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Working Memory Deficits in Students with ADHD: Implications for Developing Curriculum on Introductory Trigonometric Functions and the Unit Circle

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**Working Memory Deficits in Students with ADHD: Implications for Developing
Curriculum on Introductory Trigonometric Functions and the Unit Circle**

By

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A thesis submitted to the Department of Education and Human Development of the
College at Brockport, State University of New York, in partial fulfillment of the
requirements for the degree of Master of Science in Education

Abstract

Attention Deficit Hyperactivity Disorder (ADHD) is one of the most prevalently diagnosed disorders in children in the United States today (Zentall, 2007, p. 219; American Psychiatric Association, 2000; Faraone, Sergeant, Gillberg & Bierderman, 2003). “Teachers report that they are unprepared to work with [students with ADHD] and only those educators who have experience with students with ADHD or who have education about them [are] willing to make instructional changes” (Zentall & Javorsky, 2007, p.78; Reid, Vasa, Maag & Wright, 1994). The relatively new implementation of the Common Core State Standards (CCSS) has brought on “rigorous grade-level expectations in the area of mathematics” (Common Core State Standards Initiative, 2014, p.1) According to the guidelines of CCSS, students identified as having a disability under the Individuals with Disabilities Education Act (IDEA) will also be held to the same high standards as all students in the general classroom. The Cognitive Load Theory (CLT) lays a foundation for the following curriculum. The purpose of this curriculum project is to develop a unit in the field of introductory trigonometric functions and the unit circle that addresses specific needs of students with ADHD while still holding the high expectations implemented by the CCSS.

Keywords: Attention Deficit Hyperactivity Disorder, Cognitive Load Theory, Working Memory Deficits, Trigonometry

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Chapter 1: Introduction

Over the course of the last fifteen years, there has been a reform movement in the world of mathematic education in many western countries, including the United States (Lucangeli & Cabrela, 2006, p. 53). New York and many other states have adopted the new Common Core State Standards (CCSS) as their new form of guidance for curriculum. “The Common Core State Standards articulate rigorous grade-level expectations in the area of mathematics. These standards identify the knowledge and skills students need in order to be successful in college and careers” (Common Core State Standards Initiative, 2014, p. 1) According to the guidelines of CCSS, students identified as having a disability under the Individuals with Disabilities Education Act (IDEA) will also be held to the same high standards as all students in the general classroom. Although students with ADHD face hardships that general education students often do not, the CCSS fundamental goal is to prepared all students for success in their post-school lives, including college and/or careers (Common Core State Standards Initiative, 2014, p. 1). *How* these high standards are taught and assessed is of the utmost importance in reaching all students, including the large population of students with ADHD (Common Core State Standards Initiative, 2014, p. 1).

“Teachers report that they are unprepared to work with [students with ADHD] and only those educators who have experience with students with ADHD or who have education about them were more willing to make instructional changes (Zentall & Javorsky, 2007, p.78; Reid, Vasa, Maag & Wright, 1994). With the increasing number of students being diagnosed with ADHD in the general classroom, the importance of understanding how to work with this diverse group of students is every growing. This

curriculum project is designed for Intermediate Algebra in the unit of trigonometric functions. The purpose of this project is to develop a unit that when taught both addresses the needs of students with ADHD, specifically working memory deficits, while also reaching the high expectations implemented by the CCSS. The unit presented utilizes the various teaching styles, strategies and methods previous research has shown to be effective in educating students with ADHD in the focus of working memory deficits.

Chapter 2: Literature Review

Attention Deficit Hyperactive Disorder

“Attention deficit hyperactivity disorder (ADHD) is a chronic, neuro-behavioral disability with both genetic and environmental etiologies” (Zentall, 2007, p. 219). The diagnosis of ADHD is based on both observations of the behaviors of the subject and ratings of the major symptoms (Zentall, 2007, p. 219). ADHD is comprised of a collection of symptoms, namely, inattention, impulsivity, and overactivity (Furman, 2005, p. 999). “Even though the number of symptoms and degree of impairment vary, the majority of students with ADHD experience attention and behavior difficulties that compromise their academic success” (Zentall, 2007, p. 220). “ADHD is identified as the most prevalent disorder in children in the United States” (Zentall, 2007, p. 219; American Psychiatric Association, 2000; Faraone, Sergeant, Gillberg & Bierderman, 2003). According to recent studies, approximately 5% of children are diagnosed with ADHD (Martinussen & Major, 2011, p. 68; Polanczyk, de Lima, Horta, Bierderman & Rohde, 2007). “It is not yet clear, however, if poor academic performance that often accompanies ADHD is related more to the behavioral or the cognitive impairments associated with the disorder” (Lucangeli & Cabrele, 2006, p. 53). The effect of ADHD on mathematical

achievement is becoming a more prevalent concern given the recent reform movement in mathematics across the country (Lucangeli & Cabrele, 2006, p. 53).

Common Core State Standards

With the recent transition from NCTM standards to the Common Core State Standards (CCSS) in 2010, there has been a paradigm shift in the mathematics curriculum that is important to acknowledge in order to understand the current demands on student learning mathematics. The CCSS calls for three main shifts in mathematics; focus, coherence and rigor. “Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom” (Common Core State Standards Initiative, 2014). The idea in changing the focus is to strengthen the foundation of general mathematics and to increase the ability of students to fluently apply their knowledge. CCSS also reaches to connect mathematical topics in order to form a large body of mathematical knowledge that flows as one unit rather than disjointed information. The third shift refers to conceptual understanding, procedural skills/ fluency and application. In order to help students meet the new mathematical standards, educators will need to pursue, with equal intensity, each of these new shifts (Common Core State Standards Initiative, 2014).

With change comes struggle. It can be anticipated that the overall math population may have difficulties with these new shifts due to the extensive change in expectation on mathematic learners from the New York State Standards to the Common Core State Standards. If we can anticipate the general student to struggles with some of the new shifts, that is students who do not classify as having a disability, it can also be anticipated

that students with ADHD will also struggle with the new expectations on learning. This makes understanding students with ADHD in the classroom that much more important in order to be able to address their specific needs. It has been found that, “higher rates of math learning disabilities are reported for students with ADHD (31%) than are reported for the general population (6%-7%), and a quarter of students with arithmetic disabilities also have ADHD” (Zentall, 2007, p. 220; Mayes et al., 2000; Shalev et al., 2001).

Working Memory Deficits in Students with ADHD

“Deficits in executive functioning are proposed to play a pivotal role in explaining the problems children with ADHD encounter in daily life (Dovis, Van der Oord, Wiers & Prins, 2013, p.901; Barkley, 2006; Nigg, 2006). Executive functions play the role of regulating behaviors, thoughts and emotions. This then entails being able to enable self-control (Dovis, Van der Oord, Wiers & Prins, 2013, p.901). “Children with ADHD experience deficits in some of the abilities constituting the executive functions such as planning, organizing, maintaining an appropriate problem-solving set to achieve a future goal, inhibiting an inappropriate response or deferring a response to a more appropriate time representing a task mentally (i.e. working memory), cognitive flexibility and deduction based on limited information (Lucangeli & Cabrele, 2006, p. 53; Barry et al., 2002, p. 260). Due to the extensive nature of executive function deficits that some students with ADHD face, this paper will focus on working memory deficits. “There is evidence suggesting that the working memory impairments of children with ADHD account for their deficits in attention, hyperactivity and impulsivity” (Dovis, Van der Oord, Wiers & Prins, 2013, p.902; Burgess et al., 2010; Kofler et al., 2010; Tillman et al., 2011; Raiker et al., 2012; Rapport et al., 2009). Working memory allows people to

maintain, control and manipulate goal-relevant information. “Working memory enables skills like reasoning, planning, problem solving and goal-directed behavior” (Dovis, Van der Oord, Wiers & Prins, 2013, p.901; Baddeley, 2007; Conway et al., 2007; Martinussen et al., 2005). “Holding information in mind while ignoring external stimulation is required for the performance of mental math” (Zentall, 2007, p. 223; Carver, 1979) “For students with ADHD, difficulties sustaining attention during repetitive tasks could contribute to their failure to overlearn or automatize basic computational skills” (Zentall, 2007, p. 222). According to van Merriënboer and Sweller (2005) the Cognitive Load Theory (CLT) is a theory of particular relevance for designing instruction for target groups characterized by impaired working-memory functions, such as ADHD (p. 173).

Cognitive Load Theory

John Sweller presented the Cognitive Load Theory (CLT) in the 1980’s when working with his students on the idea of problem solving. The CLT is rooted in the idea that learning uses two types of memory: the working memory and the long-term memory. According to the theory, working memory is assumed to be limited in the amount of elements that can be processed at a given time. The working memory can store approximately seven elements but operates on only two to four. “It is able to deal with information for no more than a few seconds with almost all information lost after about twenty seconds unless it is refreshed by rehearsal” (van Merriënboer & Sweller, 2005, p. 148). Due to the nature of working memory when dealing with new information, “[when] limits are exceeded, then working memory becomes overloaded, and learning is inhibited” (Ellis, 2014, p. 12; Kalyuga, 2011).

Working memory can be broken into three aspects of cognitive loads; extraneous cognitive load, intrinsic cognitive load and germane cognitive load. “Extraneous cognitive load is not necessary for learning, and is caused by suboptimal pedagogy, which requires the learner to devote cognitive processes to tasks that are not essential for achieving instructional goals” (Ellis, 2014, p. 14; Paas, Renkl, & Sweller, 2004; Kalyuga, 2011). Extraneous cognitive load may consist of, but is not limited to elements such as, the teacher, the physical classroom or the specific types of instruction. “Working memory load may [also] be affected by the intrinsic nature of the learning tasks themselves (intrinsic cognitive load)” (van Merriënboer & Sweller, 2005, p. 149). According to van Merriënboer and Sweller (2005), the intrinsic cognitive load cannot be altered by instructional interventions. This particular cognitive load is determined mostly by level of expertise of the learner and also the interaction of the materials being learned (p. 150). “Extraneous cognitive load and intrinsic cognitive load are additive” (van Merriënboer & Sweller, 2005, p. 150). Due to the nature of a limited working memory, focus must be put on decreasing extraneous load while balancing intrinsic (element interactivity) and germane cognitive loads (van Merriënboer, Kester & Paas, 2006, p. 344). The main goal of the CLT is to help guide instruction in order to enhance transfer of learning without maximizing the elements and overloading the working memory (van Merriënboer, Kester & Paas, 2006, p. 344). “Germane [cognitive] load directly contributes to learning, that is, to the learner’s construction of cognitive structures and processes that improve performance” (van Merriënboer, Kester & Paas, 2006, p. 344).

Although, the CLT demonstrates limitations when information is new, it is important to acknowledge that when information is retrieved from the long-term memory,

there are presumably no limitations to working memory (van Merriënboer & Sweller, 2005, p. 148). “Novel information must be processed in working memory in order to construct schemata in long term memory” (van Merriënboer & Sweller, 2005, p. 150). This information illustrates the importance of utilizing and designing instruction in which focuses on strengthening the long-term memory through the idea of the germane cognitive load. “Schemata can act as a central executive, organizing information or knowledge that needs to be processed in working memory” (van Merriënboer & Sweller, 2005, p. 149). Constructed schemata and automation are both sources that help free working memory “space” for other necessary elements to occupy. Both “steer behavior without the need to be processed by working memory” (van Merriënboer & Sweller, 2005, p. 149). “Effective [CLT] instructional methods encourage learners to invest free processing resources to schema construction and automation, evoking germane cognitive load” (van Merriënboer & Sweller, 2005, p. 152).

Implications for Instruction

“The definition of learning, from a cognitive load perspective, is defined as a permanent change in long term memory” (Ellis, 2014, p. 12; Sweller et al., 1998; Sweller et al., 1991; Sweller & Candler, 1994). Essentially the goal of instructional design, per the CLT, is to stimulate the transfer of knowledge. As addressed previously, the transfer of knowledge is conducted through the germane cognitive load in the working memory. “Germane [cognitive] load directly contributes to learning [in terms of] the learner’s construction of cognitive structures and processes that improve performance” (van Merriënboer, Kester & Paas, 2006, p. 344). CLT determines instructional design by using the interactions between information structure and the knowledge of human

cognition ((van Merriënboer & Sweller, 2005, p. 147). “Well designed instruction should not only encourage schema construction but also schema automation for those aspects of a task that are consistent across problems” (van Merriënboer & Sweller, 2005, p. 149). In the mathematics classroom, the curriculum introduces student to many and various complex ideas and problems. “The most important characteristic of complex learning is that students must learn to deal with materials incorporating an enormous number of interacting elements” (van Merriënboer & Sweller, 2005, p. 156). Research has indicated traditional styles of instruction do not address the needs that CLT presents. “Methods such as blocked practice, step-by-step guidance and frequent and complete feedback may indeed have a positive effect on the acquisition curve and performance on retention tests, but *not* on problem solving and transfer of learning” (van Merriënboer, Kester & Paas, 2006, p. 346). Recent literature on the Cognitive Load Theory and ADHD has presented implications for instructional designs, known as germane-load inducing methods, which are geared towards improving specifically the transfer of knowledge. Two germane-load inducing methods that have been mentioned by van Merriënboer, Kester and Pass (2006) in recent studies include practice variability and providing guidance and feedback (p. 344-345).

Practice variability, also known as random practice, according to van Merriënboer, Kester and Paas (2006), are tasks that are of high contextual interpretation and are mixed and practiced in random order (p. 344). “Random practice of different versions of a task induces germane learning processes that require more effort than does blocked practice, but yield cognitive representation that increases later transfer test performance” (van Merriënboer, Kester & Paas, 2006, p. 345). “Performance [by students

with ADHD] on rote math calculations elicits responses, such as more activity and errors over time” (Zentall, 2007, p. 222; Bennett, Zentall, Giorgetti, Borucki & French, 2006; Lee & Zentall, 2002; Zentall & Smith, 1993). According to Zentall (2007), instructional approaches that do not take a random approach but rather a focus on memorization often lead to exacerbation of mathematical impairments (p.230).

As for providing guidance and feedback as means for inducing the germane cognitive load, research is showing that, “slightly delayed feedback is more effective than concurrent or immediate feedback” (van Merriënboer, Kester & Paas, 2006, p. 345). Van Merriënboer, Kester and Paas (2006) stress the importance, however, to acknowledge that instructional design should be assessed based on the complexity of the task (P. 345). ”The complexity of a task is largely determined by its degree of element interactivity” (van Merriënboer, Kester & Paas, 2006, p. 347). When a task is determined to reach a certain level of complexity, the intrinsic load becomes imposed leaving no processing capacity for learners to develop their own internal monitoring and feedback (van Merriënboer, Kester & Paas, 2006, p. 345). In this case, students would need further guidance and feedback, but still in a much more limited sense than traditional practice. In cases such as this, “assistive technology (e.g., calculators) can be used to reduce working memory load [in students with ADHD” (Zentall, 2007, p. 232).

According to van Merriënboer, Kester and Paas (2006), instructional learning tasks should always provide variability in practice, give limited guidance and provide infrequent and only when necessary feedback to learners (p. 350). Working with germane- inducing methods such as, “reducing element interactivity to manageable levels, chunking information based on learner expertise [and] implementing [other]

germane inducing strategies has been demonstrated to enhance the acquisition, retention and transfer of complex mathematics” (Ellis, 2014, p. 16). The use of scaffolding, explicit instructions and external aids can also support germane-inducing methods for students with ADHD. “Consequently, instructional manipulations to improve learning by diminishing extraneous cognitive load and by freeing up cognitive resources is only effective if students, even those with ADHD, are motivated and actually invest mental effort in learning processes that use freed resources” (van Merriënboer & Sweller, 2005, p. 162).

Chapter 3: Curriculum

Cognitive Load Theory (CLT) stresses that learning only happens when there is a permanent change in long-term memory (Ellis, 2014, p. 12; Sweller et al., 1998; Sweller et al., 1991; Sweller & Candler, 1994). From the previous section, it was conveyed that working memory plays a key role in the transfer and storage of that knowledge. This unit was constructed to encourage schema construction and automation in students with ADHD who face working memory deficits.

As the lessons were designed, the use of explicit instructions, external aids and scaffolding were utilized to address the working memory deficits in students with ADHD. These research based teaching practices, as discussed in chapter 2, decrease extraneous loads on the working memory in order to avoid working memory overload. As each lesson was taught, and new content was being presented, review sheets were attached to the beginning of each note packet. This allowed the students to recall the information from previous lessons prior to learning new material. These review sheets were from then on accessible by the students to use for guidance on future in-class work

and homework. The external aids were presented in the form of graphs, vocabulary sheets, tables etc.

Problems given on warm-ups, homeworks, quizzes, worksheets and test were thoughtfully layered in ways that allowed students to perform basic skills first and then progressively work on more complex problems that were grounded in the basic principles. Questions were asked in multiply ways in order to be sure that students were not building a foundation of knowledge based on procedural repetition. As previous research has indicated, students with ADHD who learn through rote math assessments, over time show greater mathematical errors (Zentall, 2007, p. 222; Bennett, Zentall, Giorgetti, Borucki & French, 2006; Lee & Zentall, 2002; Zentall & Smith, 1993). For these reasons, this curriculum does not feature questions that promote memorization. Homework was assigned from Amsco's Mathematics B (2002) textbook and specific assignments are shown on the following lessons. Due to the advancement of textbooks over the years and with the implementation of the Common Core State Standards, this particular version may not be accessible. A similar textbook, Amsco's Algebra 2 and Trigonometry, is available online for use at http://www.jmap.org/JMAP_ALGEBRA_2_AND_TRIGONOMETRY_AMSCO_RESOURCES.htm (JMAP, 2015). There are also newer versions of the text that may be available to schools that contain similar problems as those assigned in the following lessons that allow for similar evaluation of student performance.

Four learning goals that align with both the Common Core State Standards and the New York State Standards were compiled prior to the unit being taught in order to

maintain a clear focus throughout the unit. This allows both the teacher and the students to assess individual progress and continuously evaluate performance.

It should also be noted that the following lesson plans, worksheets and assessments do not follow APA formatting. In order to preserve appropriate space for student work and ensure readability, rules of APA formatting may not have been followed.

Curriculum

Learning Goal One (LG1)

Students will be able to recall and correctly identify appropriate trigonometric functions to find missing sides and/or angles (inverse functions) of a right triangle and then apply them correctly.

Alignment with standards.

Common Core

F-TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

NYS Math

A2.A.55 Express and apply the six trigonometric functions as ratios of the sides of a right triangle.

A2.A.64 Use inverse functions to find the measure of an angle, given its sine, cosine or tangent.

Learning Goal Two (LG2)

Students will be able to show understanding of the differences between degrees and radians by being able to convert radians to degrees and degrees to radians

Alignment with Standards.

Common Core

F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

NYS Math

A2.M.1 Define radian measure.

A2.M.2 Convert between radian and degree measures.

Learning Goal Three (LG3)

Students will be able to evaluate exact trigonometric function values of special right triangles angles, any of their coterminal angles and reference angles.

Alignment with Standards.

Common Core

F-TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express values of sine, cosines, and tangent for x , $\pi + x$ and $2\pi - x$ in terms of their values for x , where x is any real number.

NYS Math

A2.A.56 Know the exact and approximate values of the sine, cosine and tangent of 0° , 30° , 45° , 60° , 90° , 180° and 270° angles.

