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# Reconstruction of Planar Cam Profile Function and its Follower Displacement using B-Spline Curve based on Inverse Subdivision Method and Theory of Contact Relations - Application to Cam Mechanism of Oscillating Follower with Roller

***Le C.***
*Faculty of Technical Education and Engineering Sciences, Danang University of Technology, Danang University, Vietnam.*

***Nguyen Tan K.***
*Faculty of Informatics Technology, Danang University of Technology, Danang University, Vietnam.*

***Abstract***
*For controlling the precision of a manufacturing planar cam mechanism, one can base on a set of digital points measured on the manufactured cam profile by a coordinate measuring machine (CMM). Issued from this set of discrete digital points and by applying the well-known B-Spline least squares approach, one can reproduce the cam profile function. The given cam profile function and the theory of contact relations between cam and its follower enable us to deduce the displacement law of the follower and its derivatives. This paper presents a new method for reconstruction of a planar cam profile function from a set of digital points measuring on CMM machines, using B-Spline curve based on inverse subdivision method and theory of contact relations. An application to cam mechanism of oscillating follower with roller will be presented. To verify the feasibility and the accuracy of the proposed inverse subdivision method, a comparison between B-Spline least squares approach and inverse subdivision has been realized. The method possesses an acceptable accuracy and spends less time of calculation.*

***Keywords :*** *B-Spline curve, inverse subdivision, reconstruction, planar cam profile*

## 1 Introduction

Cam mechanisms are still being used in mechanical equipments, owing to their small dimensions, high reliability, and capability of working at high temperatures. Along with the apparition of the coordinate measuring machine, dimension and shape controlling of cam mechanisms are more convenient. From a set of digital measuring points of a planar cam profile, Tsay D. M., Tseng K. S., Chen H. P [1] have proposed a method to describe planar cam profile smooth function using B-Spline, and they have also proposed a procedure to reproduce motion law of cam follower mechanisms based on the theory of contact relations for flat translating follower and translating one with roller. The procedure contributes to accuracy controlling of manufacturing cam profile. For describing cam profile function, they have used the method of least mean square using the B-Spline in polar coordinate system. The above method has a disadvantage that when the number of vertex of B-Spline curve is important, the calculation time is too long, and that when using an inadequate number of order k of B-Spline curve, the obtained cam profile might be distorted and undesired. The aim of this paper is to apply the theory of contact relations to deduce angular displacement of oscillating follower with roller in cam mechanism and to represent a new method for describing the profile function of a planar cam from a set of digital measuring points, using the inverse subdivision [6].

## 2 Method for reproduction of motion law of cam follower from a digital cam profile

### 2.1 *Theory of contact relations between cam and follower*

Analytical method for contruction of motion, velocity, and acceleration laws of the follower with a given cam profile in planar cam mechanism using the contact cam-follower relations has been represented in reference [1]. Using the contact relations, we can deduce the displacement law of the follower and its derivatives. Let $S_c(x_c, y_c)$ and $S_f(x_f, y_f)$ be the moving coordinate system attached respectivement to the cam and to the follower, and $S_0(x_0, y_0)$ be the fixed coordinate system, $\alpha_c$ and $\mathbf{P_c}(\alpha_c)$ respectivlely denote the angular position of the cam and the cam profile at the contact point I between cam and follower in the polar coordinate system $(O_c, \mathbf{O_c x_c})$, $\alpha_f$ and $\mathbf{P_f}(\alpha_f)$ respectivlely denote the angular position of the cam and the cam profile at the contact point I in the polar coordinate system $(O_f, \mathbf{O_f x_f})$ (Figure 1).

In the fixed coordinate system $S_0$, the cam profiles $\mathbf{P_c}(\alpha_c)$ and $\mathbf{P_f}(\alpha_f)$ become $\mathbf{P_0^c}(\theta_c, \alpha_c)$ and $\mathbf{P_0^f}(\psi_f, \alpha_c)$, where $\theta_c$ and $\psi_f$ seperately are the angular cam position and the follower displacement. The vectors $\mathbf{P_0^f}(\psi_f, \alpha_c)$ and $\mathbf{P_0^f}(\psi_f, \alpha_c)$ describe the same position of the contact point I in the fixed coordinate system $S_0$, thus we have:

$$\mathbf{P_0^c}(\theta_c, \alpha_c) - \mathbf{P_0^f}(\psi_f, \alpha_c) = 0 \tag{1}$$

In a similar manner, the tangent vectors $\mathbf{T_c}$ and $\mathbf{T_f}$ on the cam and its follower profile can also be transformed to the fixed coordinate system, and they are denoted respectively by $\mathbf{T_0^c}(\theta_c, \alpha_c)$ and $\mathbf{T_0^f}(\psi_f, \alpha_f)$. When cam and its follower are in contact with each other, the relative tangent vectors at the contact point are collinear, we have [1]:

$$\mathbf{T_0^c}(\theta_c, \alpha_c) \times \mathbf{T_0^f}(\psi_f, \alpha_f) = 0 \tag{2}$$

The equations (1) and (2) describe the contact relations between the cam and its follower.

