# Helpful Homework in Geometry: A Redesigned Circles Unit 

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Bethany Joy Hooker

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## Dedication

This thesis is dedicated to my parents, who urged me to excel, inspired me to influence others, and always showered me with love and support.

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#### Abstract

Homework has been part of the educational system for many decades. During this time, public opinion has varied greatly on its usefulness in the classroom. Much of the more recent research has focused on the idea that homework can be valuable to students when the assignments are meaningful, as opposed to homework that is assigned with little or no purpose. This research was analyzed to find patterns in the various definitions of meaningful homework. It was found that meaningful homework generally contains the following qualities: brevity, choice, defined purpose, real-world connections, hands-on components, rigor through synthesis, the integration of technology or web-based activities, opportunities for family involvement, and the substantial incorporation of previously taught topics. Based on these findings, a Geometry unit on circles was redesigned to combine these elements into practical lessons, as an example to educators of how to begin making homework more constructive in the mathematics classroom.


## Chapter One: Introduction

## Rationale

A great deal of research has been done on homework, and the findings have both supported and opposed homework practices, depending on public opinion at the time (Bembenutty, 2011a; Bempechat, Li, Neier, Gillis, \& Holloway, 2011; Cooper, Robinson, \& Patall, 2006; Dettmers, Trautwein, \& Ludtke, 2009; Dettmers, Trautwein, Ludtke, Kunter, \& Baumert, 2010; Gill \& Schlossman, 2003; Kitsantas, Cheema, \& Ware, 2011; Voorhees, 2011). Recent discussions have centered on the usefulness and potential benefits of homework. It is important to critically consider if there is a difference between "homework" and "meaningful homework" in the secondary mathematics classroom. Investigating the history and recent research concerning homework may help teachers make better decisions about how to assign homework in a meaningful way.

Each specialty area, i.e., English, mathematics, and science, may use and view homework differently, yet educators can learn from the practices of others, even if they teach different subjects. For instance, mathematics may be the only content area in which homework is assigned nightly, yet some research has shown that there are potential benefits to assigning less frequent, more concentrated homework assignments, as usually seen in social studies or English Language Arts (Sallee \& Rigler, 2008). Homework is a mainstay across the k-12 education system in the United States, and most teachers start assigning homework as early as kindergarten or first grade. Thus it is necessary for teachers to develop ways to implement and assess meaningful homework from the first days of school.

While great strides have been made in the areas of homework perceptions and web-based assignments, more research needs to be done on mathematics homework. There is a significant gap in this area because homework assignments vary in design and purpose across subjects,
especially at the high school level. Students report being assigned homework most frequently in mathematics (Xu, 2005; Xu, 2010). Most mathematics teachers believe students need practice to process newly learned skills. Mathematics builds upon itself countless times throughout each level of the curriculum, so it is important that students master skills early before they are required to apply those skills to new concepts. Because of this, teachers need to reconsider and possibly redesign many of the homework assignments that are currently being used to align with the research of what constitutes meaningful homework.

## Description of the Project

There are many opinions on what meaningful homework is and is not. However, most authors would agree with Vatterott (2010) that there are five essential "hallmarks of good homework." In order to make homework meaningful, it must be purposeful, efficient, competent, aesthetically appealing, and it must allow the students to take ownership in some way. The goal of this curriculum project is to provide a complete unit plan on circles for Geometry courses that seek to assign engaging and meaningful homework. The entire unit has been re-worked to correspond with current research regarding effective homework. Students will be asked to solve problems for which there is no readily available algorithm, apply their knowledge to real-world situations, and synthesize material so that they can develop their own questions. Students will also have a choice of assignments in some cases, and they will be able to complete some homework assignments online. The purpose for each homework assignment will be clearly stated, and each task will be relatively short. Thus, the homework in the adjoining unit is likely to be different from any other unit the students have been taught.

Research presents various articles about students' perceptions, teachers' perceptions, parents' perceptions, administrators' perceptions, and even grandparents' perceptions of
homework (Bempechat, Li, Neier, Gillis, \& Holloway, 2011; Biscoglio \& Langer, 2011; Brock, Lapp, Flood, Fisher, \& Han, 2007; Cushman, 2010; Grootenboer, 2009; Hong, Wan, \& Peng, 2011; Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011; Sallee \& Rigler, 2008; Xu, 2005; Xu, 2010). Cushman (2010) stated that many students seemed to think homework was worthless. Thus a great deal of work has been done to investigate the purposes teachers have for assigning homework. For example, one survey administered by Sallee and Rigler (2008) asked questions like, "Are [teachers] being sensitive and supportive to outside interests in [their] current practice of assigning and using homework? How do the teacher's goals match (or contrast) what students gain from it and why?" to gain insight on teachers' homework practices (2008, p. 47). Answers were often ambiguous and varied. There was not a unified purpose for homework, even within a single high school. Given their findings, Sallee and Rigler (2008) suggested that educators rethink their current homework practices and only assign homework if and when it is appropriate and beneficial to student learning. This project will hopefully spark a new interest in making homework meaningful once again—not just something that teachers assign due to some long-held tradition.

## Chapter Two: Survey of Literature

## History

Homework assignments have been a central part of education since the early 1900's. In the first years of the $20^{\text {th }}$ century, homework was viewed as a way to discipline children's minds. By the 1940 's, homework had fallen from grace in the public eye. It was viewed as an intrusion into students' home lives, and greater emphasis was placed on problem-solving strategies instead of the drill and practice that once populated homework assignments. When America's educational deficiencies were highlighted by the Russian launch of the Sputnik satellite in 1957, Americans turned to homework once again. The people wanted more homework in mathematics and the sciences to prepare their students for future technological advances (Cooper, Robinson, \& Patall, 2006).

With the cultural changes in the 1960's, homework was again viewed as an imposition and an unneeded pressure. Questions were even raised as to its consequences for mental health (Gill \& Schlossman, 2003)! In the mid-1980's, with the release of "A Nation at Risk" and declining achievement test scores, change happened once again as Americans were in favor of homework as a protection against mediocrity (Cooper, Robinson, \& Patall, 2006; Gill \& Schlossman, 2003). Around the turn of the century, parents worried about stresses on their children and became more involved in the education system. Once again, homework was cast in a negative light (Cooper at al., 2006).

With all of the back-and-forth about homework, researchers and educators have explored other alternatives. Home education as it is known today, more commonly called homeschooling, first came out of a 1960's movement known as "unschooling" by a man named John Holt. While homework has gone in and out of public favor, homeschooling seems to be on the rise.

Homeschooling is seen as an alternative to homework because children can go at their own pace, allowing them to finish the lessons for the day in a relatively short amount of time. Most families involved in homeschooling reported that their students were done with work by mid-day or shortly thereafter. In the morning, they had already learned and practiced new concepts. Since one of the most commonly accepted purposes of homework—practicing newly learned skillshad already taken place, there was no need for homework in the traditional sense (Stevens, 2003).

Some opponents of homework proposed a large-scale overhaul in the way that public education was run, though extending the school day or even the entire school year. Since the 1800's, the school calendar has been changing to fit the needs of the American people. In rural communities, schools were often only run for half a year, so as to allow students to help their families with the annual harvest. However, as technology affected the amount and reasons behind homework, it also had an impact on how citizens viewed the school calendar. In the early 1900's as technological advances changed how wars were supplied and fought, year-round schools emerged as a means of accelerating students so that they could graduate early to join the work force or military. By the 1960 's, the modern school year of 180 days was basically the norm. Today, proponents of lengthening the school year have claimed that one benefit of doing so would be to decrease the amount of or need for homework. Without students taking a threemonth break every year, they predicted that retention rates would increase. Similar claims were made about extending the school day instead of, or in addition to, extending the school year as well. With more time in-school, less work would need to be done outside school. That could mean less homework assigned (Patall, Cooper, \& Allen, 2010).

Educators are now starting to think outside the homework box and talk about flipping their classrooms. The flipped classroom is inspired by the concept of switching homework and instruction; students hear or watch a lecture through various media at home, then do their homework in class the next day ("Flipped Classrooms," 2012). Not much peer-reviewed research has been done on the topic of the flipped classroom as to its effectiveness, but one school in Minnesota has seen results. The Byron School District has seen an increase in the number of students who are demonstrating proficiency on unit exams since they flipped their mathematics classrooms starting in 2010. Calculus proficiencies increased by $9.8 \%$ on average, precalculus proficiencies averaged an increase of $6.1 \%$, and overall course failure rates have decreased from $13 \%$ to $6 \%$ under flipped classrooms (Fulton, 2012). As with any new idea, there are critics. Some state that the flipped classroom is not accessible to all, especially to those students who come from low socioeconomic backgrounds (Neilson, 2012). Others teachers stated difficulties with students who did not complete traditional homework assignments before the flip took place; they certainly were not listening to or watching the lectures now either. Due to this fact, teachers had to struggle to split time between teaching those that did not do their "homework" and those that had questions on the lecture from the previous night (Ash, 2012). Still, teachers are working around these obstacles as best they can. One teacher records all of her lessons onto CD or DVD formats so that students without access to the internet at home can still experience the flipped classroom. Others are encouraging students to use resources available to them like school and public libraries, which are open after school lets out ("Flipped Classrooms," 2012). It is possible that flipped classrooms could be the new face of education, and homework, in the years to come.

## Research

Throughout the last century, as homework has fallen in and out of favor, two major views of homework have developed. Proponents of homework have cited achievement gains, and argued that students retain more knowledge when they are asked to practice skills outside of the classroom (Bembenutty, 2011b). They have also stated that being required to do homework builds good time management and promotes good study habits (2011b). Those who favor homework have also pointed to the psychological benefits of homework-that it promotes the idea that learning can take place anywhere, not just at school (2011b). Students are also taught that they can learn on their own; students develop critical thinking skills and perseverance through completing homework (2011b). Another argument for homework was that parents can be involved in the education process. They get a glimpse of what their child is doing in school, and in some cases, they are even able to offer assistance or guidance if needed (2011b).

