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A Curriculum Project on the Design and Implementation of Cognitive Load Theory in Mathematics

Kevin D. Gaydorus

The College at Brockport, kgaydorus@gmail.com

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A Curriculum Project on the Design and Implementation of Cognitive Load Theory in

Mathematics

Kevin Gaydorus

The College at Brockport

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Chapter 1: Introduction

Overview

The recent paradigm shift in national standards has decreased the amount of carry over that teachers have connecting the new standards to that of their past lessons. Teachers would benefit from curriculum plans revolving around the new Common Core State Standards (CCSS). This thesis describes the advantages of designing curricula that considers the Cognitive Load Theory (CLT). Merrienboer and Sweller (2005) state that CLT “uses interactions between information structures and knowledge of human cognition to determine instructional design” (p.147). Designing a curriculum with cognitive load theory in mind can support teachers as they work with students with disabilities. This curriculum presents a unit, connected consistently from lesson to lesson, to the cognitive load theory. This may support teachers as they teach heterogeneous classes to keep in mind the psychological aspects of learning. The amount of varied curriculum available for teachers in middle and secondary mathematics are limited. Providing a curriculum revolving around CLT may support teachers searching for lessons designed to reach students of all learning levels.

Description of Project

This curriculum project is designed for 7th grade mathematics on geometry. The

focus of this project is that students are able to solve real-world and mathematical problems involving area, surface area, volume, perimeter, and circumference. This focus will be met through the use of CLT within each lesson by decreasing extraneous information.

Rationale

The shift in standards has left many teachers with little to no unique resources to base their lessons from, especially in higher mathematics education. Newly created curriculum aligned with the CCSS will be useful to teachers who feel the need to have a change to their own lessons or the lessons they are adapting from the modules of EngageNY. EngageNY was developed by New York State to assist teachers in the implementation of new CCSS. The shift to the CCSS requires teachers to “pursue, with equal intensity, three aspects in rigor of the major work of each grade: conceptual understanding, procedural skills and fluency, and application” (Key Shifts in Mathematics, 2015). Thus, planning and implementing lessons using the CLT could be a new method that ensures better student engagement and understanding of complex mathematical concepts. It is with this understanding of the theory that students may then be able to apply their knowledge in future assessments more precisely.

Chapter 2: Survey of Literature

The Paradigm Shift from NCTM to CCSS

The shift in standards since 2010 presented teachers with challenges to recreate and edit existing instructional materials. Generally speaking, the shift from individual state standards (typically the National Council of Teachers of Mathematics (NCTM)) to the CCSS

were to focus more intently on specifics instead of a curriculum that is a “mile wide and an inch deep” (Porter, McMaken, Hwang, & Yang, 2011, p. 103). The Trends in International Mathematics and Science Study (TIMSS) measure the achievement of students in comparison to other countries using standards and data analysis (“Trends in International Mathematics and Science Study,” n.d.). In the previous TIMSS studies, the United States (US) was the only nation without national standards. CCSS have been implemented so that we have more of a nationally followed standard where all states follow similar guidelines.

The alignments from specific state standards to the CCSS in mathematics had surprisingly uncorrelated data in certain states, like Montana. According to Table 1, Montana’s standards ranged from 0.01 to 0.15 alignment with the CCSS (Porter et. al, 2011). These alignment numbers indicate that there is almost no correlation between the

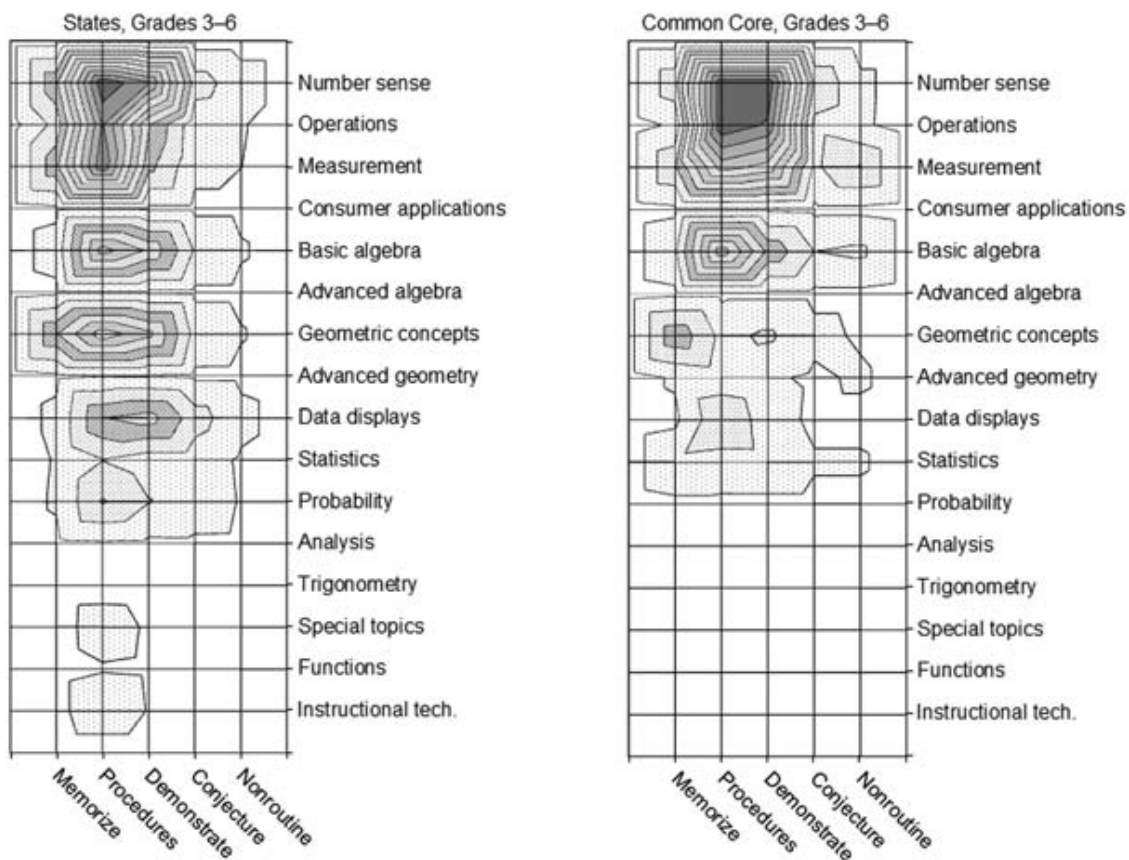
Table 1:

Alignment of State and Common Core Math Standards (Porter et. al, 2011, p. 106)

State Standards	Common Core Standards by Grade										
	K	1	2	3	4	5	6	7	8	9–12	3–6
Alabama							.19	.31	.28	.22	
California	.27	.20	.27	.21	.16	.20		.19		.31	
Delaware		.30	.38	.32	.30	.26	.25	.22	.26	.32	.47
Florida										.23	
Idaho	.25	.19	.27	.25	.21	.25	.23	.25	.20	.27	.34
Illinois		.27	.32	.25	.23	.15	.25	.34	.22	.31	.35
Indiana	.25	.26	.34	.27	.29	.27	.26	.26	.26	.25	.43
Iowa					.20				.07		
Kansas	.35	.29	.40	.31	.19	.21	.20	.30	.29	.33	.36
Maine				.18		.22	.19	.30	.30	.32	.32
Massachusetts								.19		.24	
Michigan			.46	.41	.43	.30					
Minnesota	.51	.28	.44	.41	.27	.24	.21	.33	.29	.26	.41
Mississippi					.25				.16		
Montana				.10	.12	.10	.01	.07	.07	.05	.15
New Hampshire	.23	.27	.23	.24	.19	.17	.20	.21	.23		.32
New Jersey					.15				.23		
North Carolina				.30	.21	.16	.16	.15	.22		.39
Ohio	.42	.29	.33	.22	.21	.21	.27	.17	.30	.37	.38
Oklahoma		.23	.32	.28	.28	.24	.06	.09	.16	.30	.36
Oregon	.34	.27	.41	.30	.18	.19	.24	.28	.33	.15	.38
Pennsylvania									.20		
Rhode Island										.12	
Texas							.17		.15		
Vermont	.15	.19	.21	.24	.17	.21	.21	.22	.21	.19	.30
West Virginia									.22		
Wisconsin					.17				.14	.11	
National Council of Teachers of Mathematics	.23	.27	.30	.27	.24	.27	.22	.22	.22	.38	
Minimum	.15	.19	.21	.10	.12	.10	.01	.07	.07	.05	.15
Maximum	.51	.30	.46	.41	.43	.30	.27	.34	.33	.37	.47
Average	.31	.25	.34	.27	.22	.21	.19	.23	.22	.24	.35

State standards and the CCSS. The relation between the two different standards is shown in figure 1 which represents an alignment of .45. This diagram could be interpreted as the CCSS focusing on the basics more than the higher-level mathematics that the previous state standards had at least slightly covered. In figure 1, it is very apparent that the shift from state standards to common core was switched to a much more specific curriculum instead of a broad range of each topic (Porter et. al, 2011, p. 110). Using a contour map style, the specific topics and strategies have switched from being spread evenly between topics to a

Figure 1. Comparison of Topics Taught Across Previous State (NCTM) and CCSS (Porter et. al, 2011, p. 110)



more specific focus on topics using strategies. This is apparent by comparing how the contours are in a more confined area in the common core map than the state standards map.

This shift to a more nationally focused curriculum may create a more consistent system for student learning across the US. Porter et. al (2011) described that “under a national curriculum, it would not be necessary for each state to develop its own content standards, assessments, and curriculum guides” (p. 104). This is a large advantage of the common core shift because there is more consistency in the learning of students across the States. Another advantage of common core standards over individual state standards is that “teachers place considerably less emphasis than does Common Core on ‘perform procedures’ and ‘demonstrate understanding’” (Porter et. al, 2011, p. 114). Having a better demonstration of understanding over just performing procedures meets the cognitive demand that is needed in mathematics. Although the shift is not large between procedural learning and demonstration of understanding, the change to a larger focus on understanding and lesser focus on procedure will increase student learning for the better.

Problem Solving

Students were taught abstract mathematical skills in the 1970s only to realize that they did not grasp the basic mathematical skills that were needed (Schoenfeld, 1992). Based on the complexity of problem solving, there was a time period when the US concluded that “the new math had not worked, and that we as a nation should make sure that our students had mastered the basics -- the foundation upon which higher order thinking skills were to rest” (Schoenfeld, 1992, p. 8). This experiment of teaching only the basics of mathematics was an absolute failure (Schoenfeld, 1992). If students do not grasp the basic concepts of mathematics, they have a much smaller chance of understanding the abstract mathematics presented in middle and high school.

After the back to basics era ended, the new theme was problem solving. Schoenfeld

(1992) stated that there were five roles that problems portrayed: relate mathematics to real-life situations; provide specific motivation for certain subjects; show mathematics can be fun; means for developing a new skill; and for practice. These roles play a large part in how students learn a specific mathematical skill or understand a definition/theorem.

Problem solving is an essential part of learning mathematics because it is also a part of life after school. Schoenfeld (1992) elaborated this phenomenon saying “if mathematical problem solving was to be important, it was not because it made one a better problem solver in general, but because solving mathematical problems was valuable in its own right” (p. 14). Solving mathematical problems can be a complex skill required for all students in middle and high school. So the CLT, when teaching heterogeneous classrooms, may support teachers in the development of their lessons.

Cognitive Load Theory

Cognitive load theory is defined as the “development of instructional methods that efficiently use people’s limited cognitive processing capacity to stimulate their ability to apply acquired knowledge and skills to new situations” (Paas, Tuovinen, Tabbers, & Van Gerven, 2003, p. 63). From this definition, it is obvious that the goal for teachers is to find the most efficient way to instruct students with complex tasks without overwhelming the student’s ability to comprehend the subject. It is also important to point out from this quote that students are also able to comprehend the subject matter so that they may apply this knowledge in needed situations. The fact that mathematics is continuously feeding students with new information, this theory proves vital for students compared to other subjects they learn in school.

Extraneous Cognitive Load

Van Merriënboer & Sweller (2005) stated in their article that cognitive load theory thrives for “an emphasis on instruction designed to reduce unnecessary or extraneous cognitive load” (p. 147). From this excerpt, the term extraneous is described as cognitive load that is not necessary for student learning and is usually the result of bad instructional methods (Van Merriënboer, Kester, & Paas, 2006). Because this theory’s main objective is to efficiently instruct students to the point that they are not cognitively overwhelmed, extraneous cognitive load needs to be identified and avoided. An example of avoiding extraneous cognitive load in mathematics would be to provide the students with many worked examples of a task rather than problems of the task for the students to solve (Van Merriënboer & Sweller, 2005). When students are provided with worked examples, it decreases their cognitive load but results in the same objective being accomplished which is having the students know how the task is completed. Teaching problem solving with low student cognitive load in mind decreases student perception of their own ability to solve problems. Students will then become more confident in their problem solving abilities because the extraneous cognitive load is (at best) non-existent.

Intrinsic Cognitive Load

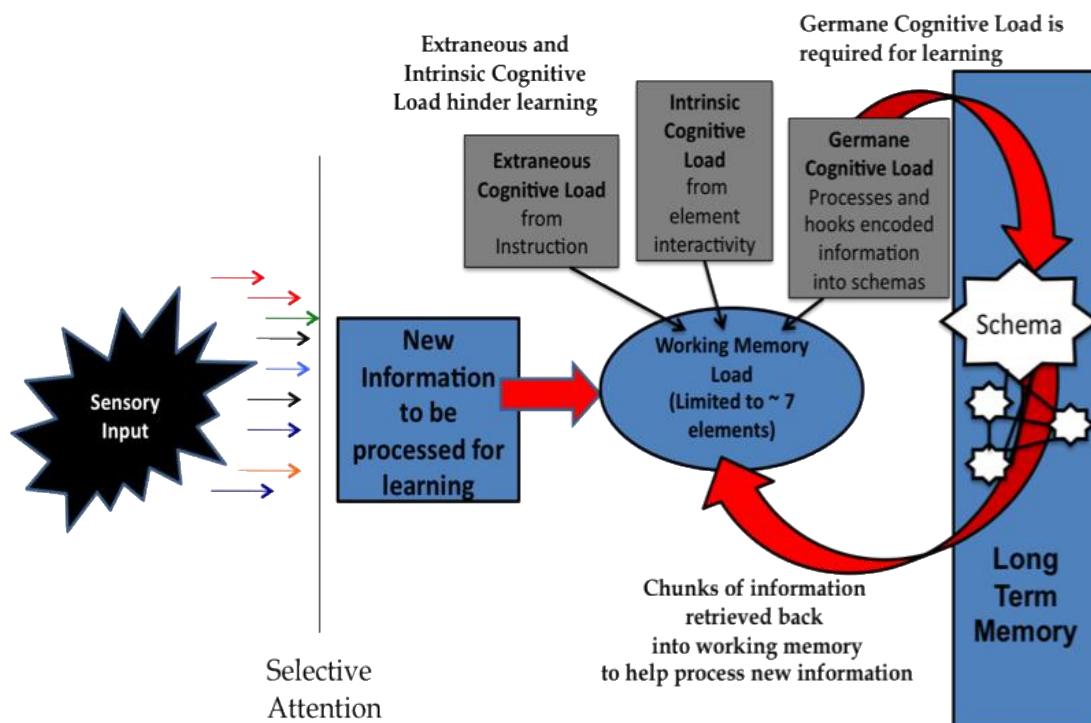
Cognitive load theory divides the types of load students have into three categories. The first was mentioned previously as extraneous load in which students do not require for learning. The second is intrinsic load, which is “determined by the interaction between the nature of the learning tasks and the expertise of the learner” (Van Merriënboer, Kester, & Paas, 2006, p. 343). In other words, intrinsic load is the correspondence that specific students have with a certain subject matter, which is mainly dependent on the student’s

background knowledge or experiences. For example, students from rural areas may have a better understanding of ratios over other student areas due to the fact that they grow up in a society where mixing oil with gasoline is required to run certain small engine equipment. It is with this specific type of cognitive load that mathematics thrives from because of all its interactive parts.

Germane Cognitive Load

Lastly, germane load is defined as the “load that directly contributes to learning, that is, to the learner’s construction of cognitive structures and processes that improve performance” (Van Merriënboer, Kester, & Paas, 2006, p. 344). It is therefore the goal for teachers to diminish extraneous load with good instruction since intrinsic load will be high because of the complexity of learning mathematics. Therefore, germane load is at its optimal value. Van Merriënboer, Kester, & Paas (2006) describe the objective as “balancing intrinsic load, which is caused by dealing with the element interactivity in the tasks, and

Figure 2: CLT Connected to Working and Long-Term Memory (Wade, 2011, p. 39)



germane load, which is caused by genuine learning processes” (p. 344). To balance these loads, teachers need to know their students and their knowledge on certain subjects (intrinsic) and use this as fuel to increase their learning potential on the subject (germane).

Along with the load categories of this theory, there are categories of memory that come into affect. Specifically speaking, working memory and long-term memory are used by students to take simple task memories and complete more complex tasks presently. According to Van Merriënboer & Sweller (2005), working memory has a limited storing capacity (about 7 elements) in which almost all information received is lost within twenty seconds, unless the information is rehearsed. Long-term memory effects working memory because long-term memory has no limitations when it is being applied to working memory. Figure 2 presents the connection between the different cognitive loads mentioned as well as how working memory and long-term memory connect. Long-term memory is organized in schemata and germane load hooks new learning, being processed in working memory, to existing schemata.

Van Merriënboer, Kester, & Paas (2006) compare the learning differences between complex tasks and simple tasks using the terms of low contextual interferences (LCI) and high contextual interferences (HCI). In LCI, “one version of a task is repeatedly practiced before another version of the task is introduced” and in HCI, “all versions of the task are mixed and practiced in a random order” (Van Merriënboer, Kester & Paas, 2006, p. 344). HCI was stated later in this article that it induces “germane learning processes that require more effort than does blocked practice, but yield cognitive representations that increase later transfer test performance” (Van Merriënboer, Kester & Paas, 2006, p. 345). From this quote, it can be said that complex tasks that require high cognitive load are more effective

for storing memory of the task and the skills needed to complete them. In simple tasks, there exists little process learning because the task is repeatedly practiced with a small cognitive load for the students before moving on to the next task.

Van Merriënboer and Sweller (2005) state in their study “in conceptual domains, there are many interacting knowledge structures that must be processed simultaneously in working memory in order to be understood” (p. 156). From this quote, they are implying that intrinsic load is a vital part of the learning process. In order to conceptually understand a topic, more is required of the students than just their germane load learning. Teaching with the supply of intrinsic load in mind was presented later in their article. Their strategy was diverted from the case where the teacher presents material to the students in two phases by including all information in both phases to thinking of the students’ cognitive load while teaching. Specifically,

In the first part, cognitive load was reduced by not presenting all information at once. Instead, isolated elements that could be processed serially were presented. In the second part, however, all information was presented at once, including the interactions among the elements (Van Merriënboer & Sweller, 2005, p. 156).

This excerpt explains that the students were presented many isolated elements that weren’t made sense of precisely when first presented. It was with this plan that the second phase of student learning was presenting all the information at once, then making connections with what was discussed in the first phase. This proved effective because students were able to conceptually learn from their intrinsic load from phase one and eventually convert to their germane load. Van Merriënboer and Sweller concluded

“presenting the full set of interacting elements in both phases resulted in less understanding than presenting isolated elements in the first phase followed by the full set in the second phase” (p. 157).

Cognitive load theory is useful for the instruction of mathematics because mathematics is difficult for many students. Understanding how to teach students knowing how their mind interacts with new information is key to their success. Knowing how to present new information and increasingly difficult information along the way will benefit students if you understand the ways to present it without increasing extraneous cognitive load. In the studies mentioned previously, success was shown when intertwining the ideas of increasing both intrinsic load and germane load together. Having these two cognitive loads work together can be very helpful when introducing complex ideas in mathematics.

It is also important to highlight that little cognitive load does not result in better student learning compared to high cognitive load. In other words, students have a better probability of learning a complex task if they are taught with the correct strategies for increasing intrinsic and germane load, with little to no extraneous load. If students were not challenged with tasks, the chance for them learning anything complex, which exists almost everywhere in mathematics, would be nearly impossible. To make complex a reachable goal, it is vital that the teacher gives worked examples for students to study themselves so that they may better complete complex mathematical problems. This technique would carry high cognitive load, less teacher dependency, and result in an increase of student self-esteem.

Chapter 3: Curriculum Project Design

Overview

This curriculum project was designed for the 7th grade heterogeneous mathematics classroom. Prior to each lesson plan, there exists a short excerpt of the connection the specific lesson has to CLT. According to the New York State CCSS (2015), the students have been introduced to the concepts of area of polygons as well as surface area and volume of three-dimensional shapes. Along with this, students have also practiced solving equations and the substitution of values into formulas. This unit plan provides the students with a deeper understanding of these concepts as well as a bridge to more complex ideologies of future mathematics classes.

Day 1 - Circles, Circumference, and Area

CLT is implemented into this first lesson in a variety of ways. To decrease extraneous load, a worked example is provided as the example for students to refer to when working in groups or individually. This lesson as provides students with intrinsic load in the beginning of the lesson with an introduction to the definitions and terms of the lesson. These definitions are given in cloze notes form so that students are not given extraneous cognitive load for writing the definitions out fully.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Internet link
- Pen/Pencil

- Lesson Handouts

Vocabulary:

- Area
- Circumference
- π (Pi)
- Radius
- Diameter

Objective:

- Identify vocabulary terms involving circles.
- Solve for the circumference and area of circles.
- Record answers in terms of pi (when applicable).

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - 4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- **Explain volume formulas and use them to solve problems**
 - 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Phase 1: (10 minutes)

- Students will be asked verbally “Raise your hand if you recognize the vocabulary terms circumference, area, diameter, radius, or pi?”
- Students will then follow along with cloze notes and fill in the blank for key words in the definitions of vocabulary terms used in the lesson.
- Students will also make inferences from the interactive radius link provided

Phase 2: (27 minutes)

- Implement facilitated discussion on worked problem
- Practice problems with partners using worked problem as reference

- Refer to vocabulary notes section if needed
- If time remains before phase 3, students will work on homework problems for independent practice

Phase 3: (3 minutes)

- TOTD: Students will describe using complete sentences why the diameter is always equal to twice the radius ($D = 2r$).

Name: _____
Day 1 Notes

Date: _____

Circles: Circumference and Area

- Circumference - The _____ around the outside of a circle.

$$C = \pi d \quad \text{"Cherry pie's delicious"}$$

- Area - The number of _____ needed to cover a given area.

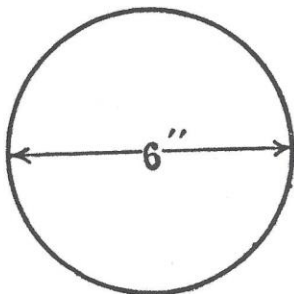
$$A = \pi r^2 \quad \text{"Apple pies are too"}$$

- Diameter - the _____ across the widest part of the circle.
- Radius - the distance from the _____ of the circle to an _____ edge (Half the diameter).
 - <http://www.mathopenref.com/radius.html>
- In terms of pi - π is still in your answer. This is an _____ answer.

Example 1: Find the **circumference** of the circle below.

a) Leave your answer in **terms of π** .

b) Round your answer to the **nearest tenth**.



$$\text{Diameter} = 6''$$

$$C = \pi d$$

$$C = \pi(6)$$

$$C = 6\pi \text{ in} \quad \leftarrow (a)$$

$$C = 18.8 \text{ in} \quad \leftarrow (b)$$

Example 2: Find the **circumference** of a circle with a **radius** of 5 cm.

- a) Leave your answer in **terms of pi**.
- b) Round your answer to the **nearest tenth**.

Example 3: What is the **diameter** of a circle with a **circumference** of 12cm? (*Hint: Work backwards*)

Example 4: Find the **area** of a circle with a **radius** of 5cm.

