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Why Common Core? Educational Standards, the Legislative Process, and Best Practices, with a Unit Design Aligned to the Common Core State Standards, by a Teacher, for Teachers

Daniel Boyd
State University of New York College at Brockport, dboy0513@brockport.edu

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by

Daniel Boyd

A thesis submitted to the Department of Education of The College at Brockport, State University of New York, in partial fulfillment of the requirements of the degree of Master of Science in Education

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ABSTRACT

This thesis project was created with two major focuses. The first is to provide insight on educational standards. This includes national versus state standards, international assessments and the implications, and the adoption of the Common Core State Standards. Second, this project shares best practices for implementing the Common Core State Standards, and includes a unit design in surface area and volume in 7^{th} grade mathematics, which is aligned to those standards.

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CHAPTER 1: INTRODUCTION

There is currently much debate regarding educational policy in the United States (US). The purpose of this thesis is to investigate issues and policies such as the types of standards that should be implemented, i.e., state standards or national standards, the manner in which those standards should be adopted and monitored, and ultimately, if said standards are successful in closing achievement gaps between the US and other countries performing at higher levels in Mathematics and Language Arts. This review will focus mainly in mathematics. International assessment data will also be examined in order to gain a better understanding of how countries are compared, and if those comparisons are reliable.

Furthermore, it is important to know what implications new standards, such as the Common Core State Standards (CCSS), may have on pedagogy. Teachers may need to learn new strategies in order to properly design and carry out lessons that align to the new standards. A second major focus of this thesis is to provide a unit design, aligned to the CCSS, in surface area and volume in 7th grade mathematics made by a teacher, for teachers. This unit is designed for post-state test implementation; and therefore includes 7th grade and 8th grade standards. This could also be implemented as a unit for a hybrid course where students are completing 7th, 8th, and 9th grade mathematics in two years. In this teacher's opinion, the modules for surface area and volume are lacking in many areas, creating a need for this unit design.

This unit serves as a curriculum for teachers to use, while implementing the standards. Standards alone state what students need to know, which helps in creating lesson objectives, but may not provide aide for teachers looking to make the best pedagogical decisions for students to achieve those lesson objectives. Thus a curriculum is provided that connects across content and the current CCSS.

The mission statement for the Common Core Initiative is as follows: "The Common Core State Standards (CCSS) provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy." – Council of Chief State School Officers and the National Governors Association Center for Best Practices ("Common Core State Standards: A Resource for SLPs, 2010). This thesis provides a unit design that embodies these ideals.

CHAPTER TWO: LITERARY REVIEW

The Obama administration has brought attention back to getting national standards in the United States of America. Those in favor of establishing these types of standards say that "fewer, higher, and clearer" benchmarks, along with assessments aligned with those benchmarks, will allow for greater accountability among schools (Burke, 2010). Many proponents of national standards argue that they will help to prepare students for college or the workforce more efficiently than the current education model of state standards (Burke, 2010).

According to (Burke, 2010), national standards may not fix the problems that the nation is facing. Burke explains that national standards may actually lessen the voice of parents and local policy makers, ultimately decreasing empowerment and ownership over education in communities. In fact, it is also plausible to believe that national standards may not target excellence, but rather cause a bar to be set for mediocrity (Burke, 2010). We will examine this further later on in the reading by comparing the results of three different states in the US.

The Common Core State Standards were built on an initiative by the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) (Burke, 2010). A major issue of these standards may be that, although they state that participation is voluntary, there are incentives attached to support the adoption of the CCSS. For example, states could only be eligible for a portion of the $4.35 billion in funding from a federal program called Race to the Top if they signed on to the CCSS (Burke, 2010). This incentive-based approach could certainly coerce states to adopt standards, regardless of efficiency. Failing to adopt the CCSS could cause school districts financial disadvantages, as well.

Another tactic used by the Obama Administration in order to get states to sign on to the CCSS was to tie Title I funds to them (Burke, 2010). Title I funds are those allocated to districts for the support of low-income students. Nearly every school district participates in this program, which accounts for $14.5 billion in federal funding (Burke, 2010). In February 2010, the announcement was made that any school that did not adopt the CCSS would not receive Title I funds. This goes a step further than the latter example by not just offering incentive, but actually penalizing states. Furthermore, the Obama Administration created a grant competition for states to create common assessments, which would replace state assessments. According to Pennsylvania Representative Glen Thompson, the Common Core is no longer a state-based initiative for volunteers, but rather a set of national standards with national assessments (Burke, 2010).

There are many possible misconceptions surrounding national standards. First, is the notion that national standards are automatically better than state counterparts, and they will make the US students more competitive globally (Burke, 2010). Assessments, such as the Trends in International Mathematics and Science Study (TIMSS), are used to evaluate students internationally. We delve deeper into the reliability of this and other international studies later in this review. Those who are in support of the US adopting national standards often note the fact that most of the countries who outperform the US on international assessments have national standards; however, this argument can be used oppositely, as many of the countries who perform below the US on international assessments have national standards, as well. Yet another counterexample of this logic is that Canada routinely outperforms the US on international assessments, yet they have no national standards (Burke, 2010).

Another possible misconception is the parents will have a better understanding of how their children are achieving academically compared to students across the country (Burke, 2010). When making a case for the implementation of national standards, proponents have not informed parents on exactly why the current standards are inadequate or unsuccessful. In addition, and perhaps more importantly, there have not been specific data to prove the proposed standards will improve the education situation in the US (Burke, 2010).

Although supporters of national standards will disagree, Burke argues that national standards are not necessary due to the variance in state standards. In fact, several states have standards that are highly regarded throughout the nation. There is still variability in state standards, but the reasons for the lack of quality at the state level may not disappear at the national level. National standards may up rigor in some states, while decreasing the quality of standards in others. This could lead to standards addressing the nation at the mean achievement level, rather than setting the bar higher (Burke, 2010).

When analyzing international assessments and standards of other nations, it may not be enough to look only at the data from the assessments. There are cultural differences that certainly may have an effect on outcomes. For example, a classroom in Korea may function much differently than one in the US. The instruction given by Korean teachers is often teacher-centered, but it is more systematic, complete, and progressive than American classroom instruction. In fact, students are typically more engaged and motivated during these lessons, and more likely to respond to the lessons with enthusiasm (Mi Choi, 2015). The fact that Korean education uses teacher-centered instruction, as well, may mean that the student-centered approach, heavily favored by many administrators, may not be the remedy. In either scenario, it may be dangerous to adopt and implement new educational standards within a nation without examining the culture and practices of that nation, and the possible outcomes based on cultural factors (Mi Choi, 2015).

Another variable that has the potential to skew data is socio-economic status (SES). According to Daniel Koretz, a professor at the Harvard Graduate School of Education, many believe that the reason US students are performing below expectations is because minority and low-income students, who are attending low-performing schools, are bringing the national average down by scoring lower than the national average on international exams; however, the results of the international assessments TIMSS and the Program for International Student Assessment (PISA) both show that American students' performance is not much different than countries with a more homogeneous sample of students, even if those countries have more equitable educational systems than the US (Koretz, 2009).

There is often controversy about just how useful international comparisons may be. International comparisons do provide data that can be analyzed, but they may not provide a clear evaluation of schools in the US (Koretz, 2009). TIMSS and PISA, the two major international assessments, differ in what, and who, they assess. PISA is an assessment which is intended to measure students' abilities to take what they have learned and use it in real-world applications. This means that the content of these exams does not closely resemble curricula within schools. The PISA exams are often measures of broad themes, rather than curricula specific, narrow topics. This means that the questions may not only be evaluating altered concepts, but that the test questions may look very different than those on typical curriculum-based tests. Furthermore, PISA assessments include results from fifteen-year-old students around the world, rather than by grade level (Koretz, 2009). The TIMSS, on the other hand, test students based upon their grade levels. In its inception year, the TIMSS assessed fourth-grade, eighth-grade, and students who were close to completion of secondary school. Questions on the TIMSS exams are designed to be closely related to school curricula, unlike PISA. For example, the TIMSS delegated approximately 25 percent of the questions to algebra on a recent test, while the PISA tested the topic throughout only 11% of the exam (Koretz, 2009).

Since the introduction of the TIMSS assessments, the United States Department of Education and other organizations have often compared the results of the US students with students of other countries by using the international mean (Koretz, 2009). For instance, on the first eighth grade exams, in 1995, American students scored slightly above the international mean in science, while scoring slightly below the international mean in mathematics (Koretz, 2009). Koretz argues that comparisons with an international average are not meaningful comparisons. The reason for this assertion is that in order for an average to be meaningful, it must be based on a clear comparison group. In national assessments, this may not be the case. Groups may change over time, which may move the average up or down (Koretz, 2009). This is evident when the TIMSS was administered again in 1999 and the US performed above the national mean in mathematics (Koretz, 2009). This begs the question of whether or not standards have anything to do with the results. The standards from 1995 to 1999 were unchanged in the US, yet there was a change in performance. Koretz suggests that comparisons should be made with nations who consistently perform at the top, such as Japan and Singapore, and with countries who have similar educational and social characteristics as the US, such as Australia and Canada. These comparisons have shown to be generally more stable over time (Koretz, 2009).

Another aspect of TIMSS that should be taken into consideration is the fact that it can measure very broad domains of achievement. For instance, this could mean a cumulative assessment of the first eight years of schooling. Due to the large domain needing to be tested, there are few test questions to decide mastery of each of the domains (Koretz, 2009). This could mean that several aspects of each domain remain untested. In addition, nations may perform better on certain domains than others. For example, the US and Australia did not perform as well in geometry than in the other areas of an international exam, while Singapore performed their best within the fractions and numbers domain. This means that if tests emphasize specific domains differently each year, then there may be variability in the results, based on the content itself (Koretz, 2009). For this reason, it is also important to consider data from several assessments, rather than relying on data from just one (Koretz, 2009).

There are trends that show reliability among test results, as well. Although, the TIMSS and the PISA can show conflicting results at times, there are also times where the results prove to be valid methods of comparison. For example, both the TIMSS and the PISA show consistent patterns when comparing the mathematics results of the US with the East Asian countries of Japan, Korea, Singapore, and Hong Kong (Koretz, 2009). While the margin of difference among each test varies slightly, the results consistently show that these countries outperform the US in mathematics on both the TIMSS and the PISA assessments (Koretz, 2009). It is important to note that countries that are similar to the US in many respects, such as England, Scotland, Sweden, and Australia had mean scores very similar to the US (Koretz, 2009).

The CCSS are both a result of and a facilitator states participating in international assessments (Stephens, 2014). Policymakers have identified a need for reform based on the results, and are looking to provide proof and accountability by continually partaking in these assessments and analyzing the results. In the past, states have been responsible for the education programs and the assessments that align to them; however, it appears as though the CCSS are attempting to replace state assessments with collaborative, national assessments. There is reason to believe that there will be positive aspects that come along with the CCSS, as well. For instance, common national assessments will represent the first time it will be possible to compare interstate results in learning and performance measures, based on the same learning standards and assessments (Stephens, 2014). It is possible that results from these assessments may provide data with less variability. The variable quality of learning standards and assessments under previous state educational systems made it very difficult to identify specific causes for high and low performance (Stephens, 2014). Based on the learning progressions of the CCSS, it is possible that with quality implementation and well-designed assessments, the US could see an increase in scores on the PISA assessment, which addresses a broader range of skills (Stephens, 2014).

When looking into the results of specific states, some conjectures may be able to be made about the implementation of the CCSS. Analysis of the data shows that students in the US performed below the Organization for Economic Cooperation and Development (OECD) average in the category of mathematical literacy (Stephens, 2014). When looking deeper into these results, Connecticut was actually a state that scored well above the US mean, and comparable to the international mean. In fact, only 12 of 68 countries/education systems participating in the assessment scored higher than students from Connecticut. Connecticut's scores were comparable to countries such as Canada, France, Germany, and the United Kingdom (Stephens, 2014). Massachusetts scored even higher than Connecticut, placing them above both the US average and the international average in the category of mathematical literacy. Only nine educational systems were able to outperform the results of Connecticut (Stephens, 2014).

Antithetically, Florida was not a strong performer on the same international assessment. Students from the state of Florida scored below both the US average and the international average in mathematical literacy. Florida was outperformed by 38 other educational systems, including Lithuania, Sweden, Hungary, Croatia, and Israel (Stephens, 2014).

The results by these three states may make it difficult to create a case for either state standards or national standards. When analyzing the results of Connecticut or Massachusetts, it seems obvious to make the claim that state standards are advantageous, and will allow students to outperform other countries on international assessments. In contrast, when analyzing the results of Florida, the proclamation that state standards are a hindrance to student success on international assessments seems relevant. There may not be a clear cut answer, but instead that there are many variables that make pinpointing the exact causes of success or failure extremely difficult. Furthermore, there may be an issue that comes with relying on the results of just one international test in order to make policy changes. For example, when analyzing the data of the TIMSS assessment in 2011, Massachusetts scored higher than Connecticut; however, the scores from the PISA assessment, just one year later, showed superior achievement by Connecticut (Stephens, 2014). Perhaps the fact that these tests assess different traits, coupled with the possibility that there is variance within state curricula provides some explanation for the contrast in results. These may all be examples of why it is important to analyze a larger sample size of data, and take into account the many other factors that go into educational research.

According to director of research projects at the National Association of Scholars, Michael Toscano, the CCSS did not take into account all of the possible positive and negative outcomes. In fact, he claims that they were implemented "quietly" and "hastily". In the Constitution, control over educational policy is regarded as a right reserved to the states, in order to serve the citizens of those states to the fullest. Yet, the shift to the CCSS did not involve states and citizens in the ways that the Constitution intended. In May 2012, a poll conducted by Achieve, a non-profit organization who was also a formal partner in creating and developing the CCSS, showed that 79 percent of American voters knew either “nothing” or “not much” about the Common Core (Toscano, 2013). Furthermore, Indiana state senator, Scott Schneider, had not even heard of the CCSS until two parents contacted him with complaints about the new curriculum. If this seems egregious, perhaps even more disturbing is the fact that Senator Schneider sat on the state Education Committee (Toscano, 2013). Schneider later commented that adoption of previous standards was transparent and comprehensive. Involvement was sought by parents, teachers, communities, and even business leaders. This was not the case for the CCSS (Toscano, 2013).

The CCSS are perhaps the most significant alterations in the history of American education. The CCSS are exactly that, standards; however, they are not a curriculum. CCSS do not offer a lot of detail as to how the standards will be tested, best practices on teaching the standards, or even sequencing of the standards. Instead, these aspects are left to states and districts to figure out on their own (Toscano, 2013). Previously in this review, the role of the NGA and CCSSO were discussed in the implementation of the CCSS. It is also important to note that the standards are largely due to influence from the private sector. The creation of the CCSS was, and still is, significantly backed by the Bill and Melinda Gates Foundation (Toscano, 2013).

Earlier in the review, Race to the Top funds and Title I funds were discussed, in the sense that the federal funds associated with them were connected to the adoption of the CCSS. When looking deeper into this situation, there is controversial evidence about the process in which the standards were accepted by states. For example, Race to the Top phase I applications were due on January 19th, 2010. The first draft of the CCSS was not posted until March 21, 2010 (Toscano, 2013). This means that states were applying for federal funds, knowing that they had to adopt the CCSS in order to receive them, before they even saw a draft of the standards. Furthermore, applications for phase II were due on June 1, 2010, yet the final draft of the CCSS was not published until the next day. In summation, phase I applicants committed to the CCSS before ever seeing a draft of what the standards entailed or even looked like, and phase II applicants adopted the standards with only two months to review and evaluate the drafted copy of them (Toscano, 2013).

There is an internal democratic process that is supposed to go on within states when education reform is in action, but this may not have been the case in the scenario with the CCSS. Many legislatures were not in session during the application period, and therefore could not weigh in on the matter (Toscano, 2013). The short time frame, coupled with this fact, took the power of advocacy away from local communities and parents. Furthermore, there is uncertainty about how state curricula will evolve. It is possible that it will not evolve to reflect the desires and ideals of communities and the parents living in them, but rather curricula may be designed based solely on each district's interpretation of the CCSS, and the assessments aligned with them (Toscano, 2013).

Another controversial topic surrounding the CCSS is the issue of accountability. The CCSS is copyrighted by private organization. These organizations, the NGA and CCSSO, are not subject to the same accountability and transparency laws and requirements as elected officials. Up until now, there is yet to be a system in place to hold states accountable for the assessments administered. Furthermore, there is no structure to provide ongoing governance of even a single aspect of the CCSS. If parents or participating states have an issue, they have no legal recourse due to the fact that the copyrighters of the standards are organizations that have no political authority, and are therefore not bound by the same laws, either (Toscano, 2013).

