# The Use of Vocabulary in the Mathematics Classroom 

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# The Use of Vocabulary in the Mathematics Classroom 

 byElizabeth A. Worthington

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ADVISOR

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## Chapter One: Introduction

Literacy in mathematics is very different than the literacy of other subjects. Students must learn the vocabulary that is used in the classroom to aide in their understanding of the topic. Most vocabulary words used in math are left in the classroom and never used again. It is comparable to that of a foreign language class. Students learn the language but never use it. By using various literacy strategies to help the students focus on what is in the mathematics problems not just the computations, it will help these students to be better readers in every subject.

As a teacher I feel that students do not understand what they are being asked to do when reading a math question, due to their misunderstanding of the vocabulary used in mathematics. I see that many of my students just skip over problems because of this; they may know the math that needs to be completed but first they need to understand what they are being asked to do.

I get very frustrated when my students don't comprehend what they are being asked to do when solving any type of word problem. They will come to class with their homework not complete because they really just don't understand what they are supposed to do. I need and want to put a stop to this. It seems that the majority of students will read something and if they don't understand it within seconds of reading it they give up. They don't have the drive to pursue it further, perhaps looking up a word, or looking in their notes.

With the push for students to become better problems solvers, they must also
become better readers of math. To guide my students in the direction of success, I feel that I need to incorporate reading in the classroom. To be more specific, students need to be mathematically literate. There needs to be a focus on content specific literacy, so my plans are to introduce my students to the reading of mathematics. To begin with I feel that they must have a better understanding of the vocabulary that is used in mathematics, as well as the writing of mathematics. I already have them write out the steps of a problem when they do their test corrections, but I feel that they need to go even further, perhaps writing a weekly journal or an occasional quiz involving writing an essay.

To incorporate literacy strategies into a classroom, the teacher must be willing to put in a little extra work when starting to use the strategies. For the mathematics student to comprehend most of the reading that is either done in class or for independent practice the teacher should incorporate the use of various vocabulary development strategies. Most of these strategies can be used before, during or after the reading and learning. During reading and learning the teacher can use various strategies to aide the students through reading of the text or notes and to also help with solving word problems. Once the reading is done there are many reflective strategies that involve literacy, this helps reinforce the learning and lets the students make it their own.

## Chapter Two: Literature Review

Carter and Dean (2006) stated that "the ultimate goal of developing good vocabulary is to aid in comprehension. Comprehension instruction should focus on reading for meaning and metacognitive reading. Reading for meaning includes activating prior knowledge. Prior knowledge helps skilled readers anticipate meanings of unknown words, make inferences about what is being read, organize new information, and connect new learning to old" (p.34).

Students need to have a good grasp on how to read and comprehend what they are reading. By the time students get to a high school mathematics class they have seen many of the vocabulary words that are being used, but they still are not sure what they mean. If they were taught how to read to understand, then perhaps they could read a question, know what to do and then do it.

Barton, Heidema and Jordon (2002) feel that "Reading mathematics and science requires special reading skills - skills that students may not have used in other content areas" (p. 24). By the time most students are in fourth or fifth grade they are no longer learning to read but reading to learn and they are never taught how to read for each discipline that they are taught. Blanton (1991) found that "one of the most common complaints about mathematics is that it is too difficult to read. Indeed, numerous concepts are often squeezed into very compact notation and symbols. That is why reading instruction should be included in mathematics classes.

Students cannot be expected to think mathematically unless they can read the material" (p. 162).

Capraro (2006) "found that teachers also encouraged better mathematical reading comprehension by encouraging students to read aloud and discuss each passage as opposed to students reading to themselves" (p. 92). Reading aloud is an excellent way to get all students involved. As we know when students are reading the others in the classroom are more attentive than if the teacher was doing the reading or if they were asked to quietly read to themselves.

Capraro and Joffrion (2006) wrote that "reading in mathematics necessitates that one understand the meaning of words. As children learn mathematics, it is essential they learn the meaning of new words that are not part of their oral vocabulary or have wholly different meanings from what they already know" ( p . 162). As research shows reading comprehension comes from the understanding of the words. Students need to have a good understanding of the words used in mathematics to be able to understand the problems.

To aid in students ability to read the language of mathematics they must learn how to decode what is being read. Decoding is the ability to understand what the various symbols mean and be able to interpret the words that go with it. Carter and Dean (2006) say that "decoding instruction occurs when the teacher draws the students' attention to a miscue in pronouncing a word or misinterpretation of a symbol while reading. Strategies may include having the student reread in order to correct
the miscue, having the student sound out the word, or providing the student with the correct word or interpretation of the symbol" (p 134). Many students will read a problem and not understand what it is asking because they lack the skills of decoding.

Reciprocal teaching is another way to aid students in the reading comprehension of mathematics. Garderen (2004) describes reciprocal teaching as "a structured strategy advocated by many reading specialists for developing comprehension skills. In reading, reciprocal teaching involves students making predictions when reading, questioning themselves about the ideas in the text, seeking clarification when confused, and summarizing content" (p. 226). This could be used as a strategy to help students in mathematics, especially word problems. As teachers we need to give our students more and different ways to approach a problem so that they can feel confident every time they are asked to solve a problem.

As we can see there are many stumbling blocks for students when it comes to reading and understanding mathematics. Teachers are one of those stumbling blocks. There has been a push for reading in the content area for secondary schools; however teachers are reluctant to take on the additional work. Jacobs (2002) found that "for subject teachers to implement principles and practices of secondary reading and writing they must first recognize reading and writing as meaning-making processes that can support their instructional goals, particularly those related to understanding content" (p. 58).

Teachers are not necessarily being asked to teach more they are just being
asked to teach differently to help their students reach a level of reading comprehension that is a necessary skill to aid in solving mathematics problems. As Fuentes (1998) puts it, "once we as teachers acknowledge that teaching students to comprehend mathematical texts is a desirable outcome of our instruction, we are well on the way to solving the problem. Answers to questions about how to teach comprehension in mathematics classrooms lies partially in the early research on reading, reading comprehension, and long-term memory" (p. 83). Carter and Dean (2006) also mentioned in their study that "mathematics teachers need to be aware of their role as teachers of reading and understand the variety of ways they can incorporate reading strategies into their lessons" (p. 144).

In a short but informative article, Sister Brandenburg (2002) who teaches precalculus and calculus stated that she "learned that students in these upper-level classes could work the problems when she taught the material, but 'putting it all together' seemed incredibly difficult. Students found it hard to combine or apply techniques. In addition, they couldn't use the material in unfamiliar situations" (p. 67). So she decided to incorporate writing into her curriculum. She found that her students learned to "formulate and express their mathematical thinking in a clear-cut and substantive manner using correct vocabulary. They became mathematical literate" (p. 68). This is what we are all striving for, to have our students become mathematical literate.

In New York State the math curriculum that mathematics teachers follow
contains key ideas, each key idea is broken down into performance indicators. When teaching my students about logic, I am covering all of the first key idea which is mathematical reasoning. My students will be taking the Math A Regents exam this year. Some of my students will take it in January and some in June. To cover all of the performance indicators for logic, I will introduce them to constructing valid arguments with the truth values of compound sentences using conjunction, disjunction, conditional, biconditional, converse, inverse and contrapositive. They will also learn the truth value of simple sentences by understanding closed and open sentences, replacement sets, solution sets and negation.