### 2.2 *Contact relations for oscillating cam-follower mechanism with roller*

Based on the theory of contact relations, we can construct the contact relations for the cam follower mechanism. As for the cam mechanism using oscillating follower with roller in Figure 1, we have:

$$\mathbf{T_c} = \begin{cases} x'(\alpha_c) \\ y'(\alpha_c) \end{cases} \tag{3}$$

$$\mathbf{P_f} = \begin{cases} r_f cos\alpha_f \\ r_f sin\alpha_f \end{cases} \tag{4}$$

$$\mathbf{T_f} = \begin{cases} -sin\alpha_f \\ cos\alpha_f \end{cases} \tag{5}$$

Where $\theta_c$ is the angular position of the cam, $\psi_f$ is the angular position of the oscillating follower, $l$, $r_f$ and $a$ denote respectively the follower length, roller radius and distance between the rotational centers of the cam and the follower.

When transforming $\mathbf{P_c}$, $\mathbf{P_f}$, $\mathbf{T_c}$, and $\mathbf{T_f}$ into the fixed coordinate system $S_0$, we obtain the following relations:

$$\mathbf{P_0^c} = \begin{bmatrix} cos\theta_c.x(\alpha_c) - sin\theta_c.y(\alpha_c) \\ sin\theta_c.x(\alpha_c) + cos\theta_c.y(\alpha_c) \end{bmatrix} \tag{6}$$

$$\mathbf{P_0^f} = \begin{bmatrix} -cos\psi_f.r_f cos\alpha_f - sin\psi_f.r_f sin\alpha_f + a - lcos\psi_f \\ sin\psi_f.r_f cos\alpha_f + cos\psi_f.r_f sin\alpha_f + lsin\psi_f \end{bmatrix} \tag{7}$$

$$\mathbf{T_0^c} = \begin{bmatrix} x'(\alpha_c)cos\theta_c - sin\theta_c y'(\alpha_c) \\ x'(\alpha_c)sin\theta_c + cos\theta_c y'(\alpha_c) \end{bmatrix} \tag{8}$$

$$\mathbf{T_0^f} = \begin{bmatrix} -sin\alpha_f \\ cos\alpha_f \end{bmatrix} \tag{9}$$

The two equations (1) and (2) which describe the contact relations become the following system of three equations:

$$\cos\theta_c.x(\alpha_c)-\sin\theta_c.y(\alpha_c)+\cos\psi_f.r_f\cos\alpha_f+\sin\psi_f.r_f\sin\alpha_f-a+l\cos\psi_f=0 \tag{10}$$

$$\sin\theta_c.x(\alpha_c)+\cos\theta_c.y(\alpha_c)-\sin\psi_f.r_f\cos\alpha_f-\cos\psi_f.r_f\sin\alpha_f-l\sin\psi_f=0 \tag{11}$$

$$\left[x'(\alpha_c).\cos\theta_c-y'(\alpha_c)\sin\theta_c\right]\cos\alpha_f+\left[x'(\alpha_c).\sin\theta_c+y'(\alpha_c)\cos\theta_c\right]\sin\alpha_f=0 \tag{12}$$