Learning Goal Four (LG4)

Students will be able to correctly identify multiple aspects of the unit circle on the coordinate plane including quadrants, angles and rotations, points as trigonometric function values and signs of trigonometric functions in each quadrant.

Alignment with Standards.

Common Core

F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express values of sine, cosines, and tangent for x , $\pi + x$ and $2\pi - x$ in terms of their values for x , where x is any real number.

NYS Math

A2.A.60 Sketch the unit circle and represent angles in standard position.

Table 1 identifies the targeted learning goals that each individual daily lesson assesses and the assessments used to measure the specified learning goals.

Table 1

Daily Lessons Aligned with Targeted Learning Goals and Correlated Assessments.

Day	Lesson	Targeted Learning Goals	Assessments
1	Basic Trigonometry, Angles as Rotations and Radian Measure.	LG1, LG2	Warm-up on review material and Homework
2	The Unit Circle and Trigonometric functions as Coordinates.	LG1, LG2, LG4	Warm-up, Quiz and Homework
3	Function Values of Special Angles and Finding Reference Angles.	LG1, LG2, LG3	Warm-up, Quiz and Homework
4	Inverse Trigonometry Functions.		

		LG1, LG2	“Up-to-now” Quiz and Homework
5	Trigonometric functions with Radian Measures	LG1, LG2	Warm-up, Quiz and Homework
6	Basic Sine and Cosine Graphs	LG4	Hands on Activity and Homework
7	Review Day	All Learning Goals	Review Packet and Homework
8	Test Day	All Learning Goals	Formal Test

Pre-Assessments

There are four quizzes used as pre-assessments throughout this unit. Three of the quizzes are quick ten-question quizzes on material from previous lessons. The fourth quiz is an “up-to now” twenty-question quiz on all material from previous lessons and other important information they should know from prior math classes. As discussed in chapter two, in regards to students diagnosed with ADHD who face working memory deficits, “slightly delayed feedback is more effective than concurrent or immediate feedback” (van Merriënboer, Kester & Paas, 2006, p. 345). The “Up-To Now” quiz, allows the students to self analyze their progress up to that point on the two previous quizzes and then allows the instructors to provide the necessary feedback before the lessons progress to more advanced content. Every question on the each quiz is worth two points. One point is awarded for correct work and one point is awarded for a correct answer. These four quizzes are used as the pre-assessments that allow for unit analysis. Based off results of pre-assessments, modifications will be adapted as seen fit. Table 2 identifies the targeted learning goals that each quiz addresses.

Table 2

Unit Quizzes Aligned with Targeted Learning Goals.

Quiz	Targeted Learning Goal
#1	LG1, LG2
#2	LG1, LG2, LG3
#3 – “Up-To Now”	LG1, LG2, LG3, LG4
#4	LG2, LG3

On each of the four quizzes, learning goals are addressed in individual questions. Learning goals may be addressed in multiple questions. Not all the learning goals are assessed in every quiz. Table 3 identifies the questions on the four quizzes that align with the unit learning goals. These questions are used to then analyze student performance in regards to the unit learning goals. Some questions are aligned with the daily standard rather than the unit learning goals therefore are not present in the table. An “X” in Table 3 indicates that the specific targeted learning goal was not present in the particular quiz.

Table 3

Specific quiz questions that Align with Targeted Learning Goals.

Targeted Learning Goals	Quiz 1 Questions	Quiz 2 Questions	Quiz 3 Questions	Quiz 4 Questions
LG1	1	5	1	1, 2, 3
LG2	4, 5	2	2	X
LG3	X	1	5, 7	4
LG4	X	X	3, 4, 6	X

Post-Assessments

There is one formal post assessment in the form of a unit test. The unit test consists of twenty short answer questions. There are fifteen two-point questions and five four-point questions. The two point questions are based on one point for correct work and one point for correct answer. The four point questions are awarded three points based on correct work and one point for correct answer. This unit test aligns with all the learning goals.

On the unit test, learning goals are addressed in individual questions. Learning goals are assessed by multiple questions. Table 4 identifies the questions that align with the unit learning goals. These questions are used to then analyze student performance in regards to the unit learning goals. Some questions are aligned with the daily standard rather than the unit learning goals therefore are not present in the table.

Table 4

Specific Unit Test Questions that Align with Targeted Learning Goals.

Targeted Learning Goals	Unit Test Questions
LG1	12, 13, 14, 18, 19
LG2	3, 4, 5, 8, 10
LG3	1, 2, 8, 9, 10, 11, 16, 17
LG4	6, 7

Informal Assessments

Students begin each day with a three to four question warm-up on review material from previous classes. All learning goals are assessed as they are introduced into the

lessons. As the lessons progress, the warm-ups will contain questions regarding materials from previous classes as well as questions regarding basic trigonometric knowledge that each student should know from previous units. Questions on warm-ups are one point each. Students will either receive one point for correct work and answer or zero points for wrong work or answer. The points awarded for warm-ups are used as extra credit participation points. Students are allowed to use notes to complete the work, but must work independently. The main goals for the daily warm-ups are to get the students to start making connections between each new lesson and the previous lessons and prior knowledge. The warm-ups are designed to keep the students thinking.

Homework is assigned every night from the given textbook. Homework is graded on a zero to three point scale evaluated based on effort. If the student shows work, and effort is evident, than that student will receive the full three points. If the homework is well done but incomplete then the student will receive two points. If very little is done, but some effort is shown the student will receive one point. If the homework is blank or it appears that no effort was put into completing it then the student will receive a zero. Each homework assignment was designed around the day's objectives and the unit learning goals.

Expectations

As stated in chapter one, “the Common Core State Standards articulate rigorous grade-level expectations in the area of mathematics” (Common Core State Standards Initiative, 2014, p. 1) The guidelines of CCSS indicates students who are identified as having a disability under the Individuals with Disabilities Education Act (IDEA) will also be held to the same high standards as all students in the general classroom, including

those diagnosed with ADHD. Although students with ADHD face hardships that general education students often do not, the CCSS fundamental goal is to prepared all students for success in their post-school lives, including college and/or careers (Common Core State Standards Initiative, 2014, p. 1). For these reasons, all students will be held to the same high standards on all assessments including the formal post-assessment.

Basic Trigonometry, Angles as Rotations, Radian Measure

Intermediate Algebra

CCSHS -11th and 12th Grade

Day 1: 2/13/13 & 2/14/13

Objectives

Students will be able to:

- Identify and evaluate coterminal angles.
- Identify and evaluate quadrant-angle relationships and quadrantal angles.
- Convert radians to degrees and degrees to radians.

Anticipatory Set:

- (1). Students will first work on a four-question warm-up on review materials in order to activate prior knowledge and to allow teacher to quickly assess student needs for the upcoming lesson.
- (2). The goals/expectation for lesson will be discussed with the class.
- (3). Students will be directed to hand in warm-up, grab a note packet, two different colored crayons and a textbook.

INPUT/Modeling:

- (1). Students will first be instructed to identify page numbers and define vocabulary words by using their textbooks.
- (2). Once class is finished with vocabulary activity, we will begin to go through the notes. I will model examples and use/explain the newly defined vocabulary.

Guided Practice:

- (1). Students will be asked to work in partners (of their own choosing).
- (2). The student will be directed to work on coterminal exercises using crayons to color code each angle.
- (3). While the students are working on their exercises, I will be walking around and assessing student progress. This will allow opportunity for quick re-teaching to partners that are struggling with the concept.
- (4). The class will be brought back together to address any questions.
- (5). We will go over the second section of the note packet on radian measure. I will again model examples and use/explain the newly defined vocabulary.
- (6). Students will work on radian measure exercise independently.

Closing:

- (1). Students will be brought back together to discuss any questions they have and to recap the day's lesson.
- (2). Students may work on homework independently until the end of class.

NYS Standards & Common Core

NYS: A.PS.3, A.PS.6, A.CM.8, A.CN.2, A.M.2, A2.A.60, A2.A.61, A2.M1, A2.M2

Common Core: F-TF.A1, F-TF.A2

Materials

- Textbook: Amsco's Mathematics B
- Calculator
- Crayons
- Note packet
- Over head projector

Differentiation

This lesson will include visual activities along with written notes. It will also have various examples that depict the concepts being taught. The lesson will be broken up into individual work, partner work and whole class instruction.

Independent Practice

Homework: From Textbook

Page 614: 7-15 (Odd)

Page 618: 3-36 (x3)

Page 667: 6-27 (x3), 33, 35, 40, 44

Assessments

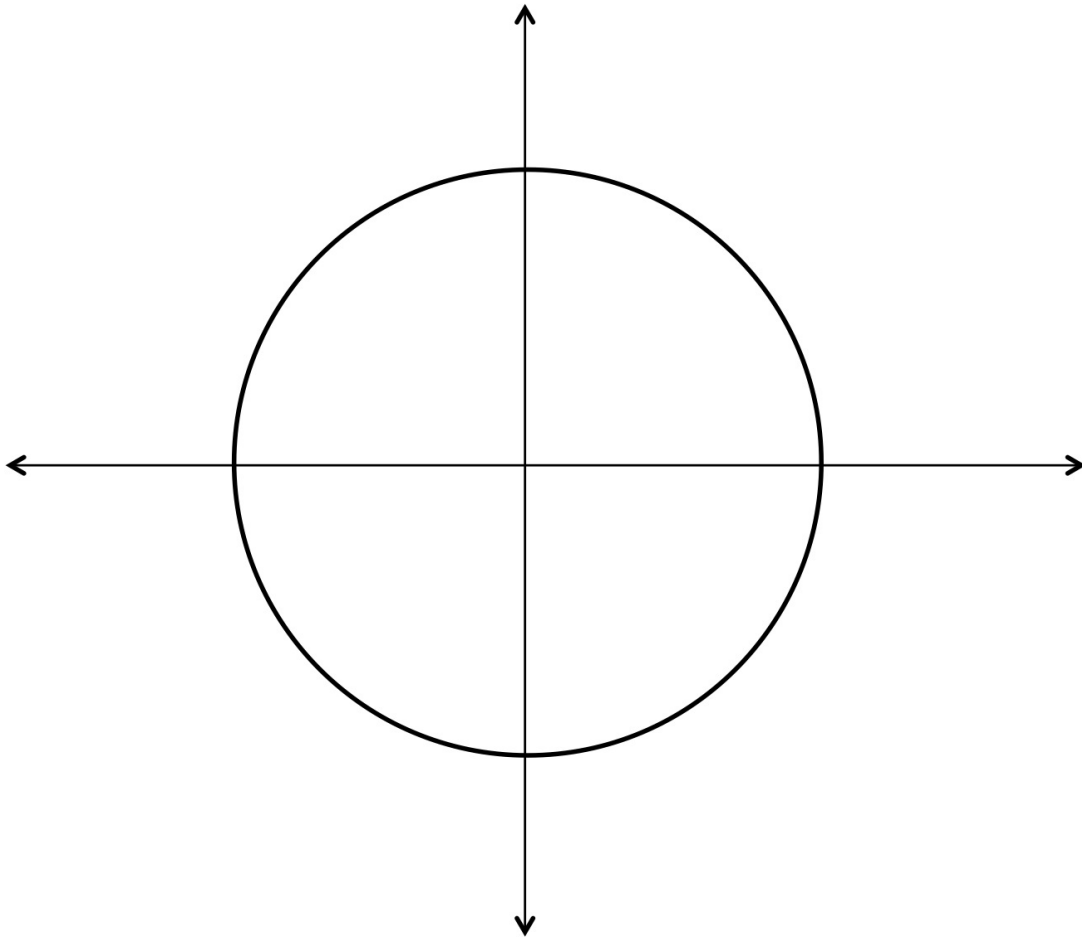
Warm-up, Participation and Discussion

NAME: _____

IA Note Packet Day 1

Homework (from textbook) → Page 614: 7-15(odd)
Page 618: 3-36 (every 3rd)
Page 667: 6-27 (every 3rd), 33, 35, 40, 44

The Unit Circle (aka your best friend)!



Lesson One Vocabulary: ☺

1. Standard Position:

Pg. _____

Definition:

2. Initial Side / Terminal Side:

Pg. _____

Definition:

3. Quadrants:

Pg. _____

Definition:

4. Coterminal Angles:

Pg. _____

Definition:

5. Quadrantal Angles:

Pg. _____

Definition:

6. Radian:

Pg. _____

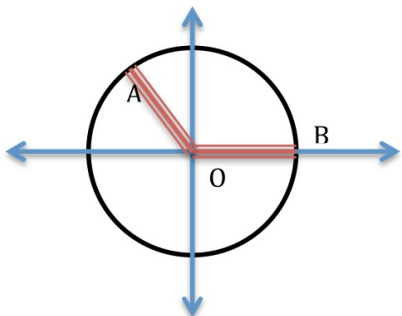
Definition:

Draw a picture to represent word



Angles and Rotations

Pgs. 614-618



Standard position is when _____.

\overline{OB} represents the _____ side.
Rephrase in your own words:

\overline{AO} represents the _____ side.
Rephrase in your own words:

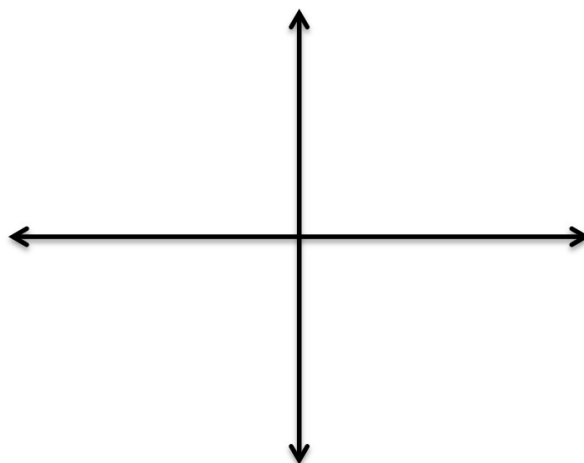
*****IMPORTANT*****

- An angle formed by a counterclockwise rotation has a _____ measure.
- An angle formed by a clockwise rotation has a _____ measure.

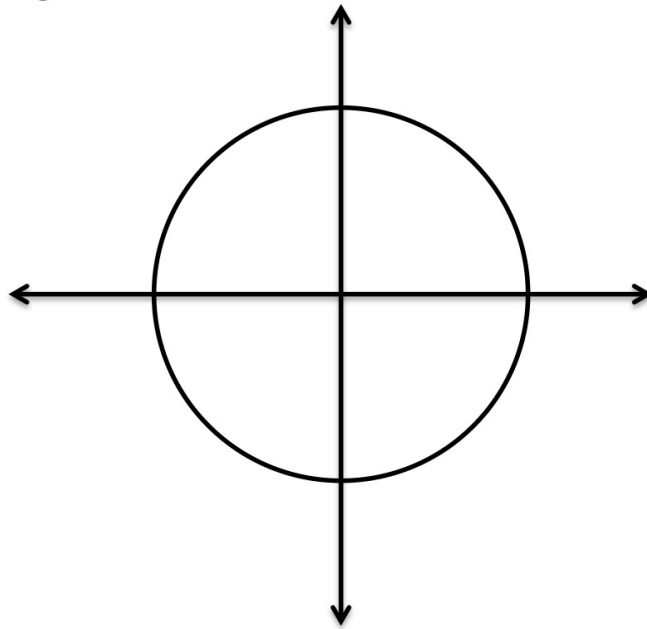


Labeling the coordinate plane:

Pg. 615



Coterminal Angles:
Pgs. 616-617



EXERCISE!

Find at least two angles that are coterminal with the given angles:

(a). 375°

(b). 580°

(c). -30°

(d). -110°

(e). -360°

(f). -75°

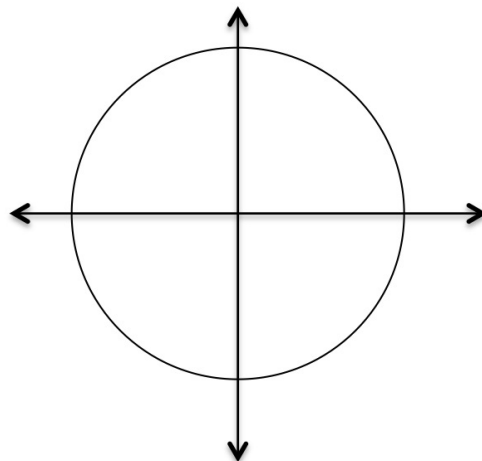
*In your own words, how do you find
coterminal angles:*

Radian Measure

Pgs. 661-668

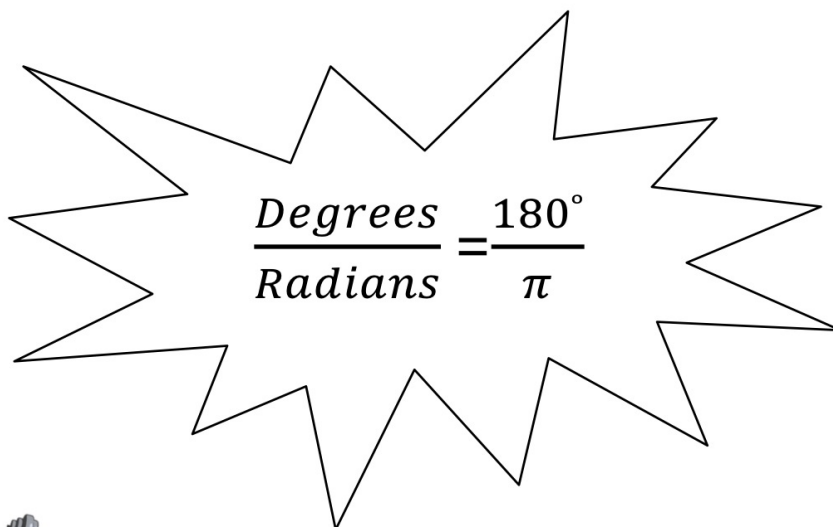
General rule:

$$\theta = \frac{(\text{intercepted arc})}{(\text{radius})} = \frac{s}{r}$$



Relationship between Degrees and Radians:

pgs. 662-663


$$\frac{\text{Degrees}}{\text{Radians}} = \frac{180^\circ}{\pi}$$

EXERCISE!



Convert from degrees to radians:

$$270^\circ$$

Convert from radians to degrees:

$$\frac{\pi}{10}$$

The Unit Circle, Sine, Cosine and Tangent as Coordinates

Objectives:

Students will be able to:

- Evaluate Sine and Cosine as coordinates using the Unit Circle.
- Identify and evaluate quadrant-sine/cosine/tangent relationship.
- Evaluate trigonometry function values of coterminal angles.

Anticipatory Set:

- (1). Students will first work on a four-question warm-up on review materials in order to activate prior knowledge and to allow teacher to quickly assess student needs for the up coming lesson and check homework.
- (2). Go over homework answers/address any question about warm-up or homework.
- (3). Students will take five-question quiz on prior lesson material.
- (4). The goals/expectation for lesson will be discussed with the class.

INPUT/Modeling:

- (1). Students will be given note pack for day-two lesson.
- (2). There will be a quick review on the unit circle and trigonometric functions to help further activate student's prior knowledge.
- (3). New material will be discussed and examples of concepts will be modeled.
- (4). Go through Note Packet.