Each article that I have read always points back to the use of vocabulary and that students need to have a strong vocabulary background to have sound reading comprehension. Kenney, Hancewicz, Heuer, Metsisto \& Tuttle (2005) feel that "teachers need to be very attentive as they encourage students to use standard vocabulary when they talk and write. We need to really listen in order to uncover misunderstandings; students are apt to parrot back definitions, thus concealing their confusion. During lessons in which students first encounter a new concept, teachers should encourage them to describe ideas in their own words before introducing the specialized terms" (p. 83). As always we as teachers need to be attentive to what are students are doing and saying to know if they comprehend what is being taught.

Carter and Dean (2006) stated that "instruction in vocabulary includes teaching students to use strategies that help them make connections between concepts
and the terminology used to describe the concepts" (p. 134). If we can take time to teach students all of the vocabulary necessary so that they can comprehend what they are reading, the schools would have a chance to improve and produce successful students. Marzano and Pickering (1997) feel that "when we understand words for which we have constructed meaning, we do not understand the definitions. We understand them because we have constructed meaning as result of our experiences with them" (p. 55).

To effectively use vocabulary strategies in teaching one should follow the principles that Readance, Bean and Baldwin (2001) have suggested in their text. The first is to be an enthusiastic model of vocabulary use. "Reach out to your students and make them believe that you believe vocabulary development is something more than a dead paragraph from your teacher's syllabus" (p. 160). Secondly, make vocabulary meaningful, "vocabulary instruction should have a long-term impact upon individual powers of communication and concept development" (p. 160). Thirdly, reinforce vocabulary, "give students an opportunity to use their new words as they read, write, speak, and listen" (p. 161). And lastly, be eclectic, "a successful vocabulary program will employ a variety of methods" (p. 161). These principles will help to develop a successful way to introduce vocabulary as part of every lesson as well as give your students another way in which to learn mathematics.

Some of the strategies that have been found to help aid students in learning and retaining vocabulary words can be found in the book "Teaching Reading in

Mathematics, 2nd ed.". Just to name a few that I could use in my own teaching are: concept circles, concept definition mapping, frayer model, semantic mapping, concentration, cue cards, etc... This list goes on, but they are all good ideas and it gives you a variety of ways to incorporate vocabulary into your teaching.

As one can gather from the information provided, there are many reasons to incorporate vocabulary usage into ones daily teaching. Many students come to math class with enough prior knowledge to get by, but what they really need is to understand what they are reading, whether in notes, homework, from the textbook, or even when taking the state exams. Many of my students need to have this extra push to understand the vocabulary used in mathematics and I am willing to take the extra step necessary to provide them with the education they deserve. Research has shown that the use of vocabulary in the mathematics classroom can help students gain a stronger reading comprehension, which in turn helps them to become better mathematicians.

Chapter Three: Unit Plan
January 2008

| Monday | Tuesday | Wednesday | Thursday | Friday |
| :--- | :--- | :--- | :--- | :--- |
|  | 1 | 2 <br> Sentences, <br> statements, and <br> truth values | 3 <br> Negation <br> ( |  |

Elizabeth Worthington
Unit Plan: Logic
Lesson 1: Sentences, Statements, Truth Values, Negations and Symbols (2 days)
Math 10 and Math AB

## Objective:

- Introduce students to the terms they will be using for this unit
- Students will have an understanding of sentences and statements used in logic
- Students will know how to negate a sentence or equation
- Students will know the symbol and word for negation


## Essential Question:

- Why is logic a part of math?
- How can it be applied to real life?


## Standards:

- Key Idea: Mathematical Reasoning - Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
- Performance Indicators:
- 1A. Construct valid arguments: Truth value of compound sentences (conjunctions, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations)
- 1B. Follow the judge of validity of arguments: Truth value of compound sentences


## DAY 1

Launch:

- Have students fill out a rating sheet to see what terms they know and then talk about the terms and what they do know about them, making sure to bring up how some of the words have different meanings depending on how it is used and in what subject.


## Lesson:

- Students will be introduced to seven of the vocabulary words
- I will model with given examples of how different types of sentences can be true or false depending on the type.
- Introduce negation and the word and symbol for it.
- Model how it is used for words and equations
- Introduce students to the use of p's and q's in logic
- Half way through the lesson I will refer back to the words that have already been introduced to the students.


## Guided Practice:

- Have students work with a neighbor on writing a sentence in symbolic form and then come back together to make sure they understand. (While students are working I will walk around the room to assist as needed).
- Have students work with a neighbor on writing complete sentences when given symbols and then come back together to make sure they understand. (While students are working I will walk around the room to assist as needed).


## Closure:

- I will pull popsicle sticks that have the student's names on them and review the words that were covered during class.


## DAY 2

Launch:

- Have an open discussion of the new terms that the students learned from the day before


## Lesson:

- Finish the examples in the note packet
- Have the students make a memory game of the terms that they have learned
- Have students play the game once to check and see if they are understanding the definitions


## Guided Practice:

- Have students begin work on their homework (While students are working I will walk around the room to assist as needed).

Independent practice:

- Students will finish the worksheet on their own time. The worksheet contains various types of problems dealing with what was taught in class as well as review type questions to help them retain past topics.

Closure:

- Students will play their memory game once they have finished making it.

Name:
Notes: 23
Date:
$\qquad$ Logic
Math $A B$ and Math 10

## Sentences, Statements, Truth Values, Negations and Symbols

 Logic: $\qquad$Truth Value: $\qquad$

Simple Sentences
Closed Sentence: $\qquad$
$\qquad$
$\qquad$

Examples:

1) The grass is green.
2) The trees grow flat.
$\qquad$
3) $3+7=10$

Open Sentence: $\qquad$
$\qquad$
$\qquad$
$\qquad$

Examples:

1) The house is blue. True or False? $\qquad$
$\qquad$
$\qquad$
2) $6+x=15$ True or False? $\qquad$
$\qquad$
$\qquad$

## Replacement Set:

$\qquad$
$\qquad$

## Solution Set:

$\qquad$
$\qquad$
$\qquad$

Examples: If the replacement set is the set of whole numbers, then

1) $6+x=15$, the solution set is? $\qquad$
2) $x>10$, the solution set is? $\qquad$

Negation: $\qquad$

Examples:

1) Original: John Kennedy was a U.S. president Negation: $\qquad$
2) Original: The post office handles mail.

Negation: $\qquad$
3) Original: $x+3=10$

Negation: $\qquad$

| NAME: | MEANING: | SYMBOL: | PROPERTIES: |
| :---: | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Truth table for NEGATION

| $p$ | $\sim p$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |

Examples1:
Symbol Statement in words Truth Value

1) $\qquad$
2) $\qquad$
$\qquad$

## Example 2:

- Write the sentence is symbolic form
- Tell whether the sentence is true, false, or open.

Let p represent "Oatmeal is a cereal."
Let q represent "She has cereal every morning."

1) Oatmeal is a cereal.
2) Oatmeal is not a cereal.
3) She has cereal every morning.
4) She does not have cereal every morning.
5) It is not true that oatmeal is not a cereal.

