Interval-valued Intuitionistic Fuzzy ELECTRE Method

Wu M.

Graduate Institute of Business and Management College of Management, Chang Gung University, Taoyuan 333, Taiwan
Email address: richwu50@gmail.com

Chen T.

Department of Industrial and Business Management College of Management, Chang Gung University, Taoyuan 333, Taiwan
Email address: tychen@mail.cgu.edu.tw

Abstract

In this study, the proposed method replaced the evaluation data from crispy value to vague value, i.e. interval-valued intuitionistic fuzzy (IVIF) data, and to develop the IVIF Elimination and Choice Translating Reality (ELECTRE) method for solving the multiple criteria decision making problems. The analyst can use IVIF sets characteristics to classify different kinds of concordance (discordance) sets using score and accuracy function, membership uncertainty degree, hesitation uncertainty index and then applied the proposed method to select the better alternatives.

Keywords*: interval-valued intuitionistic fuzzy; ELECTRE; multiple criteria decision making; score function; accuracy function*

1 Introduction

The Elimination and Choice Translating Reality (ELECTRE) method is one of the outranking relation methods and it was first introduced by Roy [3]. The threshold values in the classical ELECTRE method are playing an importance role to filtering alternatives, and different threshold values produce different filtering results. As we known that the evaluation data in classical ELECTRE method are almost exact values that can affect the threshold values. Moreover, in real world cases, exact values could be difficult to be precisely determined since analysts' judgments are often vague; for these reasons, we can find some studies [4,5,8] developed the ELECTRE method with type 2 fuzzy data. Vahdani and Hadipour [4] presented a fuzzy ELECTRE method using the concept of the interval-valued fuzzy set (IVFS) with unequal criteria weights, and the criteria values are considered as triangular interval-valued fuzzy number, and also using triangular interval-valued fuzzy number to distinguish the concordance and discordance sets, and then to solve multi-criteria decision-making (MCDM) problems. Vahdani et al. [5] proposed an ELECTRE method using the concepts of interval weights and data to distinguish the concordance and discordance sets, and then to evaluate a set of alternatives and applied it to the problem of supplier selection. Wu and Chen [8] proposed an intuitionistic fuzzy (IF) ELECTRE method that using the concept of score and accuracy function, i.e. calculated the different combinations of membership, non-membership functions and hesitancy degree, to distinguish different kinds of concordance and discordance sets, and then using the result to rank all alternatives, for solving MCDM problems.

The intuitionistic fuzzy set (IFS) was first introduced by Atanassov [1], and the IFS generalize the fuzzy set, which was introduced by Zadeh [11]. The interval-valued intuitionistic fuzzy set (IVIFS), that is combined IFS concept with interval valued fuzzy set concept, introduced by Atanassov and Gargov [2], each of which is characterized by membership function and non-membership function whose values are interval rather than exact numbers, are a very useful means to describe the decision information in the process of decision making.

As the literature review shows, few studies have applied the ELECTRE method with IVIFS to real life cases. The main purpose of this paper is to further extend the ELECTRE method to develop a new method to solve MCDM problems in interval-valued intuitionistic fuzzy (IVIF) environments. The major difference between the current study and other available papers is the proposed method, whose logic is simple but which is suitable for the vague of real life situations. The proposed method that also using the score and accuracy function, and added 2 more factors, membership and hesitation uncertainty index, i.e. applied the factors of membership, non-membership functions and hesitancy degree, to distinguish different kinds of concordance and discordance sets, and then to select the best alternatives finally. The remainder of this paper is organized as follows. Section 2 introduces the decision environment with IVIF data, the score, accuracy functions and some indices, and the construction of the IVIF decision matrix. Section 3 introduces the IVIF ELECTRE methods and its algorithm. Section 4 illustrates the proposed method with a numerical example. Section 5 presents the discussion.