The United States Department of Education has actually had little to do with the development of the CCSS, despite giving the CCSS their full support. The NGA, CCSSO, and Achieve have all received millions of dollars from the Gates Foundation to create the standards. As of February 2013, Gates had invested $163 million to develop the CCSS and to get lawmakers and business leaders to support them. Now, due to the substantial contribution of one, private sector member of society, the education of forty million American students has been altered drastically (Toscano, 2013).

The major issue many states face is the lack of research and data. For example, the CCSS was introduced with incomplete modules on the Engage NY website, and seemed to take a "learn as you go" approach. In fact, the approach seems "experimental" in nature (Toscano, 2013). Many of the pieces within the CCSS involve untested pedagogy. In February 2013, Diane Ravitch, who has been an advocate of national standards, stated that she would not support the CCSS. Her reasoning is that the standards are being implemented, even though the possible effects on students, teachers, or schools are widely unknown, due to a lack of research and information. Ravitch compares American students to guinea pigs based on this system of implementation (Toscano, 2013).

James Milgram, professor emeritus of mathematics at Stanford, sat on the Common Core Validation Committee, yet refused to sign off on the standards in 2010. He too felt that the CCSS were too experimental, and required more research. Milgram made this claim in direct relation with the new style of geometry the CCSS implemented. He stated that this type of geometry would generally be taught at the college level. A similar geometry outline was followed by Russia in the past, and it was quickly rejected, because it was unsuccessful (Toscano, 2013). A similar type of issue with the CCSS is that there are grade-sequencing problems in certain areas. Specific mathematics skills may be required at a specific grade level, but the prerequisite skills are not present in previous grade levels (Burke, 2010).

There is a possibility that the CCSS may have adverse effects on teacher autonomy. Teacher's freedom to create their own curriculum, craft their own tests, and teach their own lessons is in jeopardy. Instead, private companies, such as Pearson, are hired to create tests that teachers must align their instruction towards (Wexler, 2014). This also means that student teachers will now need tailored instruction at the college level. Due to the reform called edTPA, teacher candidates will be evaluated using certification tests, specifically aligned to the CCSS. College professors will need to teach to those tests in order to prepare teacher candidates. This may show that the reach of the CCSS goes beyond that of the high school classroom and into the college classroom. It is certainly possible that college professors may also have a loss of autonomy because they must now make ample changes to curricula in order to teach to Pearson-created assessments (Wexler, 2014).

According to Dr. Holt P. Wilson, Assistant Professor in the Department of Teacher Education and Higher Education at the University of North Carolina, the CCSS implementation method is similar to "building a plane while flying it." However, Dr. Wilson is optimistic that there will be better efforts to provide teachers with opportunities to learn about and engage with the learning progressions in the CCSS. He also believes that there will be more opportunities for teachers to learn and implement new pedagogical strategies in the classroom. Students now need to understand conceptual information, rather than just memorization. It is important for teachers to not only be masters of their content, but to be facilitators of higher level thinking. They also need to understand the progressions of the CCSS, so that they can facilitate the necessary thought processes and learning that needs to take place in order to access prior knowledge and learn new concepts (Wilson, 2014).

Differentiation is a key component of implementing the CCSS. There is not one style of teaching that is going to reach all students, nor is there one style of teaching that is going to help students to gain all types of knowledge (Wilson, 2014). Piaget defined two types of knowledge: social-conventional knowledge and logical-mathematical knowledge. For social-conventional knowledge the source of ideas is outside of the learner. Mathematics examples of this type of knowledge include things such as mathematical vocabulary and notation. Direct instruction is a great tool to teach this type of knowledge. Conversely, logical-mathematical knowledge must be obtained by students through the use of prior knowledge and problematic situations. Student-centered instruction should be utilized in order for students to achieve at the highest level in this area. Students need to develop the skills to utilize multiple approaches and make connections through investigation (Wilson, 2014).

Wilson offers many recommendations in order for the CCSS to be successful. Teachers should be provided professional development on learning progressions, and how students will gain knowledge along the way. Teachers should have time to examine students' mathematical development across grade levels, and should have conversations about how to implement strategies to make transitions seamless. Administration should support teachers in their learning of effective instructional techniques for mathematics concepts. Time should be given to teachers to collaborate with teachers in other disciplines to discuss strategies, such as literacy strategies.

Teachers should try new practices, with guidance through scaffolding. Finally, teachers should be given examples of exemplary mathematics instruction of the CCSS (Wilson, 2014).

In an interview of mathematics and English teachers, they all agreed that the CCSS have forced them to rethink their approaches to teaching, and to make necessary adjustments (VanTassel-Baska, 2016). They also point out the fact that interdisciplinary approaches are much more common with the CCSS. Differentiating instruction is reiterated as a key component of teaching the CCSS. Reaching students with disabilities requires careful thought when they are mixed in a heterogeneous classroom of students. Furthermore, reaching talented or gifted students also requires differentiation. The CCSS demand increased rigor, requiring teachers to help facilitate the needs of all learners by exposing them to higher level questioning and incorporating deeper, richer tasks (VanTassel-Baska, 2016). Co-teaching models are also becoming more popular to meet the needs of heterogeneously grouped students. This model may also make it easier to utilize tiered assignments, which assign projects of varying difficulty to each of the student groups, based on ability. This allows for differentiation, while aligning curricula to the CCSS (VanTassel-Baska, 2016).

The CCSS also provide teachers the opportunity to teach multi-disciplinary lessons (VanTassel-Baska, 2015). This means that a project can incorporate both mathematics and English. A possible example could be a project which addresses the research stand in ELA and the data representation strand in mathematics. Students could research issues, formulate questions, identify and analyze data, and create tables, graphs or other visuals (VanTassel-Baska, 2015).

Real-world applications are important for students to grasp concepts in the CCSS. Something that should be incorporated along with this idea is the use of visual supports (Fraser, 2013). This is especially true for students with special needs. Visual supports can help students by promoting independence, task completion, and increasing the number of tasks students are able to complete in a given time. Regardless of the pedagogical style, the lesson design, or the specific standards being assessed, it is important that students know exactly what the task is that they are to complete. This means that clear, concise objectives need to be set before the tasks are given (Fraser, 2013).

Only time will tell if the revolutionary change to the CCSS is a success or a failure. This review has provided insight into the controversial topics of national versus state standards, international assessments and data analysis, some of the literature pertaining to the implementation of the CCSS, the implications that come along with the adoption of the standards, and the teaching techniques necessary to provide students with the best education possible. Regardless of the ethics associated with the adoption of the standards, it is the job of teachers to make sure that all students are receiving an education which allows them to achieve at their maximum potential.

CHAPTER THREE: UNIT PLAN

This curriculum is connected tightly to the new CCSS and EngageNY, the materials that NYS generated to support the instruction of the new CCSS. This curriculum provides lessons, assessments, and other activities so that teachers can have a plan for implementing standards-based objectives. The curriculum includes warm-up questions, which engage prior knowledge necessary to apply to new situations. There are hands-on activities for teachers to support students as they work to conceptualize and make conjectures on their own. The lessons are designed to provide discussion, modeling, small group-work, and independent practice for differentiation of instruction. This curriculum includes real-world situations. When students are also exposed to real-world situations they may transfer knowledge within and across content in a more fluid way (Van Merrienboer, 2006).

LESSON 1: SURFACE AREA OF RECTANGULAR PRISMS:

Lesson designed for a 50 minute class period.

Common Core State Standards:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
    - 7.G.6 – Solve real-world and mathematical problems involving area, volume and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Objectives:

1) Students will discover the formula for calculating the surface area of a rectangular prism through a hands-on, investigation activity. They will use prior knowledge of two-dimensional shapes to aid in this discovery.

2) Students will be able to calculate the surface area of a rectangular prism given the dimensions, and will be able to find a missing dimension given the surface area and the other dimensions.

Materials for Group Activity:

- A rectangular prism for each group (jewelry box, tea box, toy box, tissue box, etc.)
- Construction Paper
- Scissors
- Tape
- Rulers
- Pre-cut box for demonstration ("net" of rectangular prism)
- SMART board (optional) to show interactive net

Outline:

1) Warm-Up. Access prior knowledge that is vital to the lesson.

2) Investigation activity to discover the surface area formula (See appendix for implementation details).

3) Two problems finding the surface area using modeling and questioning. One problem is very straight forward with a visual, and the next is a word problem with no visual. It also does not use the words surface area.

4) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

5) Two problems finding a missing dimension using modeling and questioning. Again, one problem is very straight forward with a visual, and the next is a word problem with no visual.

6) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

7) Students will have independent practice, and what they do not complete will be assigned for homework.

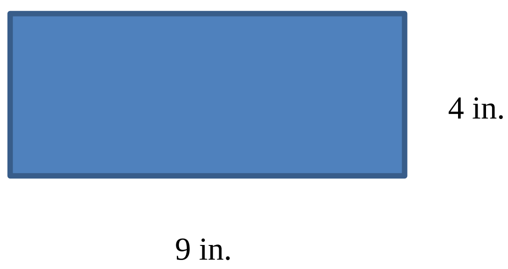
8) For a closure assessment, the teacher should choose a problem from the independent practice for students to complete and show the teacher before leaving the classroom (exit ticket).

NAME____DATE____________

LESSON 1: SURFACE AREA OF RECTANGULAR PRISMS

WARM-UP:

1. Find the area of the following rectangle. Show the formula you use, along with the substitution of values into that formula.



2. In your own words, explain the definition of a prism.

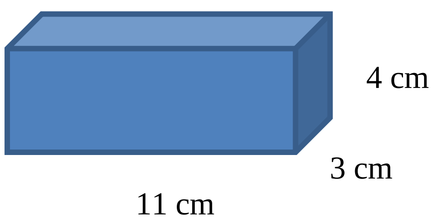
3. Give a real-life example of a three-dimensional shape that is a rectangular prism, and explain why it is a rectangular prism.

Introduction to SA of Rectangular Prisms: Group Investigation, Discussion, and Interactive Visuals to Develop the Formula

FORMULA:

Two Together:

1. Find the surface area of the following rectangular prism.

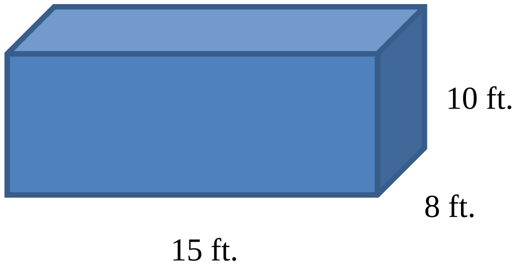


2a. Sue has just purchased a new pair of shoes for her grandmother. The measurements of the box they came in were 17” x 10” x 9”. How much wrapping paper is required to wrap the gift?

b. How much wrapping paper would Sue need if she purchased three of the same pairs of shoes for gifts?

Two with Your Partner:

3a. Find the surface area of the following rectangular prism.



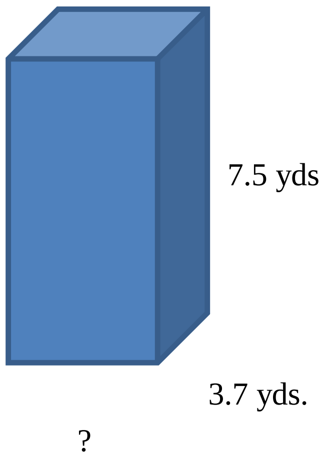
b. What could this rectangular prism represent in the real world, based on the given measurements?

4a. Rita is painting a room that is in the shape of a rectangular prism. The room has a length of 20’, width of 16’, and a height of 10’. How many square feet does Rita have to paint in order to paint the walls and the ceiling (she is not painting the floor)?

b. If one can of paint costs $28.97, and it covers an area of 400 sq. ft., then how much will Rita spend on paint in order to cover the entire room, if sales tax is 8%?

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

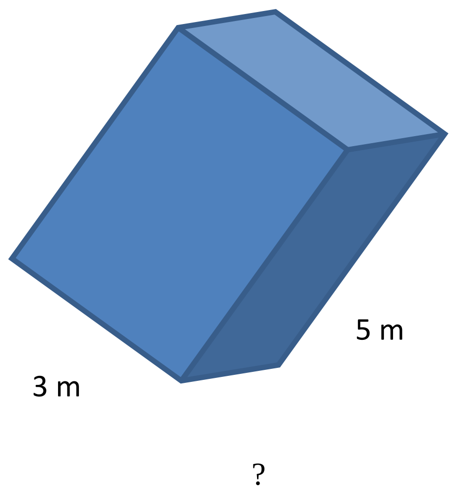
5. The surface area of the following rectangular prism is 171.98 square yards. The height is 7.5 yds. and the width is 3.7 yds. Find the length of the rectangular prism to the nearest tenth of a yard.



6. James is building a prop for his school play. Part of the prop is a wooden block that is going to be fully covered in construction paper. James needs to make the block 4 cm long and 7cm wide. If James has a total of 320 sq. cm. of construction paper, then how tall can he make the block, in order for it to be completely covered?

Two with Your Partner:

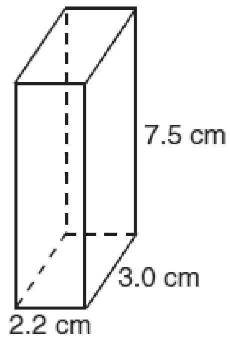
7. The surface area of the following rectangular prism is 70 square meters. Find the missing dimension.



8. Rusty makes his own cheese. He wants to put food coloring on the outside of one of his blocks of cheese, in order to make it look blue. If the amount of food coloring Rusty has will cover a surface area of 74.6 square inches, then what must the third dimension of the cheese block be if the other two dimensions are 1.5 in. and 9.8 in.? Round your answer to the nearest tenth of an inch.

Individual Practice/Homework:

1. The rectangular prism shown below has a length of 3.0 cm, a width of 2.2 cm, and a height of 7.5 cm.

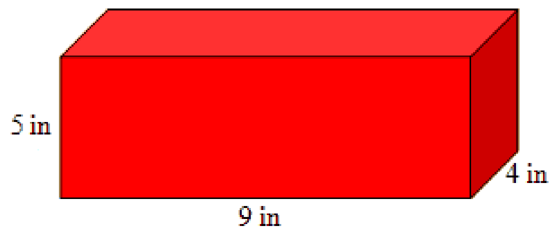


What is the surface area, in square centimeters?

1. 45.6
2. 49.5
3. 78.0
4. 91.2

2. The length and width of the base of a rectangular prism are 5.5 cm and 3 cm. The height of the prism is 6.75cm. Find the exact value of the surface area of the prism, in square centimeters.

3. Find the surface area of the prism.



1. 202 in^2
2. 180 in^2
3. 98 in^2
4. 360 in^2

4. Tony has an empty room in his house. The length of the room is 8 feet and the height is 10 feet. If he wants to wallpaper all four walls of the room, how many square feet of wallpaper should Tony buy? [Ignore the fact that there may be windows or doors in the walls.]

1. 320 ft^2

2. 640 ft^2

3. 448 ft^2

4. 256 ft^2

5. How much wrapping paper is needed to cover a rectangular box that is 18 inches by 16 inches by 4 inches?

1. 1,152 in^2 2. 76 in^2 3. 848 in^2 4. 38in^2

6. Detra has a box in the shape of a cube, with each side measuring 7 inches. What is the surface area of the cube?

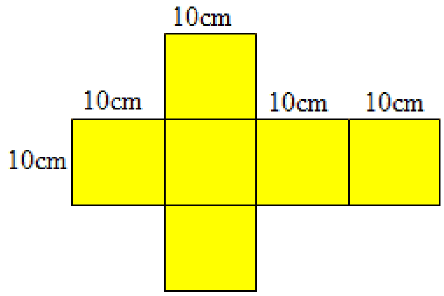
1. 42 in^2

2. 49 in^2

3. 343 in^2

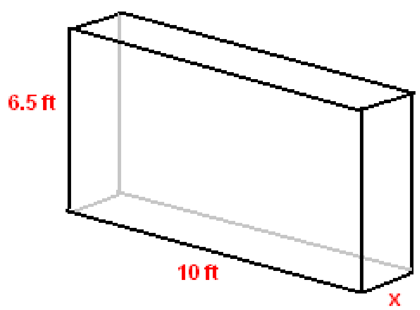
4. 294 in^2

7. Determine the surface area for the prism formed by the following net.



1. 50 cm^2
2. 10,000 cm^2
3. 600 cm^2
4. 100 cm^2

8. If the surface area of the box in the diagram is 204.25 square feet, what is the value of '*x*'?



1. 3 ft
2. 2.25 ft
3. 4.5 ft
4. 5 ft

9. Find the surface area of a storage tote that is in the shape of a rectangular prism. The tote is 108 cm long, 78 cm wide, and 52cm tall. The tote does not have a top.

