A Comparative Review of Contour and Raster Based Methods for the Prediction of Surface Water Flow from DEM Data

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Abstract
A water management strategy to cope with severe climate change needs an efficient streamline model to play the important role of predicting the direction of flood-water. Generally a model is constructed using various image processing techniques which, depending on the particular area, can be quite complicated. In this paper two conceptually different approaches for determining streamlines from DEM data, in raster and vector forms, are reviewed. The advantages and limitations of both approaches are considered and compared using the numerical results of streamlines for the geometrically complex region containing Khao Luang National Park.

Keywords: Streamline models, DEM image, Interpolation techniques, Surface water, BEM, D8 algorithm

1 Introduction

Imagine asking an expert surveyor or a local resident to predict the direction in which surface water flows. It might be possible for a small area of simple terrain, but definitely not easy to do so at all accurately for an extended area of complicated landscape. In civil engineering, the velocity vector field is used widely as a tool to describe the nature of fluid flow [1]–[3], whereas the concept of the electromagnetic field is exploited in electrical engineering to visualize streamlines related to electricity and magnetism. In these and other cases streamline models serve as useful tools which help many researchers to bring insight to problems of their interest. In hydrology these models, conventionally based on grid-based digital elevation models, are used to identify the direction of drainage in order to determine the paths followed by water, and to determine the movement of sediment and contaminants [4], [5]. In addition, streamline models are essential in water management both for optimizing the usage of limited resources, and for planning for safety and recovery from natural disaster. As a consequence of their effectiveness it follows that much attention has been paid to the development of the concepts and processes to construct efficient streamline models for all of these applications.

Initially, streamline models were mathematical models based on analytical methods and applied to simple problems. Later, numerical methods were exploited on computers for solving more complex problems involving multi-variable functions. Each particular technique has individual characteristics depending on the object of the study, surface water or ground water, and the properties of the elevation data utilised. In consequence, many different techniques have been proposed to formulate and construct a variety of different streamline models.

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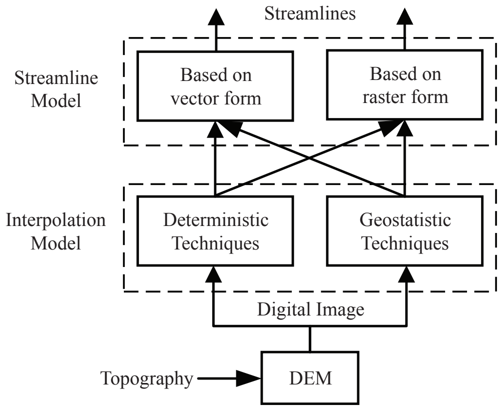


Figure 1: A process consisting of three phases: DEM, interpolation model and streamline model, for calculating from elevation data the paths along which water flows.

The overall streamline modeling process is divided into the three main phases as illustrated in Figure 1. The initial phase is to obtain measurements of the elevation or altitude data over the area of interest. This can be achieved using remote sensing techniques such as LiDAR [6] and SAR [7]. The measured data referred to the sea level is typically transformed by a digital elevation model (DEM) into a digital image. The DEM image contains pixels representing as numerical values the altitude over the landscape for a discrete set of coordinates [8]. This set of primary information is collected in either of two forms: as raster or vector data.

The second phase is to interpolate the primary information, since the raw resolution of the data is inadequate for predicting continuous streamlines. The final phase then involves constructing the model itself by combining the primary information with the interpolated information.

Individual models have constraints depending on such conditions as the type of terrain, e.g., lowland, highland, sloping ground, valley, and so on. It is difficult therefore to design a model that is able to be universally used for all regions. Calculations to make predictions of continuous streamlines [4], [9], to determine the direction of flow on flat areas or mountain tops, and to calculate the size of catchment areas are not particularly reliable. Therefore, many researchers have sought to develop modifications and improvements.

The effectiveness of alternative models is mainly influenced by the characteristics of both the primary and the interpolated data. Understanding the processing of the input data prior to designing the streamline model is absolutely essential because of the many and varied approaches adopted for use as interpolation techniques. For streamline model processing each approach, raster and vector data, have their own advantages and constraints.

Consequently, this paper presents a review of streamline modelling processes intended particularly for the benefit of those involved in applications as described above, where streamline models play an important role. The various processes are described step-by-step in section 2, starting with the concept of DEM creation and the interpolation techniques. Streamline model processing using raster style and vector style data is detailed in section 3. The model is implemented on complicated terrain in the Khao Luang National Park using both raster data and vector data to predict the paths along which surface water flows. To validate the accuracy of each approach, the numerical results from both are compared and discussed in section 4. Finally, the advantages and limitations of each approach are considered in detail.