2 Decision Environment with IVIF Data

A. Interval-valued intuitionistic fuzzy sets

Based on the definition of IVIFS in Atanassov and Gargov study [2], we have:

Definition 1: Let X be a non-empty set of the universe, and $D[0,1]$ be the set of all closed subintervals of all closed subintervals of $[0,1]$. An IVIFS $\tilde{A}$ in X is an expression defined by

$$\tilde{A}=\left\{\langle x,\tilde{M}_{\tilde{A}}(x),\tilde{N}_{\tilde{A}}(x)\rangle \mid x\in X\right\}$$
$$=\left\{\langle x,[M^L{}_{\tilde{A}}(x),M^U{}_{\tilde{A}}(x)],[N^L{}_{\tilde{A}}(x),N^U{}_{\tilde{A}}(x)]\rangle \mid x\in X\right\}, \quad (1)$$

where $\tilde{M}_{\tilde{A}}(x):X\to D[0,1]$ and $\tilde{N}_{\tilde{A}}(x):X\to D[0,1]$ denote the membership degree and the non-membership degree for any $x\in X$, respectively. $\tilde{M}_{\tilde{A}}(x)$ and $\tilde{N}_{\tilde{A}}(x)$ are closed intervals rather than real numbers and their lower and upper boundaries are denoted by $M^L{}_{\tilde{A}}(x)$, $M^U{}_{\tilde{A}}(x)$, $N^L{}_{\tilde{A}}(x)$ and $N^U{}_{\tilde{A}}(x)$, respectively, and $0\le M^U{}_{\tilde{A}}(x)+N^U{}_{\tilde{A}}(x)\le 1$.

Definition 2: [2] For each element x, the hesitancy degree of an intuitionistic fuzzy interval of $x\in X$ in $\tilde{A}$ defined as follows:

$$\tilde{\pi}_{\tilde{A}}(x)=1-\tilde{M}_{\tilde{A}}(x)-\tilde{N}_{\tilde{A}}(x)$$
$$=[1-M^U{}_{\tilde{A}}(x)-N^U{}_{\tilde{A}}(x),1-M^L{}_{\tilde{A}}(x)-N^L{}_{\tilde{A}}(x)]$$
$$=[\pi^L{}_{\tilde{A}}(x),\pi^U{}_{\tilde{A}}(x)]. \quad (2)$$

Definition 3: The operations of IVIFS [2,9] are defined as follows: for two of $A,B\in \text{IVIFS}(X)$,

(a) $A\subset B$ iff
$M^L{}_{\tilde{A}}(x)\le M^L{}_{\tilde{B}}(x)$, $M^U{}_{\tilde{A}}(x)\le M^U{}_{\tilde{B}}(x)$ and
$N^L{}_{\tilde{A}}(x)\ge N^L{}_{\tilde{B}}(x)$, $N^U{}_{\tilde{A}}(x)\ge N^U{}_{\tilde{B}}(x)$;

(b) $A=B$ iff $A\subset B$ and $B\subset A$;

(c)
$$d_1(A,B)=\frac{1}{4}\sum_{j=1}^{n}[|M^L{}_{\tilde{A}}(x_j)-M^L{}_{\tilde{B}}(x_j)|+|M^U{}_{\tilde{A}}(x_j)-M^U{}_{\tilde{B}}(x_j)|+|N^L{}_{\tilde{A}}(x_j)-N^L{}_{\tilde{B}}(x_j)|+|N^U{}_{\tilde{A}}(x_j)-N^U{}_{\tilde{B}}(x_j)|];$$

(d)
$$d_2(A,B)=\frac{1}{4n}\sum_{j=1}^{n}[|M^L{}_{\tilde{A}}(x_j)-M^L{}_{\tilde{B}}(x_j)|+|M^U{}_{\tilde{A}}(x_j)-M^U{}_{\tilde{B}}(x_j)|+|N^L{}_{\tilde{A}}(x_j)-N^L{}_{\tilde{B}}(x_j)|+|N^U{}_{\tilde{A}}(x_j)-N^U{}_{\tilde{B}}(x_j)|];$$

(e)

$$d_3(A,B)=\frac{1}{4}\sum_{j=1}^{n} w_j[|M^L{}_{\tilde{A}}(x_j)-M^L{}_{\tilde{B}}(x_j)|+|M^U{}_{\tilde{A}}(x_j)- -M^U{}_{\tilde{B}}(x_j)|+|N^L{}_{\tilde{A}}(x_j)-N^L{}_{\tilde{B}}(x_j)|+ |N^U{}_{\tilde{A}}(x_j)-N^U{}_{\tilde{B}}(x_j)|], \tag{3}$$

where $w_j=\{w_1,w_2,...w_n\}$ is the weight vector of the elements $x_j\,(j=1,2,...,n)$. The $d_1(A,B), d_2(A,B)$ and $d_3(A,B)$ are the Hamming distance, normalized Hamming distance, and weighted Hamming distance, respectively.