LESSON 2: SURFACE AREA OF CYLINDERS:

Lesson designed for a 50 minute class period.

Common Core State Standard:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
    - 7.G.4 – Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
    - 7.G.6 – Solve real-world and mathematical problems involving area, volume and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Objectives:

1) Students will discover the formula for calculating the surface area of a cylinder through a hands-on, investigation activity. They will use prior knowledge of two-dimensional shapes to aid in this discovery.

2) Students will be able to calculate the surface area of a cylinder given the dimensions, and will be able to find a missing dimension given the surface area and the other dimension.

Materials for Group Activity:

- A cylinder for each group (soup can, juice concentrate can, etc.)
- Construction Paper
- Scissors
- Tape
- Rulers
- Pre-cut cylinder for demonstration ("net" of cylinder)
- SMART board (optional) to show interactive net

Outline:

1) Warm-Up. Access prior knowledge that is vital to the lesson.

2) Go over homework. The approach to this is left to the teacher. Depending on the students, the teacher may choose to give the answers and take questions, go over each problem in detail, have students come to the board and explain their answers, have students share their answers in small groups, or collect the homework for a more formal assessment.

3) Investigation activity to discover the surface area formula (See appendix for implementation details).

4) Two problems finding the surface area using modeling and questioning. One problem is very straight forward with a visual, and the next is a word problem with no visual. It also does not use the words surface area.

5) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

6) Two problems finding a missing dimension using modeling and questioning. Again, one problem is very straight forward with a visual, and the next is a word problem with no visual.

7) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

8) Students will have independent practice, and what they do not complete will be assigned for homework.

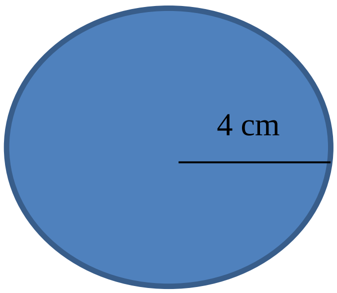
9) For a closure assessment, the teacher should choose a problem from the independent practice for students to complete and show the teacher before leaving the classroom (exit ticket).

NAME____DATE____________

LESSON 2: SURFACE AREA OF CYLINDERS

WARM-UP:

1. Find the area and circumference of the following circle. Show the formulas you use, along with the substitution of values into those formulas.



2. In your own words, explain the definition of a cylinder.

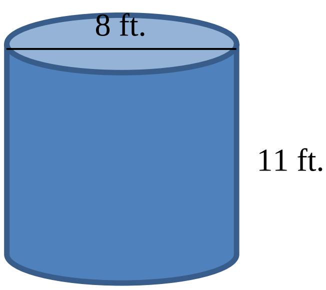
3. Give a real-life example of a three-dimensional shape that is a cylinder, and explain why it is a cylinder.

Introduction to SA of Cylinders: Group Investigation, Discussion and Interactive Visuals to Develop the Formula

FORMULA:

Two Together:

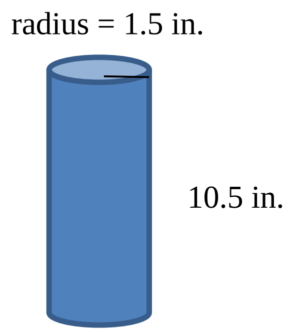
1. Find the surface area of the following cylinder. Round your answer to the nearest tenth of a square ft.



2. Vicki has one hundred identical cans of vegetables for a holiday can drive. She wants to paint the cans green and red, so that they look festive. She has enough spray paint to cover 8,000 square inches of metal. If the cans are 7 inches tall and have a 1.5 in radius, then does Vicki have enough spray paint to cover all the cans? Explain your reasoning.

Two with Your Partner:

3. Find the surface area of the following cylinder. Leave your answer in terms of π.

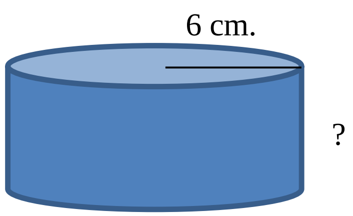


b. What could this cylinder represent in the real world, based on the given measurements?

4. Dave and three of his friends are putting a liner on his cylindrical, above ground pool. If the pool is 14' in diameter and 5.5' deep, then what is the smallest liner that Dave must purchase in order to line his pool? (Think: Would it make sense to put a liner over the top of the water.)

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

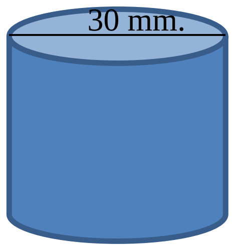
5. The surface area of the following cylinder is 528 square centimeters. The radius is 6 cm. Find the height of the cylinder. Round your answer to the nearest centimeter.



6. Kyla is covering a cylindrical object with materials she got at a craft store in order to make a carrier. She is going to cover the entire cylinder, and then cut the top, so that it acts as a cover that can be placed on and off. Kyla has a total of 800 square inches of material. If the radius of the container is 4.5 inches, what is the maximum height the container can be in order to cover the entire thing without running out of material? Round your answer to the nearest hundredth of an inch.

Two with Your Partner:

7. The surface area of the following cylinder is 4400 square millimeters. Find the height if the diameter is 30mm. Round your answer to the nearest tenth of a millimeter.

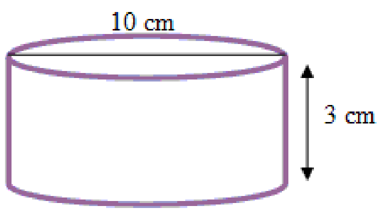


8. The surface area of a cylindrical candle is 215 square inches. If the candle has a radius of 2 inches, then what is the height of the candle? Round your answer to the nearest tenth of an inch.

Individual Practice/Homework

1.

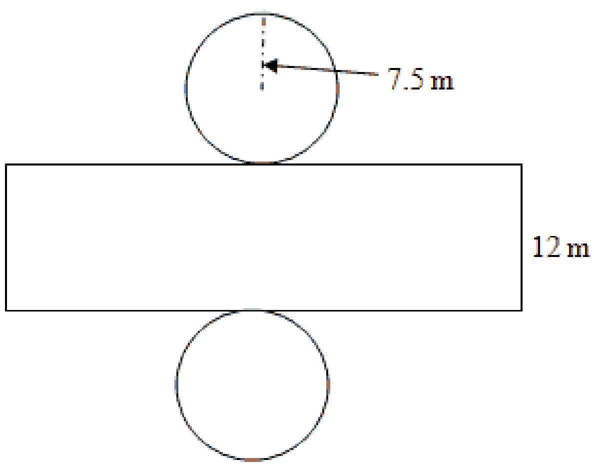
Find the surface area of the cylinder. Use 3.14 for π and round your answer to the *nearest square centimeter.*



1. 942 cm^2
2. 236 cm^2
3. 204 cm^2
4. 251 cm^2

2.

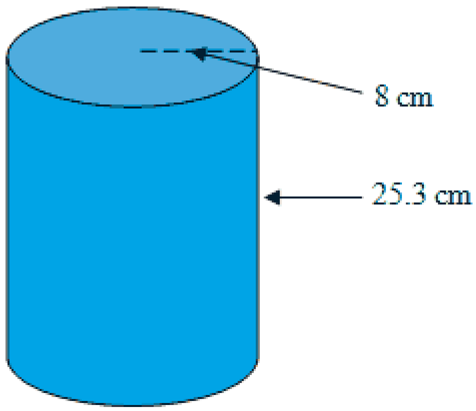
Find the surface area of the cylinder formed by the net to the nearest *tenth*. Use 3.14 for π.



1. 90 m^2
2. 282.6 m^2
3. 857 m^2
4. 918.5 m^2

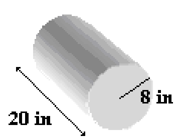
3.

Find the surface area of the cylinder to the nearest *tenth*. Use 3.14 for π.



1. 1673.0 cm^2
2. 202.4 cm^2
3. 636.0 cm^2
4. 33.3 cm^2

4. Find the surface area of the cylinder. Use the π key on your calculator and round your answer to the *nearest square inch*.



5. The surface area of a cylinder is 250 square inches. If the radius of the can is 2.5 inches, then how tall is the can to the nearest inch?

LESSON 3: VOLUME OF RECTANGULAR PRISMS:

Lesson designed for a 50 minute class period.

Common Core State Standards:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
    - 7.G.6 – Solve real-world and mathematical problems involving area, volume and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Objectives:

1) Students will discover the formula for calculating the volume of a rectangular prism through a hands-on, investigation activity. They will use prior knowledge of two-dimensional shapes to aid in this discovery.

2) Students will be able to calculate the volume of a rectangular prism given the dimensions, and will be able to find a missing dimension given the volume and the other dimensions.

Materials for Group Activity:

- 5 Post-It Note packs for each group
- Rulers
- Flat rectangular surface for demonstration
- SMART board (optional) to show interactive

Outline:

1) Warm-Up. Get students thinking about the units in which volume will be measured.

2) Go over homework. The approach to this is left to the teacher. Depending on the students, the teacher may choose to give the answers and take questions, go over each problem in detail, have students come to the board and explain their answers, have students share their answers in small groups, or collect the homework for a more formal assessment.

3) Investigation activity to discover the volume formula (See appendix for implementation details). Re-visit the warm-up activity to show the units required to measure the volume.

4) Two problems finding the volume using modeling and questioning. One problem is very straight forward with a visual, and the next is a word problem with no visual. It also does not use the word volume.

5) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

6) Two problems finding a missing dimension using modeling and questioning. Again, one problem is very straight forward with a visual, and the next is a word problem with no visual.

7) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

8) Students will have independent practice, and what they do not complete will be assigned for homework.

9) For a closure assessment, the teacher should choose a problem from the independent practice for students to complete and show the teacher before leaving the classroom (exit ticket).

NAME____DATE_____________

LESSON 3: VOLUME OF RECTANGULAR PRISMS

WARM-UP:

1. In your own words, explain the definition of a cube.

2. Based on this definition, what do you suppose a cubic foot represents?

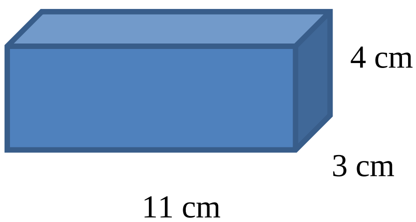
3. What about a cubic inch?

Intro. to Volume of Rectangular Prisms: Discussion and visuals to develop the formula

FORMULA:

Two Together:

1. Find the volume of the following rectangular prism.

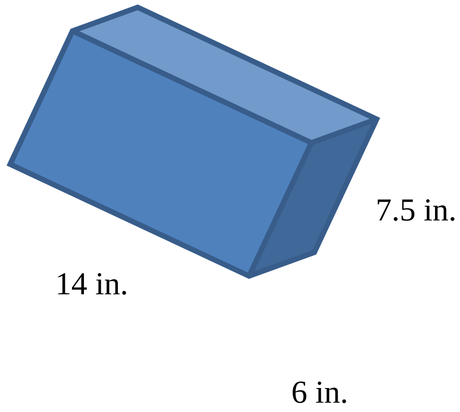


2a. A cereal box is 10” x 3.5” x 14.5”. What is the maximum amount of cereal the box can hold?

b. If a company produces 150,000 cubic inches of cereal, then how many boxes will need to be manufactured in order to package all of the cereal (we will assume boxes are filled to capacity for the sake of the exercise)?

Two with Your Partner:

3. Find the volume of the following rectangular prism.

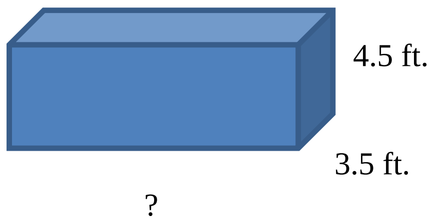


4a. Jacob wants to fill a fish tank with water. The tank is 4 feet long, 2 feet wide, and 3 feet high. How many cubic feet of water can Jacob fit into the tank if he fills it to the top?

b. If Jacob is filling the tank from a faucet with a rubber hose attached, then how long will it take to fill the tank if the faucet fills at a rate of 5 cubic feet per 2 minutes?

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

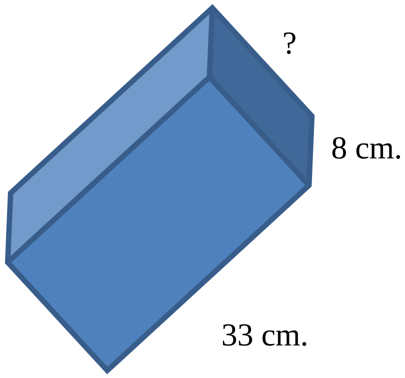
5. The volume of the following rectangular prism is 149.625 cubic cm. Find the missing dimension.



6. Mira is putting in a rectangular patio. In order to make the patio, the builders need to dig the dirt to make it flat, and pile stone evenly to the necessary height. This creates a rectangular prism with the base being the ground, and the top being where the patio will be. Stone will fill in between. If the builder has 8 cubic yards of stone, then how high up can the stone be piled evenly if the length of the patio is 9 yards and the width is 4 yards? Round your answer to the nearest hundredth of a yard.

Two with Your Partner:

7. The volume of the following rectangular prism is 3,960 cubic cm. Find the missing dimension.

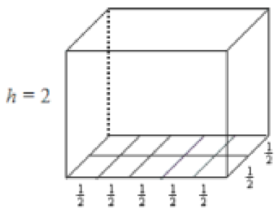


8. Jacob is now looking to buy a new fish tank. Based on how many fish he plans on placing in the fish tank, Jacob has determined that he will need 2,800 cubic inches of water. The length of the fish tank is 36 inches, and the width of the fish tank is 18 inches. Jacob has a shelf 4 feet above the table he wants to place the fish tank. Will Jacob be able to purchase the fish tank necessary to contain the required amount of water, and be placed in the spot that Jacob has chosen for the tank to stay?

Individual Practice/Homework

1.

The base of the rectangular prism has been divided into 10 squares. Each square has a side length of $\frac{1}{2}$ centimeter. The height of the prism is 2 centimeters.

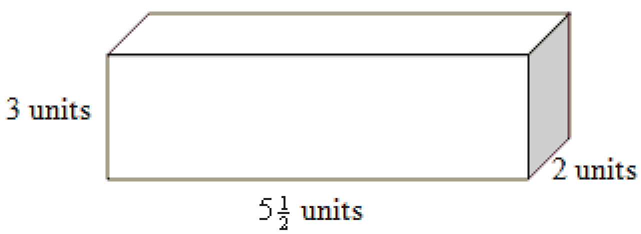


Find the area of the base and the volume of the rectangular prism.

1. Area of the base = $2\frac{1}{2}$ cm^2; Volume of the prism = 5 cm^3
2. Area of the base = 10 cm^2; Volume of the prism = 20 cm^3
3. Area of the base = 5 cm^2; Volume of the prism = 10 cm^3
4. Area of the base = $\frac{1}{4}$ cm^2; Volume of the prism = $\frac{1}{2}$ cm^3

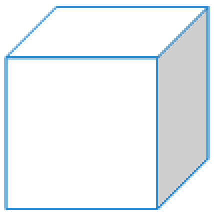
2.

Find the volume of the rectangular prism.



1. 30 $units^3$
2. $30\frac{1}{2}$ $units^3$
3. 33 $units^3$
4. 35 $units^3$

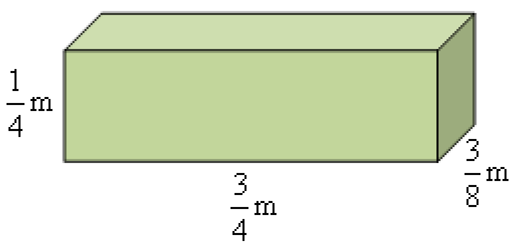
3. What are the edge lengths of a cube whose volume is $\frac{1}{27}$ cubic meter?



