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Identifying False Intuitions in Probability and Laying a Foundation for Teaching It

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Table of Contents

I. Introduction

An intuition, by definition, is a direct perception of truth that is independent of any reasoning process (Merriam-Webster's, 2005). It is viewed as something that is removed from cognitive thought and instead is "self-evident, self-consistent, and hardly questionable" (Fischbein & Gazit, 1984, p. 2). One such intuitive example in mathematics is the statement: the shortest distance between two points is a straight line. While this could be proved mathematically, it is instinctive to easily understand this notion and accept its entrance into one's cognitive intuition. However, there are other parts of mathematics that are not intuitive and require thought and even a proof to explain their existence. The question then is: how does the mind distinguish between what is intuitive and what is in need of a solid explanation? The answer lies in how the brain organizes different types of information (Fischbein & Schnarch, 1997).

The brain itself is a very cause-and-effect oriented machine. This means that if the body comes into contact with a recognizable stimulus the brain recalls how the body responded to it in the past and enacts this same response. If the stimulus is one that the body has never come into contact with, the response is logged in the brain's memory in case of reoccurrence. In the case where a stimulus is met on a regular basis, the brain becomes quicker at triggering the learned response and even creates a special schema because of the brain's cause-and-effect interpretation. While most people would agree that this is an efficient model for handling events, it becomes a poor model when the elicited response is an incorrect one. A continually wrong response can cause a person to have intuitively incorrect notions, something that can be detrimental in mathematics (Fischbein & Schnarch, 1997). When age is added into the equation, an incorrect intuition becomes even more damaging to one's cognitive state. According to Fischbein and Schnarch (1997), upon continual use of this flawed schema one's misconceptions of the specific event only worsens. A person becomes more comfortable with the schema and it is more difficult to unlearn what has been incorrectly learned.

Having studied the research on the psychological effects that incorrect intuitions can have on learning, it is safe to say that false intuitions are causing misconceptions in every mathematics classroom throughout the world. In particularly, these false intuitions are hurting students when they begin studying probability. The study of probability does not involve the teaching of a single set procedure for attacking problems, so naturally students experience difficulties when learning it. They attempt to apply old intuitive schemata to situations where the schemata is not applicable and it becomes a "square peg-round hole" scenario (Fischbein & Schnarch, 1997). The purpose of this curriculum project is, through my survey of literature, to uncover the different intuitions brought by students into the classroom and to present one method that can bring a halt to these false intuitions. Hopefully, upon the completion of this review, there will be some light shed on common misconceptions students have and one particular way to navigate through them.

The curriculum portion of this project will include a unit plan on probability that meets the New York State Standards at the Integrated Algebra level. The unit plan will include a pre-assessment, daily lesson plans, daily classwork activities, daily formative assessments, and a unit test that will serve as a summative assessment. The curriculum will be written with the proceeding research in mind and will help students dispel false intuitions. Differentiated instructional activities will also be used to best reach the widest range of students intellectually and stylistically. To help analyze and provide professional feedback, the complete unit plan will be sent to experienced high school math teachers. The teachers will fill-out a questionnaire that asks them some questions regarding the subject of probability and about the created unit plan. These responses will be used in the discussion, summary and reflection portion of this thesis project.

II. Survey of Literature

Research

In the analyzation of complex events, people involuntarily develop judgment heuristics. These heuristics are based on assessments that have been made in regards to various situations that a person has experienced. While these heuristics can sometimes be true, they can also sometimes be false. Such is the case when studying probability and more specifically, probabilistic thinking. People attempt to apply a judgment or intuition to a probabilistic situation and the particular judgment does not meet the needs of the situation. When this happens, people begin to develop one of three incorrect ways of looking at probability, in either the outcome approach, the representative heuristic, or the personalist interpretation (Konold, 1989).

The outcome approach is a heuristic that involves a person being concerned with predicting the outcome of a single trial when performing an event. While this sounds like a correct way to predict an event, the problem exists in that the outcome approach relies solely on "casual analysis." The individual uses subjective thought to predict outcomes and has no concern for frequency of occurrence, the basis of probability theory. Those following the outcome approach also tend to view probability as a prediction rather than an actual distribution or theory. This means that probability is being evaluated as right or wrong after individual trials. For example, in Konold's (1989) study he asked students, "If you were to roll this [bone], which side do you think would most likely land upright?" (p. 67). The response one student gave was, "Wow if I were a math major, this would be easy. B is nice and flat, so if D fell down, B would be up. I'd go with B" (Konold, 1989, p. 67). After the bone was rolled and the student's prediction was incorrect, the student responded with, "Wrong again. B didn't come up" (Konold, 1989, p. 67). This shows in principle, the reliance on the probability of an event in being correct on incorrect. The student did not ask to roll the bone multiple times before guessing the side that would be face up, so he is not concerning himself with the frequency or distribution of results that the bone would exhibit when it is rolled. Also, the student's comment about how the activity would be easier if he were a math major shows his subjectivity towards probabilistic thinking. He truly believes that having a better knowledge of mathematics would help him predict the part of the bone that should land face up (Konold, 1989).

Konold and his fellow authors (1993) note in their study, that through qualitative data analysis, they have found student data that is indicative of the outcome approach, but that it can be difficult to detect. For example, in testing the gambler's fallacy on subjects, they found that after four successive flips of tails many students responded that getting heads or tails was equally likely. Since the probability of flipping heads and tails is both .5, this answer on the surface seems like a correct one. Upon further questioning and review, however, they discovered that the students had said "equally likely," but in fact meant that either heads or tails could occur. The students did not determine that heads and tails had an equal probability of occurrence (Konold, Pollastek, Well, Lohmeier, & Lipson, 1993).

The representative heuristic involves a person making a judgment in accordance with how similar a sample is to its parent population (Konold et al., 1993). In simpler terms, the idea of representativeness means that the probability of an event is based on how typical the event's occurrence is (Garfield & Ahlgen, 1988). As an example of this, Konold brings to mind a description that was given to subjects in a study performed by Tversky and Kahneman back in 1974. It reads, "Steven is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail" (as cited in Konold et al., 1993, p. 394). Upon reading this, the subjects were directed to choose whether this narrative was portraying a mathematician or a lawyer. Those with the representative intuition, fell into the trap of identifying a person by the stereotypical characteristics that describe the mathematician population. While this is not totally irrelevant from making probability based judgments, it is imperfect because it focuses too much on a subjective thought process. What the subjects should have realized was that there are many more lawyers than mathematicians in the world, so there is a much greater probability for Steven to be a lawyer (Konold et al, 1993).

Like the outcome approach, the representative heuristic is rooted too much in subjective analysis. Decisions to particular probability questions are not based on frequency or distribution, but on a resemblance thought process. What is not readily known is that individuals who tend to reason in accordance with proper probabilistic thinking, sometimes resort to using the representative heuristic in certain situations. When the context of an example seems to be something the subject is familiar with, she will often resort back to using representativeness. This reversal is believed to be a result of people being "more willing to pair essential (highly visible, albeit misleading) features of evidence with outcome probabilities than they are to determine outcome probabilities on the basis of statistical rules of prediction" (Cox & Mouw, 1992, p.165).

The personalist interpretation of probability is similar to both the outcome approach and representative heuristic in that it has a heavy reliance on subjective thought. Personalists look at a situation and determine its outcome based on experiences that they have had with the event. As an example, when flipping a coin a personalist would believe that the probability of getting heads or tails is dependent upon the fairness of the coin, the technique of the flip, and the character of the person who flips the coin. This means that various personalists could come up with differing values of probability for this example (Konold, 1989).

While this preceding description sounds much like that of the outcome approach, the biggest difference is that personalists are not only concerned about a single trial of an event. Personalists will take into account the frequency of occurrence of a specific event. This means that a personalist's probability can become calibrated over a long period of time to mirror that of a frequentist's. This is a positive comparison for a personalist because frequentist ideas are widely accepted in the statistical world (Bayarri & Berger, 2004). A frequentist believes that a probability assigned to an event is justifiable, if this value is approached as n (the number of trials) approaches infinity (Batanero & Serrano, 1999). As an example, if a personalist had to decide between heads and tails on a coin flip and the personalist had information on the 100,000 flips before this specific flip, the personalist would be more inclined to base her prediction on this information. Thus, while a personalist's intuition is incorrect in terms of subjective thought, there is a degree of correctness because of its semi-reliance on frequency and distribution (Konold, 1989).