Guided Practice:

- (1). Students will be directed to get into groups of 3 or 4. They will be working on filling in the assigned chart and asked to investigate the patterns of this chart (filling in quadrants, radians and signs for tangent functions). After they will color code the chart based on where each function is positive.
- (2). Bring class together to fill in Unit Circle as much as possible (interactive Unit Circle).
- (3). Model thought process for class.
- (4). Have students finish note packet (if needed)/answer questions

Closing:

- (1). Students will be brought back together to discuss the days lesson/answer any questions
- (2). Students may work on homework.

Intermediate Algebra

CCSHS - 11th and 12th Grade

Day 2: 2/15/13 & 2/25/13

NYS Standards & Common Core

NYS: A.CM.2, A.CM.4, A.CN.4,
A2.A.55, A2.A.56, A2.A.62

Common Core: F-TF.A1, F-TF.A2

Materials

- Textbook: Amsco's Mathematics B
- Calculator
- Note packet
- Crayons
- Over head projector

Differentiation

This lesson will include visual representations of the concepts along with written notes. The class will be broken into group work and whole class instruction. Homework will be graded on a scale of 0-3 based on effort.

Independent Practice

Homework: From Textbook

Page 621: 1

Page 624: 1-7 (odd), 8

Page 630: 7-17 (odd), 27, 29

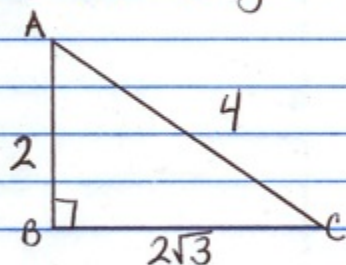
Assessments

Warm-up, Homework, Quiz,
Participation and Discussion

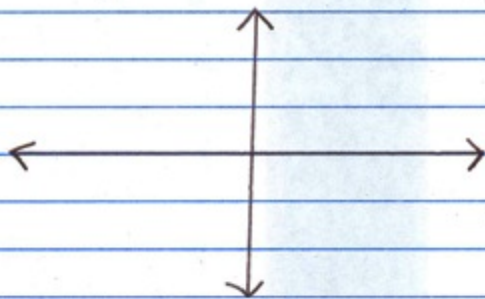
Name: _____

IA Quiz #1

1.) Solve for angle A and angle C:



2.) First label the quadrants and then assign an appropriate angle to each of the quadrants.



3.) Which set of angles are coterminal?

Ⓐ -65° and 300° Ⓑ 450° and 90° Ⓒ 25° and 380°

4.) Convert 170° to radians

5.) Convert $6\pi/7$ to degrees

NAME: _____

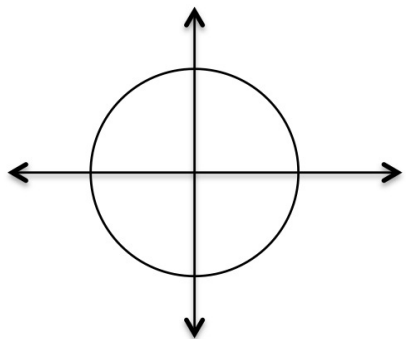
Homework (from textbook) →

Page 621: 1

Page 624: 1-7(odd), 8

Page 630: 7-17(odd), 27, 29

*** Remember The Unit Circle***

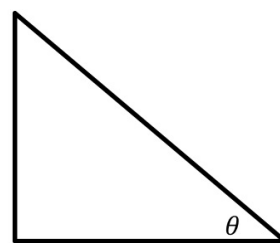


Remember Trig-Functions

Sin θ = —

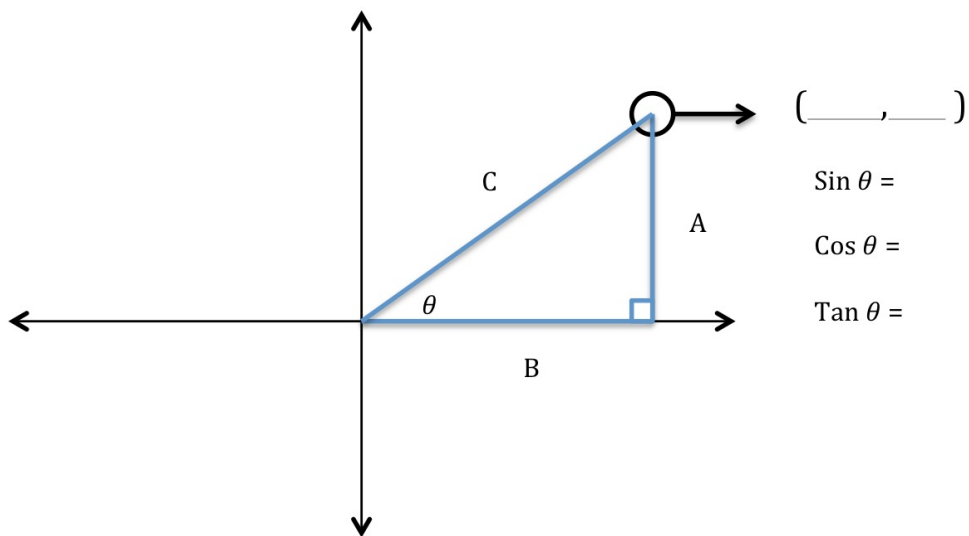
Cos θ = —

Tan θ = —

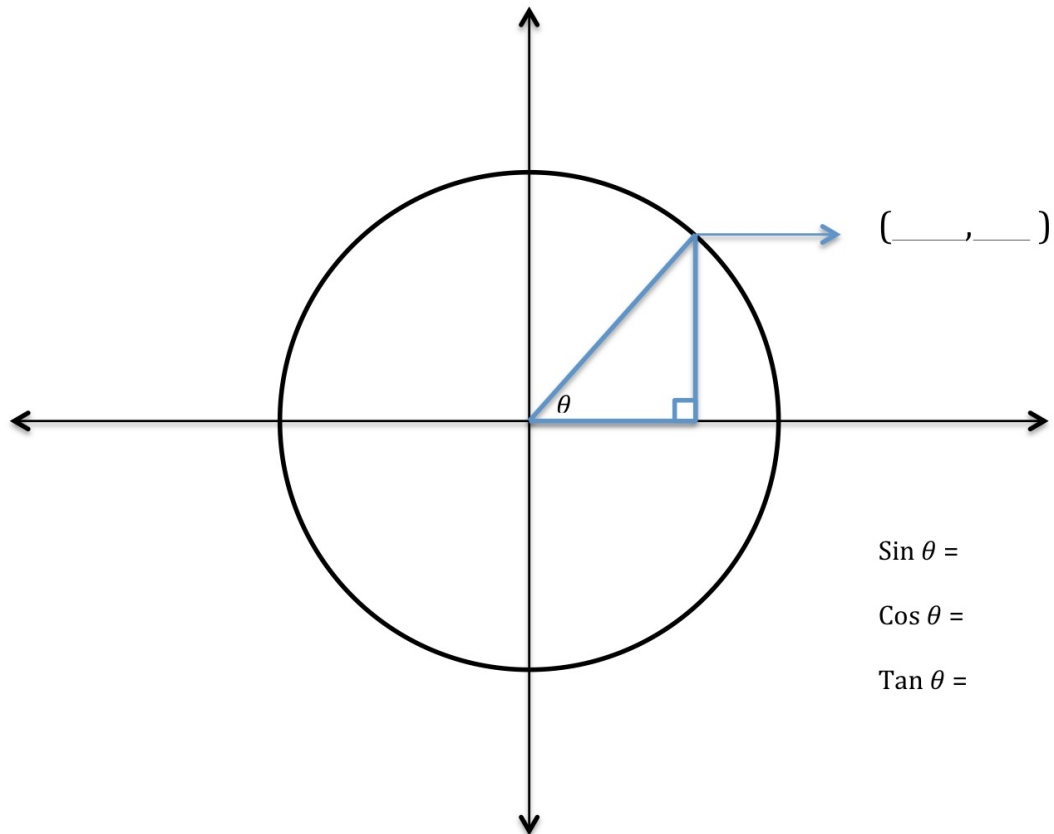


Sine and Cosine as Coordinates

Pgs. 618-620



Sine and Cosine as Coordinates on the Unit Circle:



***General Rule:
Pgs. _____.

EXERCISE!

Determine $\sin \theta$ and $\cos \theta$ when given Point P $(\frac{-3}{4}, \frac{\sqrt{7}}{4})$ on the Unit Circle:

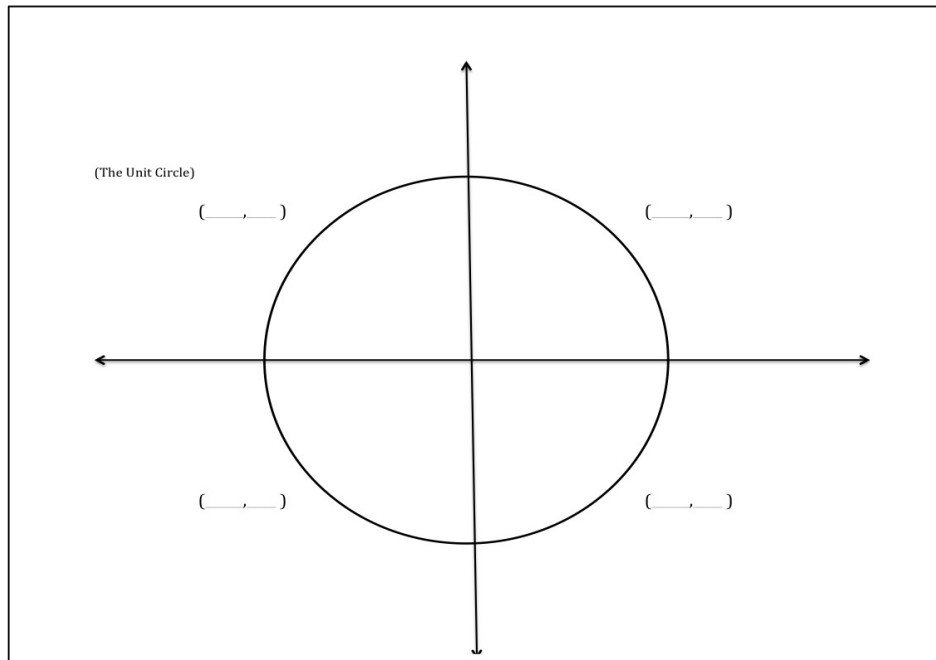
Quadrantal Angles on the Unit Circle: (No Calculators)

Pg. _____.

Degrees θ	$0^\circ, 360^\circ$	90°	180°	270°
Sine θ				
Cosine θ				
Tangent θ				
Draw angle in standard position				

External Aid: * The use of external aids can help reduce the working memory load to avoid working memory overload for students with ADHD* (see Chapter 2). The following aid was given to student to fill out to use as a guide on further work.

Degrees θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Negative Coterminal Angle																	
Reference Angles																	
Quadrant																	
Sin θ																	
Cos θ																	
Tan θ																	



Function Values of Special Angles and finding Reference Angles

Intermediate Algebra

CCSHS-11th and 12th Grade

Day 3: 2/26/13 & 2/27/13

Objectives

Students will be able to:

- Find exact numerical values of trigonometry functions with special angles on the unit circle.
- Identify and evaluate reference angles.

Anticipatory Set

- (1). Students will first work on a quick two-question warm-up on review material in order to activate prior knowledge and to allow teacher to quickly assess student needs for the up coming lesson and to check homework.
- (2). Go over homework answers/address any question about warm-up or homework.
- (3). Students will take five-question quiz on prior lesson material.
- (4). The goals/expectation for lesson will be discussed with class.

INPUT/Modeling

- (1). Students will be given a note pack for day-three lesson.
- (2). There will be a quick review on the unit circle (have students label the unit circle) and trigonometric functions involving special angles to activate student's prior knowledge.
- (3). New material will be discussed and examples of concepts will be modeled to the class.
 - a. Special triangles with hypotenuse = 1, fill in chart and add to unit circle diagram.
 - b. Reference angles, go through each quadrant ("bow-tie")

Guided Practice

- (1). Have students get into small groups/partners and generalize their understanding of reference angles in a chart format.
- (2). Have students fill in the rest of the chart from day-two lesson.
- (3). Allow time for questions and re-teaching opportunities.

Closing

- (1). Students will be brought back together to discuss the days lesson/answer any questions
- (2). Students may work on homework if time allows.

NYS & Common Core Standards

NYS: A.RP.1, A.CM.9, A.CN.1, A2.A.55, A2.A.56, A2.A.57, A2.A.60

Common Core: F-TF.A3

Materials

- Textbook: Amsco's Mathematics B
- Calculator
- Note packet
- Over head projector

Differentiation

This lesson will include visual representations of the concepts along with written notes. Class time will be split into whole class instruction and small group work. Homework will be graded on a scale of 0-3 based on effort. Textbook pages will be given for students to reference throughout the lesson. Time will be left for re-teach and questions.

Independent Practice

Homework from textbook:

Page 634: 1-15 (odd), 28-32 (even)

Page 660: 3-24 (x3)

Assessment

Warm-up, Homework, Quiz, Participation and Discussion

Name: _____

1A Quiz #2

1.) Evaluate:

a) $\sin 30^\circ$

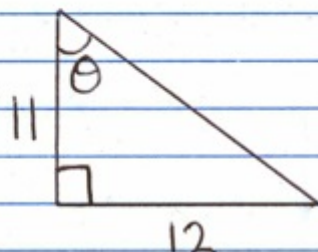
b) $\sin 210^\circ$

2.) What is 540° in radians? (show work)

3.) What quadrant is θ in if $\cos \theta < 0$ and $\sin \theta < 0$?

4.) Given the point $P(.6, .8)$ on the Unit Circle, what is $\cos \theta$ and $\sin \theta$?

5.) Solve for θ :



NAME: _____

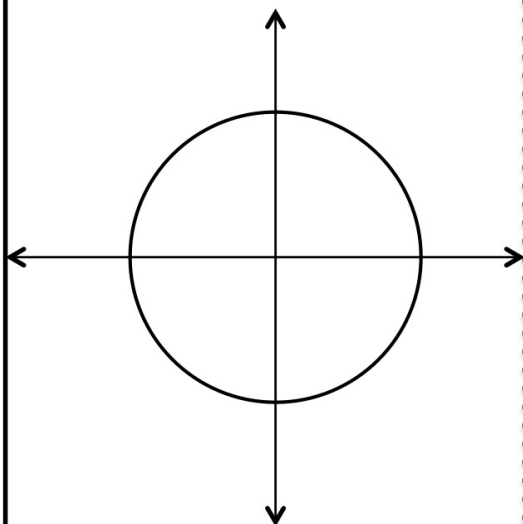
Homework (from text book) →

Page 634: 1-15(odd), 28-32(even)

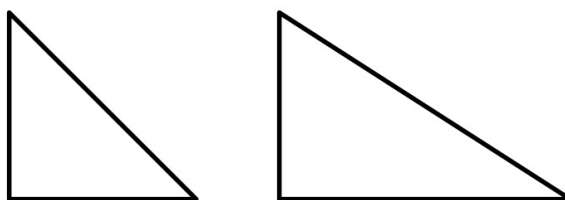
Page 660: 3-24 (every 3rd)

Review

Unit Circle!



Reference Triangles with special angles!

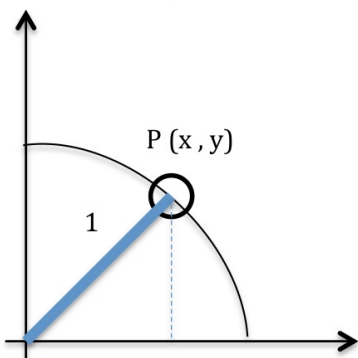
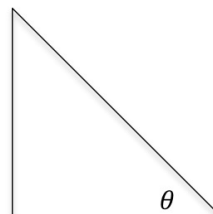


Trigonometry Functions!

Sin θ = —

Cos θ = —

Tan θ = —



Sin P =

Cos P =

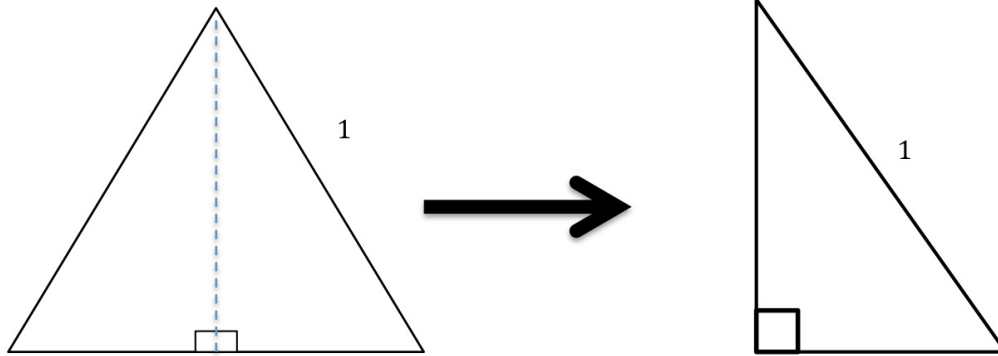
Tan P =

Function Values of Special Angles

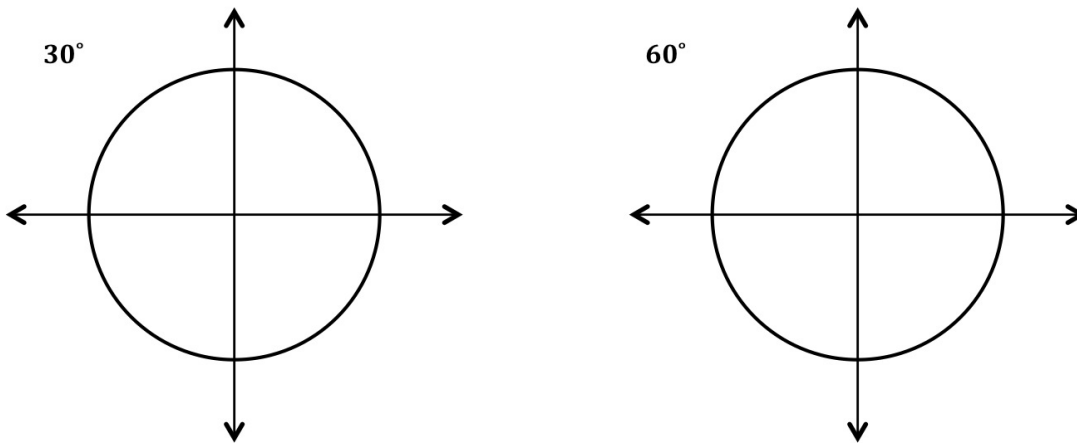
Pgs. 631-634

Angles of 30° and 60° :

From Equilateral Triangle \rightarrow

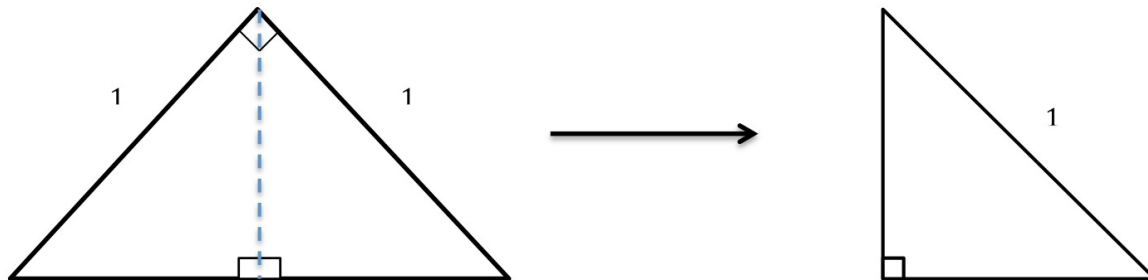


To Unit Circle \rightarrow

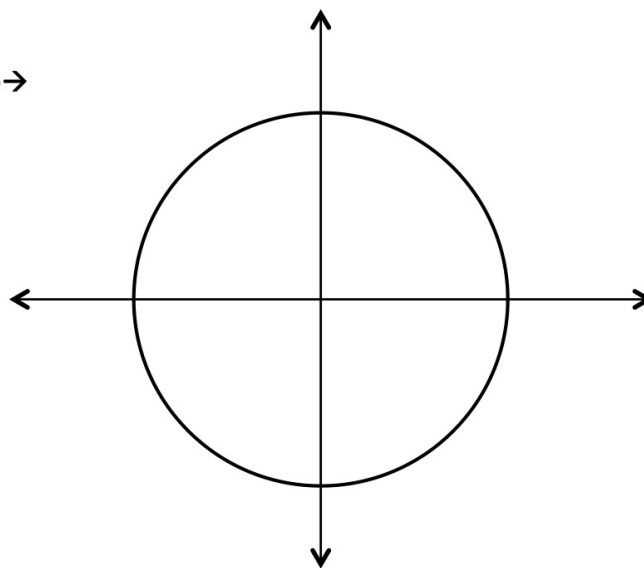


Angles of 45° :

From isosceles triangle \rightarrow



To the Unit Circle \rightarrow



θ	30°	45°	60°
Sin θ			
Cos θ			
Tan θ			

Finding Reference Angles

Pgs. 651-660

WHAT IS A REFERENCE ANGLE???????