Volume = $\frac{1}{27}$ m^3

1. $\frac{1}{3}$ m^3
2. $\frac{1}{9}$ m^3
3. $\frac{1}{3}$ m
4. $\frac{1}{9}$ m

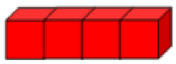
4. What is the volume of the right rectangular prism pictured below?



1. $\frac{11}{8}$ m^3
2. $\frac{3}{16}$ m^3
3. $\frac{9}{128}$ m^3
4. $\frac{9}{32}$ m^3

5.

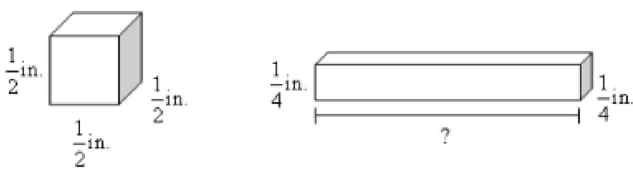
Four red cubes are arranged to form a rectangular prism. What is the volume of the rectangular prism if each individual cube has an edge length of $\frac{1}{3}$ inch?



1. 1 in.^{3}
2. $\frac{1}{27}$ in.^{3}
3. $\frac{1}{3}$ in.^{3}
4. $\frac{4}{27}$ in.^{3}

6.

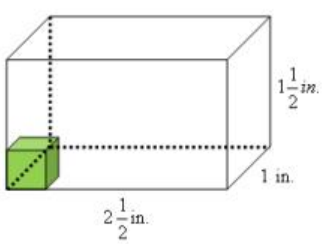
What length is the missing measurement of the rectangular prism on the right that would make its volume equal to the volume of the cube on the left?

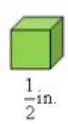


1. 1 in.
2. 2 in.
3. 3 in.
4. 4 in.

7.

The diagram shows a rectangular prism with edge lengths of $2\frac{1}{2}$ inches, 1 inch, and $1\frac{1}{2}$ inches. It also shows a green cube with edge lengths of $\frac{1}{2}$ inch.



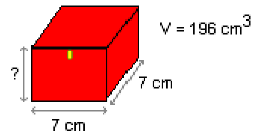


At most, how many green cubes will fit into the rectangular prism?

1. $3\frac{3}{4}$ green cubes
2. 30 green cubes
3. 15 green cubes
4. 5 green cubes

8.

Trudy knows that the volume of her jewelry case is 196 cm^3, its length is 7 cm, and its width is 7 cm. What is its height?



LESSON 4: VOLUME OF CYLINDERS:

Lesson designed for a 50 minute class period.

Common Core State Standard:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
  - 7.G.4 – Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
  - 8.G.9 – Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Objectives:

1) Students will discover the formula for calculating the volume of a cylinder through a hands-on, investigation activity. They will use prior knowledge of two-dimensional shapes to aid in this discovery.

2) Students will be able to calculate the volume of a cylinder given the dimensions, and will be able to find a missing dimension given the volume and the other dimension.

Materials for Group Activity:

- 10 to 20 poker chips for each group
- A paper circle cut to fit the size of the poker chip
- A Flintstone’s push-pop or something similar
- Rulers
- A flat circular surface for a demonstration
- SMART board (optional) to show interactive

Outline:

1) Warm-Up. Access prior knowledge that is vital to the lesson.

2) Go over homework. The approach to this is left to the teacher. Depending on the students, the teacher may choose to give the answers and take questions, go over each problem in detail, have students come to the board and explain their answers, have students share their answers in small groups, or collect the homework for a more formal assessment.

3) Investigation activity to discover the volume formula (See appendix for implementation details). Revisit the warm-up to discuss what term in the formula also shows that you will have a non-integer solution (pi).

4) Two problems finding the volume using modeling and questioning. One problem is very straight forward with a visual, and the next is a word problem with no visual. It also does not use the word volume.

5) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

6) Two problems finding a missing dimension using modeling and questioning. Again, one problem is very straight forward with a visual, and the next is a word problem with no visual.

7) Two problems for students to compute with their partners while teacher is monitoring. Then, call on students to share their processes and results with the class.

8) Students will have independent practice, and what they do not complete will be assigned for homework.

9) For a closure assessment, the teacher should choose a problem from the independent practice for students to complete and show the teacher before leaving the classroom (exit ticket).

NAME____DATE____________

LESSON 4: VOLUME OF CYLINDERS

WARM-UP:

1. Based on the previous lesson, where we found that volume is measured in cubic units, do you think that cubes will fit into a cylinder perfectly? Explain your reasoning.

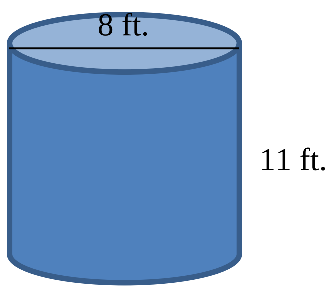
2. If volume is still measured in cubic units, regardless of the container, then what effect do you think dealing with a cylinder will have on our numerical answers, meaning, what type of numbers might you expect to get for answers? Explain your reasoning.

Introduction to Volume of Cylinders: Discussion and visuals to develop the formula

FORMULA:

Two Together:

1. Find the volume of the following cylinder. Round your answer to the nearest tenth of a cubic foot.

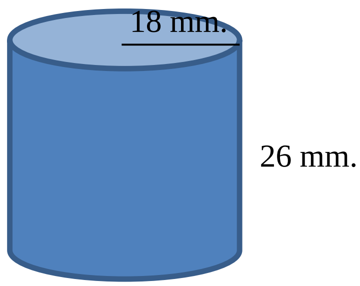


2a. Angelica has an above ground pool in the shape of a cylinder. The pool is 16’ in diameter and 5’ tall. What is the maximum amount of water that can fit in the pool? Round your answer to the nearest hundredth of a cubic foot.

b. If Angelica wants to fill the pool in order to leave 6” of space between the top of the water and the top of the sides, then how much less water would she need to fill the pool? Round your answer to the nearest cubic foot.

Two with Your Partner:

3. Find the volume of the following cylinder. Leave your answer in terms of π.

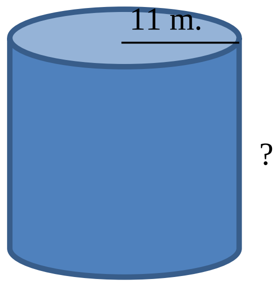


4a. An oil company fills cylindrical barrels with oil. If the barrels are 4ft. tall and 3ft. in diameter, then how much oil can each barrel hold. Leave your answer in terms of π.

b. If the company has 10,000 cubic feet of oil to distribute, then how many barrels will they need?

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

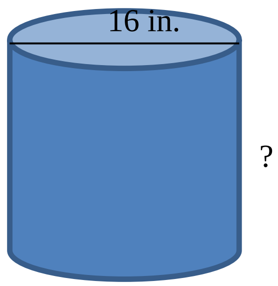
5. The volume of the following cylinder is 950.8 cubic meters. If the radius is 11 meters, then what is the height of the cylinder? Round your answer to the nearest tenth of a meter.



6. Tina has a cylindrical storage container that needs to hold 320 cm^3 worth of objects. She knows the diameter of the container is 9 centimeters, but she is unaware of the height. To the nearest tenth, what is the smallest value the height can be in order to fit her needs?

Two with Your Partner:

7. The volume of the following cylinder is 156.2 cubic in. If the diameter is 16 inches, then what is the volume of the cylinder? Round your answer to the hundredth of an inch.



8a. The volume of a cylindrical water tower is 36,000 square feet. If the radius of the water tower is 15 feet, then what is the height of the tower? Round your answer to the nearest foot.

8b. The town the tower supplies the water to has 500 people. On average, how many cubic feet of water does each person get to use from that water tower?

Individual Practice/Homework

1. Mike buys his ice cream packed in a rectangular prism-shaped carton, while Carol buys hers in a cylindrical-shaped carton. The dimensions of the prism are 5 inches by 3.5 inches by 7 inches. The cylinder has a diameter of 5 inches and a height of 7 inches.

Which container holds more ice cream? Justify your answer.

Determine, to the *nearest tenth of a cubic inch*, how much *more* ice cream the larger container holds.

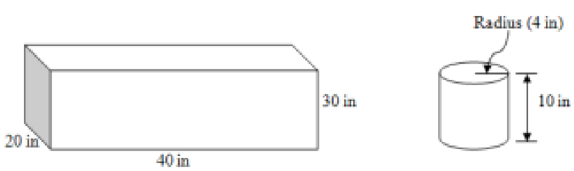
2.

A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

1. 72.3
2. 83.1
3. 180.6
4. 722.6

3.

In the diagram, a rectangular container with the dimensions 20 inches by 30 inches by 40 inches is to be filled with water, using a cylindrical cup whose radius is 4 inches and whose height is 10 inches.

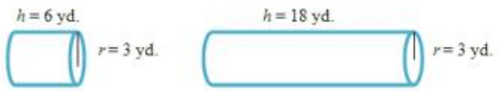


What is the maximum number of full cylindrical cups of water that can be placed into the container without the water overflowing the container?

1. 600 cups
2. 599 cups
3. 48 cups
4. 47 cups

4.

The radius of a cylinder is 3 yards and its height is 6 yards, shown below left. The height of the new cylinder is tripled but its radius remains the same, shown below right.

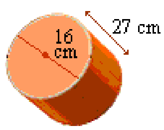


What effect will this change have on the volume of the new cylinder?

1. The new volume will be 3 times larger than the original volume.
2. The new volume will be 6 times larger than the original volume.
3. The new volume will be 9 times larger than the original volume.
4. The new volume will be 27 times larger than the original volume.

5.

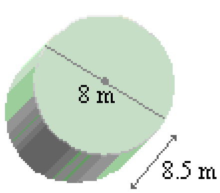
Jordan got a new water bottle to use for soccer. Approximately how much water can Jordan fit in the water bottle?



1. 1,728 cm^3
2. 5,429 cm^3
3. 1,356 cm^3
4. 21,703 cm^3

6.

Acme Pipe Corporation had to replace a section of conduit.



What is the volume of the pipe that had to be replaced, to the *nearest cubic meter*?

7.

A soup can is in the shape of a cylinder. The can has a volume of 342 cm^3 and a diameter of 6 cm. Express the height of the can in terms of π.

1. 12π cm
2. 38π cm
3. 12 cm
4. $\frac{38}{\pi}$ cm

<u>LESSON 5: CEREAL BOX LAB</u>:

Lesson designed for a 50 minute class period.

Common Core State Standards:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
    - 7.G.6 – Solve real-world and mathematical problems involving area, volume and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
    - 8.G.9 – Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Objectives:

1) Students will utilize what they have learned about surface area and volume to solve and real-world problem.

2) Students will use patterns to formulate relationships about surface area and volume of rectangular prisms.

Outline:

1) Go over homework. The approach to this is left to the teacher. Depending on the students, the teacher may choose to give the answers and take questions, go over each problem in detail, have students come to the board and explain their answers, have students share their answers in small groups, or collect the homework for a more formal assessment.

2) Investigation activity. Students will use three dimensions which result in the desired volume (teacher can provide the first example, as shown in the appendix), and then calculate the surface area. Students should work together in small groups to discover patterns, and ultimately find that a cube maximizes surface area and minimizes volume.

3) The teacher should hold a class discussion about the results and show examples of cylinders and spheres with the same volume, by even less surface area (see appendix).

4) Closure: Have students write a paragraph explaining what they learned, and turn it in before exiting the classroom.

NAME____DATE____________

LESSON 5: CEREAL BOX LAB

Wake Up cereal has asked you and your team to design a brand new cereal box for their most popular cereal, Crazy O's. The CEO wants you to design a box that will keep the volume the same as it has been, but keep the surface area of the box to a minimum (as little cardboard as possible). The CEO tells you that the volume of the box needs to be 216 in^3.

Use the following table to figure out the dimensions of the cereal box that will satisfy your CEO.

BOX #	VOLUME (in^3)	SURFACE AREA (in^2)	LENGTH (in)	WIDTH (in)	HEIGHT (in)
1	**512**				
2	**512**				
3	**512**				
4	**512**				
5	**512**				
6	**512**				
7	**512**				
8	**512**				

What will you report to the CEO? Be specific.

____

____

Discussion of results and other observations and conjectures:

Analyze two examples on the board and discuss with your partner, then as a class:

LESSON 6: MIXED PRACTICE:

Lesson designed for a 50 minute class period.

Common Core State Standard:

- 7.G.B – Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
    - 7.G.4 – Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
    - 7.G.6 – Solve real-world and mathematical problems involving area, volume and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
    - 8.G.9 – Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Objectives:

1) Students will utilize what they have learned during the unit to answer a mixture of problems.

2) Students will choose the appropriate formulas to solve for real-world problems.

Outline:

1) Students will work in small groups to determine when to use the appropriate formula. They will compute each problem, and share their findings in a class discussion.

2) The teacher may use any additional time for further review or to take questions from students.

3) The teacher could have an additional problem on the board for everyone to answer for closure, or choose one of the practice problems to be turned in before exiting the classroom.

NAME____DATE____________

LESSON 6: MIXED PRACTICE/ASSESSMENT REVIEW

1.

A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

1. 6.3
2. 11.2
3. 19.8
4. 39.8

2. Mrs. Crane has a cylinder that she would like to cover with contact paper for her classroom. How many square feet of contact paper will Mrs. Crane need if the diameter of the cylinder is 2 feet and the height is 4 feet? Use a calculator and round your answer to the nearest tenth of a square foot.

3.

Which rectangular package would require more wrapping paper to cover?

1. Package A: measures 10 in. by 6.5 in. by 12.75 in.
2. Package B: measures 9 in. by 7.25 in. by 11.5 in.

4. A planned building was going to be 100 feet long., 75 feet deep, and 30 feet high. The owner decides to increase the volume of the building by 10% without changing the dimensions of the depth and the height. What will be the new length of the building?

1. 106 ft.

2. 108 ft.

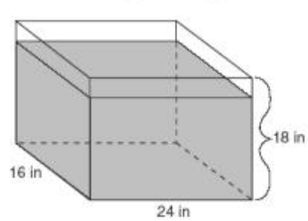
3. 110 ft.

4. 112 ft.

5. Brad wants to transfer all of his bagged coffee into identical cans. If Brad has 8,900 cubic in. of coffee to transfer, then how many cans will he need if the cans have a radius of 5 inches and a height of 9 inches?

6.

As shown in the diagram, the length, width, and height of Richard's fish tank are 24 inches, 16 inches, and 18 inches, respectively.



(Not drawn to scale)

Richard is filling his fish tank with water from a hose at the rate of 600 cubic inches per minute. How long will it take, to the *nearest minute*, to fill the tank to a depth of 15 inches?

1. 8 minutes
2. 9 minutes
3. 10 minutes
4. 11 minutes

NAME____DATE_____________

UNIT ASSESSMENT

Directions: Choose the best answer for each problem. Read each problem carefully, and be sure to check your work.

1. A cylinder has a diameter of 14 centimeters and a volume of 112π cubic centimeters. What is the height, in centimeters, of the cylinder?

   A 16

   B 4

   C $\frac{16}{7}$

   D $\frac{4}{7}$

2. An above-ground swimming pool in the shape of a cylinder has a diameter of 18 feet and a height of 4.5 feet. If the pool is filled with water to 6 inches from the top of the pool, what is the volume, to the nearest cubic foot, of the water in the pool?

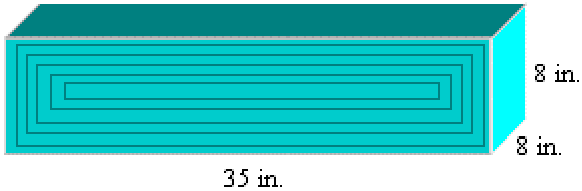
   A 226

   B 452

   C 1,018

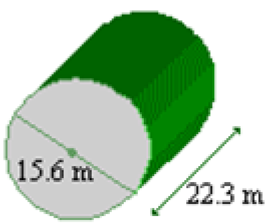
   D 4,072

3. Find the volume of the rectangular box.



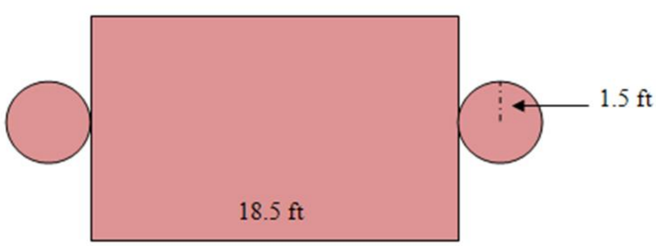
1. 51 in^3
2. 41 in^3
3. 2,240 in^3
4. 280 in^3

4. The diagram shows a silo that is about to be built on the Johnson's farm. It will be used to store corn from the farm. Approximately how much corn will be able to be stored in the silo?