- a) Leave your answer in **terms of pi**.
- b) Round to the **nearest hundredth**.

Example 5: Find the **diameter** of the circle with an **area** of 36 units².

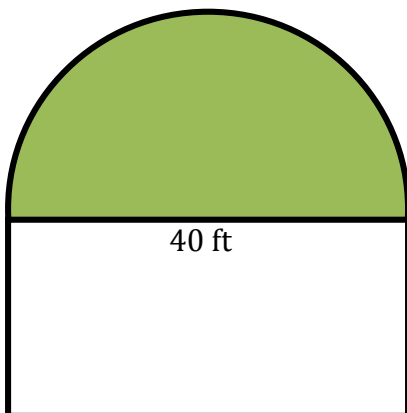
Name: _____

Date: _____

Day 1 Homework

Circles: Circumference and Area

1. What is the area of a circle with a diameter of 10 meters *in terms of pi*?
2. What is the circumference of a circle with a radius of 14 centimeters? (*Round to the nearest tenth*)
3. Find the diameter of a circle with an area of 81π . (*Hint: Radius is $\frac{1}{2}$ the diameter*)
4. Suppose your back yard is a half circle like the picture shown below. How much feet of fencing will you need to buy to line the outside of your yard if the diameter of the half circle is 40 feet? (*Round your answer to the nearest tenth*)



5. Given the circumference of a circle is 120π , find the following:

a) The diameter of the circle.

b) The radius of the circle.

c) The area of the circle (*In terms of π*).

Day 2 – Circumference and Area Stations

Day 2 uses the intrinsic cognitive load from the first day of lessons and repetition of high cognitive load examples to give students the opportunity to relay their learning. This lesson also uses real world examples to increase intrinsic cognitive load of students by providing a relatable phenomenon to their practice. Worked examples are also provided in the previous lesson's notes. To decrease extraneous cognitive load, students were provided measurements in the diagrams instead of only in the word problems.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Whiteboard
- Pen/Pencil
- Station Handouts
- Desks arranged into groups

Vocabulary:

- Area
- Circumference
- π (Pi)
- Radius
- Diameter

Objective:

- Solve for the circumference and area of circles in real life situations.
- Analyze solutions to area and circumference problems and determine parameters based off answers.

Standards:

- **Explain volume formulas and use them to solve problems**

- 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Phase 1: (5-8 minutes)

- Students will check their homework problem answers on the whiteboard as I stamp their unit outlines for completeness of homework.
- Allow time for student questions on homework (limit to a maximum of 2)
- Explain to students the outline of the lesson today with the stations
 - Students will have 9 minutes per station to answer the station problem.
 - The teacher will give students 2 minute warning per station before having students move to the next station.
 - Encourage students to work together to solve the problem as well as using the notes from their previous class.

Phase 2: (30 minutes)

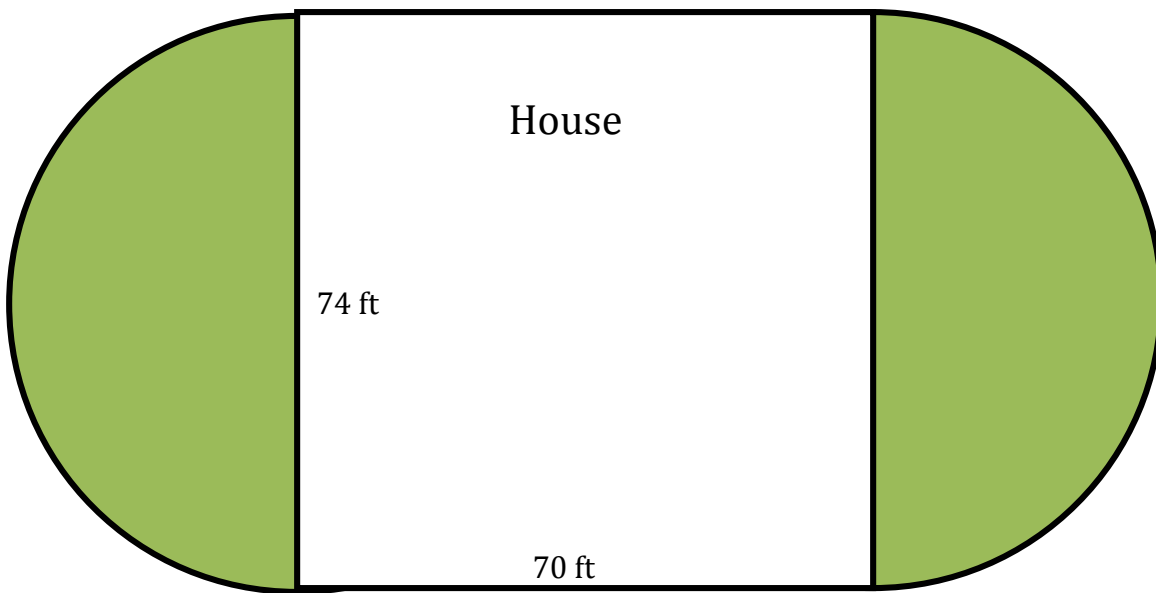
- Practice station problems with group using notes from previous class and homework as reference.
- Students will complete each station together showing all work.
- Students will move from station to station on when teacher prompts to do so.
- Time spent at stations may be modified based on how well students are communicating and solving the stations together.

Phase 3: (1 minute)

- Have students staple each station activity worksheet together and hand in on their way out the door.

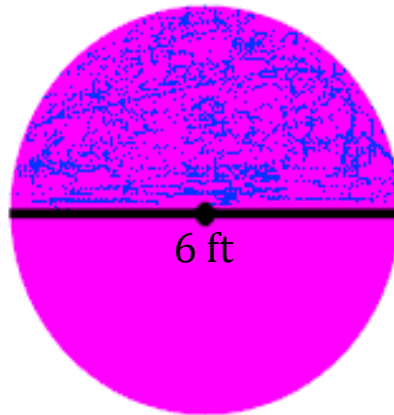
Station 1

John has to mow his lawn (green) after school today with the measurements in the picture. If his lawn mower can mow an area of 500 feet² on one tank of gas, how many tanks of gas does he need to mow his entire lawn?

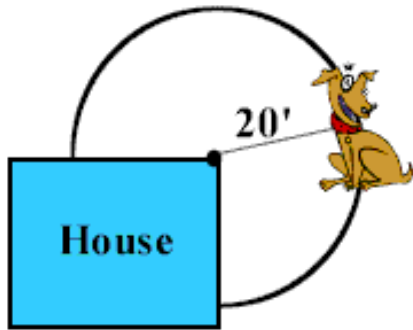


Station 2

Jessica is making a circular tablecloth for an art project. She wants half of the cloth to be a plain colored fabric and half to be a print fabric. How many square yards of each fabric (to the *nearest hundredth* of a yard) will she actually be using if the diameter of the cloth is 6 feet?



Station 3



Butch, the dog, is leashed to the corner of the house when he is outdoors alone. The leash is 20 feet long. Find the amount of ground area available to Butch when leashed outdoors.

Day 3 – Polygons: Perimeter and Area

Students will once again be provided with a cloze notes bulleted list of the next couple terms and definitions applicable to the next section of the curriculum to decrease extraneous cognitive load. Students will also be provided with the measurements of each problem provided with a figure within each figure. Decreasing the extraneous cognitive load provides students with more room for their germane cognitive load to store the information into memory.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Whiteboard
- Markers
- Lesson Handouts
- Pen/Pencil

Vocabulary:

- Area
- Perimeter
- Rectangle
- Square
- Base
- Height
- Length
- Width
- Trapezoid
- Triangle
- Parallelogram

- Polygons

Objective:

- Solve for the area and perimeter of polygons (specifically trapezoids, squares, rectangles, triangles, and parallelograms).
- Recognize the definition of area from previous lessons as the number of square units needed to cover an area.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Quiz: (15 minutes)

- Students will complete a 5 item quiz to assess student understanding of the first two lessons on circles, circumference, and area.

Phase 1: (12 minutes)

- Ask students to raise their hands if they are familiar with the terms “Polygon” and/or “Perimeter”.
 - For polygon definition, paraphrase answers from students to conclude that the definition is a straight lined shape, with at least 3 sides, and ends all connected.
- Present the 5 polygons on the smart board and have students tell you the area of each of the shapes (formulas) while writing them in the space below each shape.

Phase 2: (11 minutes)

- Have students work with a partner using the notes done in Phase 1 as a reference.
- Encourage students to draw a picture for every problem that does not already have one drawn because it lessens the difficulty when given a visual.

Phase 3: (2 minutes)

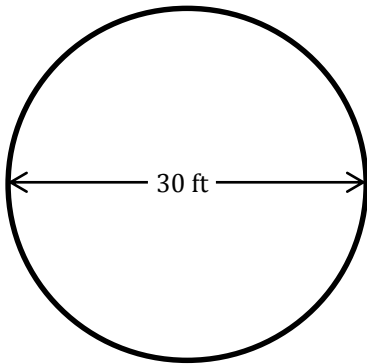
- Have students answer the ticket out the door question “How do you solve for the perimeter or area of a square when given only one side? Explain using a picture.”

Name: _____

Date: _____

Quiz 1

1. What is the circumference of a circle with a diameter of 12 cm in terms of π ?
2. What is the radius of a circle with an area of 201.6 m^2 ? (*Round your answer to the nearest tenth*)
3. Find the diameter of a circle if the circumference is 41 inches. (*Round to the nearest hundredth*)
4. Find the area of the circle shown below *to the nearest tenth*.



5. What is the area of a circle with a diameter of 12 miles? (*Round your answer to the nearest hundredth*)

Name: _____

Date: _____

Day 3 Notes

Polygons: Perimeter and Area

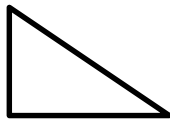
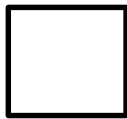
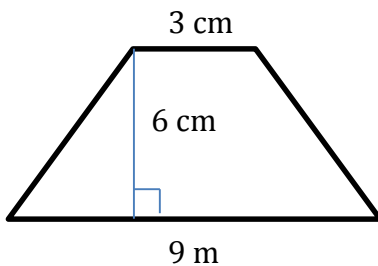
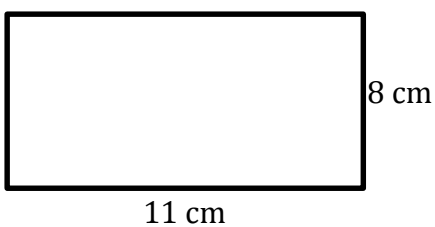
- Perimeter- The _____ around the outside of a shape.
 - (*Circumference was the distance around a circle*).
- Area- The number of _____ needed to cover a given area.

Rectangle/Square

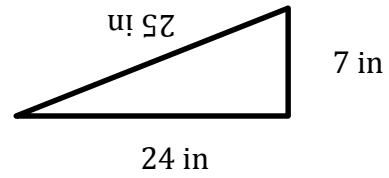
Triangle

Trapezoid

Parallelogram

Example 1: Find the **area** of the figure pictured below.Example 2: Find the **perimeter** of a right triangle with legs of length 5m and 12m and a hypotenuse of 13m.Example 3: Find the **area** of the figure pictured below.

Example 4: Find the **area** of the right triangle below.



Example 5: What is the **perimeter** of a square parking lot knowing one side measures 59 ft?

Example 6: Find the base of a parallelogram knowing that the height is 9cm and the **area** is 72 cm².

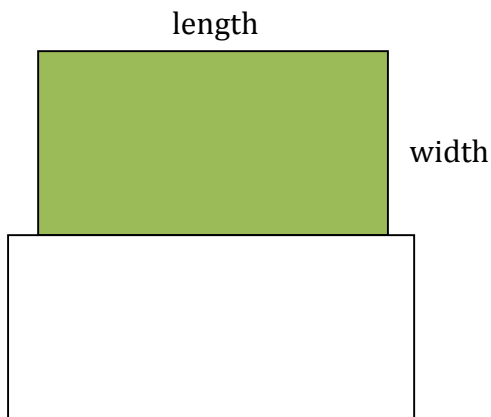
Name: _____

Date: _____

Day 3 Homework

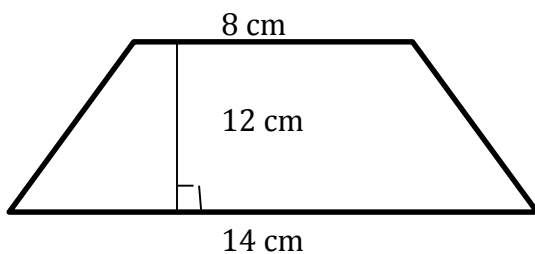
Polygons: Perimeter and Area

1. Find the height of a parallelogram knowing the base is 12 cm and the area is 144 cm².
2. Suppose you have a rectangular backyard as shown in the following picture. If the length of your yard is 42 feet and the width is 34 feet, how much fence is required to outline your yard?



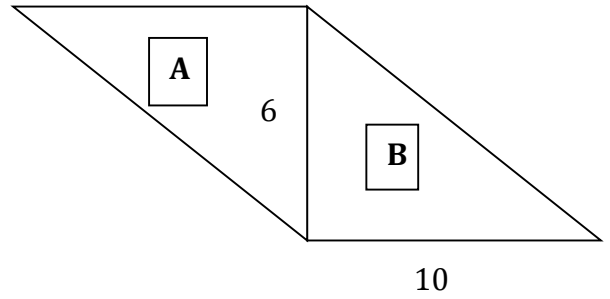
3. What is the height of a triangle with an area of 32 in² and a base of 8 in?

4. What is the area of the following trapezoid?



5. Solve the following questions based on the diagram. 10

a) Find the area of triangle A.



b) Find the area of triangle B.

c) Find the area of the parallelogram. (*Hint: $A = bh$*)

d) Add the areas of triangle A and B together.

e) What do you notice about your answers to (c) and (d)?

Day 4 – Polygons: Perimeter and Area Stations

Students will use their notes from the previous lesson (intrinsic) to assist them in completing the activity for today. The worked examples that students may refer to when completing the activity today were also in yesterday's lesson. This lesson also uses real world practices to increase intrinsic cognitive load of students by providing a relatable phenomenon to their practice.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Whiteboard
- Markers
- Lesson Handouts
- Pen/Pencil
- Rulers
- Calculators

Vocabulary:

- Area
- Perimeter
- Rectangle
- Square
- Base
- Height
- Length
- Width
- Polygons

Objective:

- Calculate the area and perimeter of the classroom.

- Compare the tile sizes of the ceiling to that of the floor and explain why the amount of tiles is different but the areas are the same.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Phase 1: (12 minutes)

- Review homework problem #5 to reinforce the fact that the two triangles that made up the parallelogram had the same exact area combined than the parallelogram's area.
- Review one other homework problem if requested together with students.
 - For polygon definition, paraphrase answers from students to conclude that the definition is a straight lined shape, with at least 3 sides, and ends all connected.
- Explain to students that yesterday we learned about area and perimeter of polygons. Today we are going to be finding the area and perimeter of the classroom.
 - Have the students notice the floor has tiles.
 - Ask the students what the dimensions of each floor tile are using a ruler (inches or feet)
 - Draw a square on the whiteboard and describe it as a "square foot".
 - Have students turn to another student and make conclusions about how the tiles will help us with area and perimeter of the classroom. (15 seconds)

- Have students think of the question “How will we find out the amount of floor tiles in the room without counting every single tile?”
- Have students guess how many tiles are actually in the room before moving on and write their guesses on the board

Phase 2: (20 minutes)

- Have students work with a partner and answer the questions given in the handout.
- Assist students as needed throughout activity.

Phase 3: (8 minutes)

- Bring student back together and record on the smart board what the determined area of the floor and ceiling was as well as the perimeter of the room.
- Facilitate a discussion revolving around question 3 of the handout:
 - Why is there a different amount of tiles on the floor than the ceiling?
 - Why isn't our answer only “2 times more”?
 - How would this be useful information to know?
 - What else could we determine the area and perimeter for in the classroom?
 - Why would that be helpful?

Day 5 – Total Area

Starting the lesson off with a group discussion by breaking down the new vocabulary term into a manageable definition decreases the potential extraneous load of new terms. Students also are provided with a worked example of how total area is determined. All required measurements for solving the problems are within each diagram to decrease extraneous cognitive load as well.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Whiteboard
- Markers
- Lesson Handouts
- Pen/Pencil
- Calculators

Vocabulary:

- Area
- Total Area
- Circle
- Rectangle
- Square
- Triangle
- Parallelogram
- Trapezoid
- π (Pi)
- Radius
- Diameter

Objective:

- Solve for the total area of irregular shapes by adding together the areas of shapes within the irregular polygon.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Phase 1: (10 minutes)

- Facilitate the breaking apart of the new vocabulary term “Total Area” with the students while recording student discussion on the whiteboard
- Using the irregular shape given in the notes handout, students will discuss with a partner how they think the total area could be found (30 seconds)
- Have students share with the class their ideas and record their responses on the smart board.
 - Show the area formulas above each section of the irregular shape and add them together.
- Stress the importance of writing the formulas being used.

Phase 2: (25 minutes)

- Have students work with a partner and determine what was done in the worked problem.
 - Have students record their observations of the worked problem in their notes.

- Choose volunteers to share what they observed in the worked problem and write it on the smart board.
- Have students work with their partner to solve the remaining problems in the notes.

Phase 3: (5 minutes)

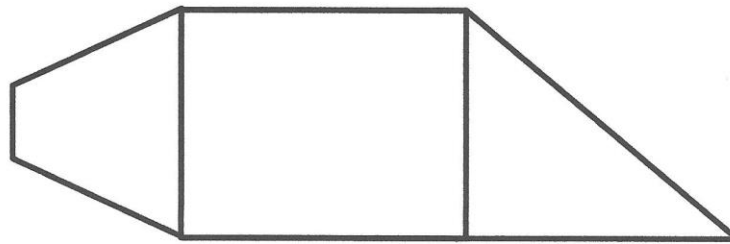
- If students are finished with the notes, they may work on the homework for the remainder of class

Name: _____
 (Day 5) Notes

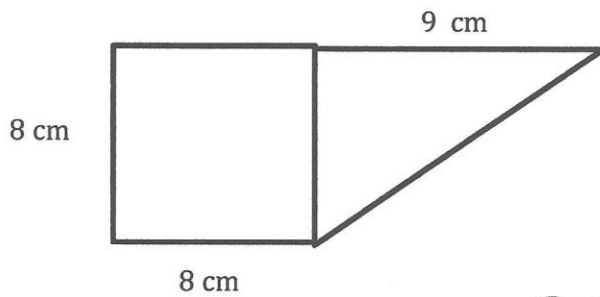
Date: _____

Total Area

- Total Area- To find the total area of a figure, add the areas of the inner figures together.



Example 1: Find the total area of the figure below.

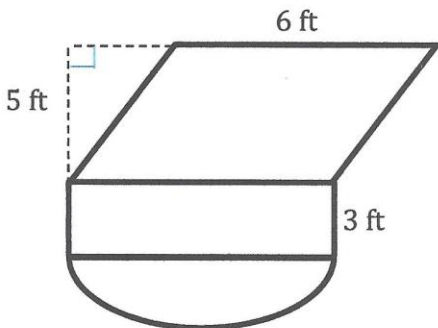


<u>Square</u>	<u>Triangle</u>
$A = bh$	$A = \frac{1}{2}bh$
$A = (8\text{ cm})(8\text{ cm})$	$A = \frac{1}{2}(9)(8)$
$A = 64\text{ cm}^2$	$A = \frac{1}{2}(72)$
	$A = 36\text{ cm}^2$

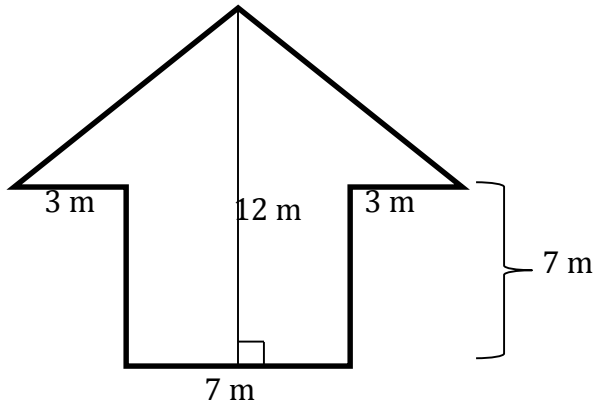
$$\text{Total Area} = 64\text{ cm}^2 + 36\text{ cm}^2$$

$$= \boxed{100\text{ cm}^2}$$

Example 2: Find the total area of the figure below (Round to the nearest tenth).



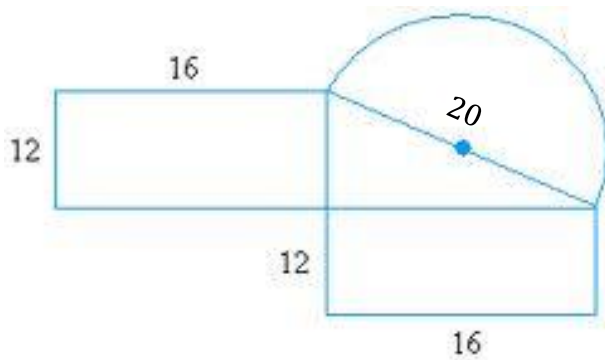
Example 3: Find the total area of the shape below.



Example 4: Consider the irregular shape shown below.

(a) Find the exact area of the irregular shape.

(b) Find the area to the nearest hundredth.



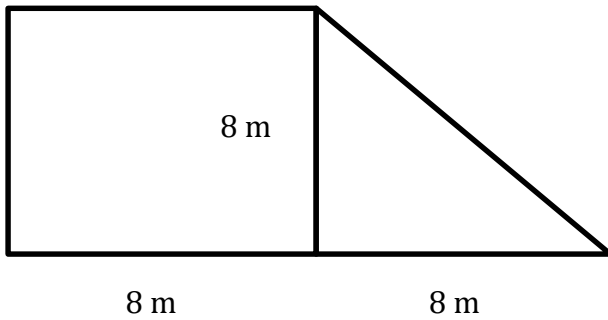
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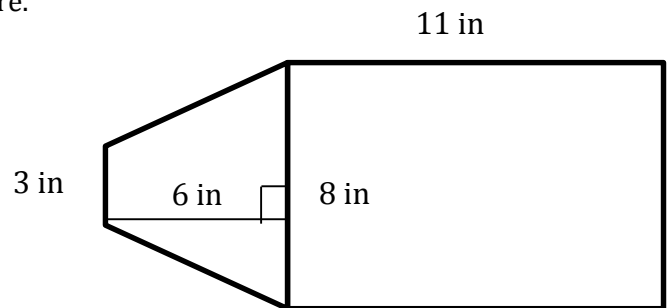
Day 5 Homework

Total Area

1. Find the total area of the figure below.

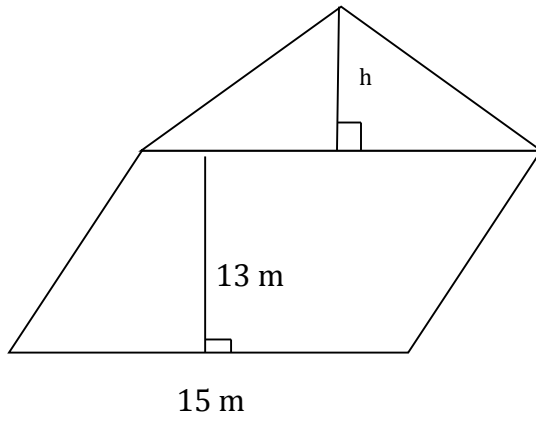


2. Find the total area of the following figure.



(One more on the back!)