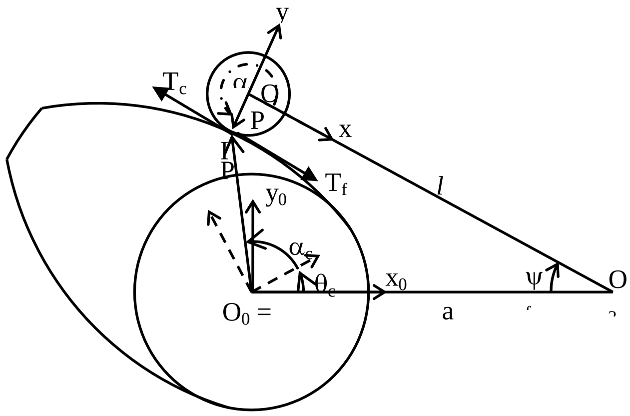

**Figure 1:** Cam with roller oscillating follower

With a given value of $\theta_c$, by solving the system of equations (10), (11), (12), we can derive the angular displacement $\psi_f=\psi_f(\theta_c)$ of the oscillating follower, function of the cam angular position $\theta_c$. Moreover, by taking total derivation of the equations (10), (11), (12) with respect to $\theta_c$, it is possible to derive the formulations for the angular velocity $\frac{d\psi_f}{d\theta_c}(\theta_c)$ and the angular acceleration $\frac{d^2\psi_f}{d\theta_c^2}(\theta_c)$ of the oscillating follower [1].

One of the problems we have to solve here is the critirions used to make the angular setting between the experimental profile and the theoretical one. For this purpose, we have to determine the minimum value of the angular displacement $\psi_f$ when designing the cam profile, by using the position vector of the initial point of contact between cam and its follower (where $\psi_f$ attains its minimum value), and determine the same angle $\psi_f$ for the experimental profile when reconstructing the angular displacement for the follower.

## 3 Method for reproduction of cam profile curve from a set of discrete digital measuring points

To reproduce the laws of angular displacement, velocity and acceleration from a manufacturing cam by using the equations (10), (11), (12), and the procedure proposed in reference [1], we have to know the functions $x(\alpha_c)$, $y(\alpha_c)$, which describe the coordinates of an arbitrary point on the cam profile with respect to a mobile orthogonal coordinating system $S_c$, attached to the cam. In other words, we have to know the equation of the cam profile. After manufacturing the planar cam on a CNC milling machine and measuring its profile by a coordinate measuring machine CMM, we obtain a set of discrete digital points which composes of m points of coordinates $(x_j, y_j)$, where $j = 1, 2, ..., m$.

We will present in this section a new B-Spline curve reconstruction method based on the inverse subdivision surface scheme (NUISS). The convenience of our reconstruction method is without solving linear systems as the traditional reconstruction one [5]. We propose a nonuniform inverse subdivision scheme (NUISS) inspired on the nonuniform bicubic subdivision [10] for polygonal curve. A curve made up from straight line segments is called a polygonal curve. The resulting reconstruction curve will be a B-Spline curve approximation.

### 3.1 *The B-Spline curve*

In the following we shall assume a B-Spline curve *C(u)* of degree *p* which is defined by *n* control points $P = \{P_0, P_1, \ldots, P_{n-1} \mid P_i \in \Re^k\}$. Let a vector known as the knot vector be defined by $\mathbf{U} = \{u_0, u_1, \ldots, u_{n+p}\}$, where **U** is a nondecreasing sequence that satisfies $u_i \in [0,1]$ and $u_i \le u_{i+1}$.

The B-Spline basis functions are defined as [5]:

$$N_{i,0}(t) = \begin{cases} 1 & t_i \le t \le t_{i-1} \\ 0 & otherwise \end{cases}$$

$$N_{i,p}(t) = \frac{t - t_i}{t_{i+p} - t_i} N_{i,p-1}(t) + \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} N_{i+1,p-1}(t)$$

Then the B-Spline curve is defined by:

$$C(t) = \sum_{i=0}^{n} P_i . N_{i,p}(t)$$

### 3.2 *Knot intervals*

Let *C(u)* be a B-Spline curve of degree p with *n* control points. The knot intervals issued from the knot vectors **U** are as follows [9,10]:

$$e_i = u_{i+3} - u_{i+2} \quad \text{for } i = -1, \ldots, n-1$$

Each edge of the control polygon is labeled with respect to a corresponding knot interval. Each edge $\overline{P_{i,j} P_{i+1,j}}$ of the control polyhedron is assigned to a knot interval $e_i : \overline{P_i P_{i+1}} \leftarrow e_i$ with *i = 0,..., n-2*. The knot intervals $e_{-1}, e_{n-1}$ are assigned to 'phantom edges' of control polygon. 'Phantom' edge is an edge immaterial which glued to the end control points using in the non-uniform subdivision scheme [9].