1. 546 m^3
2. 4,262 m^3
3. 191 m^3
4. 1,357 m^3

5. Find the surface area of the cylinder formed by the net to the nearest *tenth*.



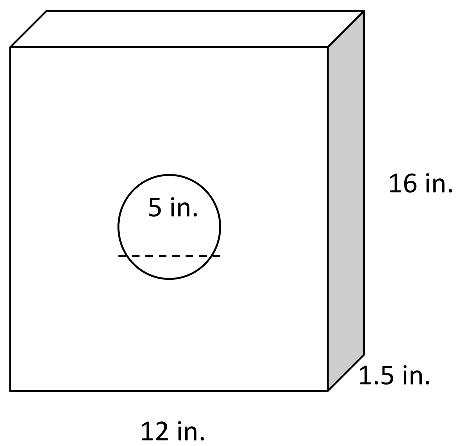
6. Carol is building a storage box for all of her gardening equipment. She wants the box to have a capacity of 12,960 cubic inches. If the box is going to be 20" in height and 18" in width, then what should the length of the box be?

7a. John wants to completely cover every wall in his room with posters. If his room is 12' wide, 16' long, and 8' high, then how many square feet do the posters need to cover?

7b. If each poster covers about 2.5 square feet, then approximately how many posters will be needed to cover the four walls? Explain your reasoning.

BONUS

Debbie is making a birdhouse. The front of the birdhouse has a length of 12 inches, a width of 1.5 inches and a height of 16 inches. She needs to drill a circular hole so that the birds can get in. The hole needs to have a diameter of 5 inches. What volume of wood will be left after Debbie cuts the hole? Round your answer to the nearest cubic inch.



CHAPTER 4: VALIDITY

This unit plan has been reviewed by a veteran teacher, who has taught many levels of mathematics, and the CCSS since their inception. This review will ensure the unit is valid in that it is properly aligned to CCSS and is well-designed from a pedagogical perspective. The observations, positive comments, and constructive criticisms are as follows:

<u>Overall Comments:</u>

- Good progression of conceptual information from concrete to abstract formula throughout individual lessons and the unit as a whole.
- Great manipulatives throughout the unit. Learners creating formulas through inquiry, discussion, and hands-on activities will help solidify concepts and increase retention. Consider using scaffolding prompts to help structure the discovery activities. You can still leave plenty of space for the students to explore the concept while providing a few access points to guide their investigation. Great job overall with the inquiry approach.
- Consider using the Standards for Mathematical Practice with students within your lesson plans. Identifying a specific practice standard (or a few) can help clarify the objectives of the lesson for learner and teacher.
- Good job anticipating student difficulties in the 'Introduction' pieces for each lesson. You have some very nice reflections from having previously taught lessons on these topics.
- Vary the style of your closure activity. Using a homework question and monitoring completion is a good day-to-day technique. Mix it up with a Pair-Share, short writing prompt, quick teacher-student debrief, etc., so that students will always be on their toes and you can assess them in different ways.
- Very thoughtful lesson designs – you have high expectations for your learners, and keep the questions and concept development moving at a brisk pace.

<u>Lesson 1 Comments:</u>

- Good use of Model → Guided → Independent sequence, with appropriate time spent at each step. The partner work allows for informal assessment through monitoring of conversation without the pressure of a teacher prompt in front of class. The Warm Up and Independent practice give a glimpse of student writing and procedural calculation.
- This lesson does a nice job facilitating the student creation of an important formula. Discovering the formula for Surface Area will help the learners internalize procedural steps more rapidly and create flexibility in problem solving.

- A scaffolding discussion/activity helping students to draw their own diagrams/"nets" given a strictly text-based problem could be useful.
- Problem 4b is a nice real-life application that involves constraints within which the student has to work.
- The wrapping paper questions 2 a and b tie in nicely to the hands-on wrapping activity (construction paper). This question presents a good opportunity to include a more complex task, such as identifying how many rolls of wrapping paper would be needed to wrap a certain number of gifts given the constraints (perhaps creating an equation in order to do so). An open-ended question about efficiency could work as a hands-on demonstration here as well. The students could discover that excess paper is usually used as a result of the wrapping process.
- Good mix of question styles in the Independent Practice. Creating a space in which a Diagram is prompted from the student could help learners who are not inherently good at visualizing, and encourages them to use multiple representations.

<u>Lesson 2 Comments:</u>

- Another good lead-in activity which develops consistency with Lesson 1 in terms of discovering the necessary formula. The conversation developed from this process will be particularly important in student confidence with the class problems.
- You have some very rich problems in the Together and Partner sections. Timing may become an issue in getting through all of these dense questions in 50 minutes. Consider leaving at least one or two of the multi-step questions for the Independent Problem Set, which will inject differentiation into your informal assessment data from the homework. Shift a few of the shorter questions into Guided Practice time to allow time for you to assess their fluency with the new formula.
- Encourage the students to draw each surface separately in completing the Guided and Independent questions.
- The rigor of your questions will prepare students to respond to various forms of prompt in testing situations, as well as encourage creative problem solving.

<u>Lesson 3 Comments:</u>

- Nice activity to build the concept of volume through stacking the cross-sections. The idea of volume being created through the product of cross-sectional area and thickness is a very big concept that can be utilized throughout mathematics.
- Good mix of questions at various levels of difficulty in both the Guided and Independent Practice sets.

- The gradual buildup to figures with a missing dimension and known Volume within the Modeling portion is well done.
- Encourage the students to produce diagrams/include a sketch as part of the solution process. This will help when dealing with more complex figures (pyramid, etc) in the future.
- A suggestion for both Lessons 2 and 3 would be to include an Anticipatory Set question in which students have to re-write a formula to solve for a given variable. This is a skill that you request of the students in each lesson, and perhaps could be used in an introductory "Warm Up" setting to ensure fluency when encountered in context.

Lesson 4 Comments:

- Once again, a nice mix of active questioning and hands-on activity to develop the key concepts. The poker chips create a nice visual, hands-on experience for putting together the volume formula. You could stress that even the thin poker chips have their own volume. Challenge the students to come up with a way to determine the volume of an individual chip.
- Good mix of computational fluency vs. problem-solving in the guided and independent questions.

Lesson 5 Comments:

- Great idea for the activity to minimize surface area given a fixed volume. The context brings a relevance to the entire sequence of lessons, especially given that the activity is not a "textbook"-style question and answer assessment.
- Nice literacy component that ties in with a real world situation wherein the students has to think analytically and defend their assertions in writing.
- Suggestion: include a rubric for assessing the student responses to the critical thinking/analysis questions after their experimentation is completed.
- Nice extension to the Grade 8 curriculum with the cylinder/sphere comparison. Students will be prepared to tackle these concepts again in Geometry thanks to the rigor and differentiation within these lessons.

Lesson 6 Comments:

- Good mix of Bloom's levels amongst the questions, although this Review is possibly a bit short.

Consider creating a graphic organizer to summarize the 5 lessons, compare/contrast information, and strengthen concepts in preparation for the assessment. You could do Surface Area vs. Surface Area for the Rectangular Prism and Cylinder, or even compare Surface Area vs. Volume for the same shape. The students need a chance to reflect on units of measurement, context, comparing procedural steps, etc.

CHAPTER 5: CONCLUSION

This thesis is intended to shed light on the Common Core State Standards and education reform in the United States. Due to the fact that the CCSS were rolled out so quickly, there is a lack of resources for teachers to utilize when implementing the new standards. The unit plan included in this thesis is meant to provide teachers with a resource to aid them in the transition from the NCTM standards to the CCSS.

In order to ensure autonomy, teachers should modify lessons as necessary. This is also important for the sake of differentiation. No classroom is the same, and therefore, no unit will work as a "one-size fits all" model. Teachers need to be certain that the units and lessons they choose will align not only with the standards, but with the individual needs of the students the provide instruction for, as well.

This unit plan is designed to encourage students to think and conceptualize at higher levels than previous standards. Students should formulate questions and make discoveries on their own. Teachers should ask guiding questions, and be facilitators of meaningful discussion. The lessons include ample real-life examples, so that students may connect their learning to real-world applications. This may encourage students to not only be more engaged in their own learning, but also equip them with the ability to transfer their knowledge to other situations they may encounter throughout their lives. It is unclear whether the CCSS will be successful or not, but this curriculum project may be a useful tool to ensure that teachers and their students are given the best opportunities to succeed.

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APPENDIX

NAME____DATE____________

LESSON 1: SURFACE AREA OF RECTANGULAR PRISMS

<u>WARM-UP</u>:

1. Find the area of the following rectangle. Show the formula you use, along with the substitution of values into that formula.

4 in.

9 in.

A = l w

A = (9)(4)

A = 36 sq. in.

2. In your own words, explain the definition of a prism.

A prism is a three-dimensional shape which has like bases on either side that are connected by rectangles.

3. Give a real-life example of a three-dimensional shape that is a rectangular prism, and explain why it is a rectangular prism.

A brick used for building houses is an example of a rectangular prism because it has rectangles for bases on the top and the bottom and they are connected by four rectangles.

Introduction to SA of Rectangular Prisms: Group Investigation, Discussion, and Interactive Visuals to Develop the Formula

Students will be given a box with construction paper. Explain that their task is to create two dimensional shapes, using the construction paper, so that each side of the box is completely covered. Then, students should detach the construction paper and compare the sides. Each group, some with more guidance than others, needs to recognize that each rectangle has another that is exactly the same. Then, students will find the area of each of the rectangles, using a ruler to get the measurements. Next, students should add the areas to get the entire surface area. Finally, students should identify the length, width, and height and create a general formula for the surface area of a rectangular prism. Finally, a class discussion with a demonstration of a pre-cut box should be done to reinforce what they should have discovered on their own. Online visual aids of three-dimensional nets can easily be accessed to further explain the concept.

FORMULA: SA = 2lw + 2lh + 2wh OR SA = 2(lw + lh + wh)

Two Together:

1. Find the surface area of the following rectangular prism.

SA = 2lw + 2lh + 2wh

SA = 2(11)(3) + 2(11)(4) + 2(3)(4)

SA = 66 + 88 + 24

SA = 178 cm^2

2a. Sue has just purchased a new pair of shoes for her grandmother. The measurements of the box they came in were 17" x 10" x 9". How much wrapping paper is required to wrap the gift?

SA = 2lw + 2lh + 2wh

SA = 2(17)(10) + 2(17)(9) + 2(10)(9)

SA = 340 + 306 + 180

SA = 826 in^2

b. How much wrapping paper would Sue need if she purchased three of the same pairs of shoes for gifts?

826 x 3 = 2,478

Sue would need 2,478 square inches of wrapping paper.

Two with Your Partner:

3a. Find the surface area of the following rectangular prism.

SA = 2lw + 2lh + 2wh

SA = 2(15)(8) + 2(15)(10) + 2(8)(10)

SA = 240 + 300 + 160

SA = 700 ft^2

b. What could this rectangular prism represent in the real world, based on the given measurements?

Based on the given measurements, this rectangular prism could represent a room in a house. The room might be narrow, because it is only 8 ft. wide, so maybe it is a bathroom.

4a. Rita is painting a room that is in the shape of a rectangular prism. The room has a length of 20', width of 16', and a height of 10'. How many square feet does Rita have to paint in order to paint the walls and the ceiling (she is not painting the floor)?

SA = lw + 2lh + 2wh

SA = (20)(16) + 2(20)(10) + 2(16)(10)

SA = 320 + 400 + 320

SA = 1,040 ft^2

b. If one can of paint costs $28.97, and it covers an area of 400 sq. ft., then how much will Rita spend on paint in order to cover the entire room, if sales tax is 8%?

1040 / 400 =2.6

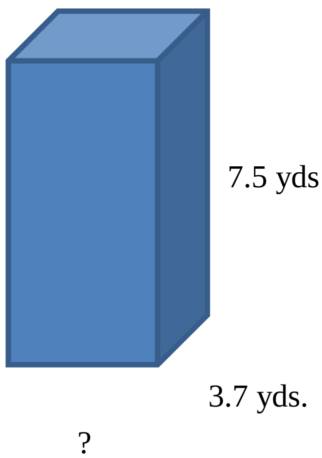
This means that Rita is going to need to buy 3 cans of paint

28.97 x 3 = $86.91

86.91 x 1.08 = $93.86

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

5. The surface area of the following rectangular prism is 171.98 square yards. The height is 7.5 yds. and the width is 3.7 yds. Find the length of the rectangular prism to the nearest tenth of a yard.



SA = 2lw + 2lh + 2wh

171.98 = 2L(3.7) + 2L(7.5) + 2(3.7)(7.5)

171.98 = 7.4L + 15L + 55.5

171.98 = 22.4L + 55.5

116.46 = 22.4L

L = 5.2 yds^2

6. James is building a prop for his school play. Part of the prop is a wooden block that is going to be fully covered in construction paper. James needs to make the block 4 cm long and 7cm wide. If James has a total of 320 sq. cm. of construction paper, then how tall can he make the block, in order for it to be completely covered?

SA = 2lw + 2lh + 2wh

320 = 2(4)(7) + 2(4)h + 2(7)h

320 = 56 + 8h + 14h

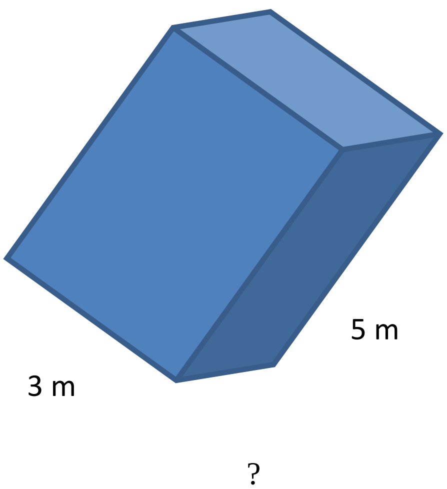
320 = 56 + 22h

264 = 22h

h = 12 cm.

Two with Your Partner:

7. The surface area of the following rectangular prism is 70 square meters. Find the missing dimension.



SA = 2lw + 2lh + 2wh

70 = 2(3)w + 2(3)(5) + 2w(5)

70 = 6w + 30 + 10w

70 = 16w + 30

40 = 16w

w = 2.5 m

8. Rusty makes his own cheese. He wants to put food coloring on the outside of one of his blocks of cheese, in order to make it look blue. If the amount of food coloring Rusty has will cover a surface area of 74.6 square inches, then what must the third dimension of the cheese block be if the other two dimensions are 1.5 in. and 9.8 in.? Round your answer to the nearest tenth of an inch.

SA = 2lw + 2lh + 2wh

74.6 = 2(9.8)(1.5) + 2(9.8)h + 2(1.5)h

74.6 = 29.4 + 19.6h + 3h

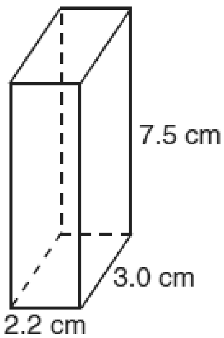
74.6 = 29.4 + 22.6h

45.2 = 22.6h

h = 2 in.

Individual Practice/Homework: (www.castlelearning.com)

1. The rectangular prism shown below has a length of 3.0 cm, a width of 2.2 cm, and a height of 7.5 cm.



$SA = 2lw + 2lh + 2wh$

$SA = 2(3.0)(2.2) + 2(3.0)(7.5) + 2(2.2)(7.5)$

$SA = 13.2 + 45 + 33$

$SA = 91.2\ cm^2$

What is the surface area, in square centimeters?

Choice 4

1. 45.6
2. 49.5
3. 78.0
4. 91.2

2. The length and width of the base of a rectangular prism are 5.5 cm and 3 cm. The height of the prism is 6.75cm. Find the exact value of the surface area of the prism, in square centimeters.