While homework seems to have many positive effects on students, there were those that were quick to point out its many flaws as well. Opponents of homework argued that students can become overexposed to academic material. Students are required to attend school all day, and then they are asked to go home and spend another few hours working on homework. Opponents claimed that this can make school uninteresting and boring, possibly even squelching independent discovery in learning (Bembenutty, 2011b). When students are presented with homework assignments that they cannot adequately complete, they can easily become frustrated and give up. Furthermore, in the middle- and high school grades, parents are often uncomfortable helping students because the material is too advanced. Students then struggle alone through assignments that are sometimes too difficult or lengthy to satisfactorily complete. Finally, homework opponents stated that other after-school activities can be just as significant to a child's
growth as homework. For instance, sports, scouts, and music lessons can all teach important life skills, but not much time is left for these activities after homework each night (2011b).

Recent trends in technological advances have shaped homework research. Several studies have been conducted in regard to the effectiveness of web-based homework (Hauk \& Segalla, 2005). In one such study, researchers found that while there were no major intellectual gains when students used web-based homework versus paper-pencil homework, it was at least not hindering student performance (2005). In addition to this fact, most students reported that they were more motivated to complete online homework assignments than paper-pencil formats (2005). Other studies have found that students prefer web-based homework, and they spend more time completing assignments (Mendicino, Razzaq, \& Heffernan, 2009; Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011).

While great strides have been made in the areas of homework perceptions and web-based assignments, some people believe more research needs to be done specifically on web-based mathematics homework ( $\mathrm{Xu}, 2005 ; \mathrm{Xu}, 2010$ ). Grouping all homework together presents a problem for researchers because homework assignments vary in design and purpose across subjects, especially at the high school level. For instance, students report being assigned homework most frequently in mathematics ( $\mathrm{Xu}, 2005$; $\mathrm{Xu}, 2010$ ). Mathematics builds upon itself countless times throughout the curriculum, so teachers want students to practice and process newly learned skills. It is important that students master skills early before they are required to apply those skills to new concepts. However, it is the author's opinion that there is not sufficient research that supports the idea that homework actually helps students acquire these new skills. If more studies were conducted to see whether or not homework really provided the practice needed to master new mathematics skills, it may greatly influence the decisions teachers make
about how and what kinds of homework to assign.
Cooper defined "homework" as tasks that are assigned to students that are meant to be carried out during non-instructional times (Bembenutty, 2011b). Most other researchers have adopted this definition as well (Bembenutty, 2011a; Bempechat, Li, Neier, Gillis, \& Holloway, 2011; Dettmers, Trautwein, \& Ludtke, 2009; Dettmers, Trautwein, Ludtke, Kunter, \& Baumert, 2010; Kitsantas, Cheema, \& Ware, 2011; Voorhees, 2011). Most students have defined homework as assignments that are disliked, but easy grades (Cushman, 2010). Other definitions, like that of author Grootenboer (2009), carry serious connotations; he stated that homework is just "schoolwork done at home" (p. 12). Beneficial homework is more complex than these descriptions, though, for definition alone is not enough. One must also explore its purpose to have a complete understanding of meaningful homework.

Homework has many different benefits. In interviews, teachers referred to homework as a vehicle through which students could practice new skills or concepts learned in class (Bembenutty, 2011b; Cooper, Robinson, \& Patall, 2006; Kitsantas et al., 2011). Many teachers assigned homework to prepare students for future lessons or supplement class activities. Especially at the elementary levels, teachers aimed to develop perseverance, work ethic, and good time management skills through homework (Bembenutty, 2011b; Cooper, Robinson, \& Patall, 2006; Kitsantas et al., 2011). In interviews conducted by Brock, Lapp, Flood, Fisher, and Han (2007), a few teachers even admitted to assigning homework solely because it was a district requirement, or because they thought parents expected it.

Homework is often compared to achievement. This term is used synonymously with "performance" in the majority of research (Dettmers, Trautwein, \& Ludtke, 2009). Usually, achievement and performance refer to scores on standardized tests. The higher a student scores
on one of these tests, the higher the achievement is perceived to be (2009). Despite this general definition, achievement can also refer to the degree of mastery on classroom quizzes or tests (Bembenutty, 2011b; Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011). In either case, achievement is generally used in reference to a quantifiable score or measurement.

## Variables and Phenomena

Some research in the field of homework has investigated the relationship between homework and achievement (Dettmers, Trautwein, \& Ludtke, 2009). One cross-cultural study conducted by Dettmers et al. (2009) examined how the amount of time spent on homework impacted student achievement on standardized tests. They found a slightly positive correlation between the two variables. However, this effect dissipated as another variable-socioeconomic status- was taken into account. This study was similar to another conducted by Dettmers, Trautwein, Ludtke, Kunter, and Baumert in 2010. Instead of comparing achievement to time spent on homework, they compared it to homework quality. The researchers found that when students believed that their homework was well-designed, they were motivated to complete it, thus predicting higher achievement. In both cases, the researchers discovered the same phenomenon that homework has some effect on increasing performance in school (2010).

Besides time spent on homework and homework quality, researchers have also been interested in how homework is assigned. Recently there has been much debate over the new method of assigning homework online. In a 2009 study, fifth grade students were given the choice between completing web-based homework or traditional paper-pencil assignments. Researchers found that the students learned significantly more when completing online assignments, pointing to the idea that web-based homework is more profitable than traditional homework (Mendicino, Razzaq, \& Heffernan). Despite this, there may have been a third variable
behind the correlation-immediate feedback. One of teaching's best practices is to provide timely feedback, and the students were getting that from the online program. Traditional homework takes time to correct, so some students typically have to wait for feedback. This suggests that even traditional homework can have the same impact, when timely feedback is given. Cushman (2010) echoed this fact when she stated that students want teachers to check their homework. Such feedback encourages participation and will likely influence achievement as well.

Timely feedback was not a variable in another study about web-based homework, because all students were completing the same online assignments. This study was not concerned with gains in academic achievement, but rather, gains in areas of students' study habits. Most students reported that their study habits had improved because online homework gave them more motivation to complete assignments. The students said they were spending more time on assignments, and that they were using a wider variety of problem-solving approaches to complete the assignments. They also reported that they would recommend continuing online homework for future classes (Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011). This seems to be a strange and exciting phenomenon because most of the research studies present student perceptions of homework as being negative. Students usually describe homework as pointless, tedious, and difficult to complete (Wilson \& Rhodes, 2010). The fact that online homework changes student point of view on the subject may be key information for educators. It suggests that online homework could be a way for teachers to entice students to complete more assignments, probably increasing academic achievement in the process.

## Common Methods

According to Hoepfl (1997), there are two main types of research, quantitative and qualitative. In most studies that investigated connections between homework and achievement, quantitative methods were employed to gather and analyze data. Quantitative techniques are well suited for this type of work. Common instruments were end-of-year state tests, the Program for International Student Assessment (PISA), or other standardized assessments. In most cases, time spent on homework and other variables requiring participants to give a number answer were gathered using simple logs or questionnaires, but analyzed using quantitative methods (Dettmers, Trautwein, \& Ludtke, 2009; Kitsantas, Cheema, \& Ware, 2011). For many studies in which the researchers wanted to get a clearer picture of student or teacher perceptions about homework purpose, questionnaires involving Likert-type scales were used almost exclusively. This type of data-gathering method was effective since the surveyed populations were rather large, and this method allowed many people to be polled quickly (Hauk \& Segalla, 2005; Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011; Xu, 2005; Xu, 2010).

Qualitative methods were used when the authors were concerned with perceptions, feelings, motivations, or attitudes toward homework. In smaller studies, personal or group interviews were conducted and recorded. The researchers then coded and analyzed the data they collected. They looked for patterns or recurring themes and grouped like responses together (Bempechat, Li, Neier, Gillis, \& Holloway, 2011; Brock, Lapp, Flood, Fisher, \& Han, 2007). Since it is clear from the research that students enjoy doing online homework more than traditional homework, authors have wanted to find out more about this connection. Strauss and Corbin (1990) stated that the qualitative process can be used to better understand any phenomenon which is relatively new or not fully understood (as cited in Hoepfl, 1997). Because
the field of online homework is only just developing, that may be why so many qualitative methods like questionnaires and interviews have been used to better understand it.

Does homework really work? Does it achieve the purposes that we want it to achieve?
Above all, what is meaningful homework? Because of such questions, Cooper (1989) conducted a meta-analysis of the research on homework and was able to find 50 studies that examined the relationship between time spent on homework and achievement. Of those, 43 studies claimed a positive correlation between the two variables at the high school level (Cooper, Robinson, \& Patall, 2006). Still, there has been little recent research to corroborate Cooper's (1989) findings. Now, researchers seem to be taking a different approach. Sallee and Rigler (2008) suggested shortening homework and allowing time for students to start assignments in class. As stated before, Vatterott (2010) claims that there are five "hallmarks of good homework": purpose, efficiency, ownership, competence, and aesthetic appeal. Finally, much of the current research advocates for online or web-based homework as a means to greater student engagement and completion (Hauk \& Segalla, 2005; Mendicino, Razzaq, \& Heffernan, 2009; Richards-Babb, Drelick, Henry, \& Robertson-Honecker, 2011; Sokolowski, Yalvac, \& Loving, 2011). All of these components form the new idea of effective, meaningful homework.

## Current Trends and Future Projections

The New York State Core Curriculum states five standards connected with circles. First, it is expected that students can prove that all circles are similar. Secondly, students must be able to identify and describe relationships among inscribed angles, radii, and chords. This may include the relationship between all of the angles formed in and around a circle: central, inscribed, and circumscribed. Next, students are asked to construct the inscribed and circumscribed circles of a triangle, as well as prove properties of angles for a quadrilateral
inscribed in a circle. The fourth standard requires students to be able to construct a tangent line from a point outside a given circle to that circle. Lastly, students need to find an arc length and sector area of circles, showing an even deeper understanding by deriving these formulas. Also included in this standard is the idea that the length of an arc intercepted by an angle is proportional to the radius, and the fact that the radian measure of the angle is the constant of proportionality (New York State P-12 Common Core Learning Standards for Mathematics, 2012).