While judgment heuristics account for a major reason as to why false intuitions about probability have been learned, there is another set of beliefs known as maxims that also account for incorrect probabilistic thinking. Maxims are a set of beliefs that are usually shared by a large portion of the population in one's culture.

They are viewed as self-evident and obvious, and are used to justify other claims (Konold et al., 1993). As an example, there are a number of maxims that are related to coin flipping because it is such a common activity in our culture. This list consists of:

> One cannot predict for certain the results of coin flipping; the outcomes of repeated trails vary unsystematically between heads and tails; a coin has no memory; heads and tails are equally likely; and heads and tails occur about equally often in a sample of flips. (Konold et al., 1993, p. 407)

The problem with these statements is that they do not hold true for every situation. Little is known about the boundary conditions for which these statements hold true and thus the statements can be contradictory when used. For example, if we take the maxim, heads and tails occur about equally often in a sample of flips and compare it to the maxim, the coin has no memory, we arrive at a contradiction. Following the first maxim, it would seem logical to think that if a person had a long run of heads, that there would be a more probable chance that the next flip would result in tails. However, the coin has no memory so how can it know that there has been a long string of heads? Another contradiction occurs in that if we belief that one cannot predict for certain the results of coin flipping, then how can heads and tails be equally likely? These contradictions are plentiful when looking at these coin maxims and it is for this reason that maxims must not be used to justify probability (Konold et al, 1993).

While the above paragraphs discuss the false intuitions that students hold, the proceeding writing will be much more positive and will give hope as to how probabilistic thought can be correctly taught. The methodology of this teaching centers around four probability constructs and a corresponding cognitive framework. The ideas are meant to be taught at an early age, but most certainly can be used for teaching this subject at any grade level (Jones, Langrall, Thornton, & Mogill, 1999).

According to Jones, the four constructs that probability can be separated into are sample space, probability of an event, comparisons amongst probabilities, and conditional probability. These serve as the four most important subtopics in probability and act as the elementary knowledge necessary for further study of the subject (Jones et al., 1999). A basic example of these constructs could involve a box that consists of one red marble, two green marbles, and four blue marbles. A question that tests knowledge of sample space would be: what possible colored marbles could be chosen? To test a student's familiarity with probability, on could ask: what is the probability of choosing a green marble? Taking this probability question one step further and asking a student: is there a higher probability for choosing a blue marble or green marble? would be an assessment of the third construct. Finally, conditional probability could be gauged by asking the following: if a green marble is chosen and then not replaced in the box, what is the probability of choosing a green marble? While each of these questions is very basic, each one helps breakdown the underlying themes that should exist in probabilistic thought.

The cognitive framework that needs to be assessed in conjunction with the constructs has four levels to it: the subjective, the transitional, the informal quantitative, and the numerical. Students need to be monitored closely so that they can be placed at the correct level and each level has specific descriptors that need to be met before an advancement up the framework can occur. The subjective level, as the name implies, centers around a student only being able to think subjectively. It is characterized by students being misled by irrelevant characteristics and having limited reasoning skills. Using the examples mentioned in the paragraph about the probability constructs, someone at the subjective level would reason that a marble would have a higher probability than all the other marbles because that specific marble is the student's favorite color. Usually this type of thinking occurs in very young children who are just beginning to experience mathematics that involves probability (Jones et al., 1999).

At the transitional level, students first begin to associate a probability with a quantitative judgment. Students are able to figure out which events are most likely to occur and least likely to occur, but still lack basic reasoning skills when comparing two different probabilities. For example, if someone at this level was given probability values, he would be unable to assign them to specific outcomes. While this level does indicate a more objective thought process, there still exists a conflict between this new concrete thinking and the skewed intuitions experienced at the subjective level (Jones et al., 1999).

The informal-quantitative stage is when students first begin to use quantitative reasoning when dealing with probability. They are able to effectively compare two events based on quantitative judgments and have developed some generative strategies for solving certain probabilistic events. For example, these students are able to create tree diagrams to figure out all the outcomes associated with flipping a coin multiple times. Students at this level also have an understanding of the sample space and can determine when it changes during a conditional probability situation (Jones et al., 1999).

Students at the numerical level are finally able to associate an exact numerical probability to a sample space. Using the marble sample space example from above, students would be capable of determining that the probability of a red marble is 1/7, the probability of a green marble is 2/7, and the probability of a blue marble 4/7. Likewise, students would also have the ability to rank probabilities from least to greatest and decide if any probabilities are equal. Lastly, at this stage a conditional probability situation not only can be identified, but numerical probabilities can be assigned to such events (Jones et al., 1999).

Once students have advanced to the numerical level of the four probability constructs, they are ready to experience what probability truly is. According to research by Gelman and Glickman (2000), the best way to do this is through participatory demonstrations that exhibit real-world examples. One such example that these two authors highlight, deals with eliminating the idea that there is usually frequent alterations between heads and tails during a sequence of random coin flips.

This demonstration starts with two students being selected as judges and the rest of the class being divided into two groups. The first group is designated as the flip group and their job is to flip a coin 100 times and record the heads-tails sequence they get on the blackboard. The second group is the fake group and their job is to randomly makeup a heads-tails sequence that resembles 100 coin flips and record it on the blackboard. While this is going on, the two judges are to leave the classroom so as not to see the sequences being written on the blackboard (Gelman & Glickman, 2000).

Once the two groups are done, the judges reenter the classroom and must decide which sequence is real. The authors claim that the judges will always select the real group's sequence because the fake group's sequence always has an orderly look to it. On the other hand, the real group's sequence will be rather streaky and will not look like a pattern. From this demonstration, the teacher can then branch off into discussions on randomness and sampling distributions, as well as point out the falseness of the representative heuristic. The authors endorse these types of examples because "they are effective in dramatizing concepts that students often find difficult" (Gelman & Glickman, 2000, p. 98). They believe that demonstrations offer a role-playing and participatory setting, that is much like that of the real statistical world. Demonstrations also serve as activities that get all students involved in the learning process. Students are able to freely think about concepts and ask questions that they may have (Gelman & Glickman, 2000).

Connections to State Standards

As mentioned in the introduction, this curriculum project involves curriculum that meets some of the New York State Standards in the Integrated Algebra course. This list includes the following: A.S.18, A.S.19, A.S.20, A.S.21, A.S.22, and A.S.23. All of these standards are content specific and are located in the statistics and probability section of the course's standards. These standards specifically deal with conditional probability, sample space, the complement of an event, empirical probabilities, the likeliness of events occurring, and dependent, independent and mutually exclusive events, respectively. Everything that is within the unit plan is based on and guided by these six standards. As can be noticed upon reading through the curriculum documents, the standards that drive each day's lesson are mentioned in each lesson plan.

While the performance standards are not listed in the unit nor in each lesson plan, upon reading through the curriculum one would definitely notice that many different performance standards are included. For example, A.PS.10 says that students should be able to "evaluate the relative efficiency of different representations and solution methods of a problem." This is exemplified by the lesson on sample spaces, in which students learn how to represent the sample spaces of compound events with both a chart and a tree diagram. While the students are expected to know both of these methods, they find out through the completion of problems which one is more efficient and ultimately more preferable for them. Looking at A.CN.6, which says that students should be able to "recognize and apply mathematics to situations in the outside world," one will again see, after looking through the proceeding curriculum, that this standard is evident in numerous problems throughout each lesson. Real world examples stimulate thought and allow people to see different aspects of life where math is needed.

The choice was made to not physically write the performance standards in each lesson plan because so many of these standards are used repetitively every day. During each activity whether it be a warm-up, notes, practice problems, group work, homework, a unit test, or simply a conversation between a teacher and a student, there are numerous performance standards being used. It did not seem necessary for the same standards to be listed in lesson after lesson. This, however, does not mean that these standards were not considered on the onset of creating the unit plan. The curriculum was definitely developed with both the content and performance standards as a blue print from which to build each lesson off of.