### 3.3 *Inverse subdivision cubic nonuniform*

In this section, we present our nonuniform inverse subdivision scheme (NUISS) based on the nonuniform subdivision scheme [10]. As the subdivision scheme is invertible [4, 8], one can restore recursively all the previous coarser polygon by using the inverse subdivision scheme. Consequently, it also permits to reconstruct the control polygon of a limit parametric curve issued from a nonuniform subdivision curve. After each step of the inverse subdivision, we retrieve curve elements of the previous subdivision curve. The knot intervals are already determined by retaking the information of input polygonal curve. We propose the formulas to compute the inverse subdivision points based on the formulas of non-uniform cubic B-Spline curve subdivision in [10]. The inverse vertex point $P_i^k$ of polygonal curve $P^k$ (Figure 2) is computed based on the points of polygonal curve $P^{k+1}$ by the following formulas [6, 7]:

$$P_i^k = -\frac{1}{2} P_{2i-2}^{k+1} + 2P_{2i-1}^{k+1} - \frac{1}{2} P_{2i+1}^{k+1} \quad \text{for } i = 1...n-2 \tag{13}$$

$$P_i^k = 2P_{2i}^{k+1} - P_{i+1}^k \quad \text{for } i = 0 \tag{14}$$

$$P_i^k = 2P_{2i-2}^{k+1} - P_{i-1}^k \quad \text{for } i = n-1 \tag{15}$$

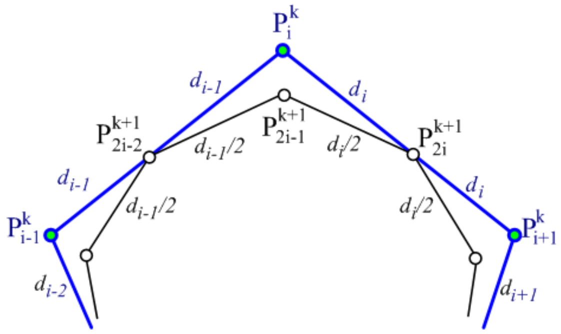


**Figure 2:** The relation of inverse points $P^k$ with the points $P^{k+1}$ [10]

### 3.4 *Construction of the knot interval vectors*

For our reconstruction NUISS method, we need to use the knot interval vectors. As these vectors are computed from knot vectors, we propose to define the knot vector from the input initial polygonal curve. Different parametric methods have been proposed. We only consider the most widely-used parametric chord length method [5].

Suppose that we are given a set of points $\{P_i\}$ *(i = 0,..., n-1)*, we will find the parametric values $\{u_i \mid 0 \le u_i \le 1, i = 0, \ldots, n-1\}$ associated with these points $\{P_i\}$.

$$\bar{u}_0 = 0, \quad \bar{u}_{n-1} = 1$$

$$\bar{u}_i = \bar{u}_{i-1} + \frac{|P_i - P_{i-1}|}{\sum_{j=1}^{n-1} |P_i - P_{i-1}|} \quad \forall i \in [1,\ldots,n-2]$$

with $|P_i - P_{i-1}|$ is the length between two consecutive points.

Our goal is to create knot interval vectors for an initial polygonal curve to implement the NUISS scheme, and also to reconstruct the B-Spline curve. The knot interval vectors for the initial polygon curve *P* can be created by using the parametric chord length method. From the initial polygonal curve *P* with initial knot interval vectors, we can recreate the coarse polygonal curve *Q* by our NUISS scheme. This coarse curve *Q* using as a control polygon of the cubic B-Spline curve *C(u)* the knot vectors of which are created from the corresponding knot interval vectors of the polygonal curve. It creates an approximate B-Spline surface reconstruction.

### 3.5 *Proposed NUISS reconstruction method*

Our curve reconstruction NUISS method of a B-Spline curve consists of the following steps:

An initial polygonal curve P=$\{P_i\}$ *(i=0,...,n-1)* is given as input.

1. Construction of the knot interval vectors for the inverse subdivision scheme. It is based on the information from the initial polygonal curve P.
2. Choice the number of inverse step *k*.
3. Reconstruction of a B-Spline curve by NUISS scheme using *k* inverse subdivision step.

The reconstructing result from an initial curve is a coarser curve (with less points) used as control polygon of a cubic B-Spline curve.

## 4 Experimental results

To verify the above method for curve reconstruction, we design a cam mechanism which realises a given motion law by using the Mechanism Design application in Pro/Engineer software, then we manufacture the cam profile on the BAZ-15 Heidenhain CNC milling machine, and finally measure the planar cam profile with the Mitutoyo Beyond Crysta 554 CMM machine at the laboratory CRePA of the Danang PFIEV. We obtain a set of 704 digital discrete points of the planar cam profile.


