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Rearranging Algebraic Equations Using Electrical Circuit Applications: A Unit Plan Aligned to the New York State Common Core Learning Standards

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A Unit Plan Aligned to the New York State Common Core Learning Standards

by

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Dedication

This thesis is dedicated to my husband, Jeff, and my daughter, Margaret, who provided a tremendous amount of love and support throughout my career change from electrical engineering to teaching mathematics.

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Abstract

As a response to both the implementation of the Common Core State Standards (CCSS) and a recent approval of a change by the New York State Board of Regents to allow multiple pathways for graduation, this curriculum project, which will be referred to as a unit plan throughout the paper, was designed to meet the need for more units of study that apply mathematical modeling in algebra to real world situations that allow students to explore applications of mathematics in careers. The unit plan on rearranging algebraic equations using electrical circuit applications is aligned to the New York State Common Core Learning Standards for mathematics (NYSCCLSM) and addresses mathematical modeling, mathematical practice standard 4. This unit plan, which may provide a method by which algebra and career & technical education (CTE) teachers can continue to work toward the Common Core State Standards Initiative (CCSSI) goal of preparing students for both college and career success ("About The Standards | Common Core State Standards Initiative," N.D.), was validated by presenting a single lesson to a small group of students as a pilot study. Responses from the post-lesson student survey indicate that there was a positive change in their average attitude toward rearranging equations with more than one variable. More investigation is required to refine the lessons and prove that this entire unit plan is useful in a larger setting and to a wider student audience.

Chapter 1: Introduction

Over the past century, the standard focus of high school education preparation has changed from preparing most students to be immediately employable in an entry-level job toward preparing most students to continue their education in college or a trade school before entering a career. This phenomenon is understandable because the quick pace of technological advance has created many new careers with a higher level of complexity.

However, there are still many traditional skilled trades, such as an electrician, that can be entered into upon the completion of a high school degree because an apprenticeship period is required. The following advertisement for electrical apprentices aired in Lansing, MI in 2000:

> “The International Brotherhood of Electrical Workers' [IBEW] apprenticeship program is an opportunity for young men and women to prepare for successful, challenging, and well-paying careers. Apprenticeship with the IBEW provides skills training and the tools you need to build a bright future. If you are at least 17, with a high school diploma or GED, have strong algebra skills, and a desire to join the electrical industry, apply for apprenticeship…” (Hill, 2002, p.450)

While there has been an emphasis on academic instruction during electrical apprenticeships, since the inception of the IBEW, this advertisement was one of the first to highlight the connection that people considering this field need a strong background in algebra before becoming apprentices.

The year 2000 was also the same year that the book entitled Principles and Standards for School Mathematics (*Standards*) (National Council of Teachers of Mathematics, 2000) was published. The National Council of Teachers of Mathematics (NCTM) clearly defined the expectations for student learning in algebra and other high school mathematics. The *Standards* gave rise to the expectation that students of algebra should be able to “write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency" (p. 296) and "use mathematical models to represent and understand quantitative relationships" (p. 303) was strongly suggested. With the legal adoption of the Common Core State Standards (CCSS) in 2010, which reiterated much of the NCTM's *Standards*, the emphasis on real-world application and mathematical modeling practices has become a necessary component of the algebra curriculum.

As Common Core standards are being implemented more rigorously in order to make students ready for demanding careers in science, technology, engineering and mathematics (STEM), more teachers of mathematics are looking for ways to engage their classes in mathematical discussions that relate to real world applications that students may encounter in their chosen career path. One need only look at the last two years of the Mathematics Teacher, a magazine published by the NCTM, to find articles relating to mathematics modeling lesson plans that include finding a cell phone by triangulation (Anhalt & Cortez, 2015) and an analysis of historical engineering events such as the completion of the first transcontinental railroad (Perham & Perham, 2015). These scenarios, when properly presented, may increase a student's engagement with mathematics because they become fascinated with the thought that mathematics relates to their ability to solve future real world problems.

Unit Plan

The purpose of this thesis is to develop a mathematical modeling exemplar unit plan that is both STEM oriented for professional development of algebra and CTE (Career and Technical Education) teachers to engage students in learning algebra by using basic electric circuit formulae. Students today are very engaged with their electronic gadgets as play and understanding circuits could lead to many different career paths in electrical engineering and electronics technology when they understand that a strong foundation in algebra is needed for that career path. This unit connects an algebra student's prior knowledge of electric circuits from the fourth grade and middle school Physical Sciences units on Energy ("Draft New York State P-12 Science Learning Standards : Next Generation Science Standards P-12 : NYSED," 2015) in order to deepen their understanding of rearranging equations. This unit plan may then form a platform for students to do well in other high school science classes such as chemistry, physics and digital electronics, because they will have a firm foundation of the models of algebraic operations necessary to transform equations so they can solve problems using complex formulae with multiple variables. It also directly supports the Next Generation Science Standard (NGSS) HS-PS3-6 which has students analyze data to "support the claim that Ohm's Law describes the mathematical relationship among the potential difference, current, and resistance of an electric circuit" ("Draft New York State P-12 Science Learning Standards : Next Generation Science Standards P-12 : NYSED," 2015) as well as the Common Core mathematical standards of A.CED.3 which states, "Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context" and A.CED.4 which states, "Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R." ("New York State P-12 Common Core Learning Standards for Mathematics | EngageNY," n.d.)

Students will have the opportunity to manipulate resistive circuits in a variety of mediums: from a very concrete representation using batteries, resistors, light emitting diodes (LEDs) and conductive dough through pictorial representations of circuits in emulators to written descriptions which are then translated into classical electrical formulae that will be manipulated to highlight variables of interest. Through the use of concrete models of circuits, students will be guided to understand the usefulness of rearranging complex equations before substituting numbers to find an optimum solution to an engineering problem.

The individual lessons in this unit plan do not have to be taught all together, but they should be taught in a sequence that is supplementary to the textbook lessons about rearranging equations. They can either be used to introduce the conceptual understanding of how rearranging equations is useful in a real-world context or they can be used to improve procedural skills that a student already knows but needs a real-world context to solidify the concept. Ideally, these lessons should be part of an iterative learning process because “conceptual and procedural knowledge appear to develop in a hand-over-hand process.” (Rittle-Johnson, Siegler, & Alibali, 2001, p. 360) Students may benefit most from this unit plan by exploring and absorbing the mathematical concepts and procedures over time throughout the year.

Terms and Definitions

BOCES - Board of Cooperative Educational Services

CCSS - Common Core State Standards

CTE - Career and Technical Education

EIA - Electronic Industries Association

ELA - English Language Arts

IBEW - International Brotherhood of Electrical Workers, an electrician’s union

NCTM - National Council of Teachers of Mathematics

NSPIE - National Society for the Promotion of Industrial Education

NYCCLS - New York Common Core Learning Standards

NGSS - Next Generation Science Standard

STEM – Science, Technology, Engineering and Mathematics

Light emitting diode - A low power substitute for a standard resistive light bulb.

Ohm’s law - The current flowing through a metallic conductor is proportional to the electromotive force applied across its ends, provided the temperature and all other conditions remain constant.(“Ohm’s law,” 2016)

Chapter 2: Literature Review

Mathematics and Career Education in New York

Over the past 150 years, the New York State Education Department and the Board of Regents has increasingly shifted the mathematical requirements for high school graduation to more rigorous standards that are appropriate for students entering higher education than entering directly into careers. The current Regents exams are aimed at students preparing to enter college, but there have historically been many mathematics-based career and technical Regents exams have been given over the years.

In 1879, there were vocational Regents exams in “Bookkeeping” and “Drawing, freehand and mechanical” along with 40 other exams that were more in keeping with a classical university education (“History of Regents Examinations,” 2012). This established both business and engineering career subjects as a valuable part of the New York State secondary school curriculum. Barlow (1992) reported:

> Around 1905, proponents of vocational education argued that a broader curriculum was needed to prepare people for the new industrial age. They wanted youth and adults to have a chance for better careers. They were unhappy that only 8 percent of youth graduated from high school, and almost all male graduates went to college while female graduates went into white-collar work. (p. 30)

New York City became the birthplace of the National Society for the Promotion of Industrial Education (NSPIE). NSPIE began to lobby Congress and New York State in 1907. By 1911 their lobbying work was felt heavily at the state level. The Regents exams expanded to include “economics and commercial arithmetic” (“History of Regents Examinations,” 2012). Representative Charles R. Davis of Minnesota proposed a bill in 1907 to “allocate federal funds to agricultural high schools for teaching agriculture and home economics, and to secondary schools in urban communities for teaching mechanical arts and home economics.” (Gordon, 2014, p. 102) In 1917, the Smith-Hughes Vocational Education act was signed into law by President Woodrow Wilson because the House Committee on Education filed a report that stated,

> It is especially designed to prepare workers for the most common occupations in which a great mass of our people find useful employment... to give training of a secondary grade to persons more than 14 years of age for... employment in the trades and industries, in agriculture, in commerce and commercial pursuits, and in callings based upon... home economics. (Barlow, 1992, pp. 30-31)

The timing of the passage of this law was propitious because it provided a trained civilian workforce to support the war effort during World War I and these skills were also put to good use in the post-war economy.

“During the 1920s, '30s, and '40s enrollments in industrial arts and business courses, part-time continuation schools, two-year trade schools, and industrial and technical high schools increased greatly” (Folts, 1996, p. 22). By 1931, the Regents exams had expanded to include “architecture, electricity, mechanical design, structural design, applied design, chemistry and dyeing, cloth construction, costume draping, marketing and salesmanship” (“History of Regents Examinations,” 2012) because there was a demand that students be ready for careers that required short post-secondary education. Students were less likely to be able to afford tuition for a four-year college due to the Great Depression and a 2-year trade program was more affordable than a 4-year college degree. Folts (1996) reported,

> The first four-year syllabus for business subjects appeared in 1925, and distributive education was introduced with federal funding in 1937. Every central rural school district

> was required to offer home economics; many also offered courses in agriculture. High school industrial arts courses were broadened to include materials and technologies beyond woodworking. A 1935 statute made all the industrial and technical schools full high schools, and increased industry presence on advisory boards. (pp. 22-23)

While these technical and career classes were not strictly mathematics classes, they included many mathematical principles of problem solving so that students would be prepared to work in agricultural, factory, or business employment directly after graduation from high school. The Electricity exam, which was aimed at students studying the electrical trades, was only given in 1935 and 1936. It was in 1940 that graduation rates reached 50% for the first time (C. B. Swanson, 2010).

Career and technical training in high school programs continued to be seen as a valuable but separate path from college preparation. “During World War II, vocational education was an integral part of the National Defense Training Program, which trained nearly 7,500,000 persons for defense and war production employment.” (Barlow, 1992, p. 32) After 1945, career and technical training within individual high school programs continued to grow, but “Federal and state aid did not keep pace with costs.” (New York State Education Department, 2008) In order to keep extending career and technical instruction without increasing costs to individual schools and school districts, “boards of cooperative educational services (BOCES), authorized by a 1948 law” (New York State Education Department, 2008), were created. The federal Vocational Act of 1963 provided the financial support to both widen the career and technical classes at BOCES along with their audience as minorities, students with disabilities, and women who wanted to enter traditionally male occupations were included. BOCES across New York State began to offer career and technical instruction in their own educational centers by 1967, a clear demarcation line between the college-preparatory and career-oriented tracks was created because students were being taken from their academic high schools to a technical instruction center for specialty training.

This led to the phasing out of many career-oriented Regents exams, including career-oriented exams that were highly mathematical. The Business Arithmetic exam was last given in June 1974 and the Bookkeeping-Accounting exam was last given in June 1987. Both of these tests had been through cycles of use and discontinuation depending on the state of the economy. It is interesting to note that the Bookkeeping exam survived for over a century which is a testament to the importance to New York State of supporting a financial career path over a career path into either electrical or mechanical trades.

The era between the 1940's and 1969 saw even more growth in the high school graduation rate. During the 1950's, it became normal for students to graduate high school. The national high school graduation rate peaked in 1969 at 77% (C. B. Swanson, 2010) because of "a steady decline in the percent of high school students taking more-rigorous mathematics courses and a steady increase in enrollments in less-demanding mathematics courses for the vast majority" (Stanic & Kilpatrick, 2003, p. 443). During the early 1970's, schools

> either ignored mathematics education or, under the guise of expanding educational freedom, advocated giving greater choice and flexibility to students in determining their school programs. This policy meant that students could more easily avoid demanding courses such as advanced algebra or trigonometry. (Stanic & Kilpatrick, 2003, p. 448)

This laissez-faire attitude within schools across the United States led to a period of slow decline in the high school graduation rate and continued the separation between college-preparatory and career-oriented tracks.

> In the 1980s legislation continued to address pressing social issues through vocational education programs. The Carl D. Perkins Vocational Education Act of 1984, which established funding authorization for a five-year period, focused on improving vocational programs and serving special populations (Barlow, 1992, p. 32).

It was reauthorized as the Carl D. Perkins Vocational and Applied Technology Education Act of 1990 and Congress

> set the stage for a three-pronged approach to better workforce preparation: (1) integration of academic and vocational education, (2) articulation between segments of education engaged in workforce preparation—epitomized by congressional support for Tech Prep, and (3) closer linkages between school and work. (Gordon, 2014, p. 114)

While the revised Perkins Act emphasized vocational education's integration with academic instruction, the funding appropriated did not match the intended purpose. The revised Perkins Act was flawed in that it required certain learning methodologies to be used for vocational education and the legislative functions of the state education boards were ignored (G. I. Swanson, 1991). The Perkins Act of 1990 became a recipe for failure and the academic and career tracking continued through the five-year appropriation period. The Perkins Act was again revised in both 1998 and 2006 with greater emphasis on diversity of the CTE student population and increased accountability of the schools and teachers for academic achievement within the CTE programs in order to mesh with the requirements of the No Child Left Behind Act of 2001 (U.S. Department of Education, 2010). The Perkins Act of 2006, also known as Perkins IV, funding has been annually reauthorized as a continuing source of funding for career and technical education through this year. It appears that Congress will continue to provide funding to the states at the current levels through

September 2015 (Hyslop, 2014) but the bill has not yet been passed and signed as of December 10, 2014.

Since the late 1990's, combined college-preparatory vocational programs have been implemented within high schools in a more academic fashion. Project Lead the Way (PLTW) is one example. It contains "a standardized set of courses designed to provide secondary school students the preparation in math, science, technology and language arts they need to succeed in college-level engineering" (Home-Douglas, 2007) and has challenged the traditional thoughts about CTE programs as being designed for those who are either not interested in or incapable of completing a course of higher education. A report on PLTW surveyed senior high school students in 2006-2007 and "90% said they knew what they wanted to study thanks in part to their PLTW experiences, and 80% said their PLTW experiences significantly increased their ability to succeed in post-secondary education" (Walcerz, 2007). The original standardized curriculum for PLTW was developed in 1997 by Dick Blais, a CTE teacher, and Richard Liebich, a Civil Engineer and prominent businessman, at Gowana Middle School in Clifton Park, New York. Liebich believes that all students need to be challenged in order to succeed and says, "We're going to lose them unless we get them excited [and] give them problems to solve and get them excited about the world they live in. It doesn't have so much to do with them as to do with the way we teach them" (Home-Douglas, 2007).

