8-2008

A Technologically Enriched Geometry Curriculum

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by

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August 1, 2008

A thesis project submitted to the

Department of Education and Human Development of the

State University of New York College at Brockport

in partial fulfillment of the requirements for the degree

of

Masters of Science in Education

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Chapter One: Introduction

Educators have adjusted the manner in which they educate their pupils as the world around them has changed in much the way mankind has adjusted the laws by which it governs itself by evaluating the needs of the times. In the past few decades the world has become inundated with technological advances to include such valuable resources as the personal computer, handheld graphing calculator and cellular phone. More recently the iPod®, video gaming systems and advances in cable and satellite television have even further separated our world from that of our parents' and grandparents'. Students of today are constantly being afforded new options in what they watch on TV, what games they play and how (not to mention where) they listen to their music.

With the flashiness of the entertainment they seek outside of school it has become increasingly harder to compete for students' attention within the walls of the classroom. For this reason I, as well as many other educators, believe that by properly integrating the available technology into the classroom we may be able to better compete for the students' time and attention. This attention will undoubtedly lead to better retention of the concepts being taught which will in turn result in better performance. It is my belief that through the proper integration of technology in the classroom students will experience a growth in their knowledge and skills. Throughout my research I tried to answer questions that have recently arisen in a secondary school geometry class. The two main questions I have attempted to focus my research on are how the use of technology affects student motivation and engagement in a geometry class and how a technologically enriched geometry curriculum can be created and implemented. Throughout this process I read numerous studies as well as reference a number of geometry textbooks in an attempt to determine what is or will be accepted as best practice in the high school geometry classroom. I also carefully planned what portions of the learning were best suited for the use of the DSP as a means of not only demonstration but an eventual understanding of the general nature of constructions and a better idea of what the meaning of proof truly is.

The National Council of Teachers of Mathematics (NCTM) said (1989) that "the mathematics classroom envisioned in the *Standards* is one in which calculators, computers, courseware, and manipulative materials are readily available and regularly used in instruction."

(Evaluation: Standard 12 - Curriculum and Instructional Resources) With the recent revision of the mathematics curriculum in New York a course in geometry has been created. It has been more than two decades since the last New York State Education Department (NYSED) sanctioned geometry course was required. As a result of this gap many teachers in New York have never even been in a high school level geometry course and are now being asked to teach one. For some teachers it is also the case that they were not required to take a geometry course in college that is similar to the one they will now be teaching. The fact that the NCTM has made a call for more use of technology coupled with the addition of geometry to the state curriculum by NYSED has led to the need of an enriched curriculum. In my opinion the best time to make this change to a technologically enriched classroom comes when the curriculum itself is changing.

Although many software packages exist to enrich the mathematics classroom few are as versatile across the board as Geometer's Sketchpad®. Geometer's Sketchpad® is a dynamic geometry software package (DSP) that contributes a technological flair to the study of mathematics. Students can use this software program to build and investigate mathematical models, objects, figures, diagrams, and graphs through virtual visualizations. Tools that mimic the capabilities of compasses and straightedges provide for a new environment in which students may perform constructions. Despite the name, Geometer's Sketchpad® can be used in all math classes and many science courses to illustrate theorems, trends, functions as well as many other applications. For this reason, many school districts can justify the purchase of a number of licenses for the software package thus making it widely available to the students. It was this availability as well as its universal acceptance by math educators that led me to use this particular DSP over the others available.

After the creation of the technologically enriched geometry curriculum it was implemented in an elective mathematics class in a rural high school located in western New York State. The class was comprised of ten students in their sophomore, junior and senior years of high school. Two of the ten students speak English as a second language. One of the ten students has an individual education plan (IEP). Each student had access to the DSP during class as well as anytime throughout the day. Although the students had all received prior instruction in logic and properties of triangles, most of the students had no experience with the use of theorems or postulates as a means of proof. The curriculum was developed keeping in mind their backgrounds as students as well as the accepted best practices as reported in research literature and classroom texts.

As I read numerous pieces of literature during the research process, I discovered that there were a number of issues that would shape how I approached the development of a curriculum that integrates technology into the geometry classroom. Two major subgroups of the literature which I have chosen to focus on are the problems associated with the use of the technology and those associated with the geometry itself. Although many pieces of literature certainly discuss both subgroups in detail, many points that were made have distinct directions formed by the area they were choosing to research and report upon.

Chapter Two: Literature Review

A. Problems in Geometry Classrooms

Much of what is known about mathematics is known because mathematicians have made conjectures and proven them to be true. A large portion of literature that investigated the problems in the geometry curricula and classrooms centered on the debate regarding the differences - and occasionally similarities - between demonstration and proof. As many teachers of mathematics will tell you, when students are initially asked to prove a statement or explain why they believe what they believe they will immediately point to the examples that work for a given conjecture and cite that as their proof. This improper use of the word "proof" may well stem from the fact that teachers use demonstration to illustrate proof in early education (Herbst & Brach, 2006).

Misuse of the word "proof" has led to a generation of students that truly believe it is enough to show that a given conjecture is true in a specific and unique instance rather than to show it is true in the general case. In my experience in the secondary mathematics classroom, the inappropriate belief that demonstration is a proof is often one of the largest obstacles to overcome. As an educator teaching students that have been raised in a world where calculations are more important than the algebraic process that leads one to the conclusion, I have found that the introduction to any general case, be it in algebra or geometry, causes a higher level of anxiety than the majority of the students are willing to cope with. In the end many students shut down when posed with this challenge to expand their knowledge. It is for this reason that proof must be introduced in the most non-aversive manner possible. I often attempt to introduce proof to my students without regard for their grade level or class discipline. The idea being that an informal introduction through the use of a story or a debate can show them the need for or beauty of a proof in a much more relatable way than can a formal introduction by way of two-column, paragraph or flow proofs regarding a mathematical conjecture or theorem. It is through this introduction that I feel an understanding of why we need to have proof is developed. Unfortunately many of the experiences our students have had in their early education have not supported this proper understanding of proof or the needs for it but rather have built up this belief that one can prove by demonstration.

The practices of "proving" a mathematical conjecture or theory through the use of numerous demonstrations is not only improper but in many ways counter productive. Unfortunately for students and educators alike this misconception of what it is to prove a statement goes unchecked as the curricula diverge from the deductive geometry that helped to develop mathematics as a respected, irrefutable science. Despite the move away from the teaching of Euclidean geometry, research has shown that the proof versus demonstration argument could be lessened through the development of a solid geometry education (Chazan, 1993; Sutherland et. al., 2004). Naturally, this leaves one to question why the study of such a deductive discipline that has influenced the greatest minds of history is being relegated to the realm of elective courses both at the secondary and post-secondary levels.

Another issue common to the literature is the need for a deductive geometry education. The development of theoretical ability is often the hidden objective built within the lessons of a geometry curriculum. Throughout the development of my curriculum I have been continually reminded of the ultimate goal of education which is to foster lifelong skills. As much as geometry may not be a life skill, forming a logical argument to support or refute a claim *is*. Exposure to deductive proof is certainly essential to the development of theoretical ability (Johnson & Ranson, 1990). A theoretical foundation provides for a knowledge base that can be utilized in a myriad of academic disciplines as well as real-world arenas. One need only look as far as their schools history books to find a number of students of geometry. Great mathematicians and philosophers like Rene Descartes and Euclid are not the only great men involved in geometry. Students find it quite interesting that world leaders and military generals have sought education in the field of geometry. Historical figures like Abraham Lincoln have studied Euclid's work and often credit this study with their success in other areas of academia and life.