**Figure 3:** Measuring cam profile on the Mitutoyo Beyond Crysta 554 CMM machine

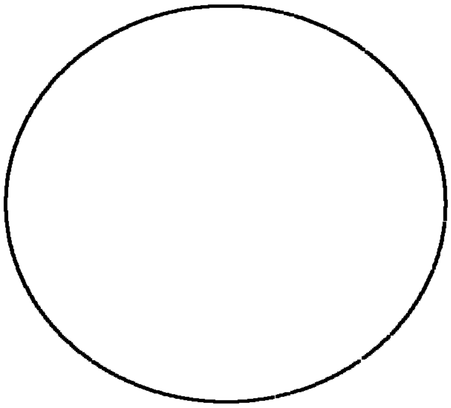

**Figure 4:** The cam profile digital points with 704 points

We use the NUISS method to construct the B-Spline curve of degree 3 from the above set of 704 digital points (Figure 4). Figure 5 illustrates the reconstructed B-Spline curve with 178 control points after *k = 2* inverse subdivision steps. Figure 6 shows the constructing results with a B-Spline degree 3, composed of *n = 90* control points after *k = 3* steps of inverse subdivision. The B-Spline curve of degree 3 in Figure 7 composes of *n = 46* control points after *k = 4* steps of inverse subdivision, the B-Spline curve of degree 3 in Figure 8 composes of *n = 24* control points after *k = 5* steps of inverse subdivision.

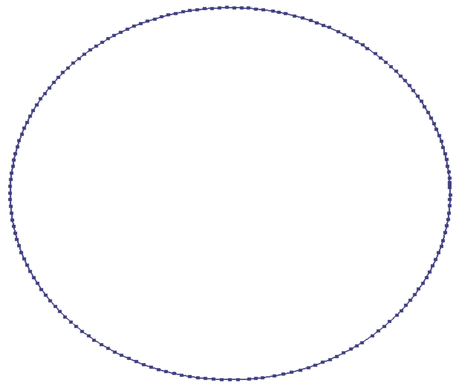

**Figure 5:** The reconstructed B-Spline curve with 178 control points after *k = 2* inverse subdivision steps

We evaluate the method by comparison of the NUISS method for construction the B-Spline curve with the traditional least squares method [5], which is actually well-known. The input is a set of 704 measured digital points (Figure 4).

Denote *Num*: number of control points of the curve, *Time*: computer calculation time, *Dmax*: maximum value of the distance from the initial point to the constructed B-Spline curve, *Davg*: mean value of these distances.

Table 1 shows us the results of the construction of the B-Spline curves degree 3 after *k=1, 2, 3, 4, 5* steps of inverse subdivision NUISS. Table 2 illustrates the results of the construction of the B-Spline curves degree 3 by using the least-squares method, the curves have the same control points as the corresponding ones constructed by our proposed method, figuring in the Table 1. The calculating times indicated in Table 1, 2 have been measured on a desktop 1.60 GHz dual core Intel with 1GB RAM. The influence of the number Num of the control points of the curve (or the step of subdivision k) over Dmax and Davg is shown in the figure 9.

**Table 1:** The evaluation of reconstruction results by our NUISS method

| k | Num | Time | Dmax | Davg |
|---|---|---|---|---|
| 5 | 24 | 31ms | 0.56912 | 0.04334 |
| 4 | 46 | 25ms | 0.12121 | 0.00619 |
| 3 | 90 | 15ms | 0.02875 | 0.00191 |
| 2 | 178 | 8ms | 0.00412 | 0.00085 |
| 1 | 354 | 5ms | 0.00205 | 0.00059 |