## Example 3:

- Write a complete sentence in words to show what the symbols represent and tell whether the sentence is true, false or open.
$p$ : Paper is made from trees.
$q$ : Trees grow everywhere.
$r$ : He likes math

1) $\sim p$
2) $\sim q$
3) $\sim(\sim r)$

Name
Homework: Lesson \#23
Date
Math AB and Math 10
Logic
*1) Tell whether each sentence is true, false, or open. If the sentence is an open sentence, identify the variable.
a) The United States of America declared independence in 1776.
b) They celebrate Independence Day on July 14 every year.
c) San Francisco is a city in New York State.
d) $5 x+2=17$
e) $5(10)+2=17$
*2) Write the negation of each sentence.
a) The school has a cafeteria. $\qquad$
b) The measure of a right angle is $90^{\circ}$. $\qquad$
c) Today is not Saturday. $\qquad$
*3) For each sentence given in symbolic form:

- Write a complete sentence in words to show what the symbols represent.
- Tell whether the sentence is true, false, or open.
$p$ : Summer follows spring $\quad q$ : Baseball is a sport $s$ : He likes baseball
a) $\sim p$
b) $\sim s$
c) $\sim(\sim q)$
*4) When $p$ is true, then $\sim p$ is
*5) $\sim(\sim(\sim p))$ has the same truth value as $\qquad$

6) Which inequality is represented in the graph below?

(1) $-4<x<2$
(3) $-4 \leq x \leq 2$
(2) $-4 \leq x<2$
(4) $-4 \leq x \leq 2$
7) The graphs of the equations $y=x^{2}+4 x-1$ and $y+3=x$ are drawn on the same set of axes. At which point do the graphs intersect?
(1) $(1,4)$
(3) $(-2,1)$
(2) $(1,-2)$
(4) $(-2,-5)$
8) A group of 148 people is spending five days at a summer camp. The cook ordered 12 pounds of food for each adult and 9 pounds of food for each child. A total of 1,410 pounds of food was ordered.
a) Write an equation or a system of equations that describes the above situation and define your variables.
b) Using your work from part a, find:
(1) the total number of adults in the group
(2) the total number of children in the group
9) A painting that regularly sells for a price of $\$ 55$ is on sale for $20 \%$ off. The sales tax on the painting is $7 \%$. Will the final total cost of the painting differ depending on whether the salesperson deducts the discount before adding the sales tax or takes the discount after computing the sum of the original price and the sales tax on \$55?

Elizabeth Worthington
Unit Plan: Logic
Lesson 2: Conjunctions and Disjunctions
Math 10 and Math $A B$

## Objective:

- Introduce students to the terms conjunction and disjunction
- Students will know the symbol and word for conjunction
- Students will know the symbol and word for disjunction
- Students will be able to apply their new knowledge to everyday

Essential Question:

- Where do we use conjunction and disjunction in the real world?

Standards:

- Key Idea: Mathematical Reasoning - Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
- Performance Indicators:
- 1A. Construct valid arguments: Truth value of compound sentences (conjunctions, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations)
- 1B. Follow the judge of validity of arguments: Truth value of compound sentences

Launch:

- Sing the School house rock song "Conjunction"
- Talk about what a conjunction is in the English language.

Lesson:

- Introduce students to the words and symbols used for conjunction and disjunction
- Guide students through examples of how conjunction and disjunction are used


## Guided Practice:

- Through out the lesson I will ask students to give assistance in completely filling in the notes correctly

Independent practice:

- Students will be given a worksheet to take and complete on their own time. The worksheet contains various types of problems dealing with what was taught in class as well as review type questions to help them retain past topics.

Closure:

- At the end of class, the students (as a whole) will tell me the symbols, words and truth values for conjunction and disjunction.

Name:
Date: $\qquad$ Logic
Math AB and Math 10
Conjunctions and Disjunctions
Conjunction $\qquad$

| NAME: MEANING: SYMBOL: PROPERTIES: <br>     <br>     |
| :--- |

Examples:
Use the following for statements to fill in the last column

| p | q |  |
| :---: | :---: | :--- |
| the sky is <br> green | coffee contains <br> caffeine |  |
|  |  |  |


| p | q |  |
| :---: | :---: | :---: |
| Snow is cold | 10 is divisible by 2 |  |
|  |  |  |

Truth table for $\qquad$

| p | q |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

When is a conjunction true? $\qquad$

## Disjunction:

$\qquad$
$\qquad$
$\qquad$

| NAME: | MEANING: | SYMBOL: | PROPERTIES: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

Examples:
Use the following for statements to fill in the last column

| P | q |  |
| :--- | :--- | :--- |
| Mrs. Nickerson <br> teaches Spanish | Bananas are <br> blue |  |
|  |  |  |


| p | q |  |
| :---: | :---: | :---: |
| Worms are <br> slimy | Mr. Gilebarto owns <br> a helicopter |  |
|  |  |  |

Truth table for : $\qquad$

| p | q | $\mathrm{p} \quad \mathrm{q}$ |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

When is a disjunction true? $\qquad$
When is it false?

Name
Date
Logic
*1) Write each sentence in symbolic form, using the given symbols.
Let p represent "It is hot"
Let q represent "It is raining"
Let $r$ represent "The sun is shining"
a) It is hot and it is raining $\qquad$
b) The sun is not shining and it is not hot. $\qquad$
c) It is raining and the sun is not shining. $\qquad$
*2) When $p$ is true and $q$ is true, then $p^{\wedge} q$ is $\qquad$ .
*3) From the given symbols, write a complete sentence.
Let s represent "I will study"
Let p represent "I will pass the test"
Let f represent "I am foolish"
a) $s \vee f$
b) $s \wedge p \vee f$ $\qquad$
c) $\sim f \wedge p$ $\qquad$
*4) When $p$ is false and $q$ is true, then $\sim(p v q)$ is $\qquad$ .
*5) When $q$ is true, then $p v q$ is $\qquad$ .
6) A locker combination system uses three digits from 0 to 9 . How many different three-digit combinations with no digit repeated are possible?
(1) 30
(2) 720
(3) 504
(4) 1,000
7) Which ratio represents $\cos A$ in the accompanying diagram of $\triangle A B C$ ?

8) Sarah hikes 7 miles north, 5 miles east, and then 4 miles north again. To the nearest tenth of a mile, how far, in a straight line, is Sarah from her starting point?

9) Dave tossed a ball in the air in such a way that the path of the ball was modeled by the equation $y=-x^{2}+6 x$. In the equation, $y$ represents the height of the ball in feet and $x$ is the time in seconds. At what time, x , is the ball at its highest point?
10) The owner of a movie theater was counting the money from 1 day's ticket sales. He knew that a total of 150 tickets were sold. Adult tickets cost $\$ 7.50$ each and children's tickets cost $\$ 4.75$ each. If the total receipts for the day were $\$ 891.25$, how many of each kind of ticket were sold?

Elizabeth Worthington
Unit Plan: Logic
Lesson 3: Conditionals and Biconditionals
Math 10 and Math AB
Objective:

- Introduce students to the terms conditional and biconditional
- Students will know the symbol and word for conditional
- Students will know the symbol and word for biconditional
- Students will be able to apply their new knowledge to everyday


## Essential Question:

- How are conditional and biconditional statements used in advertising?


## Standards:

- Key Idea: Mathematical Reasoning - Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
- Performance Indicators:
- 1A. Construct valid arguments: Truth value of compound sentences (conjunctions, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations)
- 1B. Follow the judge of validity of arguments: Truth value of compound sentences

Launch:

- I will explain to the students that "If they all do well on their next test then we can have a snack during class". I will also go into other examples of how the conditional and biconditional are used.