It is, however, difficult to make a justification for proof to students. Much of the reason students do not use proper proving processes is that they lack an understanding of the need for proof or understand any reason to perform these processes (Schoenfeld, 1986; Balcheff, 1991). Students in secondary schools often do not understand the universality of proof. These students often attempt to further validate their work after proving a statement is true by completing numerous examples to demonstrate the correctness of their findings (Fischbein & Kedem, 1982; Vinner, 1983). Although technology may provide a medium through which they may complete these demonstrations, it can also serve as a means of showing the general nature of proofs. As a student drags and drops a figure in the DSP environment they will be able to see the unchanging constructions for congruent triangles.

The Third International Mathematics and Science Study (TIMSS) data from 41 nations and our own National Assessment of Educational Progress (NAEP) data show a significant insufficiency in the geometry education provided to students (Lappan, 1999). The National Council of Teachers of Mathematics (NCTM) standards call for an increase in geometry education with an emphasis on the use of proof and theoretical development (NCTM, 1989; NCTM, 2000). The standards set forth by the NCTM were developed with the knowledge that geometry has far reaching effects. In some studies (Johnson & Ranson, 1990; Jones, 2000) it has been found that a solid geometry education fosters the development of strong problem solving skills.

The conclusion that geometry courses aid in the furthering of a students ability to solve more abstract problems is supported by the nature of mathematics. Mathematics is theoretical in and of itself (Mariotti, 2000; Herbst & Brach, 2006) and, therefore, math education should highlight this theoretical foundation upon which our knowledge has been continually built. If the recent past is any indication, the intuitive geometry which we as educators have been focusing our geometry education on causes conceptual gaps that need correction upon entering a deductive geometry classroom (Mariotti, 2000).

B. Technology in the Classroom

An integral part of Euclidean geometry is the constructions based upon the theoretical frameworks set forth by Euclid in his *Elements.* Students in today's classroom may find that the by-hand constructions performed in a traditional manner with compass and straightedge are a hindrance to the learning process (Lopez-Morteo & Lopez, 2007). Over the past few decades dynamic geometry software such as Geometer's Sketchpad® has paved the way for a much more time efficient way by which constructions may be done. Through the integration of these dynamic geometry software applications into the geometry classroom of the twenty-first century the obstacle of performing the by-hand constructions has been lessened if not removed entirely.

We must not, however, be so quick to accept these software applications as the "savior" of geometry curricula. Rather, careful consideration must be given to when, where and how these applications be integrated as an improper use of the software could have the adverse effect. Jones (2000) and Chazan (1993) feel that the misuse of the technology could lead to a superficial feeling that deductive proof need not be learned as numerous constructions can be performed to "prove" a given conjecture or theory. This result would clearly be counterproductive as it would undermine our attempts to solve the proof versus demonstration problem. Despite this seemingly paradoxical situation we have found ourselves in, proper use of the technology coupled with solid teacher-student discussion of the true explanations for what is being seen or demonstrated on the computer screen will foster the development of an understanding of how demonstration can be used to gain insight to the process of proof.

As one may argue, the purpose of the dynamic geometry software applications is to aid in the development of constructions as well as the demonstration of a theorem at work. Research, however, has shown a correlation between the success of students working in a theoretical context and the students' exposure to geometric constructions done through the use of software (Mariotti, 2000). Some have explained this correlation by highlighting the ability of the software to illustrate the solidity of a given construction during a "dragging" test whereby one or more of the vertices is digitally dragged through the software environment with the use of a mouse (Mariotti, 2000; Marrades & Gutierrez, 2000; Sutherland et. al, 2004). The very fact that these software applications provide for such a test has led many educators to accept them as a way for students to receive nearly instant feedback (Steen et. al, 2006). Further, it allows a student to make a construction in a manner that they believe will always result in a given figure and then test that belief. If the construction falls apart when dragging the figure then clearly their process was flawed and hence what they felt were logical steps to follow were in fact incorrect. Prior to the use of such technology, students would not be able to simply use the undo function to attempt to ameliorate the problem that they have created in their improper construction.

Since software helps students to link the formal with the intuitive (Haddas et. al, 2000; Sutherland et. al, 2004) it has been used more frequently. However, this link is not the only benefit. In classrooms where geometry software is used to illustrate concepts, students take a more active role in the learning process (Liu & Cummings, 2001). In the metaphorical tug-of-war between technology as entertainment and technology as *edutainment* this fact can be considered a win. Whereby the use of technology has allowed for teachers to better engage their learners, the technology has opened the door for the learning process to begin. It has been shown (Sutherland et. al, 2004) that the use of technology to teach geometric concepts increases motivation and improves attitudes toward the subject matter of geometry. Surely, higher levels of motivation and engagement lead to improved performance on assessments.

Not only have dynamic geometry software applications been shown to improve motivation, engagement and attitude, but there is also evidence that its use as a virtual manipulative can improve scores. Battista (1990) found that the ability to use these virtual manipulatives to better visualize geometric figures correlated to an increase in the success rates of students in the area of geometric problem solving. Naturally, this is not counterintuitive. For years manipulatives have been used in the math classroom to aid students in understanding such mathematical concepts as fractions, graphical transformations and algebraic equations. Now, with the greater availability of dynamic geometry software applications, geometry can be added to the long list of classroom topics made more accessible to students through the use of virtual manipulatives.

Even at early levels, software applications like Geometer's Sketchpad® can be utilized to begin the development of geometric concepts and problem solving skills (Steen et. al, 2006) Students who continually use applications in the geometry classroom often begin to use their observations to construct a scaffolding of thought that leads to generalization and proof. The connection that students make between the measurements made within the software environment and those that are of the theoretical nature of geometry can be shown to have a positive effect on their ability to recognize the need for proof as well as their ability to create generalizations that lead to proof (Kynigos, 1993).

These findings bring us full circle. The NCTM standards call for mathematics educators to make a move toward deductive geometry (Lappan, 1999; NCTM, 2000) and the use of technology has been shown to aid in this movement.

C. Summary

Many conclusions may be had through the review of all the literature dealing with the current issues of technology and geometry classrooms. Through my review of literature while preparing to create a geometry curriculum I have come to a number of interesting conclusions. First, it is the role of the educator to stress the difference between demonstration and proof. Although for years students may have been provided with demonstrations as proof for a statement, teachers of a geometry classroom must rebuild students' understandings of what proof is. The constant struggle to get students to explain why a statement is true as opposed to showing that it is true can be eased through careful construction of a curriculum that redefines what proof is. Mathematicians know that all of math is developed with and supported by deductive reasoning. It has, however, become one of the shortcomings of today's math classrooms that students are not introduced to the magnificent importance of truly proving through a logical progression of ideas that are supported by accepted theory.