Definition:

Pg. ____.

<p>Quadrant I Pg. ____.</p>	<p>Quadrant II Pg. ____.</p>
<p>Quadrant III Pg. ____.</p>	<p>Quadrant IV Pg. ____.</p>

Generalize Reference Angles!

$90^\circ < \theta < 180^\circ$	$180^\circ < \theta < 270^\circ$	$270^\circ < \theta < 360^\circ$
Quadrant:	Quadrant:	Quadrant:
$\sin \theta = \sin (180^\circ - \theta)$	$\sin \theta =$	$\sin \theta =$
$\cos \theta =$	$\cos \theta =$	$\cos \theta =$
$\tan \theta =$	$\tan \theta =$	$\tan \theta =$

EXERCISE



Find exact Numerical value of the expressions below:

- $(\sin 30^\circ)^2$
- $\tan 45^\circ + \cos 0^\circ$
- $\cos 45^\circ$

Find each exact function value:

- $\sin 100^\circ$
- $\cos 240^\circ$
- $\tan 675$

Inverse Trigonometry Functions

Intermediate Algebra

CCSHS-11th and 12th

Day 4: 2/28/13 & 3/1/13

Objectives

Students will be able to:

- Define inverse (arc) in relation to trigonometric functions.
- Evaluate values of inverse trigonometric functions
- Write expressions in terms of trigonometric functions and in terms of their inverses.

Anticipatory Set

- (1). Students will first work on a quick two-question warm-up on review material in order to activate prior knowledge and to allow teacher to quickly assess student needs for the up coming lesson and to check homework.
- (2). Go over homework answers/address any question about warm-up or homework.
- (3). Students will take an “Up to Now” ten-question quiz on all material up to this point.
- (4). The goals/expectation for lesson will be discussed with class.

INPUT/Modeling

- (1). Students will be given a note-packet for day-four lesson.
- (2). We will discuss what inverse means and look it up in the textbook.
- (3). “You have already been doing this!”
- (4). Model a few examples with explanations.

Guided Practice

- (1). Allow time for questions and re-teach opportunity.
- (2). Instruct students to get into small groups or pairs. Have them work on the page of exercises.

Closing

- (1). Students will be brought back together to discuss the days lesson/answer any questions
- (2). Students may work on homework if time allows.

NYS & Common Core Standards

NYS: A.PS.3, A.CM.1, A.CM.3, A.CN.1, A2.A.55, A2.A.62, A2.A.64, A2.A.66

Common Core: F-TF.A6, F-TF.A7

Materials

- Textbook: Amsco’s Mathematics B
- Calculator
- Note packet
- Overhead projector

Differentiation

This lesson will include visual representations of the concepts along with written notes. Class time will be split into whole class instruction and small group work. Homework will be graded on a scale of 0-3 based on effort. Textbook pages will be given for students to reference throughout the lesson. Time will be left for re-teach and questions.

Independent Practice

Homework from textbook:

Page 743: 3-21 (x3), 23, 25-28 & 32

Assessments

Warm-up, Homework, Extended Quiz, Participation and Discussion

Name: _____

1A Quiz # 3
"Up to Now Quiz!"

1.) If $\sin \theta = \frac{4}{5}$, find the value of $\cos \theta$ and $\tan \theta$:

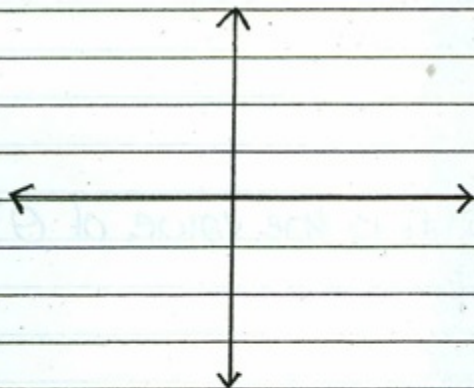
2.) Convert:

a) $\frac{3\pi}{2}$ to degrees

b) 100° to radians

3.) Label the coordinate plane: a) quadrants

b) where $\sin \theta$, $\cos \theta$
and $\tan \theta$ are
positive.



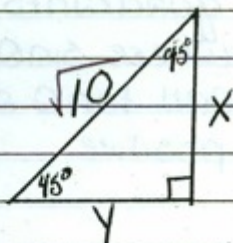
4.) Given the point $P(6, 8)$ on the unit circle, what is the value of $\sin \theta$ and $\cos \theta$?

5.) Find the exact value for $\sin 210^\circ + \cos 120^\circ$:
(must show work)

6.) In which quadrant does θ lie if $\sin < 0$ and $\tan > 0$?

7.) Identify two angles that are coterminal with 35° :

8.) Solve for x and y



9.) If $\sin \theta = \sqrt{3} \cos \theta$, what is the value of θ ?

10.) Evaluate $(\sin 390)(\cos -300)$?
(must show work)

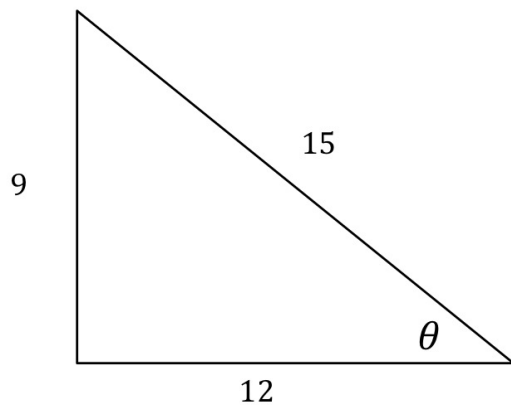
NAME: _____

IA Note Packet Day 4

Homework (from text book) → Page 743: 3-21 (every 3rd), 23, 25-28, 32

REVIEW

Determine the value of θ



WORK:

Inverse Trigonometric Functions

Pgs. 735-743

Inverse

Definition:

“When the sets of points $y = \sin x$, $y = \cos x$, and $y = \tan x$ are reflected over the line $y = x$, the images are the sets of points $y = \arcsin x$, $y = \arccos x$, and $y = \arctan x$.” - Page 735

What is the difference between $y = \arcsin$ and $y = \text{Arc Sin}$?

Pg. ____.

NOTATION

$y = \text{Arc Sin } x$ is the same thing as $y =$ _____ .

$y = \text{Arc Cos } x$ is the same thing as $y =$ _____ .

$y = \text{Arc Tan } x$ is the same thing as $y =$ _____ .

EXERCISES



Find the Value of θ in degrees:

a. $\theta = \text{Arc Cos} \left(\frac{1}{2} \right)$

b. $\theta = \text{Arc Sin} (0)$

c. $\theta = \text{Arc tan} (-\sqrt{3})$

Find the value of θ in radians:

a. $\theta = \text{Arc sin} \left(\frac{-1}{2} \right)$

b. $\theta = \text{Arc Cos} \left(\frac{-1}{2} \right)$

c. $\theta = \text{Arc tan} (\sqrt{3})$

Find the value of each expression:

a. $\text{Cos} (\text{Arc Sin } 1)$

b. $\text{Sin} (\text{Arc Cos } 0)$

Trigonometric Functions with Radian Measure

Intermediate Algebra

CCSHS-11th and 12th Grade

Day 5: 3/4/13 & 3/5/13

Objectives

Students will be able to:

- (1). Evaluate trigonometric functions with radian measures.

Anticipatory Set:

- (2). Students will work on a three-question warm-up on material from past lesson in order to help activate prior knowledge and allow time to check homework completion and assess student needs for the upcoming lesson.
- (3). Homework answers will be put up on the overhead project for students to check their work and ask questions.
- (4). Students will be given a four-question quiz on the previous lesson taught.
- (5). The goals/expectation for lesson will be discussed with the class.

INPUT/Modeling:

- (1). Students will pick up day-five note packet and begin exercise (of review material) on front page until all students are done with quizzes.
- (2). As a class, we will review converting radians to degrees and degrees to radians, reference angles, writing functions as functions of positive acute angles and using calculators to evaluate trigonometric functions.
- (3). The students will convert 0° , 30° , 45° , 60° , 90° ... 360° into radians (chart) individually.
- (4). I will introduce trigonometric functions with radian measure and model a few examples.

Guided Practice:

- (1). Have students work in groups on classwork. One page of trigonometric functions with radian measure exercises and a unit circle in which the students will be instructed to label anything they think is important to know about the unit circle.

Closing:

- (1). Bring students back together and recap the day's lesson.
- (2). Allow students to work on homework if time allows.

NYS Standards & Common Core

NYS: A.PS.2, A.PS.6, A.RP.1, A.CN.2, A.CN.4, A2.A.56, A2.A.57, A2.A.68, A2.M.2

Common Core: F-TF.A1, F-TF.A2, F-TF.A3,

Materials

- Textbook: Amsco's Mathematics B
- Calculator
- Note packet
- Over head projector

Differentiation

This lesson will include both visual representations of concepts along with written notes to explain concepts. Page numbers will be given with notes for future reference. The class time will be broken up into full class instruction, small group work and individual work. Homework will be evaluated on a scale of 0-3 based on effort.

Independent Practice

Homework: From Textbook

Page 671: 1-30 (odd)

Assessments

Warm-up, Quiz, Participation and Discussion

Name: _____

1A Quiz #4

1.) Find the value of θ in radians when
 $\theta = \text{Arc Sin}(-\frac{1}{2})$

2.) Express $\cos 305^\circ$ as a function of a positive acute angle

3.) Find the value of $\tan(\text{Arc cos } \frac{4}{5})$

4.) Determine one angle coterminal with 250°

NAME: _____

IA Note Packet Day 5

Homework (from text book)→

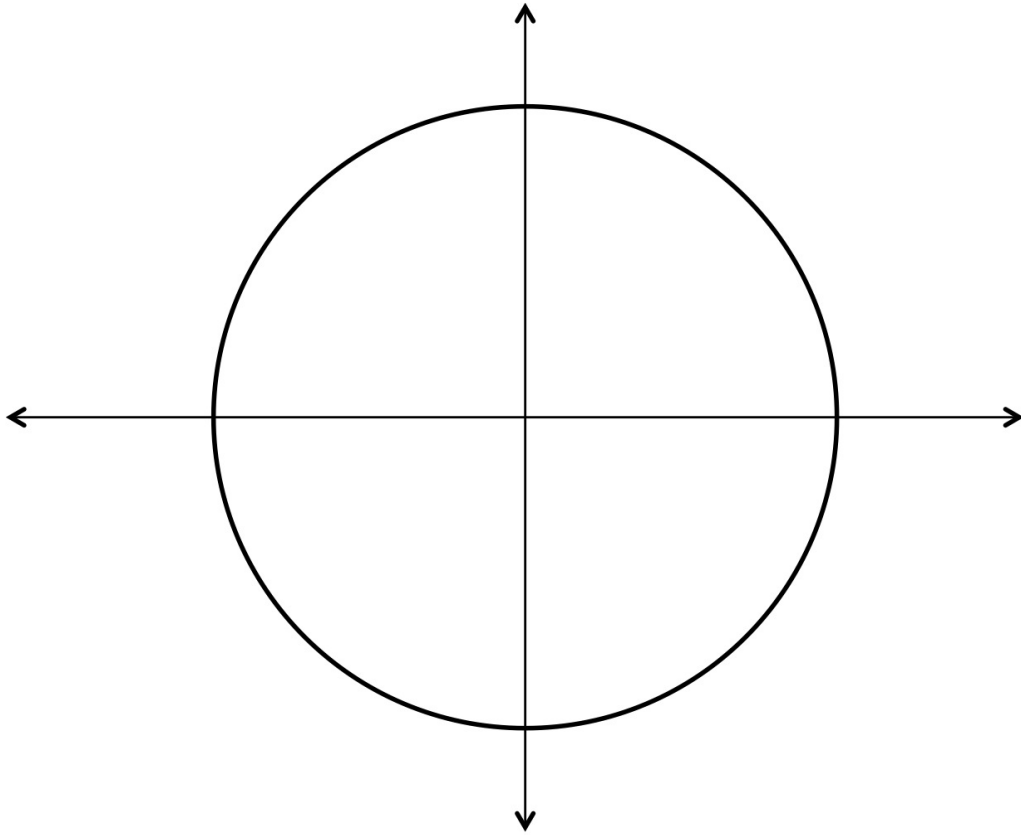
Page: 671: 1-30 (odd)

CONVERTING DEGREES TO RADIANS:

CONVERTING RADIANS TO DEGREES:

FINDING REFERENCE ANGLES:

Degree	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radian									



TRIGONOMETRIC FUNCTIONS INVOLVING RADIAN MEASURE

Pgs. 668-672

Your textbook will ask you to:

Find the exact value of each trigonometric function

Steps:

1. Convert radian measure to degrees
2. Determine which quadrant function is in.
 - a. Will the value be positive or negative?
3. Determine reference angle and write function as a positive acute angle
4. Find exact value

Example:

$$\cos \frac{4\pi}{3}$$

Work:

Given the function $f(x) = \cos\left(\frac{x}{2}\right)$

Determine $f(4\pi)$

Determine $f\left(\frac{\pi}{2}\right)$

Basic Sine and Cosine Graphs

Objectives:

Students will be able to:

- (1). Draw and evaluate the graphs of sine and cosine
- (2). Evaluate the period of the sine and cosine graphs
- (3). Make the connection between the Unit Circle and the graphs of Sine and Cosine.

Anticipatory Set:

- (1). Students will be instructed to pick up a chart to which they will convert 0° to 360° to radian measures. This will help to activate prior knowledge and allow time for a homework check and assessment of student needs for this lesson.
- (2). Homework answers will be posted for students to self check their homework and ask questions.
- (3). The goals/expectation for lesson will be discussed with the class.

INPUT/Modeling

- (1). Students will be given material for the Unit Circle activity.
 - a. Printed Unit Circle, cut out 45° , 45° , 90° and 30° , 60° , 90° triangles and check list.
- (2). I will model the first few steps of the activity and then allow the students to follow my lead.
- (3). I will assess student needs and answer questions as the activity progresses.

Guided Practice

- (1). With the finished unit circle, students will create the Sine and Cosine graphs on a separate paper.
- (2). As a class, we will analyze the graphs and make note of the period and range of each graph.

Closing:

- (3). I will pull the class back together to recap the day's lesson and answer questions.
- (4). If time, students will be allowed to work on homework.

Intermediate Algebra

CCSHS- 11th and 12th

Day 6: 3/6/13 & 3/7/13

NYS & Common Core Standards

NYS: A.CN.1, A.R.1, A.R.3, A2.A.60, A2.A.70, A2.M.2

Common Core: F-TF.A2, F-TF.A.3, F-TF.A5

Materials

- Textbook: Amsco's Mathematics B
- All note-packets
- Calculator
- Unit Circle Activity (computer paper, floss, markers)

Differentiation

This lesson will include a hands-on activity that connects two concepts of the unit (unit circle and sine/cosine graphs). Class time will be split into whole class instruction and small group work. Homework will be graded on a scale of 0-3 based on effort. Time will be left for re-teach and questions.