Current Trends and Future Projections

The most insightful part of this review was learning about the different ways people perceive the subject of probability. While probability is complicated, it is interesting to see the number of wrong ways that it is learned. The judgment heuristics and maxim beliefs mentioned, seem to be rather common and are learned and turned into intuitions long before students begin secondary education. Therefore, it is the job of the middle and high school math teacher to point out these false heuristics and rebuild the foundation with a program much like the one described. Looking towards the future, there needs to be a continual effort and push to have probability and statistics taught at the elementary level. In glancing at the New York State Mathematics Standards, one can see that probability is not introduced until the fifth grade (New York State, 2005). This is too late; especially since in the fifth grade students are expected to have the ability to do the following three things: list the possible outcomes of a single-event experiment, record experiment results using fractions and ratios, and create a sample space and determine the probability of a single event. According to the model presented in this review, this would mean that the students are expected to be at the numerical level of the four-stage cognitive framework, during the very first year they begin learning about probability. That does not seem fair for either the students or the elementary school teachers involved. Further research would need to be conducted on teaching probability at the elementary level, to see exactly what is going on. Until this is done, secondary school teachers need to be on the look out for false intuitions and they need to turn their classrooms into a participatory environment where active learning of probability can take place.

III. Body – The Curriculum Project

Integrated Algebra - Grade 9

Probability Unit Plan

Table of Contents

NYS Standards

7.S.10: Predict the outcome of an experiment

A.S.18: Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces

A.S.19: Determine the number of elements in a sample space and the number of favorable events

A.S.20: Calculate the probability of an event and its complement

A.S.21: Determine empirical probabilities based on specific sample data

A.S.22: Determine based on calculated probability of a set of events, if:

- some or all are equally likely to occur
- one is more likely to occur than another
- whether or not an event is certain to happen or not to happen

A.S.23: Calculate the probability of:

- a series of independent events
- a series of dependent events
- two mutually exclusive events
- two events that are not mutually exclusive

Unit Objectives

1. Students will be able to predict the outcomes of simple probabilities.
2. Students will be able to compare empirical results to predicted results in probability.
3. Students will be able to identify the sample spaces of different events.
4. Students will be able to calculate both the probability of an event and its complement.
5. Students will be able to recognize if an event can occur.
6. Students will be able to find the probability of dependent events.
7. Students will be able to find the probability of independent events.
8. Students will be able to find the probability of mutually exclusive events.
9. Students will be able to understand if the probability of an event is unlikely, likely, certain, or certain not to occur.
10. Students will improve at solving problems related to each probability concept that has been covered.
11. Students will be able to define conditional probability.
12. Students will be able to solve conditional probabilities when given a finite sample space.
13. Students will be prepared to take a unit test on probability.

Day to Day Schedule

Day 1: Reviewing basic probability skills.

- Objective 1: Students will be able to predict the outcomes of simple probabilities.
- Objective 2: Students will be able to compare empirical results to predicted results in probability.
- Activities:
    a) "Probability Skills" pre-assessment
- Homework: "Probability Practice" worksheet (for those students who did not get an 85% or better on the pre-assessment)

Day 2: Understanding sample spaces and the relationship between the probability of an event and its complement.

- Objective 3: Students will be able to identify the sample spaces of different events.
- Objective 4: Students will be able to calculate both the probability of an event and its complement.
- Objective 5: Students will be able to recognize if an event can occur.
- Activities:
    a) Notes on the sample space and complement of an event
    b) Warm-up
    c) Differentiated activity
- Homework: "Probability Day 2" worksheet

Day 3: Finding the probability of dependent, independent, and mutually exclusive events.

- Objective 6: Students will be able to find the probability of dependent events.
- Objective 7: Students will be able to find the probability of independent events.
- Objective 8: Students will be able to find the probability of mutually exclusive events.
- Activities:
    a) Warm-up
    b) Notes on independent, dependent, mutually exclusive, and non-mutually exclusive events
    c) Stations activity
- Homework: "Probability Day 3" worksheet

Day 4: Understanding the likeliness of an event occurring.

- Objective 9: Students will be able to understand if the probability of an event is unlikely, likely, certain, or certain not to occur.
- Objective 10: Students will improve at solving problems related to each probability concept that has been covered.
- Activities:
    a) Notes on the likeliness of an event
    b) "Practice Problems" packet
- Homework: "Practice Problems" packet (students will complete ten more problems)

Day 5: Using a finite sample space to solve conditional probabilities.

- Objective 11: Students will be able to define conditional probability.
- Objective 12: Students will be able to solve conditional probabilities when given a finite sample space.
- Objective 13: Students will be prepared to take a unit test on probability.
- Activities:
    a) Notes on conditional probability
    b) "Review Sheet" activity

Day 6: Probability Unit Test

- Activities:
    a) Test

Day 1 Instruction Plan

Grade Level: 9 **Math Course:**
Integrated Algebra

Concept: Reviewing basic probability skills.

Objective:

1. Students will be able to predict the outcomes of simple probabilities.
2. Students will be able to compare empirical results to predicted results in probability.

Assessments:

- Students will complete a pre-assessment entitled "Probability Skills" (objective 1 and 2).
- The students will complete a homework assignment that addresses each type of problem that is covered in the pre-assessment.

Standards Addressed:

NYS Learning Standards for Mathematics:

7.S.10 – Predict the outcome of an experiment

A.S.21 – Determine empirical probabilities based on specific sample data

Materials Needed:

- "Probability Skills" pre-assessment

Teaching Process:

1. *Anticipatory Set*: Students will complete a pre-assessment entitled "Probability Skills."
    - Explain to students that the pre-assessment will be corrected, but that the grade will not be recorded.
    - Tell students that if they do not get an 85% on the pre-assessment they will have homework.

2. The students will correct "Probability Skills."
    - Have each student trade papers with someone else.
    - Read off the answers to each question.
    - Have each student put the score at the top of the paper they graded.
    - Go over all of the questions on the pre-assessment.
    - Collect the pre-assessment papers to see who scored less than an 85%.

3. *Closure*: Have students answer the following question on a half sheet of paper: What is the hardest part of probability to understand? Collect this half sheet before the students leave.

Homework:

The students who did not score and 85% or better will have a worksheet to complete entitled, "Probability Practice."

Probability Skills | Name:
Pre-Assessment | Date:

1. The probability that an event will occur is always greater than or equal to ___ and less than or equal to ___. Thus, probability is always expressed as a ____________.

2. When flipping a fair coin, what is the probability of flipping heads? What is the probability of flipping tails?

3. Bill is about to roll a fair die. What is the probability that he rolls a:
   1?

   2?

   3?

   4?

   5?

   6?

4. Lisa is dealing a standard deck of cards [Note: a standard deck of cards has 52 cards; 13 cards of each suit (diamonds, hearts, spades, and clubs) and 3 face cards of each suit]. What is the probability that she will deal a:
   heart?

   7?

   black suited card?

   face card that is a diamond?

5. If there are Skittles in a bowl and 11 are purple, 5 are yellow, 14 are green, 9 are red, and 13 are orange, what is the probability of choosing a Skittle that is:
   purple?

   yellow?

   green?

   red?

   orange?

6. Henry flipped a coin 20 times and recorded his results in the table below for each trial.

Trial Number	Outcome	Trial Number	Outcome
1	T	11	T
2	T	12	T
3	T	13	T
4	T	14	T
5	T	15	T
6	T	16	H
7	H	17	H
8	H	18	T
9	T	19	H
10	H	20	H

According to his experiment, what is the probability of getting heads? Tails?

What is the predicted probability of getting heads? Tails?

7. You just lost a game of Yahtzee and think that one of the die is unfair. You know that the probability of getting any number on a fair die is 1/6, so you decide to test this. You record the results in the table below:

Trial	Outcome
1	5
2	4
3	4
4	4
5	2
6	5

According to the table, what is the probability of rolling a
1?

2?

3?

4?

5?

6?

Do you still think the die is unfair? Why or why not?

Probability

HW 1

Name:

Date:

Probability Practice

1. What is the probability of tossing a fair die and getting an odd number? an even number?

2. A standard deck of cards is dealt. What is the probability of getting a(n):
   diamond?

   10?

   club?

   ace?

   number card that is red suited?

3. Our alphabet contains 26 letters. Five to these letters are vowels (A, E, I, O, U). The remaining 21 letters are consonants. Each letter of the alphabet is written on a card and you are asked to

choose a card. What is the probability of choosing a consonant? vowel?

4. Suppose you spin a spinner that has a 1, 2, 3, and 4 on it and the area that each number covers is exactly the same. What is the probability of spinning a:
   1?

   2?

   3?

   4?

Day 2 Instruction Plan

Grade Level: 9 **Math Course:** Integrated Algebra

Concept: Understanding sample spaces and the relationship between the probability of an event and its complement.

Objective:

1. Students will be able to identify the sample spaces of different events.
2. Students will be able to calculate both the probability of an event and its complement.
3. Students will be able to recognize if an event can occur.