Secondly, the use of technology by teachers of math can lead students to the discovery of why a statement is true. The software applications provide for a familiar visual representation that can be manipulated to demonstrate the properties chosen. Although this is a demonstration and not a proof, the ease with which numerous constructions can be done can lead students to discover a generalization much the way numerous examples of an arithmetic process leads to the students' understanding of an algebraic equation. By eliminating the traditional compass and straightedge constructions in favor of the software constructions, educators have opened the door to an avenue of a much more efficient use of time. The use of the software requires the same conceptual knowledge needed to operate a compass or straightedge; however, one can easily discover problems within the construction. Since the problems that arise within the completion of a construction directly relate to a conceptual mistake made by the students, the discovery of the error in the construction will lead to the discovery of the misunderstood conceptual piece.

Lastly, it is obvious that the creation of a geometry curriculum centered on deductive reasoning is beneficial for geometry, but it can also improve student performance in other problem solving disciplines. The logical thought processes instilled in students through the implementation of a deductive geometry curriculum reaches far beyond geometry. Students with a solid background in deductive geometry show increased ability in other mathematical areas when presented with abstract problems. Moreover, these students can often translate there logical reasoning in the geometry classroom to situations outside of the academic realm.

Clearly, the literature focused my aim as I created the geometry unit I plan to implement. The way in which the technology is introduced needs to be inviting to the students as opposed to forced and artificial. By using the software applications I will create an atmosphere that is more familiar to the students. Despite all of the positives that come with the use of the dynamic geometry software applications, a proper balance between direct classroom instruction and student-centered discovery learning through technology must be struck. Carefully defining the expectations of what a proof is and how explanations should be constructed is essential to the success of my (and any) geometry unit or curriculum. The allowances I afford my students while exploring the software environment will hopefully encourage an atmosphere of exploration and discovery thus improving attitude and motivation.

Mathematics education is not and can not be viewed as a static science. As times change and students' experiences are altered by their ever changing environments, educators of all disciplines must make adjustments that reflect these changes. The current generation of students is not satisfied with the teaching styles that succeeded even one decade ago. Careful development of technologically enriched and visually stimulating lessons is essential to the success of mathematics education. Many of our students already possess capabilities to use and manipulate the technology in ways far superior to our own. Capitalizing on these capabilities is my goal. By teaching to these strengths a new age of math student will be better versed in deductive reasoning than there predecessors.

Chapter Three: Application

The literature review helped me to develop my geometry curriculum. Through my extensive research I found that the biggest problem to face would be the student's beliefs surrounding what a proof is and the need to complete a proof. As a result much of what I developed was done so with the intent to have students connect their theoretical learning with the constructions and demonstrations being done within the DSP environment. In addition to fostering this connection, I developed the curriculum keeping in mind the standards set forth by NYSED and NCTM. While creating the curriculum I was careful to balance the direct instruction regarding the postulates and theorems being investigated and the student explorations through the use of the DSP. To measure student beliefs regarding requirements for proof and the subsequent growth in their knowledge of proof as a result of the unit, I created two assessments. One assessment was given to the students on the first day of the unit prior to any instruction regarding proof and the second assessment was given during the final week of the unit. In addition to these assessments, a survey of opinion was administered following the unit of study.

The course in which this study was performed, Applied Mathematics, was comprised of ten students of varying grade levels (10-12) and differing abilities. There were eight males and two females in the class. Two of the eight students were ESL learners and one of the ten students has an IEP. The topic I chose to develop the studied unit around was congruent triangles. I chose congruent triangles as it is often the first introduction to formal proof techniques in today's secondary geometry classroom. Although many texts were referenced (Bass et. al., 2007; Boyd et. al., 2004; Gantert et. al., 2008; Larson et. al., 2001) I created any materials that were used by the students. These materials were distributed as a student workbook to be completed throughout the unit. Any assignments given to the students in the class were created by adapting appropriate questions found in the referenced texts or on prior state examinations.

Throughout the completion of this unit of study students spent approximately twelve hours actively working in the DSP environment. The remainder of the time was spent discussing outcomes as well as considering strategies that may be employed during the proof process. Students were allowed to freely explore a strategy they considered to be valid regardless of my opinion. By doing this I hoped to foster the development of skills through the constant reformulation of the student plans for proofs and their strategies. Although much of the research discussed the difference between demonstration and proof, the DSP allowed for my students to test their hypotheses regarding a given postulate or theorem by completing constructions and then considering the general case. After students had struggled with discovering the proper technique for construction I proceeded to demonstrate for them by completing the construction using the DSP which was being projected from my laptop to a large screen at the front of the room.

It should be noted that as the unit progressed more and more students became quite proficient in their exercises and were then able to replace me at the front of the room and demonstrate their work for their classmates. Each time a student was completing a construction on using the laptop projector, I encouraged the discussion that was surrounding their work and facilitated a question and answer portion regarding the whys and whats of their work. Despite an early resistance from the students to explain and justify steps they had taken, I noticed a general proficiency in justification by the final week of the unit. Many students were now able to explain their way through a proof in an elegant manner prior to using the DSP as a means of demonstration. This, to me, was the realization of a goal that had been set for them.

Although a number of students expressed frustration with the DSP at times, they all were quite actively engaged when attempting the construction. Many times students could be heard moaning and groaning under their breath about the struggles they were encountering. However, smiles, sighs and exclamations could be seen and heard when I would perform a demonstration and they saw that they too were correct. Through this process of struggling I acted as a soundboard but did not provide direct answers to their questions. I forced them to discuss their problems with one another and reflect upon the process they had performed to evaluate why something was incorrect.

For many of the students it became easy quite quickly to describe why two triangles were congruent or why two parts of a figure were congruent but to construct a figure to demonstrate that very theorem seemed to be a massive roadblock. Although they struggled with the concept of the demonstration their proofs became more and more elegant as they learned to perform the proper constructions. At one point after having the students attempt their first written proof, I asked that the students go home and write down all the instructions to making a great peanut butter and jelly sandwich. The next day when we discussed there instructions I pointed out the flaws. This led to their realization that math is a literal science and therefore each step can not be simply implied but rather explicitly explained.

In the final weeks of the unit I began to ask students more and more questions that required a higher order of thinking. Although the class had students from differing ability levels, it could be said that all of the students were math phobic in some way. The very fact that they would openly discuss and even consider volunteering their opinions to these questions that required a higher order of thinking was to me phenomenal. One of the best discussions came out of an inquiry I had regarding why constructions using Geometer's Sketchpad®

was not the same as a simple demonstration. Students actively and eagerly debated the differences between constructions and demonstrations. One quote that came out of this discussion was from a student in their junior year of high school. They said, "Constructions aren't demonstrations because they're built from other theorems and postulates. Demonstrations aren't general at all they're just using a ruler and protractor but constructions really always work no matter the lengths." Upon further questioning it was revealed to me that the students agreed with this other student and that none of them realized that constructions could be generalized until they used the DSP and were able to drag different points to change the original shape.

The two assessments I used as a way to determine growth with regards to student beliefs about proof were invaluable. It provided me insight of where they started and where they ended up. In both the pre- and post-unit assessment the students were presented with two scenarios. Each scenario explained a task that had been given to a student, a description of the process the student used to complete the task, and lastly the actual work that the student had provided the instructor. The tasks that were in question were those of proving a statement. One of the scenarios on each assessment was a proper formal proof in either a two column or paragraph proof form while on the other hand the remaining scenario made use of measurements of a specific case. The students completing the assessment would then score each of the two scenarios based on a rubric that had been provided. In addition to scoring the work they were required to defend their scores by briefly writing why they gave the student the score they had.