In addition to the Common Core content standards, there are also general mathematics and geometry standards to keep in mind as well. These strands are: (1) making sense of problems and solving them; (2) reasoning abstractly and quantitatively; (3) constructing viable arguments and critiquing the reasoning of others; (4) model with mathematics; (5) using appropriate tools strategically; (6) attending to precision; (7) look for and make use of structure; and (8) looking for and expressing regularity in repeated reasoning (New York State P-12 Common Core Learning Standards for Mathematics, 2012). Another applicable geometry standard is that students should be able to use geometric shapes, measures, and properties to describe objects (New York State P-12 Common Core Learning Standards for Mathematics, 2012). Some of these standards, as well as the ones previously mentioned, will be addressed in the unit plan and reinforced in the assigned homework. For example, students can learn in class that all circles are similar, but until they try proving that the ratio of a circle's circumference to its area is always the mathematical constant pi, they may not fully internalize the concept. Another example of how meaningful homework may help achieve these learning goals is by using geometric modeling programs to simulate inscribed and circumscribed figures, which allows students to experiment with the angles and relationships that are formed. This supports what is stated
elsewhere in the state standards: "Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena" (New York State P-12 Common Core Learning Standards for Mathematics, 2012, p. 63).

Texas, where the author resides, is one state that has not adopted the Common Core Learning Standards. Instead, the Texas educational system is directed by the Texas Essential Knowledge and Skills, or TEKS. With the change in standardized tests from the Texas Assessment of Knowledge and Skills (TAKS) test to the new State of Texas Assessments of Academic Readiness (STAAR) tests also came a new set of more rigorous TEKS. The revised Geometry TEKS involve broader concept-based standards, rather than requiring students to know isolated rules and procedures (STAAR The Next Generation, 2011).

One of the standards that will be addressed in the unit plan is that the student is expected to make conjectures about angles and circles, and then determine the validity of those conjectures. Another standard is that students are expected to be able to use numeric and geometric patterns to make generalizations about geometric properties, including but not limited to angle relationships in polygons and circles. Yet another standard that will be apparent in the unit plan is the expectation that students are able to find areas of sectors and arc lengths using proportional reasoning. Lastly, students will be asked to formulate and test conjectures about the properties and attributes of circles and the lines that intersect them (Geometry Assessment Eligible Texas Essential Knowledge and Skills, 2012).

Texas also has numerous standards that would be applicable to the unit presented below. For example, students are to be able to: (1) develop an awareness of the structure of the mathematical system; (2) connect definitions, postulates, logical reasoning, and theorems; (3) use
constructions to explore geometry and make conjectures about geometric relationships; (4) construct and justify statements about geometric figures and their properties; (5) use inductive reasoning to form a conjecture; and (6) describe geometric relationships and solve problems using a variety of approaches, including concrete, pictorial, graphical, verbal, or symbolic (Geometry Assessment Eligible Texas Essential Knowledge and Skills, 2012). The accompanying unit plan focuses on all of these standards in hopes of providing students with a learning experience that not only includes meaningful homework but also prepares them for the STAAR they will face at the end of the school year.

## Chapter Three: Body

## Explanation of Choices

The redesigned circles unit plan includes many characteristics of meaningful homework, as defined by the previously mentioned research. Several themes were repeated in many of the articles that were studied. As stated in greater detail in Chapter Two, homework that is most beneficial to students often has the following qualities: brevity, choice, defined purpose, realworld connections, hands-on components, rigor through synthesis, the integration of technology or web-based activities, opportunities for family involvement, and the substantial incorporation of previously taught topics. Using all of these attributes, the author attempted to create a unit with homework that may be unlike most mathematics classroom, while still considering the practicality of such assignments. For this reason, none of the homework assignments contain all nine characteristics of meaningful homework. It would be unrealistic to expect teachers to be able to create such multi-faceted homework every night. Additionally, when too many traits are included at one time, the task may seem disjointed or cluttered, causing it to lose student appeal. Taking all of this into consideration, the author designed nine homework assignments, each highlighting only a few qualities of meaningful homework.

Another decision the author made was to design the unit with both New York and Texas in mind. Each state represents different educational philosophies-one governed by the almost nationally accepted Common Core Learning standards and the other by its retained autonomous state standards. Both sets of standards were included in the unit plan. The content of some lessons address expectations outlined by both New York and Texas. Other days, the material may align with the standards of one state more than the other. For this reason, if the line indicating the related standards is left blank for either state, it was done intentionally.

The circles unit was created with teachers in mind. Several choices were made in the design of the unit plan that caused its appearance to differ from the rest of this thesis. Several of these decisions were made to conserve paper. With schools on increasingly tighter budgets, many teachers experience pressure to save paper in any way they can. One way the author chose to consider this concern was to use Calibri as the dominant font throughout the unit plan. Calibri usually causes documents to be a bit shorter than using another font like Times New Roman, yet it is still a clear, easily readable font. A second choice the author made was to decrease the margins on all worksheets. Obviously, this creates more room for content on the page rather than just blank space. With these two space savers, many worksheets were able to fit on two pages, a single sheet of paper if printed or copied front-to-back. Other assignments were even able to be made into half-sheet activities, dividing the number of needed copies in two.

Table 1

## Circles Unit Calendar

| Day | Topic | Day | Topic |
| :--- | :--- | :--- | :--- |
| 1 | Circle Vocabulary | 6 | Chords in Circles |
| 2 | Similarity of Circles | 7 | Segments of a Circle (A) |
| 3 | Equations and Graphs of Circles | 8 | Segments of a Circle (B) |
| 4 | Angles of a Circle | 9 | Arc Length and Sector Area |
| 5 | Tangents in Circles | 10 | Unit Assessment |

Lesson Plan: Circles Day 1
Subject: Geometry
New York State Core Curriculum Standards: Preparation for G-C1, G-C2, G-C3, G-C4, G-C5

Texas Essential Knowledge and Skills: Preparation for G.2.B, G.5.B, G.8.B, G.9.C, G.11.C
Objective: In my study of circles, I can define key vocabulary so that I will be ready to use this vocabulary in later theorems and lessons.

Anticipatory Set/Warm-Up: students pair up to try to define words they may already know and make educated guesses about the others

Lesson Main Ideas: circle vocabulary

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| define vocabulary as we go <br> through PowerPoint | $\bullet$circle terminology practice <br> worksheet | matching game with <br> partner or groups of 3 |

Closure: students give the vocabulary term that matches a picture drawn on a set of index cards as they exit the classroom

Homework: Day 1 HW - measuring real life objects in preparation for similarity

## Circle Terminology

$\qquad$
Name
Teacher
Per

Sketch
Definition


1. Circle - $\qquad$
2. Radius - $\qquad$

3. Diameter- $\qquad$

4. Secant - $\qquad$

5. Tangent - $\qquad$
6. Point of Tangency - $\qquad$

7. Common Tangent - $\qquad$

8. Tangent Circles - $\qquad$

9. Concentric Circles - $\qquad$

10. Chord -

11. Inscribed Angle - $\qquad$
$\qquad$
12. Central Angle - $\qquad$

13. Arc Measure- $\qquad$

14. Major Arc - $\qquad$

15. Minor Arc - $\qquad$

16. Semicircle - $\qquad$
$\qquad$

17. Intercepted Arc - $\qquad$

18. Arc Length - $\qquad$

19. Outside Angle - $\qquad$

20. Inside Angle -

## Circle Terminology Teacher

Name $\qquad$ Per $\qquad$
For questions 1-7 refer to the circle to the mghectace

1. Name the circle: $\qquad$
2. Name all radii: $\qquad$
3. Name a diameter: $\qquad$
4. Name a chord: $\qquad$
5. Name a tangent: $\qquad$
6. Name a secant: $\qquad$
7. Name a point of tangency: $\qquad$


For questions 8-13 refer to circle to the right.
8. $\overline{X Y}$ is a $\qquad$ of circle 0.
9. $\overline{X O}$ is a $\qquad$ of circle 0 .
10. $\overline{W Z}$ appears to be $\qquad$ to circle 0.
11. $\triangle X Y Z$ is $\qquad$ in circle 0.
(Hint: $X, Y$, and $Z$ lie on circle $O$ )
12. $\widehat{X Y Z}$ is a $\qquad$ of circle 0 .


For questions 14-16, fill in the blank or circle the appropriate word to complete the sentence.
13. A secant of a circle is $a(n)$ $\qquad$ that intersects a circle at exactly $\qquad$ point(s).
14. Concentric circles have the same $\qquad$ .
15. Two tangents drawn to a circle from an exterior point are always parallel/perpendicular.
16. If A and B are points on a circle, then $\overline{A B}$ could be a tangent/ secant line.

Determine whether each statement is true (T) or false (F). If false, circle the word(s) that make it false. 17. A chord of a circle that passes through the center of the circle is called a diameter. $\qquad$
18. If two circles are concentric, then their diameters have equal measure. $\qquad$
19. Inscribed angles are created from two chords in one circle. $\qquad$
20. Tangent circles intersect at two points. $\qquad$
the set of all points that are equidistant from a central point

## CONCENTRIC CIRCLES (kun-sen-trik)


two or more circles with the same center, like ripples in a pond

## TANGENT CIRCLES (tan-jent)


internally tangent

externally tangent
two or more circles that intersect at exactly one point

## RADIUS (ray-dee-us)

$$
\begin{aligned}
& \text { the plural is } \\
& \text { RADII } \\
& \text { (ray-dee-eye) }
\end{aligned}
$$


the distance from the center of the circle to any point on the circle; half the length of the diameter

## DIAMETER (die-am-i-ter)


a segment passing through the center with both endpoints on the circle; double the length of the radius

## CHORD (cord)


a segment with both endpoints on the circle

## SECANT (see-kant)


a line, ray, or segment that extends beyond the circle in at least one direction

## TANGENT (tan-jent)


a line, ray, or segment that intersects the circle at exactly one point, called the point of tangency

## MAJOR ARC (may-jer ark)


$\overrightarrow{A C B}$ is a major arc
$\triangle \mathrm{BCA}$ is a major arc
www.mathcaptain.com
an arc measuring more than $180^{\circ}$; named using 3 letters

## MINOR ARC (my-ner ark)

$A B$ is a minor arc
$\widehat{B A}$ is a minor arc

www.mathcaptain.com
an arc measuring less than $180^{\circ}$; named using 2 letters

## SEMICIRCLE (sem-eye-sur-kuhl)

semicircle

diameter DE
an arc measuring exactly $180^{\circ}$, cutting the circle in half

## ARC LENGTH (ark lengkth)


the distance along an arc measured in linear units (ft, in, cm, yd, etc.)