2 Pre-processing for Streamline Models

The overall streamline modelling process consists of three phases: the DEM, interpolation, and streamline model. The first two phases involve pre-processing which is essential for converting the elevation data into a suitable form and for increasing the resolution of the data. Conventionally, the DEM uses image processing technology to represent remotely sensed elevation data as three dimensional terrain surfaces. The DEM images it produces as outputs have resolutions which are too low and inadequate for direct use in the streamline model. The second phase is thus included to increase the resolution of DEM images by means of interpolation techniques. Each technique provides outputs in different forms, each form with different features. Therefore, in preparation for the discussion of the streamline models themselves, the concepts of DEM and interpolation models are briefly described in this section with emphasis on the different features of the outputs produced.

2.1 *Digital Elevation Model (DEM)*

Three dimensional terrain surfaces in a DEM image are constructed using a mixture of remote sensing and image processing techniques. Due to the time consuming nature and high cost of ground survey methods, the elevation data are generated via DEM technology instead. Two main remote sensing technologies are employed: SAR and LiDAR.

The synthetic aperture radar (SAR) technology uses a radar system [10] which can be mounted on a satellite or aircraft [11]. It is enhanced by various approaches based on techniques of interferometry [12], [13] and radargrammetry [14]. However, the uncertain nature of the electrical properties of the atmosphere is an inevitable problem with both techniques [7], [15], [16]. In order to solve this problem, the effectiveness of high technology, digital camera, is investigated [17]. This technology is based on the light detection and ranging (LiDAR) technique which, using a digital camera (Rollei H20) with the pulse rate 71 kHz mounted on a helicopter travelling at around 150km/hr, is capable of capturing high velocity moving objects.

The outputs of DEM created by various remote sensing techniques, as in described in [6], [8], are generally divided into three styles of two dimensional coordinate systems: square grids, triangular grids, and contour lines. The elevation data in the patterns of grids and digitalized contour lines are respectively called raster and vector data. The advantages and limitations of these forms of data [18]–[20], such as the roughness of data, the difficulty of increasing resolution and of evaluating slope, and the memory allocation necessary, are listed in Table 1. For instance, increasing the resolution of elevation data in raster form is more difficult than in vector form [21] since the former form must consider the entire of area of interest area to regenerate the smaller uniform grids whereas it is only necessary to insert some sampled elevation data on digitalized contour lines for the latter form. Thus, the vector data form is preferred to the raster form to simplify the complex three dimensional flow equations.

Table 1: Three patterns of elevation in DEM image

Patterns	Square	Triangular	Contour
Roughness	Abrupt	Piecewise	Smooth
Increasing resolution	Difficult	Easy	Easy
Evaluating of slope	Easy	Moderate	Difficult
Allocation of memory	More	Moderate	Less

2.2 *Interpolation model*

The interpolation model increases the resolution of elevation data in both raster and vector forms. This model is derived by the mathematical techniques whose concepts are categorized based on various methods such as geometrical proximity method, statistical method, basis function method, and artificial neural network (ANN) method [22]. In order to more readily comprehend these various methods they can be divided into two main groups: deterministic techniques and geostatistic techniques.

2.2.1 Deterministic techniques

Deterministic techniques are used to interpolate the solution of a partial differential equation (PDE) between the valid elevation data of DEM images by means of numerical methods and boundary conditions. The boundary conditions are constructed with the elevation data in both raster and vector forms.

For the raster form, the interpolation techniques are based on the extent of similarity and the degree of smoothing. The concept of extent of similarity is to detect the relationship amongst elevation data inside a small region. Many relations have been used such as IDW and TIN techniques. The concept of both techniques involves some key criteria for creating a set of sampled data to extend the similarity. The inverse distance weight (IDW) technique is a simple relation that is established with the distance between known and unknown raster data [23], [24]. The triangular irregular network (TIN) is a relation that creates the approximated function over grid cells of Voronoi subdivision [25]. It is effective and often used in representing discontinuities of the terrain's surface such as cliffs and ridges. To increase the continuity of the data, many techniques based on the degree of smoothing are also exploited. These techniques approximate the unknown functions using basis function such as polynomials [25], [26], and Fourier series [27]. For instance, the polynomial function is used to interpolate data over small square area and to evaluate the slope of terrain [26], [27].