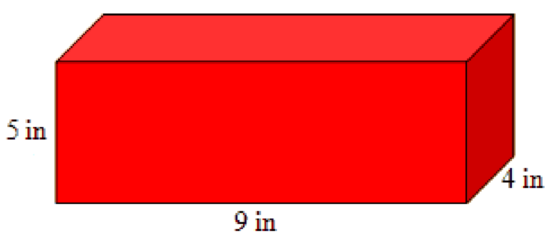
$SA = 2lw + 2lh + 2wh$

$SA = 2(5.5)(3) + 2(5.5)(6.75) + 2(3)(6.75)$

$SA = 33 + 74.25 + 40.5$

$SA = 147.75\ cm^2$

3. Find the surface area of the prism.



$SA = 2lw + 2lh + 2wh$

$SA = 2(9)(4) + 2(9)(5) + 2(4)(5)$

$SA = 72 + 90 + 40$

$SA = 202\ in^2$

1. 202 in^2
2. 180 in^2
3. 98 in^2
4. 360 in^2

4. Tony has an empty room in his house. The length of the room is 8 feet and the height is 10 feet. If he wants to wallpaper all four walls of the room, how many square feet of wallpaper should Tony buy? [Ignore the fact that there may be windows or doors in the walls.]

1. 320 ft^2
2. 640 ft^2
3. 448 ft^2
4. 256 ft^2

SA = 4Lh

SA = 4(8)(10)

SA = 320ft^2

Choice 1

5. How much wrapping paper is needed to cover a rectangular box that is 18 inches by 16 inches by 4 inches?

1. 1,152 in^2 2. 76 in^2 3. 848 in^2 4. 38in^2

SA = 2lw + 2lh + 2wh

SA = 2(18)(16) + 2(18)(4) + 2(16)(4)

SA = 576 + 144 + 128

SA = 848 in^2

Choice 3

6. Detra has a box in the shape of a cube, with each side measuring 7 inches. What is the surface area of the cube?

1. 42 in^2
2. 49 in^2
3. 343 in^2
4. 294 in^2

SA = $6s^2$

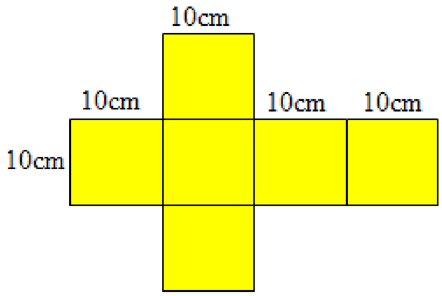
SA = $6(7)^2$

SA = 294

Choice 4

(Original SA formula will work, as well)

7. Determine the surface area for the prism formed by the following net.



1. 50 cm^2
2. 10,000 cm^2
3. 600 cm^2
4. 100 cm^2

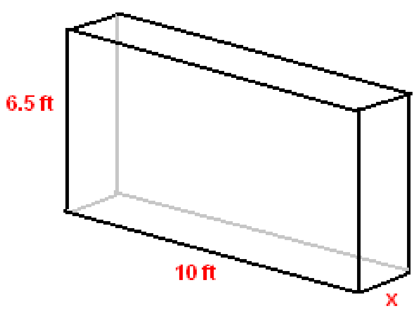
$SA = 6s^2$

$SA = 6(10)^2$

$SA = 600\ cm^2$

Choice 3

8. If the surface area of the box in the diagram is 204.25 square feet, what is the value of 'x'?



1. 3 ft
2. 2.25 ft
3. 4.5 ft
4. 5 ft

$SA = 2lw + 2lh + 2wh$

$204.25 = 2(10)x + 2(10)(6.5) + 2x(6.5)$

$204.25 = 20x + 130 + 13x$

$204.25 = 33x + 130$

$74.25 = 33x$

$X = 2.25$ ft

Choice 2

9. Find the surface area of a storage tote that is in the shape of a rectangular prism. The tote is 108 cm long, 78 cm wide, and 52cm tall. The tote does not have a top.

$SA = lw + 2lh + 2wh$

$SA = (108)(78) + 2(108)(52) + 2(78)(52)$

$SA = 8424 + 11232 + 8112$

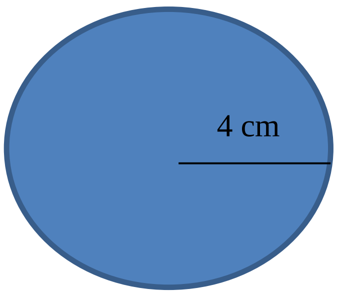
$SA = 27{,}768\ cm^2$

NAME____DATE____________

LESSON 2: SURFACE AREA OF CYLINDERS

WARM-UP:

1. Find the area and circumference of the following circle. Show the formulas you use, along with the substitution of values into those formulas. Round answers to the nearest tenth.



$A = \pi r^2$	$C = 2\pi r$
$A = \pi(4)^2$	$C = 2\pi(4)$
$A = 16\pi$	$C = 8\pi$
$A = 50.3\ cm^2$	$C = 25.1$ cm

2. In your own words, explain the definition of a cylinder.

A cylinder is a three-dimensional object that has circles for bases and a rectangle, called the lateral area, which wraps around the side.

3. Give a real-life example of a three-dimensional shape that is a cylinder, and explain why it is a cylinder.

A soup can is an example of a cylinder because it has a metal circle on the top and an identical one on the bottom. It also has a soup label, which represents the lateral area.

Introduction to SA of Cylinders: Group Investigation, Discussion and Interactive Visuals to Develop the Formula

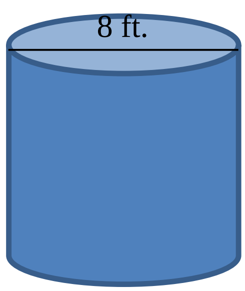
Before this activity, the teacher needs to refer to the warm-up activity when students find circumference. Have a discussion to allow students to discover that a circle's circumference is the same if you "straighten the circle's outside" to form a line segment. You can use "Fruit by the Foot" Fruit Roll-ups to demonstrate, or even "snap bracelets". This will prepare students for the type of thinking needed to find the lateral area. Students will be given a cylinder (any type of can) with construction paper. Explain that their task is to create two dimensional shapes, using the construction paper, so that each portion of the cylinder is completely covered. Then, students should detach the construction paper and examine the shapes. Each group, some with more guidance than others, needs to recognize that there are 2 circles and a rectangle. Then, students will find the area of each shape, using a ruler to get the measurements. Next, students should add the areas to get the entire surface area. Finally, students should identify the radius and height and create a general formula for the surface area of a cylinder. The lateral area will be the most difficult piece, and may require the most guidance. The pre-instruction should alleviate most confusion, or at least allow students to make a connection when offered guided questioning from the teacher. Finally, a class discussion with a demonstration of a pre-cut cylinder should be done to reinforce what they should have discovered on their own. Online visual aids of three-dimensional nets can easily be accessed to further explain the concept.

FORMULA:

$SA = 2\pi r^2 + 2\pi rh$

Two Together:

1. Find the surface area of the following cylinder. Round your answer to the nearest tenth of a square ft.



$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(4)^2 + 2\pi(4)(11)$

$SA = 32\pi + 88\pi$

$SA = 120\pi$

$SA = 377.0 \text{ ft.}^2$

2. Vicki has one hundred identical cans of vegetables for a holiday can drive. She wants to paint the cans green and red, so that they look festive. She has enough spray paint to cover 8,000 square inches of metal. If the cans are 7 inches tall and have a 1.5 in radius, then does Vicki have enough spray paint to cover all the cans? Explain your reasoning.

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(1.5)^2 + 2\pi(1.5)(7)$

$SA = 4.5\pi + 21\pi$

$SA = 25.5\pi$

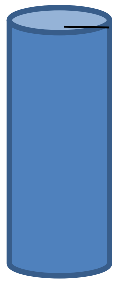
$25.5\pi \times 100 = 8011 \text{ in}^2$

Each can has a total surface area of 25.5π. After multiplying by 100, the total surface area of all the cans is 8,011 in^2. Vicki does not have enough spray paint because she can cover 8,000 in^2 and she needs more than that. 8000 – 8011 = -11 in.^{2} This means that she will be 11 in^2 short of her goal.

Two with Your Partner:

3. Find the surface area of the following cylinder. Leave your answer in terms of π.

radius = 1.5 in.



10.5 in.

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(1.5)^2 + 2\pi(1.5)(10.5)$

$SA = 4.5\pi + 31.5\pi$

$SA = 36\pi$

SA = 113.1 $in.^2$

b. What could this cylinder represent in the real world, based on the given measurements?

The measurements could represent a can of Pringles potato chips.

4. Dave and three of his friends are putting a liner on his cylindrical, above ground pool. If the pool is 14' in diameter and 5.5' deep, then what is the smallest liner that Dave must purchase in order to line his pool? (Think: Would it make sense to put a liner over the top of the water.)

$SA = \pi r^2 + 2\pi rh$

$SA = \pi(7)^2 + 2\pi(7)(5.5)$

$SA = 49\pi + 77\pi$

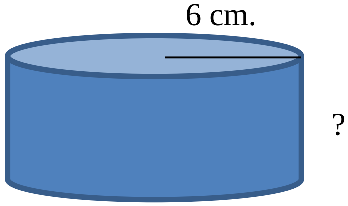
$SA = 126\pi$

SA = 395.84 $ft.^2$

Dave needs to order a liner that is at least 396 square feet.

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

5. The surface area of the following cylinder is 528 square centimeters. The radius is 6 cm. Find the height of the cylinder. Round your answer to the nearest centimeter.



$SA = 2\pi r^2 + 2\pi rh$

$528 = 2\pi(6)^2 + 2\pi(6)h$

$528 = 72\pi + 12\pi h$

$528 - 72\pi = 12\pi h$

$\frac{528-72\pi}{12\pi} = h$

h = 8 cm.

6. Kyla is covering a cylindrical object with materials she got at a craft store in order to make a carrier. She is going to cover the entire cylinder, and then cut the top, so that it acts as a cover that can be placed on and off. Kyla has a total of 800 square inches of material. If the radius of the container is 4.5 inches, what is the maximum height the container can be in order to cover the entire thing without running out of material? Round your answer to the nearest hundredth of an inch.

$SA = 2\pi r^2 + 2\pi rh$

$800 = 2\pi(4.5)^2 + 2\pi(4.5)h$

$800 = 40.5\pi + 9\pi h$

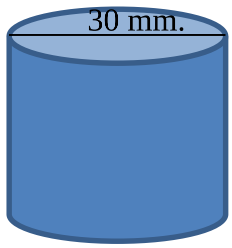
$800 - 40.5\pi = 9\pi h$

$\frac{800-40.5\pi}{9\pi} = h$

h = 23.79 in.

Two with Your Partner:

7. The surface area of the following cylinder is 4400 square millimeters. Find the height if the diameter is 30mm. Round your answer to the nearest tenth of a millimeter.



$SA = 2\pi r^2 + 2\pi rh$

$4400 = 2\pi(15)^2 + 2\pi(15)h$

$4400 = 450\pi + 30\pi h$

$4400 - 450\pi = 30\pi h$

$\frac{4400-450\pi}{30\pi} = h$

h = 31.7 mm.

8. The surface area of a cylindrical candle is 215 square inches. If the candle has a radius of 2 inches, then what is the height of the candle? Round your answer to the nearest tenth of an inch.

$SA = 2\pi r^2 + 2\pi rh$

$215 = 2\pi(2)^2 + 2\pi(2)h$

$215 = 8\pi + 4\pi h$

$215 - 8\pi = 4\pi h$

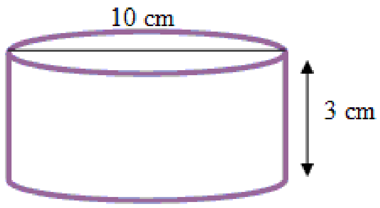
$\frac{215-8\pi}{4\pi} = h$

h = 15.1 in.

Individual Practice/Homework: (www.castlelearning.com)

1.

Find the surface area of the cylinder. Use 3.14 for π and round your answer to the *nearest square centimeter.*



1. 942 cm^2
2. 236 cm^2
3. 204 cm^2
4. 251 cm^2

SA = $2\pi r^2 + 2\pi rh$

SA = $2\pi(5)^2 + 2\pi(5)(3)$

SA = $50\pi + 30\pi$

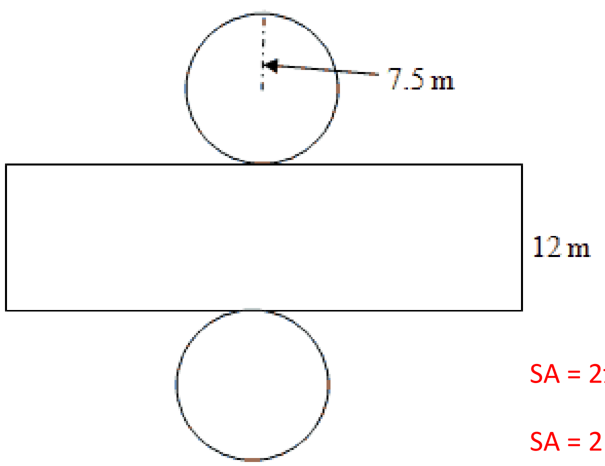
SA = 80π

SA = 251 $cm.^2$

Choice 4

2.

Find the surface area of the cylinder formed by the net to the nearest *tenth*. Use 3.14 for π.



1. 90 m^2
2. 282.6 m^2
3. 857 m^2
4. 918.5 m^2

SA = $2\pi r^2 + 2\pi rh$

SA = $2(3.14)(7.5)^2 + 2(3.14)(7.5)(12)$

SA = 353.25 + 565.2

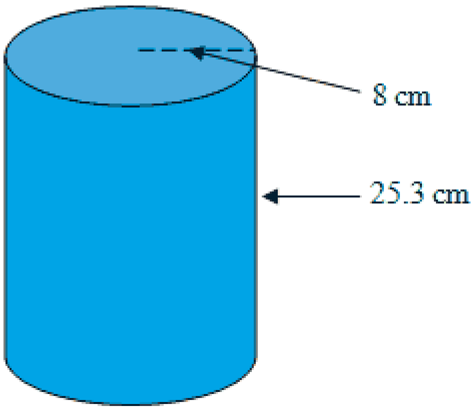
SA = 918.45

SA = 918.5 $m.^2$

Choice 4

3.

Find the surface area of the cylinder to the nearest *tenth*. Use 3.14 for π.



1. 1673.0 cm^2
2. 202.4 cm^2
3. 636.0 cm^2
4. 33.3 cm^2

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2(3.14)(8)^2 + 2(3.14)(8)(25.3)$

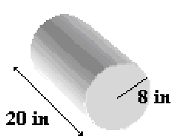
$SA = 401.92 + 1271.072$

$SA = 1672.992$

$SA = 1673.0$ in.^{2}

Choice 1

4. Find the surface area of the cylinder. Use the π key on your calculator and round your answer to the *nearest square inch*.



$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(8)^2 + 2\pi(8)(20)$

$SA = 128\pi + 320\pi$

$SA = 448\pi$

$SA = 1407$ in.^{2}

5. The surface area of a cylinder is 250 square inches. If the radius of the can is 2.5 inches, then how tall is the can to the nearest inch?

$SA = 2\pi r^2 + 2\pi rh$

$250 = 2\pi(2.5)^2 + 2\pi(2.5)h$

$250 = 12.5\pi + 5\pi h$

$250 - 12.5\pi = 5\pi h$

$\frac{250-12.5\pi}{5\pi} = h$

h = 13 inches

NAME____DATE____________

LESSON 3: VOLUME OF RECTANGULAR PRISMS

WARM-UP:

1. In your own words, explain the definition of a cube.

A cube is a rectangular prism where all sides are squares. The bases are squares, and they are connected by 4 squares. A cube has 6 sides in total.

2. Based on this definition, what do you suppose a cubic foot represents?

A cubic foot is a cube with all sides measuring 1ft.

3. What about a cubic inch?

A cubic inch is a cube whose sides are all 1 inch.

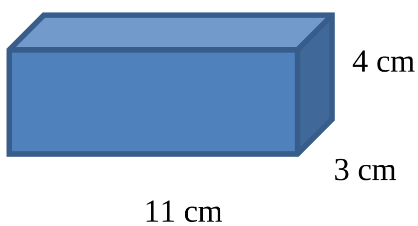
Intro. to Volume of Rectangular Prisms: Discussion and visuals to develop the formula

Students will be given an individual Post-it note. They will measure the area. Then, they will be given the whole stack to see that it is now three-dimensional. Students will measure the length, width, and height. Students will then multiply their answers. Next, students will receive another stack (doubling the height). They will repeat the process until they have done it with 4 stacks of Post-it notes. Students will then record and discuss their findings. The teacher will hold a class discussion to get students to discover the link between what they did and the volume of a rectangular prism, in order to create the formula, and recognize patterns. A visual demonstration, explaining that objects must be stacked on the base area (B), otherwise they will not be held within the container can be done with a rectangular surface and any items. Cubes can then be used to stack to show the units of measure. Finally, a class discussion with a demonstration of a pre-cut box should be done to reinforce what they should have discovered on their own.