B. The score, accuracy functions and some indices

The studies reviews of score and accuracy functions to handle multi-criteria fuzzy decision-making problems are as follows. At definition 1, an IVIFS $\tilde{A}$ in X is defined as $\tilde{A}=\left\{\langle x,[M^L{}_{\tilde{A}}(x),M^U{}_{\tilde{A}}(x)],[N^L{}_{\tilde{A}}(x),N^U{}_{\tilde{A}}(x)]\rangle \mid x\in X\right\}$, for convenience, we call $\tilde{A}_n=\langle [M^L{}_{\tilde{A}_n}(x),M^U{}_{\tilde{A}_n}(x)],[N^L{}_{\tilde{A}_n}(x), N^U{}_{\tilde{A}_n}(x)]\rangle$ an interval-valued intuitionistic fuzzy number (IVIFN) [10], where $[M^L{}_{\tilde{A}_n}(x),M^U{}_{\tilde{A}_n}(x)]\subset[0,1]$, $[N^L{}_{\tilde{A}_n}(x), N^U{}_{\tilde{A}_n}(x)]\subset[0,1]$, and $M^U{}_{\tilde{A}_n}(x)+N^U{}_{\tilde{A}_n}(x)\le 1$.

Xu [10] defined a score function s to measure the degree of suitability of an IVIFN $\tilde{A}_n$ as follows. $s(\tilde{A}_n)=\frac{1}{2}(M^L{}_{\tilde{A}_n}(x)-N^L{}_{\tilde{A}_n}(x)+M^U{}_{\tilde{A}_n}(x)-N^U{}_{\tilde{A}_n}(x))$, where $s(\tilde{A}_n)\in[-1,1]$. The larger the value of $s(\tilde{A}_n)$, the higher the degree of the IVIFN $\tilde{A}_n$. Wei and Wang [7] defined an accuracy function h to evaluate the accuracy degree of an $\tilde{A}_n$ as follows.

$h(\tilde{A}_n)=\frac{1}{2}(M^L{}_{\tilde{A}_n}(x)+M^U{}_{\tilde{A}_n}(x)+N^L{}_{\tilde{A}_n}(x)+N^U{}_{\tilde{A}_n}(x))$, where $h(\tilde{A}_n)\in[0,1]$. The larger the value of $h(\tilde{A}_n)$, the higher the degree of the IVIFN $\tilde{A}_n$. The membership uncertainty index T was proposed [6] to evaluate the membership uncertainty degree of an IVIFN $\tilde{A}_n$ as follows. $T(\tilde{A}_n)=M^U{}_{\tilde{A}_n}(x)+N^L{}_{\tilde{A}_n}(x)-M^L{}_{\tilde{A}_n}(x)-N^U{}_{\tilde{A}_n}(x)$, where $-1\le T(\tilde{A}_n)\le 1$. The larger value of $T(\tilde{A}_n)$, the smaller of the IVIFN $\tilde{A}_n$.

The hesitation uncertainty index G of a $\tilde{A}_n$ is defined as follows. $G(\tilde{A}_n)=M^U{}_{\tilde{A}_n}(x)+N^U{}_{\tilde{A}_n}(x)-M^L{}_{\tilde{A}_n}(x)-N^L{}_{\tilde{A}_n}(x)$, and the larger value of $G(\tilde{A}_n)$, the smaller of the IVIFN $\tilde{A}_n$.

In the study, we classify different types of concordance and discordance sets with the concepts of score, accuracy functions, membership uncertainty and hesitation uncertainty index at the proposed method.

C. Construction of the IVIF decision matrix

We extend the canonical matrix format to an IVIF decision matrix $\tilde{M}$. An IVIFS $\tilde{A}_i$ of the ith alternative on X is given by $\tilde{A}_i=\left\{\langle x_j,\tilde{X}_{ij}\rangle \mid x_j\in X\right\}$, where $\tilde{X}_{ij}=([M^L{}_{\tilde{A}}(x),M^U{}_{\tilde{A}}(x)],[N^L{}_{\tilde{A}}(x),N^U{}_{\tilde{A}}(x)])$. The $\tilde{X}_{ij}$ indicate the degrees of membership and non-membership interval of the ith alternative with respect to the jth criterion. The IVIF decision matrix $\tilde{M}$ can be expressed as follows:

$$\tilde{\boldsymbol{M}}=\begin{array}{c} \boldsymbol{A}_1 \\ \vdots \\ \boldsymbol{A}_m \end{array}\begin{bmatrix} \tilde{\boldsymbol{X}}_{11} & \cdots & \tilde{\boldsymbol{X}}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{\boldsymbol{X}}_{m1} & \cdots & \tilde{\boldsymbol{X}}_{mn} \end{bmatrix}$$