> The lines between academic study and career and technical skills training are blurring. State and national leaders are beginning to recognize the transformational potential that CTE offers for such issues as secondary-postsecondary transitions, remediation, and the drop-out problem. (DeWitt, 2008)

Now there is a unique opportunity for both schools and students to connect mathematics and careers.

> As states are working to align their education systems with the Common Core State Standards (CCSS) in support of the goal of graduating all students ready for college, careers and life, academic and career and technical education (CTE) leaders at the state and local levels can and should maximize this opportunity to finally break down the silos between their disciplines and collectively find ways to ensure that the new standards rigorously engage all students in both academic and CTE courses (Meeder & Suddreth, 2012)

In order to accomplish this integration, nine action steps were suggested by Meeder and Suddreth (2012): (1) Include CTE leaders and business partners in efforts to create a broader view of college and career readiness; (2) Ensure that CTE representatives are part of the state team for planning and implementing the CCSS; (3) Implement a communications plan that specifically includes CTE administrators and instructors and uses a wide variety of communication strategies: email and listserves, informational videos, local workshops and presentations, and regional and statewide conferences; (4) Engage CTE and academic educators to update CTE standards to reflect the CCSS and create crosswalks between the new CCSS standards and existing CTE standards; (5) When possible, update or create model instructional resources for both CTE and core academic teachers that have the CCSS embedded; (6) Launch new or build upon existing professional development activities to help CTE teachers integrate literacy and math strategies in their CTE classrooms; (7) Bring CTE and academic teachers together in structured professional development activities to review and reflect on the CCSS, unpack the standards to see how they can apply in the CTE context, and create model instructional resources; (8) Establish clear expectations for CCSS integration into CTE by including references to the CCSS in annual funding applications, continuous improvement planning, CTE teacher qualifications and criteria for local monitoring visits; and (9) Ensure that postsecondary CTE is also included in outreach and implementation planning. As these action plans are implemented, students and teachers in both CTE and academic curriculum tracks may benefit from the integration of real world modeling into their classrooms (pp. 9-22).

Mathematics and Career Education Today

As of 2014, the nationwide integration of CCSS into the CTE curriculums has been either sporadic or fairly ignored. While both the New York Common Core Learning Standards (NYCCLS) for English Language Arts (ELA) (New York State Education Department, 2011) and Mathematics identify what students are supposed to be able to know and do at each grade level in traditional college-preparatory courses, the integration of the NYCCLS into CTE courses have been limited to the ELA standards. The NYCCLS for Mathematics should be able to also apply to CTE curriculum since many of these courses depend heavily on mathematical modeling. However,

> developing activities relating to mathematical modeling will require exemplar resources and professional development. But this change in the math standards provides CTE educators with a valuable opportunity to better align their instruction with academic learning to help students use the higher-order skills of problem framing and problem solving. (Meeder & Suddreth, 2012, p. 6)

On October 20, 2014, the New York State Board of Regents “approved several new options for students to meet the State’s high school graduation requirements” (New York State Board of Regents, 2014). Students in CTE programs will still be required to take and pass four of the five currently required Regents exams and they will also take a “career and technical education pathway assessment approved by the Commissioner, following successful completion of an approved CTE program (CTE Pathway)" (New York State Board of Regents, 2014) to receive a Regents diploma. While this CTE pathway assessment is yet to be determined, it could include state licensing exams for particular careers or a capstone assessment from STEM programs such as Project Lead the Way. The key is that the state must "ensure that pathway assessments are of sufficient rigor, validity and reliability" (New York State Board of Regents, 2014) to assure that students are college and career ready when using these alternative pathways to graduation. Until this assessment method is clarified or state-wide and nation-wide common standards are developed for CTE programs, all students graduating from New York State public high schools will likely still be required to take and pass the current five core Regents exams of "English, science, math, as well as the U.S History and the Global Studies and Geography exams." (New York State Board of Regents, 2014)

> Regents Chancellor Merryl H. Tisch said the goals of the new options are to improve the state's 74.9 percent graduation rate, increase the percentage of students who graduate prepared for college and careers (currently 37.2 percent), and help prepare more students for success in the 21st century economy. (New York State Board of Regents, 2014)

This laudable goal still requires much work so that all students in all classes have the opportunity to learn higher level mathematical and problem solving skills in CTE classes and be able to apply them in a way that cuts across the entire curriculum.

Chapter 3: Unit Plan

Five (5) lesson plans are presented in this unit plan in order to deepen a student's conceptual and procedural understanding of algebra as it relates to real-world applications by modeling in small electrical circuits. This chapter will address the methods by which a mathematics teacher, who is not an expert in electrical circuits, can use a student's prior scientific knowledge and technological motivation to understand these algebraic concepts more deeply.

Lesson 1 will apply a student's knowledge of scientific notation & percentage calculations using the Resistor Code. Students will learn to read a resistor using the standard 4 stripe color code and find the nominal resistor value and calculate the resistor's possible minimum/maximum values using the given percent tolerance and represent them as constraints in inequality statements which is the focus of the standard A.CED.3.

Lesson 2 will have students rearrange and solve simple addition equations for electrical applications with multiple variables which is the focus of the standard A.CED.4. Students will either be provided access to several resistors, an electrical breadboard or conductive squishy circuit dough, and an ohm meter to create series resistances either physically or through emulation software to prove to themselves that the total resistance of resistors in series is the sum of those resistors ($R_S = R_1 + R_2 + \cdots + R_n$). Students will find the unknown resistor value by rearranging the formula developed during their investigation.

Lesson 3 will have students rearrange formulas for both Ohm's Law and Joule's Law to highlight a quantity of interest which is the focus of the standard A.CED.4. During the lesson, an electrical circuit that makes an LED glow, either physically or through emulation software, will prove to students that current is flowing through the circuit. Measurements of both voltage and resistance will be taken at different points of the circuit to allow students to discover the current flowing through the LED and conductive dough. Students will be encouraged to think about why current is not usually directly measured by electricians and engineers. Students will solve for the current and other variables by substitution of the measurements after rearranging the formula.

Lesson 4 will have students solve multiple step equations using parallel resistive circuits that they will investigate using the same method as they used during lesson 3. This continues the focus on the standard A.CED.4 at a deeper level. They will learn to simplify complex fractions as they multiply by the reciprocal. Students will find the unknown resistance of a resistor in a parallel circuit by rearranging the appropriate formula.

Lesson 5 is the culmination of the experience with resistive circuits which raises the complexity of the equations. Students will learn to analyze two types of more complex circuits: (1) a parallel-series resistive circuit and (2) a series-parallel resistive circuit. Students will create equations and then solve for an unknown resistance in a circuit.

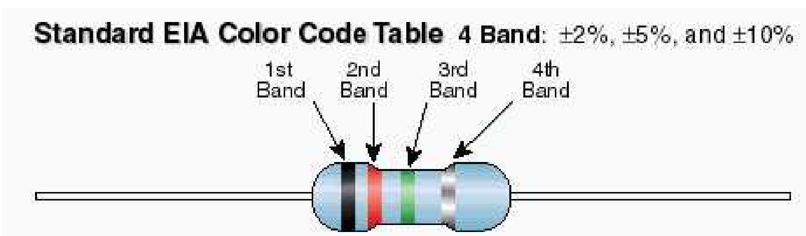
As an assessment for this unit plan, students will be given three problems that have a real-world context to assess whether they can make decisions from constraints as well as create and rearrange equations.

Lesson 1

The goal of this lesson is to enable students to write inequality statements using scientific notation by reading color coded resistors and applying tolerances.

Most resistors use a standardized color code to indicate the nominal value and tolerance of the resistor. This means that the color code will tell you the value that the manufacturer intended the resistor to be along with an acceptable amount of error. If the measured resistance of a resistor is outside of the tolerance range, it cannot be sold to a consumer.

The resistor color code (Figure 1) the students will be using in this lesson is the 4-band code defined by the international standard, IEC 60062, issued in 2004 by the International Electrical Commission. It is the same standard that was developed and issued in the 1920's by the Radio Manufacturers Association and later adopted by the Electronic Industries Association (EIA). So this code has been around a long time!



Color	1st Band (1st figure)	2nd Band (2nd figure)	3rd Band (multiplier)	4th Band (tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	±2%
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%

Figure 1: Resistor Color Code

("Resistance: Ohm´s Law. What is Electrical Resistance? | Petervaldivia," 2015)

The first color band tells the value that is in the tens place and the second color band tells the value that is in the ones place. The third color band is a multiplier of 10 with an exponent that determines how large or small the nominal resistor value is. The fourth color band is the tolerance and defines the percentage of error that an acceptably manufactured resistor could possibly have.

For instance, the resistor shown above has a black band, a red band, a green band and a silver band. This tells us that there is a 0 in the tens place, a 2 in the ones place, and it should be multiplied by 10^5 to get the nominal value. Therefore, the nominal value of this resistor is expressed in scientific notation as 2 x 10^5 ohms or 200,000 ohms in standard notation.

The fourth band is silver and tells us that the actual measured value of the resistor could be as much as 10% lower or 10% higher than the nominal value and still be acceptable. To find this measurement, multiply the nominal value by the tolerance. In this case, 200,000 ohms * 10% = 200,000 * 0.10 = 20,000 ohms or 2 x 10^4 ohms.

Since the tolerance goes both ways, we have to find both the minimum and maximum values by subtracting from and adding to the nominal value.

Minimum acceptable value = Nominal value – Tolerance

Maximum acceptable value = Nominal value + Tolerance

Our example resistor could have a minimum value as low as 200,000 ohms – 20,000 ohms = 180,000 ohms or 1.8 x 10^5 ohms and a maximum value as high as 200,000 ohms + 20,000 ohms = 220,000 ohms or 2.2 x 10^5 ohms.

If R represents the resistance, then this tolerance range can be written as a single inequality statement:

$$1.8 \times 10^5 \Omega \leq R \leq 2.2 \times 10^5\ \Omega$$

During class, the students should be given the opportunity to work with real resistors if possible. The teacher should obtain at least 5 different values of resistors which can be purchased at any local electronics store. If a teacher has a vision impaired student or cannot afford and/or obtain real resistors, a blank resistor simulation card has been provided as Figure 17 in Appendix A to facilitate this exercise. The teacher can then color the resistor stripes in the values they desire using colored markers. For students with color vision impairments, the teacher may want to provide the color name on each stripe. Depending on the time allowed for the exercise, the teacher may provide a worksheet of Table 1 along with up to 5 color coded resistors for students to analyze by using Figure 1.

Table 1

Blank Resistor Classwork

Resistor	1st Band	2nd Band	3rd Band	4th Band	Nominal Value (ohms)	Maximum Value (ohms)	Minimum Value (ohms)
1							
2							
3							
4							
5							

For each resistor, students will need to write the colors seen on each resistor's stripes in Table 1 so they can determine the nominal, maximum and minimum resistance values. Students will then write an inequality statement for each resistor that shows that resistor's tolerance using proper scientific notation. Since the first two bands in the resistor color code do not always result in a number between 1 and 10, students may need to change the value read using the color code into proper scientific notation by adjusting the exponent. For example, a resistor that has a red band, a black band, a green band and a silver band will result in the student reading 20×10^5 ohms when using the chart. Students may need to be reminded that they need to change the resistor color code reading of 20 x 10^5 ohms to 2 x 10^6 ohms to maintain proper scientific notation. Care should also be taken when calculating tolerances to maintain proper scientific notation format. Students who are less proficient with exponents may wish to change from the resistor color code reading to a standard number and then change the standard number to scientific notation.

As a realistic assessment, the teacher may provide a mismarked resistor, which can be accomplished by changing one stripe on the resistor using a small amount of nail polish, to the students as a challenge along with the actual resistance reading that was taken prior to class using an ohm meter. This will present a real-world problem for the students to solve since a person creating an electrical circuit must decide if a particular resistor is inside or outside of the tolerance band. While some students may immediately spot the changed stripe, this real-world example will allow the teacher to ask the students to interpret the data and justify their decision about what they will do with this resistor.

Students can use the resistor color codes in Figure 1 to complete Table 2 as homework so they can learn to create their own inequality statements.

Table 2

Resistor Homework

Resistor	1st Band	2nd Band	3rd Band	4th Band	Nominal Value (ohms)	Maximum Value (ohms)	Minimum Value (ohms)
1	Yellow	Violet	Brown	Silver			
2	Red	Red	Green	Gold			
3	Black	Brown	Brown	Silver			
4	Orange	Orange	Orange	Gold			
5	Red	White	Blue	Silver			
6	Red	Red	Black	Gold			
7	Green	Red	Yellow	Red			

For each resistor in Table 2, write an inequality statement that shows that resistor's tolerance in scientific notation:

1. ____

2. ____

3. ____

4. ____

5. ____

6. ____

7. ____

A homework sheet with answers for this lesson is provided in Appendix A on page 71.

Lesson 2

Now that students understand how to read the resistor code, students will be able to discover how electrical schematics enable them to create equations to find unknown variables by using the mathematical practice of modeling. The algebraic goal of this lesson is to enable students to solve single step addition equations by rearranging the variables before substituting numbers.

In order to facilitate this lesson, a teacher must have prior access to some electronic equipment or a computer with an internet connection. For a very concrete hands-on lesson, the teacher will need the following equipment for every group of students: at least 5 real resistors of different values, an electronics breadboard or Squishy Circuit conductive dough (recipe available at http://courseweb.stthomas.edu/apthomas/SquishyCircuits/conductiveDough.htm), and an ohm meter. If this equipment is not available or the students are able to work at a more pictorial level, a free circuit emulator program such as Autodesk's 123D Circuits (https://123d.circuits.io/lab) can be used on any available computers that have internet access. A circuit emulator eliminates the need for any further purchased items.

Although the resistor code allows resistors to marked with almost any value, manufacturers make certain resistors more cheaply than others, so there is a common set of resistor values that engineers know that they can readily buy. Engineers often need to create a circuit using a resistor value that they cannot buy directly from a store, so they will either put the resistors in series or in parallel to create an equivalent resistor. Students may need help understanding that an electrical schematic is a picture that helps an engineer create an equation that can lead them in the decision of what resistors to buy.

Look at a series resistance circuit with the students first using Figure 2. This circuit contains a battery and two resistors in series. From the student's previous science experience, they may know that current will flow from one terminal of the battery, through the resistors, and then back to the battery to form a circuit. While students do not yet know anything about the circuit, the teacher should ask the students what conjecture they would make about the total resistance of the two resistors. This will set the scene for their next mathematical experiment that will prove that the equivalent resistance of resistors in series is equal to the sum of the resistor values.

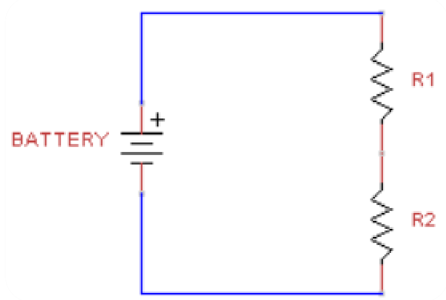


Figure 2: Series Resistance Circuit

The teacher may have the students choose and set up two resistors in series either physically in an electrical breadboard (Figure 3) or open a preconfigured circuit emulator file in Autodesk's 123D Circuits (Figure 4) or using Squishy Circuit conductive dough (Figure 5). Students should make 3 measurements without the battery connected: (1) the resistance from the top to the bottom of R_1, (2) the resistance from the top to the bottom of R_2, and (3) the resistance from the top of R_1 to the bottom of R_2. The students may then make a conjecture that the total resistance across the two resistors is equal to the sum of the two measured resistors. This is total resistance is denoted as R_S for series resistance.