Lesson:

- Introduce students to the words and symbols used for conditional and biconditional
- Guide students through examples of how conditional and biconditional are used


## Guided Practice:

- Have students work with a neighbor on writing a sentence in symbolic form and then come back together to make sure they understand. (While students are working I will walk around the room to assist as needed).
- Have students work with a neighbor on writing a complete sentence when given symbols and then come back together to make sure they understand. (While students are working I will walk around the room to assist as needed).
- Have students begin to fill in a graphic organizer of the five connectives: Negation, Conjunction, Disjunction, Conditional and Biconditional.

Independent practice:

- Students will be given a worksheet to take and complete on their own time. The worksheet contains various types of problems dealing with what was taught in class as well as review type questions to help them retain past topics.

Closure:

- Students will complete a ticket out the door that contains some of the vocabulary words that they have been learning.

Name:
Notes: 25
Date:
$\qquad$ Logic
Math $A B$ and Math 10

## Conditionals and Biconditionals

Conditional: $\qquad$
$\qquad$
$\qquad$

Hypothesis: $\qquad$
$\qquad$
$\qquad$

Conclusion: $\qquad$
$\qquad$
$\qquad$

| Name: | Meaning: | Symbol: | Properties: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

REMEMBER:

Examples:
Use the following for statements to fill in the last column

| p | $q$ |  |
| :--- | :--- | :--- |
| Today is <br> Monday | Apples are <br> red |  |
|  |  |  |


| p | q |  |
| :---: | :---: | :---: |
| Biology is cool | Boys are smart |  |
|  |  |  |
|  |  |  |

Truth table for:

| $p$ | $q$ | $p$ q |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

When is a conditional FALSE? $\qquad$

## Biconditional:

$\qquad$

| Name: | Meaning: | Symbol: | Properties: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

## Examples:

Use the following for statements to fill in the last column

| P | q |  |
| :---: | :---: | :--- |
| Girls are tall | Yellow cars are fast |  |
|  |  |  |


| p | q |  |
| :--- | :---: | :--- |
| Grapes are <br> green | Books are <br> covered |  |
|  |  |  |

Truth table for: $\qquad$

| p | q | p |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Name $\qquad$ Homework: Lesson \#25

Date $\qquad$ Math $A B$ and Math 10
Logic
*1) Write each sentence is symbolic form, using the given symbols.
$p$ : The test is easy.
$q$ : Sam studies.
$r$ : Sam passes the test.
a) If the test is easy, then Sam will pass the test. $\qquad$
b) If the test is not easy, then Sam will not pass the test. $\qquad$
c) If Sam studies, then Sam will pass the test. $\qquad$
d) Sam will not pass the test if Sam doesn't study. $\qquad$
*2) Symbols are assigned to represent sentences, and truth values are assigned to these sentences.

Let j represent "I jog"
(True)
Let d represent "I diet"
(False)
Let g represent "I feel well"
(True)
Let $h$ represent "I get hungry"
(True)
For each compound sentence in symbolic form:
a) Write a complete sentence in words to show what the symbols represent.
b) Tell whether the compound sentence is true or false.

1) $j \rightarrow g$
2) $g \rightarrow \sim h$
3) $\left(j^{\wedge} h\right) \rightarrow g$ $\qquad$
4) $(j \vee d) \rightarrow h$ $\qquad$
*3) When the conclusion $q$ is true, then $p \rightarrow q$ must be $\qquad$
*4) Write each biconditional in symbolic form, using the symbols given.
t : The triangle is a right triangle.
r : The triangle contains a right angle.
n : The triangle contains a $90^{\circ}$ angle.
a) A triangle is a right triangle if and only if it contains a right angle. $\qquad$
b) A triangle contains a $90^{\circ}$ angle if and only if it contains a right angle. $\qquad$
c) A triangle is not a right triangle if and only if it does not contain a $90^{\circ}$ angle. $\qquad$
*5) Using the symbols p and q , write each compound sentence in symbolic form.
a) pif and only if q. $\qquad$
b) If $p$ then $q$ and if $q$ then $p$. $\qquad$
c) $q$ if and only if $p$. $\qquad$
d) If $p$ then $q$ or if $q$ then $p$.
5) A cardboard box has length $x-2$, width $x+1$, and height $2 x$.
a) Write an expression, in terms of $x$, to represent the volume of the box.
b) If $x=8$ centimeters, what is the number of cubic centimeters in the volume of the box?
6) Factor completely: $3 x^{3}-6 x^{2}-24 x$
7) Given the true statement "John is not handsome" and the false statement "John is handsome or smart." Determine the truth value for the statement "John is smart."
8) A stop sign in the shape of a regular octagon is resting on a brick wall, as shown in the accompanying diagram. What is the measure of angle $x$ ?


Elizabeth Worthington
Unit Plan: Logic
Lesson 4: Inverse, Converse and Contrapositive
Math 10 and Math AB
Objective:

- Introduce students to the terms inverse, converse and contrapositive
- Students will know the symbol and word for inverse
- Students will know the symbol and word for converse
- Students will know the symbol and words for contrapositive
- Students will be able to apply their new knowledge to everyday


## Essential Question:

- How are these words used differently than the previously learned words?

Standards:

- Key Idea: Mathematical Reasoning - Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
- Performance Indicators:
- 1A. Construct valid arguments: Truth value of compound sentences (conjunctions, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations)
- 1B. Follow the judge of validity of arguments: Truth value of compound sentences

Launch:

- Discuss with students what they think inverse, converse and contrapositive mean in the English language.


## Lesson:

- Introduce students to the words and symbols used for inverse, converse and contrapositive
- Guide students through examples of how inverse, converse and contrapositive are used

Guided Practice:

- Through out the lesson I will ask students to give assistance in completely filling in the notes correctly

Independent practice:

- Students will be given a worksheet to take and complete on their own time. The worksheet contains various types of problems dealing with what was taught in class as well as review type questions to help them retain past topics.

Closure:

- Students will fill in their graphic organizer for inverse, converse and contrapositive
$\qquad$


# Inverses, Converses and Contrapostives 

## Logically Equivalent:

$\qquad$

Note: The conditional $(\rightarrow)$ is the most often connective used in reasoning. From an original conditional, there are 3 other types of conditionals. We need to identify them as well as see which one is logically equivalent to the original conditional.