$$=\begin{bmatrix} ([M_{11}{}^L,M_{11}{}^U],[N_{11}{}^L,N_{11}{}^U]) & . & ([M_{1n}{}^L,M_{1n}{}^U],[N_{1n}{}^L,N_{1n}{}^U]) \\ . & . & . \\ ([M_{m1}{}^L,M_{m1}{}^U],[N_{m1}{}^L,N_{m1}{}^U]) & . & ([M_{mn}{}^L,M_{mn}{}^U],[N_{mn}{}^L,N_{mn}{}^U]) \end{bmatrix} \tag{4}$$

An IVIFS W, a set of grades of importance, in X is defined as follows:

$$W=\left\{\langle x_j,w_j(x_j)\rangle \mid x_j\in X\right\}, \tag{5}$$

where $0\le w_j(x_j)\le 1$, $\sum_{j=1}^{n} w_j(x_j)=1$, and $w_j(x_j)$ is the degree of importance assigned to each criterion.

3 ELECTRE Method with IVIF Data

The proposed method is utilized the concept of score and accuracy function to distinguish concordance set and the discordance set from the evaluation information with IVIFS data, and then to construct the concordance, discordance, concordance (discordance, aggregate) dominance matrix, respectively, and to select the best alternative from the aggregate dominance matrix finally. In this section, the IVIF ELECTRE method and its algorithm are introduced and used throughout this paper.

A. The IVIF ELECTRE method

The concordance and discordance sets with IVIF data and their definitions are as follows.
Definition 4: The concordance set C_{kl} is defined as

$$C^1_{kl} = \{j \mid M^L_{kj} - N^L_{kj} + M^U_{kj} - N^U_{kj} > M^L_{lj} - N^L_{lj} + M^U_{lj} - N^U_{lj}\}, \tag{6}$$

$$C^2_{kl} = \{j \mid M^L_{kj} + M^U_{kj} + N^L_{kj} + N^U_{kj} > M^L_{lj} + M^U_{lj} + N^L_{lj} + N^U_{lj}\}$$

$$\text{when } s(\tilde{X}_{kj}) = s(\tilde{X}_{lj}), \tag{7}$$

$$C^3_{kl} = \{j \mid M^U_{kj} + N^L_{kj} - M^L_{kj} - N^U_{kj} < M^U_{lj} + N^L_{lj} - M^L_{lj} - N^U_{lj}\}$$

$$\text{when } h(\tilde{X}_{kj}) = h(\tilde{X}_{lj}), \tag{8}$$

$$C^4_{kl} = \{j \mid M^U_{kj} + N^U_{kj} - M^L_{kj} - N^L_{kj} \le M^U_{lj} + N^U_{lj} - M^L_{lj} - N^L_{lj}\}$$

$$\text{when } T(\tilde{X}_{kj}) = T(\tilde{X}_{lj}), \tag{9}$$

where $C_{kl} = \{C^1_{kl}, C^2_{kl}, C^3_{kl}, C^4_{kl}\}$, $J = \{j \mid j = 1, 2, ..., n\}$, and $\tilde{X}_{kj}, \tilde{X}_{lj}$ stand for the lower and upper boundaries of alternative *k* and *l* in criterion *j*, respectively.

The $s(\tilde{X}_{kj})$, $h(\tilde{X}_{kj})$ and $T(\tilde{X}_{kj})$ are score, accuracy function and membership uncertainty index, respectively, which are defined in section II. *B*.

Definition 5: The discordance set D_{kl} is defined as

$$D^1_{kl} = \{j \mid M^L_{kj} - N^L_{kj} + M^U_{kj} - N^U_{kj} < M^L_{lj} - N^L_{lj} + M^U_{lj} - N^U_{lj}\}, \tag{10}$$

$$D^2_{kl} = \{j \mid M^L_{kj} + M^U_{kj} + N^L_{kj} + N^U_{kj} < M^L_{lj} + M^U_{lj} + N^L_{lj} + N^U_{lj}\}$$

$$\text{when } s(\tilde{X}_{kj}) = s(\tilde{X}_{lj}), \tag{11}$$

$$D^3_{kl} = \{j \mid M^U_{kj} + N^L_{kj} - M^L_{kj} - N^U_{kj} > M^U_{lj} + N^L_{lj} - M^L_{lj} - N^U_{lj}\}$$

$$\text{when } h(\tilde{X}_{kj}) = h(\tilde{X}_{lj}), \tag{12}$$

$$D^4_{kl} = \{j \mid M^U_{kj} + N^U_{kj} - M^L_{kj} - N^L_{kj} > M^U_{lj} + N^U_{lj} - M^L_{lj} - N^L_{lj}\}$$

$$\text{when } T(\tilde{X}_{kj}) = T(\tilde{X}_{lj}), \tag{13}$$

where $D_{kl} = \{D^1_{kl}, D^2_{kl}, D^3_{kl}, D^4_{kl}\}$.