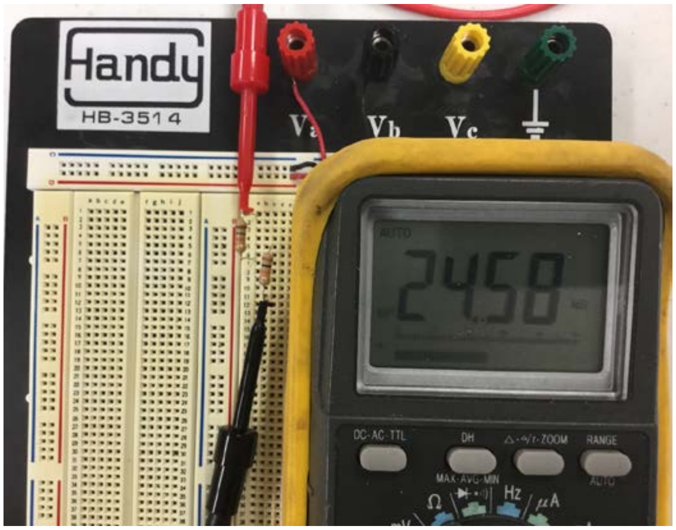


Figure 4: Physical breadboard setup for resistors in series

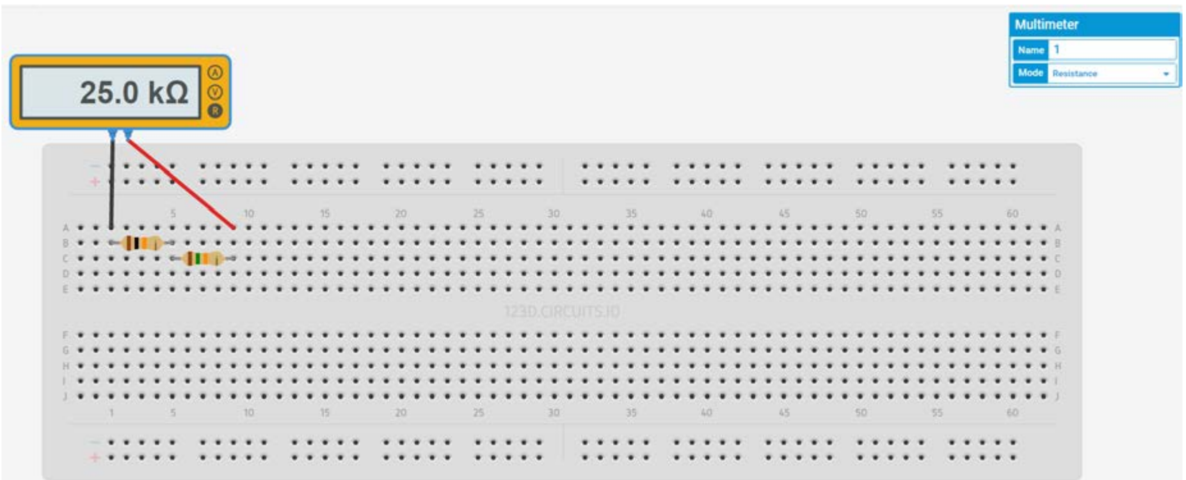


Figure 3: Emulator setup for resistors in series

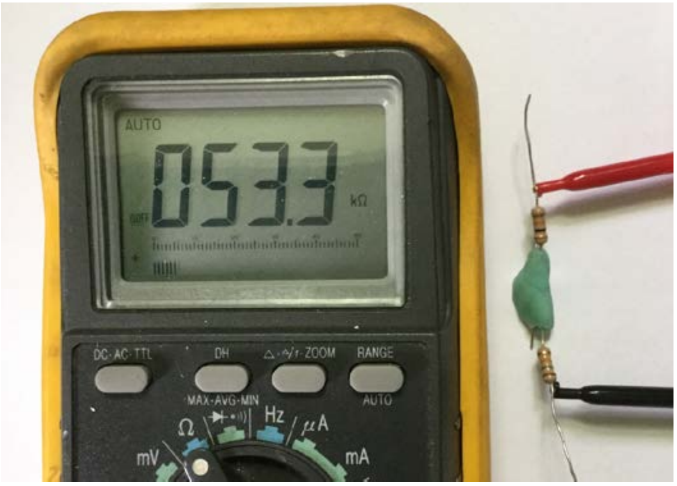


Figure 5: Squishy Circuit conductive dough setup for resistors in series

So students may prove their conjecture, the teacher may need to lead the class in learning how to swap out different resistors and make resistance measurements. In Figures 3 and 5, students will be able to physically change from one resistor to another to determine the pattern of the equation. On the emulator breadboard, students will need to highlight the resistor and change the value in the box that is in the upper right hand corner of the display. Notice that there is a definite difference between the total resistance of the physical setups and the emulated setup. The physical setup with real resistors (Figure 3) includes resistances that are within the manufacturing tolerance band while the emulated setup resistances are exactly what is called out by the color code. This difference will not affect the mathematical calculations that help students realize that the equation $R_s = R_1 + R_2 + \cdots + R_n$ is true, but the physical setup will further reinforce the idea from lesson 1 that resistors are manufactured with tolerances and more than the emulator setup (Figure 4) does.

The physical setup with real resistors connected by Squishy Circuit conductive dough (Figure 5) may present a larger challenge to students because the dough itself is yet another resistor instead of a short connecting wire with resistance that can be ignored. The same physical resistors were used in Figure 3 (24.58 kΩ) and Figure 5 (53.3 kΩ), so it is plain that the conductive dough's resistance is not negligible. If students are only working with Squishy Circuit conductive dough as their circuit connection medium, then the teacher should present the students with an opportunity to work with a standardized amount of the dough to understand how much resistance it provides.

Once a method of presenting the circuit has been chosen by the teacher, Table 3 can be used as a blank to help students record their resistance measurements if one of the physical setups is being used. This table will allow students to investigate up to 5 resistors in series on a breadboard. If the students are using the Squishy Circuit conductive dough as a connecting element between each resistor, then a maximum of 3 resistors should be used because students will need to determine the resistance of the connections which can be designated as R2 and R4. If students are using a circuit emulator, then Table 3 can be used as a review for reading the resistor color code as presented in Lesson 1.

Table 3

Blank Single Resistor Measurement Classwork

Resistor	1st Band	2nd Band	3rd Band	4th Band	Measured Resistance (ohms)
R1					
R2					
R3					
R4					
R5					

When the students know each individual resistance, they can start measuring the resistances in pairs and triples, from the beginning of one resistor to the end of the subsequent resistor, and record their measurements in Table 4.

Table 4

Blank Series Resistor Measurement Classwork

Measurement of 2 Resistors in Series	Measured Resistance (ohms)	Measurement of 3 Resistors in Series	Measured Resistance (ohms)
R1 to R2		R1 to R3	
R2 to R3		R2 to R4	
R3 to R4		R3 to R5	
R4 to R5			

At this point, it is appropriate to have students verify that the sum of the resistors in series matches the measurements they made in Table 4 and write a sentence or two about how the results either confirm or deny their conjecture. The teacher should then pose the question, “People who design circuits often know the value of one resistor and what they want the total resistance to be for two resistors in series. With what we know now, can we write an equation to help us determine that unknown resistor?” Students should be led through the following thought process to create an equation for two resistors in series:

Let R_s be the total series resistance in ohms.

Let R_1 and R_2 be the individual resistors in the series circuit.

$R_s = R_1 + R_2$

At this point, the students should choose one of the two individual resistors to solve for. This starts the process of highlighting a quantity of interest. Let us suppose that the students choose to solve for the value of the second resistor. The students may be tempted to immediately plug in the numbers given in order to solve the equation, so the teacher should give no values to any of the variables, but suggest to the students that we can rearrange the variables in the equation using the same rules that we do when we have numbers. One suggestion that a teacher may make to the students is that they can use a highlighter to indicate which variable they want to solve for in the equation. The notes the teacher gives as an example may look as follows:

$R_s = R_1 + R_2$	Highlight the variable to be found
$R_s - R_1 = R_1 - R_1 + R_2$	Subtract R_1 from both sides of the equation
$R_s - R_1 = R_2$	Simplify the equation

The teacher should then have the students independently rearrange another series resistance equation with more resistors such as $R_s = R_1 + R_2 + R_3$ to find R_1. Once students are able to accomplish that task, students can then be given values for R_S and other resistors to finish solving the problem for the highlighted variable.

As a realistic assessment at the end of the lesson, the following question may be posed: "An electrician needs a heating circuit to have a series resistance of 50 ohms. He has two resistors, R_1 and R_2, that he will use to make this circuit. He must buy another resistor, R_3, to complete the circuit. The electrician wrote down his initial equation: $50\ ohms = R_1 + R_2 + R_3$. Rearrange this equation to highlight the variable R_3." The solution to this problem should be any variation of the equation $R_3 = 50\ ohms - R_1 - R_2$.

Students who are more proficient at rearranging equations may benefit from the following challenge question: "If the electrician knows that the values of resistors R_1 and R_2 are exactly the same, how does the equation change?" The equation to answer this question should be either $R_3 = 50\ ohms - 2R_1$ or $R_3 = 50\ ohms - 2R_2$.

A homework sheet with answers for this lesson is provided in Appendix A on page 72.

Lesson 3

Now that students have experienced working with either physical or emulated circuits, they are ready to explore rearranging Ohm’s Law which states that “the current flowing through a metallic conductor is proportional to the electromotive force applied across its ends, provided the temperature and all other conditions remain constant.” (“Ohm’s law,” 2016)

The teacher must have prior access the same electrical equipment as was used in lesson 2 to facilitate this lesson. If a physical setup was used, the teacher will add an enclosed 6VDC battery pack that has an on/off switch and a 10mm 3VDC LED to each kit. The teacher should use the schematic in Figure 6 and the physical circuit pictured in Figure 7 to set up and test at least one physical circuit before class. If using Squishy Circuit conductive dough to connect the electrical elements, make sure that one connecting lump is larger than the other since it adds to the total resistance. The resistance of this circuit may be between 1kΩ and 50 kΩ depending on the current required by the LED chosen by the teacher. A higher resistance will reduce the current flowing. Students may not see the LED glowing if the resistance is too high. The resistor can be made entirely of Squishy Circuit conductive dough as shown in Figure 7. If the LED has been correctly inserted in the circuit, it should glow when the switch on the battery pack is turned on. If it does not glow, try turning the LED around since this is a directional electrical element, unlike a regular light bulb. If it still does not glow, the teacher should check for proper voltage of the battery pack.

If the emulation software setup was used, then a preconfigured emulator file using the same resistance as the physical circuit will need to be prepared to simulate the circuit as shown in Figure 8.

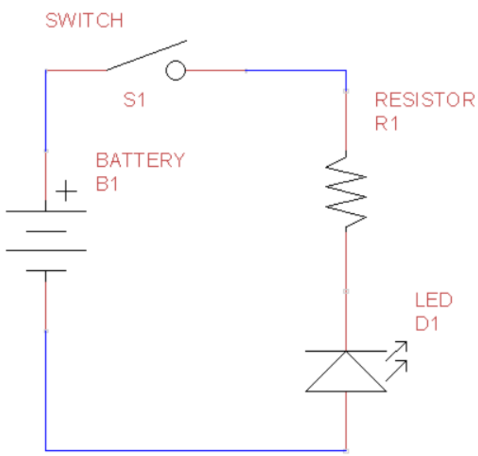


Figure 6: Electrical schematic to investigate Ohm's Law

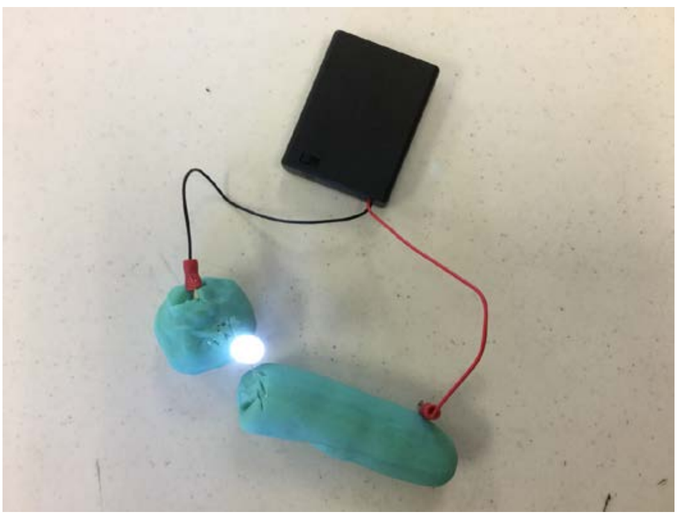


Figure 7: Squishy Circuit setup with LED on for Ohm's Law experiment

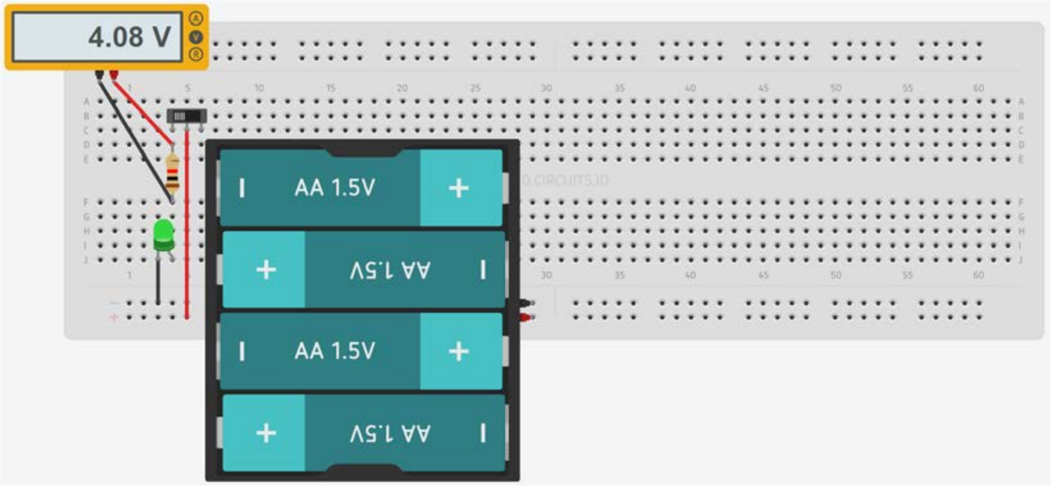


Figure 8: Active emulator setup for Ohm's Law experiment

Since this lesson is may be seen as a science experiment as well as an algebra lesson, the lesson began with an investigation of the circuit itself. Students are encouraged to draw the circuit as they see it in front of them before the schematic representation is introduced. This way they can label their own representation so that it matches the schematic. This is one way to support students who have trouble with complex tasks because they are making sense of the circuit in their own fashion before looking at the mathematics.