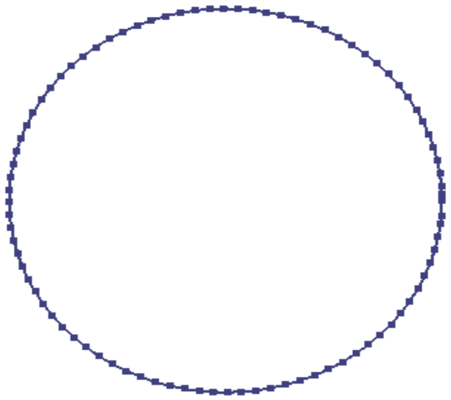

**Figure 6:** The reconstructed B-Spline curve with *n = 90*, after *k = 3* NUISS steps

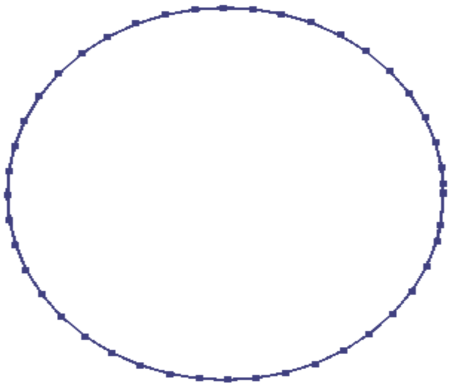

**Figure 7:** The reconstructed B-Spline curve with *n = 46*, after *k = 4* NUISS steps

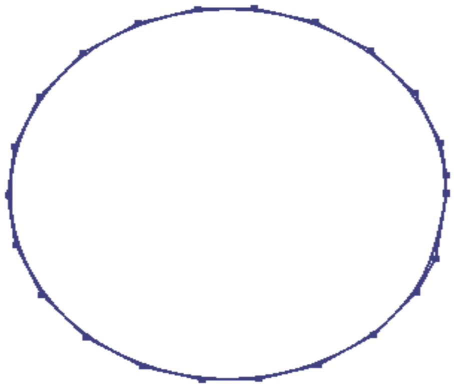

**Figure 8:** The reconstructed B-Spline curve with *n = 24*, after *k = 5* NUISS steps

**Table 2:** The evaluation of reconstruction results by least-squares method.

| Num | Time | Dmax | Davg |
|---|---|---|---|
| 24 | 47ms | 0.01800 | 0.00217 |
| 46 | 47ms | 0.00164 | 0.00029 |
| 90 | 234ms | 0.00068 | 0.00022 |
| 178 | 1422ms | 0.00064 | 0.00020 |
| 354 | 10547ms | 0.00056 | 0.00016 |

Regarding the numerical experimentations, we realize that, computational time for constructing the B-Spline curve by NUISS method is inversely proportional to the number of control points of obtained curve. However, computational time for constructing the B-Spline curve by the least-squares method is proportional to the number of control points of the curve. In other words, with the NUISS method, more important the number of control points is, shorter the computational time for curve construction, but more important with the least-squares method.

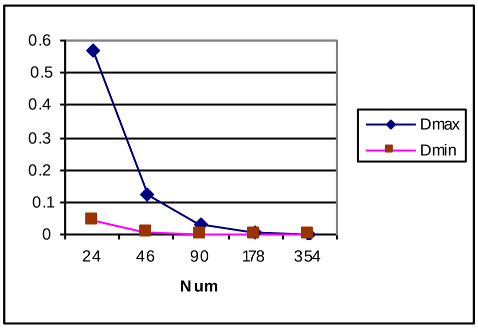


**Figure 9:** Influence of the number Num of control points of the curve over Dmax and Davg

With the NUISS method, when the number of control points is more important, the maximum value *Dmax* and the average value *Davg* are smaller (Figure 9). With small values of k (k = 1, 2 3), *Dmax* and *Davg* in Table 1 are inferior to 0.03mm. According to ISO Standard, with the cam dimension between 120-180mm, for the precision IT5, we have an IT equal to 0.018mm, for the precision IT6: IT = 0.025mm, for the precision IT7: IT = 0.040mm. In our case, the maximum cam dimension is about 150mm, with the tolerance about 0.03mm for k = 3, we obtain the precision of IT7, and with k = 2, we obtain at least the precision of IT5. So that the Dmax and Dmin in the Table 1 are acceptable for the accuracy of cam profile. We may trace a horizontal line, standing for the IT on the graph of figure 9 as a limit to choose the adequate number of control points for our need of accuracy. The NUISS method enables the B-Spline curve reconstruction using in the practical works. An important note is that the NUISS method enables the construction of B-Spline approximation curve of the original points, avoiding certain extraordinary points found out by the least-squares method because of the solution of parametric equations. Besides, the two extremities of the B-Spline curve created by the NUISS method always pass through the two extremities of the original set of points (based on the formulas of the inverse subdivision method).

## 5 Conclusions

Based on the theory of contact relations, we derive the contact equations for the case of oscillating follower cam mechanism with roller. Using the method and algorithms describing in [1], one can easily derive the displacement, velocity and acceleration laws of the oscillating follower. For the reconstruction of cam profile function, we propose in the article a new method, method of inverse subdivision, which enables us to reproduce planar cam profile from a set of digital points, measured on the cam profile by CMM machine. The method possesses an acceptable accuracy and spends less time of calculation.

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