Assessments:

- The students will complete a differentiated activity according to the way each student learns best (objective 1, 2, and 3).
- Throughout the lesson, the teacher will call on students to assist in the solving of problems.
- The students will complete a homework assignment addressing the types of problems that were completed in class (objective 1, 2, and 3).

Standards Addressed:

NYS Learning Standards for Mathematics:

A.S.19 – Determine the number of elements in a sample space and the number of favorable events

A.S.20 – Calculate the probability of an event and its complement

Materials Needed:

- chalk/white board
- "Sample Space/Complement" notes (teacher copy, underlined items will be blank on student copy)
- overhead projector
- differentiated activity directions
- "Probability Day 2" homework assignment

Teaching Process:

1. *Anticipatory Set*: The students will complete a warm-up problem that addresses basic probability knowledge.
    - Write the following question on the board: "A bag contains three red marbles, two green marbles, and four orange marbles. If one marble is selected at random, what is the probability that it will be orange?"

- Call on a student to help solve this problem.

After solving the warm-up problem, ask the students if they can name the sample space of this particular event. This will serve as the lead-in to notes on sample space.

2. The students will follow along with the teacher as the guided notes, "Sample Space/Complement," are completed using the overhead projector.
    - When the section on sample space has been completed, ask students if they can predict the probability that the marble selected will not be orange. This will serve as a lead-in to notes on the complement of an event.

3. Each student will now have the opportunity to choose which activity he can learn best from. They will choose either an auditory, kinesthetic, or visual activity and the directions for each activity are described on the proceeding sheet.

4. Once the students have completed the differentiated activities, they will hand in their answer sheets or posters.

5. *Closure*: Have the students complete problem seven from the homework assignment entitled, "Probability Day 2."
    - Go over this problem before students leave.

Homework:

The students will complete a homework assignment entitled, "Probability Day 2." This will be checked for completion the next time the class meets.

Probability Name:

Sample Space/Complement Notes Date:

sample space – <u>all of the different POSSIBLE outcomes that could occur when performing a particular event</u>

**<u>Unless stated, always put brackets around the elements of the sample space when you are listing them</u>

Examples:

1. List the sample space of a coin flip.

2. List the sample space of a die.

3. Suppose you are going to travel on a river. You have two choices of boats – a kayak or a rowboat. The river splits into three smaller streams going north, northwest, and northeast. What is the sample space for your journey? Use a tree diagram and a chart.

counting principle – if there are m ways of making one choice and n ways of making a second choice then there are $m \cdot n$ ways of making the first choice followed by the second choice.

complement – the collection of outcomes not contained in the event

In mathematics this means that for any event A,

$$\boxed{P(A) + P(\text{not } A) = 1 \text{ or } P(\text{not } A) = 1 - P(A)}$$

****The largest probability an event can be is 1****

Examples:

1. Following example 2 above, what is the probability of rolling a 6? What is the probability of the complement of rolling a 6?

2. Following example 3 above, what is the P(rowboat and going north)? What is the P(not using a rowboat and not going north)?

3. The U.S. Senate has 100 members, 2 members from each state. What is the probability of selecting a senator

from New York? What is the probability of not selecting a senator from New York?

4. You write the letters, MISSISSIPPI, on cards and mix them thoroughly in a hat. You select one card without looking.
   a) Name the sample space.

   b) Find each of the following probabilities.

P(M)	P(not M)
P(vowel)	P(not vowel)
P(P)	P(not P)

Name: Date:

In-The-Circle Activity Directions

Use the taped circle on the floor to help you name the following sample spaces and calculate the following probabilities.

1. Name the sample space of those who are boys (Hint: have all of the boys go into the taped circle and write down their names in brackets).
2. Amongst the boys, what is the P(blue eyes)?
3. Amongst the boys, what is the P(brown eyes)?
4. Amongst the boys, what is the P(not blue eyes)?
5. Amongst the boys, what is the probability of the complement of brown eyes?
6. Name the sample space of those who are girls.
7. Amongst the girls, what is the P(brown hair)?
8. Amongst the girls, what is the P(blonde hair)?

9. Amongst the girls, what is the P(not brown hair)?

10. Amongst the girls, what is the probability of the complement of blonde hair?

Name: Date:

Shout-Out Activity Directions

For this activity students need to pair-up with another student. They will take turns solving questions aloud. The person from each pairing who is not solving the question aloud needs to write down the answer their partner gave for each question. Refer to the New York Yankees roster provided.

1. Name the sample space of players born in the US.

2. Name the sample space of players born outside the US.

3. Name the sample space of players who are older than 25.

4. Name the sample space of players who are taller than 6' 2".

5. What is the probability that a player on the teams weighs more than 200 lbs?

6. What is the probability that a player on the team bats left-handed?

7. What is the probability of the complement that a player on the team weighs more than 200 lbs?

8. What is the probability of the complement that a player on the team bats left-handed?

9. What is the probability that a player on the team throws right-handed?

10. What is the probability that a player on the team has played more than 2 years?

11. What is the probability of the complement that a player on the team throws right-handed?

12. What is the probability of the complement that a player on the team has played more than 2 years?

Roster

Pitchers

No.	Player	Pos	Bat	Thw	Ht	Wt	DOB	Birthplace	Exp
62	Chamberlain, Joba	RP	R	R	6-2	230	9/23/85	Lincoln, NE	R
48	Farnsworth, Kyle	RP	R	R	6-4	235	4/14/76	Wichita, KS	9
38	Giese, Dan	RP	R	R	6-3	200	5/19/77	Anaheim, CA	R
22	Hawkins, LaTroy	RP	R	R	6-5	215	12/21/72	Gary, IN	13
35	Mussina, Mike	SP	L	R	6-2	190	12/8/68	Williamsport, PA	17
46	Pettitte, Andy	SP	L	L	6-5	225	6/15/72	Baton Rouge, LA	13

47 Ponson, Sidney	SP	R	R	6-1	258	11/2/76	Noord, Aruba	10
36 Ramírez, Edwar	RP	R	R	6-3	164	3/28/81	El Cercado, Dominican Republic	R
43 Rasner, Darrell	SP/RP	R	R	6-3	210	1/13/81	Carson City, NV	3
42 Rivera, Mariano	RP	R	R	6-2	185	11/29/69	Panama City, Panama	13
30 Robertson, David	RP	R	R	5-11	180	4/9/85	Birmingham, AL	R
61 Traber, Billy	RP	L	L	6-5	200	9/18/79	Torrance, CA	5
41 Veras, José	RP	R	R	6-5	236	10/20/80	Santo Domingo, Dominican Republic	R

Catchers

No.Player	Pos	Bat	Thw	Ht	Wt	DOB	Birthplace	Exp
19 Moeller, Chad	C	R	R	6-4	215	2/18/75	Upland, CA	8
26 Molina, José	C	R	R	6-2	235	6/3/75	Bayamon, Puerto Rico	9
20 Posada, Jorge	C	S	R	6-2	215	8/17/71	Santurce, Puerto Rico	13

Infielders

No.Player	Pos	Bat	Thw	Ht	Wt	DOB	Birthplace	Exp
14 Betemit, Wilson	1B/3B	S	R	6-3	230	11/2/81	Santo Domingo, Dominican Republic	7
24 Canó, Robinson	2B	L	R	6-0	205	10/22/82	San Pedro de Macoris, Dominican Republic	3
17 Christian, Justin	2B	R	R	6-1	188	4/30/80	Lincoln, NE	R
25 Giambi, Jason	1B/DH	L	R	6-3	235	1/8/71	West Covina, CA	13
2 Jeter, Derek	SS	R	R	6-3	195	6/26/74	Pequannock, NJ	13
13 Rodríguez, Álex	3B	R	R	6-3	225	7/27/75	New York, NY	14

Outfielders

No.Player	Pos	Bat	Thw	Ht	Wt	DOB	Birthplace	Exp
53 Abreu, Bobby	RF	L	R	6-0	210	3/11/74	Maracay, Venezuela	12
28 Cabrera, Melky	CF	S	L	5-11	200	8/11/84	Santo Domingo, Dominican Republic	3
11 Gardner, Brett	CF	L	L	5-10	180	8/24/83	Holly Hill, SC	R

Name: Date:

Poster Activity Directions

Create a poster that answers the following questions. Use lots of color and be creative.