## Remember:

A conditional $(p \rightarrow q)$ is false when it goes from true

Inverse: $\qquad$
$\qquad$

| Name: | What it means: | Original: | Result: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

Example:
Givens:
$\mathrm{p}=\mathrm{I}^{\prime} \mathrm{m} 15$

| Original statement |
| :---: |
| $p \rightarrow q$ |
| If I'm 15 then I'm a teenager |

## Converse:

$\qquad$

| Name: | What it means: | Original: | Result: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Example:
Givens: $\mathrm{p}=\mathrm{I}$ 'm 15
$\mathrm{q}=\mathrm{I}$ 'm a teenager

| Original statement | Converse of the statement |
| :---: | :---: |
| $\mathrm{p} \rightarrow \mathrm{q}$ |  |
| If I'm 15 then I'm a |  |
| teenager |  |

## Contrapositive:

$\qquad$
$\qquad$

| Name: | What it means: | Original: | Result: |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Example:

Givens: $\quad p=I^{\prime} m 15 \quad q=I^{\prime} m$ a teenager

| Original statement | Contrapositive of the statement |
| :---: | :---: |
| $\mathrm{p} \rightarrow \mathrm{q}$ |  |
| If I'm 15 then I'm a <br> teenager |  |

Truth table of all four types of conditionals:

|  |  | Conditional | Inverse | Converse | Contrapositive |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| p | q | $\sim \mathrm{p}$ | $\sim \mathrm{q}$ |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Which new type of conditional is logically equivalent to the original conditional? $\qquad$
Which two conditionals are logically equivalent? $\qquad$ and $\qquad$

Examples:
1.) Write the converse of the following statement:

If Mr. Gilebarto drives a Saturn then Mr. Mosier drives a jeep.
2.) Write the inverse of the following statement:

If students study then students will pass their exam.
3.) Write the contrapositive of the following statement: If it's cold then people go skiing.
4.) Write the converse, inverse, and contrapositive of the following statements:

| Original <br> Conditional | Converse | Inverse | Contrapositive |
| :---: | :--- | :--- | :--- |
| $\sim \mathrm{a} \rightarrow \mathrm{c}$ |  |  |  |
| $\mathrm{d} \rightarrow \sim \mathrm{f}$ |  |  |  |
| $\mathrm{x} \rightarrow \mathrm{y}$ |  |  |  |
| $\sim s \rightarrow \sim t$ |  |  |  |

5.) What is logically equivalent to: $\sim a \rightarrow b$ ?
6.) What is logically equivalent to: $c \rightarrow d$ ?
7.) What is logically equivalent to:

If I study then I'll pass my test.
8.) What is logically equivalent to:

I'll do my homework when I have time.

Name $\qquad$ Homework: Lesson \#26

Date $\qquad$

## Logic

For problems 1-3, from the given conditional write the inverse, converse and contrapositive for each.
*1) Given conditional: $p \rightarrow q$
inverse: $\qquad$
converse: $\qquad$
contrapositive: $\qquad$
*2) Given conditional: If today is Friday, then tomorrow is Saturday
inverse: $\qquad$
converse: $\qquad$
contrapositive: $\qquad$
*3) Given conditional: If a man is honest, he does not steal
inverse: $\qquad$
converse: $\qquad$
contrapositive: $\qquad$
*4) What is the inverse of the statement
"If Mike did his homework, then he will pass this test"?
(1) If Mike passes this test, then he did his homework.
(2) If Mike does not pass this test, then he did not do his homework.
(3) If Mike does not pass this test, then he only did half his homework.
(4) If Mike did not do his homework, then he will not pass this test.
*5) What is the converse of the statement
"If it is sunny, I will go swimming"?
(1) If it is not sunny, I will not go swimming.
(2) If I do not go swimming, then it is not sunny.
(3) If I go swimming, it is sunny.
(4) I will go swimming if and only if it is sunny
*6) Which statement is logically equivalent to
"If it is Saturday, then I am not in school"?
(1) If I am not in school, then it is Saturday.
(2) If it is not Saturday, then I am in school.
(3) If I am in school, then it is not Saturday.
(4) If it is Saturday, then I am in school.
7) One root of the equation $2 x^{2}-x-15=0$ is
(1) $\frac{5}{2}$
(3) 3
(2) $\frac{3}{2}$
(4) -3
8) The ratio of Tariq's telephone bill to Pria's telephone bill was 7:5. Tariq's bill was $\$ 14$ more than Pria's bill. What was Tariq's bill?
(1) $\$ 21$
(3) $\$ 35$
(2) $\$ 28$
(4) $\$ 49$
9) Which inequality is represented in the accompanying graph?

(1) $-3 \leq x<4$
(3) $-3<x<4$
(2) $-3 \leq x \leq 4$
(4) $-3<x \leq 4$
10) The height of a golf ball hit into the air is modeled by the equation $h=-16 t^{2}+48 t$, where $h$ represents the height, in feet, and $t$ represents the number of seconds that have passed since the ball was hit. What is the height of the ball after 2 seconds?
(1) 16 ft
(3) 64 ft
(2) 32 ft
(4) 80 ft

Elizabeth Worthington
Unit Plan: Logic
Lesson 5: Logic review
Math 10 and Math AB
Objective:

- Review to reinforce students learning before they take a test on Logic

Standards:

- Key Idea: Mathematical Reasoning - Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.
- Performance Indicators:
- 1A. Construct valid arguments: Truth value of compound sentences (conjunctions, disjunction, conditional, related conditionals such as converse, inverse, and contrapositive, and biconditional). Truth value of simple sentences (closed sentences, open sentences with replacement set and solution set, negations)
- 1B. Follow the judge of validity of arguments: Truth value of compound sentences

Launch:

- Students will be introduced to 5 stations that they will be asked to go to through out the period.

Lesson:

- Station 1: Students will play memory with the cards that they made
- Station 2: Students will given a crossword puzzle to complete
- Station 3: Students will correct their review packet with the key
- Station 4: Students will make cards for the knowledge wall
- Station 5: Students will complete a fill in the box game


## Guided Practice:

- Through out the period I will walk around and check to see that students are staying on task and understanding what they are to be doing.


## Closure:

- Students will be asked to study for the test and as a class we will go over all the terms out loud


## Chapter Four: Results

## Overview

As I completed this project, I can look back and see all of the different things that I learned. I wasn't expecting my students to be curious about being involved with my project. They would often ask me how things are going with it. It was more work than usual to implement the vocabulary stratagies into my everyday teaching. I am not saying that I don't emphasis vocabulary, but I felt that I was over emphasis zing. It was fun trying new strategies with the kids and seeing their reactions. I had used the same unit last year with the exception of my emphasis on vocabulary. My initial thought was that this year's group would do much better on the test than last year's group. I found that I was wrong. The students knew the meaning of the words better, but they weren't any stronger knowing the concepts. The focus on vocabulary helped the students with the definitions, but it didn't reinforce the concepts enough. One of my students had asked after we were done with the unit how my project was going. I told her that the results weren't where I thought they would be and she stated that "I felt as if we knew the words, but lost track of what the concepts were".

As I looked at the data, the test scores didn't vary much from last year to this year. I felt the only way to see if there was a significant difference would be to run a t-test.

## Summary of Unit Plan

Upon receipt of the parental permissions slips we were ready to begin the chapter on Logic. Of the 87 students that I teach 67 of them returned their permission slips. This may not be as accurate to my data as I would have liked. Some of the students stated that they just kept forgetting to bring it in while others said that their parents didn't want them to be involved.

