 $= -30 + 6$

$$a_{10} = -24$$

2. $a_{10} = 2\frac{1}{2}(10) - 2$
 $= 25 - 2$

$$a_{10} = 23$$

3. $a_{10} = -5(10) + 19$
 $= -50 + 19$

$$a_{10} = -31$$

Extensions/Homework

1. Use the equation of an arithmetic sequence show below to answer the following questions.

$$a_n = 6 + (n - 1)(4)$$

- a. What is the first term, a_1 ? What is the common difference, d ?

$$a_1 = 6 \quad d = 4$$

- b. Simplify the equation completely.

$$a_n = 6 + 4n - 4$$

$$a_n = 4n + 2$$

- c. What is the value of the y -intercept of this relationship?

$$2$$

- d. What is the value of a_4 ?

$$a_4 = 4(4) + 2 = 16 + 2 = 18$$

For # below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n - 1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

2. y -intercept of 2, slope of 6.

$$y = 6x + 2$$

$$a_1 = 8$$

$$a_n = 8 + (n-1)(6)$$

$$a_n = 8 + 6n - 6$$

$$a_n = 6n + 2$$

3. $a_0 = 3, d = -2$

$$y = -2x + 3$$

$$a_1 = 1$$

$$a_n = 1 + (n-1)(-2)$$

$$a_n = 1 - 2n + 2$$

$$a_n = -2n + 3$$

- 4.

x	0	1	2	3	4	5
y	3	7	11	15	19	23

$$y = 4x + 3$$

$$a_n = 7 + (n-1)(4)$$

$$a_n = 7 + 4n - 4$$

$$a_n = 4n + 3$$

Lesson 3: Recursive Formulas

In the previous two lessons we discussed how both Michelle's and Michael's daily income situations were examples of arithmetic sequences. We used the general linear equation $y = mx + b$ and the arithmetic sequence equation $a_n = a_1 + (n - 1)d$ to represent these relationships.

Michelle's Daily Pay

Lemonades Sold	0	1	2	3	4
Daily Pay (in dollars)	13	15.5	18	20.5	23

Michael's Daily Pay

- Initial daily pay of \$20
- \$1 earned for each iced tea sold

- A. We can use *NOW* – *NEXT* statements to illustrate how to create a sequence of values **recursively**. The *NOW* variable represents the current term and the *NEXT* variable represents the very next term. An example is shown below.

Sequence: 4, 7, 10, 13, ...

$$\text{NEXT} = \text{NOW} + 3$$

Starting at: 4

This example says to start at a value of 4 and then add 3 to the current term in order to get the next term.

1. Write a *NOW* – *NEXT* statement for both Michelle and Michael's daily pay. Use the y-intercept (for 0 drinks sold) as the starting value for each.

Michelle
 $\text{NEXT} = \text{NOW} + 2.5$
 starting at: 13

Michael
 $\text{NEXT} = \text{NOW} + 1$
 starting at: 20

2. Which number in each of your equations would be the slope, m , if the equation was in the form $y = mx + b$?

Michelle
 $m = 2.5$

Michael
 $m = 1$

3. Which number in each of your equations would be the y-intercept, b , if the equation was in the form $y = mx + b$?

Michelle
 $b = 13$

Michael
 $b = 20$

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- B.** Another similar method of writing these equations recursively uses different notation to represent the same situation. An example of this notation is shown below.

Sequence: 4, 7, 10, 13, ...

$$a_{n+1} = a_n + 3 \text{ for } n \geq 1$$

$$a_1 = 4$$

- Match up each part of this new formula with the previous one.
 - $NOW = \underline{a_n}$
 - $NEXT = \underline{a_{n+1}}$
 - Starting at = $\underline{a_1}$
- Why does this formula need to include the $n \geq 1$ part? Would you be able to create the sequence without this? Explain.

The $n \geq 1$ says the domain of the formula. It shows the sequence values are a_1, a_2, a_3, \dots

- Rewrite your $NOW - NEXT$ statements from **A1** using this new format.

$$\begin{array}{l} \text{Michelle} \\ a_{n+1} = a_n + 2.5 \text{ for } n \geq 0 \\ a_0 = 13 \end{array}$$

$$\begin{array}{l} \text{Michael} \\ a_{n+1} = a_n + 1 \text{ for } n \geq 0 \\ a_0 = 20 \end{array}$$

- What did you label your starting values in part 3 as? Explain your decision.