3. Find the height of the triangle given the following measurements and the total area of the figure being 240 ft^2 .



Day 6 – Total Area Stations

Students will be working in groups, given worked examples from the previous class notes (intrinsic) and homework (if completed), and measurements of all problems written in with each diagram. All these strategies used to decrease extraneous load and potentially increase germane load.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Whiteboard
- Markers
- Pen/Pencil
- Station Handouts
- Desks arranged into groups

Vocabulary:

- Area
- Total Area
- Circle
- Rectangle
- Square
- Triangle
- Parallelogram
- Trapezoid
- π (Pi)
- Radius
- Diameter

Objective:

- Solve for the total area of irregular shapes by adding together the areas of shapes within the irregular polygon in real-world application word problems.

Standards:

- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Phase 1: (8 minutes)

- Students will check their homework problem answers on the whiteboard as I stamp their unit outlines for completeness of homework.
- Allow time for student questions on homework (limit to a maximum of 1)
- Explain to students the outline of the lesson today with the stations
 - Students will have 6 minutes per station to solve the station problem.
 - The teacher will give students 2 minute warning per station before having students move to the next station.
 - Encourage students to work together to solve the problem as well as using the notes from their previous class.

Phase 2: (31 minutes)

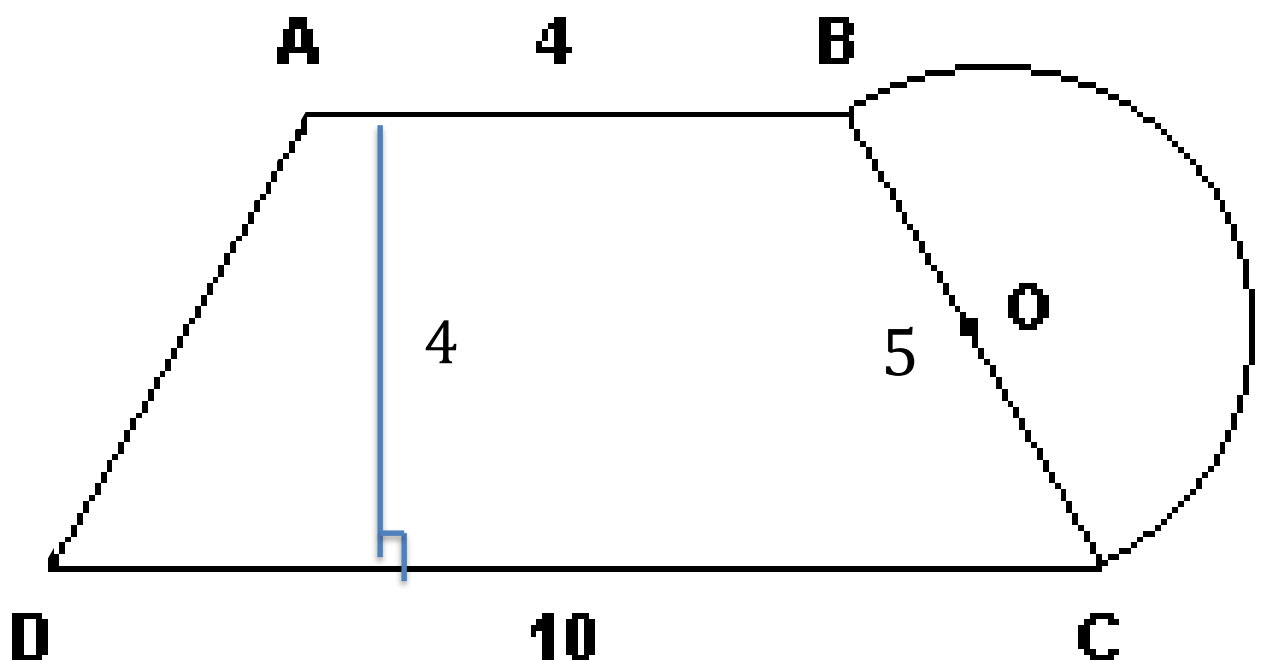
- Practice station problems with group using notes from previous classes and homework as additional assistance.
- Students will complete each station together showing all work.
- Students will move from station to station when teacher prompts to do so.
- Time spent at stations may be modified based on how well students are communicating and solving the stations together.

Phase 3: (1 minute)

- Have students staple each station activity worksheet together and hand in on their way out the door.

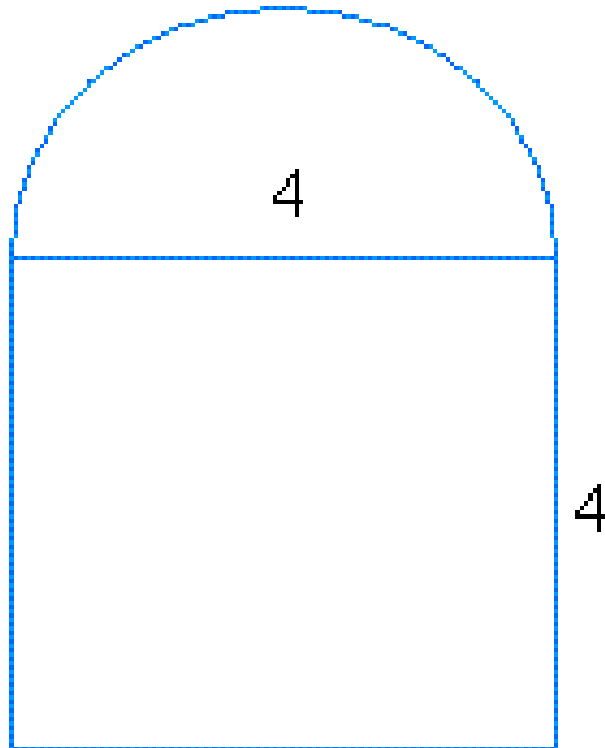
STATION 1

- Find the total area.
- Leave your answer in terms of π .



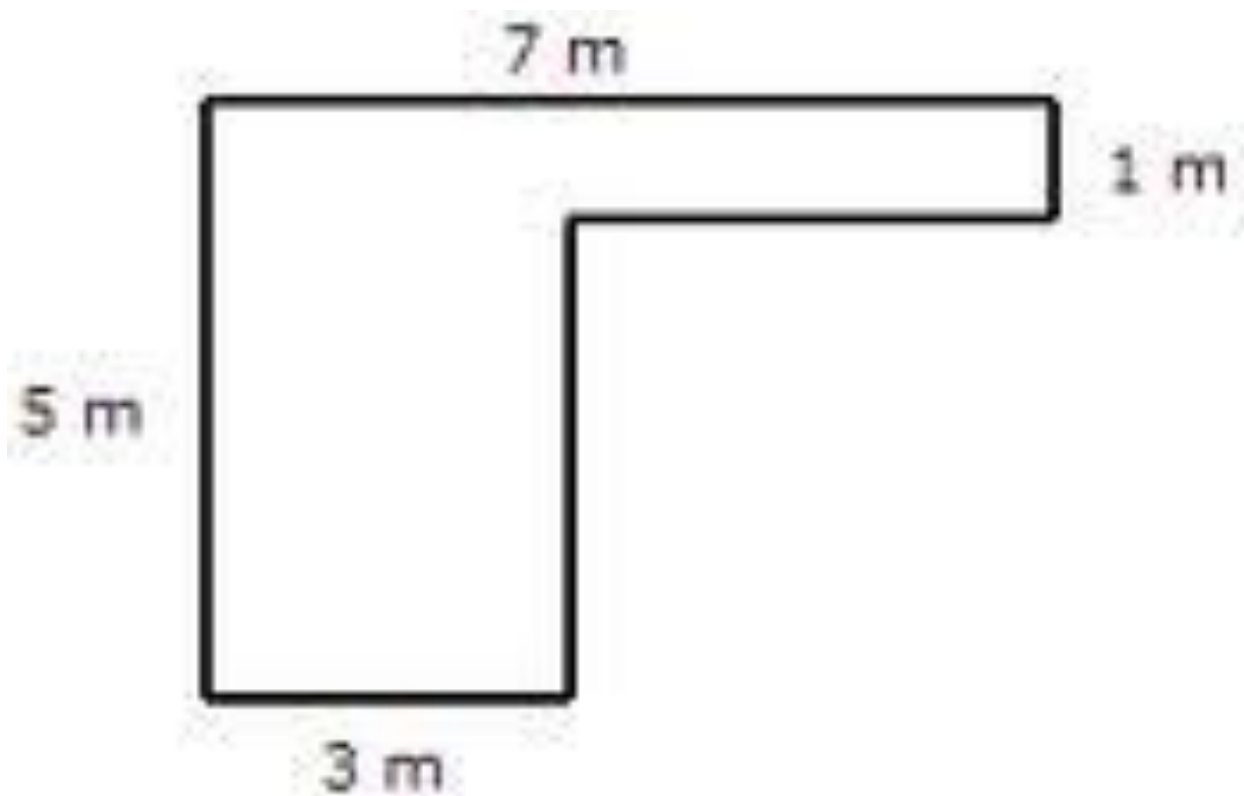
STATION 2

- Find the total area.
- Leave your answer in terms of π .



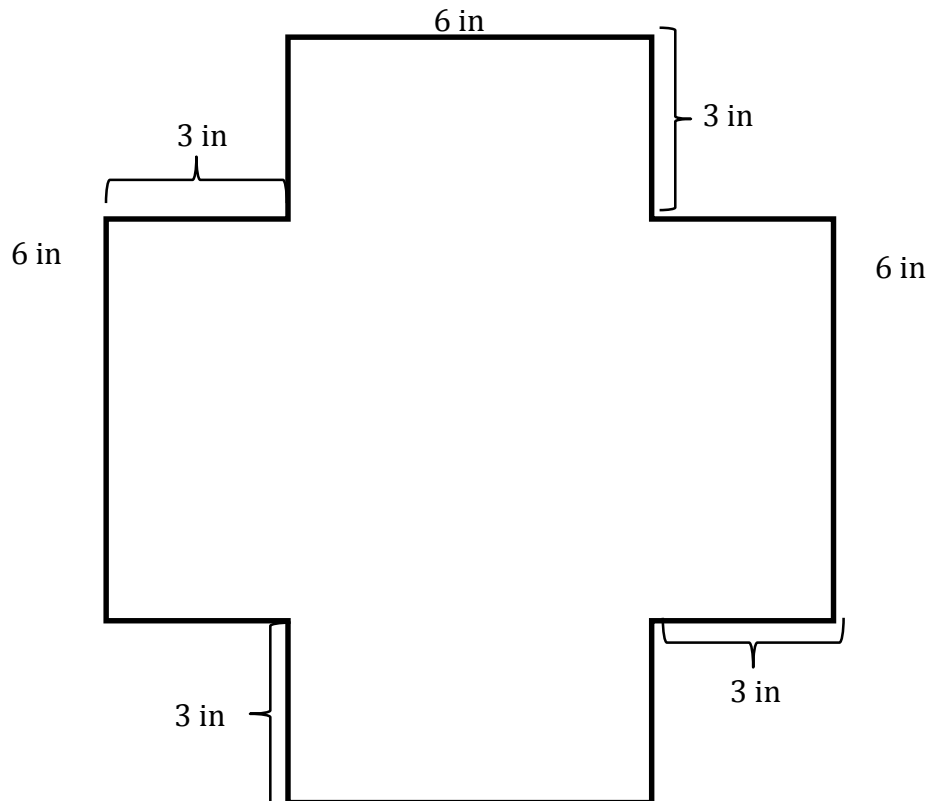
STATION 3

- Find the total area.



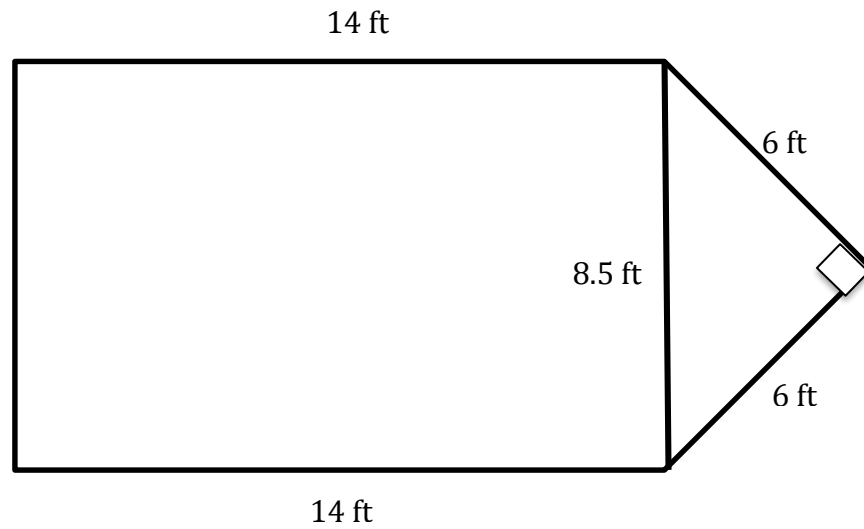
STATION 4

- Find the total area.



STATION 5

- Find the total area.
- Round your answer to the nearest tenth



Day 7 – Area of Shaded Regions

Using notes from previous classes (intrinsic), measurements within each problem diagram (decrease extraneous cognitive load), and assisting each other in partner work all potentially increase germane cognitive load.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Whiteboard
- Markers
- Individual Student Whiteboards
- Erasers
- Lesson Handouts

Vocabulary:

- Area
- Shaded
- Shaded Area
- π (pi)
- Radius
- Diameter
- Circle
- Rectangle
- Square
- Triangle
- Parallelogram
- Trapezoid

Objective:

- Determine procedure for calculating shaded areas of polygons and solve.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Phase 1: (15 minutes)

- Before passing out the notes, students will use their whiteboards to solve the problems presented on the smart board.
- Using individual whiteboards, students will be prompted to solve for the area of the square. Once they have written their work individually on their whiteboards, they will put the board against their chest until directed to show the teacher.
- Continuing this, have students do the next two problems one at a time to make an informal assessment of where each student is at with solving for area and their ability to use formulas in the directions.
- Have students check their whiteboard answers and discuss where some students might have made a mistake.

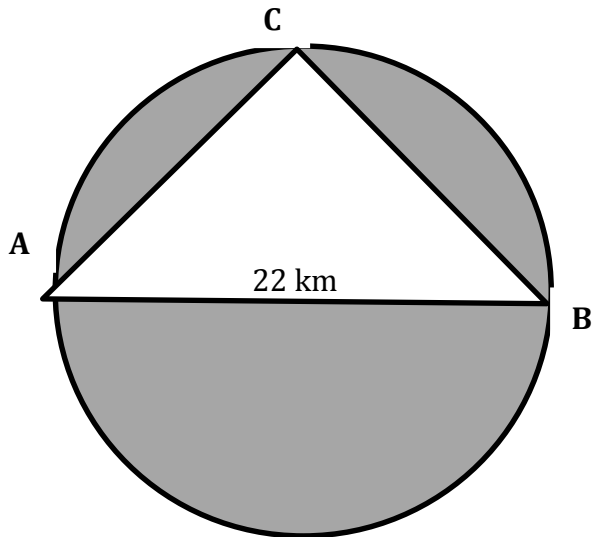
Phase 2: (20 minutes)

- With partners, students will work on example 2 in the notes
- Using complete sentences, students will answer the question on today's lesson about the process in solving problems with shaded regions.
- Bring students back together and review example 2.
- If students finish the notes, they may work independently on their homework for the night.

Phase 3: (5 minutes)

- TOTD: Students will compare and contrast their knowledge of the process of Total Area problems to that of Shaded Area problems.

Example 2: In this figure, **AB is the diameter** of the circle. Find the **area** of the **shaded region**.



1.) Find the **area** of the **circle**.

2.) Find the **area** of the **triangle**.

3.) Using the results from part 1 and 2, express the **area** of the **shaded portion**.

Question! Why does the method of subtracting the area of the smaller figure from the area of the larger figure determine the area of the shaded region?

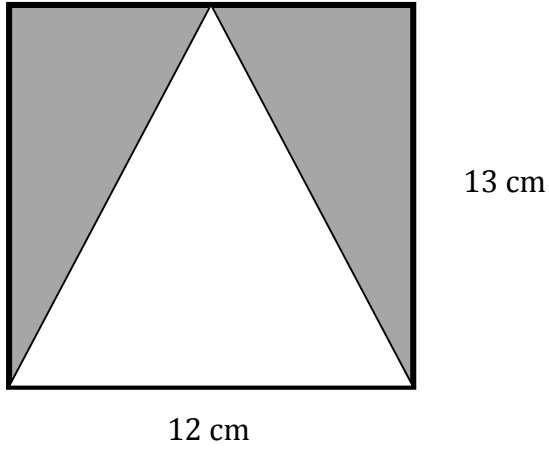
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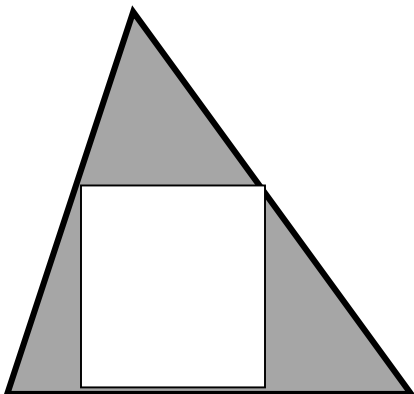
Day 7 Homework

Area of Shaded Regions

1. Find the area of the following shaded region.



2. Find the area of the shaded shape below if the height of the triangle is 19 inches, the base is 18 inches, and the area of the square is 121 inches².



Day 8 – Area of Shaded Regions Stations

Students will be provided with the measurements of each problem within each diagram (decrease extraneous cognitive load), work with a group to solve each station problem, and be encouraged to use notes from previous classes (intrinsic) to assist them in solving each station problem.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Whiteboard
- Markers
- Pen/Pencil
- Station Handouts
- Desks arranged into groups
- Quiz

Vocabulary:

- Area
- Shaded
- Shaded Area
- π (pi)
- Radius
- Diameter
- Circle
- Rectangle
- Square
- Triangle
- Parallelogram
- Trapezoid

Objective:

- Solve for the area of shaded regions and determine the process differences between shaded region area and total area.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Phase 1: (8 minutes)

- Students will check their homework problem answers on the whiteboard as I stamp their unit outlines for completeness of homework.
- Allow time for student questions on homework (limit to a maximum of 1)
- Explain to students the outline of the lesson today with the stations
 - Students will have 6 minutes per station to solve the station problem.
 - The teacher will give students 2 minute warning per station before having students move to the next station.
 - Encourage students to work together to solve the problem as well as using the notes from their previous class.

Phase 2: (17 minutes)

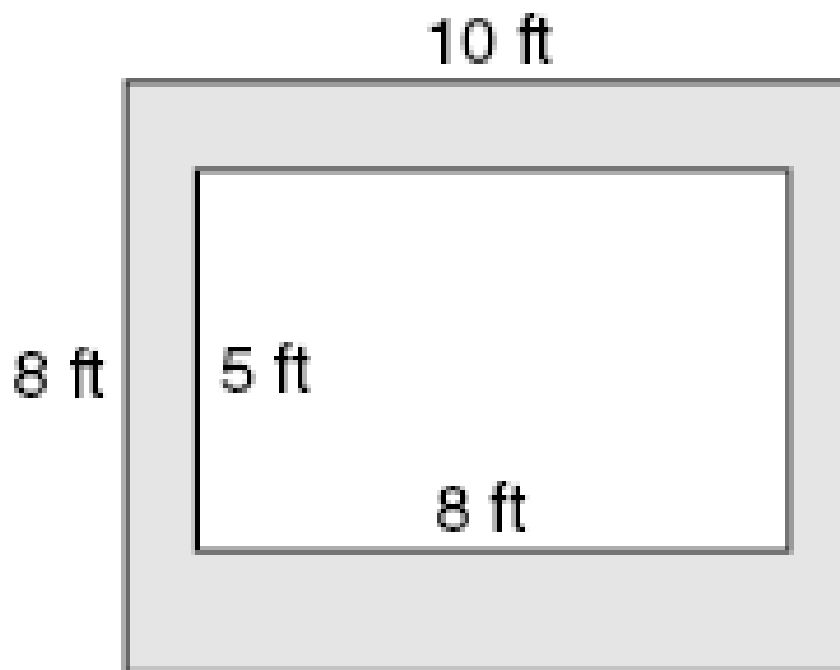
- Practice station problems with group using notes from previous classes and homework for additional assistance.
- Students will complete each station together showing all work.
- Students will move from station to station when teacher prompts to do so.
- Time spent at stations may be modified based on how well students are communicating and solving the stations together.

Phase 3: (15 minutes)

- Administer quiz on Total Area and Shaded Region Area for remainder of class.

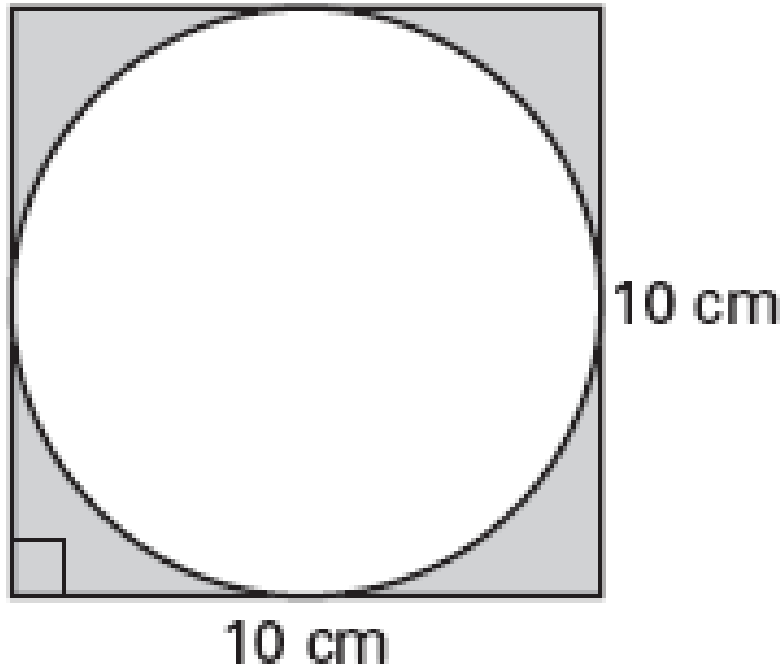
STATION 1

- Find the area of the shaded region.



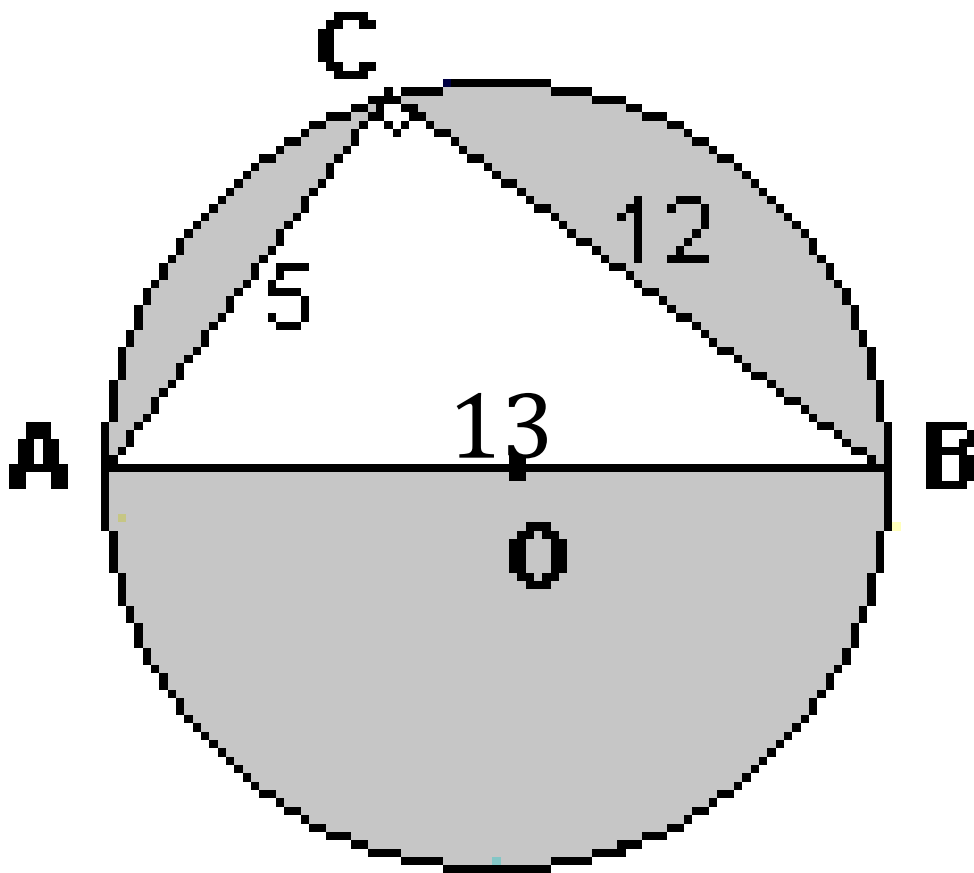
STATION 2

- Find the area of the shaded region.
- Leave your answer in terms of π .