With the 67 students that did participate, I first had them fill out a Rating Terms sheet which contained all of the new vocabulary words that they would be using through out this chapter. The results are as follows:

| Rating Terms | I've never heard <br> of the word <br> before | I've heard of the <br> term, but I don't <br> know how it applies <br> to mathematics | I understand the <br> meaning of this <br> term and can apply <br> it to a mathematics <br> problem |
| :--- | :---: | :---: | :---: |
| Biconditional | $77 \%$ | $20 \%$ | $3 \%$ |
| Closed sentence | $43 \%$ | $48 \%$ | $9 \%$ |
| Conclusion | $5 \%$ | $46 \%$ | $49 \%$ |
| Conditional | $26 \%$ | $68 \%$ | $6 \%$ |
| Conjunction | $26 \%$ | $70 \%$ | $4 \%$ |
| Contrapositive | $77 \%$ | $22 \%$ | $1 \%$ |
| Converse | $20 \%$ | $63 \%$ | $17 \%$ |
| Disjunction | $55 \%$ | $37 \%$ | $8 \%$ |
| Hypothesis | $1 \%$ | $32 \%$ | $67 \%$ |
| Inverse | $11 \%$ | $41 \%$ | $48 \%$ |
| Logic | $5 \%$ | $61 \%$ | $34 \%$ |
| Logically equivalent | $43 \%$ | $43 \%$ | $14 \%$ |
| Negation | $51 \%$ | $39 \%$ | $10 \%$ |
| Open sentence | $37 \%$ | $48 \%$ | $15 \%$ |
| Replacement set | $49 \%$ | $41 \%$ | $10 \%$ |
| Solution set | $14 \%$ | $48 \%$ | $38 \%$ |
| Truth value | $51 \%$ | $38 \%$ | $11 \%$ |

As one can see from the above data, $45 \%$ of the terms were under the category "I've heard of the terms, but don't know how it applies to mathematics. The words that students chose most are those that can be applied to other subjects. In the category "I've never heard of the word before, $35 \%$ of the terms were under this choice. The terms with a higher percentage were those terms students would have not heard of until they have been taught logic. While $20 \%$ of the terms were under the category "I understand the meaning of this terms and can apply it to a mathematics problem". These terms are used in mathematics and science classrooms from early on in their schooling.

This data helped me to understand my student's knowledge of the terms that we were going to be working with over the next few weeks. Although they said they knew what some of the terms meant, they weren't sure how it would apply to Logic.

During the first two days of this chapter, we learned many of the vocabulary terms and how they can apply to mathematics and to real life. At the end of the second day we made cards for a memory game for the students to use for review later on in the chapter. I had the students put a term on one card and the definition on another card. I had them keep the cards in the back of the room and as more terms were added to their notes they were also added to their memory game.

As we added new material each day, I also tried various strategies to help the students understand and remember all of their new terms. On day three I asked the students to recall the terms they had learned up to and including that day. We
made it a game. All of the kids stood up and then I pulled popsicle sticks which had the student's names on them. Each student had to state something new they had learned over the last few days and if they did they could sit down. If they weren't sure they could phone a friend. I felt that this helped those students who are auditory learners.

As the week progressed I gave the students a ticket out the door, which needed to be completed before they could leave. I was surprised to hear that many of them had never heard of a "Ticket out the Door". I knew that this would be a great strategy to use in the future. A graphic organizer was the next vocabulary strategy used. We did this as a whole class and then they could add their own examples. I explained that they would want to put something in the example box that will help them remember that term, it could be a picture, words or phrase.

In my classroom I have a knowledge wall made by the students. It contains large index cards that contain terms, definitions, formulas, what ever the students need to help them remember all they need to know. Through out this chapter they added more to the wall than what was up there since the beginning of the school year. We were starting to run out of room, so we had a contest to see who could be the most creative. I give the students various ways to remember everything. For example when we were going over converse, I asked the kids what they think of when they hear converse and they all said shoes. So I said what do we have to do when we use converse, they said switch, so when you think of converse think shoes
and shoes starts with " $s$ " which will help you remember switch. My point is one of the students drew a nice picture of a converse shoe and put that on the wall.

At this point in the chapter I felt the students were going to do a great job on the test. To finish up with review, we watch a clip from "Alice and Wonderland" and then completed a worksheet that I found on Math Bits website. The clip was from the Mad Hatters Tea Party. I was interesting listening to the logic and how the kids responded to it. They knew exactly where the logic was and how to complete the worksheet. That was the first time I had used a video clip in my teaching. It was another tool that I will use again. The students also received a review packet to complete, this packet was made similar to that of the test and it was the same packet that I used last year. I felt that it would help the data if the kids were given the same work.

The review that was done in class the day before the test included stations for the students to use. At station one, the students were asked to complete a crossword puzzle. To aid the students the terms were written on the board. Station two was a fill in the table, which consisted of two different sheets. Sheet one dealt with the connectives, the students were given an envelope that contained the terms, symbols, words, and properties for each one. They had to fill the table in correctly before they could move on to the second sheet, where they were given an envelope and again they needed to fill in the table for the different conditionals. I was amazed how fast some of the students completed this task. Station three the students played memory with
the game that they had made through out this chapter. The last station was called check your answers, this was where they would have their review packet and check their answers with my key. As the students were working on the various stations I walked around to keep them on task and clear up any misunderstandings. I had never used stations as a review before and I felt that it was successful and I feel that the students enjoyed it while they were still learning.

## Test Results

The current test scores were compared to a similar test from last year's class to determine if there was a significant difference in student achievement due to implementing the various vocabulary strategies. From 67 students from this year, in all levels of ability, the classes the mean was 84.57 , the mode was 100 , the median was 85 , the maximum score was 100 and the minimum score was 59 , the range was 42 and there was a standard deviation of 11.10 . To compare to the class from 20062007 school year, there were 78 students, in all levels of ability, the classes mean was 86.65 , the mode was 100 , the median was 89 , the maximum score was 100 and the minimum score was 52 , the range was 49 and there was a standard deviation of 11.04.

Although there was decrease of 2.08 in the average of the test from last years class to this years class. I felt it needed further investigation due to the fact that the mode and maximum were 100 and that the standard deviation had a difference of .06 . To compare the data further a t-test was completed to see if there was a significant difference between the two years the test was given. To no surprise the class from last year had a significant difference from that of this year's class of 3.86. I still felt I needed to investigate further, so I looked at the student's averages for the first 10 weeks of school. Here I found that the class from 2006-2007 school year had a course average of 85.11, while the class from 2007-2008 school year had a course average of 84.94 . The difference was only .17 , which made me realize that I was
working with similar subjects, but that the extra work on vocabulary did not make a significant difference for one unit. However, we don't know if over the course of time if there would be a significant difference. I do feel that it did help the students to see the various ways to learn material and I know that they ask for these different tasks to help make math more fun.

Questionnaire
After the students had taken the test and we went over it together I gave them a questionnaire to complete. The following bar graph represents the results from the questionnaire.

## Vocabulary Questionnaire



Question 1: An emphasis on vocabulary words helps me to understand the material being learned.

Question 2: Seeing the vocabulary words before we started to use them in class helped me to understand the material being learned.

Question 3: I have trouble knowing what to do when solving math problems due to the lack of understanding the words within the problem.

Question 4: There is an importance to learning the vocabulary words that are used
Question 5: I care about my grades in school.

When looking at the results of the questionnaire. Question 1,38 percent of the students felt the emphasis on vocabulary words somewhat helped them, while 43 percent agreed that it did help them to understand the material better. For question 2, 36 percent somewhat agreed that seeing the vocabulary words before we started to use them helped in understanding the material, while 30 percent agreed that it did indeed help them. Question 3, 30 percent felt a disagreement with this question stating that they do not have trouble knowing what to do when solving math problems due to the lack of understanding the words, while 23 percent felt that they somewhat agreed that they do have trouble knowing what to do. Question 4, 29 percent somewhat agreed to the importance of learning the vocabulary words used in math class, while 38 agreed that it is important. Question 5, asked if the students care about their grades and 65 percent strongly agreed to this question, while 23 agreed.

The results of the questionnaire gives me hope that even though the students say they dislike math and they think its not relevant to study the vocabulary words, I know that they really do know it will help them, just as it helps for them to know the words they are using when writing an essay for a different class.