$a_0 \rightarrow$ the "first" term is when 0 drinks are sold, not 1 drink. $a_0 \rightarrow$ initial daily pay

- Using your formula for Michelle's daily pay from part 4, list the steps needed to find the value of a_4 .

$a_0 = 13$
 Find a_1 by adding 2.5 to a_0 (13)
 Find a_2 by adding 2.5 to a_1 (15.5)
 Find a_3 by adding 2.5 to a_2 (18)
 Find a_4 by adding 2.5 to a_3 (20.5)
 $\rightarrow a_4 = 23$

ARITHMETIC SEQUENCES USING SITUATED TASKS

C. Using your work from this lesson and previous lessons, answer the following questions.

1. If you had to find the value of a_{20} , which formula would you use? Explain.

Option 1

$$\begin{aligned} a_{n+1} &= a_n + 3 \text{ for } n \geq 1 \\ a_1 &= 5 \end{aligned}$$

Option 2

$$a_n = 3n + 2$$

Option 2. Can plug in 20 for n and solve in a couple steps.

$$a_{20} = 3(20) + 2$$

$$a_{20} = 62$$

2. Was your choice in part 1 an explicit or a recursive relationship?

Explicit

3. When is an explicit formula for an arithmetic sequence (in the form $y = mx + b$ or $a_n = a_1 + (n - 1)d$) more helpful than a recursive one?

Explicit easier to find any given sequence value without having to find previous values.

4. When is a recursive formula more useful than an explicit one?

Recursive formula easily shows the starting value and the common change between values.

D. Write an explicit and a recursive equation for each situation described below.

1. Joan makes \$10 per day plus an additional \$1.30 for each basket she sells.

$$J = 10 + 1.3b$$

$$\begin{aligned} a_{n+1} &= a_n + 1.3 \text{ for } n \geq 0 \\ a_0 &= 10 \end{aligned}$$

2. A stack of boxes at a grocery store consists of 22 boxes on first row on the bottom, 18 in the next row up, and so on forming an arithmetic sequence until the top has 2 boxes.

$$\begin{aligned} B &= 22 + (n-1)(-4) \\ B &= 22 - 4n + 4 \\ B &= 26 - 4n \end{aligned}$$

$$\begin{aligned} a_{n+1} &= a_n - 4 \text{ for } n \geq 1 \\ a_1 &= 22 \end{aligned}$$

Extensions/Homework

1. At a large grocery store they are having a promotion to give away free samples of pasta sauce. On the first day, they start with 200 cans of sauce. They plan on giving away 12 cans of sauce per day.



- a. Write a recursive formula to represent this situation.

$$a_{n+1} = a_n - 12 \text{ for } n \geq 1$$

$$a_1 = 200$$

- b. Write an explicit formula to represent this situation.

$$C = 200 + (n-1)(-12)$$

$$C = 200 - 12n + 12$$

$$C = 212 - 12n$$

- c. Find how many cans they will have left after 8 days.

$$C = 212 - 12(8)$$

$$C = 212 - 96$$

$$= 116 \text{ cans}$$

- d. Determine on which day of the promotion they will run out of sauce.

$$C = 0 = 212 - 12n + 12n \quad \text{On the 18}^{\text{th}} \text{ day}$$

$$\frac{12n}{12} = \frac{212}{12} \quad n = 17.6$$

Write an explicit and a recursive equation for each situation described below.

2. To help study for an upcoming test, Julia decides to study each night more than she studied the previous night. She plans on studying for 10 minutes the first night and will increase by 5 minutes each night.

Explicit

$$S = 10 + (n-1)(5)$$

$$S = 10 + 5n - 5$$

$$S = 5n + 5$$

Recursive

$$a_{n+1} = a_n + 5 \text{ for } n \geq 1$$

$$a_1 = 10$$

3. James is paying off his student loans every month. In the first month, his loans start at a value of \$30,000. He pays them off by writing checks for \$250 each month.