For the vector form, the interpolation techniques are based on barriers such as locations of the river, road, canal, and so on. These locations are used as the boundary conditions to provide the solution of a PDE with numerical methods such as the finite difference method (FDM) [28], the thin plate splines method (TPS) [29], kernel smoothing barriers (KSB) [30], diffusion kernel with barriers (DKB) [31], and boundary element method (BEM) [9]. The solution obtained to a PDE based on Laplace's equation is often used to describe the static elevation data behavior; then is formulated with various artificial relations. However, the implementation of this concept is still found wanting so that it has been developed to enhance the interpolation performance for more smooth streamlines.

2.2.2 Geostatistic techniques

Geostatistic techniques involve analysing the features of geography by means of statistical techniques [32]. Due to its simplicity and effectiveness, a simple technique known as Kriging is traditionally used as the central element of many geostatistic processing techniques. The process of interpolating elevation data is divided into two steps. The first step is to identify the similarity of random data by spatial covariance and variogram parameters, and to measure dissimilarity by a semi-variance parameter. The last step is to configure the weight functions using the previous three parameters.

The concept of the Kriging family [33] presents the general form to determine the interpolated elevation with

$$\mathbf{z}\left(x_0\right)-\mu=\sum_{i=1}^{n}\lambda_i\left[z\left(x_i\right)-\mu\left(x\right)\right] \quad (1)$$

where $\mathbf{z}(x_0)$ is the estimated elevation at location x_0. μ and n are respectively a known stationary mean, and the number of sampled points for estimation of the size of search window control. $\mu(x_0)$ is the mean of samples within the search window. The size of search window can be validated after the range of influence of the semivariogram is considered; then the formula is used to estimate the residuals from reference value μ with appropriate weight function λ_1.

The accuracy of the simple Kriging method is influenced by the assumed reference and types of weight functions. In the ordinary Kriging interpolator [34] the reference is replaced by the mean of some sampled data $\mu(x_0)$. There are many extensions to the simple Kriging method [35] such as universal Kriging, block Kriging, factorial Kriging, dual Kriging, simple Kriging with varying local means, and Kriging with an external drift. One advanced extension of methods in the Kriging family is known as the Cokriging interpolator [35]. The spatial cross correlation between two sets of some elevation data are analyzed in a process called cross-semivariance. Due to its higher accuracy, it has become the most popular technique. Improvements of Cokriging [36], [37] are similar to the Kriging interpolator. They have been implemented in the main modules in the ArcGIS simulation [38].

In this section, the two models that serve as pre-processing phases for streamline models are described briefly. Various techniques based on both vector and raster forms are used to increase resolution of a DEM image. We found that the concepts to increase the resolution by using geostatistic techniques in raster form are simpler than using deterministic techniques in either vector or raster forms. However, the application of deterministic techniques with the numerical method to solve a PDE is more effective for maintaining continuity of the streamlines.

3 Streamline Models

Streamline models are the models in which the paths along which water flows on the earth's surface are illustrated as lines by using the valid elevation data from the aforementioned DEM and interpolation models. The interpolation models produce output in either raster or vector forms. Each output form is taken as the input of the streamline model, and thereby influences the development of that model. The streamline models produced by the raster data form involve cells connected in series using the image processing techniques, whereas the vector data form relies on analytical techniques to provide continuous flow paths. The process of generating streamline models by each input data form is detailed in the following sections.

3.1 *Input as raster data form*

The elevation data in the form of raster data is collected as a rectangular structure of pixels in a digital image. The data is located on a network of grid cells, with the number of elevation data points directly proportional to the image resolution. The concept of the streamline models using input data as the raster data form is to draw the non-smoothing path from a starting pixel to

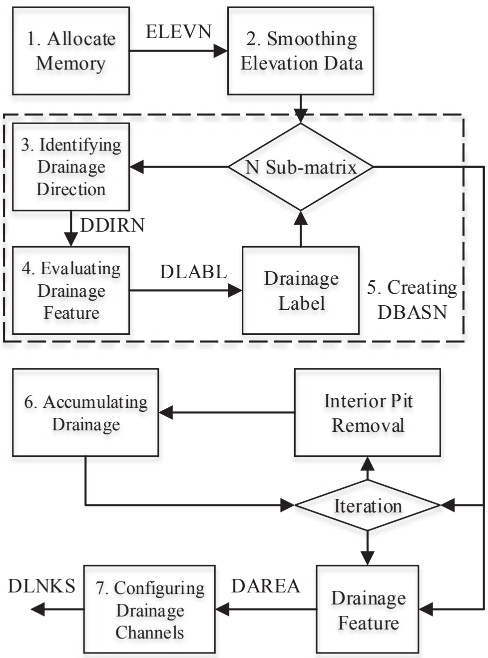
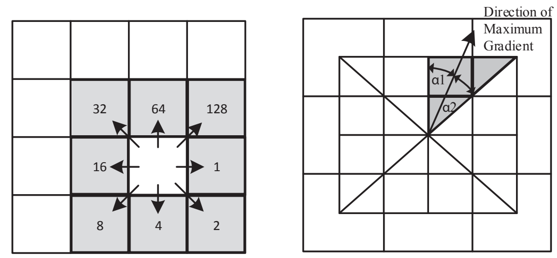