## Reflection

Now that I have completed this curriculum project I feel that I have learned much about how to implement various methods to help students better understand vocabulary words, but there is so much more to learn. I know I will continue to find new ways to help my students understand the terms we use in math class.

While finishing up this project it made me realize focusing on one method of teaching isn't they way to go as we need to use all different methods to reach all of the different students that we work with. I know that my results were not where I was hoping they would be, but I feel that the students did learn more, it just didn't show through their test scores.

As with anything the more you practice the better you get, so the more I practice various ways to incorporate vocabulary into my teaching the more my students will learn.

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## Chapter Six: Appendices

## Rate the following logic terms as follows:

1. I've never heard of the word before.
2. I've heard of the term, but I don't know how it applies to mathematics.
3. I understand the meaning of this term and can apply it to a mathematics problem.

|  | Biconditional |
| :--- | :--- |
| Closed Sentence |  |
| Conclusion |  |
| Conditional |  |
| Conjunction |  |
| Contrapositive |  |
| Converse |  |
|  | Disjunction |
| Hypothesis |  |
| Inverse |  |
| Logic |  |
| Logically Equivalent |  |
|  | Negation |
| Open sentence |  |
|  | Replacement set |
|  | Solution set |
| Truth value |  |




| Name | $T$ |
| :---: | :---: |
| Choose four of the six terms and give a definition in your own words. | C |
| 1) Logic | T |
| 2) Open sentence | 0 |
| 3) Negation | T |
| 4) Closed sentence | T |
| 5) Conjunction | E |
| 6) Disjunction | D |
| If you have any questions please write them down here: | 0 |

## LOGIC VOCABULARY CROSSWORD PUZZLE •



## ACROSS

3 The phrases in the hypothesis and conclusion of the conditional change places.

6 A compound sentence formed by using the words, "if then" or "implies" or other wording so the phrases can be rearranged to be interpreted using those words.

13 A set or group of items that is available to be used to define the variable.

14 Two sentences are this if they have the same truth value when compared with each other.

15 A compound sentence formed by combining two simple sentences using the word "or".

16 Contains a variable (either a word or a symbol). The truth value cannot be determined until the variable is defined.

## DOWN

1 The study of reasoning.
2 The phrase that forms the "if" part of the conditional statement.

3 The phrase that forms the "then" part of the conditional statement.

4 The members of the replacement set that make the sentence true when they are substituted in place of the variable.

5 Whether a statement is true or false.

7 A compound sentence formed by using the words "if and only if" to combine two simple sentences.

8 A compound sentence formed by combining two simple sentences using the word "and".

9 The converse and inverse are both performed. The phrases change places and both are negated.

10 The phrases in the hypothesis and conclusion of the conditional are negated.

11 The reader determines the truth value based on the information in the sentence.

12 The opposite of the sentence given. It also refers to the use of the word "not".

NAME: $\qquad$
Period: $\qquad$

Date:
Math AB and Math 10
Logic Review

1) Truth Tables for Connectives:
-below is a truth table you need to fill in using your knowledge of connectives.
-make sure your T's look like T's and F's look like F's (not both)

| $\mathbf{p}$ | $\mathbf{q}$ | $\sim \mathbf{p}$ | $\sim \mathbf{q}$ | $\mathbf{p} \wedge \mathbf{q}$ | $\mathbf{p} \vee \mathbf{q}$ | $\mathbf{p} \rightarrow \mathbf{q}$ | $\mathbf{p} \leftrightarrow \mathbf{q}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

2) Conditionals:

- there are 3 other types of conditionals, fill in the table below with the 3 other types when given the original conditional of $\mathbf{p} \rightarrow \mathbf{q}$

| NAME: | SYMBOLIC FORM: | IN WORDS: |
| :--- | :---: | :---: |
| Original <br> Conditional | $\mathrm{p} \rightarrow \mathrm{q}$ |  |
|  |  |  |
|  |  |  |
|  |  |  |

Vocabulary:
3) Closed sentence
4) Conclusion $\qquad$
$\qquad$
5) Hypothesis $\qquad$
$\qquad$
6) Logic $\qquad$
7) Logically equivalent
8) Open sentence $\qquad$
$\qquad$
9) Replacement set $\qquad$
$\qquad$
10) Solution set
11) Truth value

For 12-15: A) Write each of the following sentences in symbolic form.
B) Determine the truth value of each sentence.

Let p represent "A square is a rectangle." (true)
Let q represent "A square is a polygon." (true)
Let r represent "A circle is a polygon." (false)
12) If a square is not a rectangle, then a square is not a polygon.
A.
B. $\qquad$
13) If a circle is a polygon or a circle is not a polygon, then a square is a rectangle.
A.
B. $\qquad$
14) A square is a polygon if and only if a circle is a polygon.
A.
B. $\qquad$
15) If it is not true that a square is a rectangle and a polygon, then a circle is a polygon.
A.
B. $\qquad$
16) Using the sentence: "If a carrot is a vegetable, then vegetables do not grow on trees."
Write, in each case: a) the inverse
b) the converse
c) the contrapositive
A) (Inverse) $\qquad$
B) (Converse) $\qquad$
C) (Contrapositive)
17) Given the true statements: " $t$ is a multiple of 3 " and " $t$ is even." What could be a value of $t$ ?
(1) 8
(3) 15
(2) 9
(4) 24
18) Which statement is logically equivalent to "If the team has a good pitcher, then the team has a good season"?
(1) If the team does not have a good season, then the team does not have a good pitcher.
(2) If the team does not have a good pitcher, then the team does not have a good season.
(3) If the team has a good season, then the team has a good pitcher.
(4) The team has a good pitcher and the team does not have a good season.
19) Which statement is logically equivalent to "If a triangle is an isosceles triangle, then it has two congruent sides"?
(1) If a triangle does not have two congruent sides, then it is an isosceles triangle.
(2) If a triangle does not have two congruent sides, then it is not an isosceles triangle.
(3) If a triangle is not an isosceles triangle, then it has two congruent sides.
(4) If a triangle is an isosceles triangle, then it does not have two congruent sides.
20) Which statement is the converse of "If it is a 300 ZX , then it is a car"?
(1) If it is not a 300 ZX , then it is not a car.
(2) If it is not a car, then it is not a 300 ZX .
(3) If it is a car, then it is a 300 ZX .
(4) If it is a car, then it is not a 300 ZX .
21) Which statement is expressed as a biconditional?
(1) Two angles are congruent if they have the same measure.
(2) If two angles are both right angles, then they are congruent.
(3) Two angles are congruent if and only if they have the same measure.
(4) If two angles are congruent, then they are both right angles.
22) What is the inverse of the statement "If Julie works hard, then she succeeds"?
(1) If Julie succeeds, then she works hard.
(2) If Julie does not succeed, then she does not work hard.
(3) If Julie works hard, then she does not succeed.
(4) If Julie does not work hard, then she does not succeed.