Explicit

$$L = 30000 + (n-1)(-250)$$

$$L = 30000 - 250n + 250$$

$$L = 30250 - 250n$$

Recursive

$$a_{n+1} = a_n - 250 \text{ for } n \geq 1$$

$$a_1 = 30000$$

Lesson 4: Explicit and Recursive Using Various Syntax

Just like how we can use the variables M and t in place of y and x in $y = mx + b$, we can also use different variables in place of a_n and n in $a_n = a_1 + (n - 1)d$.

- A. Josh has a job selling tie-dye T-shirts at the beach. His pay is represented by the equation show below:

$$a_n = 8 + (n - 1)(3)$$

1. Simplify this equation completely.

$$a_n = 8 + 3n - 3$$

$$a_n = 3n + 5$$



2. Rewrite the equation using different variables that better align with the context. Briefly explain why you chose your variables.

$$J = 3t + 5$$

$J \rightarrow$ stands for Josh's earnings
 $t \rightarrow$ stands for number of T-shirts sold

3. Could this equation represent this situation? Explain.

$$J_t = 3t + 5$$

Yes, it is almost identical to $J = 3t + 5$.

J_t just shows that J , Josh's earnings, are related to the number of T-shirts sold, t .

- B. Function notation is also often used to represent a linear relationship. The form $f(x) = mx + b$ can be used in place of $y = mx + b$.

1. Rewrite your equation from A2 using function notation.

$$f(t) = 3t + 5$$

2. Explain how a_n and $f(n)$ notations are alike.

Both show that n is the input. The outputs are just labeled differently.

$$a_1 = f(1) \rightarrow \text{first term}$$

$$a_5 = f(5) \rightarrow \text{fifth term}$$

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- C. Recursive sequences can also be written using various forms. Two recursive formulas for Josh's T-shirt income are shown below.

(a)

$$J_{n+1} = J_n + 3 \text{ for } n \geq 0$$

$$J_0 = 5$$

(b)

$$J_n = J_{n-1} + 3 \text{ for } n \geq 1$$

$$J_0 = 5$$

1. Compare these formulas. Are they both accurate ways of showing Josh's income relationship?

Yes. They both say start with \$5 for 0 shirts sold and then add 3 for each additional shirt sold.

2. Using both formulas, generate the first five terms of each sequence. Does this support or change your answer from part 1?

a	
n	J _n
0	5
1	8
2	11
3	14
4	17

b	
n	J _n
0	5
1	8
2	11
3	14
4	17

This supports my answer to part 1 since the two sequences are the same.

3. Rewrite the equations using *NEXT* – *NOW* statements.

$$\text{NEXT} = \text{NOW} + 3$$

Starting at 5

4. a. If J_n is the *NOW*, what is the *NEXT*?

$$J_{n+1}$$

- b. If J_n is the *NEXT*, what is the *NOW*?

$$J_{n-1}$$

5. Rewrite formula (a) using function notation like $J(n + 1)$.

$$J(n+1) = J(n) + 3 \text{ for } n \geq 0$$

$$J(0) = 5$$

ARITHMETIC SEQUENCES USING SITUATED TASKS

D. Nicole works for the same T-shirt company as Josh and has to put up posters around town to advertise for an upcoming sale. She starts the day with 240 posters and is able to put up 3 every minute.

1. Write an explicit equation for this sequence using p_m as the number of posters left after m minutes.

$$p_m = 240 - 3m$$

2. Rewrite your equation from part 1 using function notation.

$$f(m) = 240 - 3m$$

3. Write a recursive formula for this sequence using p_m as the number of posters left after m minutes.

$$p_{m+1} = p_m - 3 \text{ for } m \geq 0$$

$$p_0 = 240$$

4. Rewrite your recursive formula in a different but equivalent way. Use part C as an example.

$$p_m = p_{m-1} - 3 \text{ for } m \geq 1$$

$$p_0 = 240$$

5. Rewrite your formula from part 3 using function notation.

$$f(m+1) = f(m) - 3 \text{ for } m \geq 0$$

$$f(0) = 240$$

6. Rewrite your formula from part 4 using function notation.

$$f(m) = f(m-1) - 3 \text{ for } m \geq 1$$

$$p_0 = 240$$

7. Out of all equations written in parts 1-6, which is the easiest one for you to understand. Justify your choice.

$$\#2 \quad f(m) = 240 - 3m$$

Easy to see initial amount (240), change (-3), and can be used to find any term in sequence.