STATION 3

- Find the area of the shaded region
- Leave your answer in terms of π .

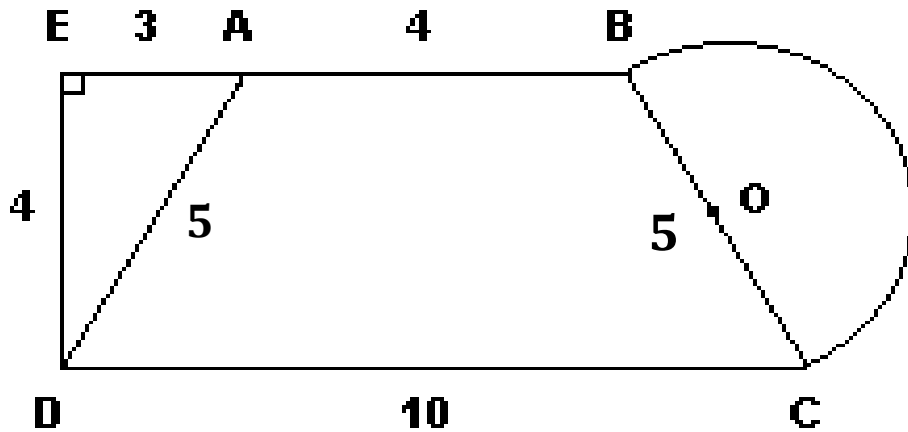


Name: _____

Date: _____

Quiz 2

1. Use the following figure to answer the questions **in terms of π** .



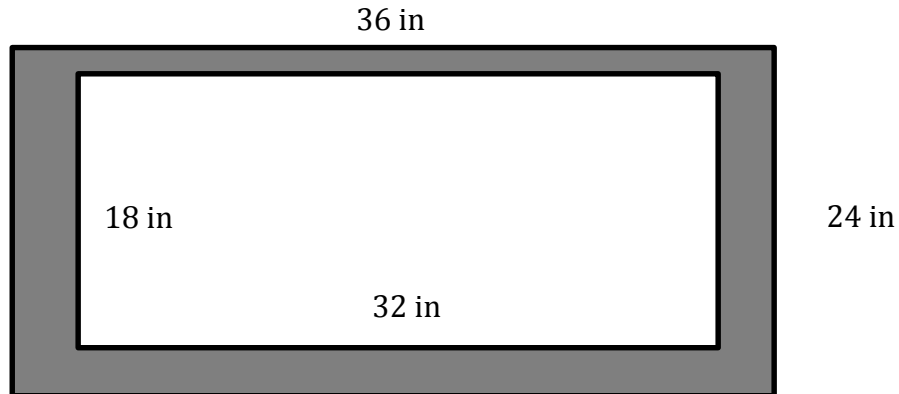
(a) Find the area of the triangle.

(b) Find the area of the trapezoid.

(c) Find the area of the **half** circle. *(In terms of π)*

(d) Find total area of the figure. *(In terms of π)*

2. In this figure, a rectangle is drawn within another rectangle. Solve the following:



(a) Find the area of the outer rectangle.

(b) Find the area of the inner rectangle.

(c) Using the results from part (a) and (b), find the area of the shaded region.

Day 9 – Surface Area Without Variables

Students will be given cloze notes as well as diagrams with measurements and labels within the diagram (decrease extraneous cognitive load). They will also be encouraged to use worked examples from previous lessons as well as real-world applications of today's topic using manipulatives (intrinsic).

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Whiteboard
- Markers
- Cardboard Box
- Soda Can
- Lesson Handouts

Vocabulary:

- Surface Area
- Rectangular Prism
- Cylinder
- Length
- Width
- Height
- Radius
- Cube
- 3-dimensional
- Variable

Objective:

- Solve for the surface area of given solids with actual numbered measurements.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Phase 1: (5-8 minutes)

- Students will attempt the warm up to the best of their ability.
- After a given amount of time, review the warm up with the students by facilitating a class discussion of the requirements of the problems.
- Pass out the notes for the day as well as an index card.

Phase 2: (30 minutes)

- Have students fill in the cloze notes together with the teacher for the vocabulary of the day.
- Using the manipulatives presented in front of the class (cardboard box), have students discuss with a partner how they may solve for the surface area of the box (3 minutes). Facilitate a whole class discussion in determining what the **surface area** of the box would be using student discussions as a basis.
- Similarly, facilitate the same procedure for the surface area of the soda can.
- As discussions occur, take notes on the front of the board for discoveries of the formula for both 3-dimensional objects.
- When formulas are made, guide students through first example.
- Have students attempt example 2 with a partner. Review after as a class.
- Have students attempt example 3 with a partner. Review after as a class.

Phase 3: (2 minutes)

TOTD: Have students write on their index card the answer to this question: "Why does the surface area of a rectangular solid have 3 terms added together?"

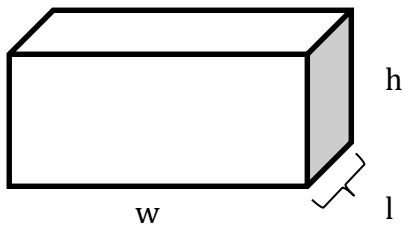
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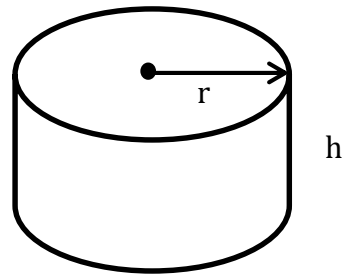
Day 9 Notes

Surface Area (without Variables)

- Surface Area- The _____ of the surface of a three-dimensional object.

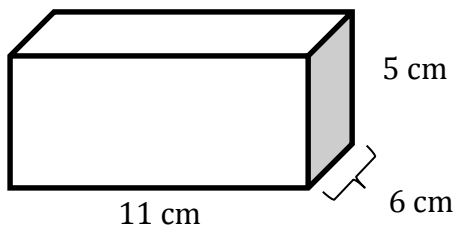


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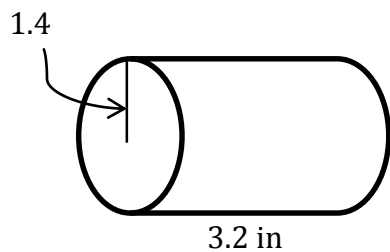


SA =

Example 1: What is the **surface area** of the solid below?



Example 2: What is the **surface area** of the solid below to the **nearest hundredth**?



Example 3: How many square feet of wrapping paper are needed to entirely cover a box that is 2 feet by 3 feet by 4 feet?

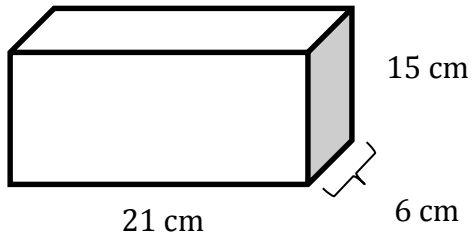
Name: _____

Date: _____

Day 9 Homework

Surface Area

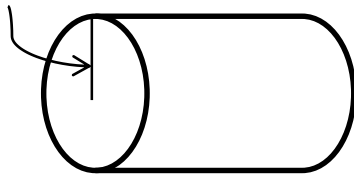
1. Find the surface area of the following rectangular solid.



2. A can of corn has a height of 6 inches and a diameter of 3 inches. Find to the nearest tenth the surface area of the can.

3. Find the surface area of the following lateral cylinder in terms of π .

8.1 cm



14.6 cm

Day 10 – Surface Area With Variables

Students will be provided with cloze notes, worked examples from previous classes, and measurements within the diagrams of each problem (decreasing extraneous cognitive load, building intrinsic load). This lesson also uses multiple real-world examples (intrinsic) to potentially increase germane load. Important vocabulary terms have been highlighted.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Whiteboard
- Markers
- Cardboard Box
- Soda Can
- Lesson Handouts

Vocabulary:

- Surface Area
- Rectangular Prism
- Cylinder
- Length
- Width
- Height
- Radius
- Cube
- 3-dimensional
- Variable

Objective:

- Solve for the surface area of given solids with variable measurements.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Phase 1: (5-8 minutes)

- Have students get into groups of 3-4 and discuss each other's homework as I walk around and check. The homework answers will be written on the whiteboard before class begins.
- Review any problems that may have risen from group discussions.
- Pass out the notes for the day as well as an index card.

Phase 2: (30 minutes)

- Have students fill in the cloze notes together with the teacher for the vocabulary of the day.
- Have students discuss with a partner how they may solve for the surface area of the box in example 1(a) (3 minutes). Facilitate a whole class discussion in determining what the **surface area** of the box would be using student discussions as a basis.
- When 1(a) is completed, ask students to put their thumbs up or down if they think our answer is in "terms of x". Have a volunteer explain why after giving students time to think.
- Have students pair back up with their partner to discuss what they must do for solving example 1(b). Review after as a class.
- Have students attempt example 2 with a partner. Review after as a class.

- Have students attempt example 3 with a partner. Review after as a class.

Phase 3: (2 minutes)

TOTD: Have students write on their index card the answer to this question: “Why is it that we must leave answers ‘in terms of x ’?”

Name: _____

Date: _____

Day 10 Notes

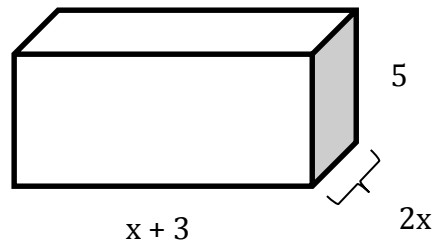
Surface Area (with Variables)

Variable- A letter that represents an _____ number.

In terms of x- Like leaving answers "in terms of π ", you leave the _____ in your answer.

Example 1: A plastic storage box in the shape of a **rectangular solid** has a **length** of $x + 3$, a **width** of $2x$, and a **height** of 5.

(a) Find the **surface area** of the box in terms of x .



(b) What is the **surface area** of the box if $x = 3$?

Example 2: A crate in the shape of a **rectangular solid** has a **length** of $x + 7$, a **width** of 2, and a **height** of $3x$. Find the **surface area** of the crate **in terms of x** .

Example 3: A **paint can** has a **height** of $x - 4$ and a **diameter** of $2x$.

(a) Find the **surface area** of the paint can **in terms of x** .

(b) What is the **surface area** of the paint can if $x = 4$? (**Round to the nearest tenth**)

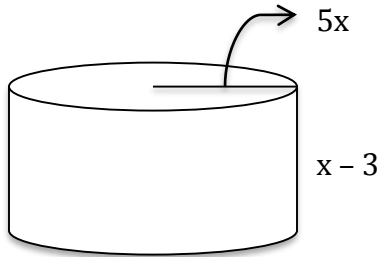
Name: _____

Date: _____

Day 10 Homework

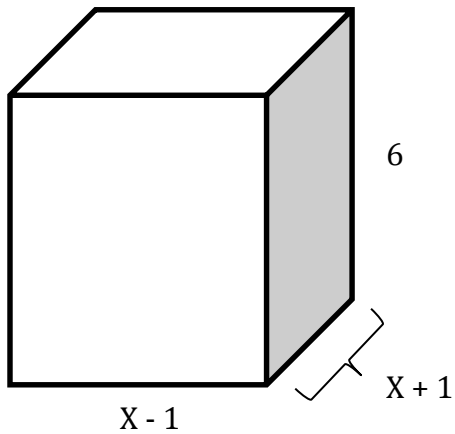
Surface Area (with Variables)

1. Find the surface area of the diagram shown below in terms of x . (*Round to the nearest tenth*)



2. What is the surface area of the diagram above to the nearest tenth if $x = 13$?

3. What is the surface area of the following shape? (Optional – BONUS)



Day 11 – Volume

Students will be provided with a worked example (decrease extraneous load), measurements labeled in the diagrams given, a list of the procedure to follow when solving for volume (which has been used in previous lessons), the formulas for solving for volume, and manipulatives to enforce the concept. Students may use the information from the first phase to solve examples given in the second phase (intrinsic).

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Smart Board
- Whiteboard
- Markers
- Cardboard Box
- Soda Can
- Lesson Handouts

Vocabulary:

- Surface Area
- Rectangular Prism
- Cylinder
- Length
- Width
- Height
- Radius
- Cube
- 3-dimensional
- Variable

Objective:

- Solve for the volume of 3-dimensional objects such as the rectangular solid, the cylinder, and the cube.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - Solve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Phase 1: (5-8 minutes)

- Have students check their homework answers on the board as the teacher checks for completeness.
- Review any problems through informal observation of student's homework.

Phase 2: (31 minutes)

- Ask students if they have ever heard of the term "volume".
- With a partner, have them describe to each other what they each think volume is (1 minute).
- Have students share with the class their definitions and modify their thinking so that you may form a formal definition. Write this on the whiteboard.
- When students have determined a sufficient definition, have some students read the procedure for solving word problems for volume.
- Students will then group with a partner and discuss the worked example provided in example 1. Discussion points to be addressed are:
 - Is the procedure stated before followed?
 - Why does the problem require solving for volume when the word "volume" isn't stated?
 - Were any extra steps taken in addition to the procedure? Why?
- After listening to student discussions, share what some students had said with the class anonymously.
- With partners, have students use example 1 as a template for solving example 2.

- Have students attempt example 3 with a partner. Review after as a class using cubic inch manipulatives to enforce the concept.

Phase 3: (1 minute)

TOTD: Have students hand in notes to formally assess their ability to solve example 2 using worked example 1 as a reference.

Name: _____
(Day 11) Notes

Date: _____

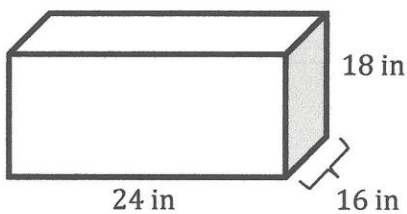
Volume

Rectangular Prism: $V = \text{length}(\text{width})(\text{height})$ Cylinder: $V = \pi r^2 h$

Procedure:

1. Always start with your **formula**.
2. **Substitute** in values that are given in the problem.
3. **Solve** for the value that is not given. This may require multiple steps or require you to work backwards.
4. **Labels** for solving volume are always cubed. (units³)

Example 1: Steve is filling his fish tank with water from a hose at a rate of **500 cubic inches per minute**. How long will it take, to the nearest minute, to **fill the tank**?



Volume = how much space something takes up

$$V = l \cdot w \cdot h$$

$$V = (24 \text{ in})(16 \text{ in})(18 \text{ in})$$

$$V = 6912 \text{ in}^3$$

6912 in^3 needs to be filled...

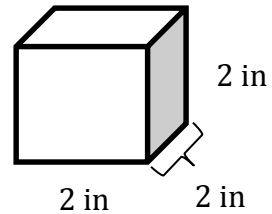
$$\frac{6912 \text{ in}^3}{500 \frac{\text{in}^3}{\text{min}}} = 13.824 \frac{\text{in}^3}{\text{in}^3/\text{min}} = 13.824 \text{ min}$$

14 minutes

Example 2: The length of a fuel hose for a string trimmer is 12 centimeters and its radius is 2 centimeters.

- (a) Calculate the total **volume** that the fuel hose can hold.
- (b) On average, a string trimmer gas tank **holds 3800 cubic centimeters**. If the trimmer uses the **volume of part (a)** worth of gas **per minute, how long** will it take to use all the gas?
(Round to the nearest minute)

Example 3: Consider a cube whose sides measure 2 inches.



- (a) What is the volume of this cube?
- (b) If the length of the sides of the cube were doubled, what would be the new volume of the cube?
- (c) What is the ratio of the larger cube's volume to that of the smaller cube?
- (d) Why did the cube's volume not double when the length of its sides was doubled?

Day 12 – Volume Stations

Students may use their previous day notes to assist them using worked examples for the station activity today. The station activities are real-world examples (intrinsic) that students can relate to and potentially lessen extraneous cognitive load while solving. Students will be working in groups and also provided with measurements within the diagrams (decreasing extraneous cognitive load).

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Pen/Pencil
- Station Handouts
- Desks arranged into groups

Vocabulary:

- Area
- Rectangle
- Square
- Cube
- Rectangular Solid
- Volume
- Length
- Width
- Height
- 3-dimensional

Objective:

- Using volume formulas, evaluate given/concluded data and solve conceptual questioning.

Standards:

- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Phase 1: (2 minutes)

- Explain to students the outline of the lesson today with the stations
 - Students will have 10 minutes per station to solve the station problem.
 - The teacher will give students 2 minute warning per station before having students move to the next station.
 - Encourage students to work together to solve the problem as well as using the notes from their previous class.

Phase 2: (32 minutes)

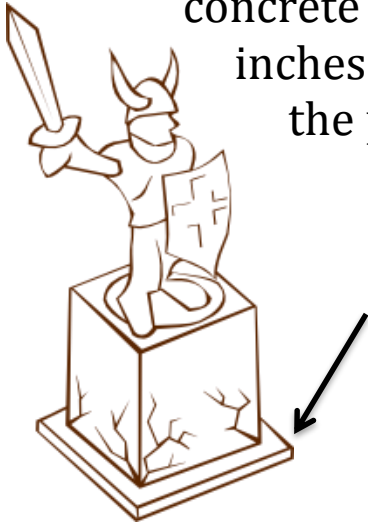
- Encourage students to refer to notes from previous classes if confused.
- Students will complete each station together showing all work.
- Students will move from station to station when teacher prompts to do so.
- Time spent at stations may be modified based on how well students are communicating and solving the stations together.

Phase 3: (6 minutes)

- Group discussion on applications of station problems:
 - Were you solving for just the volume/surface area of objects? Why or why not?
 - How does knowing the volume of an object apply to money management?
 - Does configuring volume of shapes help determine how much of an item you can fit inside it? Give examples (besides the station activity)

Station 1

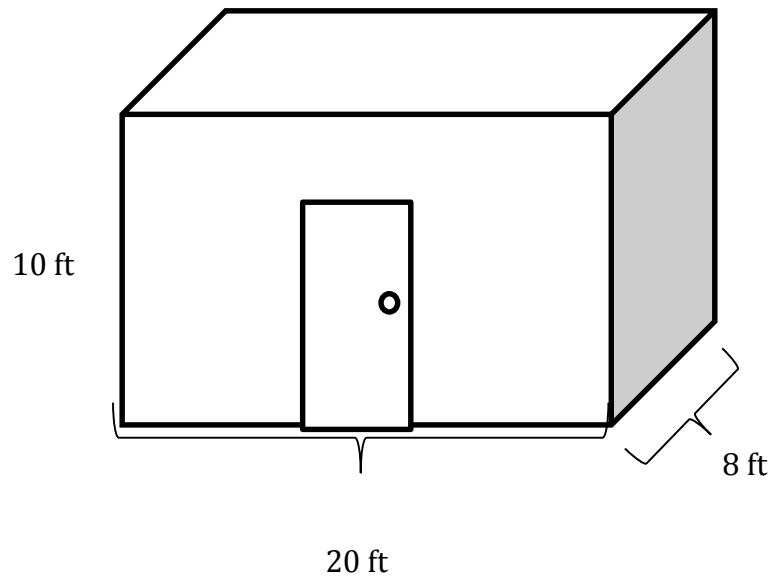
The pedestal on which a statue is raised is a rectangular concrete solid measuring 9 feet long, 9 feet wide and 6 inches high. How much is the cost of the concrete in the pedestal, if concrete costs \$8 per cubic foot?
(Hint: Use volume)



Station 2

Josh is painting the walls of his bedroom, as shown below.

- Josh will not be painting the floor, ceiling, or door.
- Josh's door is 3 feet by 8 feet.
- If one quart of paint covers 100 square feet, how many quarts will Josh need to buy? (Hint: Use surface area)



Station 3

Leroy Jenkins delivers muffins packed in their own little boxes in the shape of a cube, measuring 3 inches on each side. Leroy packs the individual muffin boxes into a larger box. The larger box is also in the shape of a cube, measuring 2 feet on each side. How many of the individual muffin boxes can fit into the larger box? (Hint: Use volume)

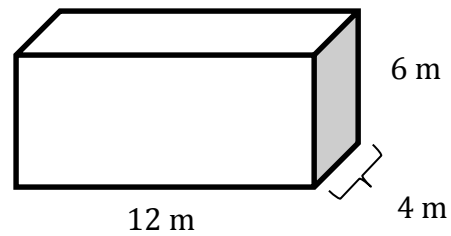
Name: _____

Date: _____

Day 12 Homework

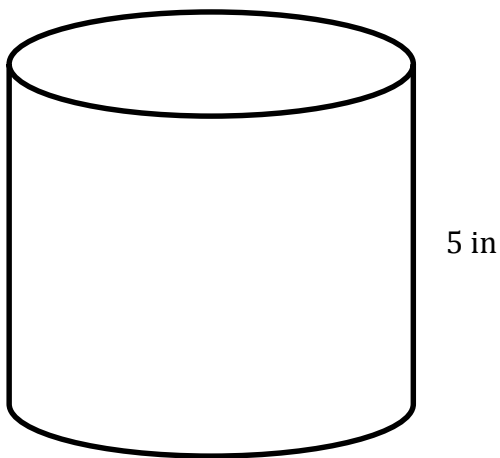
Volume

1. Find the volume of the following rectangular prism.



2. If a cylinder has a volume of $144\pi \text{ ft}^3$ and a radius of 6 ft, find the height of the shape.

3. Find the diameter of the following cylinder given that the volume is $125\pi \text{ inches}^3$.



Day 13 – Perimeter, Area, Surface Area, and Volume Review Day 1

To decrease extraneous cognitive load, students are provided with measurements within the diagrams given in the review game. Students are to work in groups and utilize their class notes from the unit (intrinsic). Students will have a better probability of reviewing the unit topics and being able to apply the concepts on the assessment with extraneous cognitive load lessened.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Pen/Pencil
- Whiteboard
- Student Whiteboards
- Markers
- Erasers
- Smartboard
- Calculator
- Desks arranged into groups

Vocabulary:

- Perimeter
- Circumference
- Surface Area
- π (pi)
- Radius
- Diameter
- Polygon
- Variable
- Cylinder

- Area
- Rectangle
- Square
- Cube
- Rectangular Solid
- Volume
- Length
- Width
- Height
- 3-dimensional

Objective:

- Draw a geometric picture when given information about a geometric shape with missing values.
- Solve for the circumference and area of circles.
- Solve for the perimeter and area of polygons.
- Solve for total area and shaded region area.
- Solve for surface area of 3-dimensional objects.
- Solve for volume of 3-dimensional objects.
- Identify missing value of a word problem and choose the correct formula for determining its value.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - 4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
 - 6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- **Explain volume formulas and use them to solve problems**
 - 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 - 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Phase 1: (5 minutes)

- Have students check homework answers written on the whiteboard as the teacher checks for completeness and possible common errors of the class.
- Review any common mistakes made by students if applicable.