FORMULA: $V = Bh$ OR $V = lwh$

Two Together:

1. Find the volume of the following rectangular prism.



$V = lwh$

$V = (11)(3)(4)$

$V = 132 \text{ cm}^3$

2a. A cereal box is 10” x 3.5” x 14.5”. What is the maximum amount of cereal the box can hold?

$V = lwh$

$V = (10)(3.5)(14.5)$

$V = 507.5 \text{ in}^3$

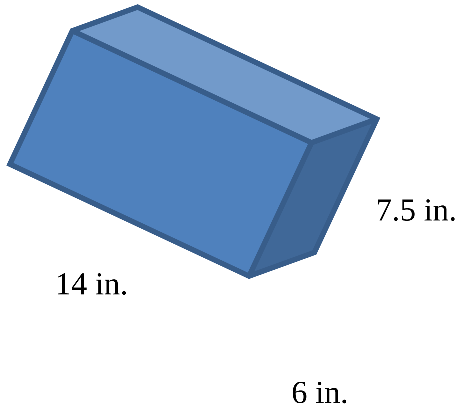
b. If a company produces 150,000 cubic inches of cereal, then how many boxes will need to be manufactured in order to package all of the cereal (we will assume boxes are filled to capacity for the sake of the exercise)?

150,000 / 507.5 = 295.57

The company will manufacture 295 boxes, and have some leftover cereal, or they will make 296 boxes, and have one box smaller than the rest.

Two with Your Partner:

3. Find the volume of the following rectangular prism.



V = lwh

V = (14)(6)(7.5)

V = 630 in.^{3}

4a. Jacob wants to fill a fish tank with water. The tank is 4 feet long, 2 feet wide, and 3 feet high. How many cubic feet of water can Jacob fit into the tank if he fills it to the top?

V = lwh

V = (4)(2)(3)

V =24 ft^3

b. If Jacob is filling the tank from a faucet with a rubber hose attached, then how long will it take to fill the tank if the faucet fills at a rate of 5 cubic feet per 2 minutes?

2 / 5 = 0.4 minutes per cubic foot

0.4 x 24 = 9.6 minutes

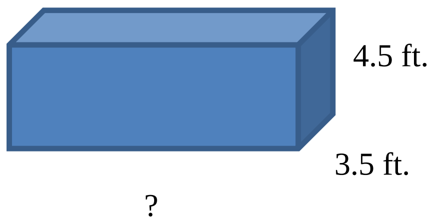
OR

0.6 x 60 = 36

9 minutes and 36 seconds.

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

5. The volume of the following rectangular prism is 149.625 cubic cm. Find the missing dimension.



V = lwh

149.625 = L(3.5)(4.5)

149.625 = 15.75L

L = 9.5 ft.

6. Mira is putting in a rectangular patio. In order to make the patio, the builders need to dig the dirt to make it flat, and pile stone evenly to the necessary height. This creates a rectangular prism with the base being the ground, and the top being where the patio will be. Stone will fill in between. If the builder has 8 cubic yards of stone, then how high up can the stone be piled evenly if the length of the patio is 9 yards and the width is 4 yards? Round your answer to the nearest hundredth of a yard.

V = lwh

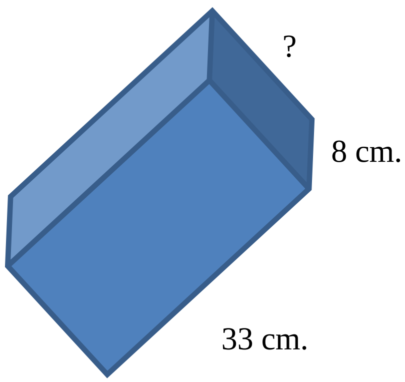
8 = (9)(4)h

8 = 36h

h = 0.22 yds.

Two with Your Partner:

7. The volume of the following rectangular prism is 3,960 cubic cm. Find the missing dimension.



V = lwh

3960 = (33)(8)h

3960 = 264h

h = 15cm

8. Jacob is now looking to buy a new fish tank. Based on how many fish he plans on placing in the fish tank, Jacob has determined that he will need 2,800 cubic inches of water. The length of the fish tank is 36 inches, and the width of the fish tank is 18 inches. Jacob has a shelf 4 feet above the table he wants to place the fish tank. Will Jacob be able to purchase the fish tank necessary to contain the required amount of water, and be placed in the spot that Jacob has chosen for the tank to stay?

V = lwh

2,800 = (36)(18)h

2,800 = 648h

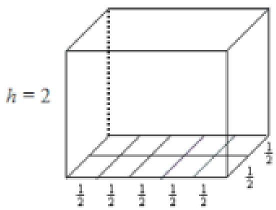
h = 4.32 ft.

No, Jacob is 0.32 ft. short. He either needs to buy less fish or move the tank to a spot that gives him at least 0.32 ft. more room. In fact, he will likely need more room than that in order to feed the fish and get air in the

Individual Practice/Homework: (www.castlelearning.com)

1.

The base of the rectangular prism has been divided into 10 squares. Each square has a side length of $\frac{1}{2}$ centimeter. The height of the prism is 2 centimeters.



Find the area of the base and the volume of the rectangular prism.

1. Area of the base = $2\frac{1}{2}$ cm^2; Volume of the prism = 5 cm^3
2. Area of the base = 10 cm^2; Volume of the prism = 20 cm^3
3. Area of the base = 5 cm^2; Volume of the prism = 10 cm^3
4. Area of the base = $\frac{1}{4}$ cm^2; Volume of the prism = $\frac{1}{2}$ cm^3

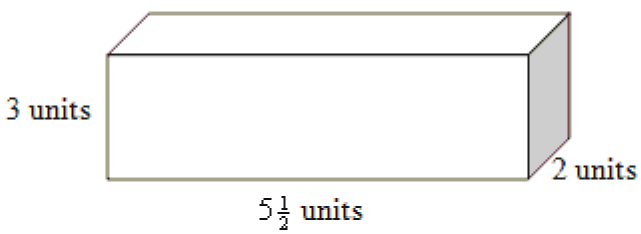
Students can visualize this problem.

OR

A of Base = lw	V = lwh
A of Base= (2.5)(1)	V = (2.5)(1)(2)
A of Base = 2.5 cm^2	V = 5 cm^3

2.

Find the volume of the rectangular prism.



1. 30 $units^3$
2. $30\frac{1}{2}$ $units^3$
3. 33 $units^3$
4. 35 $units^3$

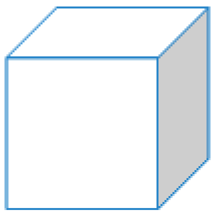
V = lwh

V = (5.5)(2)(3)

V = 33 $units^3$

Choice 3

3. What are the edge lengths of a cube whose volume is $\frac{1}{27}$ cubic meter?



Volume = $\frac{1}{27}$ m^3

1. $\frac{1}{3}$ m^3
2. $\frac{1}{9}$ m^3
3. $\frac{1}{3}$ m
4. $\frac{1}{9}$ m

V = lwh

$V = s^3$

$\frac{1}{27} = s^3$

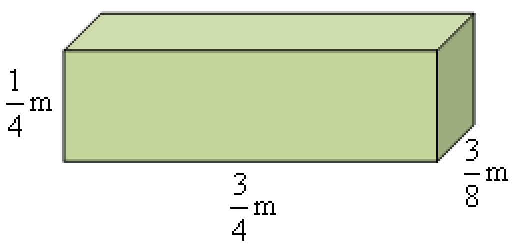
$\sqrt[3]{1/27} = s$

$s = \frac{1}{9}$ m

Choice 4

Students could still figure this out without necessarily knowing how to take the cubed root. They just need to think of what number can multiply by itself three times to get 1/27. This could be a good time to discuss cubed root with an advanced group

4. What is the volume of the right rectangular prism pictured below?



1. $\frac{11}{8}$ m^3
2. $\frac{3}{16}$ m^3
3. $\frac{9}{128}$ m^3
4. $\frac{9}{32}$ m^3

V = lwh

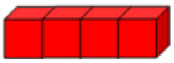
V = (1/4)(3/4)(3/8)

V = 9/128 m^3

Choice 3

5.

Four red cubes are arranged to form a rectangular prism. What is the volume of the rectangular prism if each individual cube has an edge length of $\frac{1}{3}$ inch?



1. 1 in.^3
2. $\frac{1}{27}\text{ in.}^3$
3. $\frac{1}{3}\text{ in.}^3$
4. $\frac{4}{27}\text{ in.}^3$

V = lwh

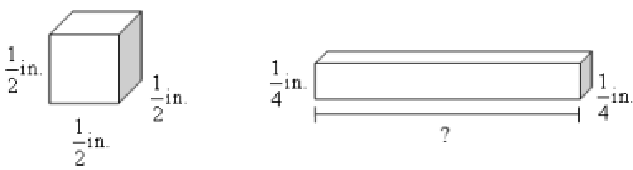
V = (4/3)(1/3)(1/3)

V = 4/27 in.^{3}

Choice 4

6.

What length is the missing measurement of the rectangular prism on the right that would make its volume equal to the volume of the cube on the left?



1. 1 in.
2. 2 in.
3. 3 in.
4. 4 in.

V = lwh

V = (.5)(.5)(.5)

V = 1/8 in.^{3}

V = lwh

1/8 = L (1/4)(1/4)

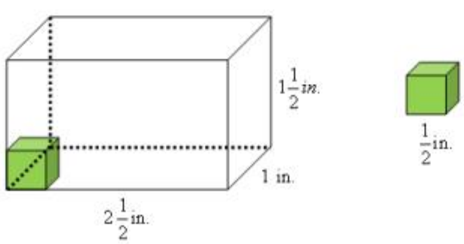
1/8 = 1/16 L

L = 2 in.

Choice 2

7.

The diagram shows a rectangular prism with edge lengths of $2\frac{1}{2}$ inches, 1 inch, and $1\frac{1}{2}$ inches. It also shows a green cube with edge lengths of $\frac{1}{2}$ inch.



At most, how many green cubes will fit into the rectangular prism?

1. $3\frac{3}{4}$ green cubes
2. 30 green cubes
3. 15 green cubes
4. 5 green cubes

Students could complete this visually.

OR

V = lwh

V = (2.5)(1)(1.5)

V = 3.75

V = lwh

V = (0.5)^3

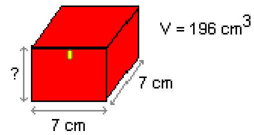
V = .125

3.75/.125 = 30 cubes

Choice 2

8.

Trudy knows that the volume of her jewelry case is 196 cm^3, its length is 7 cm, and its width is 7 cm. What is its height?



V = lwh

196 = (7)(7)h

196 = 49h

h = 4 cm.

NAME____DATE____________

LESSON 4: VOLUME OF CYLINDERS

WARM-UP:

1. Based on the previous lesson, where we found that volume is measured in cubic units, do you think that cubes will fit into a cylinder perfectly? Explain your reasoning.

No, cubes will not fit perfectly into a cylinder because a cylinder does not have straight edges. There will be space in between the edges of the cubes and the rounded portion of the cylinder. You would have to cut the cubes up in order to fill the cylinder completely.

2. If volume is still measured in cubic units, regardless of the container, then what effect do you think dealing with a cylinder will have on our numerical answers, meaning, what type of numbers might you expect to get for answers? Explain your reasoning.

Since we can't fit cubes perfectly into a cylinder, there will be portions of cubes cut up to fill the rest of the space. This means that our answers will be irrational numbers/non-terminating decimals.

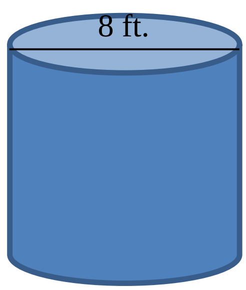
Introduction to Volume of Cylinders: Discussion and visuals to develop the formula

Students will be given an individual, circular piece of paper that was cut from the outline of a poker chip. They will measure the area. Then, they will be given many poker chips to stack to see that it is now three-dimensional. Students will multiply the area by the height for different numbers of poker chips and share their findings. The teacher will hold a class discussion to get students to discover the link between what they did and the volume of a cylinder, in order to create the formula, and recognize patterns. A visual demonstration, explaining that objects must be stacked on the base area (B), otherwise they will not be held within the container can be done with a circular surface and any items. This can be related to Flintstone push-pops ice cream containers. The plastic circle actually travels through the tube, making it a great visual. Cubes can then be used to stack to show the units of measure, and why they will not fit perfectly, irrational solutions.

FORMULA: $V = Bh$ OR $V = \pi r^2 h$

Two Together:

1. Find the volume of the following cylinder. Round your answer to the nearest tenth of a cubic foot.



$V = \pi r^2 h$

$V = \pi(4)^2 \cdot 11$

$V = 16\pi \cdot 11$

$V = 176\pi$

$V = 552.9 \text{ ft.}^3$

2a. Angelica has an above ground pool in the shape of a cylinder. The pool is 16' in diameter and 5' tall. What is the maximum amount of water that can fit in the pool? Round your answer to the nearest hundredth of a cubic foot.

$V = \pi r^2 h$

$V = \pi(8)^2 \cdot 5$

$V = 64\pi \cdot 5$

$V = 320\pi$

$V = 1005.31 \text{ ft.}^3$

b. If Angelica wants to fill the pool in order to leave 6" of space between the top of the water and the top of the sides, then how much less water would she need to fill the pool? Round your answer to the nearest cubic foot.

$V = \pi r^2 h$

$V = \pi(8)^2 \cdot 4.5$

$V = 64\pi \cdot 4.5$

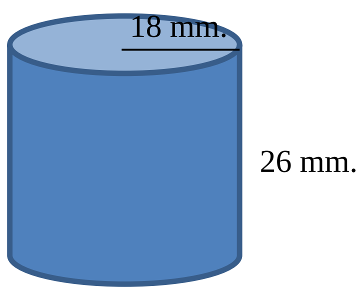
$V = 288\pi$

$V = 904.78 \text{ ft.}^3$

$1005.31 - 904.78 = 100.53$

Two with Your Partner:

3. Find the volume of the following cylinder. Leave your answer in terms of π.



$V = \pi r^2 h$

$V = \pi(18)^2 \cdot 26$

$V = 324\pi \cdot 26$

$V = 8424\pi \text{ mm.}^3$

4a. An oil company fills cylindrical barrels with oil. If the barrels are 4ft. tall and 3ft. in diameter, then how much oil can each barrel hold. Leave your answer in terms of π.

$V = \pi r^2 h$

$V = \pi(1.5)^2 \cdot 4$

$V = 2.25\pi \cdot 4$

$V = 9\pi$ ft.^{3}

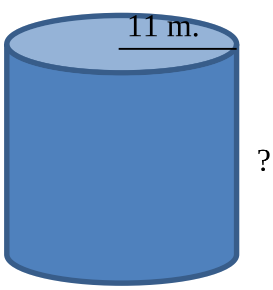
b. If the company has 10,000 cubic feet of oil to distribute, then how many barrels will they need?

$10000 / 9\pi = 353.68$ barrels

The company will be able to fill 353 full barrels, and they will have 1 barrel that is about 2/3 full.

What about these? Read each of the following problems, and discuss with your partner what is different about them, compared to the first four that have been completed. Then, we will solve them together.

5. The volume of the following cylinder is 950.8 cubic meters. If the radius is 11 meters, then what is the volume of the cylinder? Round your answer to the nearest tenth of a meter.



$V = \pi r^2 h$

$950.8 = \pi(11)^2 h$

$950.8 = 121\pi h$

$h = 2.5$ m.

6. Tina has a cylindrical storage container that needs to hold 320 cm^3 worth of objects. She knows the diameter of the container is 9 centimeters, but she is unaware of the height. To the nearest tenth, what is the smallest value the height can be in order to fit her needs?

$V = \pi r^2 h$

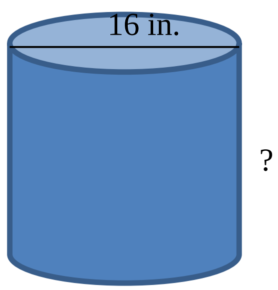
$320 = \pi(4.5)^2 h$

$320 = 20.25\pi h$

h = 5.0 m.

Two with Your Partner:

7. The volume of the following cylinder is 156.2 cubic in. If the diameter is 16 inches, then what is the height of the cylinder? Round your answer to the hundredth of an inch.