## ARC MEASURE (ark meh-zher)


the size of an arc measured in degrees; equal to the central angle

## SECTOR AREA (sek-ter)


the area of a slice of the circle

## CENTRAL ANGLE (sen-truhl)


a central angle is EQUAL TO the measure of its intercepted arc
an angle with its vertex at
the center of the circle

## INSCRIBED ANGLE (in-skrybed)


an angle with its vertex on the circle

## "INSIDE" ANGLE (in-syde)


an angle with its vertex somewhere inside the circle; made by two chords or secants

## "OUTSIDE" ANGLE (out-syde)


an angle with its vertex somewhere outside the circle; made by a combination of tangents and/or secants

# INTERCEPTED ARC (in-ter-sept-ed) 

$B A C$ is inscribed in@X
$\overparen{B C}$ is an intercepted arc

the part of the circle that lies between the two lines that cross through it

## COMMON TANGENT <br> (kom-un tan-jent)



4 common tangents



1 common tangent
a line that is tangent to more than one circle

AREA OF A CIRCLE (air-ee-uh)

the amount of space inside a circle

## CIRCUMFERENCE (ser-kuhm-fer-ense)


the distance around a circle

## EQUATION OF A CIRCLE (ee-qway-zhun)

standard:

example:

$$
(x-h)^{2}+(y-k)^{2}=r^{2}
$$

$$
(x-2)^{2}+(y--1)^{2}=4^{2}
$$

$$
(x-2)^{2}+(y+1)^{2}=4^{2}
$$

a way to express a circle on the coordinate plane


| tangent | secant | chord |
| :---: | :---: | :---: |
| major arc | minor arc | semicircle |
| radius | diameter | central <br> angle |



Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Introduction to Similarity Homework

Why am I doing this? Tomorrow we are going to show that all circles are similar. The work you do tonight with objects from your own home will help us get enough data to show that this is true!

Directions:

1. Use your ruler to measure 2 feet of string. Cut the string.
2. Lay it out next to your ruler and use a marker to mark points along your string at every half inch.
3. Find 5 cylindrical objects or circles in your home and measure the circumference and the diameter of each one. Record those numbers in the table on the back of this paper.
4. Try pairing the circumference and the diameter in different ways (add them, subtract them, multiply them, divide them). There is one special pairing that gives an interesting result. See if you can find it!

Name $\qquad$ Teacher $\qquad$ Per $\qquad$ Introduction to Similarity Homework

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| Object | Circumference | Diameter | C + d | C - d | d - C | (C)(d) | C/d | d/C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |

Did you find it? What is the special pairing, and what is special about it? $\qquad$

What exact number do you think you should be getting for each number in this column? $\qquad$
Why is your data probably a little bit "off"? $\qquad$
$\qquad$

| Object | Circumference | Diameter | C + d | C - d | d - C | (C)(d) | C/d | d/C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

Did you find it? What is the special pairing, and what is special about it? $\qquad$

What exact number do you think you should be getting for each number in this column? $\qquad$
Why is your data probably a little bit "off"? $\qquad$

Lesson Plan: Circles Day 2
Subject: Geometry
New York State Core Curriculum Standards: G-C1

Texas Essential Knowledge and Skills: G.1.A, G.3.B
Objective: In my study of circles, I can prove that all circles are similar and use this fact to solve problems about similar circles.

Anticipatory Set/Warm-Up: recall definition of similar figures; discuss how homework connects to this definition of similarity

Lesson Main Ideas: circle similarity

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| formalize connection <br> between homework and <br> circle similarity | $\bullet$worksheet on similarity <br> (odds) | finish practice sheet <br> (evens) <br> ticket out the door |

Closure: students will complete a ticket out the door (TOTD)
Homework: Day 2 HW - choice of 3 tasks
$\qquad$
$\qquad$ Per $\qquad$

## ALL CIRCLES ARE SIMILAR

In general, two figures are similar if there is a set of transformations that will move one figure exactly covering the other. To prove any two circles are similar, only a translation (slide) and dilation (enlargement or reduction) are necessary. This can always be done by using the differences in the center coordinates to determine the translation and determining the quotient of the radii for the dilation.

## Example 1

Show that circle C with center $(-1,2)$ and radius 3 is similar to circle D with center $(3,4)$ radius 5 .

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and two units up. To enlarge circle C to the same radius as D , the enlargement ratio is the quotient of the radii: $\frac{5}{3}$.

## Example 2

Show that circle C with center $(0,2)$ and radius 6 is similar to circle D with center $(0,-6)$ radius 2 .

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center eight units down. To reduce circle C to the same radius as D , the reduction ratio is the quotient of the radii: $\frac{2}{6}=\frac{1}{3}$.

## Problems

Show the two given circles are similar by stating the necessary transformations from C to D .

1. C : center $(2,3)$ radius 5 ; D : center $(-1,4)$ radius 10 .
2. $C$ : center $(0,-3)$ radius $2 ; D$ : center $(-2,5)$ radius 6 .
3. $C$ : center $(-2,8)$ radius $4 ; D$ : center $(0,4)$ radius 9 .
4. C : center $(2,8)$ radius 5 ; D : center $(-2,4)$ radius 1 .
5. C : center $(12,32)$ radius 15 ; D : center $(-1,4)$ radius 10 .
6. C : center $(2,0)$ radius 7 ; D : center $(-1,0)$ radius 4 .

## Ticket Out the Door

## Solve the following problem on an

 index card. Put your name on it and hand it in when done.Show the two circles are similar by stating the necessary transformations from C to D .

C: center $(3,4) r=4, D:(4,1) r=2$

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Similarity Homework

For tonight's homework, you will have 3 choices. You must pick one assignment to do, although you may do more if you wish. All assignments may be done on this worksheet.

Option \#1: Write a short paragraph (5-10 sentences) explaining circle similarity and its implications.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Option \#2: Solve these two practice problems and provide step justifications for everything you do.

1) Show the two circles are similar by stating the necessary transformations from $C$ to $D$.

C: center $(-2,4) r=2, D:(3,2) r=6$

Justifications:
2) Show the two circles are similar by stating the necessary transformations from $C$ to $D$.
C: center $(2,2) r=7, D:(-6,-2) r=3$

Justifications:

Option \#3: Write and solve three of your own similarity problems.
1)
2)
3)

Lesson Plan: Circles Day 3
Subject: Geometry
New York State Core Curriculum Standards:
Texas Essential Knowledge and Skills: G.4.A
Objective: In my study of circles, I can graph circles given their equations and write the equations of circles given their graphs.

Anticipatory Set/Warm-Up: similarity problems
Lesson Main Ideas: equations and graphs of circles

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| 3-4 examples of each type <br> (graph $\rightarrow$ equation and <br> equation $\rightarrow$ graph) on <br> board as student follow <br> along in notes | $\bullet$practice worksheet - <br> 1 problem on front and <br> 1 problem on back | finish remainder of <br> practice worksheet |

Closure: students do a think-pair-share about what the two main ideas were in the lesson
Homework: Day 3 HW - equations practice and review of vocabulary

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Equations of Circles

Graph the following circles on the grids using the equations given.

1. $(x-4)^{2}+(y+2)^{2}=16$

2. $(x)^{2}+(y)^{2}=6.25$

3. $(x-5)^{2}+(y-1)^{2}=2.25$

4. $(x+1)^{2}+(y)^{2}=25$

5. $(x+2)^{2}+(y+2)^{2}=1$

6. $(x)^{2}+(y+6)^{2}=36$


Write the equation that matches the circles graphed below.
7.

www.math-aids.com
8.

www.math-aids.com
9.

www.math-aids.com
10.

11.

12.

$\qquad$ Teacher $\qquad$ Per $\qquad$

## Equations of Circles Homework

Write the equation of the circle shown or graph the circle whose equation is given.
1.

www.math-aids.com
3. $(x+2)^{2}+(y+1)^{2}=9$

2. $\qquad$

www.math-aids.com
4. $(x-6)^{2}+(y+4)^{2}=1$


Figure $A$
Use Figure A to answer questions 5-8.
5. Name the circle.
6. Name the radii of the circle.
7. Name the chords of the circle.
8. Name the diameters of the circle.


Use Figure B to answer questions 9-13.
9. Name the circle.
10. Name a line tangent to the circle.
11. Name the secant.
12. Name the chord.
13. Name the diameter.


Figure C


Lesson Plan: Circles Day 4
Subject: Geometry
New York State Core Curriculum Standards: G-C2, Math Practices 5

Texas Essential Knowledge and Skills: G.5.B, G.3.D, G.2.B
Objective: In my study of circles, I can identify and describe relationships among central angles, inscribed angles, "inside" angles, and "outside" angles so that I can solve problems involving the angles of circles.