Extensions/Homework

1. Wood is often stacked like the picture shown at right with the bottom layer having the most and the top layer having the least. Suppose the first layer on the bottom has 48 logs. Each additional layer has 4 less logs than the layer below it.



- a. Write an explicit equation for this relationship. Explain why you chose the variables you did.

$$W_l = 48 + (l-1)(-4)$$

$$W_l = 48 - 4l + 4$$

$$W_l = 52 - 4l$$

$W \rightarrow$ number of logs (wood)
 $l \rightarrow$ layer number
 The variables easily connect to the words they represent.

- b. If your answer in part a is not in function notation, rewrite it so that it is. Otherwise, rewrite it so that the equation is in a_n notation.

$$W(l) = 52 - 4l$$

- c. Write a recursive formula for this situation.

$$W_{l+1} = W_l - 4 \quad \text{for } l \geq 1$$

$$W_1 = 48$$

- d. Write a different, yet equivalent, recursive formula for this situation.

$$W_l = W_{l-1} - 4 \quad \text{for } l \geq 2$$

$$W_1 = 48$$

- e. Using any of your formulas, find the number of logs in the first 6 layers.

l	1	2	3	4	5	6
W_l	48	44	40	36	32	28

Lesson 5: Applying the Various Syntax

- A. Jeff creates a plan to raise money to clean up the beach. He receives an initial donation and then receives a donation for each T-shirt sold at Josh's T-shirt store. The store offers Jeff a few options to choose from.

For each function below, define what each variable represents. Also, determine how much the initial donation and how much the donation per T-shirt would be.



1. $f(0) = 40$

$$f(x+1) = f(x) + 2 \text{ for } x \geq 0$$

$f(x)$ → total in donations
for x shirts

x → number of T-shirts
sold

\$2 per T-shirt

\$40 initial donation

2. $d_t = 30 + 2.25t$

d_t → total in donations
for t shirts sold

t → number of T-shirts
sold

\$2.25 per T-shirt

\$30 initial donation

3. $x_n = x_{n-1} + 2.75$

$$x_0 = 25$$

x_n → total in donations
for n shirts sold

n → number of T-shirts
sold

\$2.75 per T-shirt

\$25 initial donation

4. $g(y) = 35 + 2.1y$

$g(y)$ → total in donations
for y shirts sold

y → number of T-shirts
sold

\$2.10 per T-shirt

\$35 initial donation

5. For this context, is the first term of the sequence the 0th term or the 1st term? Explain.

The 0th term. In this case, 0 T-shirts could be sold so the first term is the initial donation.

ARITHMETIC SEQUENCES USING SITUATED TASKS

- B.** Upon hearing about Jeff's fundraiser, the town decides to help with cleanup on the first day. As a result, the group was able to remove 15 barrels of garbage on the first day. After that, the remaining cleaners were able to remove 4 barrels of garbage a day.

1. Complete the table below to represent this situation.

Day	1	2	3	4	5	6
Barrels of Garbage Removed	15	19	23	27	31	35

2. One of Jeff's workers, Molly, determines that a recursive formula could represent this relationship. Describe what her formula represents.

$$G(d + 1) = G(d) + 4 \text{ for } d \geq 0$$

$$G(0) = 15$$

Day 0 \rightarrow 15 barrels of garbage removed
 Each day, 4 more barrels are collected.

3. Does her equation accurately depict this situation? If yes, explain. If no, fix her formula so it is accurate.

No. It should say $G(1) = 15$ since the first term is day 1. And then it would say $d \geq 1$ for the first part.

4. Write an explicit formula using g_d notation for this relationship.

$$g_d = 15 + (d-1)(4)$$

$$\text{or } g_d = 11 + 4d$$

5. Rewrite your formula from part 4 using function, $f(x)$, notation.

$$f(x) = 11 + 4x$$

6. For this context, is the first term of the sequence the 0th term or the 1st term? Explain.

The 1st term. Since there is no day 0 for this situation. The first term is day 1.