Figure 2: Process of D8 algorithm.

adjacent pixels that are surrounding the starting pixel under specific criteria to represent the water flow direction. One of the most important criteria is derived by the D8 algorithm [4], [39]. It is a classical method which evaluates the appropriate water flow direction by considering the difference of elevation of each adjacent pixel surrounding the starting pixel. This method consists of seven procedures as illustrated in Figure 2. These procedures are described sequentially in following paragraphs.

1. Allocating memory: The first step is to access the elevation data in DEM image; then collect them in memory units of matrix ELEVN (elevation). The data collected on each pixel are discrete samples so the data is unknown in the gaps between the pixels. The unknown data is typically assumed to have no information. The gaps between pixels often cause the discontinuity of streamlines or artificial pits in the process. This problem was primarily solved in previous work by decreasing the gap to increase the DEM image resolution. It is obviously seen that the efficiency of streamline models is affected by the amount of missing data or the discontinuity in DEM model.

2. Smoothening elevation data: This step is to compensate for missing data or the discontinuities in



(a) eight possible directions (b) infinite directions

Figure 3: D8 algorithm on a sub-region.

the elevation data from the previous step by dividing the whole grid into many submatrices composed of 3 × 3 grids. There are 9 pixels with a central pixel surrounded by eight adjacent pixels as shown in Figure 3 (a). The elevation data of eight pixels is used to form a weight function for determining the data value. This method is a basic data smoothing technique.

3. Identifying drainage direction: The direction of water flow on each pixel is identified in this step. There are eight possible directions: North, East, South, West, North East, South East, South West, and North West that are encoded into numerical values and stored in the DDRIN (drainage direction) matrix shown in Figure 3 (a). Note that the values of each surrounded pixel are symbol numbers which are taken to represent each direction, such as 64 for the north direction, 1 for the east direction, 4 for the south direction, and 16 for the west direction. The possible directions of flow radiating from the central pixel are either cardinal or diagonal directions. The direction of flow is predicted by choosing the minimum value of elevation data of surrounding pixels. The drawback of this method is the unnatural or non-smooth direction of flow that it produces. Although this technique provides significant errors in determining streamlines and in the size of the areas contributed with dispersal landscapes, it is nevertheless appropriate for the cases of valleys or steep landscapes [40]. Moreover, it has been extensively exploited in analyzing more complicated areas such as flowing divergence model for ridge areas.

The concept of D8 algorithm has been extended and published as the D-infinity flow assignment model [41]–[45]. This method formulates the direction of flow by means of the function of both elevation and slope as shown in Figure 3 (b). The direction of flow is determined by an angle α defined by the maximum gradient of eight triangular facets. Many alternate extensions of the D8 algorithm have been proposed, such as the D16 algorithm [5]. This method increases the number of pixels under consideration from 3 × 3 submatrices to 5 × 5 submatrices. Its results were attractive to develop solutions for ridge areas. Another interesting method is the Rho8 method [46], which was augmented with a stochastic component to get a higher accuracy for the case of an area composed of hills with steep aspect. With this approach the correct streamlines are obtained uniquely in case they are parallel with other flowing paths. However, the randomness sometimes produces deviant streamlines which contain cross-over points between paths of flow.

4. Evaluating drainage feature: In this step the features of each cell are evaluated in combination with the flowing direction analysis. Nine cells of the 3 × 3 submatrices are considered whose central cell will respectively be a pit or a fork depending on the direction of eights surrounded pixels as either inward or outward. These features are represented by the symbols and are stored in the DLABL (drainage label) matrix. This matrix is used to evaluate the size of basin drainage. During the process, lots of symbols are generated. It is a time consuming process. To eliminate lots of generated symbols, the labeling techniques were proposed [47].