Anticipatory Set/Warm-Up: review of equations/graphs of circles
Lesson Main Ideas: central angles, inscribed angles, "inside" angles, "outside" angles

| I Do* | We Do | You Do |
| :---: | :---: | :---: |
| - formalize rules: central angle = arc inscribed angle $=1 / 2$ arc "inside" angle $=1 / 2$ sum of arcs "outside" angle $=1 / 2$ difference of arcs <br> *this section is after the "We Do" section today | - measure 2 of each type of angle with a protractor given the arc measures and discover angle rules <br> - notes sheet on angles | - practice worksheet on angles |

Closure: students pair up, each creates a problem, then they switch and try to do each other's problems

Homework: Day 4 HW - use Geometer's Sketchpad to create each type of angle learned in lesson
$\qquad$
$\qquad$ Per $\qquad$

1. What is the equation of the circle graphed on this grid?

www.math-aids.com
2. Graph the circle given by the following equation: $(x+3)^{2}+(y-2)^{2}=9$


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

Warm-Up!

1. What is the equation of the circle graphed on this grid?

www.math-aids.com
2. Graph the circle given by the following equation: $(x+3)^{2}+(y-2)^{2}=9$


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Angle Relationships in a Circle - Notes

Central Angles: Central Angle = Intercepted Arc

1. Find the measure of arc YX .

Inscribed Angles: Inscribed Angle = ½ Intercepted Arc
5. Find the value of $x$.
9. Find the value of $x$.

## "Outside" Angles: Outside Angle = $1 / 2$ Difference of the Intercepted Arcs

13. Find the measure of $<\mathrm{P}$.

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## 4 Basic Formulas to Remember:

## Central Angle =

$\qquad$
$m \Varangle A O B=m \overparen{A B}$

Inscribed Angle =
$\qquad$ $m \Varangle A B C=\frac{1}{2} m \overparen{A C}$


Inside Angle formed by 2 Chords $=1 / 2$ $\qquad$ of Intercepted Arcs

$$
m \ltimes B E D=\frac{1}{2}(m \overparen{A C}+m \overparen{B D})
$$



Find the measure of each angle.

1. $\mathrm{m}<\mathrm{STU}=$ $\qquad$
2. $m<A B D=$ $\qquad$ 3. $\mathrm{m}<\mathrm{RNM}=$ $\qquad$

3. $\mathrm{m}<\mathrm{FGH}=$ $\qquad$

$$
\mathrm{m} \angle \bar{J}=
$$


6. $\mathrm{m}<\mathrm{YUV}=$ $\qquad$

7. $m<A B E=$
$m \overparen{B C}=$ $\qquad$

8. $\mathrm{m}<\mathrm{LKI}=$ $\qquad$
$\mathrm{m} \overparen{J}=$ $\qquad$
9. $\mathrm{m}<\mathrm{YUX}=$


## And the FOURTH one is...

Angle Formed Outside of a Circle by the Intersection of:
"Two Tangents" or "Two Secants" or "a Tangent and a Secant"
Outside Angle $=1 / 2$ Difference of Intercepted Arcs

| 1) Find $x$. | 2) Find $x$. |
| :--- | :--- |

Find the measure of $x$.
10. $x=$ $\qquad$

11. $x=$

12. $x=$

13. $x=$ $\qquad$

14. $x=$ $\qquad$

15. $x=$ $\qquad$

16. $x=$ $\qquad$

17. $x=$ $\qquad$

18. "?" = $\qquad$


## Mixed Review:

19. An angle formed by 2 chords that meet at 1 point on the circle: $\qquad$ .
20. An angle formed by the center and 2 radii of a circle is called a $\qquad$ .
21. If $Q R=24$, and $Q P=7$, what is the measure of $C N$ ? $\qquad$


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Angles Homework

Why am I doing this? Geometer's Sketchpad is a great program because of how accurately it displays geometric figures. When you design shapes in Sketchpad, they are pictures you can actually trust! The measurements that you will display alongside your pictures will reinforce the concepts we learned in class.

## Directions:

1. Open Geometer's Sketchpad and make a circle.
2. Construct segments to create a central angle.*
3. Have the program show the measure of the central angle and its intercepted arc next to the circle.
4. Print out the figure you created along with the measurements. Glue them below. You may also use the back of the paper if you need to.
5. Underneath the picture, write 1-2 sentences about how this supports the rule we learned today.
*You will repeat steps 1-5 with each angle type (inscribed angles, "inside" angles, and "outside" angles). You need to have all 3 scenarios for the "outside" angles - made by 2 secants, 2 tangents, or 1 secant and 1 tangent - for a total of 6 circle/angle printouts.*

Lesson Plan: Circles Day 5
Subject: Geometry
New York State Core Curriculum Standards: G-C2, G-C4

Texas Essential Knowledge and Skills: G.11.C, G.3.D, G.2.A
Objective: In my study of circles, I can apply the theorem that a tangent is perpendicular to a radius or diameter it intersects to solve problems involving right triangles.

Anticipatory Set/Warm-Up: have students make a conjecture about the relationship between a tangent and a radius/diameter

Lesson Main Ideas: a tangent meets a radius/diameter perpendicularly

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| define theorem as we go <br> through PowerPoint | $\bullet$practice worksheet on <br> tangents | •use compass and <br> straightedge to construct a <br> tangent that intersects a <br> radius; confirm that it is <br> perpendicular with a <br> protractor |

Closure: students complete a ticket out the door; they find someone with a different form and compare answers - they should match

Homework: Day 5 HW - teach family/friends how to do tangent problems
$\qquad$ Teacher $\qquad$ Period $\qquad$

## Practice with Tangents

1) Determine if $T P$ is a tangent to the circle.

2) Find $x$. Assume lines that appear to be tangent are tangent.

3) Find "?". Assume lines that appear to be tangent are tangent.

4) Find $x$. Assume lines that appear to be tangent are tangent.

5) Determine if $A B$ is a tangent circle $O$.

6) Find $x$. Assume lines that appear to be tangent are tangent.

7) Find "?". Assume lines that appear to be tangent are tangent.

8) Find the perimeter of the polygon.


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Tangents TOTD - Partner A

Directions: You and a partner will complete your TOTD individually. You have different problems, but if you each solve correctly, your answers should match! So try them and then compare.

1. $\overline{A B}$ and $\overline{B C}$ are tangents. Solve for x .
2. All segments are tangents. Find the perimeter

Y. $P X=12, X Y=30$, and $Z Y=40$.


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Tangents TOTD - Partner B

Directions: You and a partner will complete your TOTD individually. You have different problems, but if you each solve correctly, your answers should match! So try them and then compare.

1. $\overline{A B}$ and $\overline{B C}$ are tangents. Solve for x .
2. All segments are tangents. Find the perimeter $f \triangle A B C$. $A B=32, C B=42$, and $A N=30$.


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Tangents Homework

Why am I doing this? When you are able to solve problems on your own, that is a huge step toward learning a new topic. However, when you can TEACH someone else how to solve those problems, then you know you have truly mastered the skill!

Directions: Teach a family member or friend (who is not currently taking Geometry) how to do the questions below. You may teach one person or choose a different person for each question. Make sure you answer any questions they have along the way, and then assess them at the end of the process (by making up a similar problem, asking them to explain the process back to you, etc.). When they know how to solve the problems by themselves, you have completed your homework.

1. Find the perimeter of triangle ACE if $\mathrm{AC}=28, \mathrm{CD}=12$, and $\mathrm{FE}=14$.

2. Assuming a tangent intersecting a diameter, is it possible to have a triangle with the given side lengths? Explain why or why not.


Lesson Plan: Circles Day 6
Subject: Geometry
New York State Core Curriculum Standards:
Texas Essential Knowledge and Skills: G.2.B, G.3.B
Objective: In my study of circles, I can describe theorems involving special scenarios with chords.

Anticipatory Set/Warm-Up: students pair up to compare experiences from the homework from the previous night

Lesson Main Ideas: theorems involving parallel chords, congruent chords, chords equidistant from the center, and a chord perpendicular to a radius/diameter

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| $\bullet$describe 4 chord theorems <br> using PowerPoint | $\bullet$last slide of PowerPoint <br> links to regentsprep.org <br> for practice problems | create 1 of each type of <br> chord theorem practice <br> problem with a partner |

Closure: students give a valid conclusion to a scenario on an index card on the way out the door (given: the hypothesis of each theorem, conclude: the conclusion of the theorem)

Homework: Day 6 HW - find examples of chords in real life and the jobs that use chord theorems
 $\qquad$
Theorem 1: Parallel chords in the same circle always create congruent arcs. In the figure below, $\operatorname{arcs} A B$ and $C D$ have equal measures.


## Converse of Theorem 1:

$\qquad$

## Ex 2:



$$
\begin{aligned}
& \overline{A B} \| \overline{C D} \\
& \overparen{A B B}=60^{\circ} \\
& m \overparen{A D}=80^{\circ}
\end{aligned}
$$

Find $m \overparen{C D}=$ $\qquad$

## Ex 1:


www.regentsprep.org
$\overline{C D}$ diameter
$\overline{A B} \| \overline{C D}$

$$
m \overparen{A B}=80^{\circ}
$$

Find $m \overparen{C A}=$ $\qquad$

Ex 3: Given the circle with 2 parallel chords and central angle $40^{\circ}$, find the value of $x$.

www.regentsprep.org
Theorem 2: Chords equidistant from the center of $\boldsymbol{a}$
circle are congruent. In the figure below, chords WX
and YZ are congruent.
$\overline{G B} \cong \overline{G E}$

Theorem 3: Chords of the same length in the same circle create congruent arcs. In the figure below, $\operatorname{arcs} A B$ and $C D$ are congruent.


## Converse of Theorem 3:

$\qquad$

Ex 2:

$m \overparen{B E}=210^{\circ}$
Find $m \overparen{C D}$ $\qquad$

Ex 1: Find the measure of the arcs indicated below.


Find $\overparen{A B}$
Find $\overparen{A B F}$ $\qquad$
Find $\overparen{A B D}$ $\qquad$

Ex 3: Find the measure of arc CE.