Phase 2: (35 minutes)

- With whiteboards already distributed to every desk with the desks prearranged before class, students will sit at their predetermined seat.
- Remind students according to their unit outline that they have a test in two days.
- Students will be working in groups playing a Jeopardy review game for unit practice.
- Explain the following items to the students for the review game:
 - Every student in the class needs to solve the problem selected because:

- Every student in the group must have the correct answer to receive the points.
- If every player in the group does **not** have the correct answer, then **every other** team has the chance to get the points under the same conditions (no extra time will be allowed).
- If the team that chose the question obtains the correct answer, they will be able to receive a bonus \$100 if a randomly chosen student can verbally **explain** how they obtained their answer.
- The team with the most points after two days will receive bonus points on their exam.
- Normal class rules apply.

Day 14 – Perimeter, Area, Surface Area, and Volume Review Day 2

To decrease extraneous cognitive load, students are provided with measurements within the diagrams given in the review game. Students are to work in groups and utilize their class notes from the unit (intrinsic). Students will have a better probability of reviewing the unit topics and being able to apply the concepts on the assessment with extraneous cognitive load lessened.

Grade Level: 7th Grade

Duration: 40 minutes

Materials:

- Pen/Pencil
- Whiteboard
- Student Whiteboards
- Markers
- Erasers
- Smartboard
- Calculator
- Desks arranged into groups

Vocabulary:

- Perimeter
- Circumference
- Surface Area
- π (pi)
- Radius
- Diameter
- Polygon
- Variable
- Cylinder

- Area
- Rectangle
- Square
- Cube
- Rectangular Solid
- Volume
- Length
- Width
- Height
- 3-dimensional

Objective:

- Draw a geometric picture when given information about a geometric shape with missing values.
- Solve for the circumference and area of circles.
- Solve for the perimeter and area of polygons.
- Solve for total area and shaded region area.
- Solve for surface area of 3-dimensional objects.
- Solve for volume of 3-dimensional objects.
- Identify missing value of a word problem and choose the correct formula for determining its value.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - 4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
 - 6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- **Explain volume formulas and use them to solve problems**
 - 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 - 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Phase 1: (5 minutes)

- Have students collaboratively restate the rules of the review game.
- If some rules were not mentioned, simply remind the students of what is expected during today's class.
 - Every student in the class needs to solve the problem selected because:
 - Every student in the group must have the correct answer to receive the points.
 - If every player in the group does **not** have the correct answer, then **every other** team has the chance to get the points under the same conditions (no extra time will be allowed).

- If the team that chose the question obtains the correct answer, they will be able to receive a bonus \$100 if a randomly chosen student can verbally **explain** how they obtained their answer.
- The team with the most points after two days will receive bonus points on their exam.
- Normal class rules apply.

Phase 2: (30 minutes)

- With whiteboards already distributed to every desk with the desks prearranged before class, students will sit at their predetermined seat as they had sat yesterday.
- Remind students according to their unit outline that they have a test tomorrow.
- Students will be working in groups playing a Jeopardy review game for unit practice.

Phase 3: (5 minutes)

- If not reached already, determine a fair place to stop so that the students may complete the final jeopardy question.
- Give students 30 seconds to determine how much they want to wager, giving the total points of each team on the whiteboard.
- Present the final jeopardy question and determine the team winner.

Day 15 – Unit Test**Grade Level:** 7th Grade**Duration:** 40 minutes**Materials:**

- Pen/Pencil
- Calculator

Vocabulary:

- Perimeter
- Circumference
- Surface Area
- π (pi)
- Radius
- Diameter
- Polygon
- Variable
- Cylinder
- Area
- Rectangle
- Square
- Cube
- Rectangular Solid
- Volume
- Length
- Width
- Height
- 3-dimensional

Objective:

- Draw a geometric picture when given information about a geometric shape with missing values.

- Solve for the circumference and area of circles.
- Solve for the perimeter and area of polygons.
- Solve for total area and shaded region area.
- Solve for surface area of 3-dimensional objects.
- Solve for volume of 3-dimensional objects.
- Identify missing value of a word problem and choose the correct formula for determining its value.

Standards:

- **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
 - 4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
 - 6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **Explain volume formulas and use them to solve problems**
 - 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- **Solve real-world and mathematical problems involving area, surface area, and volume.**
 - 1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 - 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
 - 8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

- 1 Calculate the perimeter of the rectangle shown in the diagram.



- (1) 104.4 cm
 (2) 598.4 cm
 (3) 52.2 cm
 (4) 69.2 cm
- 2 The perimeter of a parallelogram is 32 meters and the two shorter sides measure 4 meters. What is the length of the longer sides?

- (1) 4 meters
 (2) 6 meters
 (3) 10 meters
 (4) 12 meters

- 3 What is the area of a trapezoid that has bases measuring 8 inches and 10 inches and a height of 3 inches?

- (1) 27 in^2
 (2) 18 in^2
 (3) 9 in^2
 (4) 120 in^2

- 4 The area of the larger circle is about 254.5 m^2 . The area of the smaller circle is about 78.5 m^2 . What is the area of the shaded region?



- (1) 333 m^2
 (2) 176 m^2
 (3) 4 m^2
 (4) 17 m^2
- 5 What is the diameter of a circle whose circumference is 5 in terms of π ?

- (1) 5π
 (2) $\frac{5}{\pi}$
 (3) $\frac{5}{\pi^2}$
 (4) $\frac{2.5}{\pi}$

6 What is the approximate circumference of a circle whose radius is 3?

(1) 7.07

(2) 9.42

(3) 18.85

(4) 28.27

7 Express in terms of π , the area of a circle whose radius is 6.

(1) 36π

(2) 12π

(3) 6π

(4) 36

8 The volume of a rectangular solid is 80 cubic centimeters, the length is 2 centimeters, and the width is 4 centimeters. What is the height of the rectangular solid?

(1) 5 centimeters

(2) 6 centimeters

(3) 10 centimeters

(4) 20 centimeters

9 Henry made a cube in technology class. Each edge measured 1.5 cm. What is the volume of the cube in cubic centimeters?

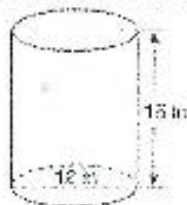
(1) 2.25

(2) 3.375

(3) 9.0

(4) 13.5

10 What is the volume of this container to the nearest tenth of a cubic inch?



(Not drawn to scale)

(1) $6,785.8 \text{ in}^3$

(2) $4,241.2 \text{ in}^3$

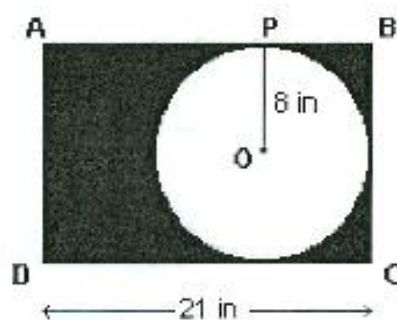
(3) $2,160.0 \text{ in}^3$

(4) $1,696.5 \text{ in}^3$

PART III

Answer all questions in this part. Each correct answer will receive 3 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

- 11 In the diagram, circle O is inscribed in rectangle $ABCD$. Radius OP is drawn to AB . Find the area of the shaded region. *Leave your answer in terms of π .*

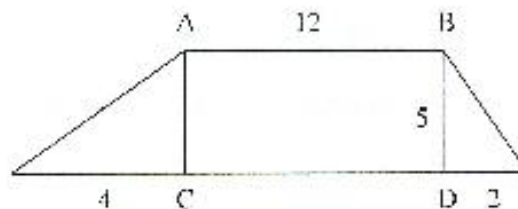


- 12 Mrs. Ayer is painting the outside of her son's toy box, including the top and bottom. The length, width, and height of the toy box are $x - 4$, 5, and 3 binomial in terms of x ?

PART IV

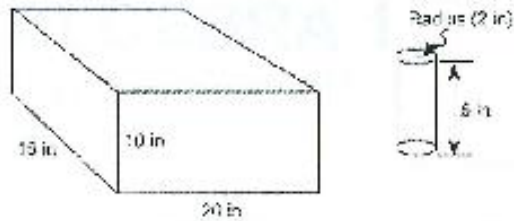
Answer all questions in this part. Each correct answer will receive 4 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

- 13 In the diagram, $ABCD$ is rectangle with a right triangle at either side.



Find the area of the entire figure. (Hint: What shapes make up this figure?)

14 In the diagram below, the rectangular container is to be filled with water using the cylindrical cup.



- Find the volume of the rectangular container.
- Find the volume of the cylindrical cup the *nearest tenth*.
- What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?

Chapter 4: Validity of Curriculum Project

This curriculum project was submitted to a veteran teacher currently teaching the specified grade level. The mathematics teacher has been working with the grade level of the curriculum for most of his career and is adequately valid to make such feedback. It is with this that the author is certain that the feedback the teacher portrays will provide content validity of the curriculum project.

The teacher provided feedback in two sections. The first was explaining the overall curriculum design itself as the second section portrayed the teacher's feedback on specific lessons of the curriculum. The first section is as follows:

Overall, I feel this is a solid well-thought out unit plan, with many positives and I offer a few suggestions for improvement as well. The unit plan emphasizes scaffolding and cloze notes, which are important as lead-ins to each lesson. I would suggest beginning with a general "What does area mean" lesson or half-lesson, where the students are introduced to area. This may help their overall success for this plan for the lessons incorporating area, because I have noticed that many 7th grade students do not have a solid concept of what area is. They just want to multiply numbers in a lot of cases, but this "concept" on their part leads to confusion when they are asked to solve for perimeter, and even volume.

The unit plan positively stresses practicing the prior lessons by using stations the following days. This is a good way to see first-hand if the students do grasp the important concepts. A suggestion that I believe might help, regarding Day 1's lesson would be to move the area of rectangular shapes prior to the area of a circle, since 7th graders seem to remember that concept much more than anything relative to a circle. Once the area of rectangular shapes are mastered, then move on to circular areas. Another suggestion for that transition, from rectangles to circles, is to review the differences between those shapes and discuss the derivation of the area of the circle and how pi was derived. For question number 4 on the homework, I would suggest to make clear what region that you are asking for the area of. I have found that students tend to think, when there is any shaded region in a figure, that the shaded region is the specific region or shape that you are asking about.

The second section of feedback was more specific to each lesson with a concluding paragraph. This section of feedback is as follows:

For Day 2, I would suggest scaffolding a couple examples, where students have to do conversions prior to using conversions in Day 2's exercises. For Day 3's quiz, I would include a problem then on conversion, since Day 2 included that skill. Also, it may be a good idea to scaffold where the formula for a right triangle comes from (even something as simple as a right triangle is half of a rectangle). As a side note, for the figure for number 2 on the homework, it shows 2 rectangles. I would suggest labeling specifically which rectangle you are referring to.

For Day 5's lesson, I liked how the lesson built upon using the area of the individual shapes that comprise the composite figures. I suggest in addition to include an example that shows how the composite figures can be sliced up into different shapes, different ways, and the total area can still be calculated. This would show the students that there is not just one way to solve each problem.

For the stations activity in Day 6, I like that the students have a variety of composite shapes to solve for. This will give the students a much better base of understanding for the topic. In Day 6, I see 5 stations planned where they have an overall time of 31 minutes. I would suggest paring down to perhaps 4, just so that students have enough time to spend on each station.

For Quiz #2, as part of Day 8, I recommend adding a few questions at the beginning of the quiz, where students are asked to fill in the formulas for area of a triangle, circle, and semi-circle. I have found this to be helpful, so that prior to the students proceeding with the quiz, that you check their answers for the formulas, so you can make sure they are off to a good start.

Another positive about this plan, is that you mix types of problems well on both the lesson examples/exercises and on the quiz. Too often do teachers tend to overemphasize problems where there are just values given, instead of mixing in real-life word problems, where students can see the importance of math AND you can assess their understanding of such problems.

For Day 11, in addition to the good level of real-life problem emphasis, I would recommend demonstrating what volume is to the class, with some sort of small demo that should help those confused with the concept understand what volume really is. Also, for this lesson I would recommend cutting it down a bit time-wise based on the need for the students to think through each aspect of the exercises.

In summary, as I said, this unit plan has many positives, including its overall structure (proceeding from the more basic – area of circles, to volume of solids) as well as the mix of lecture/cloze notes and then the use of stations on days following those lectures, and the good mix of basic problems (problems based on given values) and real-world problems. Having already covered most of this material myself this year, I found both useful examples/exercises for my future lessons in reviewing this plan and it helped me to consider and evaluate my own teaching and approach to this topic and I will be able to use some of this plan as new ideas for future plans of my own.

Chapter 5: Discussion, Summary, and Reflection

The inspiration in the design of this curriculum project was to have a developed unit that aligned with the CCSS; contained lessons designed with CLT in mind, and used the research of problem solving to develop future mathematical thinkers. Developing this curriculum aligned with the CCSS and a focus on CLT can provide teachers of the specified grade level an opportunity for a variety of content. Connecting each lesson of the unit to its use of the ideas of CLT may provide teachers with the psychological aspects of learning. Implementing CLT with every lesson can support teachers in their search for curriculum that reaches individual student needs.

This curriculum is different from the resources used to design the curriculum in a variety of ways. The most notable difference in this curriculum from the resources used to create it is the implementation of CLT in every lesson. Designing a curriculum with CLT provides teachers with another strategy of designing and implementing lessons. The resources and content of the unit were designed solely from the new CCSS. The paradigm shift of national standards from NCTM to CCSS has lessened the amount of resources available to teachers pertaining to the new standards. As a result, it is imperative that more curriculums are designed with the CCSS.

CLT is especially important in mathematics because of the amount of content and complex tasks presented within each topic. Mathematics requires higher-level thinking and the “cognitive load imposed by such tasks is often excessive for novice learners and may seriously hamper learning” (Van Merriënboer & Sweller, 2005, p. 152). Thus developing a curriculum with CLT recognizes the load of complex tasks and minimizes the extraneous load to optimize student learning.

For future consideration, this unit could be improved in a variety of ways. Taking in the feedback from the mathematics teacher in Chapter Four, a main concern brought to the author's attention is the prerequisite knowledge required of students at the specific grade level. The teacher stated that the prerequisite skills required by students to complete some of the complex tasks in the unit may need more attention for students to successfully apply them. Additionally, the order at which the unit plan was scaffolded per lesson was in question. The teacher suggested that some lessons be switched before others for better student understanding of some of the future lesson topics.

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Appendix

Name: _____
Day 1 Notes

Date: _____

Circles: Circumference and Area

- Circumference - The distance around the outside of a circle.

$$C = \pi d \quad \text{"Cherry pie's delicious"}$$

- Area - The number of square units needed to cover a given area.

$$A = \pi r^2 \quad \text{"Apple pies are too"}$$

- Diameter - the distance across the widest part of the circle.
- Radius - the distance from the center of the circle to an outside edge (Half the diameter).

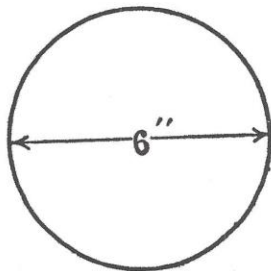
o <http://www.mathopenref.com/radius.html>

- In terms of pi - π is still in your answer. This is an exact answer.

Example 1: Find the **circumference** of the circle below.

a) Leave your answer in **terms of π** .

b) Round your answer to the **nearest tenth**.



$$\text{Diameter} = 6''$$

$$C = \pi d$$

$$C = \pi(6)$$

$$C = 6\pi \text{ in} \quad \leftarrow \text{(a)}$$

$$C = 18.8 \text{ in} \quad \leftarrow \text{(b)}$$

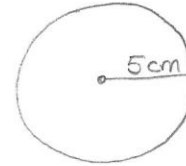
Example 2: Find the **circumference** of a circle with a **radius** of 5 cm.

a) Leave your answer in **terms of pi**.

$$C = 10\pi \text{ cm}$$

b) Round your answer to the **nearest tenth**.

$$C = 31.4 \text{ cm}$$

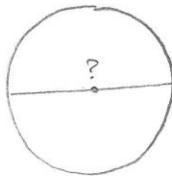


$$C = \pi d$$

$$C = \pi(10)$$

$$C = 31.4$$

Example 3: What is the **diameter** of a circle with a **circumference** of 12cm? (*Hint: Work backwards*)



$$C = \pi d$$

$$\frac{12}{\pi} = \frac{\pi d}{\pi}$$

$$d = \frac{12}{\pi} \text{ cm}$$

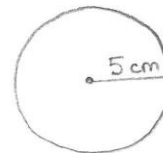
Example 4: Find the **area** of a circle with a **radius** of 5cm.

a) Leave your answer in **terms of pi**.

$$A = 25\pi \text{ cm}^2$$

b) Round to the **nearest hundredth**.

$$A = 78.54 \text{ cm}^2$$



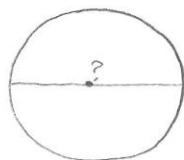
$$A = \pi r^2$$

$$A = \pi(5)^2$$

$$A = 25\pi$$

$$A = 78.54$$

Example 5: Find the **diameter** of the circle with an **area** of 36 units².



$$A = \pi r^2$$

$$36 = \pi r^2$$

$$\sqrt{\frac{36}{\pi}} = \sqrt{r^2}$$

$$3.39 = r$$

$$d = 2r = 6.78 \text{ units}$$

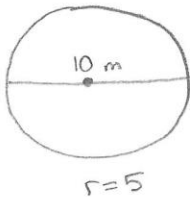
Name: _____

Date: _____

Day 1 Homework

Circles: Circumference and Area

1. What is the area of a circle with a diameter of 10 meters *in terms of π* ?

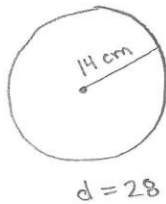


$$A = \pi r^2$$

$$A = \pi(5)^2$$

$$A = 25\pi \text{ m}^2$$

2. What is the circumference of a circle with a radius of 14 centimeters? (*Round to the nearest tenth*)

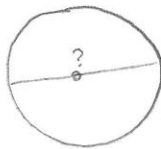


$$C = \pi d$$

$$C = \pi(28)$$

$$C = 88.0 \text{ cm}$$

3. Find the diameter of a circle with an area of 81π . (*Hint: Radius is $\frac{1}{2}$ the diameter*)



$$A = \pi r^2$$

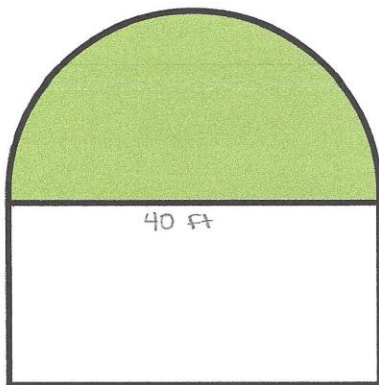
$$81\pi = \pi r^2$$

$$81 = r^2$$

$$r = 9$$

$$d = 18 \text{ units}$$

4. Suppose your back yard is a half circle like the picture shown below. How much feet of fencing will you need to buy to line the outside of your yard if the diameter of the half circle is 40 feet? (*Round your answer to the nearest tenth*)



$$C = \pi d$$

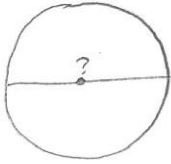
$$C = \pi(40)$$

$$C = 125.7 \text{ ft}$$

$$\frac{125.7}{2} = 62.8 \text{ ft}$$

5. Given the circumference of a circle is 120π , find the following:

a) The diameter of the circle.



$$\begin{aligned}C &= \pi d \\120\pi &= \pi d \\d &= 120 \text{ units}\end{aligned}$$

b) The radius of the circle.

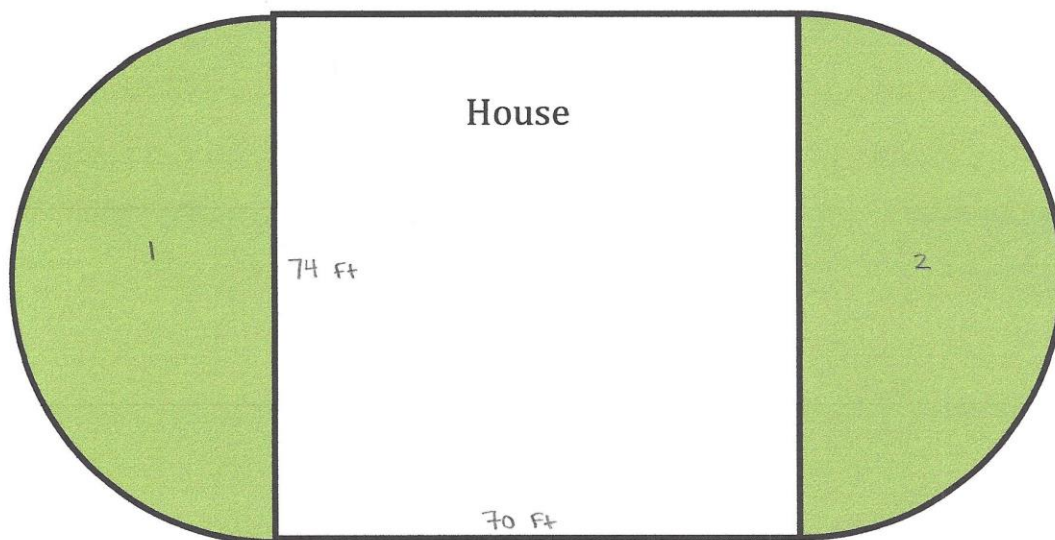
$$\frac{120}{2} = 60 \text{ units}$$

c) The area of the circle (*In terms of π*).

$$\begin{aligned}A &= \pi r^2 \\A &= \pi(60)^2 \\A &= 3600\pi \text{ units}^2\end{aligned}$$

Station 1

John has to mow his lawn after school today with the measurements in the picture. If his lawn mower can mow an area of 500 feet² on one tank of gas, how many tanks of gas does he need to mow his entire lawn?



$$\begin{aligned} \frac{1}{A} &= \pi r^2 \cdot \frac{1}{2} \\ A &= \pi(37)^2 \cdot \frac{1}{2} \\ A &= (4300.8) \cdot \frac{1}{2} \\ A &= 2150.4 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \frac{2}{A} &= \pi r^2 \cdot \frac{1}{2} \\ A &= \pi(37)^2 \cdot \frac{1}{2} \\ A &= (4300.8) \left(\frac{1}{2}\right) \\ A &= 2150.4 \text{ ft}^2 \end{aligned}$$

$$\text{Total Lawn to mow} = 2150.4 (2) = 4300.8 \text{ ft}^2$$

$$\frac{4300.8}{500} = 8.6 \text{ tanks}$$

9 tanks

Station 2

Jessica is making a circular tablecloth for an art project. She wants half of the cloth to be a plain colored fabric and half to be a print fabric. How many square yards of each fabric (to the *nearest hundredth* of a yard) will she actually be using if the diameter of the cloth is 6 feet?

Print

$$A = \pi r^2 \cdot \frac{1}{2}$$

$$A = \pi(1)^2 \cdot \frac{1}{2}$$

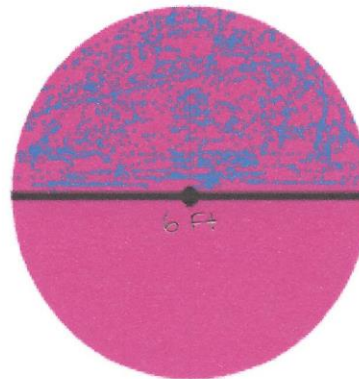
$$A = 1.57 \text{ yd}^2$$

Plain

$$A = \pi r^2 \cdot \frac{1}{2}$$

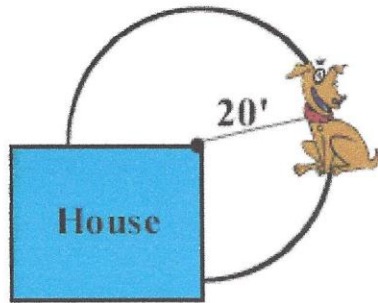
$$A = \pi(1)^2 \cdot \frac{1}{2}$$

$$A = 1.57 \text{ yd}^2$$



$$\underline{6 \text{ Ft} = 2 \text{ yards}}$$

Station 3



$$r = 20 \text{ ft}$$

$$A = \pi r^2$$

$$A = \pi \cdot (20)^2$$

$$A = 400\pi \text{ ft}^2$$

$$A = 1256.64 \text{ ft}^2$$

Butch, the dog, is leashed to the corner of the house when he is outdoors alone. The leash is 20 feet long. Find the amount of ground area available to Butch when leashed outdoors.