5. Creating DBASN matrix: This step is to identify the line of direction from one pixel to the adjacent pixel by joining some cells together with connections called drainage links. The drainage links are unbranched paths that are divided into two types. The first type is an upstream link consisting of some source points called an "exterior link", whereas the other type is a link consisting of some fork (junction) points called "interior link". They are determined with the numerical values in DDIRN of the second step and in DLABL matrix of the fourth step; then the symbol is collected in the drainage basin matrix (DBASN). For the case of a flat area, the processing can be complicated since there are only a few pits or overflow points.

6. Accumulating drainage: This step is to combine the drainage basins in order to investigate the connection of drainage links and to configure the drainage channel location. Connections are identified by implementing an iterative technique [48] on each pixel using the information of drainage basins in DLABL matrix. The pixels used to represent the connections are labeled into the DAREA matrix. In principle, in a basin, the drainage links should converge at a point. However in some instances they do not do so, and cross over one another instead.

7. Configuring drainage channel: The last step is to revise the features of each pixel since some pixels may have irregular features after combining drainage basins. The irregular features are corrected by analyzing the information in DAREA matrix. This matrix collects the direction of each pixel after combining. The output of this step is the location of drainage paths called drainage channels, whose size is larger than conventional drainage paths.

3.2 *Input as vector data form*

The concept of streamline models using the vector data form as input is to draw the continuous flowing path by determining certain information from the solution of a differential equation. This information includes the unknown elevation and slope of each position inside the region considered, as determined from the particular solutions of the partial differential equations (PDE) with the contour lines of the digital image for the boundary conditions. All points along a contour line have the same elevation. The numbers of contour lines are directly proportional to the slope of the terrain, i.e., a hill landscape gives many more contour lines than a flat area.

In recent years, analytical techniques: separation of variables, eigenfunction expansion, similarity transform, Laplace transform, Fourier transform, and Green's functions, have been used to solve the particular solution of the PDE. Only some problems involving the basic geometrical shape of contour lines are able to be carried out analytically. However, computer technology has increasingly been used to assist in generating streamlines from more complex geometrical shapes of contour lines using numerical techniques [49]. Numerical techniques are mainly divided into two groups. The first one is the reformulation of the numerical solutions on the differential forms of fluid transport equations, whereas the second one is derived from the integral forms.

For the differential form, the finite difference method (FDM) was the first method that was used to study the flow of oil and geothermal water. This method formed the unknown variables with many

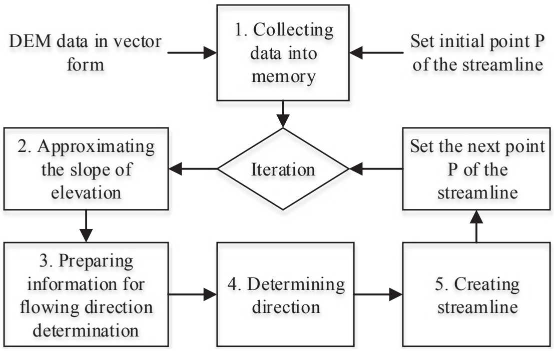


Figure 4: Process of semi-analytical solution.

rectangular segments of the desired domain on contour lines. The unknown variables relating to each other in equations of difference, were then rearranged into a matrix. This matrix is sparse and banded so that the inversion of linear equation system can be fast and simple. The limitation of this method is the inability of rectangular segments modeling the region between contour lines. An interesting method proposed to eliminate this problem was the finite element method (FEM) [50]. This method constructed the unknown variables with many triangular segments instead. It made the FEM method more accurate than the FDM method [2]. However, as the number of segments increases, the process consumes a significantly greater amount of time [3].

For the integral forms, numerical techniques are applied to solve the boundary integral equation using the Boundary Element Method (BEM). The advantage of this method compared with FDM and FEM is the absence of any need for rectangular or triangle modeling in the region between contour lines, since it is able to formulate functions of elevation from the contour lines directly. The PDE of this method is in Laplace form, which is not the time-varying function and is consistent with the fact that the direction of flow changes gradually in response to the shape of the terrain. The combination of Laplace's equation and the BEM gives solutions which are semi-analytical [9], [51]–[53]. The process of flowing direction model with this method which is consisted of five steps is detailed in Figure 4.

1. Collecting the elevation data into memory: The first step is to rearrange the elevation data collected in the vector form of sampled coordinates and to select which samples are used. The contour lines representing the earth's surface over the region are typically overly dense, and also oversampled. Some of the contour lines can be discarded entirely, and some of the samples on the remaining contours can also be discarded. It is only necessary to retain enough samples to reconstruct the contour smoothly from them. For sharp curves more samples are required than for gentle curves. Carefully selecting which samples to keep decreases the amount of memory required and increases the efficiency of the model.