Table 3.1 - Pre-assessment Rubric and Scenarios

Please answer all of the questions below to the best of your ability. Your answers to the questions that follow should be those of opinion. There are no right or wrong answers to these questions. Use the rubric provided as well as your knowledge of math to aid in your response. Please be sure to be as thorough as you can. Thank you for your cooperation.

Score of 1 – The student shows limited or no knowledge of the concept being assessed.	Score of 2 – The student's response has some validity but still contains many gaps in the understanding of what is being assessed.	Score of 3 – The student's response is mostly valid but falls short of providing sufficient evidence to support the claim they are proving.	Score of 4 – The student's response is completely valid but is not clearly communicated.	Score of 5 – The student's response is clear, concise and complete. Evidence is provided that supports the claim and that evidence is appropriate.

1. A math teacher asks a student to prove to him that if two angles of a triangle are equal in measure then the two corresponding sides of that triangle are also equal. Immediately the student neatly draws a diagram being sure to use a protractor to create two congruent angles and labels the triangle appropriately. Upon completion of the triangle the student measures each side and indicates that she has drawn a triangle with two congruent angles and it resulted in two congruent sides

2. Jonathon's teacher has asked to class to prove that two triangles formed by a combination of parallel and intersecting lines are congruent. He begins to prepare his proof on his assignment paper by drawing the diagram and labeling all the parts of the triangles. Following this step Jonathon writes a paragraph explaining what theorems and postulates can be used to prove the two triangles congruent. Jonathon uses no ruler or protractor and lists no lengths or angle measures in his explanation. What follows is the work Jonathon provided to his teacher.

The pre-assessment results showed what I had believed to be true. Each of the students participating in the study scored the scenario that relied solely upon demonstration by way of measurement and a single specific case higher than they did the scenario where an appropriate formal proof was performed. Furthermore, the mean score given to the student that utilized demonstration was 4.3 of a possible 5 whereas the mean score given to the student that utilized proof was 2.9. Following the pre-assessment it was clear to me that there were some definite misconceptions regarding the word proof. I now knew I would need to reinvent the concept of proof for these students through a steady progression throughout the unit. Therefore, I started simply by asking students questions like the difference between showing something is true and explaining it. As we progressed I began to use the words explanation and proof interchangeably so that students could understand or see the link between the two. Ultimately, this improved their conceptual understanding which was seen throughout the unit and finally in the post-assessment. The post-assessment was identical in concept to the pre-assessment but posed two new scenarios. Here again there was a dramatic difference in scoring and a clear cut favorite among the students in the study. However, unlike the pre-assessment the students in the study all ranked the formal proof response as being more complete and satisfying the requested task better than the demonstration response. The mean scores on the post assessment were 3.0 of a possible 5 for the demonstration and a 4.1 for the formal proof.

Table 3.2 - Post-assessment Rubric and Scenarios

Please answer all of the questions below to the best of your ability. Your answers to the questions that follow should be those of opinion. There are no right or wrong answers to these questions. Use the rubric provided as well as your knowledge of math to aid in your response. Please be sure to be as thorough as you can. Thank you for your cooperation.

Score of 1 – The student shows limited or no knowledge of the concept being assessed.	Score of 2 – The student's response has some validity but still contains many gaps in the understanding of what is being assessed.	Score of 3 – The student's response is mostly valid but falls short of providing sufficient evidence to support the claim they are proving.	Score of 4 – The student's response is completely valid but is not clearly communicated.	Score of 5 – The student's response is clear, concise and complete. Evidence is provided that supports the claim and that evidence is appropriate.

1. Jordan's teacher has asked the class to prove that a quadrilateral formed by a series of intersecting lines is a parallelogram. She begins to prepare her proof on her assignment paper by drawing the diagram and labeling all the parts of the quadrilaterals. She includes all of the given information that her teacher provided. Following this step Jordan writes a paragraph explaining what theorems and postulates may be applied to show this quadrilateral is a parallelogram. Jordan uses no ruler or protractor, she does not use graph paper or find the slopes of the sides of the quadrilateral, and she lists no lengths or angle measures in her explanation. What follows is the solution she provided to her teacher.

2. A teacher asks his student to prove to him that given that two angles of a triangle are complimentary the triangle is right. The student neatly draws a diagram being sure to use a protractor to create two complimentary angles. He labels the triangle appropriately. Upon completion of the triangle the student measures each leg and uses the Pythagorean Theorem to find the hypotenuse. He indicates that he has drawn a triangle with two complimentary angles and it resulted in a right triangle. What follows is the answer he provided to his teacher.

In addition to the pre- and post-assessments a post-unit survey designed to gain feedback from the students regarding the specific use of the DSP as well as their attitudes toward the use of the DSP in this capacity. It also provided the students the chance to explain why they did or did not feel that the DSP was helpful in their studies. I used this piece of assessment as a reflection tool to help me better build the unit in the future. It provided me with great insight into what the students felt about the computer program as well as their level of engagement. The responses ranged in tone and in opinion but the majority of the students expressed a desire to take more math classes that had been integrated with technology. All of the students noted that they had felt much more engaged as a result of the use of the DSP then they would have been otherwise. One student said, "At the beginning I really thought the computer would be a distraction and I would want to surf the net or play a game but I ended up using Sketchpad as my entertainment. I wouldn't say I was on task 100% of the time but even when I wasn't doing the requested construction or proof I was working on another proof or construction that I had just figured out." As a teacher I find this to be encouraging. Certainly it would be nice if each student was on the same step as his or her neighbor and that that step would be the one the teacher wanted them to be on but this can clearly never be true all of the time. As a result I feel that this student's honesty has shown me that the use of the DSP becomes so enjoyable and puzzle-like that even in times of distraction the students were engaged in a meaningful mathematical task. This is clearly better than the alternative whereby students are fidgeting with their cellular phones attempting to text one another or doodling in their notebooks instead of taking notes. Now the doodling is being done on the sketchpad and it has a purpose.

Table 3.3 - Post-Unit Survey Questions

Post-Unit Survey
1. Do you believe that the use of a software program like Geometer's Sketchpad® (GSP) was appropriate for the context of this class? Why or why not?
2. In what ways was this unit made easier by the use of the computer? In what ways do you think it was made more difficult? In other words what were some benefits and what were some drawbacks.
3. When considering how to prove a geometric statement through the use of logical reasoning, why would a student want to use Geometer's Sketchpad? Explain.
4. In what ways did the use of GSP change how you participate in class? Explain.
5. If you were given the opportunity to complete an entire geometry course that was created with the intent of using technology like GSP would you choose to do so? Why or why not?

Chapter Four: Conclusion and Recommendations

Throughout my research to prepare for this unit I was made keenly aware of the fact that one of the premier problems facing geometry teachers today is that their students do not understand the meaning, purpose or necessity of proof. After the realization that this was such a prevalent issue in the geometry classroom, I decided to focus the majority of my energy on improving this understanding through the use of technology. Upon further investigation I found that in order to tackle this problem, I would first need to properly engage and motivate students in the geometry class. Much of the literature I read pointed to lack of motivation and engagement as a reason for the misunderstanding of proof. By integrating the technology into the curriculum I hoped to bridge a gap with the students where a happy medium could be reached and therefore I could begin the process of reinventing the definition of proof for these misinformed students.