Name: _____

Date: _____

Quiz 1

1. What is the circumference of a circle with a diameter of 12 cm in terms of
- π
- ?

$$C = \pi d$$

$$C = \pi(12)$$

$$C = 12\pi \text{ cm}$$

2. What is the radius of a circle with an area of 201.6 m
- ²
- ? (Round your answer to the nearest tenth)

$$A = \pi r^2$$

$$201.6 = \pi r^2$$

$$\sqrt{r^2} = \sqrt{64.2}$$

$$r = 8.0 \text{ m}$$

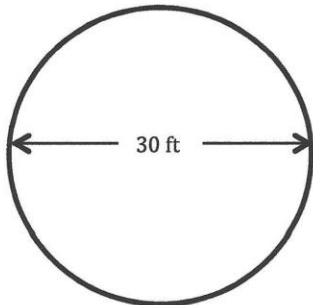
3. Find the diameter of a circle if the circumference is 41 inches. (Round to the nearest hundredth)

$$C = \pi d$$

$$41 = \pi d$$

$$d = 13.05 \text{ in}$$

4. Find the area of the circle shown below to the nearest tenth.



$$A = \pi r^2$$

$$A = \pi(15)^2$$

$$A = 225\pi$$

$$A = 706.9 \text{ ft}^2$$

5. What is the area of a circle with a diameter of 12 miles? (Round your answer to the nearest hundredth)

$$A = \pi r^2$$

$$A = \pi(6)^2$$

$$A = 36\pi$$

$$A = 113.10 \text{ mi}^2$$

Name: _____
 (Day 3) Notes

Date: _____

Polygons: Perimeter and Area

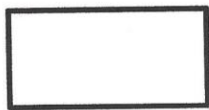
- **Perimeter**- The distance around the outside of a shape.
 - (Circumference was the distance around a circle).
- **Area**- The number of square units needed to cover a given area.

Rectangle/Square

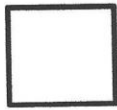
Triangle

Trapezoid

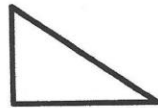
Parallelogram



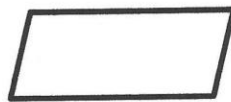
$A = bh$



$A = s^2$



$A = \frac{1}{2}bh$

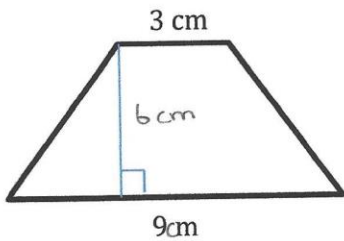


$A = bh$



$A = \frac{1}{2}(b_1 + b_2)h$

Example 1: Find the **area** of the figure pictured below.



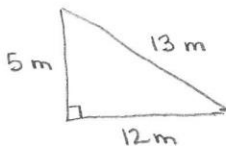
$A = \frac{1}{2}(3+9) \cdot 6$

$A = \frac{1}{2}(12) \cdot 6$

$A = 6 \cdot 6$

$A = 36 \text{ cm}^2$

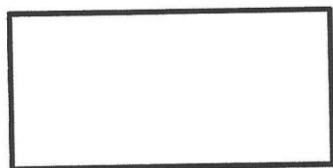
Example 2: Find the **perimeter** of a right triangle with legs of length 5m and 12m and a hypotenuse of 13m.



$P = 5 \text{ m} + 12 \text{ m} + 13 \text{ m}$

$P = 30 \text{ m}$

Example 3: Find the **area** of the figure pictured below.



8 cm

11 cm

$A = bh$

$A = 11(8)$

$A = 88 \text{ cm}^2$

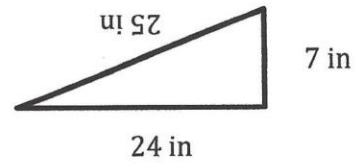
Example 4: Find the **area** of the right triangle below.

$$A = \frac{1}{2}bh$$

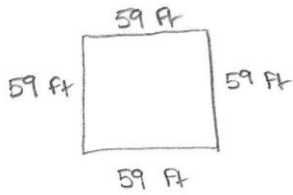
$$A = \frac{1}{2}(24)(7)$$

$$A = (12)(7)$$

$$A = 84 \text{ in}^2$$



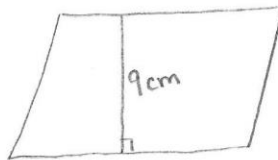
Example 5: What is the **perimeter** of a square parking lot knowing one side measures 59 ft?



$$P = 4(59)$$

$$P = 236 \text{ ft}$$

Example 6: Find the base of a parallelogram knowing that the height is 9cm and the **area** is 72 cm².



$$A = bh$$

$$72 = b(9)$$

$$b = 8 \text{ cm}$$

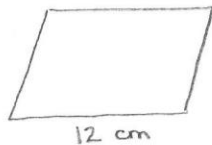
Name: _____

Date: _____

(Day 3) Homework

Polygons: Perimeter and Area

1. Find the height of a parallelogram knowing the base is 12 cm and the area is 144 cm².

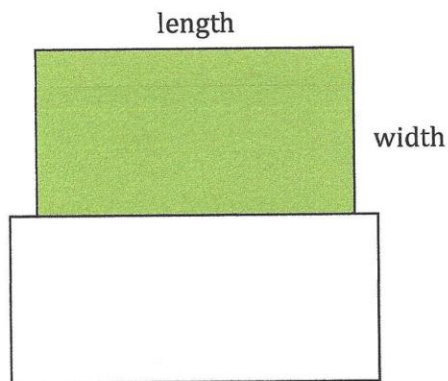


$$A = bh$$

$$144 = 12h$$

$$h = 12 \text{ cm}$$

2. Suppose you have a rectangular backyard as shown in the following picture. If the length of your yard is 42 feet and the width is 34 feet, how much fence is required to outline your yard?



$$P = \text{Perimeter of yard} - \text{house side}$$

$$= 42 + 34 + 34$$

$$= 110 \text{ Ft of Fence}$$

3. What is the height of a triangle with an area of 32 in² and a base of 8 in?



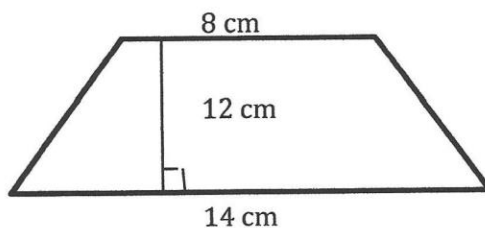
$$A = \frac{1}{2}bh$$

$$32 = \frac{1}{2}(8)h$$

$$32 = 4h$$

$$h = 8 \text{ in}$$

4. What is the area of the following trapezoid?



$$A = \frac{1}{2}(b_1 + b_2)h$$

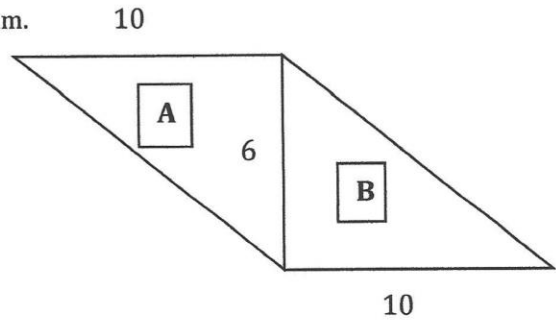
$$A = \frac{1}{2}(14 + 8) \cdot 12$$

$$A = \frac{1}{2}(22) \cdot 12$$

$$A = 11 \cdot 12$$

$$A = 132 \text{ cm}^2$$

5. Solve the following questions based on the diagram.



a) Find the area of triangle A.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(10)$$

$$A = 30 \text{ units}^2$$

b) Find the area of triangle B.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10)(6)$$

$$A = 30 \text{ units}^2$$

c) Find the area of the parallelogram. (Hint: $A = bh$)

$$A = bh$$

$$A = 10(6)$$

$$A = 60 \text{ units}^2$$

d) Add the areas of triangle A and B together.

$$\triangle A + \triangle B = 30 + 30$$

$$= 60 \text{ units}^2$$

e) What do you notice about your answers to (c) and (d)?

They're equal.

(Results vary on class)

Name: _____

Name: _____

1. Calculate the area of the room. (Hint: Each floor tile is 1x1 foot)

$$\begin{aligned} l &= 20 \text{ Ft} & A &= lw \\ w &= 32 \text{ Ft} & A &= 20(32) \\ & & A &= 640 \text{ Ft}^2 \end{aligned}$$

2. Calculate the area of the ceiling. (Hint: Each ceiling tile is 2x2 feet)

$$\begin{aligned} l &= 10 \text{ tiles} \cdot 2 = 20 \text{ Ft} & A &= lw \\ w &= 16 \text{ tiles} \cdot 2 = 32 \text{ Ft} & A &= 20(32) \\ & & A &= 640 \text{ Ft}^2 \end{aligned}$$

3. How many times more tiles are there on the floor than the ceiling. (Hint: Divide answer 1 by answer 2)

Two times more tiles on the floor than the ceiling.

4. Calculate the perimeter of the room.

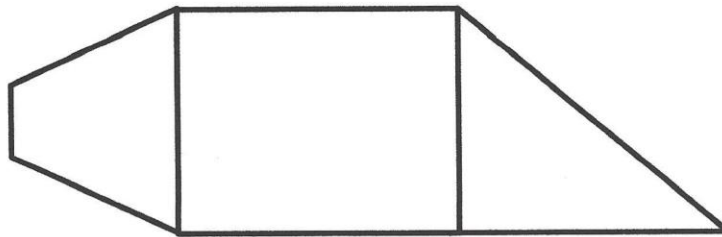
$$\begin{aligned} P &= 20 + 32 + 20 + 32 \\ &= 104 \text{ Ft} \end{aligned}$$

Name: _____
 (Day 5) Notes

Date: _____

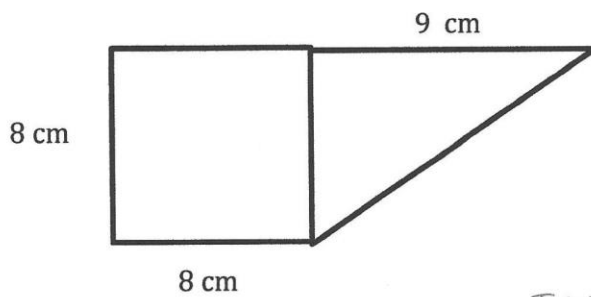
Total Area

- Total Area- To find the total area of a figure, add the areas of the inner figures together.



$$TA = A = \frac{1}{2}(b_1 + b_2)h + A = bh + A = \frac{1}{2}bh$$

Example 1: Find the total area of the figure below.



Square

Triangle

$$A = bh$$

$$A = \frac{1}{2}bh$$

$$A = (8\text{ cm})(8\text{ cm})$$

$$A = \frac{1}{2}(9)(8)$$

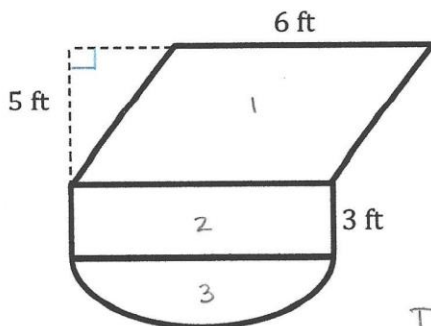
$$A = 64\text{ cm}^2$$

$$A = \frac{1}{2}(72)$$

$$A = 36\text{ cm}^2$$

$$\begin{aligned} \text{Total Area} &= 64\text{ cm}^2 + 36\text{ cm}^2 \\ &= \boxed{100\text{ cm}^2} \end{aligned}$$

Example 2: Find the total area of the figure below (Round to the nearest tenth).



1

2

3

$$A = bh$$

$$A = bh$$

$$A = \pi r^2 \cdot \frac{1}{2}$$

$$A = 6(5)$$

$$A = 6(3)$$

$$A = \pi(3)^2 \cdot \frac{1}{2}$$

$$A = 30\text{ ft}^2$$

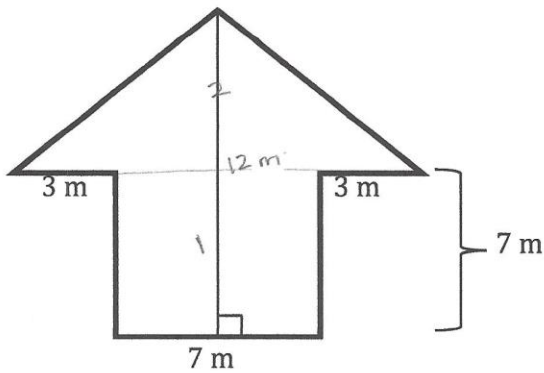
$$A = 18\text{ ft}^2$$

$$A = 4.5\pi$$

$$A = 14.14\text{ ft}^2$$

$$\begin{aligned} TA &= 30 + 18 + 14.14 \\ &= 62.14\text{ ft}^2 \end{aligned}$$

Example 3: Find the total area of the shape below.



1

$$A = bh$$

$$A = 7(7)$$

$$A = 49 \text{ m}^2$$

2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(7)(12)$$

$$A = 32.5 \text{ m}^2$$

$$TA = 49 + 32.5$$

$$= 81.5 \text{ m}^2$$

Example 4: Consider the irregular shape shown below.

(a) Find the exact area of the irregular shape.

1

$$A = bh$$

$$A = 12(16)$$

$$A = 192 \text{ u}^2$$

2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(16)(12)$$

$$A = 96 \text{ u}^2$$

3

$$A = bh$$

$$A = 12(16)$$

$$A = 192 \text{ u}^2$$

4

$$A = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2}\pi(10)^2$$

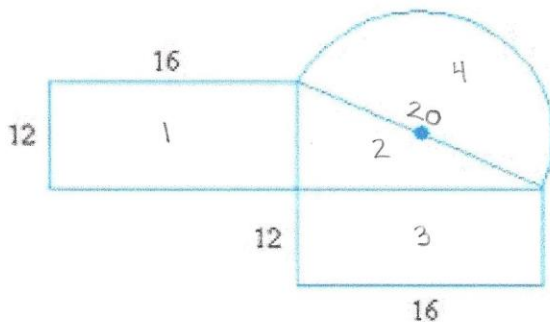
$$A = 50\pi \text{ u}^2$$

$$TA = 192 + 96 + 192 + 50\pi$$

$$= (480 + 50\pi) \text{ u}^2$$

(b) Find the area to the nearest hundredth.

$$TA = 637.08 \text{ u}^2$$

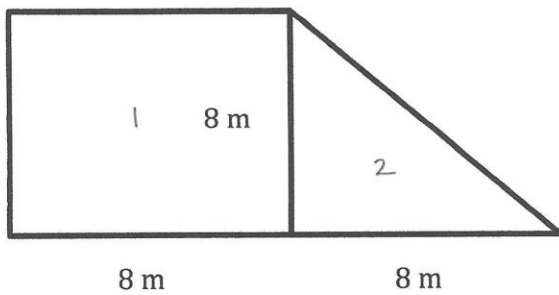


Name: _____
 (Day 5) Homework

Date: _____

Total Area

1. Find the total area of the figure below.



$$\begin{aligned} \underline{1} \\ A &= bh \\ A &= 8 \cdot 8 \\ A &= 64 \text{ m}^2 \end{aligned}$$

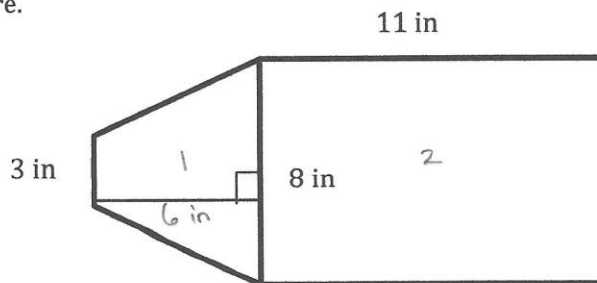
$$\begin{aligned} \underline{2} \\ A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(8)(8) \\ A &= 32 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} TA &= 64 + 32 \\ &= 96 \text{ m}^2 \end{aligned}$$

2. Find the total area of the following figure.

$$\begin{aligned} \underline{1} \\ A &= \frac{1}{2}(b_1 + b_2)h \\ A &= \frac{1}{2}(8 + 3)(6) \\ A &= \frac{1}{2}(11)(6) \\ A &= 33 \text{ in}^2 \end{aligned}$$

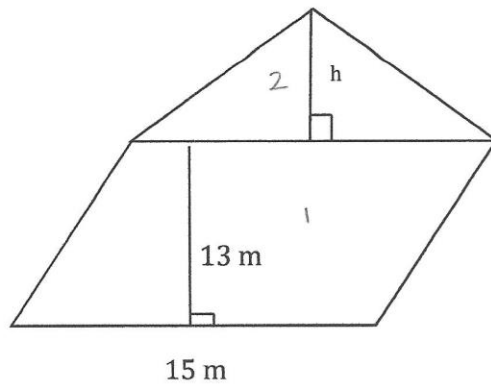
$$\begin{aligned} \underline{2} \\ A &= bh \\ A &= 8(11) \\ A &= 88 \text{ in}^2 \end{aligned}$$



$$\begin{aligned} TA &= 88 + 33 \\ &= 121 \text{ in}^2 \end{aligned}$$

(One more on the back!)

3. Find the height of the triangle given the following measurements and the total area of the figure being 240 ft².



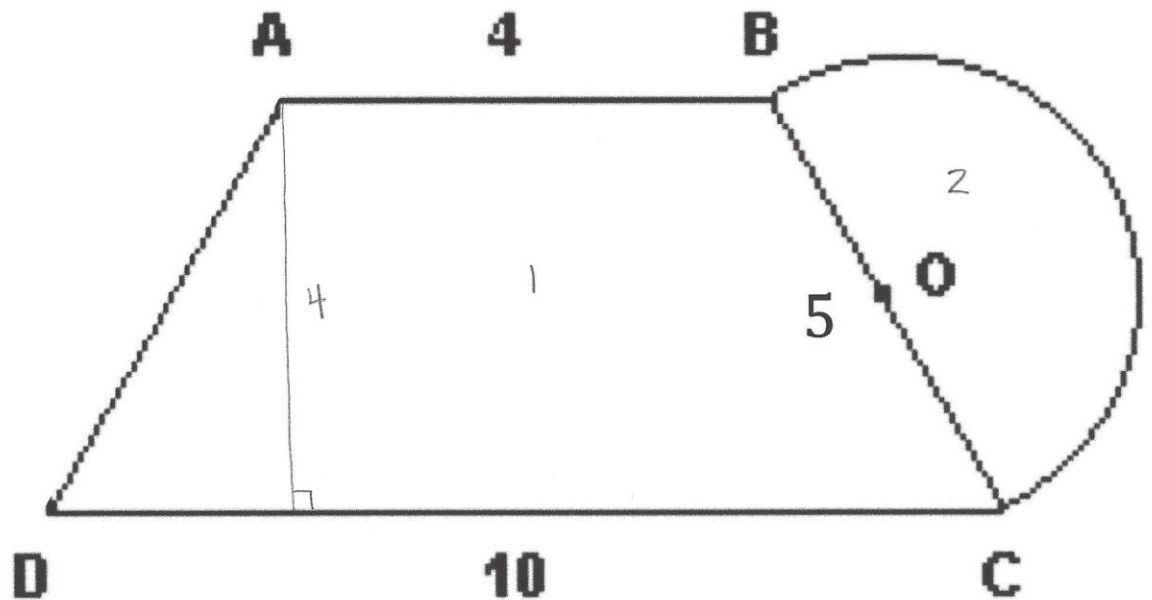
$$\begin{aligned} \underline{1} \\ A &= bh \\ A &= 13(15) \\ A &= 195 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \underline{2} \\ A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(15)h \\ A &= 7.5h \end{aligned}$$

$$\begin{aligned} TA &= A1 + A2 \\ 240 &= 195 + 7.5h \\ 45 &= 7.5h \\ h &= 6 \text{ m} \end{aligned}$$

STATION 1

- Find the total area.
- Leave your answer in terms of π .



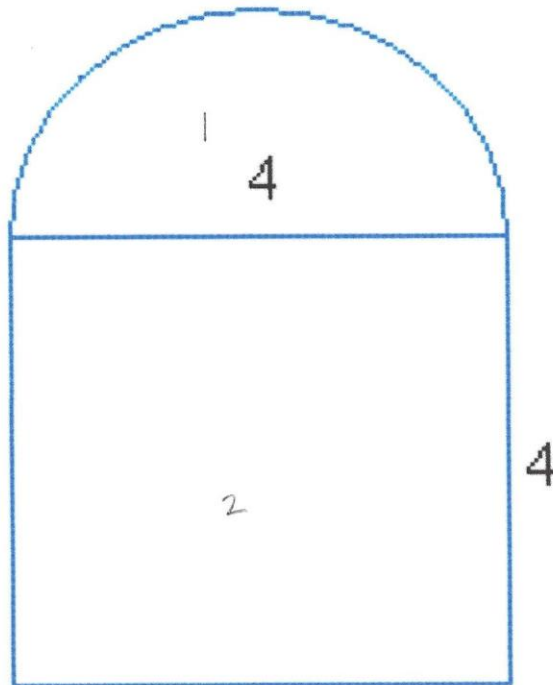
$$\begin{aligned} \underline{1} \\ A &= \frac{1}{2}(b_1 + b_2)h \\ A &= \frac{1}{2}(10 + 4) \cdot 4 \\ A &= \frac{1}{2}(14) \cdot 4 \\ A &= 28 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} \underline{2} \\ A &= \pi r^2 \cdot \frac{1}{2} \\ A &= \pi(2.5)^2 \cdot \frac{1}{2} \\ A &= \pi(6.25) \cdot \frac{1}{2} \\ A &= 3.125\pi \text{ units}^2 \end{aligned}$$

$$TA = 28 + 3.125\pi \text{ units}^2$$

STATION 2

- Find the total area.
- Leave your answer in terms of π .