The relative value of the concordance set of the IVIF ELECTRE method is measured through the concordance index. The concordance index g_{kl} between A_k and A_l is defined as:

$$g_{kl} = \omega_C \times \sum_{j \in C_{kl}} w_j(x_j), \tag{14}$$

where ω_C is the weight of the concordance set, and $w_j(x_j)$ is defined in (5).

The concordance matrix G is defined as follows:

$$\boldsymbol{G} = \begin{bmatrix} - & g_{12} & \cdots & \cdots & g_{1m} \\ g_{21} & - & g_{23} & \cdots & g_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ g_{(m-1)1} & \cdots & \cdots & - & g_{(m-1)m} \\ g_{m1} & g_{m2} & \cdots & g_{m(m-1)} & - \end{bmatrix}, \tag{15}$$

where the maximum value of g_{kl} is denoted by g^*.

The evaluation of a certain A_k are worse than the evaluation of competing A_l.

The discordance index is defined as follows:

$$h_{kl} = \frac{\max\limits_{j \in D_{kl}} \omega_D \times d(X_{kj}, X_{lj})}{\max\limits_{j \in J} d(X_{kj}, X_{lj})}, \tag{16}$$

where $d(X_{kj}, X_{lj})$ is defined in (3), and ω_D is the weights of discordance set on IVIF ELECTRE method.

The discordance matrix H is defined as follows:

$$\boldsymbol{H} = \begin{bmatrix} - & \boldsymbol{h}_{12} & \cdots & \cdots & \boldsymbol{h}_{1\boldsymbol{m}} \\ \boldsymbol{h}_{21} & - & \boldsymbol{h}_{23} & \cdots & \boldsymbol{h}_{2\boldsymbol{m}} \\ \cdots & \cdots & - & \cdots & \cdots \\ \boldsymbol{h}_{(\boldsymbol{m}-1)1} & \cdots & \cdots & - & \boldsymbol{h}_{(\boldsymbol{m}-1)\boldsymbol{m}} \\ \boldsymbol{h}_{\boldsymbol{m}1} & \boldsymbol{h}_{\boldsymbol{m}2} & \cdots & \boldsymbol{h}_{\boldsymbol{m}(\boldsymbol{m}-1)} & - \end{bmatrix}, \tag{17}$$

where the maximum value of h_{kl} is denoted by h^* that is more discordant than the other cases.

The concordance dominance matrix K is defined as follows:

$$\boldsymbol{K} = \begin{bmatrix} - & \boldsymbol{k}_{12} & \cdots & \cdots & \boldsymbol{k}_{1\boldsymbol{m}} \\ \boldsymbol{k}_{21} & - & \boldsymbol{k}_{23} & \cdots & \boldsymbol{k}_{2\boldsymbol{m}} \\ \cdots & \cdots & - & \cdots & \cdots \\ \boldsymbol{k}_{(\boldsymbol{m}-1)1} & \cdots & \cdots & - & \boldsymbol{k}_{(\boldsymbol{m}-1)\boldsymbol{m}} \\ \boldsymbol{k}_{\boldsymbol{m}1} & \boldsymbol{k}_{\boldsymbol{m}2} & \cdots & \boldsymbol{k}_{\boldsymbol{m}(\boldsymbol{m}-1)} & - \end{bmatrix}, \tag{18}$$

where $k_{kl} = g^* - g_{kl}$, and a higher value of k_{kl} indicates that A_k is less favorable than A_l.