NAME: $\qquad$ Date: $\qquad$
Period: Math $\mathrm{AB} / 10$

For 1-5: Below are boxes to put in your connectives names, symbol, meaning, and properties. Use the sample model for reference on what to do. There are 5 connectives. (4 points each)

|  | Symbol |
| :---: | :---: |
| Connective |  |
| Name | Properties <br> (When is it <br> true/false) |
| Connective <br> "Meaning" | Seal |



| 4) |  |
| :--- | :--- |
|  |  |
|  |  |
|  |  |


6) Truth Table:

- Below is a truth table you need to fill in using your knowledge of connectives.
- Make sure your T's look like T's and F's look like F's (8 points)

| $\mathbf{p}$ | $\mathbf{q}$ | $\sim \mathbf{p}$ | $\sim \mathbf{q}$ | $\mathbf{p} \wedge \mathbf{q}$ | $\mathbf{p} \vee \mathbf{q}$ | $\mathbf{p} \rightarrow \mathbf{q}$ | $\mathbf{p} \leftrightarrow \mathbf{q}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

For 7-9: Other forms of conditionals, there are 3 other forms of a conditional $p \rightarrow q \quad$ (you are to fill in the table below) (3 points each)

| NAME: | SYMBOLIC FORM: | IN WORDS: |
| :--- | :---: | :---: |
| Original Conditional | $\mathrm{p} \rightarrow \mathrm{q}$ |  |
| 7$)$ |  |  |
| 8$)$ |  |  |
| 9$)$ |  |  |

For 10-13 fill in the blank to complete the statement. ( 2 points each)
10) Logically equivalent statements have the same $\qquad$ value.
11) The original conditional and its $\qquad$ are logically equivalent.
12) The converse and the $\qquad$ are logically equivalent.
13) $\qquad$ is the study of reasoning.

For 14-18: A) Write each of the following sentences in symbolic form. [1pt] B) Determine the truth value of each sentence. [1pt]

Let p represent "Peter is a rabbit."
(true)
Let q represent "Peter is an animal."
Let $r$ represent "Peter lives in a pond." (false)
14) Peter does not live in a pond.
A. $\qquad$
B. $\qquad$
15) Peter is not a rabbit and Peter is an animal.
A.
B. $\qquad$
16) Peter is not an animal or Peter lives in a pond.
A.
B. $\qquad$
17) If Peter is not a rabbit, then Peter is not an animal.
A.
B. $\qquad$
18) Peter is not a rabbit if Peter is an animal.
A.
B. $\qquad$
19) Using the sentence: "If today is Wednesday, then tomorrow is Thursday."

Write, in each case:
a) the inverse [2pts]
b) the converse [2pts]
c) the contrapositive [2pts]
A) (Inverse)
B) (Converse)
C) (Contrapositive) $\qquad$

## Regents Logic Questions:

## Part I: Multiple Choice (2 points each)

20) What is the contrapositive of the statement "If I study, then I pass the test"?
(1) I pass the test if I study.
(2) If I do not study, then I do not pass the test.
(3) If I do not pass the test, then I do not study.
(4) If I pass the test, then I study.
21) Which statement is logically equivalent to "If it is Saturday, then I am not in school"?
(1) If I am not in school, then it is Saturday.
(2) If it is not Saturday, then I am in school.
(3) If I am in school, then it is not Saturday.
(4) If it is Saturday, then I am in school.
22) Given the statement: "A right angle measures $90^{\circ}$." How is this statement written as a biconditional?
(1) If an angle is a right angle, then it measures $90^{\circ}$.
(2) An angle is a right angle if, and only if, it measures $90^{\circ}$.
(3) An angle measures $90^{\circ}$ and it is a right angle.
(4) If an angle does not measure $90^{\circ}$, then it is not a right angle.
23) What is the converse of the statement "If the Sun rises in the east, then it sets in the west"?
(1) If the Sun does not set in the west, then it does not rise in the east.
(2) If the Sun does not rise in the east, then it does not set in the west.
(3) If the Sun sets in the west, then it rises in the east.
(4) If the Sun rises in the west, then it sets in the east.
24) What is the inverse of the statement "If Bob gets hurt, then the team loses the game"?
(1) If the team loses the game, then Bob gets hurt.
(2) Bob gets hurt if the team loses the game.
(3) If the team does not lose the game, then Bob does not get hurt.
(4) If Bob does not get hurt, then the team does not lose the game.

## Part IV: (4 points)

25) Given the statement: "If I live in Albany, then I am a New Yorker."

In the spaces provided below, write the inverse, the converse, and the contrapositive of this statement.

Inverse: $\qquad$
Converse: $\qquad$
Contrapositive: $\qquad$
Which conditional is logically equivalent to its original statement?
inverse converse contrapositive

## Bonus:

How many weeks, days, hours, minutes and seconds are in $1,000,000$ seconds?

## Worksheet

Logic with Lewis Carroll
Name $\qquad$

Lewis Carroll, the author of Alice's Adventures in Wonderland and Through the Looking Glass, was actually a mathematician. As a hobby, Carroll wrote stories which contained amusing examples of logic and which reflected his passion for mathematics.

Consider this quote from a conversation which occurred during the Mad Hatter's Tea Party in Alice in Wonderland.
"Then you should say what you mean." the March Hare went on.
"I do," Alice hastily replied; "at least -- at least I mean what I say -- that's the same thing, you know."
"Not the same thing a bit!" said the Hatter, "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"
"You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"
"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"
"It is the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute.

Examine the last statements made by the Dormouse:
(1) I breathe when I sleep.
(2) I sleep when I breathe.

1. Write each of the statements in "If .... then" form.
$\square$
2. Write the converse of each statement.
$\square$
3. Write the inverse of each statement.
$\square$
4. Write the contrapositive of each statement.

|  |  |
| :--- | :--- |

5. Examine the truth value of each of your statements (True or False)

|  | I breathe when I sleep. | I sleep when I breathe. |
| :---: | :---: | :---: |
| Original <br> Statement: |  |  |
| Inverse: |  |  |
| Converse: |  |  |
| Contrapositive: |  |  |

6. What relationships do you notice between the truth values and the different statements?

## Let's examine what the calculator tells us about True or False statements

7. Examine the truth value of each of the following statements (True or False)

|  | $5>3$ | $-1<-2$ |
| :---: | :---: | :---: |
| Original |  |  |
| Statement: |  |  |

8. Enter $5>3$ into your graphing calculator and hit ENTER. The " $>$ " symbol is under the TEST menu. What does the calculator tell you?
9. Enter $-1<-2$ into your graphing calculator and hit ENTER. What does the calculator tell you?
10. Draw a conclusion about how the calculator deals with TRUE and FALSE statements.

## Vocabulary Questionnaire

1) An emphasis on vocabulary words helps me to understand the material being learned.

| Strongly <br> Disagree | Disagree | Somewhat <br> Disagree | Somewhat <br> Agree | Agree | Strongly <br> Agree |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

2) Seeing the vocabulary words before we started to use them in class helped me to understand the material being learned.

| Strongly <br> Disagree | Disagree | Somewhat <br> Disagree | Somewhat <br> Agree | Agree | Strongly <br> Agree |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

3) I have trouble knowing what to do when solving math problems due to the lack of understanding the words within the problem.

| Strongly <br> Disagree | Disagree | Somewhat <br> Disagree | Somewhat <br> Agree | Agree | Strongly <br> Agree |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

4) There is an importance to learning the vocabulary words that are used in math class.

| Strongly <br> Disagree | Disagree | Somewhat <br> Disagree | Somewhat <br> Agree | Agree | Strongly <br> Agree |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

5) I care about my grades in school.

| Strongly <br> Disagree | Disagree | Somewhat <br> Disagree | Somewhat <br> Agree | Agree | Strongly <br> Agree |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 |