Independent Practice

Homework from textbook:

Page 710: 1-7, 9

Page 713: 1-8

Assessments

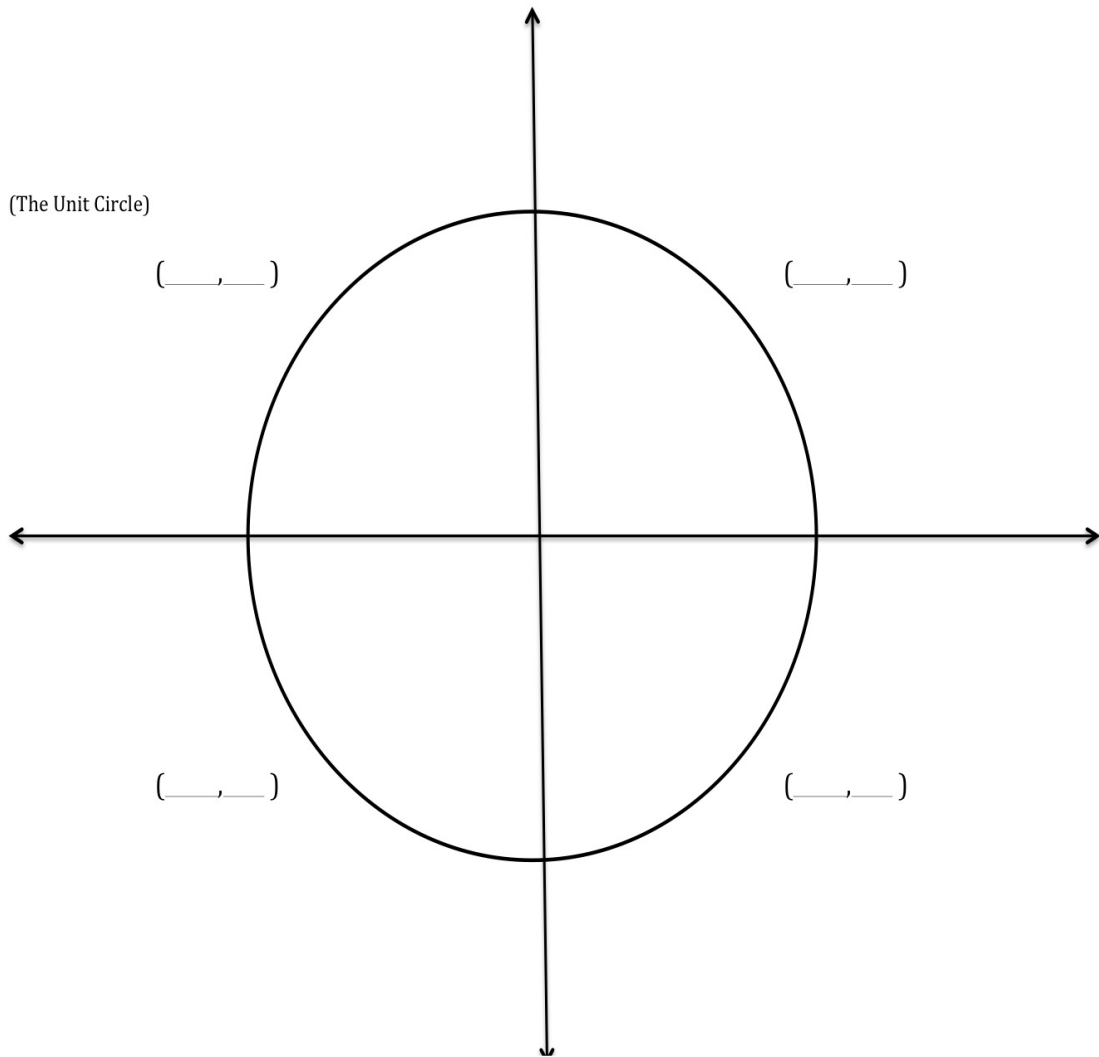
Chart, Homework, Activity, Participation and Discussion

Quiz #5

Label:

- Quadrants
- 0, 90, 180, 270 & 360 degrees
- Coordinate points on x and y-axis'
- 30 degrees and one coterminal angle
- Show where sine, cosine & tangent are positive

(The Unit Circle)



Unit Circle Floss Activity

*External Aid *

Degree θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Radian Measure																	

Formula for converting Degrees \leftrightarrow Radians:

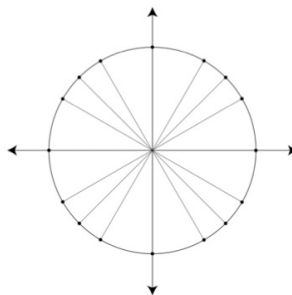
Materials needed for activity:

- Blank Computer Paper
- Blank Unit Circle Paper
- 30, 60 45 degree Triangle cut outs
- Piece of Floss (length = 1.5 ft.)
- Colored Marker
- Scissors
- Tape

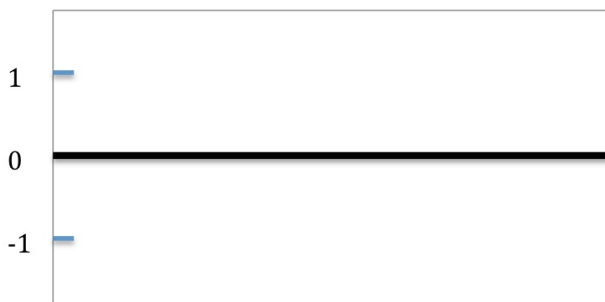
Unit Circle –Sine & Cosine Curves – Floss Activity

Follow Instructions:

- I. Cut out 30, 60 and 45 degree triangles
- II. On Unit circle, use special triangles to mark the angles around the unit circle as seen below.



- III. Using external aids, label the unit circle in both degrees and radian measures
- IV. Using one end of the floss (mark that end of floss with red dot), wrap the floss around the unit circle starting at 0 degrees and ending at 360 degrees.
- V. Using your colored marker, make a mark at each point on the unit circle that you already labeled.
- VI. Put floss to the side
- VII. Take blank piece of computer paper and fold it in half horizontally. Open the paper back up and use a pen to make the line created darker. Mark the Left edge of the paper as shown below



- VIII. Take floss and starting with the same end you started at 0 degrees, place the floss over the line you just created at the far left side of the paper.
- IX. Use your marker to transfer your marks from the floss to the line on the computer paper.
- X. Mark the line where you created marks for 90, 180, 270 and 360 degrees
- XI. Use external aid to make marks at 1, 0 and -1 in accordance to the degrees labeled for SINE functions.
- XII. Take your pencil and now trace the graph.
- XIII. YOU HAVE CREATED THE SINE GRAPH!

REPEAT STEPS WITH COSINE FUNCTIONS TO CREATE THE COSINE GRAPH

Review: Chapter 7- Trigonometric Functions

Intermediate Algebra

CCSHS-11th and 12th Grade

Day 7: 3/8/13 & 3/11/13

Objectives

Students will be able to:

- Recall basic trigonometry, angles as rotation, radian measure/conversions, the unit circle, sine/cosine/tangent as coordinates, function values of special triangles, reference angles, inverse trigonometry functions, trigonometric functions of radian measures and the basic sine and cosine graphs.
- Apply/combine concepts correctly when necessary.

Anticipatory Set

- (1). Students will be given a four-question warm-up on past lesson materials to activate student's prior knowledge and to allow time for homework check and overall student assessment of needs for the day.
- (2). Students will be given review packet. They will be instructed to work independently for twenty minutes before asking any questions.

INPUT/Modeling

- (1). Students will be instructed to get into small groups or pairs to discuss questions that arose while they worked on the review packet.
- (2). If many students struggle with a certain problem or concept, I will pull the class together and model the example and explain my reasoning.

Guided Practice

- (1). Students will be given the opportunity to work in pairs or small groups to finish the review packet.

Closing

- (1). Students will be brought back together to discuss class questions.
- (2). If time, students may begin working on their homework independently.

NYS & Common Core Standards

NYS: A.PS.2, A.PS.5, A.PS.6, A.PS.8, A.RP.1, A.CM.1, A.CM.3, A.CN.1, A.CN.2, A.CN.4, A2.A.55 --> A2.A.57, A2.A.60 --> A2.A.66

Common Core: F-TF.A1 --> F.TF.A5

Materials

- Textbook: Amsco's Mathematics B
- Calculator
- Note packet(s)
- Over head projector

Differentiation

This lesson will allow for one on one instruction/re-teach along with small group instruction/re-teach. Students will have access to textbooks and notes for reference. Homework will be graded on a scale of 0-3 based on effort.

Independent Practice

Homework: From Textbook

Page 699: 2, 5, 10, 11, 12-21(x3), 36, 40-42

Page 747: 1, 5, 23, 24, 50

Assessments

Warm-up, Homework, Participation and Discussion

Test: Chapter 7 - Trigonometric Functions

Intermediate Algebra

CCSHS-11th and 12th Grade

Day 8: 3/12/13 & 3/13/13

Objectives

Students will be able to:

- Recall basic trigonometry, angles as rotation, radian measure/conversions, the unit circle, sine/cosine/tangent as coordinates, function values of special triangles, reference angles, inverse trigonometry functions, trigonometric functions of radian measures and the basic sine and cosine graphs.
- Apply/combine concepts correctly when necessary.

Anticipatory Set

- (1). Students will be given a blank unit circle and special triangles to fill in for a warm-up. This will help activate prior knowledge and allow time for homework check and for me to assess student needs.

INPUT/Modeling

- (1). As a class we will discuss the important parts of the unit circle and the sides of the special triangles. They will be able to use this on their unit test as a reference.

Guided Practice

- (1). Students will take the unit test independently.

Closing

- (1). Once students are done with the test, they will be given a cumulative review packet to work on. That packet must be completed for homework.

NYS & Common Core Standards

NYS: A.PS.2, A.PS.5, A.PS.6, A.PS.8, A.RP.1, A.CM.1, A.CM.3, A.CN.1, A.CN.2, A.CN.4, A2.A.55 --> A2.A.57, A2.A.60 --> A2.A.66

Common Core: F-TF.A1 --> F.TF.A5

Materials

- Calculator
- Over head projector

Differentiation

Students will be given time to look over notes, ask questions about the homework before the test begins. Some students will be taken to a separate room with another teacher to complete the exam

Independent Practice

Homework: Cumulative review packet

Assessments

Warm-up, Homework, Participation, Discussion and test.

INTERMEDIATE ALGEBRA
TEST CHAPTER 7

NAME _____
DATE _____

PART 1 - ANSWER ALL QUESTIONS AND SHOW ALL WORK! (2 PTS EA)

1) Name an angle coterminal with a 240 degree angle.

2) Name an angle coterminal with a -210 angle.

3) Express a 45 degree angle in radians.

4) Express a 120 degree angle in radians.

5) Express $\frac{7\pi}{4}$ radians in degrees.

6) What quadrant is θ in if $\cos \theta > 0$?

7) What quadrant is θ in if $\sin \theta < 0$ and $\cos \theta > 0$?

8) Evaluate $\cos \frac{\pi}{3}$

9) Evaluate $\sin 150$

10) Evaluate $\sin \frac{-\pi}{4}$

11) Evaluate $\cos 300$

12) Find $\text{Arc sin } 1$

13) Find Arc sin $(-1/2)$

14) Find Arc cos -1

15) Sketch the sine curve between 0 and 2π

PART 2 - ANSWER ALL QUESTIONS AND SHOW ALL WORK! (4 PTS EA)

16) Evaluate $\sin 30 + \cos 60$

17) Evaluate $\sin 210 + \cos 120$

18) Find $\sin (\text{Arc cos } (-1))$

19) If $\cos \theta = \frac{4}{5}$ and $\sin \theta > 0$, find the value of $\sin \theta$.

20) Sketch the sine and cosine curve on the same set of axes from 0 to 2π .
Where are cosine and sine both > 0 ?

Chapter 4: Validity

The trigonometric unit presented in this thesis was developed with the main focus of aiding working memory deficits in students with ADHD. Two classes of Intermediate Algebra were instructed using this developed unit. Class One did not consist of any students who were diagnosed with ADHD. However, Class Two was an inclusive classroom, in which multiple students were classified as having ADHD. Based on the philosophy of the Common Core State Standards that all students, including those diagnosed with ADHD, must be held to the same high standards, the analysis of student work is based on whole class performance rather than individual students performance.

The following four learning goals, also previously presented in Chapter 3, are aligned with both the Common Core State Standards and the New York State Standards. They were compiled prior to the unit being developed and taught in order to maintain a clear focus throughout the unit for all individuals involved. The following data analysis reflects the student performance on assessments based on the four unit learning goals set as student learning parameters.

Learning Goal One (LG1)

Students will be able to recall and correctly identify appropriate trigonometric functions to find missing sides and/or angles (inverse functions) of a right triangle and then apply them correctly.

Learning Goal Two (LG2)

Students will be able to show understanding of the differences between degrees and radians by being able to convert radians to degrees and degrees to radians.

Learning Goal Three (LG3)

Students will be able to evaluate exact trigonometric function values of special right triangles angles, any of their coterminal angles and reference angles.

Learning Goal Four (LG4)

Students will be able to correctly identify multiple aspects of the unit circle on the coordinate plane including quadrants, angles and rotations, points as trigonometric function values and signs of trigonometric functions in each quadrant.

Data Analysis of Pre-Assessments

The statistical results of the pre-assessment show that students struggled most with learning goal one (LG1) and learning goal two (LG2). Table 5 shows the percentage of student performance in regards to the targeted learning goals in correlation to the pre-assessment quizzes. As indicated in Table 5, the average performance results for all students showed the lowest percentage of success on these two learning goals.

Table 5

Pre-Assessments Results of Whole Group in correlation to the Targeted Learning Goals (Percent Correct per Learning Goals)

Learning Goals	Quiz 1	Quiz 2	Quiz 3	Quiz 4	Average
LG1	57.1%	45.7%	48.6%	36.2%	46.9%
LG2	25.7%	40%	11.4%	X	25.7%
LG3	X	62.7%	X	40%	51.4%
LG4	X	X	64.3%	X	64.3%

This suggested that the students required more practice on basic principles of trigonometric functions. For this reason, lessons included vocabulary support, various

external aids for self-use and a variety of differentiated strategies to reach all the needs of individual learners including those with ADHD. An “X” in Table 5 indicates that the specific targeted learning goal was not present in the particular quiz.

The results of the pre-assessments also indicated that the students were strongest with learning goal four (LG4). This particular learning goal is visually based. Lessons were then taught with many forms of visual aids to represent the concepts being taught to further support the successful performance. However, because this pattern of performance also remained consistent among the pre-assessment averages for both Class One and Class Two, as shown in Table 6, higher focus remained on the learning goals that showed the weakest performance on the pre-assessments. Table 6 shows the performance of Class One and Class Two on Quiz 1, Quiz 2, Quiz 3, Quiz 4 and the average of all quizzes. An “X” in Table 6 indicates that the specific targeted learning goal was not present in the particular quiz.

Table 6

Pre-Assessments Results of Class One and Class Two in correlation to the Targeted Learning Goals (Percent Correct per Learning Goals)

Targeted Learning Goals	Class One Quiz 1	Class Two Quiz 1	Class One Quiz 2	Class Two Quiz 2	Class One Quiz 3	Class Two Quiz 3	Class One Quiz 4	Class Two Quiz 4	Class One Average	Class Two Average
LG1	68.8%	47.4%	68.8%	26.3%	62.5%	36.8%	35.4%	36.8%	58.9%	36.8%
LG2	40.6%	13.2%	62.5%	21.1%	18.8%	5.3%	X	X	40.6%	13.2%
LG3	X	X	68.8%	57.9%	X	X	56.3%	26.3%	62.6%	42.1%
LG4	X	X	X	X	75%	55.3%	X	X	75%	55.3%

In comparison of the two classes presented in Table 6, the data shows that Class One's performance based on the unit learning goals was consistently higher than Class Two throughout all of the pre-assessment quizzes. However they both show consistent patterns in strength and weaknesses amongst the learning goals.

After analyzing quizzes, warm-ups, homeworks and in-class discussions, it became evident that many students lacked the basic principles needed to be successful in the upcoming lessons. The use of explicit instructions, external aids and scaffolding was implemented to address these student needs. As previously addressed in Chapter 2 and Chapter 3, all three of these tools are also helpful in aiding those with ADHD, who face working memory deficits, to be successful.

All assessments were created on a cumulative basis. Students were presented questions on basic trigonometric knowledge along with questions on new content as it was presented. Questions on assessments were formulated to combine all content up to that particular point in the unit. As discussed previously, students with ADHD show more signs of error on rote math assessments over time. These unit assessments fostered the variability that students with ADHD, especially those who face working memory deficits, require for performance success.

Data Analysis of Post-Assessment

In evaluating the post-assessments, in comparison to the pre-assessments, there shows some evidence of student learning in regards to the unit learning goals. Table 7 shows a direct comparison of the pre-assessment average scores against the post-assessment average scores in regards to the learning goals of Class One, Class Two and a combination of both classes in the Whole Group category.

Table 7

Comparison of Pre-Assessment and Post-Assessment Results (Average Percent Correct per Unit Learning Goals)

Learning Goals	Class One Quizzes Average	Class One Unit Test Average	Class Two Quizzes Average	Class Two Unit Test Average	Whole Group Average for Quizzes	Whole Group Average for Unit Test
LG1	58.9%	50%	36.8%	75.8%	46.9%	64%
LG2	40.6%	47.5%	13.2%	61.1%	25.7%	54.9%
LG3	62.6%	31.3%	42.1%	53.2%	51.4%	43.2%
LG4	75%	40.6%	55.3%	47.4%	64.3%	44.3%

Overall, as a whole group, the students increased their percentage of correct answers on LG1 by 17.1% and on LG2 by 29.2% and decreased their percentage of correct answers on LG3 by 8.2% and on LG4 by 20%. While analyzing the individual classes, Class One only showed growth in LG 2 while Class Two showed significant growth in LG1, LG2 and LG3. It is important to note that Class Two is the inclusive classroom that contains multiple students with ADHD. Although the results cannot conclude that all students showed improvement in understanding of the unit, the data does provide evidence that indicates the average student showed overall improvement by the end of the unit.

Validity.

Although the data shows a standard outline for the growth of the classes both individually and as a whole, there are many variables that skew the overall data. For example, LG4 is only evaluated on two questions for the Unit exam where LG3 is

evaluated on eight questions. Both LG1 and LG2 are evaluated on five questions. If the test was more evenly divided among the four learning goals, there may have been a more accurate analysis data.

Homework performance also showed to play a key role in the students' achievement and may have skewed the data presented. There showed to be a correlation between the homework average and the performance on pre and post-assessments. "Consequently, instructional manipulations to improve learning by diminishing extraneous cognitive load and by freeing up cognitive resources is only effective if students, even those with ADHD, are motivated and actually invest mental effort in learning processes that use freed resources" (van Merriënboer & Sweller, 2005, p. 162).

Chapter 5: Conclusion

With the relatively new implementation of the Common Core State Standards, there has been a need for advancements in both research and curriculum development that corresponds to the high demands of the CCSS in relation to mathematics. The purpose of this curriculum project was to develop a unit on introductory trigonometric functions and the unit circle that, when taught, addressed the needs of students with ADHD while also reaching the high expectations implemented by the CCSS. The unit presented utilized the various teaching styles, strategies and methods research had shown to be effective in educating students with ADHD in the focus of working memory deficits.

"Students with attention deficit-hyperactivity disorder (ADHD) now represent a large number of children with significant behavioral challenges within general education" (Zentall & Javorsky, 2007, p. 78). Although students with ADHD face hardships that

general education students often do not, the CCSS fundamental goal is to prepared all students for success in their post-school lives, including college and/or careers (Common Core State Standards Initiative, 2014, p. 1). *How* these high standards are taught and assessed was said to be of the utmost importance in reaching all students (Common Core State Standards Initiative, 2014, p. 1).

The curriculum developed in this thesis was surrounded by the ideas brought upon by the Cognitive Load Theory. “The definition of learning, from a cognitive load perspective, is defined as a permanent change in long term memory” (Ellis, 2014, p. 12; Sweller et al., 1998; Sweller et al., 1991; Sweller & Candler, 1994). Essentially the goal of instructional design, per the CLT, is to stimulate the transfer of knowledge from the working memory. According to research, “deficits in executive functioning are proposed to play a pivotal role in explaining the problems children with ADHD face (Dovis, Van der Oord, Wiers & Prins, 2013, p.901; Barkley, 2006; Nigg, 2006). Thus, the theory behind Cognitive load showed importance in understanding how to address these deficits in the classroom.

The revisions for this curriculum project that should be kept in mind for future use include revisions on how both the pre-assessments and post-assessments were scored. In the current project, only questions that where answered completely correct were factored into the data. Questions that got partial credit where considered incorrect in regards to meeting unit learning goals. Now as a researcher and author of this thesis, the consideration of flawed reasoning is important. Mathematical reasoning is founded on four constructs; the development, justification and use of mathematic generalizations, the idea that mathematical reasoning that leads to an interconnected web of mathematical

knowledge, the development of “mathematical memory” and, “an emphasis on mathematical reasoning in the classroom that incorporates the study of flawed or incorrect reasoning as an avenue towards deeper development of mathematical knowledge” (Stiff, L. & Curcio, F., 1999, p.1). Partial credit can be considered as flawed reasoning on the part of the student and therefore teachers should consider flawed reasoning as being on the path to learning ((Stiff, L. & Curcio, F., 1999, p.2). Students with ADHD are often on the path to learning, but with additional supports to reduce extraneous cognitive load, that path to learning can become more evident.