The discordance dominance matrix L is defined as follows:

$$\boldsymbol{L} = \begin{bmatrix} - & \boldsymbol{l}_{12} & \cdots & \cdots & \boldsymbol{l}_{1\boldsymbol{m}} \\ \boldsymbol{l}_{21} & - & \boldsymbol{l}_{23} & \cdots & \boldsymbol{l}_{2\boldsymbol{m}} \\ \cdots & \cdots & - & \cdots & \cdots \\ \boldsymbol{l}_{(\boldsymbol{m}-1)1} & \cdots & \cdots & - & \boldsymbol{l}_{(\boldsymbol{m}-1)\boldsymbol{m}} \\ \boldsymbol{l}_{\boldsymbol{m}1} & \boldsymbol{l}_{\boldsymbol{m}2} & \cdots & \boldsymbol{l}_{\boldsymbol{m}(\boldsymbol{m}-1)} & - \end{bmatrix}, \tag{19}$$

where $l_{kl} = h^* - h_{kl}$, a higher value of l_{kl} indicates that A_k is preferred over A_l.

The aggregate dominance matrix R is defined as follows:

$$\boldsymbol{R} = \begin{bmatrix} - & \boldsymbol{r}_{12} & \cdots & \cdots & \boldsymbol{r}_{1\boldsymbol{m}} \\ \boldsymbol{r}_{21} & - & \boldsymbol{r}_{23} & \cdots & \boldsymbol{r}_{2\boldsymbol{m}} \\ \cdots & \cdots & - & \cdots & \cdots \\ \boldsymbol{r}_{(\boldsymbol{m}-1)1} & \cdots & \cdots & - & \boldsymbol{r}_{(\boldsymbol{m}-1)\boldsymbol{m}} \\ \boldsymbol{r}_{\boldsymbol{m}1} & \boldsymbol{r}_{\boldsymbol{m}2} & \cdots & \boldsymbol{r}_{\boldsymbol{m}(\boldsymbol{m}-1)} & - \end{bmatrix}, \tag{20}$$

where

$$r_{kl} = \frac{l_{kl}}{k_{kl} + l_{kl}}, \tag{21}$$

k_{kl} and l_{kl} are defined in (18) and (19), and r_{kl} is in the range from 0 to 1. A higher value of r_{kl} indicates that the alternative A_k is more concordant than the alternative A_l; thus, it is a better alternative. In the best alternatives selection process,

$$\overline{T}_k = \frac{1}{m-1} \sum_{l=1, l \neq k}^{m} r_{kl}, \; k = 1, 2, \ldots, m, \tag{22}$$

and $\overline{T}_k$ is the final value of the evaluation. All alternatives can be ranked according to the value of $\overline{T}_k$. The best alternative $A*$ with $\overline{T}_k^{\,*}$ can be generated and defined as follows:

$$\overline{T}_k^{\,*}(A*) = \max\{\overline{T}_k\}, \tag{23}$$

where $\overline{T}_k^{\,*}$ is the final value of the best alternative and $A*$ is the best alternative.

B. Algorithm

The algorithm and decision process of the IVIF ELECTRE method can be summarized in the following four steps, and there are calculate the concordance, discordance matrices, construct the concordance dominance, discordance dominance matrices and determine the aggregate dominance matrix in the Step 3. Figure 1 illustrates a conceptual model of the proposed method.

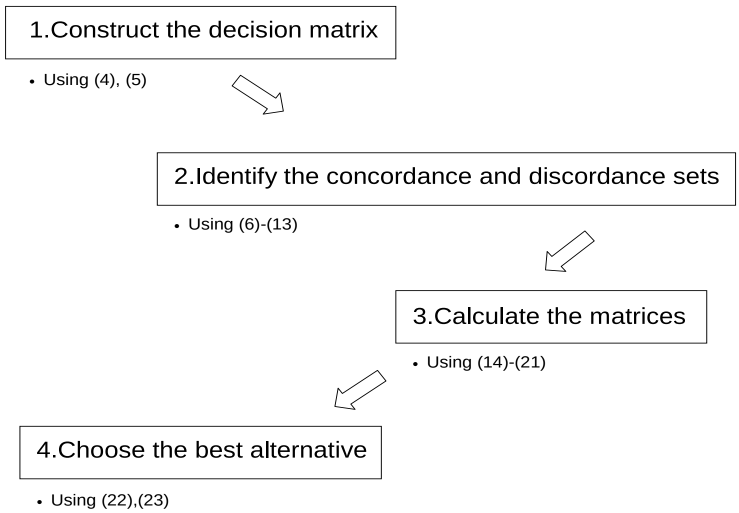

Figure 1: The process of the IVIF ELECTRE method algorithm.