It was my feeling that during this unit the students were much more engaged than they had been in past units of study. There are clearly multiple causes that one could site for this improved engagement. One might claim that my attitude changed toward the material as geometry is among my favorite topics. Others may say that it could be that the students themselves enjoy geometry more than the other topics covered in their applied mathematics course. I personally believe that, although these causes could play a role in the engagement, the technology is the main component. My justification for this comes in the form of student feedback. The answers that students provided on the *Post-Unit Survey* (see table 3.3) all pointed to the fact that they enjoyed working with the computers more than they had with a graphing calculator or textbook and notebook lecture based class. During a classroom discussion that occurred after the survey was completed, a number of students said that calculators are okay and they are at least a form of technology that teachers are comfortable using in the classroom but computers and DSP provide the students with even more capabilities. One particular student commented that, "Computers are what we're comfortable with and that's what teachers should care about, not that they are comfortable with calculators or lecture."

Although it was difficult to integrate the technology I did feel that it was well worth the energy and effort. Teachers should carefully plan their lessons as I found that those that I had spent less time planning and less time preparing for quickly fell apart. It is not easy to think on your feet and redirect a lesson that goes awry when you are dealing with technology. When using the DSP to guide a class you can not expect to just be spontaneous as it is far too difficult to do the same things you've done in other lecture based lessons that have strayed from the objective or fallen apart.

Geometer's Sketchpad® is software best learned by diving in and working with it. At the beginning of the unit I felt quite proficient in aiding the students when they had questions or problems arose with their constructions. Unfortunately, I soon learned I knew less about the program than I had anticipated. It was at this point where I found that being humble was an extremely valuable quality. When a problem or question arose that I could not solve immediately or was unsure about, I simply admitted that I didn't know the answer but would be sure to figure out what was causing the problem by the next time we met. By taking this time to find what was causing the problem I often learned more about the software but more importantly my students would often try to help the faltering student through the exercise without my request.

Naturally as educators we can recognize the value in peer instruction. This unit was among the best I have taught in this regard. As I mentioned, when problems occurred students would help one another and by doing so would improve their ultimate understanding. Once again this peer instruction also pointed to an improved engagement and upon successful instruction of their neighbor in the classroom the students felt a sense of accomplishment that in turn led to an outwardly visible motivation that had previously not been seen. Students would enter the classroom asking what we were going to be doing that day or immediately open their laptop and begin working in the DSP environment without my cues. Some students commented that the reasons they would work without my request was it was like a game to see if they could remember the steps to creating a particular construction.

Motivation and engagement were only the beginning of the process to promoting a stronger understanding of proof. Once the students were clearly engaged and motivated I began to push them further than they had been in the past. At times they were clearly uncomfortable when being pushed to write down the steps they felt that were necessary to create a construction. However, their motivation to succeed at this point had overcome their fear or discomfort with the situation. After they had become more comfortable with writing the steps to the constructions I pushed them a bit further and had them write why they believed these steps led to congruent triangles. Again their motivation overcame all other discomfort. By the conclusion of the unit the students had a much clearer understanding of proof as was evidenced by the responses to the *Post-Assessment* (see table 3.2). In addition to their understanding of what proof was their ability to complete a logical, coherent, and sometimes elegant proof had definitely improved. Further, the displayed improvement in proving was even clearer upon noting that they had been utilizing the DSP as a means to construct a demonstration which they then used as a plan for their proof. I myself do not recall writing a plan of a proof until my first proof based undergraduate mathematics course yet these students who admittedly had struggled in their prior math experiences were doing so often times without my urging.

Despite the fact that the majority of my focus was on improving student motivation and engagement, I spent much time outside the classroom developing the curriculum to be conducive to the improvement of the motivation and engagement. The curriculum was developed using what most texts would regard as the best progression of ideas and concepts. After numerous days of browsing accepted secondary geometry texts I found that nearly every text progressed through the triangle congruence unit in the same way. With the exception of tasks they asked of students, each text introduced the concepts in the same order and hence I felt that this was also the best way to progress through the unit. Following this step in my curriculum development I wrote down each concept that would be taught and then developed my own set of notes and tasks to present to the students. The final step of the process was to reflect upon my experiences teaching the curriculum and determine its effectiveness.

Upon reflection I feel that more time should have been put into the development of some of the tasks I required the students to complete. Because it had been a while since I had used the DSP I had not expected some of the problems that were encountered by the students. It is for this reason I believe that if I had spent more time developing step by step instructions for the students to follow it may have led to a much more streamlined process. If I were to do this unit again I would provide step by step instructions for the use of the DSP for at least the first two weeks of the unit. Following the first two weeks the students would be allowed to work freely without a set of instructions to guide them. This may have alleviated some of the initial problems the students had while performing the constructions.

Although the students in this class seemed to be more motivated and engaged I feel I could have developed the curriculum to further safeguard against the occasional distraction caused by working with the laptops. At times, the more advanced students would complete a task early leaving them only to help other students. Although this was a welcome sign of motivation and engagement, more careful development and planning would create meaningful tasks that could be easily modified for those students who were more proficient than the rest. By developing these tasks differently, differentiation could be accomplished more easily. As this particular class is a class that meets the third year requirement for students not inclined to take a regents level course, it could be assumed that in the future the class will again have a wide variety of students with differing interests and abilities. Therefore, by putting more time into the development of the tasks, assessments and lecture notes a wider variety of students could be reached and challenged despite their differing capabilities.

I think that in the future if one were to implement a curriculum that integrated technology in the secondary geometry classroom that they should attempt to do so in a lab where the computers are a permanent fixture and a projector be connected to the teacher computer. I encountered a number of problems with the technology as a result of the transience of the fact that they were merely laptops. These problems led to the loss of valuable instruction time that could not be recaptured. A computer lab as opposed to a laptop cart poses less risk of technology failure due to battery power loss, poor or no wireless connection, or bad resolution from an improvised projector. In addition to these problems there was the lost time due to the students needing to remove their laptop from the cart, move to their seats and then log on. A lab would have provided me the ability to have the computers logged on prior to the students entering class and eliminated the five minutes needed for set up and tear down of the classroom.

In the end I feel that this unit provided a significant amount of growth both for me and my students.

My students learned that math could be fun and they could be motivated, engaged and interested in the subject that had caused them so much trouble in the past. Further they learned one of the most valuable lessons of a math education. They learned what proof is and how to string together a series of logical ideas and thoughts. Myself, I learned how to better integrate technology into my classroom. The unit allowed me the freedom to create a curriculum through which I could develop my own thoughts and ideas. It allowed me to revisit the curriculum and evaluate it as both the creator and implementer. This provided me with two perspectives that were unique. This reflection process has made me a better planner and given me a better insight into the problems that arise through the development of a new curriculum. Therefore, the experience for me was invaluable and irreplaceable.

It is my sincere belief that through a constant state of redevelopment and recreation of a technologically enriched curriculum that one can vastly improve engagement and motivation of students in a geometry classroom like the one in which this study was conducted. Although I do not want to over generalize my observations, I believe that the integration of technology would have a similar effect in other geometry classes even those for advanced placement students and those for students of lesser abilities. Students' lives are constantly bombarded with new ways to be entertained and yet, despite what some call educational reform and *new* curriculum, students are asked to learn through the same media that they have for years - textbooks and lectures. By stepping outside of their own comfort zone, geometry educators could see a positive impact within their classroom that is more than worth the discomfort they feel through the development process. Furthermore, students will most assuredly enjoy the class on a higher level and participate in classroom activities in a more meaningful manner. These things put together will ultimately lead to the improved education of students and their better understanding of the geometry concepts being studied.