$V = \pi r^2 h$

$156.2 = \pi(8)^2 h$

$156.2 = 64\pi h$

h = 0.78 m.

8a. The volume of a cylindrical water tower is 36,000 square feet. If the radius of the water tower is 15 feet, then what is the height of the tower? Round your answer to the nearest foot.

$V = \pi r^2 h$

$36000 = \pi(15)^2 h$

$36000 = 225\pi h$

$h = 51$ ft.

8b. The town the tower supplies the water to has 500 people. On average, how many cubic feet of water does each person get to use from that water tower?

$36000 / 500 = 72\ ft^3$

Individual Practice/Homework: (www.castlelearning.com)

1. Mike buys his ice cream packed in a rectangular prism-shaped carton, while Carol buys hers in a cylindrical-shaped carton. The dimensions of the prism are 5 inches by 3.5 inches by 7 inches. The cylinder has a diameter of 5 inches and a height of 7 inches.

Which container holds more ice cream? Justify your answer.

$V = lwh$

$V = (5)(3.5)(7)$

$V = 122.5 \text{ in.}^3$

$V = \pi r^2 h$

$V = \pi(2.5)^2(7)$

$V = 137.4446786 \text{ in.}^3$

The cylinder holds more ice cream because it has a larger volume.

Determine, to the *nearest tenth of a cubic inch*, how much *more* ice cream the larger container holds.

$137.4446786 - 122.5 = 14.9 \text{ in}^3$

2.

A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

1. 72.3
2. 83.1
3. 180.6
4. 722.6

$V = \pi r^2 h$

$V = \pi(5)^2(2.3)$

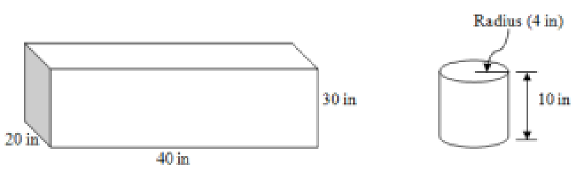
$V = 57.5\pi$

$V = 180.6 \text{ in.}^3$

Choice 3

3.

In the diagram, a rectangular container with the dimensions 20 inches by 30 inches by 40 inches is to be filled with water, using a cylindrical cup whose radius is 4 inches and whose height is 10 inches.



What is the maximum number of full cylindrical cups of water that can be placed into the container without the water overflowing the container?

1. 600 cups
2. 599 cups
3. 48 cups
4. 47 cups

$V = lwh$

$V = (40)(20)(30)$

V = 24,000 in.3

$V = \pi r^2 h$

$V = \pi(4)^2(10)$

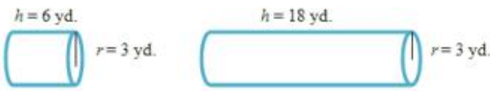
$V = 160\pi$

$24{,}000 / 160\pi = 47.7$ cups

The maximum number of full cups that can fit inside the

4.

The radius of a cylinder is 3 yards and its height is 6 yards, shown below left. The height of the new cylinder is tripled but its radius remains the same, shown below right.



What effect will this change have on the volume of the new cylinder?

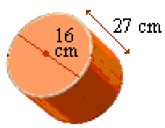
1. The new volume will be 3 times larger than the original volume.
2. The new volume will be 6 times larger than the original volume.
3. The new volume will be 9 times larger than the original volume.
4. The new volume will be 27 times larger than the original volume.

The base is the exact same area, so tripling the height will also triple the volume. Students could also prove this mathematically.

Choice 1

5.

Jordan got a new water bottle to use for soccer. Approximately how much water can Jordan fit in the water bottle?



1. 1,728 cm^3
2. 5,429 cm^3
3. 1,356 cm^3
4. 21,703 cm^3

$V = \pi r^2 h$

$V = \pi(8)^2(27)$

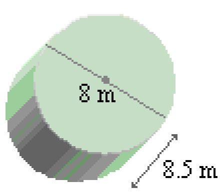
$V = 1728\pi$

h = 5429 cm^3

Choice 2

6.

Acme Pipe Corporation had to replace a section of conduit.



What is the volume of the pipe that had to be replaced, to the *nearest cubic meter*?

$V = \pi r^2 h$

$V = \pi(4)^2(8.5)$

$V = 136\pi$

h = 427 $m.^3$

7.

A soup can is in the shape of a cylinder. The can has a volume of 342 cm^3 and a diameter of 6 cm. Express the height of the can in terms of π.

1. 12π cm
2. 38π cm
3. 12 cm
4. $\frac{38}{\pi}$ cm

$V = \pi r^2 h$

$342 = \pi(3)^2 h$

$342 = 9\pi h$

$38/\pi = h$

Choice 4

NAME____DATE____________

LESSON 5: CEREAL BOX LAB

Wake Up cereal has asked you and your team to design a brand new cereal box for their most popular cereal, Crazy O's. The CEO wants you to design a box that will keep the volume the same as it has been, but keep the surface area of the box to a minimum (as little cardboard as possible). The CEO tells you that the volume of the box needs to be 512 in^3.

Use the following table to figure out the dimensions of the cereal box that will satisfy your CEO.

BOX #	VOLUME (in^3)	SURFACE AREA (in^2)	LENGTH (in)	WIDTH (in)	HEIGHT (in)
1	**512**	1032	128	2	2
2	**512**				
3	**512**				
4	**512**				
5	**512**				
6	**512**				
7	**512**				
8	**512**	384	8	8	8

What will you report to the CEO? Be specific.

The company should build a cube with each dimension measuring 8 inches in order to minimize the surface area and maximize the volume. I found this by trying dimensions that made the volume 512 in^3, and I noticed a pattern. As all the dimensions got closer together, the surface area kept going down. It was lowest when the side lengths were all the same.

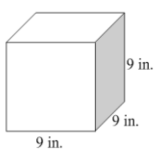
Discussion of results and other observations and conjectures:

What were your conclusions?

If a cube is the best, why don't they use cubes for cereal boxes?

- Answers: storing, pouring, advertisement, making a box with more surface area gives the illusion that the customer is getting more cereal. Cardboard is cheaper than cereal, so this is still more profitable, even though you are using more cardboard.

Analyze two examples on the board and discuss with your partner, then as a class:



$V = s^3$

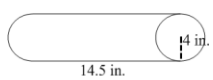
$V = (9)^3$

$V = 729 \text{ in.}^3$

$SA = 6(s)^2$

$SA = 6(9)^2$

$SA = 486 \text{ in.}^2$



$V = \pi r^2 h$

$V = (3.14)(4)^2(14.5)$

$V = 728.5 \text{ in.}^3$

$SA = \pi r^2 + 2\pi rh$

$SA = (3.14)(4)^2 + 2(3.14)(4)(14.5)$

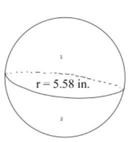
$SA = 50.24 + 364.24$

$SA = 414.48 \text{ in.}^2$

Based on these calculations, what conclusions can you make?

Answer: Cylinders are better at maximizing volume while minimizing surface area.

Why doesn't cereal come in cylinders?



$V = (4/3)\pi r^3$

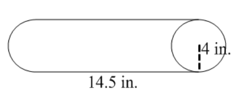
$V = (4/3)\pi(5.58)^3$

$V = 727.8 \text{ in.}^3$

$SA = 4\pi r^2$

$SA = 4\pi(5.58)^2$

$SA = 391.27 \text{ in.}^2$



$V = \pi r^2 h$

$V = (3.14)(4)^2(14.5)$

$V = 728.5 \text{ in.}^3$

$SA = \pi r^2 + 2\pi rh$

$SA = (3.14)(4)^2 + 2(3.14)(4)(14.5)$

$SA = 50.24 + 364.24$

$SA = 414.48 \text{ in.}^2$

Based on these calculations, what conclusions can you make?

Answer: Spheres are even better at maximizing volume while minimizing surface area.

Why doesn't cereal come in spheres?

NAME____DATE____________

LESSON 6: MIXED PRACTICE/ASSESSMENT REVIEW

1.

A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

1. 6.3
2. 11.2
3. 19.8
4. 39.8

$1000 = \pi r^2 h$

$950.8 = \pi(11)^2 h$

$950.8 = 121\pi h$

h = 2.5 m.

2. Mrs. Crane has a cylinder that she would like to cover with contact paper for her classroom. How many square feet of contact paper will Mrs. Crane need if the diameter of the cylinder is 2 feet and the height is 4 feet? Use a calculator and round your answer to the nearest tenth of a square foot.

$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(1)^2 + 2\pi(1)(4)$

$SA = 2\pi + 8\pi$

$SA = 10\pi$

$SA = 31.4 \text{ ft.}^2$

3.

Which rectangular package would require more wrapping paper to cover?

1. Package A: measures 10 in. by 6.5 in. by 12.75 in.
2. Package B: measures 9 in. by 7.25 in. by 11.5 in.

$SA = 2lw + 2lh + 2wh$

$SA = 2(10)(6.5) + 2(10)(12.75) + 2(6.5)(12.75)$

$SA = 130 + 255 + 165.75$

$SA = 550.75 \text{ in}^2$

$SA = 2lw + 2lh + 2wh$

$SA = 2(9)(7.25) + 2(9)(11.5) + 2(7.25)(11.5)$

$SA = 130.5 + 207 + 166.75$

$SA = 504.25 \text{ in}^2$

4. A planned building was going to be 100 feet long., 75 feet deep, and 30 feet high. The owner decides to increase the volume of the building by 10% without changing the dimensions of the depth and the height. What will be the new length of the building?

1. 106 ft.
2. 108 ft.
3. 110 ft.
4. 112 ft.

V = lwh

V = (100)(75)(30)

V = 225,000 ft.^{3}

225,000 x 1.10 = 247,500 ft.^{3}

V = lwh

247500 = L(75)(30)

247500 = 2250L

L = 110 ft.

Choice 3

5. Brad wants to transfer all of his bagged coffee into identical cans. If Brad has 8,900 cubic in. of coffee to transfer, then how many cans will he need if the cans have a radius of 5 inches and a height of 9 inches?

$V = \pi r^2 h$

$V = \pi(5)^2 \cdot 9$

$V = 25\pi \cdot 9$

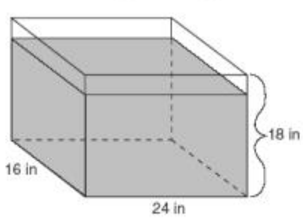
$V = 225\pi$ in.^{3}

8900 / 225π = 12.59 cans

Brad will need 13 cans.

6.

As shown in the diagram, the length, width, and height of Richard's fish tank are 24 inches, 16 inches, and 18 inches, respectively.



(Not drawn to scale)

V = lwh

V = (24)(16)(15)

V = 5,760

5760 / 600 = 9.6 minutes

It will take 10 minutes.

Choice 3

Richard is filling his fish tank with water from a hose at the rate of 600 cubic inches per minute. How long will it take, to the *nearest minute*, to fill the tank to a depth of 15 inches?

1. 8 minutes
2. 9 minutes
3. 10 minutes
4. 11 minutes

* Problems 1 – 4 and 6 (www.castlelearning.com)

NAME____DATE____________

UNIT ASSESSMENT

Directions: Choose the best answer for each problem. Read each problem carefully, and be sure to che4ck your work.

1. A cylinder has a diameter of 14 centimeters and a volume of 112π cubic centimeters. What is the height, in centimeters, of the cylinder?

A 16

B 4

C $\frac{16}{7}$

D $\frac{4}{7}$

$V = \pi r^2 h$

$112\pi = \pi(7)^2 h$

$112\pi = 49\pi h$

$h = 16/7$ cm.

Choice C

2. An above-ground swimming pool in the shape of a cylinder has a diameter of 18 feet and a height of 4.5 feet. If the pool is filled with water to 6 inches from the top of the pool, what is the volume, to the nearest cubic foot, of the water in the pool?

A 226

B 452

C 1,018

D 4,072

$V = \pi r^2 h$

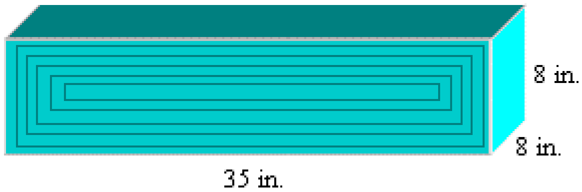
$V = \pi(9)^2(4)$

$SA = 324\pi$

$SA = 1018$ ft.^{3}

Choice C

3. Find the volume of the rectangular box.



$V = lwh$

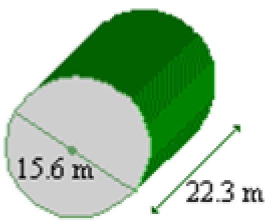
$V = (35)(8)(8)$

$V = 2{,}240 \text{ in.}^3$

Choice 3

1. 51 in^3
2. 41 in^3
3. 2,240 in^3
4. 280 in^3

4. The diagram shows a silo that is about to be built on the Johnson's farm. It will be used to store corn from the farm. Approximately how much corn will be able to be stored in the silo?



$V = \pi r^2 h$

$V = \pi(7.8)^2(22.3)$

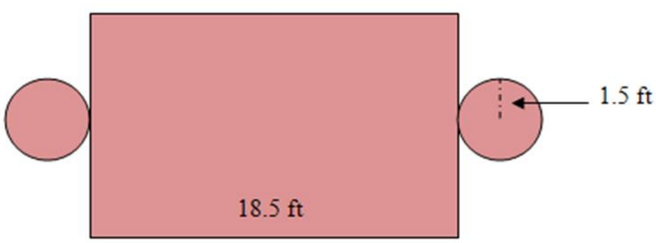
$V = 1356.732\pi$

$V = 4{,}262 \text{ m.}^3$

Choice 2

1. 546 m^3
2. 4,262 m^3
3. 191 m^3
4. 1,357 m^3

5. Find the surface area of the cylinder formed by the net to the nearest *tenth*.



$SA = 2\pi r^2 + 2\pi rh$

$SA = 2\pi(1.5)^2 + 2\pi(1.5)(18.5)$

$SA = 4.5\pi + 55.5\pi$

$SA = 60\pi$

$SA = 188.5 \text{ ft.}^2$

6. Carol is building a storage box for all of her gardening equipment. She wants the box to have a capacity of 12,960 cubic inches. If the box is going to be 20" in height and 18" in width, then what should the length of the box be?

V = lwh

12960 = L(18)(20)

12960 = 360L

L = 36 in.

7a. John wants to completely cover every wall in his room with posters. If his room is 12' wide, 16' long, and 8' high, then how many square feet do the posters need to cover?

SA = 2lh + 2wh

SA = 2(16)(8) + 2(12)(8)

SA = 265 + 192

SA = 457 $ft.^2$

7b. If each poster covers about 2.5 square feet, then approximately how many posters will be needed to cover the four walls? Explain your reasoning.

457 / 2.5 = 182.8 posters

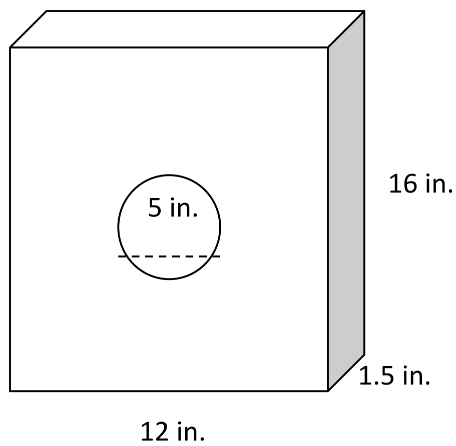
I divided 457 by 2.5 to see how many 2.5's it takes to get to 457. If John wants to cover the whole wall, he should buy 183 posters, and maybe have a little bit of overlap because I rounded up.

*Problems 1, 2, and 6 (www.engageny.org) (Previous state exam questions)

Problems 3, 4, 5, and 7 (www.castlelearning.com)

BONUS

Debbie is making a birdhouse. The front of the birdhouse has a length of 12 inches, a width of 1.5 inches and a height of 16 inches. She needs to drill a circular hole so that the birds can get in. The hole needs to have a diameter of 5 inches. What volume of wood will be left after Debbie cuts the hole? Round your answer to the nearest cubic inch.



$V = lwh$

$V = (12)(1.5)(16)$

$V = 288$ in.^{3}

$V = \pi r^2 h$

$V = \pi(2.5)^2(1.5)$

$V = 9.375\pi$ in.^{3}

$288 - 9.375\pi = 259$ in.^{3}