Billy is in his first year of college and he needs to register for one more class. His subject choices are math, history, or psychology. His time choices are morning, afternoon, or evening. His professor choices are female or male. His building choices are Bailey Hall, Donovan Hall, or Smith Hall.

1. Name the sample space of his class choices.

2. What is the probability that his last class will be math, in the evening, taught by a male, in Smith Hall?

3. What is the probability of the complement of question 2?

Probability

HW 2

Name:

Date:

Probability Day 2

1. Name the sample space of the states that begin with the letter M.

2. A bag has five green marbles and four blue marbles. If one marble is drawn at random, what is the probability that it is not green?

3. During a half hour of television programming, eight minutes is used for commercials. If a television set is turned on at a random time during the half hour, what is the probability of the complement of a commercial being shown?

4. If a letter is chosen at random from the word "BASEBALL," what is the probability that the letter chosen is not an "L"?

5. You are at The Sandwich Shop and are trying to figure out what to get for lunch. Your bread choices are wheat or white. Your meat choices are ham, turkey, Italian assorted, or tuna. Your cheese choices are American, provolone, or Swiss. What is your sample space of sandwich choices? How many possible outcomes are there? (Solve using a tree diagram)

6. You have an important interview coming up and are not sure what outfit to wear. Your shirt choices are white, blue, black, or red. Your pant choices are khaki or black and your shoe choices are black or brown. Using a chart, what is your sample space of outfit choices? How many possible outcomes are there?

7. If the probability that an event will occur is p, what is the probability that the complement of the event will occur?

8. Using example six, what is the probability that the outfit will be blue shirt, khaki pant, and brown shoes?

9. The probability that Tim will be elected president of the freshman class is 0.7. What is the probability that Tim will not be elected president?

10. If the probability that it will rain is 0.65, what is the P(not rain)?

11. The probability that an event will occur is 4/9. What is the probability that the event will not occur?

12. The probability that the event will occur is 5/8. What is the P(not occur)?

13. If the probability of snow tomorrow is 2/5, what is the probability of no snow tomorrow?

14. From example five, show the number of outcomes by using the counting principle.

Day 3 Instruction Plan

Grade Level: 9 **Math Course:** Integrated Algebra

Concept: Finding the probability of dependent, independent, and mutually exclusive events.

Objective:
1. Students will be able to find the probability of dependent events.
2. Students will be able to find the probability of independent events.
3. Students will be able to find the probability of mutually exclusive events.

Assessments:
- The students will complete a stations activity that assesses knowledge of the day's topic (objective 1, 2, and 3).
- Throughout the lesson, the teacher will call on students to assist in the solving of problems.
- The students will complete a homework assignment addressing the types of problems that were completed in class (objective 1, 2, and 3).

Standards Addressed:

NYS Learning Standards for Mathematics:

A.S.23 – Calculate the probability of:
- a series of independent events
- a series of dependent events
- two mutually exclusive events
- two events that are not mutually exclusive

Materials Needed:
- chalk/white board
- "Independent, Dependent and Mutually Exclusive" notes (teacher copy, underlined items will be blank on student copy)
- overhead projector
- stations activity directions
- stations answer sheet
- "Probability Day 3" homework assignment

Teaching Process:
1. *Anticipatory Set*: Students will complete a warm-up problem that addresses finding the probability of independent events.
    - Write the following question the board: "A bag contains three red marbles, two green marbles, and four orange marbles. If one

marble is selected at random and replaced, and then a second marble is selected, what is the probability that both the first and second marble selected is red?"

- Call on a student to help the teacher solve this problem after going over the homework.
- This will serve as a lead-in to the day's lesson.

2. Check and go over the previous night's homework on sample space and complement.
    - Check the homework while the students are completing the warm-up.
    - Go over the homework once the students have completed the warm-up.

3. The students will follow along with the teacher as the guided notes, "Independent, Dependent, and Mutually Exclusive," are completed using the overhead projector. The notes will cover pertinent vocabulary, detailed explanations, and key examples. The underlined portions of the notes will be blank on the student copy.

4. The students will break-up into groups of four to five for the stations activity. Each group will have approximately five minutes to complete each station. Each student will fill out an answer sheet individually.

5. *Closure*: Have the students finish up the last station they are at and hand-out the night's homework. Remind students to hold on to the stations answer sheet, as the teacher will go over the answers the next day.

Homework:

The students will complete a homework assignment entitled, "Probability Day 3." This will be checked for completion the next time the class meets.

Probability | Name:

Independent, Dependent and Mutually Exclusive Notes | Date:

Independent Events

1. Outcomes of events that DO NOT have an affect on each other

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B)}$$

2. **Keyword: AND, WITH REPLACEMENT**

Examples:

1. You have three $1 bills, two $5 bills, and one $20 bill in your pocket. What is the probability of:
   a. choosing a $5 bill and a $20 bill, with replacement?

   b. choosing a $1 bill and a $20 bill, with replacement?

   c. choosing a $1 bill, $5 bill, and a $20 bill with replacement?

2. If two cards are drawn from a standard deck of 52 cards with replacement, what is the probability that both cards will be black kings?

Dependent Events

1. <u>Outcomes of events that DO have an affect on each other</u>

$$\boxed{\underline{P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)}}$$

2. **<u>Keyword: AND, WITHOUT REPLACEMENT</u>**

Examples:

1. You have three $1 bills, two $5 bills, and one $20 bill in your pocket. What is the probability of:
   d. choosing a $5 bill and a $20 bill, without replacement?

   e. choosing a $1 bill and a $20 bill, without replacement?

   f. choosing a $1 bill, $5 bill, and a $20 bill without replacement?

2. If two cards are drawn from a standard deck of 52 cards without replacement, what is the probability that both cards will be black kings?

Mutually Exclusive Events

1. Outcomes of events that could not occur at the same time
2. Think of it as this.....2 events together (mutually) that agree to exclude (not allow) each others' elements

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

3. **Keyword: OR**

Example

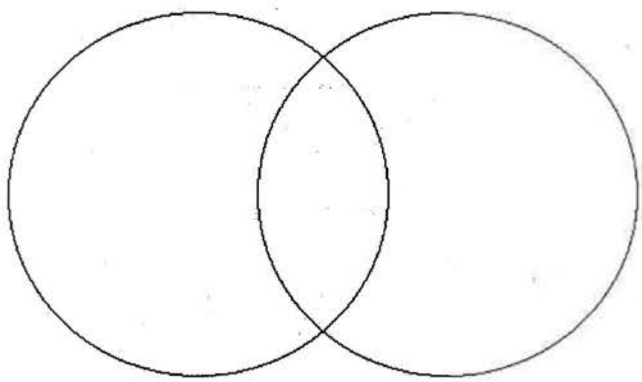
1. You have three $1 bills, two $5 bills, and one $20 bill in your pocket. What is the probability of:
    a. choosing a $5 bill or a $20 bill?

    b. choosing a $1 bill or a $20 bill?

c. choosing a $1 bill, $5 bill or a $20?

If events A and B are **not mutually exclusive then the probability that an outcome will be in one event or the other event is

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$



Example:

1. If a card is drawn from a standard deck of 52 cards what is the probability that the card will be a number card or a multiple 4?

Probability Unit Name:

Stations Activity Date:

Station 1

a. b.

c. d.

Station 2

a. b.

c. d.

Station 3

1)

2)

3)

Station 4

a)

b)

Station 5

Station 1

An urn contains 5 black marbles, 2 red marbles, and 3 orange marbles. One marble is selected at random, its color is noted, and it is returned to the urn. Another marble is then selected. What is the probability that:

a) one is red and one is black?

b) one is orange and one is green?

c) both are black?

d) the second marble selected is red?

Station 2

If two cards are drawn from a standard deck of 52 cards without replacement, what is the probability that:

a) the first card is red and the second card is black?

b) the first card is a king and the second card is a queen?

c) both cards are 7's?

d) that the second card is a 10?

Station 3

Are the two events dependent, independent, or mutually exclusive? Explain.

1. Toss a coin twice.

2. Pick a vowel at random and do not replace it. Then pick a different vowel.

3. Roll a pair of dice. What is the probability that the sum of the dice is 8 or 10?

Station 4

It is known that the probability of obtaining zero defectives in a sample of 40 items is 0.34, while the probability of obtaining 1 defective item in the sample is 0.46. What is the probability of:

a) obtaining not more than 1 defective item in a sample?

b) obtaining more than 1 defective item in a sample?