ARITHMETIC SEQUENCES USING SITUATED TASKS

C. For each arithmetic sequence defined below, do the following:

- State the first term and the common difference
- Write the first five terms of the sequence
- State whether the formula is explicit or recursive.
- If the formula is explicit, write a recursive formula. If it is recursive, write an explicit equation.
- Write the relationship as a *NEXT* – *NOW* statement.

1. $f(1) = 12$

$$f(x) = f(x-1) + 4 \text{ for } x \geq 2$$

← Recursive

First term: 12

Common
d. difference: 4

x	1	2	3	4	5
f(x)	12	16	20	24	28

Explicit: $f(x) = 12 + (x-1)(4)$

$$f(x) = 8 + 4x$$

$$\text{NEXT} = \text{NOW} + 4$$

Starting at 12

2. $b_i = 4 + (i-1)(-2)$

← Explicit

First term: 4

Common
d. difference: -2

i	1	2	3	4	5
b_i	4	2	0	-2	-4

Recursive: $b_{i+1} = b_i - 2$ for $i \geq 1$

$$b_1 = 4$$

$$\text{NEXT} = \text{NOW} - 2$$

Starting at 4

3. $x_{n+1} = x_n - 2$ for $n \geq 1$

$$x_1 = -5$$

← Recursive

First term: -5

Common
d. difference: -2

n	1	2	3	4	5
x_n	-5	-7	-9	-11	-13

Explicit: $x_n = -5 + (n-1)(-2)$

$$x_n = -2n - 3$$

$$\text{NEXT} = \text{NOW} - 2$$

Starting at -5

ARITHMETIC SEQUENCES USING SITUATED TASKS

- D. Throughout these problems you have seen some sequences that start with the first term as a_1 and others that start with a_0 . Describe the differences between these two types of sequences. Use examples to show how each one may be used.

a_1 sequences have a first term when $n=1$.

Example: Garbage collected each day

a_0 sequences have a first term when $n=0$.

Example: Cost with an initial price (a_0)

Extensions/Homework

Jane decides to do her own fundraiser to help with the beach cleanup. Her results from the cleanup form an arithmetic sequence and are shown in the table.

Day	1	2	3	4	5	6
Barrels of Garbage Removed	10	16	22	28	34	40

1. What is the initial value of the sequence? What is the common difference?

Initial Value Common
 $a_1 = 10$ difference: 6

2. Write an explicit equation for this relationship. Explain why you chose the variables you did.

$$a_n = 10 + (n-1)(6)$$

$$a_n = 6n + 4$$

3. Josh defines Jane's sequence recursively as shown below. Does his sequence accurately create her results? If yes, explain. If not, what should he fix?

$$j_{n+1} = j_n + 10 \text{ for } n \geq 1$$

$$j_1 = 6$$

No, he needs to switch the 6 & 10. The first term is 10 and the common difference is 6.

Summative Assessment

Name: _____

Multiple Choice #1-5

1. What is the common difference of the sequence 5, 2, -1, -4, ... ?

(1) 3

(2) -3

(3) 5

(4) -5

2. Which formula accurately represents the sequence from question 1?

(1) $a_{n+1} = a_n + 3$ for $n \geq 1$
 $a_1 = 5$

(2) $a_{n+1} = a_n - 3$ for $n \geq 1$
 $a_1 = 5$

(3) $a_{n+1} = a_n + 5$ for $n \geq 1$
 ~~$a_1 = -3$~~

(4) $a_{n+1} = a_n + 5$ for $n \geq 1$
 ~~$a_1 = -3$~~

3. What is the fifth term of the sequence described below?

$$f(n) = f(n-1) + 4 \text{ for } n \geq 2$$

$$f(1) = -3$$

(1) 13

(2) 5

(3) 1

(4) 17

4. Which equation would produce the same sequence as
- $a_n = 2 + (n-1)(-3)$
- ?

(1) $y = 2x + 3$

(2) $y = -3x + 2$

(3) $y = -3x - 1$

(4) $y = -3x + 5$

5. Which explicit equation would represent the sequence defined below?

$$x_{n+1} = x_n + 3 \text{ for } n \geq 1$$

$$x_1 = 7$$

(1) $x_n = 3n + 7$

(2) $x_n = 3n + 4$

(3) $x_n = 7n + 3$

(4) $x_n = 7n - 4$

$$7 + (n-1)(3)$$

$$3n - 3 + 7$$

$$3n + 4$$

$$7, 10, 13, 16, 19$$

ARITHMETIC SEQUENCES USING SITUATED TASKS

Short Answer – Show all work to support your solutions.