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Chapter 6: Appendix A

A Technologically Enriched Geometry Curriculum:

Curriculum Resources

by

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August 1, 2008

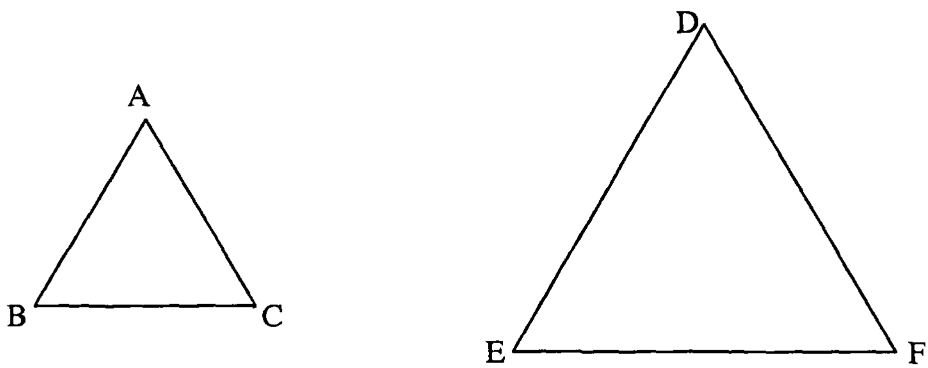
Department of Education and Human Development of the

State University of New York College at Brockport

Name:_______________________________ Date:___________

Theorem 1-1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$

Define the terms that follow.

Congruent
Polygons:____

Corresponding
Parts:____

Key Question: *How do we know that Theorem 1-1 is true?*

Name:________________________________ Date:___________

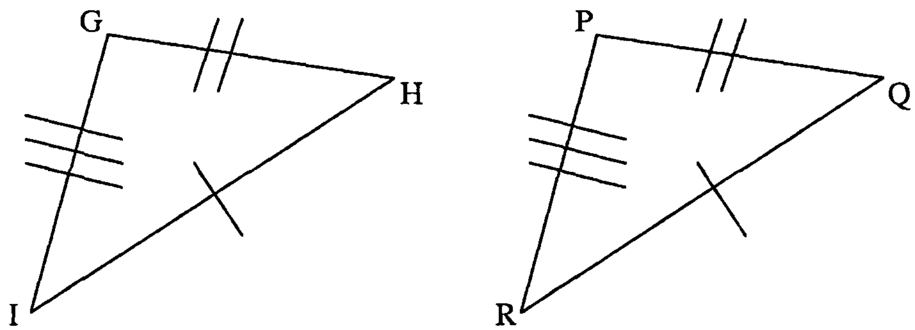
We know that if each part of one triangle is congruent to its corresponding part of another triangle then the two triangles are congruent. By constructing congruent parts of triangles, construct a pair of congruent triangles with Geometer's Sketchpad. Be sure to use the measuring utilities to show that your corresponding parts are congruent. By dragging a point in your original triangle is your constructed congruent triangle still congruent? In the next days we will be learning how to truly construct congruent triangles through the use of proven postulates and theorems. Save your constructions.

Now that your constructions have been completed think about how you might use Theorem 1-1 to complete this task. Write your response below.

Name:______________________________ Date:___________

Fortunately, it is not necessary to always know that all six corresponding parts of a pair of triangles (3 sides and 3 angles) are congruent to determine that the triangles are congruent.

Postulate 1-1 Side-Side-Side (SSS) Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.



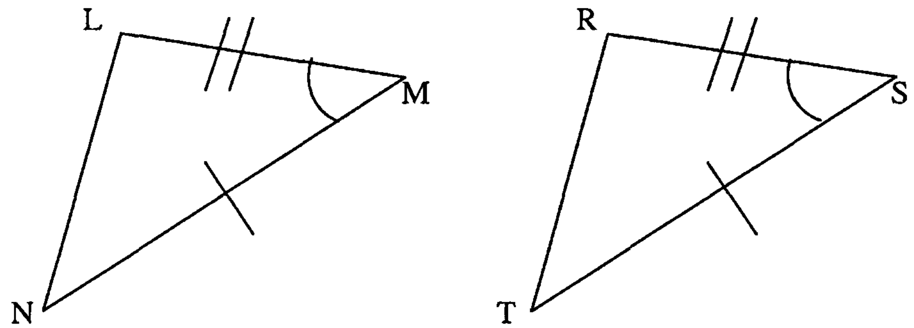
$\Delta GHI \cong \Delta PQR$

Using the SSS Postulate and Geometer's Sketchpad, construct a pair of congruent triangles and save your construction.

Key Question: *Although we know that postulates are the self-evident truths, try your best to explain why the SSS postulate is true.*

Name:______________________________ Date:____________

Postulate 1-2 Side-Angle-Side (SAS) Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.



$\Delta LMN \cong \Delta RST$

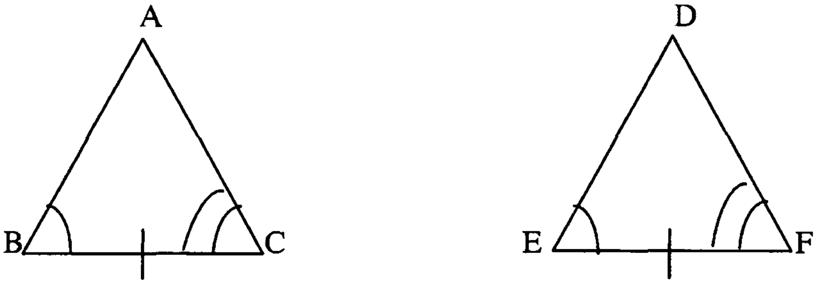
The word included refers to something, in this case an angle, between two other objects, in this case the sides. It is frequently used in the language of the postulates and theorems involving the angles and sides of a triangle.

Using the SAS Postulate and Geometer's Sketchpad, construct a pair of congruent triangles and save your construction.

Key Question: *Does it matter what the size of the included angle is? Length of the sides? What do your answers to that question lead you to believe about the application of the SAS Postulate? (Hint: Recall your knowledge of conditional statements and the fact that postulates are accepted as being truths.)*

Name:______________________________ Date:__________

Postulate 1-3 Angle-Side-Angle (ASA) Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

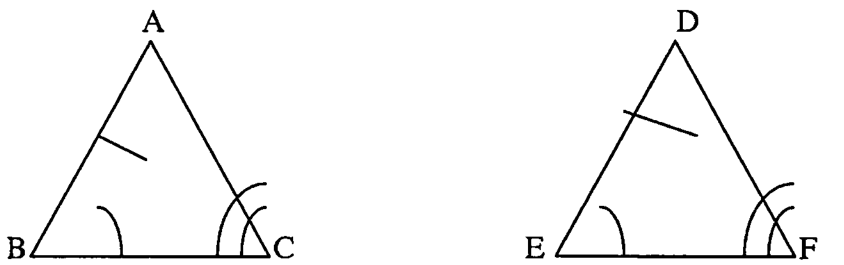


If $\angle ABC \cong \angle DEF$, $\angle ACB \cong \angle DFE$, and $\overline{BC} \cong \overline{EF}$ then $\Delta ABC \cong \Delta DEF$.