Station 5

The probability that a student passes Math is 2/3. The probability that this same student passes English is 4/9. Since the probability that he will pass both subjects is 8/27, what is the probability of him passing at least one subject?

Probability

HW 3

Name:

Date:

Probability Day 3

A die is rolled twice. Find each probability.

1. Does this situation indicate dependence or independence?

2. P(1, then 2)

3. P(3, then even)

4. P(greater than 2, then odd)

Row	Student			
A	1	2	3	4
B	5	6	7	8

An arrangement of 8 students is shown. The names of all the students are in a basket. The teacher draws one name and replaces it. Then the teacher draws a second name. Find each probability.

5. Does this situation indicate dependence or independence?

6. P(student 1, then student 8)

7. P(student 10, then a student in row B)

8. P(a student in row A, then student 6, 7, or 8)

A box contains 20 cards, numbered 1-20. You draw a card. Without replacing the first card, you draw a second card. Find each probability.

9. P(2, then 10)

10. P(4, then odd)

11. P(even, then 14)

A pair of dice is rolled and their numbers are summed. Two possible sums are a number greater than 7 and an odd number.

12. Are these two events mutually exclusive?

13. Find the probability that the sum is greater than 7 or that the sum is an odd number?

A pair of dice is rolled and their numbers are summed. Two possible sums are a multiple of 3 and a multiple of 5.

14. Are these two events mutually exclusive?

15. Find the probability that the sum is a multiple of 3 or a multiple of 5?

Day 4 Instruction Plan

Grade Level: 9 **Math Course:** Integrated Algebra

Concept: Understanding the likeliness of an event occurring.

Objective:
1. Students will be able to understand if the probability of an event is unlikely, likely, certain, or certain not to occur.
2. Students will improve at solving problems related to each probability concept that has been covered.

Assessments:
- The students will complete problems that address today's concept as well as all other probability topics that have been covered in this unit (objective 1 and 2).
- Throughout the lesson, the teacher will call on students to assist in the solving of problems.

Standards Addressed:

NYS Learning Standards for Mathematics:

A.S.22 – Determine based on calculated probability of a set of events, if:
- Some or all are equally likely to occur
- One is more likely to occur than another
- Whether or not an event is certain to happen or not to happen
- two events that are not mutually exclusive

Materials Needed:
- chalk/white board
- "Lesson Four" notes
- "Practice Problems" packet

Teaching Process:
1. *Anticipatory Set*: Check and go over the previous night's homework on dependence, independence, and mutually exclusive events.
2. Go over the stations activity from the class before. Be sure to answer all questions before moving on.
3. On the board/overhead projector, give brief notes and have students copy down a practice problem on the day's topic.

4. The students will be given a packet of 22 practice problems that are divided into three sections: easy, average, and hard. They will be instructed to choose ten problems to complete in class, two of which must be from the hard section.

5. *Closure*: Have each student compare answers with another student. Remind students that they may have completed different problems, but that they can check those problems that they both completed.

Homework:

The students will complete ten more problems from the packet. This will be checked for completion the next time the class meets.

Lesson Four Notes

- As an event becomes more likely to occur, its probability gets closer to 1
- Thus, the probability that an event is........
  - certain not to happen = 0
  - likely not to happen is < .5
  - likely to happen is > .5
  - certain to happen = 1

Example

1. One card is randomly selected from an ordinary deck of 52 playing cards. The following six events are possible outcomes for the selection:
   - the 3 of hearts
   - a club
   - the queen of diamonds
   - a red card
   - an ace
   - a face card

   From the six events listed above:

   Which is/are most likely to occur?

   Which is/are least likely to occur?

   Which events are equally likely to occur, if any?

Which event(s) is/are more likely to occur than selecting a face card?

Which event(s) is/are less likely to occur than selecting a club?

Probability | Name:

Practice Problems | Date:

Directions: You must show your work on a separate sheet of paper.

Easy

1. Name the sample space of math classes taught at the high school level.

2. What is probability of the complement of flipping heads when flipping a coin?

3. When rolling a die, are the chances of rolling a multiple of 7 likely, unlikely, certain, or certain not to happen?

4. A box contains 5 coins from different countries (U.S., Canada, Mexico, France, and Germany). George reaches in and takes a coin, puts that coin back, and then reaches in and takes another coin. What is the probability that both coins are from Germany?

5. Name the sample space of continents in the world.

6. A drawer contains 3 black and 2 white socks. A sock is drawn at random and not replaced. What is P(2 black socks)?

7. When rolling a die, are the chances of rolling a multiple of 1 likely, unlikely, certain, or certain not to happen?

Average

8. If two events are independent and both have a probability of 1/8, what is P(A, then B)?

9. A fair die is rolled and the outcome noted. Calculate the probabilities and determine whether each of the following outcomes is:

    certain to happen
    certain not to happen
    likely to happen (probability is > 50%)
    likely not to happen (probability is < 50%)

    rolling a 2?
    rolling a number less than 3?
    rolling a 7?
    rolling a number less than 10?

10. Following the same situation as in question 9, find the probability and determine the likeliness of the following outcomes:
    rolling a number greater than 1?
    rolling a factor of 6?
    rolling a number that is an integer?
    rolling a negative number?

11. Draw a Venn Diagram of two non-mutually exclusive events?

12. Today, the school's cafeteria is offering a choice of pizza or spaghetti. You can get milk or juice to drink. For

dessert you can get pudding or an apple. You must take one of each choice. Use a tree diagram or chart find the sample space of possibilities.

13. If the probability of an event is .125, what is the probability of the complement of this event?

14. A clothing store sells shirts in three sizes: small, medium, and large. The shirts come with buttons or with snaps. The colors available are blue or beige. Use the counting principle to show all possibilities.

15. Draw a Venn diagram of two mutually exclusive events.

Hard

16. One card is selected out of a standard deck of 52 cards. Find the probability and determine the likeliness of the following outcomes:
    Drawing a queen or king of spades?
    Drawing a black suited card or an ace?
    Drawing a number card that is a multiple of 2 or a number card that is a multiple of 4?

17. Of the events in question 16, list in order of most likely to occur.

18. A bag contains an unknown number of marbles. You know that P(red) = $\frac{1}{4}$ and P(green) = $\frac{1}{4}$.
    Are all the marbles in the bag red or green?
    What is P(not red or green)?

How many marbles of each color might be in the bag?
Are there other correct answers? Explain.

19. It is time to vote for student council. There are 7 nominees that need to fill 3 positions.
How many people are eligible for the first position?
After the first position is filled, how many people are eligible for the second position?
After the second position is filled, how many people are eligible for the third position?
How many possible outcomes are there?

20. Airport codes are made by any 3-letter arrangement in any order. How many possible outcomes are there?

21. A pair of dice is rolled. What is the probability that the sum of the numbers is either an even number or a multiple of 3?

22. If the independent probabilities that three teams, A, B, and C will win are 0.5, 0.4, and 0.7, respectively. Find the probability that:
all teams will win?
no teams will win?
only one team will win?
at least one will win?

Day 5 Instruction Plan

Grade Level: 9 **Math Course:** Integrated Algebra

Concept: Using a finite sample space to solve conditional probabilities.

Objective:

1. Students will be able to define conditional probability.
2. Students will be able to solve conditional probabilities when given a finite sample space.
3. Students will be prepared to take a unit test on probability.

Assessments:

- Students will participate in a review activity that reviews all of the topics covered in this unit (objective 1, 2 and 3).

Standards Addressed:

NYS Learning Standards for Mathematics:

A.S.18 – Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

Materials Needed:

- conditional probability notes
- "Probability Review"
- individual white boards
- "Probability Review" Answer Key

Teaching Process:

1. *Anticipatory Set*: Check each student to make sure that they have completed the previous night's homework.

2. Call out the answer for each question in the "Practice Problems" packet. Go over any questions students have.

3. Have students copy down notes on the board on conditional probability.

4. After the notes, the students will complete a review activity that aims at getting them prepared for the next day's unit test. The students will compete against a partner using individual white boards to answer various questions from a review sheet. More specific directions will be provided on the review sheet.

5. *Closure*: The teacher will ask students if there are any questions about the test or the topics it will cover.

Homework:

The students will be encouraged to complete the remainder of the review sheet as a studying tool. However, the review sheet will not be checked at the beginning of the next day's class. Each student will be given an answer key to the review sheet.

Lesson Five Notes

conditional probability – the probability of an event is conditional if the event relies on the occurrence of another event.