As with the previous lessons, the teacher should help activate the students' prior knowledge about electrical circuits by talking about how the battery is measured in volts (V) and the resistor is measured in ohms (Ω). Students will recognize the switch as the same as the one on the outside of the battery pack. The LED may be a new electrical element to students, but it can be compared to a regular light bulb that has a very low resistance, uses very little power, and has a very long life. It may be helpful to ask students what happens when they turn the switch on to complete the circuit. They should see the LED glowing if the circuit is properly assembled and working and identify that as a sign that current is flowing through the circuit. It may be helpful for the students to add the current to their drawing or schematic by showing it as an arrow that goes from the battery, through the switch, resistor and LED and returns to the battery (*Figure 9*).

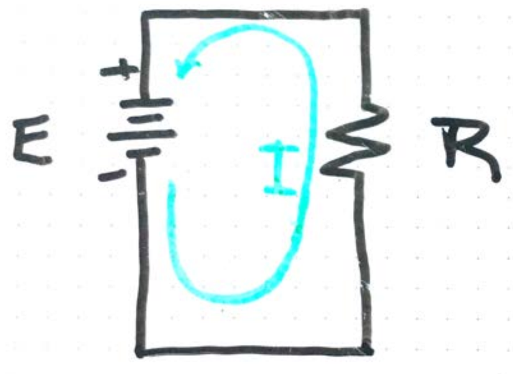


Figure 9: Hand drawn resistive circuit schematic showing current flow

Once students understand that the current can only go one direction in a series circuit, they should make the connection with the series resistances they investigated in the previous lesson. The teacher may want to introduce the formal definition of Ohm's Law, as stated earlier, and help the students parse out the common variables as follows:

Let E be the electromotive force measured in volts (V).

Let I be the current measured in amps (A).

Let R be the total resistance in ohms (Ω) that the current is flowing through.

Students should then be able to construct the standard equation $E = IR$ but they may more likely think of it as $E = RI$ if they are used to seeing the equation $y = kx$ as a proportional equation. It is essential that an algebra teacher use the standard format of the equation for the formula's definition because students will encounter it in that arrangement during science and technology classes.

After defining the standard equation, the teacher should lead the students through rearranging the equation to highlight the current variable (I). The reason for this is that there is an inherent difficulty in measuring current directly in a battery powered circuit because the circuit must be broken and a current meter inserted into the circuit. When working with circuits on breadboards or made with Squishy Circuit conductive dough, this may seem like a trivial problem. Students need to realize that electrical professionals may not be able to break into a circuit on a printed circuit board and so they rely on calculations rather than measurements to determine the amount of current flowing. The student notes will look as follows:

$E = IR$	Highlight the variable to be found.
$\frac{E}{R} = \frac{IR}{R}$	Divide both sides of the equation by R.
$\frac{E}{R} = I$	Simplify the equation.

The teacher should then have the students independently rearrange the equation to solve for the resistance (R) after showing this example. Their rearranged equation should either look like $\frac{E}{I} = R$ or $R = \frac{E}{I}$.

Measurements should now be made with the circuit powered on. Students should be advised to turn off the circuit for safety each time they move the connections to the meter. Students who are using the emulator software will have to start and stop the simulation to make their measurements. If there are multiple meters that can be used, this will avoid any problems that students might encounter. Typically, students may have problems with the wires coming out of the Squishy Circuit conductive dough and thus having the LED glowing during this process is a big help to them when making measurements. Students can fill in

Table 5 which has been designed to minimize necessary movements of the meter. They will need to move the switch on the meter from volts to ohms each time they move to measuring the next electrical element.

Table 5

Measurements for Ohm’s Law investigation

Element	Measurement	Units
Battery		V DC
Resistance across Resistor (R1)		Ω
Voltage across Resistor (R1)		V DC
Resistance across the LED (D1)		Ω
Voltage across LED (D1)		V DC

When the measurements are accomplished, the teacher should provide the students with the information about the current that the LED (D1) requires to glow. The 10mm 3VDC white LED in Figure 7 required between 0.065 A to 0.075 A to glow and the green emulated LED in Figure 8 required 0.00408 A at 1.91 VDC. This information should be available when purchasing the LEDs since there are many different styles available. Then the students can be asked, “Does Ohm’s Law generally hold true for the circuit we measured? Use the data collected to justify your answer.” They should recognize that they will use the rearranged equation for current (I) to calculate their current and check their answer against the current that the teacher gives.

To add to the complexity of the problem, the equation for power should be investigated as well. Electrical power is defined as “The rate at which energy is delivered to a light bulb by a circuit is related to the electric potential difference established across the ends of the circuit (i.e., the voltage rating of the energy source) and the current flowing through the circuit.” (“Power: Putting Charges to Work,” n.d.) This sentence can be parsed similarly to the Ohm’s Law formula:

Let P be the power, the rate at which energy is delivered measured in watts (W).

Let E be the electric potential difference measured in volts (V).

Let I be the current measured in amps (A).

Students should notice that Ohm’s Law and the power formula, $P = IE$, contain two of the same variables so that the two equations can be combined by substituting one into the other. It is then possible to find the power of a circuit when we know voltage and resistance. A good joke that the teacher may make at this point is, “Power is as easy as PIE,” which may help students remember the formula in the future. The student notes will look as follows:

$P = IE$	Highlight the variable to be replaced in the power equation
$\frac{E}{R} = I$	Recall the rearranged equation for Ohm’s Law with current highlighted
$P = \frac{E}{R} * E$	Substitute this into the power equation
$P = \frac{E^2}{R}$	Simplify the equation

The students can then be asked the practical question, “How much power is being used by the resistor we measured? How much power is being used by the LED?”

Students should then be encouraged to independently combine these two equations, but they should substitute for the voltage instead. The rearrangement of the equations should look as follows:

$P = IE$	Highlight the variable to be replaced in the power equation
$E = IR$	Recall the original equation for Ohm’s Law with voltage (E) highlighted
$P = I * IR$	Substitute Ohm’s Law into the power equation
$P = I^2R$	Simplify the equation

Students can then be asked, “How much current is calculated to be flowing through the resistor and the LED given the power information found in the last question? Does this answer make sense given what you already know about the current needed for the LED to glow?” This may lead to a lively discussion about the changes that have occurred with light bulbs in their homes. As higher wattage carbon filament light bulbs become obsolete they are being replaced with either compact fluorescent bulbs or multiple LED bulbs. The LED light bulbs provide the same amount of light at a much lower energy cost. This creates a personal connection to real life issues that students encounter in their everyday life and provides motivation to learn more about circuits and algebra.

As a realistic assessment at the end of the lesson, the following question may be posed: “LED light bulbs are used in battery-operated outdoor lighting. If the LED bulbs use a total power of 15 watts from a battery of 12 volts, answer the following questions: (a) What is the total resistance? (b) What is the total current used by the bulbs? Be certain to identify the formula you used before substituting to solve the equation.” Students who took good notes should be able to rearrange the teacher’s examples to get the following answers:

(a) $R = \frac{E^2}{P} = \frac{144}{15} = 9.6\ ohms$; (b) $I = \frac{E}{R} = \frac{12}{9.6} = 1.25\ amps$

Students who are less proficient in rearranging equations may take more steps and use the Ohm’s Law and power equations independently, but they have begun comprehending that equations that use the same variables can be combined in ways that may be new to them to solve real-world problems.

A homework sheet with answers for this lesson is provided in Appendix A on page 74.

Lesson 4

In this lesson, students will be exploring parallel resistor circuits as they rearrange equations. They are called parallel because there are at least two resistors that share the same two connection points in the circuit as shown in Figure 10. This means that the have the same voltage across the two ends and that the current coming through the switch is the same as the sum of the currents going through the two resistors.

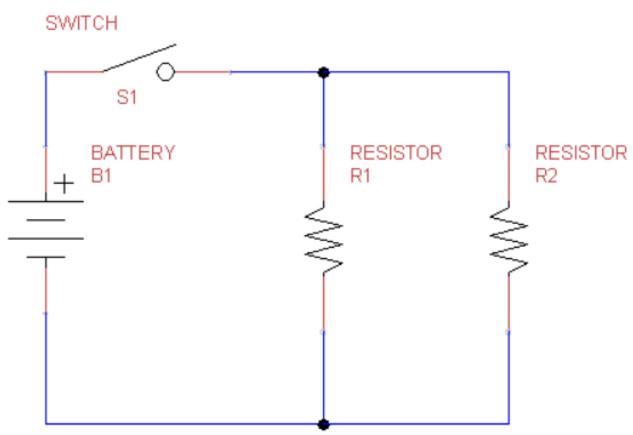


Figure 10: Electrical schematic to investigate parallel circuits

At this point, working with the Squishy Circuit conductive dough becomes a liability because it would add series resistance elements that are undesirable for students investigating parallel circuits. Therefore, this lesson should only be worked on using either a physical breadboard or a circuit emulator. Figure 11 shows the basic setup of the circuit emulator based on the schematic in Figure 10 where $R_1 = R_2$. Since this is the simplest case for students to understand, it should be investigated first. The teacher may also want to discuss the current flowing in the circuit as similar to water flowing in pipes. When the water flows from a main pipe into two smaller branches, the water splits in proportion to the size of the pipes. When the same water comes back from the two branches, the volume of the water adds back together. Electric current also splits proportionally between the parallel branches and the general formula for any two parallel resistors can be derived through the use of Ohm's Law.

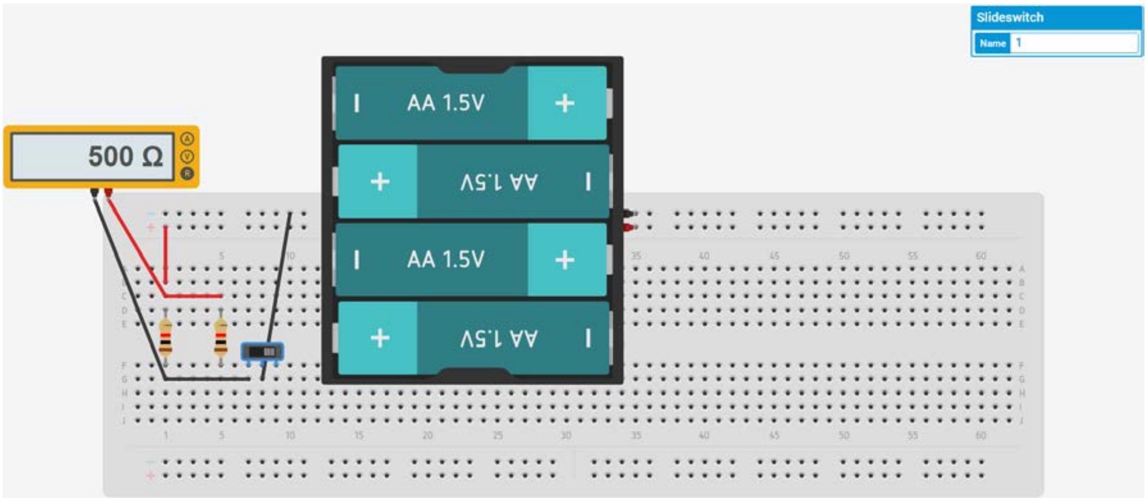


Figure 11: Parallel circuit emulator setup

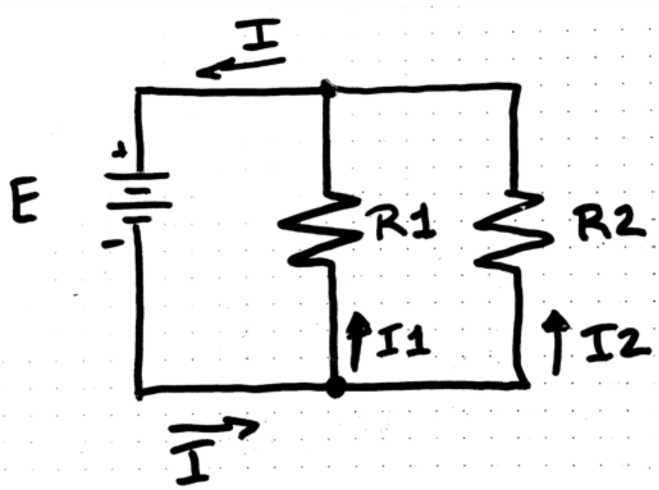


Figure 12: Hand drawn parallel resistive circuit schematic showing current flow

Students should be able to translate their understanding of parallel current flows as described in the water pipe example and mark up their schematic similar to Figure 12 into the equation $I_P = I_1 + I_2$ which should remind them of the series resistance equation covered in Lesson 2. Students can measure that the voltage is the same across the ends of the parallel resistors and recall that Ohm's Law rearranged for current is $I = \frac{E}{R}$ so the teacher may want to give the following notes to help the students understand the more complex equation for parallel resistance:

$I_P = I_1 + I_2$ The total current is the sum of the currents flowing through each resistor

$\frac{E_P}{R_P} = \frac{E_1}{R_1} + \frac{E_2}{R_2}$ Substitute the rearranged Ohm's Law arranged for current (I) in each case

$\frac{E}{R_P} = \frac{E}{R_1} + \frac{E}{R_2}$ Since $E_P = E_1 = E_2$, we can drop the subscript on E.

$\frac{1}{E}\left(\frac{E}{R_P}\right) = \frac{1}{E}\left(\frac{E}{R_1}\right) + \frac{1}{E}\left(\frac{E}{R_2}\right)$ Multiply both sides by the reciprocal of E to simplify it further.

$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$ This is the standard form of the parallel resistance equation.

Students should now be encouraged to explore this equation to see if it holds true with actual resistors and measurements. Students can use Table 6 to record different resistances and find an unknown parallel resistance. If the resistances are low, students can be encouraged to look at the equation in fractional form.

Table 6

Exploring parallel resistances – Find the total parallel resistance

Sample	R_1 Measured (ohms)	R_2 Measured (ohms)	R_T Calculated (ohms)	R_T Measured (ohms)
1				
2				
3				
4				

As an example, if $R_1 = R_2 = 1000\ \Omega$, then students may substitute straight into the equation as follows:

$\frac{1}{R_T} = \frac{1}{1000} + \frac{1}{1000}$ Substitute $R_1 = R_2 = 1000\ \Omega$

$\frac{1}{R_T} = \frac{2}{1000}$ Add the fractions since the denominators are already common

$2R_T = 1000$ Use cross product rule

$\frac{2R_T}{2} = \frac{1000}{2}$ Divide both sides by 2 to get R_T alone

$R_T = 500$ Simplify

If $R_1 \neq R_2$, then students will need to find a common denominator to solve the equation with fractions. Let us define $R_1 = 1000\ \Omega$ and $R_2 = 250\ \Omega$ for this example:

$\frac{1}{R_T} = \frac{1}{1000} + \frac{1}{250}$ Substitute $R_1 = 1000\ \Omega$ and $R_2 = 250\ \Omega$

$\frac{1}{R_T} = \frac{1}{1000} + \frac{4}{1000}$ Change the fractions to common denominators

$\frac{1}{R_T} = \frac{5}{1000}$ Add the fractions with common denominators

$5R_T = 1000$ Use cross product rule

$\frac{5R_T}{5} = \frac{1000}{5}$ Divide both sides by 5 to get R_T alone

$R_T = 200$ Simplify

Once the students have completed Table 6 using whatever physical resistors or emulated resistor values the teacher provides, they can be asked, "Does the equation for parallel resistances generally hold true for the resistors we measured? Use the data collected to justify your answer." This will allow students time to compare their calculated values to the measured values and they should recognize that there are limitations on their ohm meters that may not give them as precise an answer as their calculations.