Theorem 4: If a radius (or diameter) is
Ex 1: perpendicular to a chord, then it bisects the chord and its arcs. In the figure on the left, $\mathrm{AE}=\mathrm{EB}$ and arcs $A D$ and $B D$ have equal measures.


AB is a diameter and $A B \perp C D$. That means that $A B$ bisects $C D$ and so... CE=DE. It also means that $\mathrm{m} \overparen{C B}=\mathrm{m} \widehat{\mathrm{DB}}$ and $\mathrm{mCA}=\overline{\mathrm{DA}}$.

## Converse of Theorem 4:

$\qquad$

Ex 2: Given
$C D=16$
$A B=16$
$O B=10$
Find $O F=$ $\qquad$


Ex 3:

$A C \perp B D$
$\mathrm{m} \overparen{A B C}=94^{\circ}$
Find $\overparen{A B}$ $\qquad$

$A C \perp B D$
$\mathrm{m} \overline{A C}=12$
$\mathrm{m} \overline{D E}=8$
Find the radius $\qquad$

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Chords Homework

Directions: Research chords using internet search engines. Find at least three examples of everyday uses of chords in real life. Focus on careers that require knowledge of circles and their properties. Summarize your findings below, and include a printout of any pictures you find that show chords in the real world.

Chord Use \#1: $\qquad$

Career/Job (if applicable): $\qquad$

Brief Explanation: $\qquad$
$\qquad$

Picture (if applicable):

Chord Use \#2: $\qquad$

Career/Job (if applicable): $\qquad$

Brief Explanation: $\qquad$
$\qquad$

Picture (if applicable):

Chord Use \#3:
Career/Job (if applicable): $\qquad$
Brief Explanation: $\qquad$
$\qquad$

Picture (if applicable):

Lesson Plan: Circles Day 7
Subject: Geometry
New York State Core Curriculum Standards:

Texas Essential Knowledge and Skills: G.9.C, G.4.A
Objective: In my study of circles, I can use segment rules to solve problems involving chords, tangents, and secants in circles.

Anticipatory Set/Warm-Up: chord theorem practice problems
Lesson Main Ideas: rules involving 2 chords, 2 secants, 2 tangents (review), and a tangent/secant

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| introduce rules using notes <br> sheet with examples | $\bullet$practice worksheet on <br> chords, secants, and <br> tangents (odds - not 13) | finish practice worksheet <br> (evens - not 12) |

Closure: students complete \#12 and \#13 on practice worksheet and get their papers stamped if they are correct

Homework: Day 7 HW - create 2 of each type of segment problem

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Chords Warm-Up

1. Solve for $x$.
$\widehat{A D}=5(x+12), \widehat{B C}=8 x$

2. Find the measure of arc LK.
$\overline{\mathrm{LM}}$ is a diameter, $\widehat{\mathrm{KN}}=100^{\circ}$

3. Solve for $x$. Assume point $O$ is the center.

$$
\overline{\mathrm{FG}}=4 x+2.5, \overline{\mathrm{EH}}=x+17.5
$$


4. Find $\overline{O Q}$. Assume point $O$ is the center.


Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Segments in a Circle- Notes

## Two Chords

the product of the segments of the first chord $=$,
the product of the segments of the second chord

## Two Tangents (review!)

two tangents drawn from the same external point
are equal
Ex 1: $\mathrm{AC}=8$. Find the length of BC .
tangent = tangent
$A C=B C$


## One Tangent and One Secant



Name $\qquad$ Teacher $\qquad$ Period $\qquad$

## Chords, Secants, and Tangents Dractice

Find $x$ to the nearest tenth if necessary. Assume segments that appear to be tangent are tangent.

1. $\mathrm{x}=$ $\qquad$


2. $x=$ $\qquad$
3. $x=$ $\qquad$

4. $x=$ $\qquad$

5. $x=$ $\qquad$
6. $x=$ $\qquad$

7. $x=$ $\qquad$
8. $x=$ $\qquad$
9. $x=$ $\qquad$
10. $x$ _
11. $\mathrm{PT}=$ $\qquad$ $\mathrm{PN}=$ $\qquad$ Given the circle with secants and chords below, find PT, PN, LR, RT. (Note: LT $\neq P M$ )

12. $x \approx$ $\qquad$ Given a circle with a tangent, secant and chord, find x . (Hint: Find y first)


Circles Day 7
Homework

Name $\qquad$ Teacher $\qquad$ Per $\qquad$


Directions: This is your chance to show your creative side! Your mission is to create two examples of each segment scenario we learned about today. You must draw pictures, include a variable to solve for, and provide enough numbers and/or expressions for something to be able to solve your problem.

## 2 Chords:

\#1

2 Secants:
\#1
\#2

2 Tangents:
\#1
\#2

1 Secant and 1 Tangent:
\#1

Lesson Plan: Circles Day 8
Subject: Geometry
New York State Core Curriculum Standards:

Texas Essential Knowledge and Skills: G.9.C, G.4.A
Objective: In my study of circles, I can use segment rules to solve problems involving chords, tangents, and secants in circles.

Anticipatory Set/Warm-Up: practice problems about the 4 rules learned yesterday
Lesson Main Ideas: rules involving 2 chords, 2 secants, 2 tangents (review), and a tangent/secant

| I Do | We Do | You Do |
| :--- | :--- | :--- |
|  | $\bullet$quickly go through warm- <br> up to refresh rules and <br> ensure students are <br> remembering them <br> correctly | segments "BING" (like <br> BINGO) |

Closure: students solve problems the teacher writes on the board as time allows after BING activity

Homework: Day 8 HW - preview for arc length and sector area (research online and watch videos)

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Segments Warm-Up

1. In the accompanying diagram, $\overline{\mathrm{PA}}$ is tangent to circle $O$ at $A, P B C$ is a secant, $P B=4$, and $B C=8$. What is the length of PA?

2. The accompanying diagram shows two intersecting paths within a circular garden. What is the length of the portion of the path marked x ?

3. The circle is circumscribed by the pentagon as shown. If $Q Z=10, Y X=9, X W=9$, $U W=17$, and $S U=10$. Find the perimeter of the pentagon.

4. Find the value of $x$ if $A B=17, B C=10$, and $C D=11$.

$\qquad$ Teacher $\qquad$ Per $\qquad$

## Segments "BING"

You've heard of BINGO, right? This is basically the same idea, except you only need to get 4 in a row to win.

1. In the accompanying diagram, PA is tangent to circle $O$ at $A$, secant $P B C$ is drawn, $P B=4$, and $B C=12$. Find $P A$.

2. In the diagram below, $\mathrm{AC}=11, \mathrm{CD}=4$, and the perimeter of triangle ACE is 32 . Find FE.

3. In the accompanying diagram of circle O , chords $A B$ and $C D$ intersect at $E, A E=5$, $C D=18$, and $E D=8$. Find the length of $E B$.

4. In the accompanying diagram of a circle, chords $A B$ and $C D$ intersect at $E$. $C E=8, C D=17$, and $A E=4$. Find the length of $B E$.

5. Randy and his family are going fishing. The circle represents where each person of his family was standing around the pond. Randy, his brother Tim, and his mother Jill are standing in a straight line. There is 20 ft between Randy $(R)$ and $\operatorname{Tim}(T)$. Randy's father Frank ( $F$ ) is standing at the side of the pond which is tangent to Randy's mother Jill (J) who is standing in the parking lot. If there is 6 ft between Tim and Jill, how far is Frank from Jill, to the nearest foot?

6. In the accompanying diagram of circle O , diameter $A B$ is perpendicular to chord $C D$ and intersects $C D$ at $E, A E=9$, and $E B=4$. What is ED?

7. In the accompanying diagram of circle O , chords $A B$ and $C F$ intersect at $E$. If $E B=14$, $A E=5$, and $C E=10$, find $E F$.

8. In the accompanying diagram, PAB and PCB are secants drawn to circle $O, P A=8, P B=30$, and $P D=16$.

9. In the diagram below, $\mathrm{EC}=15, \mathrm{DC}=\mathrm{x}, \mathrm{BC}=5$, and $A B=x+3$. Find the value of $x$.

10. In the diagram of circle $O$ below, secants CBA and CED intersect at $C$. If $A C=39, B C=3$, and $D C=9$, find $E C$.

11. In the accompanying diagram of circle $O$, diameter $A B$ is perpendicular to chord $C D$ at $E$, $C D=8$, and $E B=2$. What is the length of the diameter of circle O ?

12. In the accompanying diagram, $\mathrm{LM}=2 \mathrm{x}+3$, $M N=3, N P=4$, and $P Q=x+2$. Find the length of LM.

13. In the diagram below, tangent $C D$ and secant CBA are drawn to circle $O$ from external point $C$. If $D C=10$, and $B C=4$, find the length of $A B$.

14. In the accompanying diagram, segments RS, ST, and $T R$ are tangent to the circle at $\mathrm{A}, \mathrm{B}$, and C , respectively. If $S B=3, B T=5$, and $T R=13$, what is the measure of RS?

15. In the accompanying diagram $A D$ is tangent to circle $O$ at $D$ and $A B C$ is a secant. If $A D=10$ and $A C=50$, find $A B$.


## "BING" Board

Fill in the squares below with answers from the answer bank (some answers occur more than once). As you solve the problems in the "Segments 'BING'" packet, cross out the corresponding square.

When you get 4 in a row, tell Mrs. Hooker and claim your prize!
There is also a bigger prize for being the first one to cross off every square, so keep working!

|  | Answer Bank |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
|  |  |  |  |  |
| 2 | 7 | 11 | 16 |  |
| 4 | 8 | 12 | 18 |  |
| 5 | 9 | 13 | 21 |  |
| 6 | 10 | 15 | 39 |  |



Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## ArG Lenģin Pin Sector Area Prewiew

Directions: Open your favorite search engine and research the phrases "arc length" and "sector area." Find the best websites that explain these concepts. Record what sites you found most helpful. When you feel you have a good grasp of the material, visit www.brightstorm.com to watch the following videos. (You will need to use your username and password to log in.) Once you have watched the videos, answers the 3 problems indicated by the tabs at the top of the video. Your homework is complete when you are comfortable enough with arc length and sector area to be able to explain the concepts to someone else in class tomorrow. Feel free to jot down some notes below if you need to do so.