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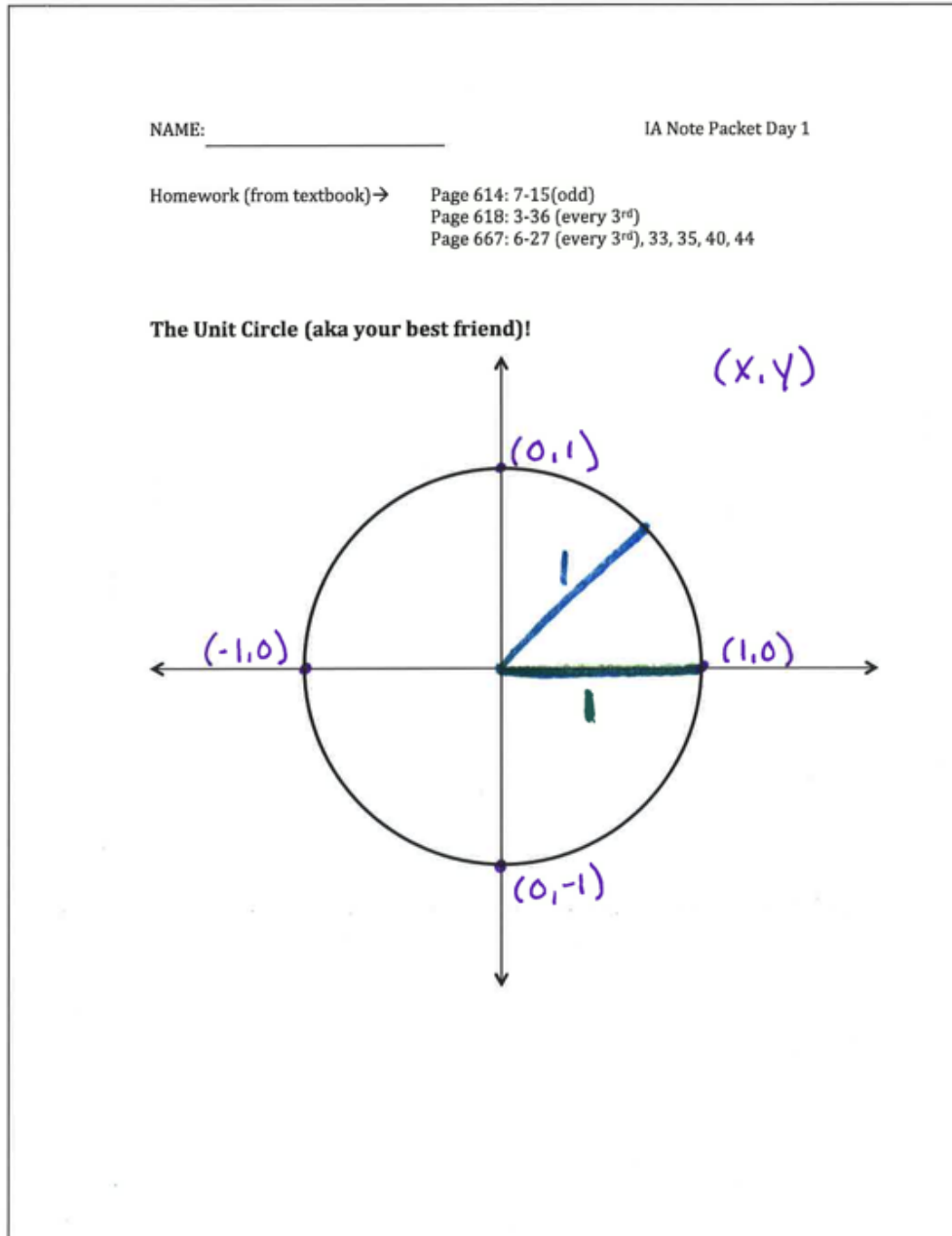
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Appendix

The answer keys for the worksheets, quizzes and unit test can be found on pages 78 to 112.



Lesson One Vocabulary: ☺

Draw a picture to represent word

1. **Standard Position:**

Pg. 615

Definition: An angle whose vertex is at the origin and its initial side is the non-negative ray of the x-axis.

2. **Initial Side / Terminal Side:**

Pg. 615

Definition: Initial: The ray at which an angle of rotation begins.
Terminal: The ray at which an angle of rotation ends.

3. **Quadrants:**

Pg. 615

Definition: The four plane created by the intersection of the x and y-axis.

4. **Coterminal Angles:**

Pg. 616

Definition: Angles in standard position that have the same terminal side.

5. **Quadrantal Angles:**

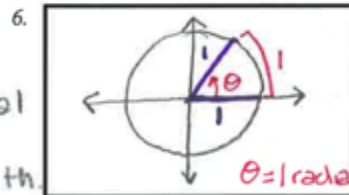
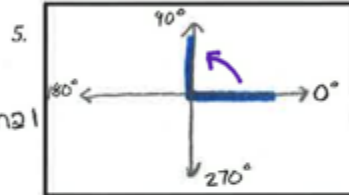
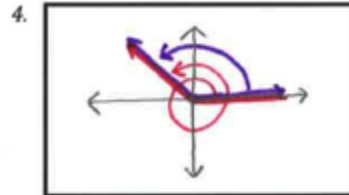
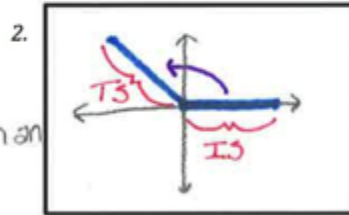
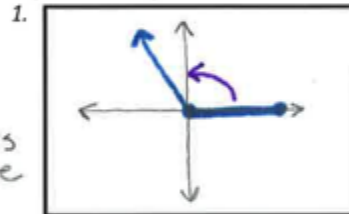
Pg. 616

Definition: An angle whose terminal side lies on either the x-axis or the y-axis.

6. **Radian:**

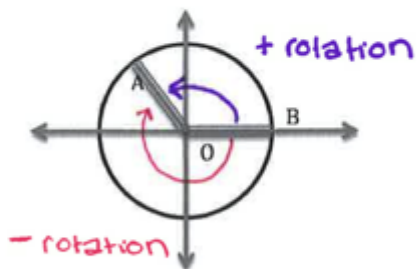
Pg. 661

Definition: A unit of angle, equal to an angle at the center of a circle whose arc is equal in length.



Angles and Rotations

Pgs. 614-618



Standard position is when vertex at origin & initial side at nonnegative x-axis

\overline{OB} represents the initial side.
Rephrase in your own words:

\overline{OA} represents the terminal side.
Rephrase in your own words:

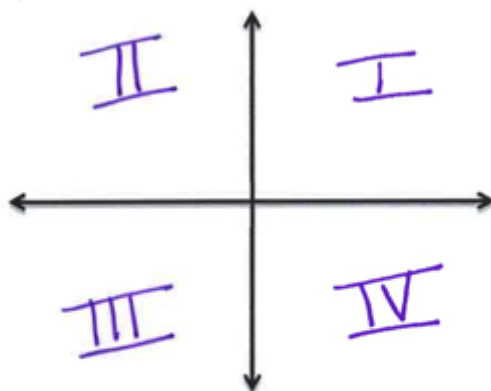
IMPORTANT

- An angle formed by a counterclockwise rotation has a + measure.
- An angle formed by a clockwise rotation has a - measure.



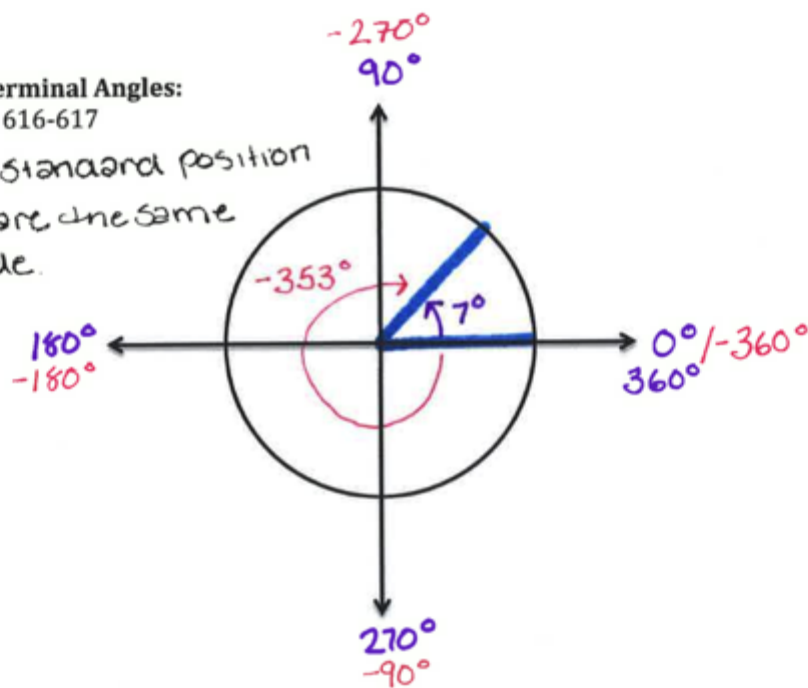
Labeling the coordinate plane:

Pg. 615



Coterminal Angles:
Pgs. 616-617

- * Angles in standard position
- * Angles share the same terminal side.



EXERCISE!



Find at least two angles that are coterminal with the given angles:

(a). 375° 15°
 735°

(b). 580° 220°
 940°

(c). -30° 330°
 690°

(d). -110° 250°
 610°

(e). -360° 0°
 360°

(f). -75° 285°
 645°

In your own words, how do you find coterminal angles:

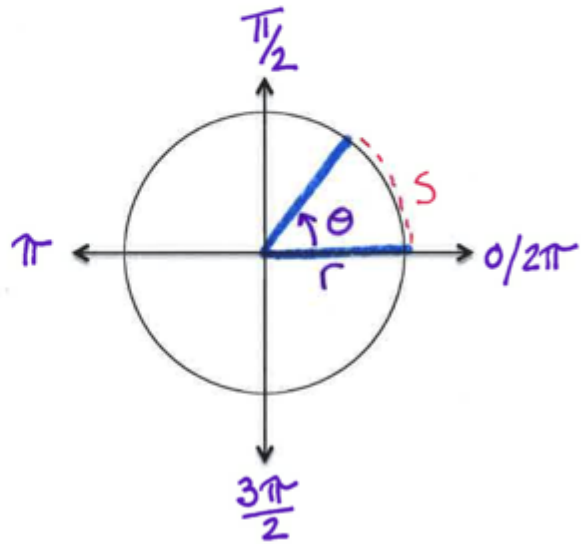
add/subtract 360°
to/from given angle.

Radian Measure

Pgs. 661-668

General rule:

$$\theta = \frac{(\text{intercepted arc})}{(\text{radius})} = \frac{s}{r}$$



Relationship between Degrees and Radians:
 pgs. 662-663

$$\frac{\text{Degrees}}{\text{Radians}} = \frac{180^\circ}{\pi}$$

EXERCISE!



Convert from degrees to radians:

270°
 $\frac{270}{R} = \frac{180}{\pi}$
 $\frac{270\pi}{180} = R$
 $\frac{270\pi}{180} = \frac{180R}{180}$
 $\left[R = \frac{3\pi}{2} \right]$

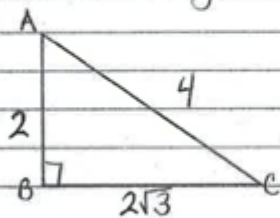
Convert from radians to degrees:

$\frac{\pi}{10}$
 $\frac{D}{\pi} = \frac{180}{\pi}$
 $D\pi = \frac{180\pi}{10}$
 $\frac{D\pi}{\pi} = \frac{18\pi}{\pi}$
 $\left[D = 18^\circ \right]$

Name: _____

1A Quiz #1

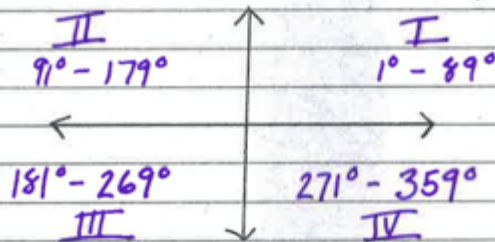
1.) Solve for angle A and angle C:



$$\cos(A) = \frac{2}{4} \quad \cos^{-1}\left(\frac{1}{2}\right) = A \quad A = 60^\circ$$

$$\sin(C) = \frac{2}{4} \quad \sin^{-1}\left(\frac{1}{2}\right) = C \quad C = 30^\circ$$

2.) First label the quadrants and then assign an appropriate angle to each of the quadrants:



3.) Which set of angles are coterminal?

Ⓐ -66° and 300° Ⓑ 450° and 90° Ⓒ 25° and 380°

4.) Convert 170° to radians

$$\frac{170}{R} = \frac{180}{\pi} \quad \frac{170\pi = 180R}{180} \quad \frac{170\pi = R}{180} \quad \left[R = \frac{17\pi}{18} \right]$$

5.) Convert $\frac{6\pi}{7}$ to degrees

$$\frac{6\pi}{7} = \frac{180}{\pi} \quad \frac{6\pi = 1,080}{\pi} \quad \left[D = \frac{1,080}{7} \approx 154.29^\circ \right]$$

NAME: _____

IA Note Packet Day 2

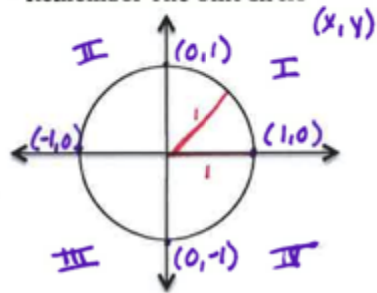
Homework (from textbook) →

Page 621: 1

Page 624: 1-7(odd), 8

Page 630: 7-17(odd), 27, 29

*** Remember The Unit Circle***

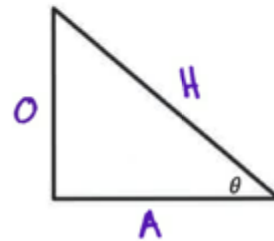


Remember Trig-Functions

$$\sin \theta = \frac{O}{H}$$

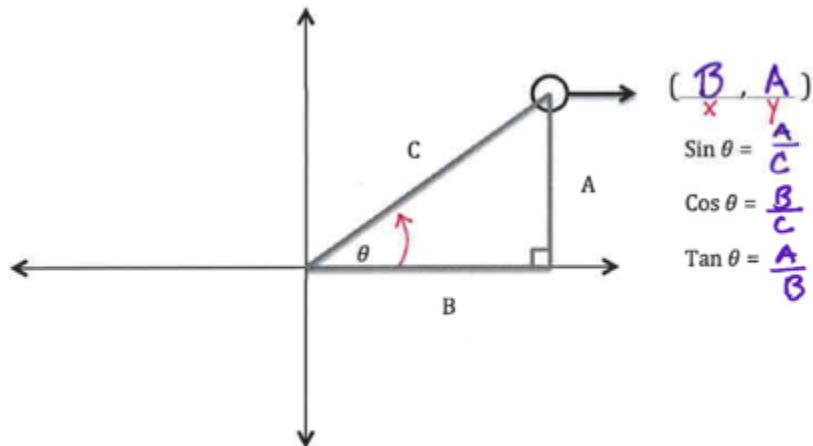
$$\cos \theta = \frac{A}{H}$$

$$\tan \theta = \frac{O}{A}$$



Sine and Cosine as Coordinates

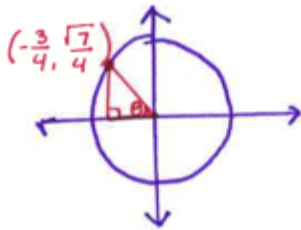
Pgs. 618-620



EXERCISE!



Determine $\sin \theta$ and $\cos \theta$ when given Point P $(-\frac{3}{4}, \frac{\sqrt{7}}{4})$ on the Unit Circle:



$$(x, y) = (\cos \theta, \sin \theta)$$

$$\sin \theta = \frac{\sqrt{7}}{4}$$

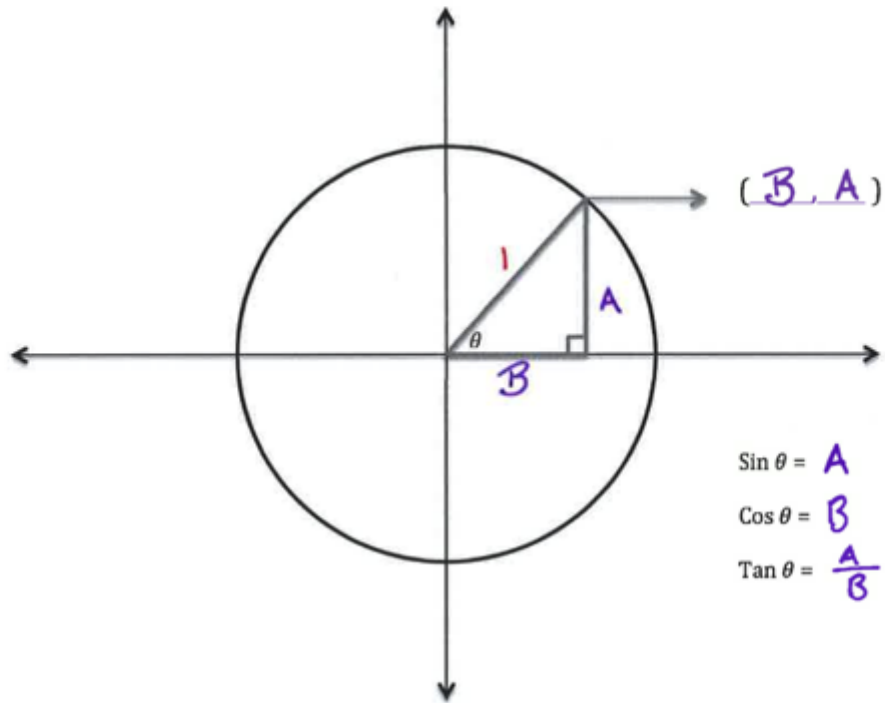
$$\cos \theta = -\frac{3}{4}$$

Quadrantal Angles on the Unit Circle: (No Calculators)

Pg. _____

Degrees θ	$0^\circ, 360^\circ$	90°	180°	270°
Sine θ	0	1	0	-1
Cosine θ	1	0	-1	0
Tangent θ $= \frac{\sin \theta}{\cos \theta}$	0	Undefined	0	Undefined
Draw angle in standard position				