Students who understand this may go on to understand that they can find an unknown resistor if they know the total parallel resistance (R_P) and either resistor R_1 or R_2. The teacher may challenge the students to do this rearrangement by posing the following question, "Suppose an electrical engineer knows he needs to put two resistors in parallel to make his circuit work correctly. He has one resistor (R_1) that is 150 ohms and he needs the total resistance (R_P) should be

100 ohms. What resistor should he buy for the second resistor (R_2)? Rearrange the equation first to make the problem easier to solve." The solution may be presented as follows by the teacher and students working collaboratively:

$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$	This is the standard form of the parallel resistance equation.
$\frac{1}{R_P} - \frac{1}{R_1} = \frac{1}{R_1} - \frac{1}{R_1} + \frac{1}{R_2}$	Subtract $\frac{1}{R_1}$ from both sides of the equation
$\frac{1}{R_P} - \frac{1}{R_1} = \frac{1}{R_2}$	Simplify the equation
$\frac{1}{100} - \frac{1}{150} = \frac{1}{R_2}$	Substitute the known values into the equation
$\frac{15}{15}\left(\frac{1}{100}\right) - \frac{10}{10}\left(\frac{1}{150}\right) = \frac{1}{R_2}$	Multiply each term by a form of 1 to make the lowest common denominator
$\frac{15}{1500} - \frac{10}{1500} = \frac{1}{R_2}$	Simplify the equation
$\frac{5}{1500} = \frac{1}{R_2}$	Subtract the fractions with common denominators
$5R_2 = 1500$	Use cross product rule
$\frac{5R_2}{5} = \frac{1500}{5}$	Divide both sides by 5 to get R_T alone
$R_2 = 300\ \Omega$	Simplify and add units

If this lesson is used with students taking Algebra 2, the teacher may wish students to take the rearrangement of the equation further by multiplying by the reciprocal of each of the variables before allowing students to substitute values:

$\frac{1}{R_P} - \frac{1}{R_1} = \frac{1}{R_2}$	Simplify the equation
$R_P R_1\left(\frac{1}{R_P} - \frac{1}{R_1}\right) = R_P R_1\left(\frac{1}{R_2}\right)$	Multiply both sides by the reciprocal of the lowest common denominator

$\frac{R_P R_1}{R_P} - \frac{R_P R_1}{R_1} = \frac{R_P R_1}{R_2}$ Distribute the LCD

$R_1 - R_T = \frac{R_P R_1}{R_2}$ Simplify

$R_2(R_1 - R_T) = R_P R_1$ Use the cross product rule to get R_2 into the numerator

$\frac{R_2(R_1 - R_P)}{R_1 - R_P} = \frac{R_P R_1}{R_1 - R_P}$ Divide both sides by $R_1 - R_P$ to get R_2 alone

$R_2 = \frac{R_P R_1}{R_1 - R_P}$ Simplify the equation

The answer to the previous problem will be the same, but will be much easier for students to calculate since the common denominator issue has been dealt with prior to substitution.

As a realistic assessment at the end of the lesson, the following question may be posed: "An electrician knows she needs to put two resistors in parallel to make her circuit work correctly. She knows that R_2 is 470 ohms and she needs the total resistance (R_T) to be as close to 100 ohms as possible. Which standard resistor should she buy for resistor R_1?

a. 110 ohms

b. 120 ohms

c. 130 ohms

d. 140 ohms"

While each answer of b, c, and d plausible if the question had included a 10% tolerance on the total resistance, c provides an answer ($R_T = 127.\overline{027}\ \Omega$) that is closest to the desired total resistance when checked with the standard form of the equation.

A homework sheet for this lesson is provided in Appendix A on page 76.

Lesson 5

This lesson is the culmination of the student experience with rearranging equations. The teacher may want to take two days for it because of the complexity of creating and rearranging the equations. Students have learned about simplifying resistors that are in series and in parallel during lesson 2 and lesson 4. Since very few electrical circuits are that simple, it would be beneficial for students to experience rearranging the equations for both a parallel-series circuit and a series-parallel circuit as shown in Figure 13 to find the total equivalent resistance which combines all the resistors into one regardless of the initial schematic layout.

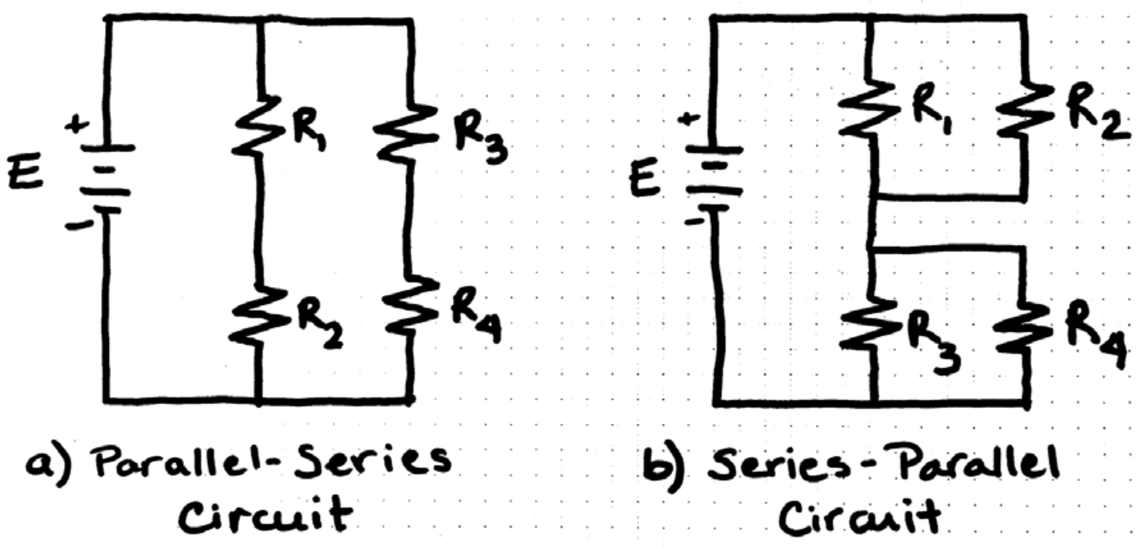


Figure 13: Hand drawn schematics for complex circuits

For student who have trouble with complex mathematics, the rules for dealing with parallel-series circuits and series-parallel can be likened to the rules for order of operations because they are both meant to make evaluating the circuit or equation easier.

When dealing with parallel-series circuits (Figure 13a), the resistors that are in series are analyzed first because they are simple to add together to make an equivalent resistor, as was done in lesson 2. When the series resistors have been reduced, then the parallel resistors can be reduced using the formula from lesson 4.

When dealing with series-parallel circuits (Figure 13b), the resistors that are in parallel are analyzed first. When the parallel resistors have been reduced, then the resistors that are left will be in series and can be added together.

So when the teacher is discussing the differences in the circuits, students may notice that the second part of the name given to the complex circuit is the first thing that they will do to simplify the circuit. If the teacher does not want to go into the complexity of having students create a single equation for a complex circuit, then it is suggested to scaffold each problem into several stages for the students as discussed in this lesson plan.

First we remind the students of the standard equations we used to find equivalent resistances for two resistors:

$R_s = R_1 + R_2$ The standard form of the series resistance equation.

$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$ The standard form of the parallel resistance equation.

Next, we remind the students that the variables R_s and R_P can be any two resistors that are in series or parallel and that they can be named differently. So we ask the students the resistors that are in series in Figure 13a and how they can be represented as equations to make one resistor in each parallel path as shown in *Figure 14*. The students should come up with the equation $R_s = R_1 + R_2$ because those variables are familiar. They can be easily led to see that $R_3 + R_4$ is another series resistance, but may need to be told that we need to have another variable name for that equivalent series resistance. The author's suggestion is that the teacher propose they begin combining the resistor numbers to create a new variable name in a way that helps them relate to the circuit, thus the two equations for the equivalent series resistors in each parallel path become $R_{1+2} = R_1 + R_2$ and $R_{3+4} = R_3 + R_4$. This method of naming is often used when engineers are working in complex situations so that they immediately see where they have already simplified part of the circuit.

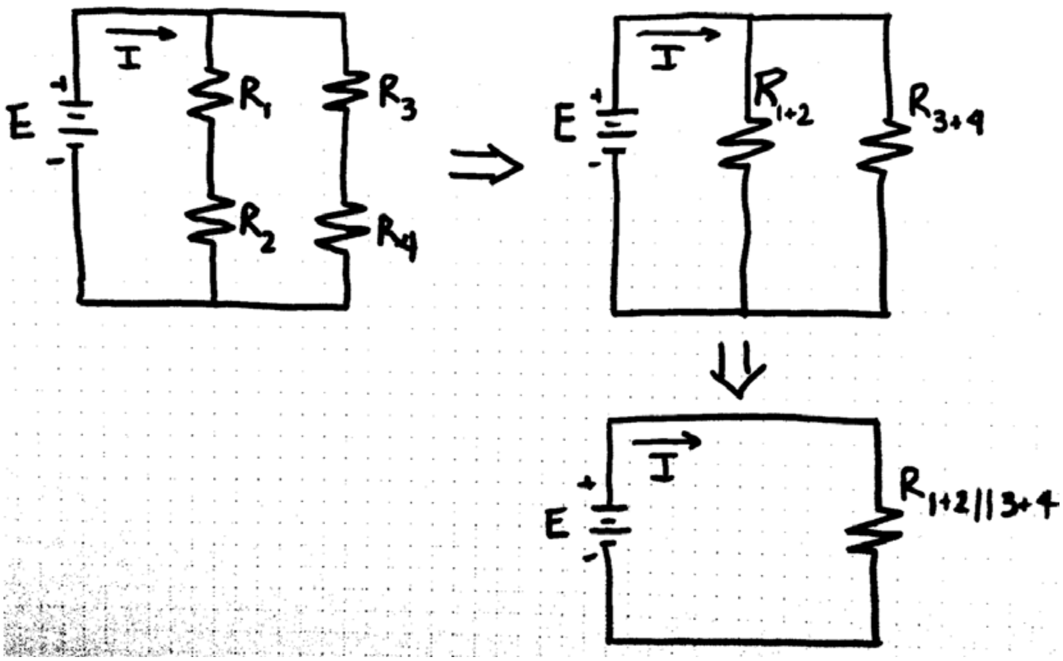


Figure 14: Schematic strategy for solving a parallel-series circuit

Then the students will need to combine the equivalent resistors R_{1+2} and R_{3+4} using the equation for parallel resistors and this can be represented as $\frac{1}{R_{1+2\|3+4}} = \frac{1}{R_{1+2}} + \frac{1}{R_{3+4}}$. Students may recall from middle school geometry that the "‖" symbol means parallel, so this is a way to help them remember what portion of the circuit they are analyzing. If the teacher or student wishes to use a simpler method of naming the equivalent resistor variables, they may, but the teacher should realize that another method will then require explicitly defining the variables where this method actually shows what series and parallel analysis has been done.

So let us look at the combined equation where we substitute the original resistor variables into the newly developed equation:

$$\frac{1}{R_{1+2\|3+4}} = \frac{1}{R_{1+2}} + \frac{1}{R_{3+4}}$$

$$\frac{1}{R_{1+2\|3+4}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$$

While this equation appears complex, it is actually easier for a student to analyze for a single missing resistor value.

The teacher may wish to pose the following scenario, “If $R_1 = R_3$ and $R_2 = R_4$, how can we simplify this equation?” Students should be able to substitute the variables and understand the following simplification:

$\frac{1}{R_{1+2\|3+4}} = \frac{1}{R_1+R_2} + \frac{1}{R_1+R_2}$	Substitute the variables
$\frac{1}{R_{1+2\|3+4}} = \frac{2}{R_1+R_2}$	Add the fractions because the denominators are common
$2R_{1+2\|3+4} = 1(R_1 + R_2)$	Use the cross product rule
$\frac{2}{2}(R_{1+2\|3+4}) = \frac{1}{2}(R_1 + R_2)$	Divide both sides by 2
$R_{1+2\|3+4} = \frac{1}{2}(R_1 + R_2)$	Simplify the equation

Teachers should understand that students may choose to use the opposite variables so the final equation could be expressed as $R_{1+2\|3+4} = \frac{1}{2}(R_3 + R_4)$, $R_{1+2\|3+4} = \frac{1}{2}(R_1 + R_4)$, or $R_{1+2\|3+4} = \frac{1}{2}(R_3 + R_2)$, but the formulas are equivalent and the final answer when evaluated will be the same.

Now students are ready to create an equation for the series-parallel circuit in Figure 13b. The teacher can have students look at *Figure 15* to help them understand how to create the equation that determines the equivalent resistance of this circuit. Students will see that the circuit is reduced by analyzing each of the parallel resistances first and then series resistances.

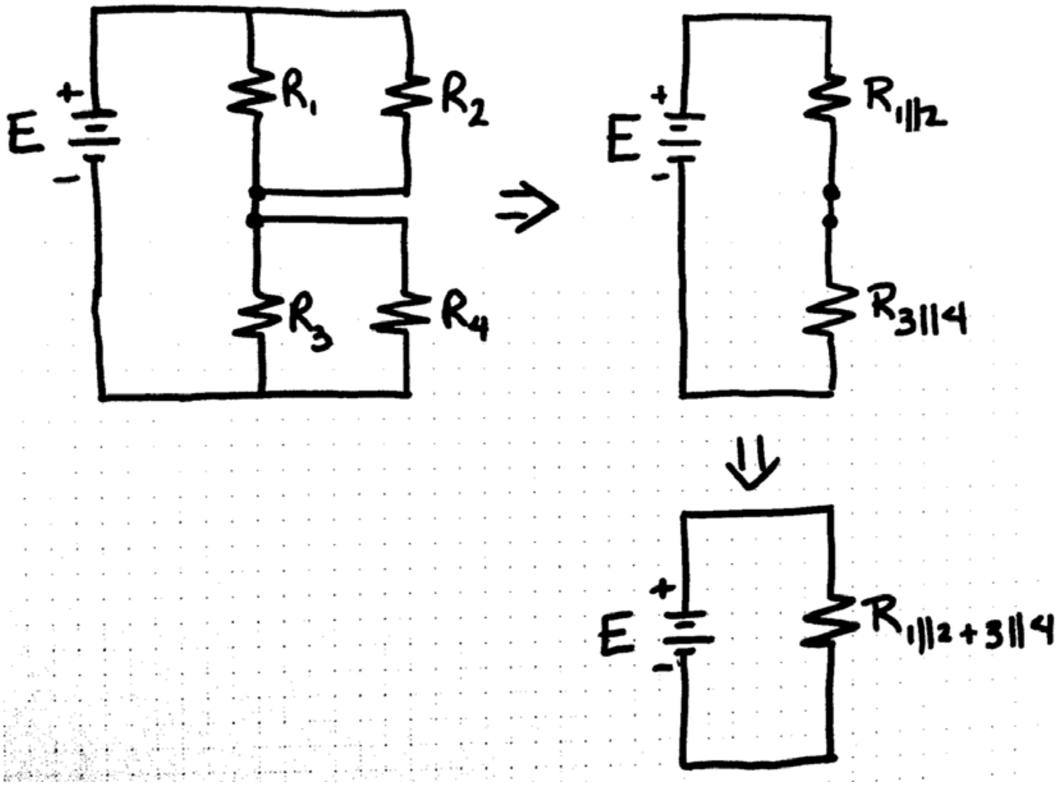


Figure 15: Schematic strategy for solving a series-parallel circuit

Depending on where the teacher took the parallel circuit analysis in lesson 4, students should recall the equation in standard form, $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$. At this point it will be helpful for students to think about how to get to the final reduction in Figure 15 and might contemplate thinking about creating the equation using a backwards strategy. The following notes sequence may be used to help guide students toward a generalized formula:

$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$	The standard form of the parallel resistance equation.
$R_S = R_1 + R_2$	The standard form of the series resistance equation.
Where do we want to get to?	$R_{1\parallel 2+3\parallel 4}$
$R_{1\parallel 2+3\parallel 4} = R_{1\parallel 2} + R_{3\parallel 4}$	We know that the final resistance equation must contain the sum of the parallel resistors.