Link for arc length: http://www.brightstorm.com/math/geometry/circles/arc-length/ Link for sector area: http://www.brightstorm.com/math/geometry/area/area-of-a-sector/

Name $\qquad$ Teacher $\qquad$ Per $\qquad$

## Arc Length and Sector Area Preview

Directions: Open your favorite search engine and research the phrases "arc length" and "sector area." Find the best websites that explain these concepts. Record what sites you found most helpful. When you feel you have a good grasp of the material, visit www.brightstorm.com to watch the following videos. (You will need to use your username and password to log in.) Once you have watched the videos, answers the 3 problems indicated by the tabs at the top of the video. Your homework is complete when you are comfortable enough with arc length and sector area to be able to explain the concepts to someone else in class tomorrow. Feel free to jot down some notes below if you need to do so.

Link for arc length: http://www.brightstorm.com/math/geometry/circles/arc-length/ Link for sector area: http://www.brightstorm.com/math/geometry/area/area-of-a-sector/

Lesson Plan: Circles Day 9
Subject: Geometry
New York State Core Curriculum Standards: G-C5

Texas Essential Knowledge and Skills: G.8.B, G.2.B
Objective: In my study of circles, I can calculate the length of an indicated arc and the sector it makes.

Anticipatory Set/Warm-Up: students do a think-pair-share about last night's homework
Lesson Main Ideas: arc length and sector area

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| -facilitate class discussion <br> about last night's <br> homework <br> formalize formulas for arc <br> length and sector area | •discuss last night's <br> homework <br> go through PowerPoint <br> with students helping <br> explain concepts | practice worksheet on arc <br> length and sector area |

Closure: students write the formula for arc length and the formula for sector area with a 1 sentence explanation of where each comes from

Homework: Day 9 HW - review homework on regentsprep.org

# Arc Length \& Segment/Sector Area 

## Name

$\qquad$
$\qquad$ Period $\qquad$

www.coolmath.com


Arc Length $=\frac{\theta}{360} \cdot 2 \pi r$
www.coolmath.com

Find the length of each arc (L). Leave your answer in terms of $\pi$.

1. $L=$ $\qquad$

2. L= $\qquad$

$$
r=16 \mathrm{~m}, \theta=75^{\circ}
$$

5. L= $\qquad$


When she is outdoors, Tasha, the dog, is tied to a stake in the center of a circular area of radius 24 feet. The angle between her dog house and her favorite hydrant is 165 degrees. To the nearest foot, what is the length of the arc from her dog
3. L= $\qquad$

4. $L=$ $\qquad$

$$
r=18 \mathrm{~cm}, \theta=60^{\circ}
$$

6. L= $\qquad$
Alison is jogging on a circular track that has a radius of 140 feet. She runs along the track from point $R$ to point $N$, a distance of 230 feet. Find to the nearest degree, the measure of minor arc RN.

7. $\mathrm{L}=$

8. $L=$ $\qquad$



A sector of a circle is a pie shaped portion of the area of the circle.
www.coolmath.com


| Area of a Sector | Area of a Segment |
| :---: | :---: |
| $A=\pi r^{2}\left(\frac{m^{\circ}}{360^{\circ}}\right)$ <br> Find the area of sector $D E F$. $\begin{aligned} A & =\pi(6)^{2}\left(\frac{40^{\circ}}{360^{\circ}}\right) \\ & =36 \pi\left(\frac{1}{9}\right) \\ & =4 \pi \mathrm{~cm}^{2} \\ & \approx 12.57 \mathrm{~cm}^{2} \end{aligned}$ | $A=\pi r^{2}\left(\frac{m^{\circ}}{360^{\circ}}\right)-\frac{1}{2} b h$ <br> Find the area of segment $A C B$ $\begin{aligned} A & =\pi(3)^{2}\left(\frac{90^{\circ}}{360^{\circ}}\right)-\frac{1}{2}(3)(3) \\ & =9 \pi\left(\frac{1}{4}\right)-4.5 \\ & =2.25 \pi-4.5 \\ & \approx 2.57 \mathrm{in}^{2} \end{aligned}$ |

- Directions: Find the area of the unshaded sector ( $\mathrm{A}_{\text {sector }}$ ) and the area of the shaded segment ( $\mathrm{A}_{\text {segment }}$ ) for each. Round your answers to the nearest tenth. Show all work.

1. $\mathrm{A}_{\text {sector }}=$ $\qquad$

2. $\mathrm{A}_{\text {sector }}=$ $\qquad$
$\mathbf{A}_{\text {segment }}=$ $\qquad$

3. $\mathrm{A}_{\text {sector }}=$
$\mathrm{A}_{\text {segment }}=$

$\qquad$
$A_{\text {segment }}$
$\qquad$
4. $A_{\text {sector }}=$ $\qquad$
$\mathrm{A}_{\text {segment }}=$ $\qquad$
$r=8$ in.; central angle $=30^{\circ}$
5. $\mathrm{A}_{\text {sector }}=$ $\qquad$
$\mathrm{A}_{\text {segment }}=$ $\qquad$
$r=25 \mathrm{mi}$.; central angle $=80^{\circ}$
6. $\mathrm{A}_{\text {sector }}=$ $\qquad$
$\mathbf{A}_{\text {segment }}=$ $\qquad$
$r=10 \mathrm{~km} . ;$ central angle $=100^{\circ}$
7. In the diagram below AC and BD are arcs of circles with centers at $O$. The radius, $O A$, is 10 centimeters and the radius, $O B$, is $\mathbf{1 6}$ centimeters. Find the shaded area.

8. In the diagram PQ and RS are arcs of circles with center $\mathbf{O}$. The radius, $O Q$, is 30 centimeters long and the radius, $O S$, is 20 centimeters long. Calculate the perimeter of the shape

Name $\qquad$ Teacher $\qquad$ Per $\qquad$
CIRCLES
CIRCLES
CIRCLES REVIEW

HOME
REVIEW HOME
REVIEW HOME

Directions: Type the following web address in your browser and answer the questions in the table below. Include all necessary work, as well as the correct answer to receive full credit. Happy studying!
http://www.regentsprep.org/Regents/math/geometry/MultipleChoiceReviewG/Circles.htm

| Question | Work | Answer |
| :---: | :---: | :---: |
| 1 |  |  |
| 2 |  |  |
|  |  |  |
|  |  |  |
|  |  |  |


| 4 |  |  |
| :---: | :---: | :---: |


| 10 |  |  |
| :---: | :--- | :--- |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |
| 13 |  |  |


| 16 |  |  |
| :---: | :--- | :--- |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 10 |  |  |

Lesson Plan: Circles Day 10
Subject: Geometry
New York State Core Curriculum Standards: G-C1, G-C2, G-C3, G-C4, G-C5
Texas Essential Knowledge and Skills: G.1.A, G.2.A, G.2.B, G.3.B, G.4.A, G.5.B, G.8.B, G.9.C, G.11.C

Objective: In my study of circles, I can use my knowledge of the material taught during this unit to successfully complete my unit exam.

Anticipatory Set/Warm-Up: go over homework
Lesson Main Ideas: circle unit

| I Do | We Do | You Do |
| :--- | :--- | :--- |
| go over homework from <br> previous night |  | complete unit exam |
|  |  |  |

Closure: none
Homework: none
$\qquad$
$\qquad$
$\qquad$

## Circles Test - A

Using the following diagram, correctly identify and name each of the following. Use correct labeling/notation.

1. one chord $\qquad$


O is the center of the circle.
3. one secant
$\qquad$
4. one radius $\qquad$
5. one diameter $\qquad$
6. one central angle $\qquad$
7. one inscribed angle $\qquad$

In the problems below, solve for the missing variable(s) or answer the question. Show all your work.
8.

80
11.

9.


10. Write the equation of the circle shown here.
13. Graph the following circle on the grid.


14. $\overline{\mathrm{DL}}=12, \overline{\mathrm{LC}}=14, \overline{\mathrm{AL}}=3+3 \mathrm{x}$, and $\overline{\mathrm{LB}}=4 \mathrm{x}+1$.

Find x .

18. $\overline{\mathrm{AB}} / / \overline{\mathrm{CD}}$. What conclusion can you draw from this information?

19. $\overline{\mathrm{AB}} \cong \overline{\mathrm{CD}}$. Find x .

www.regentsprep.org
20. $\overline{O R} \perp \overline{A B}$. Find $x$.

www.regentsprep.org
17. Find x .

21. Find x .

22. $\overparen{A B} \cong \overparen{C D}$. What conclusion can you draw from this information?

23. Find the length of major arc $A B C$.

24. Find the area of the shaded sector.

$\qquad$ Teacher $\qquad$ Period $\qquad$

## Circles Test - B

Using the following diagram, correctly identify and name each of the following. Use correct labeling/notation.

1. one inscribed angle $\qquad$
2. one central angle $\qquad$
3. one chord $\qquad$ O is the center of the circle.
4. one secant
$\qquad$
5. one tangent $\qquad$
6. one diameter $\qquad$

.

In each problem below, solve for the missing variable(s) or answer the question. Show all your work.
8.



12.

10. Graph the following circle on the grid.

$$
(x+3)^{2}+(y-2)^{2}=9
$$


questions, showing all work.
13. Write the equation of the circle shown here.

14. $\overline{\mathrm{TC}}=14, \overline{\mathrm{UC}}=12, \overline{\mathrm{CV}}=2 \mathrm{x}+2$, and $\overline{\mathrm{CW}}=2 \mathrm{x}+5$.

Find x .

18. $\widehat{B C} \cong \widehat{A D}$. What conclusion can you draw from this information?

15. Find "?"

19. $\overline{\mathrm{OR}} \perp \overline{\mathrm{AB}}$. Find x .