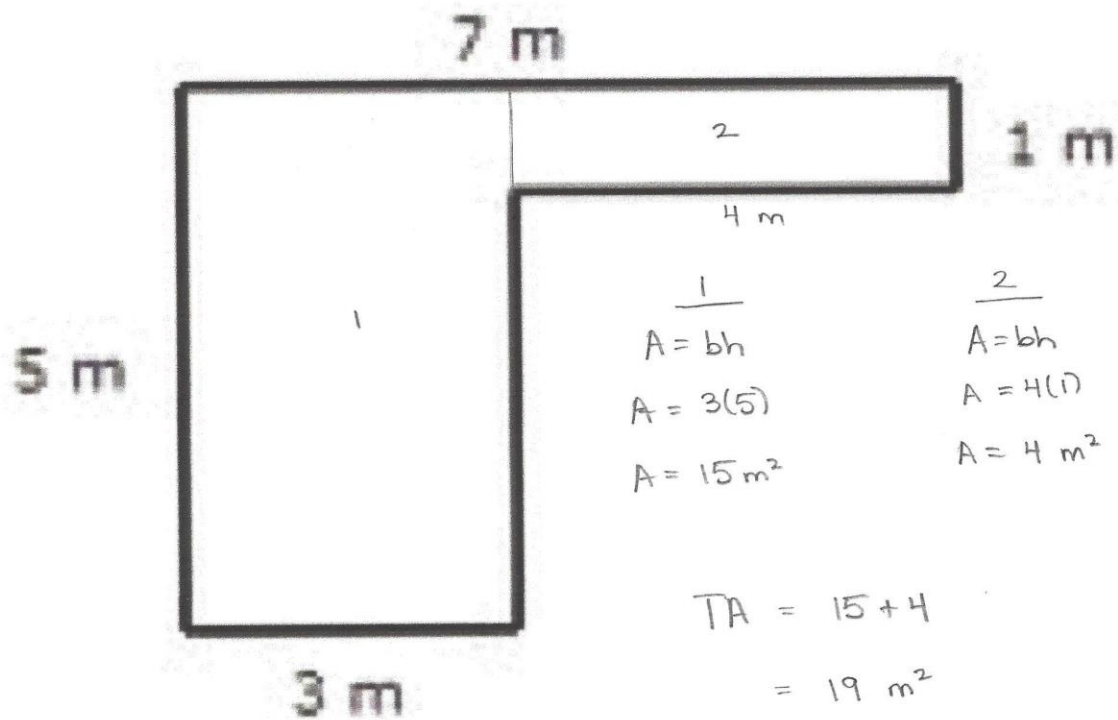
$$\begin{aligned} \underline{1} \\ A &= \pi r^2 \cdot \frac{1}{2} \\ A &= \pi (2)^2 \cdot \frac{1}{2} \\ A &= 2\pi \text{ u}^2 \end{aligned}$$

$$\begin{aligned} \underline{2} \\ A &= bh \\ A &= 4(4) \\ A &= 16 \text{ u}^2 \end{aligned}$$

$$TA = (16 + 2\pi) \text{ units}^2$$

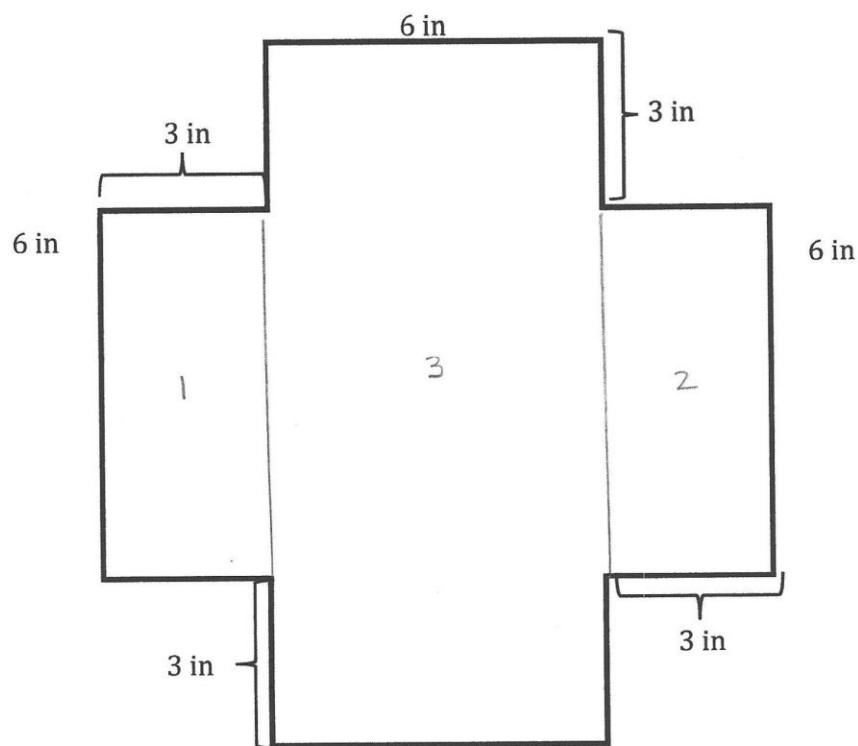
STATION 3

- Find the total area.



STATION 4

- Find the total area.



$$\begin{aligned} \underline{1} \\ A &= bh \\ A &= 3(6) \\ A &= 18 \text{ in}^2 \end{aligned}$$

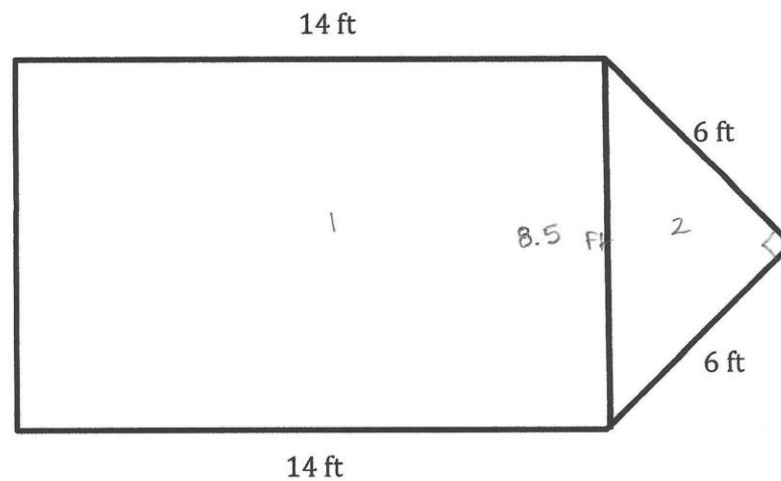
$$\begin{aligned} \underline{2} \\ A &= bh \\ A &= 3(6) \\ A &= 18 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \underline{3} \\ A &= bh \\ A &= 6(3+3+6) \\ A &= 6(12) \\ A &= 72 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} TA &= (1) + (2) + (3) \\ &= 18 + 18 + 72 \\ &= \boxed{108 \text{ in}^2} \end{aligned}$$

STATION 5

- Find the total area.
- Round your answer to the nearest tenth



$$\begin{aligned} \frac{1}{A} &= bh \\ A &= 14(8.5) \\ A &= 127.5 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} \frac{2}{A} &= \frac{1}{2}bh \\ A &= \frac{1}{2}(6)(6) \\ A &= 18 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} TA &= (1) + (2) \\ &= 127.5 + 18 \\ &= 145.5 \text{ ft}^2 \end{aligned}$$

Name: _____
(Day 7) Notes

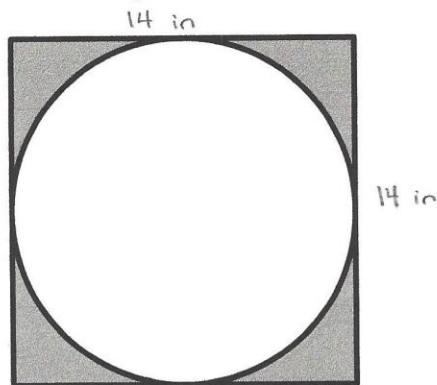
Date: _____

Area of Shaded Regions

Area of shaded region = Area of larger figure - Area of smaller figure

Find the **area** of the **shaded region** in each of the following figures.

Example 1: In this figure, a circle is drawn in a square whose side has a length of 14 inches.



- 1.) Find the **area** of the **square**.

$$A = bh$$

$$A = 14(14)$$

$$A = 196 \text{ in}^2$$

- 2.) Find the **area** of the **circle**.

$$A = \pi r^2$$

$$A = \pi(7)^2$$

$$A = \pi(49) = 49\pi \text{ in}^2$$

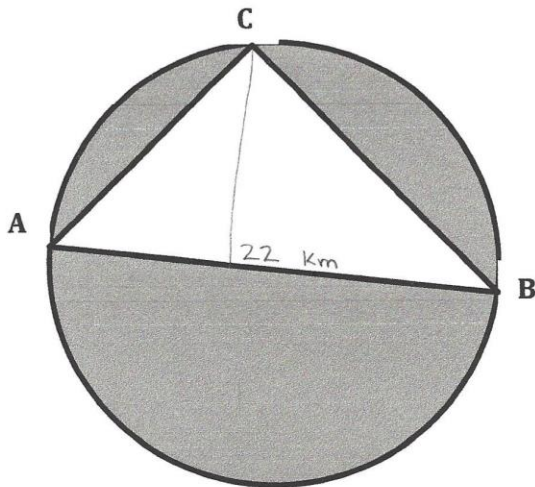
- 3.) Using the results from part 1 and 2, express the **area** of the **shaded portion**.

$$A = (1) - (2)$$

$$A = 196 - 49\pi$$

$$A = 42.1 \text{ in}^2$$

Example 2: In this figure, **AB is the diameter** of the circle. Find the **area of the shaded region**.



1.) Find the **area of the circle**.

$$A = \pi r^2$$

$$A = \pi (11)^2$$

$$A = 380.1 \text{ km}^2$$

2.) Find the **area of the triangle**.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(22)(11)$$

$$A = 121 \text{ km}^2$$

3.) Using the results from part 1 and 2, express the **area of the shaded portion**.

$$\begin{aligned} A &= (1) - (2) = 380.1 - 121 \\ &= 259.1 \text{ km}^2 \end{aligned}$$

Question! Why does the method of subtracting the area of the smaller figure from the area of the larger figure determine the area of the shaded region?

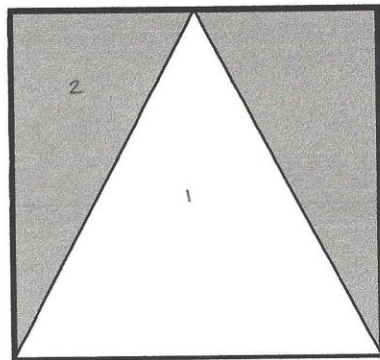
This method determines the area of the shaded region because the smaller figure is completely within the larger shaded shape therefore subtracting the smaller from the larger takes away the square units that are not shaded.

Name: _____
 (Day 7) Homework

Date: _____

Area of Shaded Regions

1. Find the area of the following shaded region.



12 cm

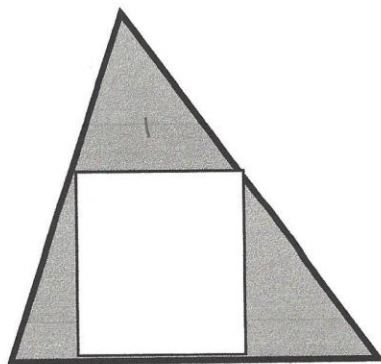
13 cm

$$\begin{aligned} \underline{1} \\ A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(12)(13) \\ A &= 6(13) \\ A &= 78 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \underline{2} \\ A &= bh \\ A &= 12(13) \\ A &= 156 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Shaded } A &= (2) - (1) \\ &= 156 - 78 \\ &= 78 \text{ cm}^2 \end{aligned}$$

2. Find the area of the shaded shape below if the height of the triangle is 19 inches, the base is 18 inches, and the area of the square is 121 inches².

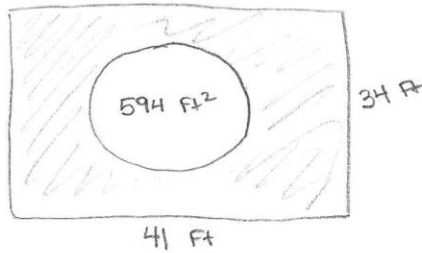


18 in

$$\begin{aligned} A_{\Delta} &= \frac{1}{2}bh \\ &= \frac{1}{2}(18)(19) \\ &= 9(19) \\ &= 171 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\Delta} - A_{\square} &= 171 - 121 \\ &= 50 \text{ in}^2 \end{aligned}$$

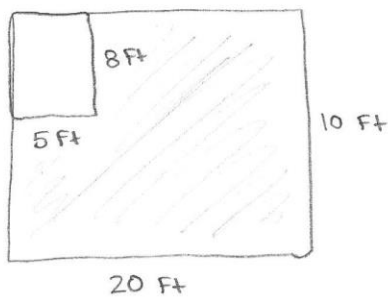
3. Suppose you want to install a pool into your backyard. If your backyard has a length of 34 feet and a width of 41 feet, and the pool has an area of 594 feet², how many square feet will your yard have left with the pool installed? (Draw a picture!)



$$\begin{aligned} A_{\square} &= bh \\ &= 41(34) \\ &= 1394 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} A_{\square} - A_{\circ} &= 1394 - 594 \\ &= 800 \text{ ft}^2 \end{aligned}$$

4. Suppose you decide to get a larger bed for your bedroom. Your bedroom is 20 ft by 10 ft and your new bed is 8 ft by 5 ft. How much room will you have left in your bedroom with your new bed? (Draw a picture!)



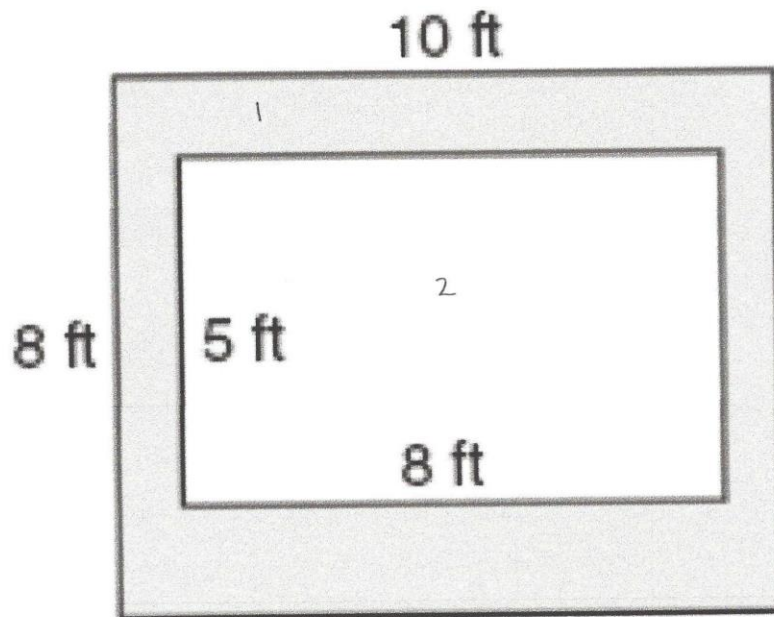
$$\begin{aligned} A_R &= bh \\ &= 20(10) \\ &= 200 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} A_B &= bh \\ &= 8(5) \\ &= 40 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} A_R - A_B &= 200 - 40 \\ &= 160 \text{ ft}^2 \end{aligned}$$

STATION 1

- Find the area of the shaded region.



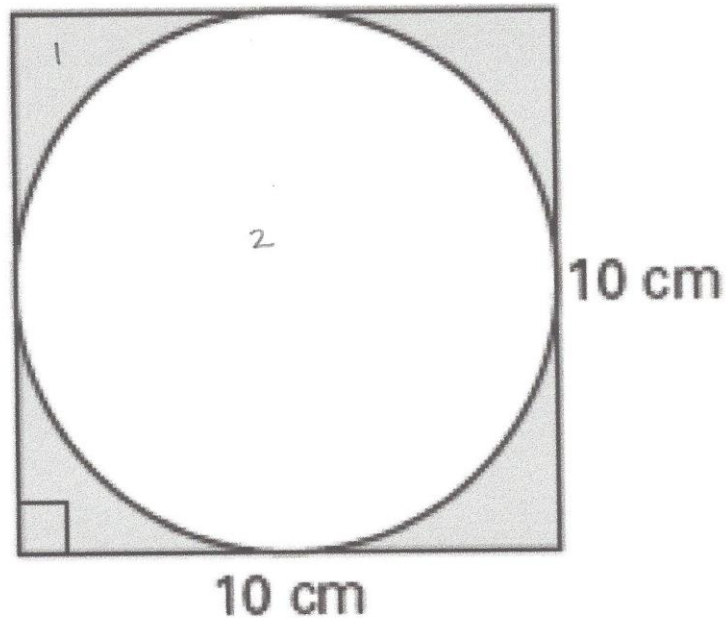
$$\begin{aligned} & \underline{1} \\ A &= bh \\ A &= 8(10) \\ A &= 80 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} & \underline{2} \\ A &= bh \\ A &= 8(5) \\ A &= 40 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} A_s &= (1) - (2) \\ &= 80 - 40 \\ &= 40 \text{ ft}^2 \end{aligned}$$

STATION 2

- Find the area of the shaded region.
- Leave your answer in terms of π .



$$\begin{aligned} \underline{(1)} \\ A &= bh \\ A &= 10(10) \\ A &= 100 \text{ cm}^2 \end{aligned}$$

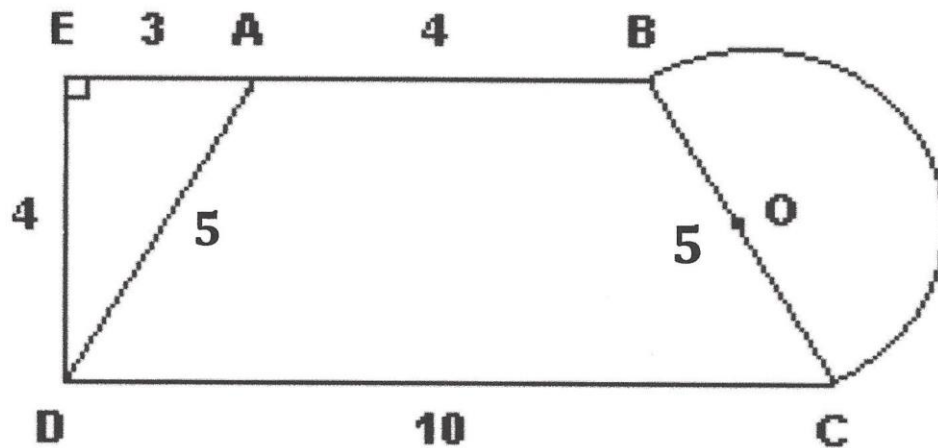
$$\begin{aligned} \underline{(2)} \\ A &= \pi r^2 \\ A &= \pi(5)^2 \\ A &= \pi(25) \\ A &= 25\pi \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A_s &= (1) - (2) \\ &= (100 - 25\pi) \text{ cm}^2 \end{aligned}$$

Name: _____
 Quiz 2

Date: _____

1. Use the following figure to answer the questions **in terms of π** .



- (a) Find the area of the triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(3)(4) \\ &= 6 \text{ units}^2 \end{aligned}$$

- (b) Find the area of the trapezoid.

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(10 + 4)(4) \\ &= 28 \text{ units}^2 \end{aligned}$$

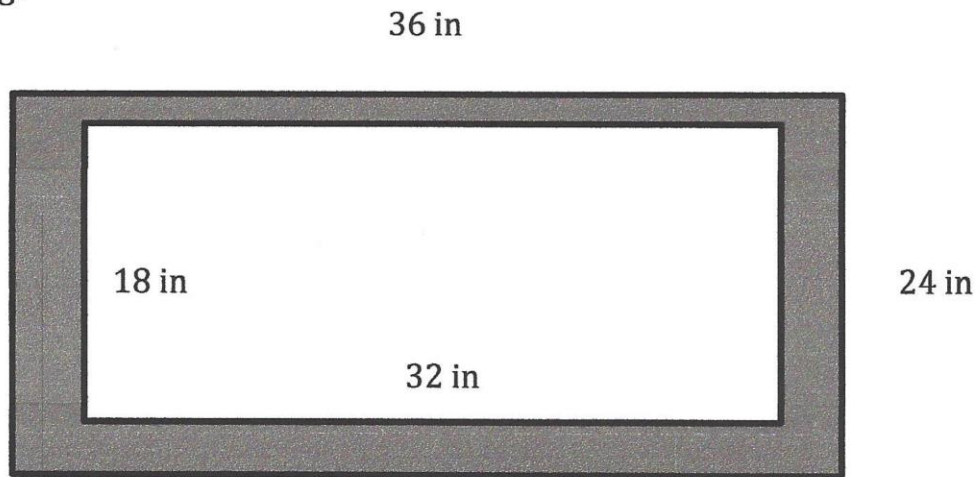
- (c) Find the area of the **half** circle. (In terms of π)

$$\begin{aligned} A &= \frac{1}{2}\pi r^2 \\ A &= \frac{1}{2}\pi(2.5)^2 \\ A &= 3.125\pi \text{ units}^2 \end{aligned}$$

- (d) Find total area of the figure. (In terms of π)

$$\begin{aligned} A_T &= 6 + 28 + 3.125\pi \\ &= 34 + 3.125\pi \text{ units}^2 \end{aligned}$$

2. In this figure, a rectangle is drawn within another rectangle. Solve the following:



- (a) Find the area of the outer rectangle.

$$\begin{aligned} A &= 36(24) \\ &= 864 \text{ in}^2 \end{aligned}$$

- (b) Find the area of the inner rectangle.

$$\begin{aligned} A &= bh \\ &= 32(18) \\ &= 576 \text{ in}^2 \end{aligned}$$

- (c) Using the results from part (a) and (b), find the area of the shaded region.

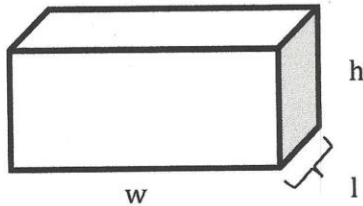
$$\begin{aligned} A_s &= 864 - 576 \\ &= 288 \text{ in}^2 \end{aligned}$$

Name: _____
(Day 9) Notes

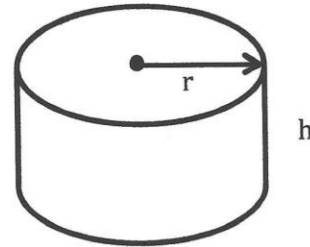
Date: _____

Surface Area (without Variables)

- **Surface Area**- The total area of the surface of a three-dimensional object.



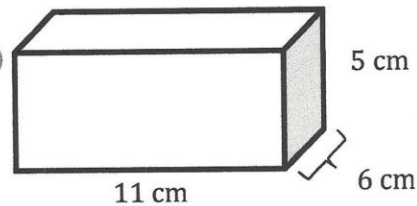
$$SA = 2lw + 2wh + 2lh$$



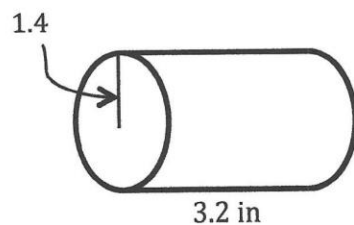
$$SA = 2\pi r^2 + 2\pi rh$$

Example 1: What is the **surface area** of the solid below?

$$\begin{aligned} SA &= 2lw + 2wh + 2lh \\ &= 2(11)(6) + 2(6)(5) + 2(11)(5) \\ &= 132 + 60 + 110 \\ &= 302 \text{ cm}^2 \end{aligned}$$

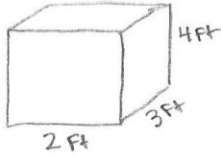


Example 2: What is the **surface area** of the solid below to the **nearest hundredth**?



$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(1.4)^2 + 2\pi(1.4)(3.2) \\ &= 12.32 + 28.15 \\ &= 40.47 \text{ in}^2 \end{aligned}$$

Example 3: How many square feet of wrapping paper are needed to entirely cover a box that is 2 feet by 3 feet by 4 feet?



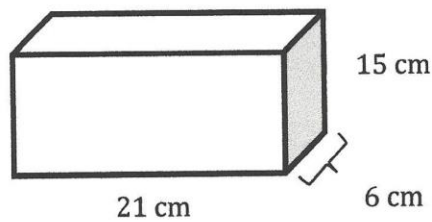
$$\begin{aligned} SA &= 2lw + 2lh + 2wh \\ &= 2(2)(3) + 2(2)(4) + 2(3)(4) \\ &= 12 + 16 + 24 \\ &= 52 \text{ ft}^2 \\ &\quad \downarrow \\ &\quad \text{of wrapping paper} \end{aligned}$$

Name: _____
(Day 9) Homework

Date: _____

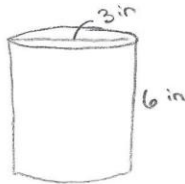
Surface Area

1. Find the surface area of the following rectangular solid.



$$\begin{aligned} SA &= 2lw + 2lh + 2wh \\ &= 2(21)(6) + 2(21)(15) + 2(6)(15) \\ &= 252 + 630 + 180 \\ &= 1062 \text{ cm}^2 \end{aligned}$$

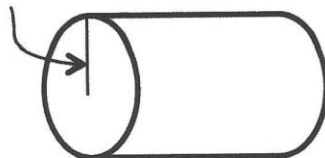
2. A can of corn has a height of 6 inches and a diameter of 3 inches. Find to the nearest tenth the surface area of the can.