Using Postulate 1-3 we can construct two congruent triangles in Geometer's Sketchpad. Below write a plan that describes the steps you will perform to create the congruent triangles. Be specific about what you are constructing. After you have made a list of steps attempt to complete your construction in GSP.

Name:______________________________ Date:___________

Theorem 1-2 Angle-Angle-Side (AAS) Theorem
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the two triangles are congruent.



If $\angle ABC \cong \angle DEF$, $\angle ACB \cong \angle DFE$, and $\overline{BA} \cong \overline{ED}$ then $\Delta ABC \cong \Delta DEF$.

Recall that if something is labeled as a theorem it must be possible to prove that statement true. Although in the past we have been using postulates to show congruence of triangles we now have a theorem. Before constructing to congruent triangles using this theorem, attempt to explain that it must be true. (Hint: Theorem 1-1)

Now using Geometer's Sketchpad construct two congruent triangles using the AAS Theorem. It may be best to first write a plan as you did prior to the ASA Postulate construction.

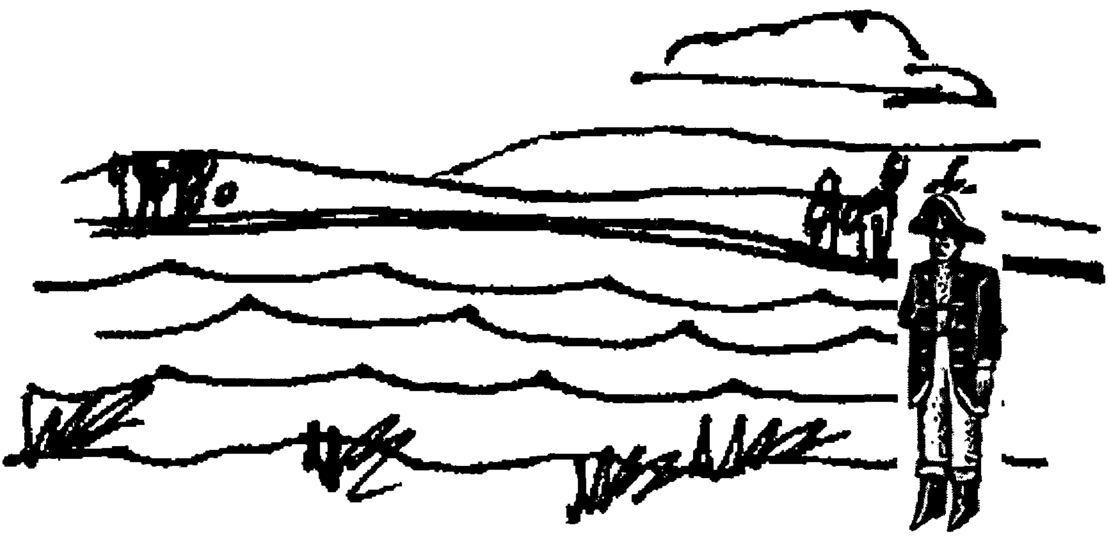
Name:______________________________ Date:___________

With SSS, SAS, ASA and AAS, you can use three corresponding congruent parts of two triangles to show that those two triangles are congruent. Once the two triangles are congruent you can make conclusions regarding the remaining parts of the triangles. These conclusions can be made because, by definition, corresponding parts of congruent triangles are congruent. This is often abbreviated as **CPCTC**.

According to a legend, one of Napoleon's officers used congruent triangles to estimate the width of a river. On the riverbank the officer tilted the visor of his cap until the angle at which he was forced to look placed his eyes directly at the other shoreline. The officer then turned his body and noted the spot his eyes landed upon on his side of the river. By pacing the distance between his origin and the spot he noted, he was able to estimate the distance across the river. Explain below by use of two column proof, paragraph proof or flow proof why the officer was correct.

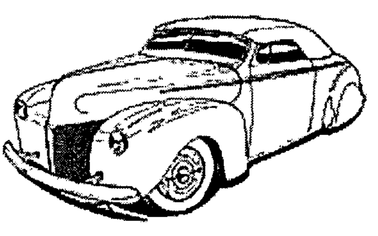
Name:______________________________ Date:___________

Yesterday you wrote a plan for a proof and then proved that the legend of Napoleon's officer was a completely plausible situation. Today you must attempt to complete a sketch using GSP to construct a diagram that demonstrates an example of your proof. You may choose to make your demonstration as elaborate or basic as you see fit.



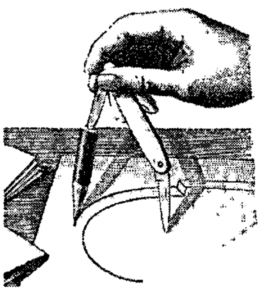
Name:______________________________ Date:____________

Two cars of the same model have hood braces tare identical, connect to the body of the car in the same place, and fit into the same slot in the hood. Drawing a diagram of your own below label all parts with letters for vertices and write out you have been given. Then complete a proof that the hood braces hold the hoods open at the same angle. After your proof has been completed make a construction based upon your given statements that will demonstrate this situation.



Name:________________________________ Date:___________

In the construction for the bisector of $\angle BAC$, $\overline{AB} \cong \overline{AC}$ because they are radii of the same circle. $\overline{BX} \cong \overline{CX}$ because both arcs had the same radius. In the space below draw a diagram that shows the situation as described. Then tell why you can conclude that $\overrightarrow{AX}$ bisects $\angle BAC$.



Name:________________________________ Date:__________

Isosceles triangles are a common part of art, design and engineering. We see these special triangles everywhere. The congruent sides of an isosceles triangle are called its **legs** and the remaining side is called the **base.** The two congruent sides form the **vertex angle** and the angles formed by the legs and the base are called **base angles.** Below you will find three theorems about isosceles triangles that we will investigate using Geometer's Sketchpad.

Theorem 1-3 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Theorem 1-4 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Theorem 1-5
The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

Name:______________________________ Date:___________

Using the software we can easily create a sketch that will help us to write a proof of the Isosceles Triangle Theorem. To do this we must first construct an isosceles triangle. There are two ways to do this but in this case we must start with the base and construct two congruent circles at the end points of the base. By connecting the intersection of these circles with the endpoints both of the legs will be the same length. The next step in the construction so that we can better prove the theorem is to construct the bisector of the vertex angle. This can be down by simply selecting each of the points that determine the vertex angle and then constructing the angle bisector. Now you have constructed an isosceles triangle with its bisector. From here we can prove that the two base angles are congruent. How can you prove this? Below write a proof for the theorem. *(Remember the construction alone is not the proof but merely an aid to help you plan and visualize your proof.)*

Name:______________________________ Date:___________

Now that you have proven theorem 1-3, we can attempt to prove its converse (theorem 1-4). This time we will construct our isosceles triangle by starting with our base and constructing two congruent angles at the endpoints of this base. Where the two rays that form these angles intersect will be the vertex of our isosceles triangle. By constructing the triangle in this manner and then constructing the bisector of the vertex angle we will be starting with the initial condition of the theorem. That is we will have the two base angles congruent and know nothing of the two legs. Now that you have your diagram constructed, use it to help you formulate a proof. Below write a proof of theorem 1-4. *(Remember the construction alone is not the proof but merely an aid to help you plan and visualize your proof.)*

Name:______________________________ Date:___________

In this exercise you are required to prove theorem 1-5. You are being asked to prove this theorem in any manner you see fit (two column, paragraph or flow). You must, however, use the software to construct a diagram that is appropriate for the proof. Be sure that your diagram only satisfies the given conditions and does not assume things to be true that have not been given to you in the conditions of the theorem. Below, first explain the steps you took to construct your diagram and then write your proof.