4 Numerical Example

In this section, we present an example that is connected to a decision-making problem with the best alternative selection. Suppose a potential banker intends to invest the money from four possible alternatives (companies), named A_1, A_2, A_3, and A_4. The criteria of a company is x_1 (risk analysis), x_2 (the growth analysis), and x_3 (the environmental impact analysis) in the selection problem. The subjective importance levels of the different criteria W are given by the decision makers:

$W = [w_1, w_2, w_3] = [0.35, 0.25, 0.4]$. The decision makers also give the relative weights as follows:

$W' = [w_C, w_D] = [1,1]$. The IVIFS decision matrix decision $\tilde{M}$ is given with cardinal information:

$$\tilde{\boldsymbol{M}} = \begin{bmatrix} ([\boldsymbol{M}_{11}{}^L, \boldsymbol{M}_{11}{}^U], [\boldsymbol{N}_{11}{}^L, \boldsymbol{N}_{11}{}^U]) & . & ([\boldsymbol{M}_{1n}{}^L, \boldsymbol{M}_{1n}{}^U], [\boldsymbol{N}_{1n}{}^L, \boldsymbol{N}_{1n}{}^U]) \\ . & . & . \\ ([\boldsymbol{M}_{m1}{}^L, \boldsymbol{M}_{m1}{}^U], [\boldsymbol{N}_{m1}{}^L, \boldsymbol{N}_{m1}{}^U]) & . & ([\boldsymbol{M}_{mn}{}^L, \boldsymbol{M}_{mn}{}^U], [\boldsymbol{N}_{mn}{}^L, \boldsymbol{N}_{mn}{}^U]) \end{bmatrix}$$

$$= \begin{bmatrix} ([0.4, 0.5], [0.3, 0.4]) & ([0.4, 0.6], [0.2, 0.4]) & ([0.1, 0.3], [0.5, 0.6]) \\ ([0.4, 0.6], [0.2, 0.3]) & ([0.6, 0.7], [0.2, 0.3]) & ([0.4, 0.7], [0.1, 0.2]) \\ ([0.3, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.3, 0.4]) & ([0.5, 0.6], [0.1, 0.3]) \\ ([0.7, 0.8], [0.1, 0.2]) & ([0.6, 0.7], [0.1, 0.3]) & ([0.3, 0.4], [0.1, 0.2]) \end{bmatrix}$$

(Step 1 has completed.)

Applying Step 2, the concordance and discordance sets are identified using the result of Step 1.

The concordance set, applying (6) - (9), is:

$$\boldsymbol{C_{kl}} = \begin{bmatrix} - & 1,3 & 1,3 & 1,3 \\ 1,2,3 & - & 1,2,3 & 2,3 \\ 2,3 & 1,2,3 & - & 2,3 \\ 1,2,3 & 1,2,3 & 1,2,3 & - \end{bmatrix}.$$

For example, C_{24}, which is in the 2nd (horizontal) row and the 4th (vertical) column of the concordance set, are “2,3”.

The discordance set, obtained by applying (10) - (13), is as follows:

$$\boldsymbol{D_{kl}} = \begin{bmatrix} - & 2 & 2 & 2 \\ - & - & - & 1 \\ 1 & - & - & 1 \\ - & - & - & - \end{bmatrix}.$$

Applying Step 3, the concordance matrix is calculated.

$$\boldsymbol{G}=\begin{bmatrix} - & 0.8 & 0.8 & 0.8 \\ 1 & - & 1 & 0.5 \\ 0.5 & 1 & - & 0.5 \\ 1 & 1 & 1 & - \end{bmatrix}$$. For example,

$$g_{21}=w_C\times w_1+w_C\times w_2+w_C\times w_3$$
$$=1\times 0.35+1\times 0.25+1\times 0.40=1.0\,.$$

The discordance matrix is calculated:

$$\boldsymbol{H}=\begin{bmatrix} - & 0.267 & 0.143 & 0.357 \\ 0 & - & 0 & 1 \\ 0.143 & 0 & - & 1 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

For example:

$$h_{12}=\frac{\max\limits_{j\in D_{12}} w_D\times d(X_{1j},X_{2j})}{\max\limits_{j\in J} d(X_{1j},X_{2j})}=\frac{0.100}{0.375}=0.267\,,$$

where

$$d(X_{13},X_{23})=\frac{1}{4}(|0.1-0.4|+|0.3-0.7|+$$
$$|0.5-0.1|+|0.6-0.2|)=0.375\,,$$

and

$$w_D\times d(X_{12},X_{22})=1\times(\frac{1}{4}(|0.4-0.6|+|0.6-0.7|$$
$$+|0.2-0.2|+|0.4-0.3|))=0.100\,.$$