16. Find $x$.

17. Find $x$.

20. $\overline{A B} \cong \overline{C D}$. Find $x$.


22. $\mathrm{AB} \cong \mathrm{CD}$. What conclusion can you draw from this information?

23. Find the length of major arc $A B C$.

24. Find the area of the shaded sector.


## Chapter Four: Discussion

## How to Use the Unit Plan

Each day in the new circles unit involves some sort of warm-up or anticipatory set, notes designed to aid the teacher in instruction, guided and independent practice, lesson closure, and a homework assignment. Occasionally, a lesson does not have an explicit worksheet for one of the above lesson parts. In this case, the information is either located in a PowerPoint accompanying this paper in the Appendices, or left up to the teacher to decide how best to fill in the missing material. Sometimes the author intended for activities such as matching games, BINGO, or a think-pair-share to be used as part of a lesson. These are simply recommendations; teachers may replace the planned activities with ones which they think will best suit the needs of their students and classroom. Lastly, the unit culminated with an exam. Two different forms of the test were created, designed to look very similar to one another. These exams may also be broken up into two smaller quizzes, the first half placed in the middle of the unit, and the second half given at the end.

## Highlighted Characteristics of Meaningful Homework

Each lesson concludes with a homework assignment. These were carefully created to exude the characteristics of meaningful homework. All homework assignments were designed to be brief-about 15 minutes of less-with the exception of the review homework included in Day 9. The homework assignments accompanying Day 1 and Day 8 were an introduction to the next day's topic. This method of using homework to introduce a new topic is reminiscent of the flipped classroom theory, where a student's "homework" is to learn a new concept. The Day 1 assignment also contained another common characteristic of meaningful homework: defined purpose. The purpose of the work was clearly expressed in the box titled "Why am I doing this?"

This box was also seen on assignments from Days 4 and 5 as well. Day 1 was the only assignment to showcase hands-on activities. Students were asked to gather objects and physically measure them, thus increasing the level of student engagement. The homework assignment in Day 2 highlighted the power of choice. There were three options they had to choose from. In the first two options, it would be clear to a teacher how well a student understood what was taught because they both involve some sort of written explanation. The third option provides students with an indirect way of showing what they comprehend by allowing them to create their own problems. Days 4 and 7 also increased the level of rigor by requiring students to produce their own problems or examples, mimicking those seen in class.

A misconception about part of a new process or skill can become ingrained if it is practiced repeatedly. For this reason, one of the hallmarks of beneficial homework is that it largely dwells on previously learned topics. When the homework is graded or discussed the next day, the students will be able to see if they truly understand the objectives from the previous day. If they answered correctly, it is probably safe for them to practice this skill more until it is fully learned. If they answered incorrectly, the student is able to seek help to amend any misunderstanding before it is deeply rooted. The homework included in Day 3 showed this by limiting the practice of newly learned skills to just four problems. The bulk of the worksheet was a review of circle vocabulary. Obviously Day 9 also followed this theory since it was entirely review. Days 4,6 , and 8 focused on the integration of technology or online resources. Many students today are highly motivated when using technology, and it was the author's attempt to harness this motivation through the use of modeling programs, online videos, and other websites. Day 5 allowed students the opportunity to involve people close to them. They were asked to teach family members or friends how to solve problems similar to what they learned in class. Not
only does teaching a concept reinforce it in the mind of the teacher, this assignment added another layer of reinforcement by utilizing interpersonal relationships. Lastly, the students were asked to use everyday objects and research careers in Day 1 and Day 6, respectively. This connected what was learned in class to the real world.

## Conclusion

It is the author's sincere hope that educators find this circles unit to be useful. Homework is a critical part of the learning process. Sadly it is usually viewed as just another expected routine rather than being perceived as the tool it can and should be. However, when teachers unlock the potential of meaningful homework, they can expect to see a change in student comprehension, motivation, and engagement.

## References

Ash, K. (2012). Educators View 'Flipped' Model With a More Critical Eye. Education Week, 32(2), S6-S7.

Bembenutty, H. (2011a). The First Word: Homework's Theory, Research, and Practice. Journal of Advanced Academics, 22(2), 185-192.

Bembenutty, H. (2011b). The Last Word: An Interview with Harris Cooper--Research, Policies, Tips, and Current Perspectives on Homework. Journal of Advanced Academics, 22(2), 340-349.

Bempechat, J., Li, J., Neier, S. M., Gillis, C. A., \& Holloway, S. D. (2011). The Homework Experience: Perceptions of Low-Income Youth. Journal of Advanced Academics, 22(2), 250-278.

Biscoglio, J., \& Langer, N. (2011). Grandparents Against Homework. Educational Gerontology, 37(2), 154-163.

Brock, C. H., Lapp, D., Flood, J., Fisher, D., \& Han, K. (2007). Does Homework Matter? An Investigation of Teacher Perceptions about Homework Practices for Children from Nondominant Backgrounds. Urban Education, 42(4), 349-372.

Cooper, H., Robinson, J., \& Patall, E. A. (2006). Does Homework Improve Academic Achievement? A Synthesis of Research, 1987-2003. Review of Educational Research, 76(1), 1-62.

Cushman, K. (2010). Show Us What Homework's For. Educational Leadership, 68(1), 74-78.
Dettmers, S., Trautwein, U., \& Ludtke, O. (2009). The Relationship between Homework Time and Achievement Is Not Universal: Evidence from Multilevel Analyses in 40 Countries. School Effectiveness and School Improvement, 20(4), 375-405.

Dettmers, S., Trautwein, U., Ludtke, O., Kunter, M., \& Baumert, J. (2010). Homework Works If Homework Quality Is High: Using Multilevel Modeling to Predict the Development of Achievement in Mathematics. Journal of Educational Psychology, 102(2), 467-482.

Flipped Classrooms Making a Splash in American Schools. (2012). Curriculum Review, 52(3), 2-3.

Fulton, K. P. (2012). 10 reasons to flip. Phi Delta Kappan, 94(2), 20-24.
Geometry Assessment Eligible Texas Essential Knowledge and Skills. (2012). Retrieved from http://www.tea.state.tx.us/student.assessment/staar/ac/

Gill, B. P., \& Schlossman, S. L. (2003). A Nation at Rest: The American Way of Homework. Educational Evaluation and Policy Analysis, 25(3), 319-337.

Grootenboer, P. (2009). Homework and Learning Mathematics. Australian Primary Mathematics Classroom, 14(4), 11-15.

Hauk, S., \& Segalla, A. (2005). Student Perceptions of the Web-Based Homework Program WeBWorK in Moderate Enrollment College Algebra Classes. Journal of Computers in Mathematics and Science Teaching, 24(3), 229-253.

Hoepfl, C.M. (1997). Choosing qualitative research: A primer for Technology Education. Journal of Technology Education, 9(1), 47-63.

Hong, E., Wan, M., \& Peng, Y. (2011). Discrepancies between Students' and Teachers' Perceptions of Homework. Journal of Advanced Academics, 22(2), 280-308.

Kitsantas, A., Cheema, J., \& Ware, H. W. (2011). Mathematics Achievement: The Role of Homework and Self-Efficacy Beliefs. Journal of Advanced Academics, 22(2), 310-339.

McReynolds, K. (2007). Homeschooling. Encounter, 20(2), 36-41.

Mendicino, M., Razzaq, L., \& Heffernan, N. T. (2009). A Comparison of Traditional Homework to Computer-Supported Homework. Journal of Research on Technology in Education, 41(3), 331-359.

Nielsen, L. (2012). Five Reasons I'm Not Flipping Over the Flipped Classroom. Technology \& Learning, 32(10), 46.

New York State P-12 Common Core Learning Standards for Mathematics. (2012). Retrieved from http://engageny.org/resource/new-york-state-p-12-common-core-learning-standards-for-mathematics

Patall, E. A., Cooper, H., \& Allen, A. (2010). Extending the School Day or School Year: A Systematic Review of Research (1985-2009). Review of Educational Research, 80(3), 401-436.

Richards-Babb, M., Drelick, J., Henry, Z., \& Robertson-Honecker, J. (2011). Online Homework, Help or Hindrance? What Students Think and How They Perform. Journal of College Science Teaching, 40(4), 81-93.

Sallee, B., \& Rigler, N. (2008). Doing Our Homework on Homework: How Does Homework Help?. English Journal, 98(2), 46-51.

Sokolowski, A., Yalvac, B., \& Loving, C. (2011). Science Modelling in Pre-Calculus: How to Make Mathematics Problems Contextually Meaningful. International Journal of Mathematical Education In Science And Technology, 42(3), 283-297.

Stevens, M. L. (2003). The Normalisation of Homeschooling in the USA. Evaluation \& Research in Education, 17(2/3), 90-100.

Texas Education Agency (2011). STAAR the next generation [Brochure]. Retrieved from $\underline{\text { http://www.tea.state.tx.us/WorkArea/linkit.aspx?LinkIdentifier=id\&ItemID=2147502688\&libID= }}$ 2147502682.

Vatterott, C. (2010). Five Hallmarks of Good Homework. Educational Leadership, 68(1), 10-15.
Voorhees, S. (2011). Why the Dog Eats Nikki's Homework: Making Informed Assignment Decisions. Reading Teacher, 64(5), 363-367.

Wilson, J., \& Rhodes, J. (2010). Student Perspectives on Homework. Education, 131(2), 351358.

Xu, J. (2005). Purposes for Doing Homework Reported by Middle and High School Students. Journal of Educational Research, 99(1), 46.

Xu, J. (2010). Homework Purposes Reported by Secondary School Students: A Multilevel Analysis. Journal of Educational Research, 103(3), 171-182.

Most pictures/activities are original but some have been copied from the following sites:

- www.bugforteachers.com
- www.coolmath.com
- www.cpm.org
- www.math-aids.com
- www.mathcaptain.com
- www.mathwarehouse.com
- www.regentsprep.org
- www.winpossible.com