Sine and Cosine as Coordinates on the Unit Circle:



$$\sin \theta = A$$

$$\cos \theta = B$$

$$\tan \theta = \frac{A}{B}$$

***General Rule:

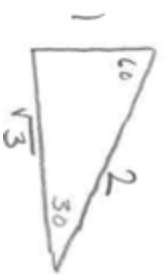
Pgs. _____,

$(x, y) = (\cos \theta, \sin \theta)$ on unit circle

$\tan \theta = \sin \theta / \cos \theta$

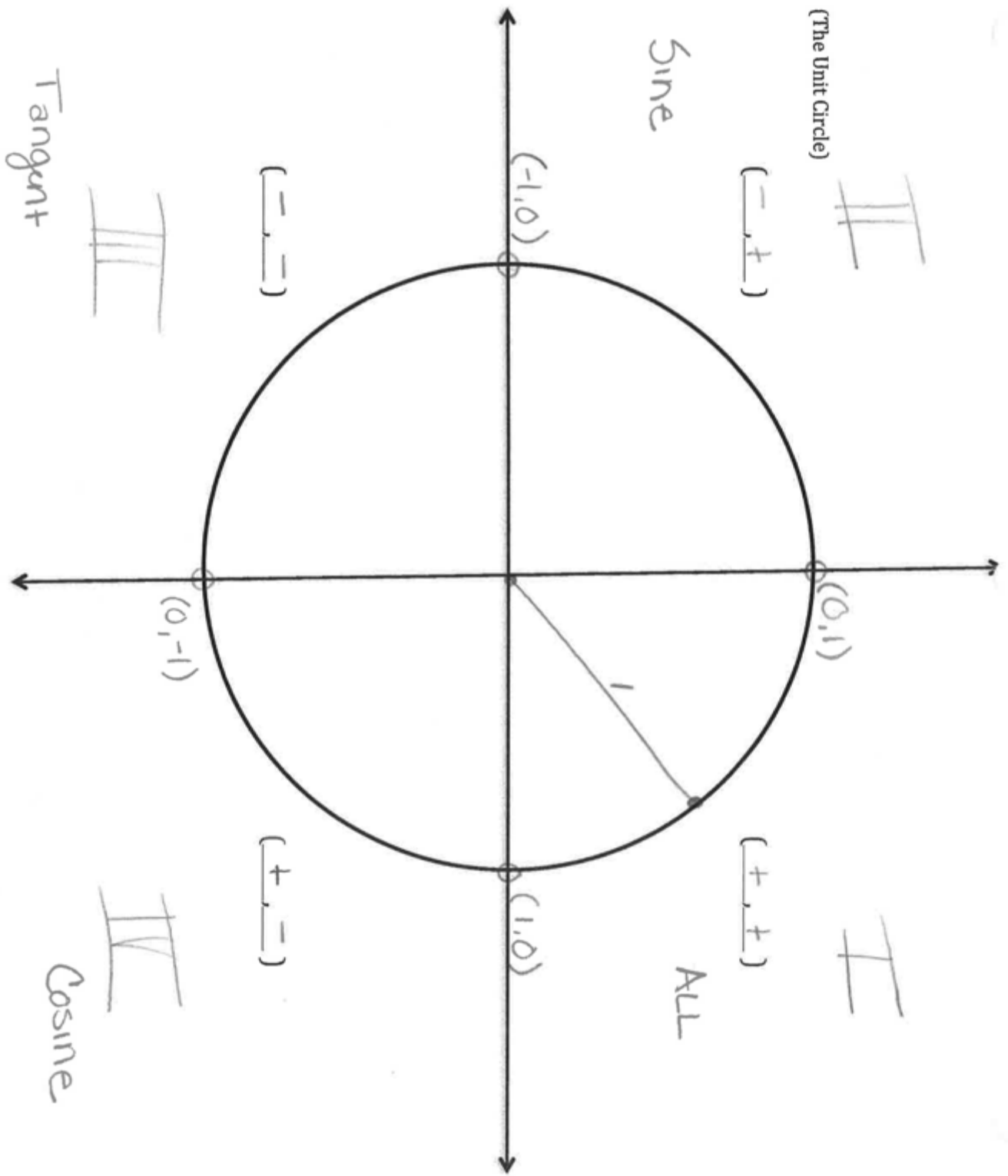
Degrees θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Negative Coterminal Angle	-360	-330°	-315°	-300°	-270°	-240°	-225°	-210°	-180°	-150°	-135°	-120°	-90°	-60°	-45°	-30°	0°
Reference Angles						60°	45°	30°	90°	30°	45°	60°	90°	60°	45°	30°	0°
Quadrant Day 2	I/IV	I	I	I	I/IV	II	II	III	IV/III	III	IV	III	IV/III	IV	IV	IV	IV/I
* (Sign) Day 2 Sin θ Value Day 3	± 0	$+\frac{1}{2}$	$+\frac{\sqrt{2}}{2}$	$+\frac{\sqrt{3}}{2}$	$+$	$+\frac{1}{\sqrt{3}}$	$+\frac{\sqrt{2}}{2}$	$+\frac{1}{2}$	± 0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	± 0
* Cos θ	$+$	$+\frac{\sqrt{3}}{2}$	$+\frac{\sqrt{2}}{2}$	$+\frac{1}{2}$	± 0	$-\frac{1}{\sqrt{3}}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	± 0	$+\frac{1}{\sqrt{3}}$	$+\frac{\sqrt{2}}{2}$	$+\frac{\sqrt{3}}{2}$	$+$
* Tan θ	$+$	$+\frac{\sqrt{3}}{3}$	$+$	$+$	± 0	$-\frac{1}{\sqrt{3}}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{3}$	± 0	$+\frac{\sqrt{3}}{3}$	$+$	$+\frac{\sqrt{3}}{3}$	± 0	$-\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$	± 0

Remember reference Δ 's



Questions:

Where do the changes in signs occur?
Patterns?



Name: _____

IA Quiz #2

1.) Evaluate:

a) $\sin 30^\circ$

$$\frac{1}{2}$$

b) $\sin 210^\circ$

$$-\frac{1}{2}$$

2.) What is 540° in radians? (show work)

$$\frac{540^\circ}{R} = \frac{180}{\pi} \quad \frac{180R}{180} = \frac{540\pi}{180} \quad \left[R = \frac{54\pi}{18} = \frac{6\pi}{2} \right]$$

3.) What quadrant is θ in if $\cos\theta < 0$ and $\sin\theta < 0$?

$$(\cos\theta, \sin\theta)$$
$$(-, -)$$

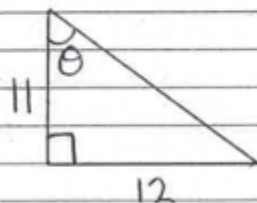
[III]

4.) Given the point $P(.6, .8)$ on the Unit Circle, what is $\cos\theta$ and $\sin\theta$?

$$\cos\theta = .6$$

$$\sin\theta = .8$$

5.) Solve for θ :



$$\tan\theta = \frac{12}{11}$$

$$\tan^{-1}\left(\frac{12}{11}\right) \approx 82.88^\circ$$

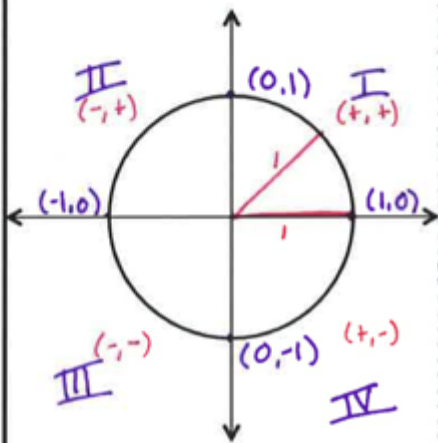
NAME: _____

IA Note Packet Day 3

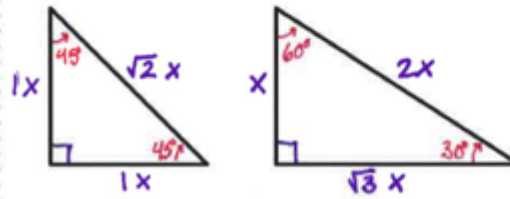
Homework (from text book) → Page 634: 1-15(odd), 28-32(even)
Page 660: 3-24 (every 3rd)

Review

Unit Circle!

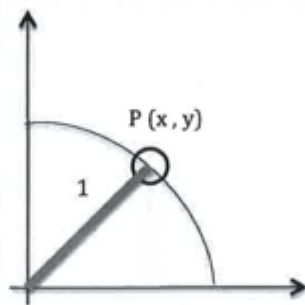
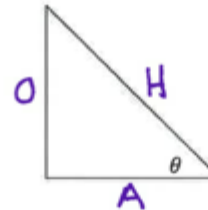


Reference Triangles with special angles!



Trigonometry Functions!

$$\begin{aligned}\sin \theta &= \frac{O}{H} \\ \cos \theta &= \frac{A}{H} \\ \tan \theta &= \frac{O}{A}\end{aligned}$$



$$\sin P = y$$

$$\cos P = x$$

$$\tan P = \frac{y}{x}$$

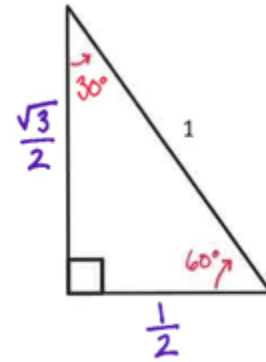
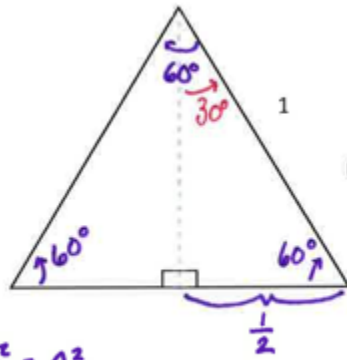
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Function Values of Special Angles

Pgs. 631-634

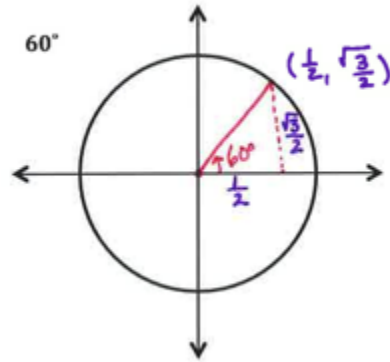
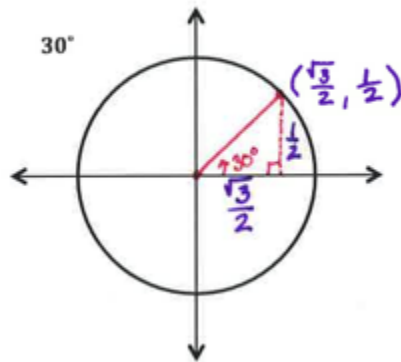
Angles of 30° and 60°:

From Equilateral Triangle →



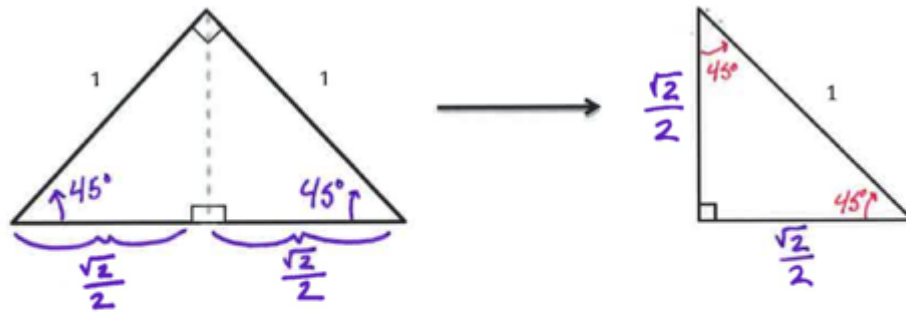
* $a^2 + b^2 = c^2$

To Unit Circle →

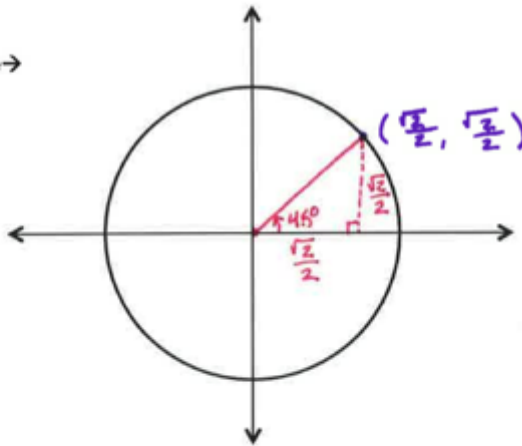


Angles of 45°:

From isosceles triangle →



To the Unit Circle →



θ	30°	45°	60°
Sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tan θ	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Finding Reference Angles

Pgs. 651-660

WHAT IS A REFERENCE ANGLE???????

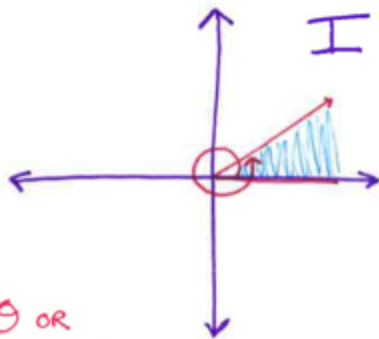
Definition:

Pg. ____.

The trigonometric function values of angles with degree measure greater than 90° or less than 0° can be found from their values at corresponding acute angles called reference angles.

Quadrant I

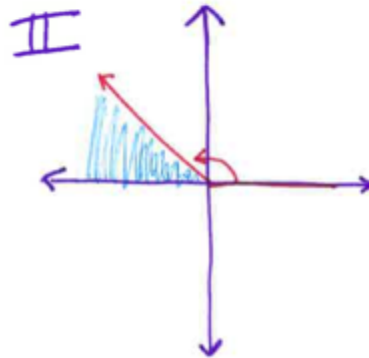
Pg. ____.



θ or
 $\theta - 360^\circ$ when $m \neq > 360^\circ$

Quadrant II

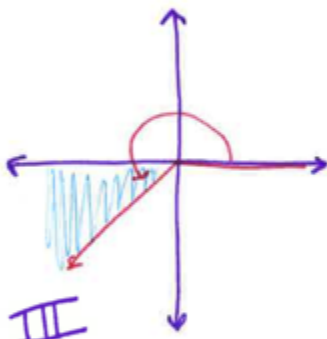
Pg. ____.



$180^\circ - \theta$

Quadrant III

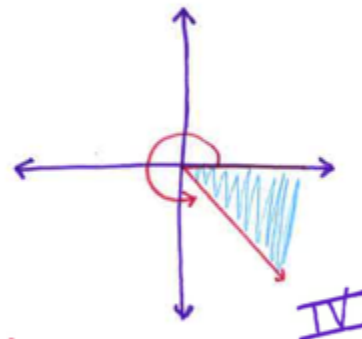
Pg. ____.



$\theta - 180^\circ$

Quadrant IV

Pg. ____.



$360^\circ - \theta$

Generalize Reference Angles!

$90^\circ < \theta < 180^\circ$	$180^\circ < \theta < 270^\circ$	$270^\circ < \theta < 360^\circ$
Quadrant: II	Quadrant: III	Quadrant: IV
$\sin \theta = \sin(180^\circ - \theta)$	$\sin \theta = \sin(\theta - 180)$	$\sin \theta = \sin(360 - \theta)$
$\cos \theta = -\cos(180 - \theta)$	$\cos \theta = -\cos(\theta - 180)$	$\cos \theta = \cos(360 - \theta)$
$\tan \theta = -\tan(180 - \theta)$	$\tan \theta = \tan(\theta - 180)$	$\tan \theta = -\tan(360 - \theta)$

EXERCISE



Find exact Numerical value of the expressions below:

a. $(\sin 30^\circ)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

b. $\tan 45^\circ + \cos 0^\circ = 1 + 1 = 2$

c. $\cos 45^\circ = \frac{\sqrt{2}}{2}$

Find each exact function value:

a. $\sin 100^\circ = \sin(180 - 100) = \sin(80) \approx .98$

b. $\cos 240^\circ = \cos(360 - 240) = \cos(120) = -\frac{1}{2}$

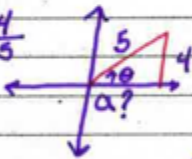
c. $\tan 675^\circ = \tan(360 - 315) = \tan(45) = 1$
 $675 - 360 = 315^\circ$

Name: _____

1A Quiz # 3
"Up to Now Quiz!"

1.) If $\sin \theta = \frac{4}{5}$, find the value of $\cos \theta$ and $\tan \theta$:

$$\sin \theta = \frac{O}{H} = \frac{4}{5}$$
$$\cos \theta = \frac{A}{H} = \frac{3}{5}$$
$$O = 4$$
$$H = 5$$



$$a^2 + b^2 = c^2 \rightarrow a^2 + 4^2 = 5^2$$
$$a^2 + 16 = 25$$
$$a^2 = 9$$
$$a = 3$$

2.) Convert:

$$[\cos \theta = \frac{3}{5}]$$

~~$\frac{0}{2} = \frac{180}{2}$~~ @ $\frac{3\pi}{2}$ to degrees

$$[\theta = 270^\circ]$$

~~$\frac{100}{R} = \frac{180}{\pi}$~~ (b) 100° to radians

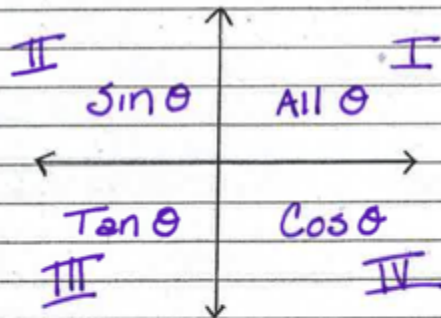
$$\frac{100\pi}{180} = R$$
$$[R = \frac{5\pi}{9}]$$

$$0\pi = 270\pi$$

$$100\pi = 180R$$

3.) Label the coordinate plane: @ quadrants

@ where $\sin \theta$, $\cos \theta$
and $\tan \theta$ are
positive.



$$\sin \theta = + \text{ in I, II}$$
$$\cos \theta = + \text{ in I, IV}$$
$$\tan \theta = + \text{ in I, III}$$

4.) Given the point $P(.6, .8)$ on the unit circle,
what is the value of $\sin \theta$ and $\cos \theta$?

$$\sin \theta = .8 \quad \cos \theta = .6$$

5.) Find the exact value for $\sin 210^\circ + \cos 120^\circ$:
(must show work)

$$\sin(210-180) + \cos(180-120)$$

$$-\sin(30) + -\cos(60) = -\frac{1}{2} + -\frac{1}{2} = -1$$

6.) In which quadrant does θ lie if $\sin < 0$ and $\tan > 0$?

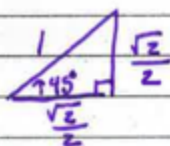
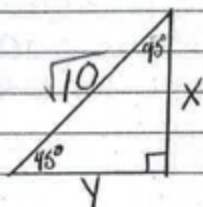
+ III

7.) Identify two angles that are coterminal with 35° :

$$35^\circ + 360^\circ = 395^\circ + 360^\circ = 755^\circ$$

395° and 755°

8.) Solve for x and y



$$\frac{1 \cdot \sqrt{10}}{2} = \frac{\sqrt{10}}{2}$$

$$\frac{\sqrt{2}}{2} \cdot \sqrt{10} = \frac{\sqrt{20}}{2}$$

$$\frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$[x = \sqrt{5} \quad y = \sqrt{5}]$$

9.) If $\sin \theta = \sqrt{3} \cos \theta$, what is the value of θ ?

$$\frac{\sin \theta}{\cos \theta} = \sqrt{3} \frac{\cos \theta}{\cos \theta} \quad \frac{\sin \theta}{\cos \theta} = \tan \theta = \sqrt{3} \quad \tan^{-1}(\sqrt{3}) = \theta$$

$$[\theta = 60^\circ]$$

10.) Evaluate $(\sin 390)(\cos -300)$?
(must show work)

$$\frac{(\sin(390-360))(\cos(-300+360))}{\sin(30) \cdot \cos(60)} = \left(\frac{1}{2}\right) \left(\frac{1}{2}\right) = \left[\frac{1}{4}\right]$$

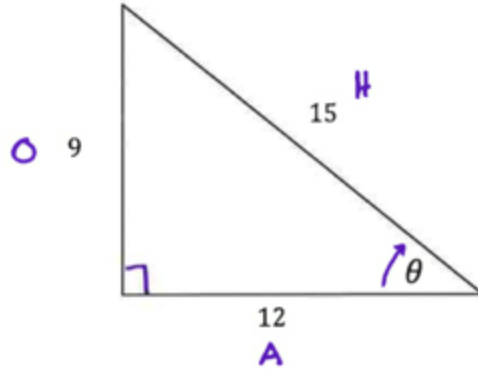
NAME: _____

IA Note Packet Day 4

Homework (from text book) → Page 743: 3-21 (every 3rd), 23, 25-28, 32

REVIEW

Determine the value of θ



WORK:

$$\sin \theta = \frac{9}{15} = \frac{3}{5} \quad \sin^{-1}\left(\frac{3}{5}\right) \approx 36.87^\circ$$

$$\cos \theta = \frac{12}{15} = \frac{4}{5} \quad \cos^{-1}\left(\frac{4}{5}\right) \approx 36.87^\circ$$

$$\tan \theta = \frac{9}{12} = \frac{3}{4} \quad \tan^{-1}\left(\frac{3}{4}\right) \approx 36.87^\circ$$

Inverse Trigonometric Functions

Pgs. 735-743

Inverse

Definition:

• an element, when combined with a given element in an operation, produces the identity element for that operation.