- In math, we write this as $P(A|B)$. We say, "the probability of event A given event B."
- If A and B are independent events, $P(A|B) = P(A)$
- If A and B are dependent events, $P(A \text{ and } B) = P(A) \cdot P(B|A)$.....so $P(B|A) = P(A \text{ and } B)/P(A)$
- Figure out if events are somehow related

Class Examples:

1. Two dice are rolled. What is the probability of rolling two fours? What is the probability of rolling two fours knowing that the first toss is a four?

2. A bag contains 9 red Skittles, 13 orange Skittles, and 11 purple Skittles. What is the probability of drawing an orange skittle if one has already been drawn?

3. There are 900 kids in a high school. Of these 900 kids, 200 take Chemistry, 200 are juniors, and 110 are juniors who take Chemistry. What is the probability that a randomly chosen student, who is a junior, is taking Chemistry?

Probability Name:

Review Sheet Date:

Directions: This review activity will help you get prepared for the unit test. After you select a partner, each of you needs to get a white board. You and your partner will select what order you do the questions in. You will choose a question and each of you will attempt to answer the question individually. After each person has arrived at an answer, you will check with the answer key. If both people have a correct answer, move on to another question. If one person has the answer right, but the other person does not, the person who got the correct answer should explain how the problem is done. If both have gotten the problem wrong, you may raise your hand to get help from the teacher. Remember to hit as many different topics as you can and keep track of which partner gets more questions correct. The winner in each twosome will receive a bonus point on the test.

Vocabulary

Define the following terms using words.

1. sample space
2. counting principle
3. complement
4. independent events
5. dependent events

Define the following terms using mathematical symbols

6. mutually exclusive events
7. non-mutually exclusive events
8. conditional probability

Lesson 2 Questions

9. Name the sample space of rolling a dice.
10. A local store sells 4 different kinds of napkins, 2 different kinds of plastic silverware, and 3 different kinds of paper plates. Use a tree diagram to show how many choices Jeff has if he chooses one of each product? Support this answer using the counting principle.
11. A restaurant has 3 different soups, 5 different entrées, and 6 different drinks to choose from. Use a tree diagram to show how many elements are in the sample space if one soup, one entrée, and one drink must be chosen? Support this answer using the counting principle.
12. If the probability of an event, E is .37, what is the P(not E).
13. There are 18 different books on a shelf. What is the probability of choosing one book? What is the complement of this?

Lesson 3 Questions

14. If two cards are drawn with replacement, what is the probability that the first card will be a heart and the second card will have a suit that is black?
15. In question 14, replace the words "with replacement" with the words "without replacement" and solve the problem.
16. If a coin is flipped 7 times, what is the probability that it will land on heads every time? Is this an independent or dependent situation?

17. There are 7 blue marbles, 5 yellow marbles, and 8 green marbles in a bag. If a marble is drawn and not replaced, what is the probability of drawing a blue marble? If 5 consecutive marbles are drawn and none are replaced, what is the probability that all are blue? Is this an independent or dependent situation?
18. Two dice are rolled and their number is summed. What is the probability that the sum will be greater than 10 or a multiple of 5?
19. A card is dealt. What is the probability that the card is red suited or is a face card?

Lesson 4 Questions

20. One card is selected out of a standard deck of 52 cards. Calculate the probabilities and determine whether each of the following outcomes is:
    certain to happen
    certain not to happen
    likely to happen (probability is > 50%)
    likely not to happen (probability is < 50%)

    Drawing a queen?
    Drawing a black suited card?
    Drawing a black faced card or a red faced card?

Lesson 5 Questions

21. A history teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

22. At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?
23. In New England, 84% of the houses have a garage and 65% of the houses have a garage and a back yard. What is the probability that a house has a backyard given that it has a garage?

Probability Name:

Unit Test Date:

Fill in the blanks in the following sentences.

1. When talking about conditional probability, in

   mathematics we write P(A|B) =

2. Mutually exclusive events are outcomes of events that ______________ ____.

3. If events A and B are **not mutually exclusive** then the probability that an outcome will be in one event or the other event is
   P(A or B) =

4. The counting principle states that if there are m ways of making one choice and n ways of making another choice, then there are _______ ways of making the first choice followed by the second choice.

5. Independent events are events whose outcomes _______ have an affect on each other

Solve.

6. What is the probability of getting an 8 when rolling a die?

   ______

   What is P(not an 8)?

   ______

7. There are 5 tootsie rolls, 3 starbursts, and 4 dum-dum suckers in a bowl. If three pieces of candy are drawn with replacement, what is the probability that you draw a tootsie roll, then a starburst, and then a dum-dum?

8. A pet store has 3 kinds of goldfish, 2 kinds of algae eaters, and 4 kinds of tropical fish. In order to create an aquarium, you want to buy one of each kind of fish. Using a tree diagram, show how many different choices there are?

   ______

Support this answer with the counting principle.

9. Two dice are rolled and their sum is found. What is the probability of rolling a multiple of 3 or a multiple of 4?

Is this a mutually exclusive or non-mutually exclusive situation?

____

10. There are 5 tootsie rolls, 3 starbursts, and 4 dum-dum suckers in a bowl. If three pieces of candy are drawn without replacement, what is the probability that you draw a tootsie roll, then a starburst, and then a dum-dum?

11. A fair die is rolled and the outcome noted. Calculate the probabilities and determine whether each of the following outcomes is:
certain to happen
certain not to happen
likely to happen (probability is > 50%)
likely not to happen (probability is < 50%)

rolling a 4?

rolling a 2 or 6?

rolling a number divisible by 2?

rolling a number less than 10?

12. There is a survey conducted in the city of Rochester that asks: What is your favorite season. The following lays out the specifics of survey: 500 people, 300 females, 200 males, 25 like winter and are males, 50 like fall and are female, and 75 like summer and are males. What is the probability that a randomly chosen person likes:
summer, given that he is a male?

winter, given that he is a male?

fall, given that she is a female?

IV. Discussion, Summary, Reflection

<u>**PROBABILITY QUESTIONNAIRE**</u>

Please answer the following three questions before reading through the included unit plan.

1. On a scale from 1 to 10, 10 being the most difficult, how difficult do you think probability is for students to learn? Please justify rating.

2. On a scale from 1 to 10, 10 being the most difficult, how difficult do you think probability is for teachers to teach? Please justify rating.

3. What are common misconceptions students have when learning probability?

Please answer the following three questions after reading through the included unit plan.

1. On a scale from 1 to 10, 10 being the best, how would you rate the included unit plan?

2. Please describe some positive features of the included unit plan.

3. Please provide some feedback as to how to improve the included unit plan.

Figure 1

Of the eight questionnaires (figure 1 is an example of the questionnaire) that were sent out to experienced teachers, six were filled out and returned to me. The first three questions of the questionnaire were aimed at finding out how difficult probability is for teachers to teach, how difficult probability is for students to learn, and some common misconceptions students have about probability, respectively. These three questions were directed to be answered before the teachers read through the unit plan. The experienced teachers gave a range of 5 – 8, and a mean score of 6.8 as to how difficult this subject is for students to learn (10 being most difficult). On the second question, the range of scores was from 4 – 8 and the mean was 5.8 (10 being most difficult). Translating these scores into words, I would say that in regards to these teachers' experiences, most believe that probability is a subject that is at an above average difficulty level for both students to learn and for teachers to teach. These scores are definitely around the levels that I would have thought, as I too believe that probability is challenging for students and teachers alike.

Looking at the answers given by the teachers in response to the common misconceptions of students question was very interesting. One answer that stood out to me, stated that probability was unlike other subjects because it is not nearly as straight forward as other topics in that there is not one set of rules that can be used in each probabilistic situation. This is definitely supported by the research on judgment heuristics because students are applying judgment heuristics even if they do not pertain to the particular situation. Students are not thinking critically enough to understand that certain questions do not meet the specifications needed to apply a distinct judgment heuristics. They are relying too much on intuition and not enough on an evaluative thought process.

Another response to the misconception question that related to the research portion of this project, was that students come into the classroom with negative feelings towards probability prior to being taught the unit. This relates to the literature discussed because these negative feelings are intuitive to certain individuals when the word probability is heard. It may be that the student had only one bad experience with this topic, but this experience and feeling that comes with it, envelopes all future probability encounters. This means that teachers must do their best at uncovering the reasons as to why a student is so scared of probability. If it is a specific problem that scares the student, the teacher must work together with the student to make sure that he understands it. If it is an ambiguous subtopic, go in depth into the subtopic to expose the difficulties she may be having. If it is a lack of confidence, provide the student with extra practice to increase his familiarity with different problems and help develop a more positive attitude.