Let's rearrange the standard equation to get R_P out of the denominator and into the numerator.

$\frac{R_P}{1}\left(\frac{1}{R_P}\right) = \frac{R_P}{1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ Multiply both sides of the equation by the reciprocal of $\frac{1}{R_P}$.

$1 = R_P\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ Simplify the equation by cancellation.

Multiply both sides of the equation by the reciprocal of $\frac{1}{R_1} + \frac{1}{R_2}$ since it is inside parenthesis.

$$1\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) = R_P\left(\frac{\frac{1}{R_1}+\frac{1}{R_2}}{1}\right)\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right)$$

$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} = R_P$ Simplify the equation by cancellation.

Now this form highlights the equivalent parallel resistance, but it looks ugly to most teachers and students because it is the inverse of the sum of two inverses. Students who are adept at using a graphing calculator to solve complex fractions will have no problem evaluating the equation for R_P when given actual resistance values.

If this lesson is used with students taking Algebra 2, the teacher may wish students to take the rearrangement of the equation further before allowing students to substitute values:

$\frac{R_1R_2}{R_1R_2}\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) = R_P$ Multiply the left side of the equation by a form of 1 that makes a common denominator for the fractions added in the denominator of the overall fraction.

$\frac{R_1R_2}{\frac{R_2+R_1}{R_1R_2}} = R_P$ Simplify the equation.

$R_1R_2\left(\frac{R_1R_2}{R_2+R_1}\right) = R_P$ Recall that dividing by a fraction is the same as multiplying by the reciprocal.

$\frac{{R_1}^2{R_2}^2}{R_2+R_1} = R_P$ Simplify the equation.

This simplified general form for parallel resistors looks less ugly, but requires students to understand much more about fractions and rearranging equations.

We can use our results of the rearrangement to create our required portions of the equation using our standard notation.

For Algebra 1: $R_{1\|2+3\|4} = R_{1\|2} + R_{3\|4}$

$$R_{1\|2+3\|4} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} + \frac{1}{\frac{1}{R_3}+\frac{1}{R_4}}$$

For Algebra 2: $R_{1\|2+3\|4} = \frac{{R_1}^2{R_2}^2}{R_2+R_1} + \frac{{R_3}^2{R_4}^2}{R_4+R_3}$

The teacher may wish to pose the same scenario as before by saying, "If $R_1 = R_3$ and $R_2 = R_4$, how can we simplify this equation?" Students should be able to substitute the variables and understand the following simplification:

For Algebra 1: $R_{1\|2+1\|2} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} + \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$ Notice the common denominator

$R_{1\|2+1\|2} = \frac{2}{\frac{1}{R_1}+\frac{1}{R_2}}$ Add the fractions

For Algebra 2: $R_{1\|2+1\|2} = \frac{{R_1}^2{R_2}^2}{R_2+R_1} + \frac{{R_1}^2{R_2}^2}{R_2+R_1}$ Notice the common denominator

$R_{1\|2+1\|2} = \frac{2{R_1}^2{R_2}^2}{R_2+R_1}$ Add the fractions

As a realistic assessment at the end of the lesson, the following question may be posed: "You are a computer repair person. You need to build the circuit as shown in portion of the schematic below (Figure 16), but a water drop has made it impossible to read the value of resistor R_1. You need the total resistance $(R_{1+2\|3})$ to be as close to 1200 ohms as possible.

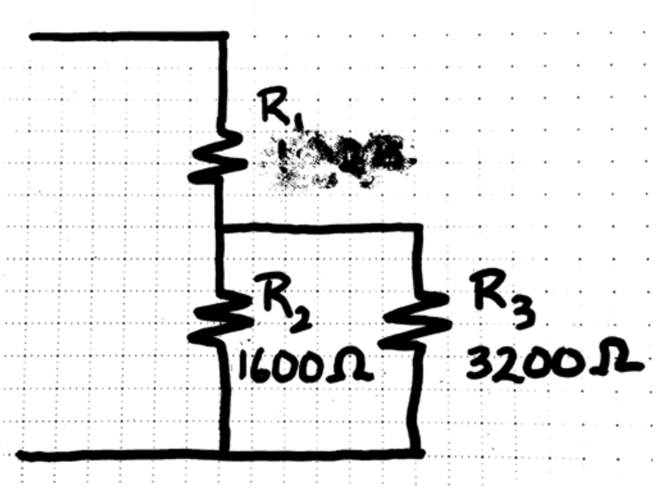


Figure 16: Schematic portion for Lesson 5 assessment

1. Write either a single equation or two equations that completely describes the total resistance.

$R_{1+2\|3} = R_1 + R_{2\|3}$

If a student gives this as their complete answer, it shows a beginning understanding of the concept of creating equations because they understand the series portion of the circuit. This answer earns partial credit.

$\frac{1}{R_{2\|3}} = \frac{1}{R_2} + \frac{1}{R_3}$

A student may write this to help themselves understand the parallel portion of the circuit. A student should be able to solve the problem posed, but they do not yet show understanding of rearranging fractional equations. If this and the previous equation are shown, the answer earns partial credit.

$R_{1+2\|3} = R_1 + 1066.\overline{6}$

If a student gives this as their complete answer, they understand the concept of creating equations and calculating with fractions because they used the parallel equation to obtain the value. This answer earns full credit.

$R_{2\parallel3} = \frac{R_2R_3}{R_3+R_2}$	Students who can rearrange the fractions may get to this form of the parallel circuit. This shows a fuller conceptual understanding of rearranging equations. This answer earns partial credit even if the equation, $R_{1+2\parallel3} = R_1 + R_{2\parallel3}$, is missing.
$R_{1+2\parallel3} = R_1 + \frac{R_2R_3}{R_3+R_2}$	If a student gives this as their complete answer, it shows a complete understanding of creating and rearranging equations. This answer earns full credit.

2. Which standard resistor should you buy for resistor R_1? Show your work.
   a. 110 ohms
   b. 120 ohms
   c. 130 ohms
   d. 140 ohms”

   Answer c provides an answer ($R_1 = 133.\overline{3}\ \Omega$) that is closest to the desired total resistance.

   A homework sheet for this lesson is provided in Appendix A on page 76.

Unit Plan Assessment

In order to properly assess student learning from this unit plan, a 3 question assessment is provided. The answers are in Appendix A on page 77. Each question addresses one or more of the lesson plans to give an integrated overview of what students know and can do when presented with electrical circuit models that have constraint requirements and equations with multiple variables that require rearrangement in order to solve the problem in a simple manner.

Algebra teachers may allow students to use a reference sheet with the resistor color code (Figure 1) and the standard form of equations for series and parallel resistances, Ohm's Law and power. This is so that the teacher can assess student's mathematical abilities rather than their scientific knowledge.

1. A resistor manufacturer is making 3 large bins of resistors with different nominal resistances. Fill in the chart below to tell the inspector what the tolerances for each resistor should be using scientific notation. Show your work!

   a.

Resistor	Band 1	Band 2	Band 3	Band 4	Nominal Resistance	Minimum Resistance	Maximum Resistance
A	Black	Red	Red	Gold			
B	Orange	Orange	Green	Silver			
C	Green	Black	Yellow	Red			

   b. The inspector finds an unmarked resistor lying on the floor. The manufacturing manager doesn't want anything thrown out if it measures within tolerance for one of the resistor values they are making that day. The resistor measures 352,000 ohms. Should it be thrown out or go into a resistor bin into after it has been properly marked? Justify your answer using an inequality statement.

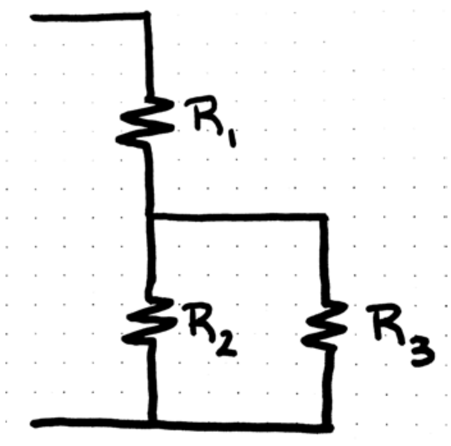
2. A fisherman hooked up a resistive heater to an automotive battery that is 12 volts to warm up his cabin. The heater requires 13.2 watts of power to run for one hour.

a. Rearrange the power equation to solve for current (I).

b. How much current, in amps, does the heater draw?

c. What is the heater's resistance in ohms? Rearrange Ohm's law as needed before solving and round to the nearest tenth of an ohm.

3. An electrician made a schematic drawing of a portion of a circuit with both parallel and series resistances. She knows that the equivalent resistance ($R_{1+2\|3}$) is 9850 Ω.



The electrician wrote down this equation to describe the circuit:

$$R_{1+2\|3} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

a. Rearrange the equation to highlight the variable R_1.

b. Given that $R_2 = 20{,}000\ \Omega$ and $R_3 = 12000\ \Omega$, find the value of R_1.

c. Rearrange the equation below to highlight the variable R_2

$$\frac{1}{R_{2\|3}} = \frac{1}{R_2} + \frac{1}{R_3}$$

d. Given that $R_{2\|3} = 7500\ \Omega$ and $R_3 = 30000\ \Omega$, find the value of R_2.

Chapter 4: Validation of the Unit Plan

As a validation strategy, a portion of this unit plan, Lesson 3, was presented to a combined class of 7th and 8th grade students in a small private school during May 2016 to find out whether electrical circuit applications would be a real-world problem that is engaging to these students. The reason this lesson was chosen was because Ohm's Law is mentioned specifically as an example in the standard A.CED.4, but the author was unable to find any current Algebra I textbook that used Ohm's Law as a motivation for students to learn rearranging equations.

The fifteen students included in this validation study are studying either pre-algebra or algebra. Eleven of the students completed both a pre-lesson and post-lesson survey to determine if the electrical circuit lesson had a significant effect on either their attitude toward mathematics and science in general or their motivation to learn more complex algebra.

Of the eleven students surveyed, four students are seventh grade females studying pre-algebra and seven students, three females and four males, are studying algebra. All the students are Caucasian and studying the New York Regents Living Environment course for science.

The four pre-algebra students began the year in the algebra class but were transferred to a pre-algebra class, taught by the author, in November 2015. Pre-Algebra (Larson, Boswell, Kanold, Stiff, & McDougal, 2012), a standard Common Core textbook with many different types of remediation activities, was chosen because these students had significant gaps in their foundational mathematical knowledge that impeded their ability to understand how to solve equations and do complex mathematical tasks (Merriënboer, Kester, & Paas, 2006) at that time. Many of the gaps in their mathematical knowledge have been remediated, but these students still have difficulties comprehending complex mathematical tasks, especially when dealing with equations and multiple variables.

The seven algebra students are being taught using Algebra I (Burger et al., 2012), a standard Common Core textbook, to prepare to take the New York State Common Core Algebra I Regents Examination in June 2016. Two of these students are in eighth grade and have already taken and passed the Algebra I Regents examination in June 2015, but they did not score at or above a mastery level of 85% so they are being encouraged to retake the exam. The other five students are in seventh grade and taking Algebra I for the first time and approximately half of them are expected to attempt the Algebra 1 Regents examination in June 2016. All of the algebra students understand how to solve equations, but their teacher had previously informed me that several of the students resist rearranging variables even when working with equations that contain only two variables.

On the day prior to presenting the lesson on Ohm’s Law and Power, each class had experienced a textbook lesson on displaying data for statistical analysis. This was a fortunate circumstance as the two classes are normally taught in separate locations and without coordination of lessons between the two teachers. When the lesson from the unit plan was presented, all the students were in one room where they worked in mixed teams at tables where the experiments were physically setup using Squishy Circuit conductive dough. Students studying pre-algebra sat with students studying algebra and the discussion was lively since using algebra to solve electrical circuit problems was an unfamiliar topic for most students.

After the mathematics lesson for each day, the students that had a signed parental consent form, a signed student assent form, and were present for the lesson were asked to complete a survey entitled “Middle School Math & Science Interest Survey” that had been assembled by this author. The inspiration for the survey statements came from two sources: (1) the LISD Student Interest Survey – Math Motivation (6-8) (“LISD Student Interest Survey - Math Motivation (6-8),” n.d.) and (2) the Factors Influencing College Success in Mathematics (FICS-Math) project (Wade, 2011). Neither instrument was used in its' entirety to keep the survey to a reasonable length and 10 statements were created by the author to reflect the nature of normal mathematical practices within the middle school classroom. The blank survey instrument can be found in Appendix B on page 83.

Each time the survey was given, students were instructed to choose the category that most closely matched their attitude toward the statements about mathematics and science on that day. "Strongly Disagree" was coded as 1 point, "Disagree" was rated as 2 points, "Agree" was rated as 3 points, and "Strongly Agree" was rated as 4 points. Unfortunately, a couple of the students marked their attitude toward a few statements with a checkmark on the line between Agree and Disagree, despite instructions to only make marks in the boxes so those answers were excluded and deemed as missing when analyzing the data through a two-tailed paired t-test using IBM SPSS Statistics for Windows (Version 23.0). The data was analyzed using the following five filters: (1) all students (Table 7); (2) pre-algebra students only (Table 8); (3) algebra students only (Table 9); (4) males only (Table 10); and (5) females only (Table 11). The data was not analyzed by grade level because the population for eighth grade was so small that there is a risk that a particular student could be identified and because it is a normal practice at this school not to differentiate between the two grades which are taught in a single room.

Because the total population was small, few of the statements held any significant change between the two lessons with a paired-samples t-test significance test level of $p < 0.05$ at the 95% confidence level, an increased significance test level of $p < 0.10$ at the 95% confidence level was used for all analysis.

When the statement “I don’t like doing math when there is more than one variable” was analyzed for all students (Table 7), the results show that all student beliefs about the difficulty of doing mathematics when there is more than one variable changed from pre-lesson to post-lesson by an average of .37 points. When the algebra student responses were analyzed (Table 9), the results show that student beliefs about the difficulty of doing mathematics when there is more than one variable differs from pre-lesson to post-lesson by an average of .43 points. This indicates that the average student beliefs changed from agreeing to disagreeing with the statement as written. This leads the author to conclude that most of these students either like or don’t mind working with more than one variable which was the posited outcome of this lesson. It is interesting to note that all of the algebra students in this sample disagreed with the statement after the lesson which is evidence that this lesson effected a positive change in their mindset about equations with multiple variables.