"When the sets of points $y = \sin x$, $y = \cos x$, and $y = \tan x$ are reflected over the line $y = x$, the images are the sets of points $y = \arcsin x$, $y = \arccos x$, and $y = \arctan x$." - Page 735

What is the difference between $y = \arcsin$ and $y = \text{Arc Sin}$?

Pg. ____.

• arcsine is an inverse trig relation whose range is all real numbers.

• Arcsine is an inverse trig function whose range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

NOTATION

$y = \text{Arc Sin } x$ is the same thing as $y = \underline{\sin^{-1}x}$.

$y = \text{Arc Cos } x$ is the same thing as $y = \underline{\cos^{-1}x}$.

$y = \text{Arc Tan } x$ is the same thing as $y = \underline{\tan^{-1}x}$.

EXERCISES



Find the Value of θ in degrees:

a. $\theta = \text{Arc Cos} \left(\frac{1}{2} \right)$ 60°

b. $\theta = \text{Arc Sin} (0)$ 0°

c. $\theta = \text{Arc tan} (-\sqrt{3})$ 300°

Find the value of θ in radians:

a. $\theta = \text{Arc sin} \left(\frac{-1}{2} \right)$ $330^\circ = \frac{11\pi}{6}$

b. $\theta = \text{Arc Cos} \left(\frac{-1}{2} \right)$ $120^\circ = \frac{2\pi}{3}$

c. $\theta = \text{Arc tan} (\sqrt{3})$ $60^\circ = \frac{\pi}{3}$

Find the value of each expression:

a. $\text{Cos} (\text{Arc Sin } 1)$ $\text{Cos}(90) = 0$

b. $\text{Sin} (\text{Arc Cos } 0)$ $\text{Sin}(90) = 1$

Name: _____

1A Quiz #4

- 1.) Find the value of θ in radians when
 $\theta = \text{Arc Sin}(-\frac{1}{2})$

$$\theta = 330$$

$$\frac{330}{R} = \frac{180}{\pi}$$

$$\frac{330\pi}{180} = \frac{180R}{180}$$

$$\frac{33\pi}{18} = R \quad \left[R = \frac{11\pi}{6} \right]$$

- 2.) Express $\cos 305^\circ$ as a function of a positive acute angle

$$\cos(360 - 305) = [\cos(55)]$$

- 3.) Find the value of $\tan(\text{Arc cos } \frac{4}{5})$

$$[.75]$$

- 4.) Determine one angle coterminal with 250°

$$250 + 360 = [610^\circ]$$

NAME: _____

IA Note Packet Day 5

Homework (from text book) →

Page: 671: 1-30 (odd)

CONVERTING DEGREES TO RADIANS:

$$\frac{D}{R} = \frac{180}{\pi}$$

ex: 3° $\frac{3}{R} = \frac{180}{\pi}$ $\frac{3\pi}{180} = \frac{180R}{180}$

$$\left[R = \frac{\pi}{60} \right]$$

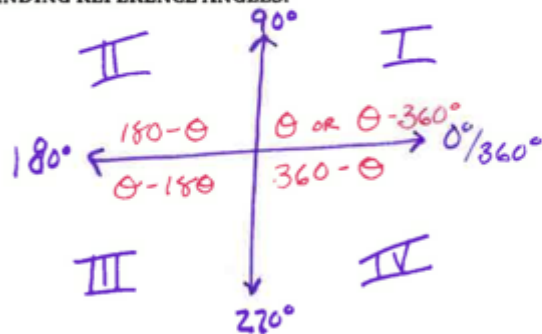
CONVERTING RADIANS TO DEGREES:

$$\frac{D}{R} = \frac{180}{\pi}$$

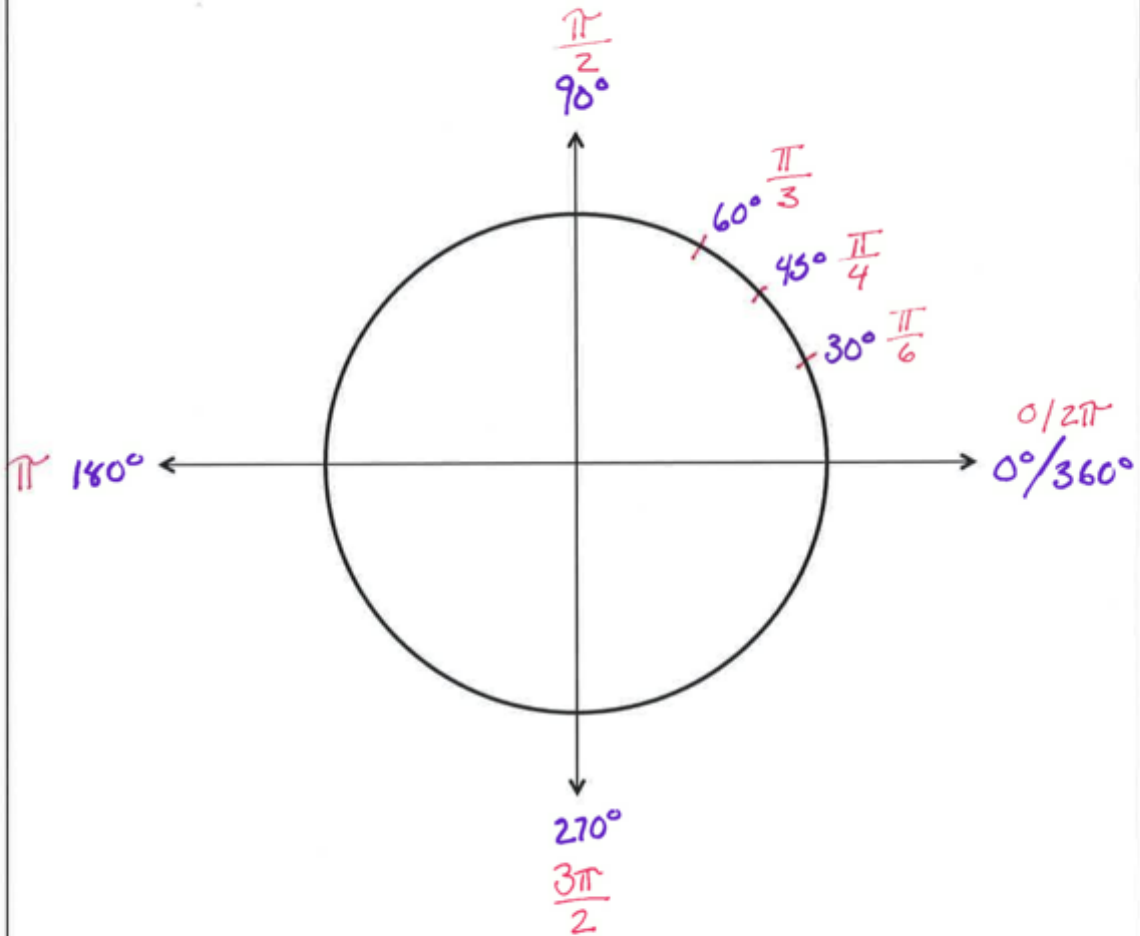
ex: $\frac{\pi}{60}$ $\frac{D}{(\frac{\pi}{60})} = \frac{180}{\pi}$ $D = \frac{180\pi}{\pi} = \frac{180\pi}{\pi}$

$$\left[D = 3^\circ \right]$$

FINDING REFERENCE ANGLES:



Degree	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{2\pi}{6}$ OR $\frac{\pi}{3}$	$\frac{2\pi}{4}$ OR $\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π



TRIGONOMETRIC FUNCTIONS INVOLVING RADIAN MEASURE
Pgs. 668-672

Your textbook will ask you to:

Find the exact value of each trigonometric function


Steps:

1. Convert radian measure to degrees
2. Determine which quadrant function is in.
 - a. Will the value be positive or negative?
3. Determine reference angle and write function as a positive acute angle
4. Find exact value

Example:

$$\cos \frac{4\pi}{3}$$

Work: ① $\frac{4\pi}{3} \Rightarrow \left(\frac{4\pi}{3}\right) = \frac{150}{\pi} \quad \frac{0\pi}{\pi} = \frac{240\pi}{\pi} \quad 0 = 240^\circ$

② $\cos(240^\circ) \Rightarrow$  \cos is $(-)$ in III

③ $-\cos(240-180) = -\cos 60$

④ $-\cos 60 = \left[-\frac{1}{2}\right]$

Given the function $f(x) = \cos\left(\frac{x}{2}\right)$

Determine $f(4\pi)$

$$x = 4\pi \quad \cos\left(\frac{4\pi}{2}\right) = \cos(2\pi) = \cos 360^\circ = [1]$$

Determine $f\left(\frac{\pi}{2}\right)$

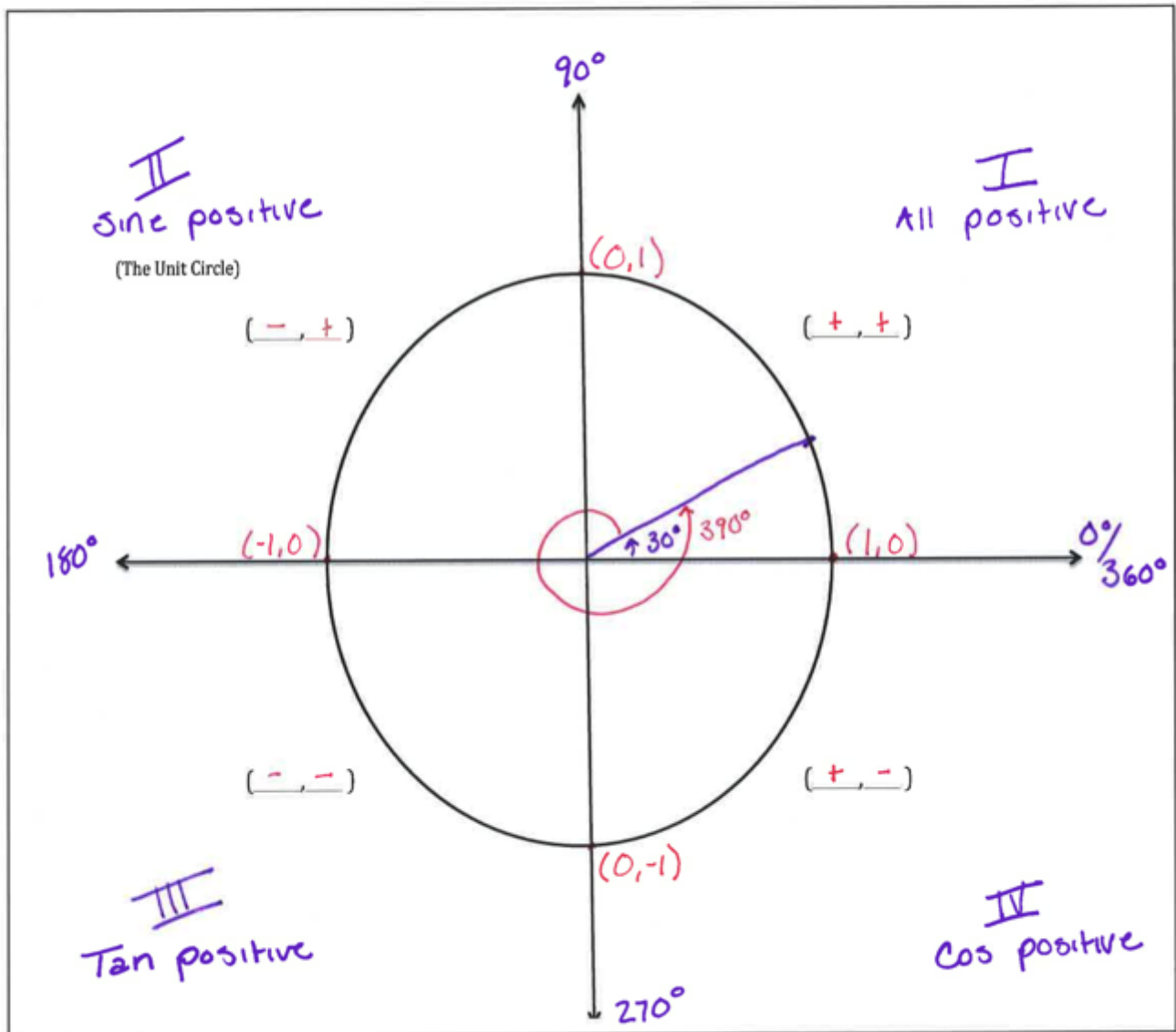
$$x = \frac{\pi}{2}$$

$$\cos\left(\frac{\frac{\pi}{2}}{2}\right) = \cos\left(\frac{\pi}{4}\right) = \cos 45^\circ = \left[\frac{\sqrt{2}}{2}\right]$$

Quiz #5

Label:

- Quadrants
- 0, 90, 180, 270 & 360 degrees
- Coordinate points on x and y-axis
- 30 degrees and one coterminal angle
- Show where sine, cosine & tangent are positive



Unit Circle Floss Activity

*External Aid *

Degrees θ	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Radian Measure	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{6\pi}{4}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

Formula for converting Degrees \leftrightarrow Radians:

$$\frac{D}{R} = \frac{180}{\pi}$$

Materials needed for activity:

- Blank Computer Paper
- Blank Unit Circle Paper
- 30, 60 45 degree Triangle cut outs
- Piece of Floss (length = 1.5 ft.)
- Colored Marker
- Scissors
- Tape

PART 1 - ANSWER ALL QUESTIONS AND SHOW ALL WORK! (2 PTS EA)

1) Name an angle coterminal with a 240 degree angle.

$$240 + 360 = [600^\circ]$$

2) Name an angle coterminal with a -210 angle.

$$-210 + 360 = [150^\circ]$$

3) Express a 45 degree angle in radians.

$$\frac{45}{R} = \frac{180}{\pi} \Rightarrow \frac{45\pi}{180} = \frac{180R}{180} = \left[\frac{\pi}{4} \right]$$

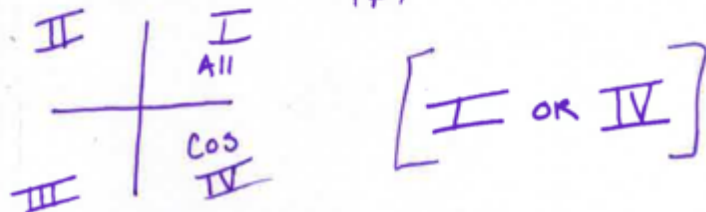
4) Express a 120 degree angle in radians.

$$\frac{120}{R} = \frac{180}{\pi} \Rightarrow \frac{120\pi}{180} = \frac{180R}{180} \left[R = \frac{2\pi}{3} \right]$$

5) Express $\frac{7\pi}{4}$ radians in degrees.

$$\frac{\theta}{\left(\frac{7\pi}{4}\right)} = \frac{180}{\pi} \Rightarrow \frac{\theta\pi}{\pi} = \frac{315\pi}{\pi} \quad [\theta = 315^\circ]$$

6) What quadrant is θ in if $\cos \theta > 0$?
(+)



7) What quadrant is θ in if $\sin \theta < 0$ and $\cos \theta > 0$?
(-) (+)

[IV]

8) Evaluate $\cos \frac{\pi}{3}$

$$\frac{\theta}{\frac{\pi}{3}} = \frac{180}{\pi} \Rightarrow \frac{\theta\pi}{\pi} = \frac{60\pi}{\pi} \quad \theta = 60^\circ$$

$$\cos 60^\circ = \left[\frac{1}{2}\right]$$

9) Evaluate $\sin 150$

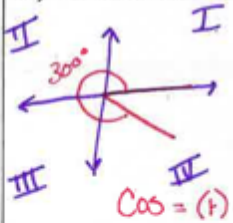
$$+\sin(180-150) = \sin(30) = \left[\frac{1}{2}\right]$$

10) Evaluate $\sin \frac{-\pi}{4}$

$$\left(\frac{-\pi}{4}\right) = \frac{180}{\pi} \Rightarrow \frac{0\pi}{\pi} = -\frac{45\pi}{\pi} \quad \theta = -45^\circ$$

$$-\sin 45^\circ = \left[-\frac{\sqrt{2}}{2}\right]$$

11) Evaluate $\cos 300$



$$\cos(360-300) = \cos 60 = \left[\frac{1}{2}\right]$$

12) Find Arc $\sin 1$

$$\left[90^\circ\right]$$

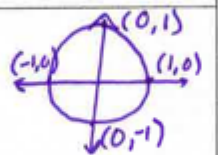
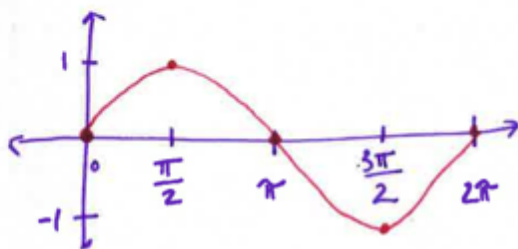
13) Find Arc sin $(-1/2)$

$$[-30^\circ \text{ OR } 330^\circ]$$

14) Find Arc cos -1

$$[180^\circ]$$

15) Sketch the sine curve between 0 and 2π



$(\cos\theta, \sin\theta)$

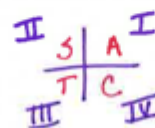
PART 2 - ANSWER ALL QUESTIONS AND SHOW ALL WORK! (4 PTS EA)

16) Evaluate $\sin 30 + \cos 60$

$$\frac{1}{2} + \frac{1}{2} = [1]$$

17) Evaluate $\sin 210 + \cos 120$

(-) (-)



$$-\sin(210-180) + \cos(180-120)$$

$$-\sin(30) + \cos(60)$$

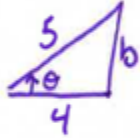
$$-\frac{1}{2} + \frac{1}{2} = [-1]$$

18) Find $\sin(\text{Arc cos}(-1))$

$$\sin(180) = [0^\circ]$$

19) If $\cos \theta = \frac{4}{5}$ and $\sin \theta > 0$, find the value of $\sin \theta$.

$\cos \theta = \frac{A}{H}$
 $\left[\sin \theta = \frac{3}{5} \right]$



$a^2 + b^2 = c^2$
 $4^2 + b^2 = 5^2$
 $16 + b^2 = 25$
 $b^2 = 9 \quad b = 3$

20) Sketch the sine and cosine curve on the same set of axes from 0 to 2π .
 Where are cosine and sine both > 0 ?