Save a copy of your construction to your student folder and title it "Theorem_1_5."

Name:______________________________ Date:___________

A **corollary** is a statement that follows directly from a theorem. One can think of this statement as an addendum or addition to the original thought. Below you will find two corollaries to the Isosceles Triangle Theorem and its converse. These corollaries identify statements that can be made in a special case of isosceles triangles.

Corollary to Theorem 1-3
If a triangle is equilateral, then the triangle is equiangular.

Corollary to Theorem 1-4
If a triangle is equiangular, then the triangle is equilateral.

Your task is to make two constructions that demonstrate these corollaries. Be sure to construct the first so that each side is equal and then use measuring utilities to show each angle is equal. On the second construction you must construct the triangle so that each angle is congruent (this may be more difficult as each angle must be exactly 60 degrees) and use measuring utilities to show each side is equal.

Name:______________________________ Date:___________

Another special category of triangles are right triangles. In a right triangle the side opposite the right angle is called the **hypotenuse** and the other two sides are called the **legs**. Right triangles are the only type of triangles where congruence between two triangles can be determined by a SSA congruence rule. The rule only occurs in right triangles and therefore has a name that helps to remind us that it only occurs then.

Theorem 1-6 Hypotenuse-Leg (HL) Theorem
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Using the given conditions below, create a diagram using the software that demonstrates this theorem. After you have constructed the diagram write a proof for the theorem. When you have finished your proof discuss with a friend. Discuss the differences and similarities in the proofs and if there are other ways this theorem can be proved.

GIVEN: ΔPQR and ΔXYZ are right triangles, with right angles Q and Y respectively. $\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$.

PROVE: $\Delta PQR \cong \Delta XYZ$

(NOTE: There is more than one way to complete this proof.)

Name:______________________________ Date:___________

Throughout this unit you have completed numerous constructions and proofs. Below, write an explanation of what a proof is. Discuss the difference between proof and demonstration as well as any likeness. You may also choose to make mention of how you used demonstration in your proving process. Do not forget to include a discussion about Geometer's Sketchpad.

Chapter 6: Appendix B

A Technologically Enriched Geometry Curriculum:

Assessment Tools

by

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August 1, 2008

Department of Education and Human Development of the

State University of New York College at Brockport

Pre-Assessment

Please answer all of the questions below to the best of your ability. Your answers to the questions that follow should be those of opinion. There are no right or wrong answers to these questions. Use the rubric provided as well as your knowledge of math to aid in your response. Please be sure to be as thorough as you can. Thank you for your cooperation.

Score of 1 – The student shows limited or no knowledge of the concept being assessed.	Score of 2 – The student's response has some validity but still contains many gaps in the understanding of what is being assessed.	Score of 3 – The student's response is mostly valid but falls short of providing sufficient evidence to support the claim they are proving.	Score of 4 – The student's response is completely valid but is not clearly communicated.	Score of 5 – The student's response is clear, concise and complete. Evidence is provided that supports the claim and that evidence is appropriate.

1. A math teacher asks a student to prove to him that if two angles of a triangle are equal in measure then the two corresponding sides of that triangle are also equal. Immediately the student neatly draws a diagram being sure to use a protractor to create two congruent angles and labels the triangle appropriately. Upon completion of the triangle the student measures each side and indicates that she has drawn a triangle with two congruent angles and it resulted in two congruent sides.

If the student were receiving a grade from 1 to 5 (5 being the highest) on this assignment, what would you give the student and why?

2. Jonathon's teacher has asked to class to prove that two triangles formed by a combination of parallel and intersecting lines are congruent. He begins to prepare his proof on his assignment paper by drawing the diagram and labeling all the parts of the triangles. Following this step Jonathon writes a paragraph explaining what theorems and postulates can be used to prove the two triangles congruent. Jonathon uses no ruler or protractor and lists no lengths or angle measures in his explanation.

If the student were receiving a grade from 1 to 5 (5 being the highest) on this assignment, what would you give the student and why?

Post-Assessment

Please answer all of the questions below to the best of your ability. Your answers to the questions that follow should be those of opinion. There are no right or wrong answers to these questions. Use the rubric provided as well as your knowledge of math to aid in your response. Please be sure to be as thorough as you can. Thank you for your cooperation.

Score of 1 – The student shows limited or no knowledge of the concept being assessed.	Score of 2 – The student's response has some validity but still contains many gaps in the understanding of what is being assessed.	Score of 3 – The student's response is mostly valid but falls short of providing sufficient evidence to support the claim they are proving.	Score of 4 – The student's response is completely valid but is not clearly communicated.	Score of 5 – The student's response is clear, concise and complete. Evidence is provided that supports the claim and that evidence is appropriate.

1. Jordan's teacher has asked the class to prove that a quadrilateral formed by a series of intersecting lines is a parallelogram. She begins to prepare her proof on her assignment paper by drawing the diagram and labeling all the parts of the quadrilaterals. She includes all of the given information that her teacher provided. Following this step Jordan writes a paragraph explaining what theorems and postulates may be applied to show this quadrilateral is a parallelogram. Jordan uses no ruler or protractor, she does not use graph paper or find the slopes of the sides of the quadrilateral, and she lists no lengths or angle measures in her explanation.

If the student were receiving a grade from 1 to 5 (5 being the highest) on this assignment, what would you give the student and why?

2. A teacher asks his student to prove to him that given that two angles of a triangle are complimentary the triangle is right. The student neatly draws a diagram being sure to use a protractor to create two complimentary angles. He labels the triangle appropriately. Upon completion of the triangle the student measures each leg and uses the Pythagorean Theorem to find the hypotenuse. He indicates that he has drawn a triangle with two complimentary angles and it resulted in a right triangle.

If the student were receiving a grade from 1 to 5 (5 being the highest) on this assignment, what would you give the student and why?

Post-Unit Survey

The following is a small questionnaire of 5 questions. This questionnaire should take you approximately 30 minutes to complete. Be honest and complete with all responses. Do NOT write your name on the questionnaire as this is to be anonymous. The answers from your questions will be used to modify the unit of study that you have just completed. Thank you for your cooperation.

1. Do you believe that the use of a software program like Geometer's Sketchpad® (GSP) was appropriate for the context of this class? Why or why not?

2. In what ways was this unit made easier by the use of the computer? In what ways do you think it was made more difficult? In other words what were some benefits and what were some drawbacks.

3. When considering how to prove a geometric statement through the use of logical reasoning, why would a student want to use Geometer's Sketchpad? Explain.

4. In what ways did the use of GSP change how you participate in class? Explain.

5. If you were given the opportunity to complete an entire geometry course that was created with the intent of using technology like GSP would you choose to do so? Why or why not?