An unexpected change seen in all students (Table 7) and female students (Table 11) was the response to the statement “I like doing math when money is involved.” Money was not at the heart of this lesson, but only mentioned as a short aside when the class discussed the cost of power at the end of the period and why people might want to use lower power LED and fluorescent light bulbs as replacements for standard resistive bulbs. When analyzing the responses of all students (Table 7), the results show that student beliefs about this statement differs about .5 points from pre-lesson to post-lesson. When analyzing the responses of female students (Table 11), the results show that student beliefs about this statement differs about .43 points from pre-lesson to post-lesson. This indicates to the author that the students surveyed liked when they could connect the mathematical concept we discussed to money.

When the statement “Learning algebra is important for success in high school” was analyzed for all students (Table 7), the results show that all student beliefs about the importance of algebra differs from pre-lesson to post-lesson by an average of .45 points. While most students still agree with the statement, more of the students surveyed were unsure that learning algebra was important for high school success.

When the statement “I like learning about math” was analyzed for all students (Table 7), the results show that the group of all students differs from pre-lesson to post-lesson by an average of .27 points. While most students still agree with the statement, more of the students surveyed were unsure that they liked learning about math post-lesson. This could be because of their unfamiliarity with context of the lesson when the pre-lesson of statistical data displays was a very familiar topic.

When the statement “Math is important to the world around us” was analyzed for all students (Table 7), the results show that the group of all students differs from pre-lesson to post-lesson by an average of .28 points and the group of algebra students (Table 9) differs from pre-lesson to post-lesson by an average of .43 points. While most students still agree strongly with the statement, more of the students surveyed simply agreed with the statement about math post-lesson. While this statement was not expected to show a change, it does show how strongly some students feel about the importance of math in a real-world context.

Table 7

All students

Statement	Pre-lesson M	Pre-lesson SD	Post-lesson M	Post-lesson SD	n	95% Confidence Interval of the Difference Lower	95% Confidence Interval of the Difference Upper	R	t	df	Sig. (2-tail)
I like learning about math.	3.00	.447	2.73	.647	11	-.041	.587	.692	1.936	10	.082
Math is important to the world around us.	3.73	.467	3.45	.688	11	-.041	.587	.736	1.936	10	.082
I like doing math when money is involved.	2.30	.823	2.80	.789	11	-1.006	.006	.616	-2.236	9	.052
I don't like doing math when there is more than one variable.	2.73	.647	2.36	.674	11	.025	.703	.709	2.390	10	.038
Learning algebra is important for success in high school.	3.36	.505	2.91	.701	11	-.007	.916	.386	2.193	10	.053

The most interesting change analyzed is the response of pre-algebra students (Table 8) and all female students (Table 11) to the statement "I would get respect as a science major in college." When analyzing the responses of all female students (Table 11), the results show that student beliefs about their ability to be respected as a science major differs from pre-lesson to post-lesson by an average of .57 points. When analyzing the responses of the pre-algebra students (Table 8), who happen to all be female, the results show that student beliefs about their ability to be respected as a science major differs from pre-lesson to post-lesson by an average of .75 points. This suggests that the female students sampled, regardless of which math they are studying, doubt their ability to be respected as a science major in college although they have good role models. Their female pre-algebra teacher was previously an electrical engineer and the female algebra teacher also teaches science to all the seventh and eighth grade students. Therefore this author believes that more exposure to concepts and careers presented in this unit plan is needed for female students to consider a STEM career because the "mathematics preparation that students bring with them into engineering programs has been found to be central to their success" (Cass, Hazari, Cribbs, Sadler, & Sonnert, 2011).

Table 8

Pre-algebra students

	Pre-lesson		Post-lesson			95% Confidence Interval of the Difference					Sig.
Statement	M	SD	M	SD	n	Lower	Upper	R	t	df	(2-tail)
I would get respect as a science major in college.	2.25	.957	1.50	.577	4	-.046	1.546	.905	3.000	3	.058

Table 9

Algebra students

	Pre-lesson		Post-lesson			95% Confidence Interval of the Difference					Sig.
Statement	M	SD	M	SD	n	Lower	Upper	R	t	df	(2-tail)
Math is important to the world around us.	3.86	.378	3.43	.787	7	-.066	.923	.801	2.121	6	.078
I don't like doing math when there is more than one variable.	2.43	.535	2.00	.000	7	-.066	.923		2.121	6	.078

When the statement "I am confident when I answer questions in my math class" was analyzed for male students (Table 10), the results show that the male students sampled about their confidence level differs from pre-lesson to post-lesson by an average of .75 points. This result suggests that they were less confident in answering questions during this lesson than during the pre-lesson. This could be because of their unfamiliarity with context of the lesson when the pre-lesson of statistical data displays was a very familiar topic.

Table 10

Male students

	Post-lesson		Pre-lesson			95% Confidence Interval of the Difference					
Statement	M	SD	M	SD	n	Lower	Upper	R	t	df	Sig. (2-tail)
I am confident when I answer questions in my math class.	3.50	.577	2.75	.500	4	-.046	1.546	.577	3.000	3	.058

Table 11

Female students

	Pre-lesson		Post-lesson			95% Confidence Interval of the Difference					
Statement	M	SD	M	SD	n	Lower	Upper	R	t	df	Sig. (2-tail)
I like doing math when money is involved.	2.14	.900	2.57	.787	7	-.923	.066	.807	-2.121	6	.078
I would get respect as a science major in college.	2.71	.951	2.14	.900	7	.077	1.066	.835	2.828	6	.030

Students surveyed also created statements students about what they liked about the lesson. Four of the students specifically liked the experimental aspect of the activity. A female algebra student liked that “we used equations to see what would work and switching the equations around.” A male algebra student liked that “it involved circuits and real world problems.” Another male algebra student liked “being able to fly through questions that used to be hard.” This shows that students value real world problems that challenge their thoughts about math and science.

Students surveyed also created statements students about what they didn’t like about the lesson. Only one student mentioned not liking the experiment. Two students remarked that this lesson only applied to certain professions, which is true of many textbook algebra lessons in rearranging equations. A female pre-algebra student didn't like that "it was hard at first." A male algebra student "was confused when we were combining formulas." Another male algebra student didn't like "getting stuck on a problem and having to have help with a problem." The negative comment that the author found most helpful was that a female algebra student wanted "graphs or pictures to explain it better." The author had not previously considered graphing as an option for any of these lessons, but it could help students who are more visual learners understand the how the rearranged equations may be evaluated for different values.

Special work needs to be done with the Squishy Circuit conductive dough recipe and measurements to make this connecting element have a less variable resistance. The experimental circuits were setup as shown in Figure 7 (p. 30) and students noticed that the resistance measurement of the conductive dough kept changing while the circuit was powered. Making the resistance measurements with the circuit unpowered showed a similar problem, but to a smaller degree. This is probably due to the temperature of the dough changing as current flowed because a heating of resistive elements is a real consequence of Ohm's Law. The students solved the problem by turning the circuit on and then taking an average reading over a period of 5 seconds. This is a practical solution that the students chose which enhanced the lesson's real-world nature of mathematical modeling and problem solving.

Chapter 5: Conclusion

This unit plan has been created to guide teachers as they have students represent constraints using resistors, create equations from electrical schematics and rearrange equations to highlight a variable of interest to solve real-world problems. It is created as a model instructional resource for both CTE and core academic teachers so that the CCSS and NGSS are embedded in the lesson (Meeder & Suddreth, 2012, p. 16). Cross-curriculum experiences inside the mathematics classroom can prepare students to be successful in their high school science courses and be key in developing a student's mathematical identity which may lead to an interest in STEM related careers. By creating an intentional classroom connection between algebra, science and careers, students will realize that all of their courses are preparing them for careers as well as college, one of the stated goals of the Common Core. This unit plan particularly encourages students to investigate the electrical trades and engineering.

The author pilot tested the third lesson successfully with a small group of students and is aware that more testing that needs to be done with this unit plan to refine it for use with larger groups of students. There were some difficulties noted with using the Squishy Circuit conductive dough as a connecting element so further work must be done to create a recipe that has a lower and more consistent resistance. The author will also create a full set of circuit emulator files at a future date for all of the lessons so that teachers who cannot obtain physical electronic equipment can still use this unit plan to enable students to complete the investigations without any additional expense.

The survey form used to evaluate the student response lesson plan also requires some reformatting for use with a larger audience. Changing the contiguous boxes to individual bubbles may avoid future problems for students in marking this survey.

While the students surveyed experienced the expected positive change in attitude towards equations with more than one variable after the lesson, there were other consequences of the pilot lesson that make the creation and implementation of this entire unit plan critical. Female students surveyed particularly had a negative response to the statement about getting respect as a science major in college which is concerning to the author as a woman who has pursued a STEM career for over 30 years. Both girls and boys need to be presented with opportunities in middle and high school to experience science, technology, engineering and mathematics in a positive, playful, and connected manner or they may not see a STEM pathway to high school graduation as a viable path for themselves. Granted, not all students want to pursue STEM careers, but it is important for students to understand how science and mathematics are integrated into almost every career they may choose to pursue. It is this author's conclusion that more unit plans that show how algebra is applied in many different professions need to be created and implemented in the classroom.

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Appendix A

Lesson 1 Resistor Alternative

The simulated resistor in Figure 17 can be used as an alternative to working with real resistors by a teacher who is unable to obtain them. This simulated resistor may be especially helpful for students with color vision impairments because the teacher can write the name of the color on each stripe rather than relying on the student's interpretation of the color.

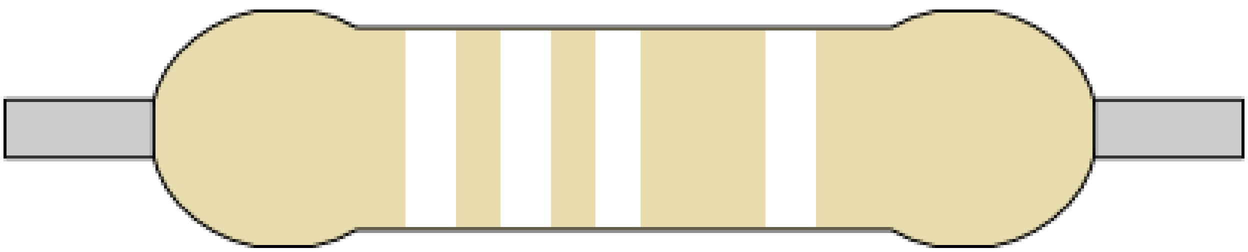


Figure 17: Simulated Resistor Card

Lesson 1 Homework with Answers

Table 12

Solution to Table 2 Resistor Homework

Resistor	1st Band	2nd Band	3rd Band	4th Band	Nominal Value (ohms)	Maximum Value (ohms)	Minimum Value (ohms)
1	Yellow	Violet	Brown	Silver	$4.7x10^2$	$5.17x10^2$	$4.23x10^2$
2	Red	Red	Green	Gold	$2.2x10^6$	$2.31x10^6$	$2.09x10^6$
3	Black	Brown	Brown	Silver	$1x10^1$	$1.1x10^1$	$9x10^0$
4	Orange	Orange	Orange	Gold	$3.3x10^4$	$3.465x10^4$	$3.135x10^4$
5	Red	White	Blue	Silver	$2.9x10^7$	$3.19x10^7$	$2.61x10^7$
6	Red	Red	Black	Gold	$2.2x10^1$	$2.31x10^1$	$2.09x10^1$
7	Green	Red	Yellow	Red	$5.2x10^5$	$5.304x10^5$	$5.096x10^5$

For each resistor in the table above, write an inequality statement that shows that resistor's tolerance in scientific notation:

1. $4.23 \times 10^2\Omega \leq R \leq 5.17 \times 10^2\Omega$

2. $2.09 \times 10^6\Omega \leq R \leq 2.31 \times 10^6\Omega$

3. $9 \times 10^0\Omega \leq R \leq 1.1 \times 10^1\Omega$

4. $3.135 \times 10^4\Omega \leq R \leq 3.465 \times 10^4\Omega$

5. $2.61 \times 10^7\Omega \leq R \leq 3.19 \times 10^7\Omega$

6. $2.09 \times 10^1\Omega \leq R \leq 2.31 \times 10^1\Omega$

7. $5.096 \times 10^5\Omega \leq R \leq 5.304 \times 10^5\Omega$

Lesson 2 Homework with Answers

1. An electrical engineer needs a circuit inside of toy with 2AA batteries to have a series resistance of 25000 Ω. She already has two resistors, R_1 and R_2, that she will use to make this circuit. She must buy another resistor, R_3, to complete the circuit. The electrical engineer wrote down her initial equation: $25000\ \Omega = R_1 + R_2 + R_3$.
   a. Rearrange this equation to highlight the variable R_3.

      $R_3 = 25000 - R_1 - R_2$

   b. If $R_1 = 470\ \Omega$ and $R_2 = 15000\ \Omega$, what is the value of R_3?

      $R_3 = 25000 - 470 - 15000 = 9530\ \Omega$

   c. What is the 4 color code for a standard resistor nearest to this value with a 2% tolerance?

      White, Green, Red, Red

2. An electrician knows that the sum of the voltages across each of three resistors in a series circuit equals the voltage of the battery. Let V_B be the voltage of the battery and V_1, V_2, and V_3 be the voltages across the three different resistors.
   a. Create an equation from the statement above.

      $V_1 + V_2 + V_3 = V_B$

   b. Rearrange this equation to highlight V_2.

      $V_2 = V_B - V_1 - V_3$

   c. If the battery voltage is 12 volts, V_1 is 5 volts, and V_3 is 1.5 volts, what is the measurement of V_2?

      $V_2 = 12 - 5 - 1.5 = 5.5\ volts$

3. A computer repair person found a burned out resistor on a bad circuit board in the middle of 3 resistors in series. He measured the total resistance of the same 3 resistors on a working board

and found that the total resistance was 4790 Ω. Then he drew the following schematic portion and made notes of the color code of the resistors on the working board.

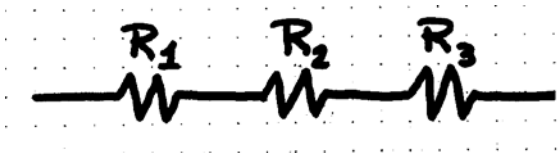


Figure 18: Schematic portion of series circuit for Lesson 2 homework

Color Code: R_1 is Orange, Orange, Red, Gold R_3 is White, Black, Black, Gold

a. What are the nominal resistances of R_1 and R_3?

$R_1 = 3300\ \Omega, R_3 = 90\ \Omega$

b. Write an equation and solve for the nominal resistance of R_2 on the working board.

$R_2 = R_S - R_1 - R_3$

$R_2 = 4790 - 3300 - 90$

$R_2 = 400\ \Omega$

c. If R_1 measures 3200 Ω and R_3 measures 90 Ω on the bad board, what nominal resistance should R_2 be?