The concordance dominance matrix is constructed as follows.

$$\boldsymbol{K}=\begin{bmatrix} - & 0.2 & 0.2 & 0.2 \\ 0 & - & 0 & 0.5 \\ 0.5 & 0 & - & 0.5 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

The discordance dominance matrix is constructed as follows.

$$\boldsymbol{L}=\begin{bmatrix} - & 0.733 & 0.857 & 0.643 \\ 1 & - & 1 & 0 \\ 0.857 & 1 & - & 0 \\ 1 & 1 & 1 & - \end{bmatrix}.$$

The aggregate dominance matrix is determined:

$$\boldsymbol{R}=\begin{bmatrix} - & 0.786 & 0.811 & 0.763 \\ 1 & - & 1 & 0 \\ 0.632 & 1 & - & 0 \\ 1 & 1 & 1 & - \end{bmatrix}.$$

Applying Step 4, the best alternative is chosen:

$$\overline{T}_1=0.786\,,\overline{T}_2=0.667\,,\overline{T}_3=0.544\,,\overline{T}_4=1.000\,.$$

The optimal ranking order of alternatives is given by $A_4\succ A_1\succ A_2\succ A_3$. The best alternative is A_4.

5 Discussion

In this study, we provide a new method, the IVIF ELECTRE method, for solving MCDM problems with IVIF information. A decision maker can use the proposed method to gain valuable information from the evaluation data provided by users, who do not usually provide preference data. Decision makers utilize IVIF data instead of single values in the evaluation process of the ELECTRE method and use those data to classify different kinds of concordance and discordance sets to fit a real decision environment. This new approach integrates the concept of the outranking relationship of the ELECTRE method. In the proposed method, we can classify different types of concordance and discordance sets using the concepts of score function, accuracy function, membership uncertainty degree, hesitation uncertainty index, and use concordance and discordance sets to construct concordance and discordance matrices. Furthermore, decision makers can choose the best alternative using the concepts of positive and negative ideal points. We used the proposed method to rank all alternatives and determine the best alternative. This paper is the first step in using the IVIF ELECTRE method to solve MCDM problems. In a future study, we will apply the proposed method to predict consumer decision making using a questionnaire in an empirical study of service providers selecting issue.

This research is supported by the National Science Council (No. NSC 99-2410-H-182-022-MY3).

References

[1] K. T. Atanassov, “Intuitionistic fuzzy sets,” Fuzzy sets and Systems, vol. 20, pp. 87-96, 1986.

[2] K. Atanassov and G. Gargov, “Interval valued intuitionistic fuzzy sets,” Fuzzy sets and Systems, vol. 31, pp. 343-349, 1989.

[3] B. Roy, “Classement et choix en présence de points de vue multiples (la méthode ELECTRE),” RIRO, vol. 8, pp. 57-75, 1968.

[4] B. Vahdani and H. Hadipour, “Extension of the ELECTRE method based on interval-valued fuzzy sets,” Soft Computing, vol. 15, pp. 569-579, 2011.

[5] B. Vahdani, A. H. K. Jabbari, V. Roshanaei, and M. Zandieh, “Extension of the ELECTRE method for decision-making problems with interval weights and data,” International Journal of Advanced Manufacturing Technology, vol. 50, pp. 793-800, 2010.

[6] Z. Wang, K. W. Li, and W. Wang, “An approach to multiattribute decision making with interval-valued intuitionistic fuzzy assessments and incomplete weights,” Information Sciences, vol. 179, pp. 3026-3040, 2009.

[7] G. W. Wei, and X. R. Wang, “Some geometric aggregation operators on interval-valued intuitionistic fuzzy sets and their application to group decision making,” International conference on computational intelligence and security, pp. 495-499, December 2007.

[8] M.-C. Wu and T.-Y. Chen, “The ELECTRE multicriteria analysis approach based on Atanassov's intuitionistic fuzzy sets,” Expert Systems with Applications, vol. 38, pp. 12318-12327, 2011.

[9] Z. S. Xu, “On similarity measures of interval-valued intuitionistic fuzzy sets and their application to pattern recognitions,” Journal of Southeast University, vol. 23, pp. 139 -143, 2007a.

[10] Z. S. Xu, “Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making,” Control and Decision, vol. 22, pp. 215 -219, 2007b.

[11] L. A. Zadeh, “Fuzzy Sets,” Information and Control, vol. 8, pp. 338-353, 1965.