2. Evaluating slope: The slope of each coordinate inside a region of interest is evaluated by the differential function of the elevation function, which itself is the semi-analytical solution of boundary integral equation along the contour line. The elevation function is

$$h(\mathbf{P}) = \sum_{j=1}^{N} \int_{S_j} a(\mathbf{P},\mathbf{Q})\big(\mathbf{J}(\mathbf{Q})\cdot\mathbf{n}(\mathbf{Q})\big)ds_Q - \sum_{j=1}^{N} \int_{S_j} b(\mathbf{P},\mathbf{Q})h(\mathbf{Q})ds_Q \quad (2)$$

where $h(\mathbf{P})$ and $h(\mathbf{Q})$ are respectively the particular solution of the interpolated elevation at point **P** and the sampled elevation at point **Q** along the contour S_j. $\mathbf{J}(\mathbf{Q})\cdot\mathbf{n}(\mathbf{Q})$ is the slope along the unit normal vector $\mathbf{n}(\mathbf{Q})$ (for more detail see [9]). $a(\mathbf{P}, \mathbf{Q})$ and $b(\mathbf{P}, \mathbf{Q})$ are the shape functions of the distance between **P** the **Q** and points that are equal to

$$a(\mathbf{P},\mathbf{Q}) = \frac{1}{2\pi}\ln|\mathbf{P}-\mathbf{Q}|$$

$$b(\mathbf{P},\mathbf{Q}) = \frac{1}{2\pi|\mathbf{P}-\mathbf{Q}|^2}(\mathbf{P}-\mathbf{Q})\cdot\mathbf{n}(\mathbf{Q})$$

The slope at point **P** is obtained by the differentiation of equation (2) as

$$\nabla_{\mathbf{P}} h(\mathbf{P}) = \sum_{j=1}^{N} \int_{S_j} c(\mathbf{P},\mathbf{Q})\big(\mathbf{J}(\mathbf{Q})\cdot\mathbf{n}(\mathbf{Q})\big)ds_Q - \sum_{j=1}^{N} \int_{S_j} d(\mathbf{P},\mathbf{Q})h(\mathbf{Q})ds_Q \quad (3)$$

and the rate of change of the slope at point **P** is further obtained by the differentiation of equation (3) as

$$\nabla_{\mathbf{P}}\left[\nabla_{\mathbf{P}} h(\mathbf{P})\right] = \sum_{j=1}^{N} \int_{S_j} e(\mathbf{P},\mathbf{Q})\big(\mathbf{J}(\mathbf{Q})\cdot\mathbf{n}(\mathbf{Q})\big)ds_Q - \sum_{j=1}^{N} \int_{S_j} f(\mathbf{P},\mathbf{Q})h(\mathbf{Q})ds_Q \quad (4)$$

where, $c(\mathbf{P}, \mathbf{Q})$, $d(\mathbf{P}, \mathbf{Q})$, $e(\mathbf{P}, \mathbf{Q})$ and $f(\mathbf{P}, \mathbf{Q})$ are the shape functions that are equal to

$$c(\mathbf{P},\mathbf{Q}) = \frac{1}{2\pi}\frac{\mathbf{P}-\mathbf{Q}}{|\mathbf{P}-\mathbf{Q}|^2}$$

$$d(\mathbf{P},\mathbf{Q}) = \frac{\mathbf{n}(\mathbf{Q})}{|\mathbf{P}-\mathbf{Q}|^2} - 2\frac{(\mathbf{P}-\mathbf{Q})\bullet\mathbf{n}(\mathbf{Q})}{|\mathbf{P}-\mathbf{Q}|^4}(\mathbf{P}-\mathbf{Q})$$

$$e(\mathbf{P},\mathbf{Q}) = \frac{1}{2\pi}\left\{\frac{\mathbf{I}}{|\mathbf{P}-\mathbf{Q}|^2} - 2\frac{(\mathbf{P}-\mathbf{Q})^2}{|\mathbf{P}-\mathbf{Q}|^4}\right\}$$

$$f(\mathbf{P},\mathbf{Q}) = \frac{1}{2\pi}\frac{-2}{|\mathbf{P}-\mathbf{Q}|^3}\left\{\frac{\mathbf{n}(\mathbf{Q})(\mathbf{P}-\mathbf{Q})}{|\mathbf{P}-\mathbf{Q}|} + \frac{(\mathbf{P}-\mathbf{Q})\mathbf{n}(\mathbf{Q})}{|\mathbf{P}-\mathbf{Q}|} + \frac{(\mathbf{P}-\mathbf{Q})\mathbf{n}(\mathbf{Q})}{|\mathbf{P}-\mathbf{Q}|}\left(\mathbf{I} - 4\frac{(\mathbf{P}-\mathbf{Q})^2}{|\mathbf{P}-\mathbf{Q}|^2}\right)\right\}$$