The remaining misconceptions that the teachers gave included insufficient background knowledge and deficiencies in reading comprehension. Both of these are common in all school subjects and mathematics is no different. Teachers need to continually stress the importance of recalling prior knowledge and remembering the current knowledge that is being taught. It is also important for teachers to encourage students to become better readers as this will aid in achieving a higher-functioning life.

The next three questions of the survey were completed after the teacher read through the included unit plan. The questions instructed the teachers to rate the unit plan (scale: 1 – 10, 10 being the best), select some of its positive features, and provide suggestions for possible improvement, respectively. The scores of the rating of the unit plan ranged from 7 – 9 with the mean score being 8.5. To me this means that the overall unit plan, barring minor changes, is useable in a real classroom environment. Looking at the positive feedback given to me, I am most proud of the fact that the teachers recognized the differentiated instruction, the real-world examples, and the group work activities throughout the unit plan. All three of these features are important for a teacher to utilize, but at the same time can be difficult to implement.

Differentiated instruction was specifically used in two of the five lessons. I say specifically because a teacher differentiates his instruction on a daily basis without ever noting it in a formal written lesson plan. For example, when a teacher asks students to work independently on problems during class, he will tend to gravitate towards those kids that he knows struggle with the particular topic and provide little to no help to those that excel. The specific differentiated activities aimed at students utilizing their best learning style and at allowing, and in some cases challenging, students to attempt problems at varying difficulty levels. I chose to use this method of instruction because it is imperative that each student feel like learning probability can be geared specifically for them. While specific activities cannot be used in every single lesson, it is a good practice to attempt to incorporate different learning styles and different levels of difficulty wherever possible.

The real-world examples that were included in the unit plan, definitely reflect Gelman and Glickman's (2000) research on creating a more participatory environment. While their writings were endorsing class-wide demonstrations, their underlying theme is rooted in trying to spark the interest of the students. This is what I have attempted to do by having problems that deal with cards, dice, flipping coins, sports, geography, food, and other topics of interest to teenagers. These topics are especially present in the classwork activity that is differentiated by ability level. I made sure to try and come up with realistic situations because this day is all about getting enough practice so that each student becomes more comfortable with the types of problems that will be on the unit test. If the problems are dull and reflect nothing that is current in their lives, the students will not be inclined to improve. While they might complete the necessary number of problems, they almost certainly will not retain what they have learned.

Being able to work cooperatively with other people is a skill that must be acquired in order to be successful at any type of job. With this in mind, I made sure to include various group work activities throughout the unit. In some lessons, the activity was as simple as having students compare each others answers, so as to decrease the amount of problems in an assignment that needed to be addressed as a whole class. While this is a minor type of interaction it still helps students develop teamwork skills. In other lessons, group work most certainly was the central theme of the activity. For example, in lesson three, the students were instructed to break-up into groups of four to five and complete a stations activity. The goal here was to get students to collaborate in order to become familiar with problems related to dependence, independence, and mutually exclusive events.

While the previously mentioned feedback items are what I am most proud of, there were two other characteristics throughout the unit plan that the teachers noted as being essential to successful curriculum writing. The first characteristic was that my objectives of each lesson were clear and consistent with New York State Standards. In today's world of standardized education, a teacher must align her teachings with the standards that each state requires of a particular course. Whether a teacher agrees with the standards or not, it remains her job to make sure that each standard is covered so that her students will be prepared to succeed on the mandatory summative assessment supplied by the state's department of education. I made sure that the objectives and standards were noted in each lesson so that my reasons for teaching a subtopic of probability were understood. The second characteristic was that there was other subject integration in the unit. While it was minor, there were problems related to social studies ("Sample Space/Complement Notes" – question 6, "Practice Problems" – question 4). It may not seem like a big deal, but for those students who have a lack of interest in math but enjoy a subject like social studies, it could make all of the difference in the world. Incorporating more content areas into math cannot hurt and can only help in developing a classroom full of students who want to be involved in the material being taught.

Perhaps the most important question on the questionnaire was the last question that asked the teachers to provide some feedback as to how to improve the unit plan. There were a number of different suggestions the teachers gave and while none of them were citing major improvements, all were helpful. This list included: providing more opportunities for struggling students, using a rubric for certain activities, making sure that students have correct background knowledge, and using various methods so that students better understand vocabulary. Providing more opportunities for struggling students is definitely a skill that I will develop as I begin teaching. It takes years of experience to understand where the students will have their misconceptions, so it is difficult right now as to where extra help and practice should be built in. By creating lessons that included different levels of ability and different leaning styles, my goal was to make sure that struggling students could gain confidence by completing problems at their ability level, as well as gain an interest in the topic of probability because of the way they are learning it. However, I did not include activities like this in each lesson and could have noted where and when I would have to provide more support for struggling students. Again, this is a skill that I aim to improve at and it was an excellent reminder that all students are not going to "get it" right away.

A rubric is a great assessment tool for providing some parameters for students to follow, when a project has directions that are less structured. I needed to include a rubric for the poster in the differentiated activity in lesson two. The categories of the rubric in no particular order would have been creativity, neatness, and correctness. With this rubric, I also should have included an example poster that shows what is needed for a poster to receive a perfect score. This would have helped the students gain some insight into my expectations for this activity. A rubric also would have helped me to be more consistent in grading the posters. Without a list of set constraints, grading can be more subjective than it should be.

Background knowledge, or prior knowledge as it is sometimes called, is the basis of learning new things. Students must have certain acquired knowledge in probability in order to understand what is being taught in this unit plan. It is exactly for this reason, why I included a pre-assessment at the beginning of the unit plan. A teacher needs to know what his students are familiar with and what needs to be refreshed. What I did a poor job at, was assuming that students had the necessary non-mathematical background knowledge for answering certain questions. For example, there are many questions throughout the unit about playing cards and dice. I did not specifically discuss the breakdown of colors, suits, or kinds of cards in a deck, nor did I discuss the digits on the face of a die. I should not assume that all students have interacted with playing cards or with dice. Not assuming background knowledge is a teaching trait that I will look to improve upon.

Making sure students understand vocabulary in mathematics, is something that is often overlooked. Students must understand the central terms to the unit before they can be successful at solving problems. While in most other subjects this means simply memorizing a definition, in math there is sometimes a symbolic notation that goes with the definition. I was given two helpful hints for ways to increase the comprehension of vocabulary. The first hint deals with stressing vocabulary using non-linguistic representations. As an example, the teacher could ask students to represent a sample space with a diagram, or create a drawing to show what replacement means. Both of these representations help students associate a visual with a definition, which can be very helpful to a person who is trying to retain information. The other hint is geared toward getting students to become comfortable with symbolic notation. On one of the questionnaires, a teacher noted that sometimes students become overwhelmed by the notation and completely shutdown because it looks too confusing to understand. To prevent this, the teacher insisted that I explain the notation using words. For example, in the "Independent, Dependent, and Mutually Exclusive Notes" the notation for a non-mutually exclusive event is written as P(A or B) = P(A) + P(B) – P(A and B). Underneath this notation I should have written, "the probability of event A or event B occurring, is equal to the probability of event A occurring, plus the probability of event B occurring, minus the probability of event A and event B occurring." This would have made the notation less obscure and would have created a higher sense of comfort for students.

The response I received from the teachers was overwhelmingly positive and helpful in determining the strengths and weaknesses of my unit plan on probability. I am glad that I decided to incorporate their feedback into my thesis project and feel that it compliments the research portion of this paper very well. Upon reading the unit plan and then reading this section, you may have noticed that I did not make any of the corrections to the unit plan that I talked about. I chose to do it this way because it is more beneficial for the reader to be able to see what I am talking about when I analyze the teacher feedback and point-out, in the unit plan, where the improvements need to be made. I most certainly will make the proper adjustments before implementing this unit plan in the classroom, but I wanted the reader to read exactly what the teachers read, when this unit plan was sent to them.

While I currently do not plan to research this topic in depth anytime soon, I undoubtedly will remain interested in teaching the subject of probability. As mentioned, I truly believe it is one of the most challenging mathematics topics to learn and teach and I will always be on the lookout for ways to improve my teaching of it.

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