6. Bailey creates a pattern using blocks. In the first design, she uses 5 blocks. In the second, she uses 11 blocks. In the third, she uses 17 blocks.

- a. Assuming the pattern continues in the same way. Explain why this situation represents an arithmetic sequence.

5, 11, 17, ... There is a common difference of 6.
 $\begin{array}{c} \underbrace{5, 11, 17, \dots} \\ \quad \uparrow \quad \uparrow \\ \quad +6 \quad +6 \end{array}$ The domain is whole numbers since she cannot create a fractional design #.

- b. Write an explicit equation to model this sequence.

$$\begin{aligned} a_n &= 5 + 6(n-1) \\ a_n &= 6n - 6 + 5 \\ a_n &= 6n - 1 \end{aligned}$$

- c. Write a recursive formula to model this sequence.

$$\begin{aligned} a_{n+1} &= a_n + 6 \text{ for } n \geq 1 \\ a_1 &= 5 \end{aligned}$$

- d. Determine the number of blocks Bailey would use in the 50th design.

$$\begin{aligned} a_{50} &= 6(50) - 1 \\ a_{50} &= 300 - 1 \\ a_{50} &= 299 \end{aligned}$$

- e. Which design number would need 35 blocks? Show your work to support your answer.

$$\begin{aligned} 35 &= 6n - 1 \\ \underline{+1} & \quad \underline{+1} \\ \frac{36}{6} &= \frac{6n}{6} \quad n = 6 \end{aligned} \quad \text{The 6th design}$$

ARITHMETIC SEQUENCES USING SITUATED TASKS

7. Josie is collecting donations for a walkathon. Her donors pledge a certain amount up front and then more for each mile she walks. Her donors provide her with the equations below to represent their pledges. For each equation, determine the initial donation, amount pledged per mile, and first five amounts for walking 1-5 miles.

a. $f(x) = 4x - 2$

Initial Donation	Pledge Per Mile	First Five Terms					
$f(0) = -2$	\$4	Mile	1	2	3	4	5
		Donation Total	2	6	10	14	18

b. $b_{n+1} = b_n + 3$ for $n \geq 1$
 $b_1 = 5$

Initial Donation	Pledge Per Mile	First Five Terms						
\$2	\$3	Mile	0	1	2	3	4	5
		Donation Total	2	5	8	11	14	17

c. $a_n = 4 + (n - 1)(3) = 3n + 1$

Initial Donation	Pledge Per Mile	First Five Terms					
\$1	\$3	Mile	1	2	3	4	5
		Donation Total	4	7	10	13	16

- d. Are any of the plans from a-c unrealistic? Explain.

Plan (a) \rightarrow initial donation of $-\$2$ is unrealistic since they would not donate negative dollars.

ARITHMETIC SEQUENCES USING SITUATED TASKS

8. If Y_n represents the current term of a sequence, what notation would be used to represent the **next** term?

$$Y_{n+1}$$

9. If $f(n)$ represents the current term of a sequence, what notation would be used to represent the **previous** term?

$$f(n-1)$$

10. Robert is planning on running a marathon of about 42 kilometers. To build up his endurance, he plans on running 1.5 km the first day and then running an additional 0.75 km each day.

- a. Write an explicit formula to represent the distance he runs each day D_n after n days.

$$D_n = 1.5 + 0.75(n-1)$$

$$D_n = 0.75n + 0.75$$

- b. Will Robert meet his goal of 42 km after a month (30 days)? If not, how close will he be?

$$D_{30} = 0.75(30) + 0.75$$

$$D_{30} = 23.25$$

$$42 - 23.25 = 18.75$$

No, he is 18.75 km away from his goal.

- c. On what day will Robert reach a distance of 42 km?

$$42 = 0.75n + 0.75$$

$$\begin{array}{r} 42 \\ -0.75 \\ \hline 41.25 \end{array} = \begin{array}{r} 0.75n \\ -0.75 \\ \hline 0.75n \end{array}$$

$$55 = n$$

55th day