3. Preparing information for the flowing direction determination: This step is to generate the information to identify the direction of water flow at a local origin (or starting) point **P** by using the calculated results from previous step: interpolated elevation $h(\mathbf{P})$, slope $\nabla_{\mathbf{P}} h(\mathbf{P})$, and differentiation of slope $\nabla_{\mathbf{P}}\nabla_{\mathbf{P}} h(\mathbf{P})$.This information represents the continuous function $h(r, \theta)$ of elevation data in the neighbourhood (r, θ) surrounding point **P**. The function h is a solution to Laplace's equation, which can be expanded in polar coordinates as an infinite series $h(r,\theta) = a_0 + \sum_{m=1}^{\infty} r^m (a_m \cos m\theta + b_m \sin m\theta)$ and expressed in an approximate finite series expansion as

$$h(r,\theta) = a_0 + r(a_1 \cos\theta + b_1 \sin\theta) + r^2(a_2 \cos 2\theta + b_2 \sin 2\theta);\ r < r_0 \quad (5)$$

where a_0, a_1, a_2, b_1 and b_2 are coefficients. The accuracy of $h(r, \theta)$ depends on the size of the circle of radius r.

4. Determining direction: This step is to determine the continuous direction of flowing path from point **P**. This continuous direction specifies the connection of flowing path from the origin point to any points along the edge of circle with radius r. The criteria used to judge the flowing direction is based on the steepest slope that means the absolute minimum of elevation function $h(r, \theta)$. Therefore, θ min can be solved by the differentiation of $h(r, \theta)$ with respect to θ as

$$\frac{\partial h(r,\theta)}{\partial\theta} = r_0(-a_1 \sin\theta + b_1 \cos\theta) + 2r_0(-a_2 \sin 2\theta + b_2 \cos 2\theta) = 0 \quad (6)$$

Note that the possible roots obtained from equation (6) consist of 4 values: the absolute maximum,

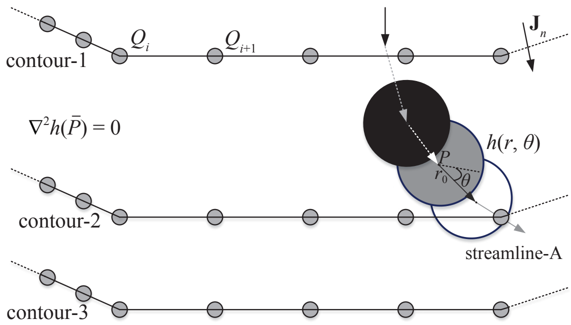


Figure 5: Flowing direction model by vector data input.

the absolute minimum, the relative maximum, and the relative minimum. The flowing direction from point **P** can feature as the forks when the absolute minimum and the relative minimum occur at different angles.

5. Creating streamline: The final step is to construct the line of flowing path from many continuous connection points. It is an iterative process in identifying the next angle of flowing from the starting point $\mathbf{P}(x, y)$ coordinate by θ min. The identified point is the starting point for the next iteration. The coordinate of the next flowing point is calculated in

$$x' = x + r_0 \cos\theta_{\min}$$
$$y' = y + r_0 \sin\theta_{\min} \quad (7)$$

where r_0 is taken as the distance between point $\mathbf{P}_i$ and next point $\mathbf{P}_{i+1}$. It can be clearly seen that the streamline continuity depends on the magnitude of r_0 and the rate of change of the angle θ. According to all five steps, the flowing direction is thus obtained as illustrated in Figure 5.

Two different concepts in modeling the flowing direction lines by using the elevation data in raster and vector data forms are briefly detailed. The model whose input data are in raster form is constructed based on image processing techniques. Thus, the elevation data of each pixel is used to continuously draw the segment of line from the starting pixel to the adjacent pixel to form a line representing the flowing direction by means of D8 algorithm. The model whose input data are in vector form is constructed based on the solution of PDE. The flowing direction of this model is controlled by the shape of contour lines. The advantage and drawback of each model is further discussed by comparing the numerical outputs of the two methods.