$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(1.5)^2 + 2\pi(1.5)(6) \\ &= 14.1 + 56.5 \\ &= 70.5 \text{ in}^2 \end{aligned}$$

3. Find the surface area of the following lateral cylinder in terms of π .

8.1 cm



14.6 cm

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(8.1)^2 + 2\pi(8.1)(14.6) \\ &= 131.22\pi + 236.52\pi \\ &= 367.74\pi \text{ cm}^2 \end{aligned}$$

Name: _____
(Day 10) Notes

Date: _____

Surface Area (with Variables)

Variable- A letter that represents an unknown number.

In terms of x- Like leaving answers "in terms of π ", you leave the variable in your answer.

Example 1: A plastic storage box in the shape of a **rectangular solid** has a **length** of $x + 3$, a **width** of $2x$, and a **height** of 5.

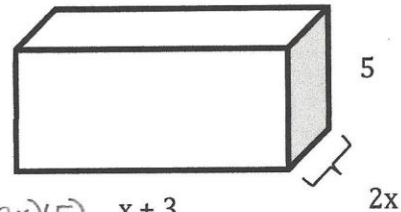
(a) Find the **surface area** of the box in terms of x .

$$SA = 2lw + 2lh + 2wh$$

$$= 2(x+3)(2x) + 2(x+3)(5) + 2(2x)(5) \quad x+3$$

$$= 4x^2 + 12x + 10x + 30 + 20x$$

$$= (4x^2 + 42x + 30) \text{ units}^2$$



(b) What is the **surface area** of the box if $x = 3$?

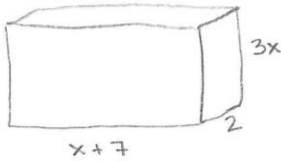
$$SA = 4x^2 + 42x + 30$$

$$= 4(3)^2 + 42(3) + 30$$

$$= 36 + 126 + 30$$

$$= 192 \text{ units}^2$$

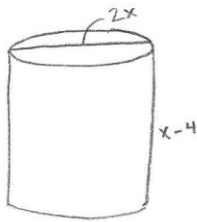
Example 2: A crate in the shape of a **rectangular solid** has a **length** of $x + 7$, a **width** of 2, and a **height** of $3x$. Find the **surface area** of the crate **in terms of x** .



$$\begin{aligned}
 SA &= 2lw + 2lh + 2wh \\
 &= 2(x+7)(2) + 2(x+7)(3x) + 2(2)(3x) \\
 &= 4x + 28 + 6x^2 + 42x + 12x \\
 &= (6x^2 + 58x + 28) \text{ units}^2
 \end{aligned}$$

Example 3: A paint can has a **height** of $x - 4$ and a **diameter** of $2x$.

(a) Find the **surface area** of the paint can **in terms of x** .



$$\begin{aligned}
 SA &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi x^2 + 2\pi(x)(x-4) \\
 &= 2x^2\pi + (2x^2 - 8x)\pi \\
 &= (4x^2\pi + 8x\pi) \text{ units}^2
 \end{aligned}$$

(b) What is the **surface area** of the paint can if $x = 4$? (**Round to the nearest tenth**)

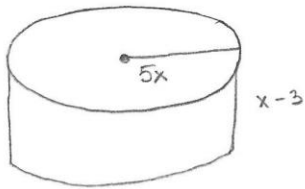
$$\begin{aligned}
 SA &= 4x^2\pi + 8x\pi \\
 &= 4(4)^2\pi + 8(4)\pi \\
 &= 201.1 + 100.5 \\
 &= 301.6 \text{ units}^2
 \end{aligned}$$

Name: _____
(Day 10) Homework

Date: _____

Surface Area (with Variables)

1. Find the surface area of the diagram shown below in terms of x . (Round to the nearest tenth)

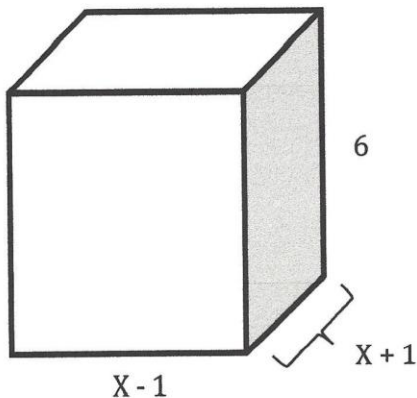


$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(5x)^2 + 2\pi(5x)(x-3) \\ &= 50x^2\pi + (10x^2 - 30x)\pi \\ &= 157.1x^2 + 31.4x^2 - 94.2x \\ &= (188.5x^2 - 94.2x) \text{ units}^2 \end{aligned}$$

2. What is the surface area of the diagram above to the nearest tenth if $x = 13$?

$$\begin{aligned} SA &= 188.5x^2 - 94.2x \\ &= 188.5(13)^2 - 94.2(13) \\ &= 30631.9 \text{ units}^2 \end{aligned}$$

3. What is the surface area of the following shape? (Optional)



$$\begin{aligned} SA &= 2lw + 2wh + 2lh \\ &= 2(x-1)(x+1) + 2(x+1)(6) + 2(x-1)(6) \\ &= 2x^2 - 2 + 12x + 12 + 12x - 12 \\ &= (2x^2 + 24x - 2) \text{ units}^2 \end{aligned}$$

Name: _____
(Day 11) Notes

Date: _____

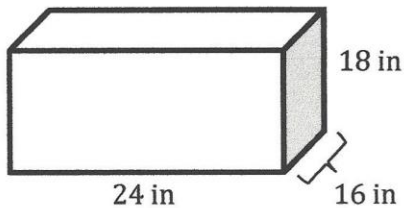
Volume

Rectangular Prism: $V = \text{length}(\text{width})(\text{height})$ Cylinder: $V = \pi r^2 h$

Procedure:

1. Always start with your **formula**.
2. **Substitute** in values that are given in the problem.
3. **Solve** for the value that is not given. This may require multiple steps or require you to work backwards.
4. **Labels** for solving volume are always cubed. (units^3)

Example 1: Steve is filling his fish tank with water from a hose at a rate of **500 cubic inches per minute**. How **long** will it take, *to the nearest minute*, to **fill the tank**?



Volume = how much space something takes up

$$V = l \cdot w \cdot h$$

$$V = (24 \text{ in})(16 \text{ in})(18 \text{ in})$$

$$V = 6912 \text{ in}^3$$

6912 in^3 needs to be filled...

$$\frac{6912 \text{ in}^3}{500 \frac{\text{in}^3}{\text{min}}} = 13.824 \frac{\text{in}^3}{\text{in}^3/\text{min}} = 13.824 \text{ min}$$

14 minutes

Example 2: The length of a fuel hose for a string trimmer is 12 centimeters and its radius is 2 centimeters.

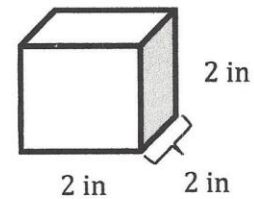
- (a) Calculate the total **volume** that the fuel hose can hold.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi (2)^2 (12) \\ &= 48\pi = 150.8 \text{ cm}^3 \end{aligned}$$

- (b) On average, a string trimmer gas tank **holds 3800 cubic centimeters**. If the trimmer uses the **volume of part (a)** worth of gas **per minute**, **how long** will it take to use all the gas? (*Round to the nearest minute*)

$$\frac{3800}{150.8} = 25.2 \approx 25 \text{ minutes}$$

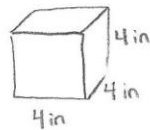
Example 3: Consider a cube whose sides measure 2 inches.



- (a) What is the volume of this cube?

$$\begin{aligned} V &= lwh \\ &= 2^3 \\ &= 8 \text{ in}^3 \end{aligned}$$

- (b) If the length of the sides of the cube were doubled, what would be the new volume of the cube?



$$\begin{aligned} V &= lwh \\ &= 4^3 \\ &= 64 \text{ in}^3 \end{aligned}$$

- (c) What is the ratio of the larger cube's volume to that of the smaller cube?

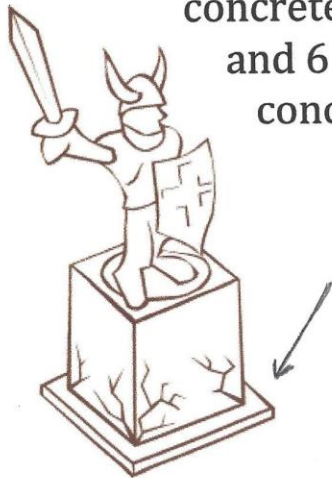
$$64 : 8 \quad \rightarrow \quad 8 : 1$$

- (d) Why did the cube's volume not double when the length of its sides was doubled?

Because you're doubling three measurements and since the volume is calculated by multiplying these dimensions, the volume changes by $(2)(2)(2) = 8$ times more.

Station 1

The pedestal on which a statue is raised is a rectangular concrete solid measuring 9 feet long, 9 feet wide and 6 inches high. How much is the cost of the concrete in the pedestal, if concrete costs \$8 per cubic foot? (Hint: Use volume)



$$\begin{aligned}V &= lwh \\ &= (9)(9)(.5) \\ &= 40.5 \text{ ft}^3\end{aligned}$$

$$(40.5)(8) = 324$$

$$\text{\$ } 324$$

Station 2

Josh is painting the walls of his bedroom, as shown below.

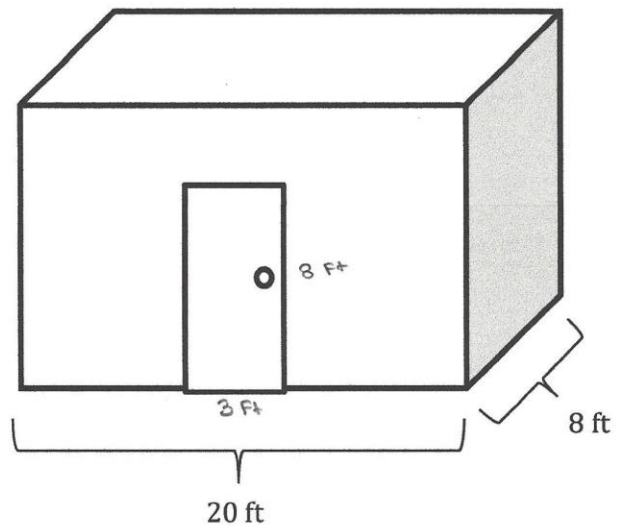
- Josh will not be painting the floor, ceiling, or door.
- Josh's door is 3 feet by 8 feet.
- If one quart of paint covers 100 square feet, how many quarts will Josh need to buy? (Hint: Use surface area)

$$\begin{aligned}
 \text{SA}_{\text{r}} &= \cancel{2lw} + 2lh + 2wh \\
 &= \cancel{2(20)(8)} + 2(20)(10) + 2(8)(10) \\
 &= \cancel{320} + 400 + 160 \\
 &= 560 \text{ ft}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{D}} &= bh \\
 &= 3(8) \\
 &= 24 \text{ ft}^2
 \end{aligned}$$

$$560 - 24 = 536 \text{ ft}^2$$

$$\frac{536}{100} = 5.36 \text{ quarts} \approx \boxed{6 \text{ quarts to cover}}$$

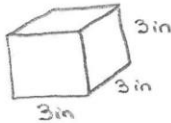


Station 3

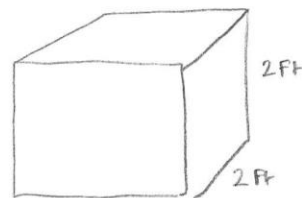
Leroy Jenkins delivers muffins packed in their own little boxes in the shape of a cube, measuring 3 inches on each side. Leroy packs the individual muffin boxes into a larger box. The larger box is also in the shape of a cube, measuring 2 feet on each side. How many of the individual muffins boxes can fit into the larger box?

(Hint: Use volume)

muffin box →



$$\begin{aligned} V_s &= lwh \\ &= 3(3)(3) \\ &= 27 \text{ in}^3 \end{aligned}$$



← Large box

→ Convert to inches

$$\begin{aligned} V_L &= lwh \\ &= (24)(24)(24) \\ &= 13824 \text{ in}^3 \end{aligned}$$

$$\frac{13824}{27} = 512 \text{ muffin boxes}$$

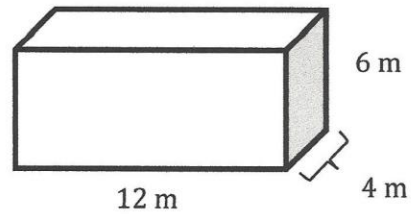
Name: _____
(Day 12) Homework

Date: _____

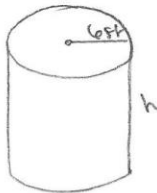
Volume

1. Find the volume of the following rectangular prism.

$$\begin{aligned} V &= lwh \\ &= 12(4)(6) \\ &= 288 \text{ m}^3 \end{aligned}$$

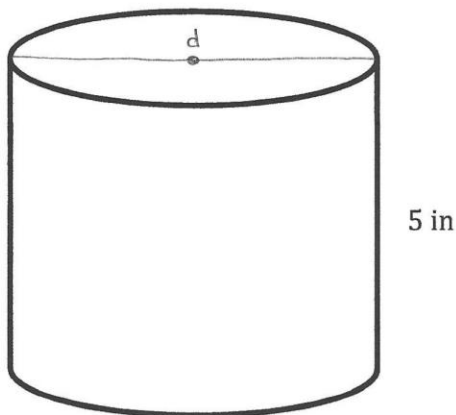


2. If a cylinder has a volume of $144\pi \text{ ft}^3$ and a radius of 6 ft, find the height of the shape.



$$\begin{aligned} V &= \pi r^2 h \\ 144\pi &= \pi(6)^2 h \\ 144 &= 36h \\ h &= 4 \text{ ft} \end{aligned}$$

3. Find the diameter of the following cylinder given that the volume is $125\pi \text{ inches}^3$.



$$\begin{aligned} V &= \pi r^2 h \\ 125\pi &= \pi r^2 \cdot 5 \\ 25 &= r^2 \\ r &= 5 \\ d &= 2r = 10 \text{ in} \end{aligned}$$

Days 13 and 14 follow a power point template with categories divided into the unit topics. Each category dollar amount contains an item that becomes increasingly more difficult to solve as the price increases. Listed below are some slides from the power point displaying the categories and examples of items.

Jeopardy

Circumference, Perimeter, and Area	Total Area and Shaded Region Area	Surface Area	Volume
<u>\$100</u>	<u>\$100</u>	<u>\$100</u>	<u>\$100</u>
<u>\$200</u>	<u>\$200</u>	<u>\$200</u>	<u>\$200</u>
<u>\$300</u>	<u>\$300</u>	<u>\$300</u>	<u>\$300</u>
<u>\$400</u>	<u>\$400</u>	<u>\$400</u>	<u>\$400</u>
<u>\$500</u>	<u>\$500</u>	<u>\$500</u>	<u>\$500</u>

\$300

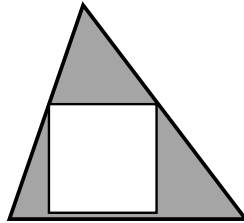
A square-shaped room measures 7 meters on one side. What is the perimeter of the room?

P = 28 meters



\$200

What are the **steps** to finding the area of the shaded region for the following figure? (Be specific)



Find the area of the triangle and the area of the square, then subtract the area of the square **from** the area of the triangle.



\$500

A plastic storage box in the shape of a rectangular prism has a length of $5x$, a width of $x + 4$, and a height of 2. Find the surface area of this box **in terms of x** .

$$SA = (10x^2 + 64x + 16) \text{ units}^2$$



\$100

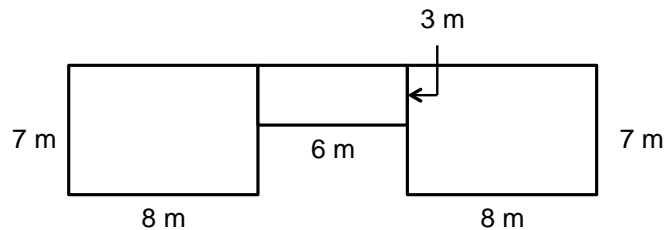
What is the volume formula for a rectangular prism?

$$V = lwh$$



\$400

Find the total area of the shape below.



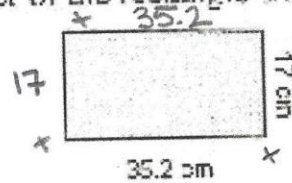
$$\text{Total Area} = 130 \text{ m}^2$$



PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

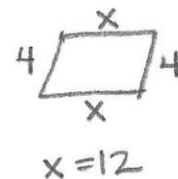
1 Calculate the perimeter of the rectangle shown in the diagram.



- (1) 104.4 cm
- (2) 598.4 cm
- (3) 52.2 cm
- (4) 69.2 cm

2 The perimeter of a parallelogram is 32 meters and the two shorter sides measure 4 meters. What is the length of the longer sides?

- (1) 4 meters
- (2) 6 meters
- (3) 10 meters
- (4) 12 meters



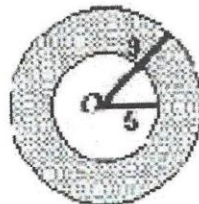
3 What is the area of a trapezoid that has bases measuring 8 inches and 10 inches and a height of 3 inches?

- (1) 27 in²
- (2) 18 in²
- (3) 9 in²
- (4) 120 in²

$$\frac{1}{2}(18)(3)$$

$$9(3)$$

4 The area of the larger circle is about 254.5 m². The area of the smaller circle is about 78.5 m². What is the area of the shaded region?



- (1) 333 m²
- (2) 176 m²
- (3) 4 m²
- (4) 17 m²

$$\begin{array}{r} 254.5 \\ - 78.5 \\ \hline 176.0 \end{array}$$

5 What is the diameter of a circle whose circumference is 5 in terms of π ?

- (1) 5π
- (2) $\frac{5}{\pi}$
- (3) $\frac{5}{\pi^2}$
- (4) $\frac{2.5}{\pi}$

$$C = \pi d$$

$$5 = \pi d$$

$$d = \frac{5}{\pi}$$

6 What is the approximate circumference of a circle whose radius is 3?

- (1) 7.07
- (2) 9.42

- (3) 18.85
- (4) 28.27

$$C = \pi d$$

$$C = \pi(6)$$

$$C = 9.42$$

7 Express in terms of π , the area of a circle whose radius is 6.

- (1) 36π
- (2) 12π

- (3) 6π
- (4) 36

$$A = \pi r^2$$

$$= \pi(6)^2$$

8 The volume of a rectangular solid is 80 cubic centimeters, the length is 2 centimeters, and the width is 4 centimeters. What is the height of the rectangular solid?

- (1) 5 centimeters
- (2) 6 centimeters

- (3) 10 centimeters
- (4) 20 centimeters

$$V = lwh$$

$$80 = 2(4)h$$

$$80 = 8h$$

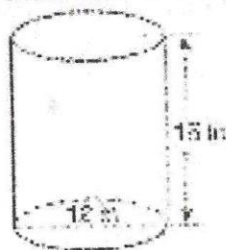
9 Lenny made a cube in technology class. Each edge measured 1.5 cm. What is the volume of the cube in cubic centimeters?

- (1) 2.25
- (2) 3.375

- (3) 9.0
- (4) 13.5

$$V = (1.5)^3$$

10 What is the volume of this container to the nearest tenth of a cubic inch?



(Not drawn to scale)

- (1) $6,785.8 \text{ in}^3$
- (2) $4,241.2 \text{ in}^3$

- (3) $2,160.0 \text{ in}^3$
- (4) $1,696.5 \text{ in}^3$

$$V = \pi r^2 h$$

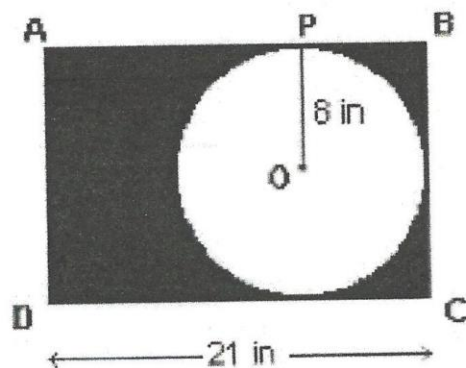
$$= \pi(6)^2 15$$

$$=$$

PART III

Answer all questions in this part. Each correct answer will receive 3 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

- 11 In the diagram, circle O is inscribed in rectangle $ABCD$. Radius OP is drawn to AB . Find the area of the shaded region. Leave your answer in terms of π .



$$A_{\square} = lw$$

$$= 21(16)$$

$$= 336 \text{ in}^2$$

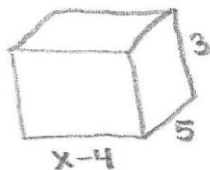
$$A_{\circ} = \pi r^2$$

$$= \pi(8)^2$$

$$= 64\pi \text{ in}^2$$

$$A_s = (336 - 64\pi) \text{ in}^2$$

- 12 Mrs. Ayer is painting the outside of her son's toy box, including the top and bottom. The length, width, and height of the toy box are $x - 4$, 5, and 3 binomial in terms of x ?



$$SA = 2lw + 2wh + 2hl$$

$$= 2(x-4)(5) + 2(5)(3) + 2(3)(x-4)$$

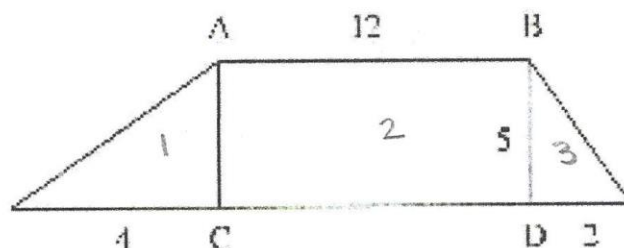
$$= 10x - 40 + 30 + 6x - 24$$

$$= 16x - 34$$

PART IV

Answer all questions in this part. Each correct answer will receive 4 credits. No partial credit will be allowed. For each question, write in the space provided the numeral preceding the word or expression that best completes the statement or answers the question.

13 In the diagram, $ABCD$ is rectangle with a right triangle at either side.



Find the area of the entire figure. (Hint: What shapes make up this figure?)

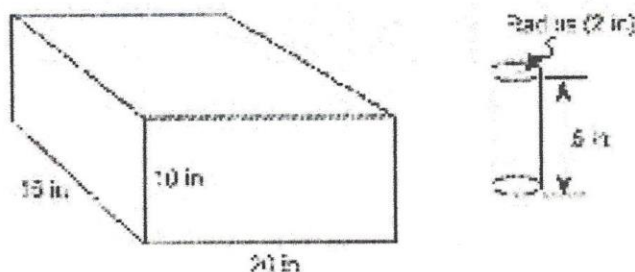
$$\begin{aligned} A_1 &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(5) \\ &= 10 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} A_2 &= bh \\ &= 12(5) \\ &= 60 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} A_3 &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(5) \\ &= 5 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{Total}} &= 10 + 60 + 5 \\ &= 75 \text{ units}^2 \end{aligned}$$

14 In the diagram below, the rectangular container is to be filled with water using the cylindrical cup.



- a. Find the volume of the rectangular container.

$$\begin{aligned} V &= lwh \\ &= 20(10)(15) \\ &= 3000 \text{ in}^3 \end{aligned}$$

- b. Find the volume of the cylindrical cup the *nearest tenth*.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(2)^2(5) \\ &= 62.8 \text{ in}^3 \end{aligned}$$

- c. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?

$$3000 \div 62.8 = 47.77$$

47 Full cups