$R_2 = 500\ \Omega$

Lesson 3 Homework with Answers

Rearrange and combine the equations, if necessary, to more easily answer the following questions.

Ohm's Law: $\boldsymbol{E = IR}$ **Power:** $\boldsymbol{P = IE}$

E is for Voltage. I is for Current. R is for Resistance. P is for Power.

1. How much current (I) flows through a lamp that has a resistance (R) of 240 ohms and is connected to a 120 volt (E) source?

Rearrange Ohm's Law to solve for current (I).

$$I = \frac{E}{R} = \frac{120\ volts}{240\ ohms} = 0.5\ amps$$

2. An LED requires a current of 0.025 amps to turn on when connected to a 3 volt battery. What is the resistance of the LED?

Rearrange Ohm's Law to solve for resistance (R).

$$R = \frac{E}{I} = \frac{3\ volts}{0.025\ amps} = 120\ ohms$$

3. A hair dryer is connected to a voltage (E) of 120 volts and has a current (I) of 12 amps. How many watts of power are used?

No rearrangement of the Power equation is required.

$$P = IE = (12\ amps)(120\ volts) = 1440\ watts$$

4. A circuit uses 216.2 watts and draws a current of 1.88 amps. What is the resistance? (Round your answer to the nearest hundredth ohm.)

Power is $P = IE$ but we don't know voltage (E) so we substitute Ohm's Law $(E = IR)$ into the power equation to get resistance into the equation.

$$P = I(IR) = I^2R$$

$$216.2\ watts = (1.88\ amps)^2R$$

$$\frac{216.2\ watts}{3.5344\ amps^2} = \frac{3.5344\ amps^2 R}{3.5344\ amps^2}$$

$$\frac{216.2\ watts}{3.5344\ amps^2} = \frac{3.5344\ amps^2 R}{3.5344\ amps^2}$$

$$76350.11\ ohms \approx R$$

Lesson 4 Homework with Answers

1. An electrical engineer needs a circuit inside of toy with 2AA batteries to have a parallel resistance of 8000 **Ω**. He already has resistor, R_1, and needs to buy R_2 to complete the circuit. The electrical engineer wrote down his initial equation: $\frac{1}{8000\ \Omega} = \frac{1}{R_1} + \frac{1}{R_2}$.

   a. Rearrange this equation to highlight the variable R_2. Show all your work.

$$\frac{1}{8000} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{8000} - \frac{1}{R_1} = \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_1}$$

$$\frac{1}{8000} - \frac{1}{R_1} = \frac{1}{R_2}$$

$$R_2\left(\frac{1}{8000} - \frac{1}{R_1}\right) = R_2\left(\frac{1}{R_2}\right)$$

$$R_2\left(\frac{1}{8000} - \frac{1}{R_1}\right)\left(\frac{1}{\left(\frac{1}{8000} - \frac{1}{R_1}\right)}\right) = 1\left(\frac{1}{\left(\frac{1}{8000} - \frac{1}{R_1}\right)}\right)$$

$$R_2 = \frac{1}{\left(\frac{1}{8000} - \frac{1}{R_1}\right)}\ \Omega$$

   Algebra 2 students could rearrange this further to the following equation:

$$R_2 = \frac{8000R_1}{R_1 - 8000}\ \Omega$$

   b. If $R_1 = 16000\ \Omega$, what is the value of R_2?

$$R_2 = \frac{1}{\left(\frac{1}{8000} - \frac{1}{16000}\right)} = 16000\ \Omega$$

   c. What is the 4 color code for a standard resistor nearest to this value with a 2% tolerance?

   Black, Blue, Orange, Red

Lesson 5 Homework with Answers

You are part of a team building a robot for the FIRST Robotics competition. Everyone works together during the build and you've been assigned to build a part of the portion of the electrical circuit as shown in schematic below (*Figure 19*). You need the total resistance, $R_{(1+2)\parallel 3}$, to be as close to 1500 ohms as possible.

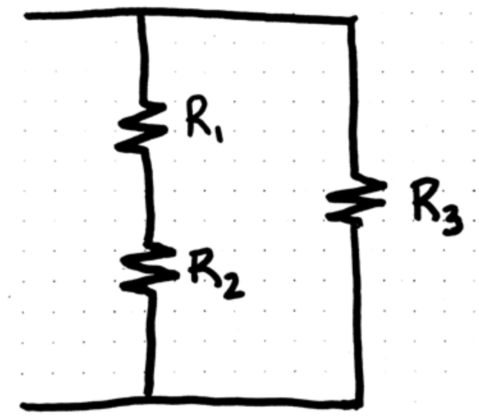


Figure 19: Parallel-series schematic portion

a. Write an equation that describes the series resistance, R_{1+2}.

$R_{1+2} = R_1 + R_2$ By definition

b. Use the series equation from part a to write an equation that describes the parallel resistance, $R_{(1+2)\parallel 3}$, in standard form.

$\frac{1}{R_{(1+2)\parallel 3}} = \frac{1}{R_1+R_2} + \frac{1}{R_3}$ By definition

$\frac{1}{1200} = \frac{1}{R_1+R_2} + \frac{1}{R_3}$ Acceptable substitution with given information

c. Rearrange the equation to highlight resistor R_3.

$$\frac{1}{R_{(1+2)\parallel 3}} - \frac{1}{R_1 + R_2} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} - \frac{1}{R_1 + R_2}$$

$$\frac{1}{R_{(1+2)\parallel 3}} - \frac{1}{R_1 + R_2} = \frac{1}{R_3}$$

$$R_{(1+2)\|3}(R_1+R_2)\left(\frac{1}{R_{(1+2)\|3}}-\frac{1}{R_1+R_2}\right)=R_{(1+2)\|3}(R_1+R_2)\left(\frac{1}{R_3}\right)$$

$$R_3\left((R_1+R_2)-R_{(1+2)\|3}\right)=R_3\left(\frac{R_{(1+2)\|3}(R_1+R_2)}{R_3}\right)$$

$$R_3\left((R_1+R_2)-R_{(1+2)\|3}\right)=R_{(1+2)\|3}(R_1+R_2)$$

$$\frac{R_3\left((R_1+R_2)-R_{(1+2)\|3}\right)}{(R_1+R_2)-R_{(1+2)\|3}}=\frac{R_{(1+2)\|3}(R_1+R_2)}{(R_1+R_2)-R_{(1+2)\|3}}$$

$$R_3=\frac{R_{(1+2)\|3}(R_1+R_2)}{(R_1+R_2)-R_{(1+2)\|3}}$$

If the student substituted the given value $R_{(1+2)\|3}$ into part b before rearranging the equation, their answer may appear as:

$$R_3=\frac{1500(R_1+R_2)}{(R_1+R_2)-1500}$$

d. If $R_1 = 3300\Omega$ and $R_2 = 4700\Omega$, solve for resistance R_3 to the nearest hundred ohms.

$$R_3=\frac{1500(3300+4700)}{(3300+4700)-1500}=\frac{1500(8000)}{8000-1500}$$

$$R_3=\frac{1500(8000)}{6500}=\frac{12{,}000{,}000}{6500}=1846.153846$$

$$R_3\approx 1800\ \Omega$$

Even if the student was not able to rearrange the equation in part c, they can still solve for the resistance R_3 by substituting values into part b.

$$\frac{1}{1500}=\frac{1}{3300+4700}+\frac{1}{R_3}$$

$$\frac{1}{1500}=\frac{1}{8000}+\frac{1}{R_3}$$

$$\frac{1}{1500}-\frac{1}{8000}=\frac{1}{R_3}$$

$$\frac{1}{1500} - \frac{1}{8000} = \frac{1}{R_3}$$

$$\frac{8}{12000} - \frac{1.5}{12000} = \frac{1}{R_3}$$

$$\frac{6.5}{12000} = \frac{1}{R_3}$$

$$6.5R_3 = 12000$$

$$\frac{6.5R_3}{6.5} = \frac{12000}{6.5}$$

$$R_3 = 1846.153846$$

$$R_3 \approx 1800\ \Omega$$

e. What is the most appropriate color code for your final choice for resistor R_3. Justify your choice by writing an inequality statement showing that the actual resistor value is contained within the tolerance.

Brown, Grey, Red, Gold

The gold stripe has a $\pm 5\%$ tolerance, so the entire tolerance is $1710\ \ \Omega \leq R_3 \leq 1890\ \Omega$.

Since $1846.153846\ \Omega \leq 1890\ \Omega$, therefore +/- 5% tolerance is appropriate.

A $\pm 10\%$ tolerance would also be acceptable as long as the 4th band is Silver and the justification is correct.

This question reinforces the information from Lesson 1 which will help students review for the unit assessment.

Unit Plan Assessment with Answers

1. A resistor manufacturer is making 3 large bins of resistors with different nominal resistances. Fill in the chart below to tell the inspector what the tolerances for each resistor should be using scientific notation. Show your work!

   a.

Resistor	Band 1	Band 2	Band 3	Band 4	Nominal Resistance	Minimum Resistance	Maximum Resistance
A	Brown	Red	Red	Gold	1.2×10^3	1.14×10^3	1.26×10^3
B	Orange	Orange	Green	Silver	3.3×10^6	2.97×10^6	3.63×10^6
C	Green	Black	Yellow	Red	5.0×10^5	4.9×10^5	5.1×10^5

   b. The inspector finds an unmarked resistor lying on the floor. The manufacturing manager doesn't want anything thrown out if it measures within tolerance for one of the resistor values they are making that day. The resistor R measures 3,520,000 ohms. You need to decide if it should it be thrown out or go into a resistor bin into after it has been properly marked? Justify your answer using an inequality statement.

   Since the resistor value of 3,520,000Ω is nearest to the nominal resistance of Resistor B (3,330,000 Ω), the resistor fits in the tolerance band of $2.97 \times 10^6\ \Omega \leq R \leq 3.63 \times 10^6\ \Omega$ and it should be put in bin for Resistor B.

2. A fisherman hooked up a resistive heater to an automotive battery that is 12 volts to warm up his cabin. The heater requires 13.2 watts of power to run for one hour.

   a. Rearrange the power equation to solve for current (I).

   $\frac{P}{E} = I$ To receive full credit, students must show a one-step equation rearrangement starting from the standard form of $P = IE$.

   b. How much current, in amps, does the heater draw?

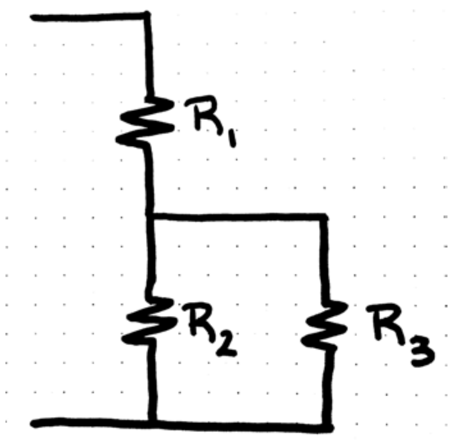
$$I = \frac{13.2}{12} = 1.1\ amps$$

c. What is the heater's resistance (R) in ohms? Rearrange Ohm's law as needed before solving and round to the nearest tenth of an ohm.

$\frac{E}{I} = R$ To receive full credit, students must show a one-step equation rearrangement starting from the standard form of $E = IR$.

$R = \frac{12}{1.1} \approx 10.9\ \Omega$

3. An electrician made a schematic drawing of a portion of a circuit with both parallel and series resistances. She knows that the equivalent resistance ($R_{1+2\|3}$) is 9850 Ω.



The electrician wrote down this equation to describe the circuit:

$$R_{1+2\|3} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

a. Rearrange the equation to highlight the variable R_1.

$R_1 = R_{1+2\|3} - \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}}$ To receive full credit, students must show a one-step equation rearrangement.

b. Given that $R_2 = 20{,}000\ \Omega$ and $R_3 = 12000\ \Omega$, find the value of R_1.

$$R_1 = 9850 - \frac{1}{\frac{1}{20000} + \frac{1}{12000}} = 9850 - 7500 = 2350\ \Omega$$

c. Rearrange the equation below to highlight the variable R_2

$$\frac{1}{R_{2\|3}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{2\|3}} - \frac{1}{R_3} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3}$$

$$R_2\left(\frac{1}{R_{2\|3}} - \frac{1}{R_3}\right) = R_2\left(\frac{1}{R_2}\right)$$

$$R_2\left(\frac{1}{R_{2\|3}} - \frac{1}{R_3}\right)\left(\frac{1}{\frac{1}{R_{2\|3}} - \frac{1}{R_3}}\right) = 1\left(\frac{1}{\frac{1}{R_{2\|3}} - \frac{1}{R_3}}\right)$$

$$R_2 = \frac{1}{\frac{1}{R_{2\|3}} - \frac{1}{R_3}}$$

If a student takes this rearrangement further, another acceptable final answer is:

$$R_2 = \frac{R_3 R_{2\|3}}{R_3 - R_{2\|3}}$$

d. Given that $R_{2\|3} = 7500\ \Omega$ and $R_3 = 30000\ \Omega$, find the value of R_2.

$$R_2 = \frac{1}{\frac{1}{7500} - \frac{1}{30000}} = 10000\ \Omega$$

Appendix B

Middle School Math & Science Survey Instrument

What grade are you in?	7^{th}	8th
What is your gender?	Male	Female
What math class are you currently taking?	Prealgebra	Algebra
What science class are you currently taking?	Earth Science	Living Environment

		Strongly Agree	Agree	Disagree	Strongly Disagree
1	I like learning about math.				
2	Math is important to the world around us.				
3	Doing math is a lot of hard work.				
4	Learning new things in math is fun.				
5	I am good at math.				
6	Math is important to get a good job.				
7	I like the challenge of a difficult math problem.				
8	Math is difficult for me.				
9	I don't like to try new things in math.				
10	Math is important to me.				
11	I like doing math when graphs are involved.				
12	I like doing math that includes geometry.				
13	I like doing math when money is involved.				
14	I like doing math that uses real-world measurements.				
15	I like explaining how I do math.				
16	I like solving equations.				
17	I like doing math when an experiment is involved.				
18	I like drawing pictures to explain a math problem.				
19	I like doing math when I can use a calculator or computer.				
20	I like playing math games.				
21	I don't mind doing math when I can do an experiment.				
22	I don't like doing math when there is more than one variable.				
23	I am confident I can be successful in science in high school.				
24	I am confident I can be successful in math in high school.				
25	I am confident when I answer questions in my science class.				

		Strongly Agree	Agree	Disagree	Strongly Disagree
26	I am confident when I answer questions in my math class.				
27	I can learn the skills necessary to be in a scientific career.				
28	My math classes are preparing me for a science classes in high school.				
29	Learning algebra is important for success in high school.				
30	I solve real life problems in my math class.				
31	My current science class is preparing me for science in high school.				
32	I get satisfaction from doing well in my science class.				
33	I like learning about science.				
34	I would get respect as a science major in college.				
35	I enjoy learning in my math class.				
36	It is important that I understand mathematical concepts.				
37	I can conduct experiments.				
38	I get satisfaction when I do well in my math class.				
39	I want to take more science classes in high school.				
40	I like solving math problems that involve real life situations.				
41	My math classes are preparing me for college.				
42	My math classes are preparing me for a career.				

43. What I liked about today's math class was ______________________________

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44. What I didn't like about today's math class was __________________________

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