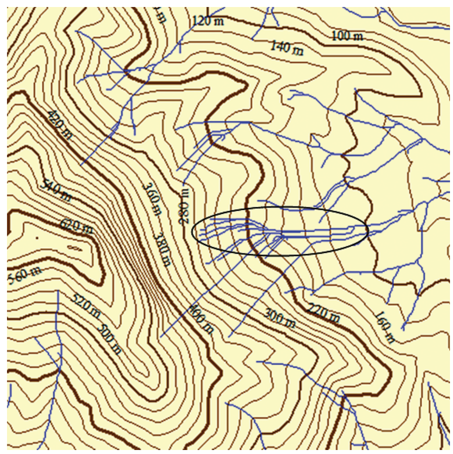
4 Numerical Results and Discussion

To investigate the accuracy of streamline models provided by two concepts: raster and vector forms of the input data, streamline models using both the D8 algorithm and semi-analytical methods have been applied to the same terrain. The results from both methods are compared and discussed for their important features. For the raster form, the process based on D8 algorithm in the Global Mapper program is used. It is a geographic information system (GIS) software package developed by Blue Marble Geographics on Microsoft Windows [54]. For the vector form, the semi-analytical method exploits the numerical solution of the BEM method [9]. The input data used in this investigation are the elevation data of DEM images of Khao Luang Mountain, which is situated in the same general region as the authors' university. This area is chosen since it covers the complex landscape as illustrated in Figure 6(a). Moreover, this area is often confronted by flooding events varying from severe to full disaster, and is of particular interest to the office of the Royal Development in Nakhon Si Thammarat province through a project, to help the people who live and encounter danger in the area, established by King Bhumibol Adulyadej in 1992 [55]. The DEM image is transformed as a planar slope of elevation data as shown in Figure 6(a). The small site of Khlong Tha Di (Tha Di canal) is selected to demonstrate the numerical results of streamlines since it has a problem associated with the discontinuity of streamlines.

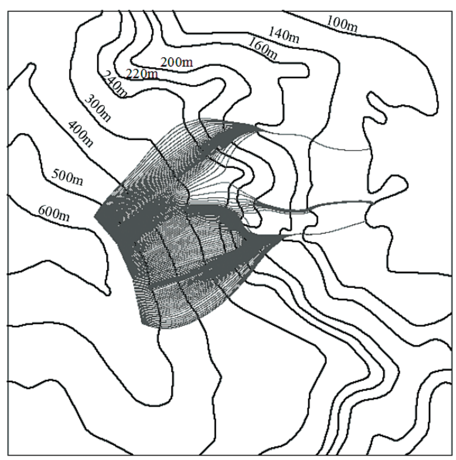
The simulated results from D8 algorithm shown in Figure 6(b) present the locations of many drainage and fork links. It is clearly seen that the streamlines which run between the contour lines 200 to 140 meters correspond to the same location of canal as in Figure 6(a). Further upstream, between 260 to 200 meters, the streamlines show more detail of the small drainage links. The information of these small drainage links disappears in Figure 6(a) since they cannot be directly sensed by the technique of DEM creation. The Global Mapper program is widely accepted as the standard tool for predicting streamlines in these hilltop areas.

For the numerical results from semi-analytical method as illustrated in Figure 6(c), the streamlines between the contour lines of 200m to 140m represent the canal location corresponding to Figure 6(a) while the greater detail of small drainage links are evident

(a) DEM image



(b) Based on D8 algorithm



(c) Based on BEM

Figure 6: Input data and output data of the streamline models.

between 400 and 260 meters. The results also provides the low density of streamlines on the contour lines of 400 to 260 meters that are suitable for using this model in the top and bottom of hill landscape.

The results from both streamline models are compared to each other to investigate their features. It can be clearly seen that they can generate similar paths from the top of hill to the same canal. However, the model using D8 algorithm is unable to provide the very small drainage links approaching the ridge of the hill. Moreover, the drawbacks of this model compared with another model are: (i) consuming lots of memory in process of flat terrain case, (ii) difficult to adjust the different degree of elevation to lower complexity, and (iii) providing exaggerated flowing direction along the axes of grid.

The streamline models based on semi-analytical methods can provide very fine drainage links over the hill region and more continuous streamlines than the models of D8 algorithm. Thus, the semi-analytical method or the method using input data in vector form is suggested to be used in enhancing the smoothness and continuity of streamlines models, for the size of catchment area estimation, and situations where the flow rate varies over time.

5 Conclusions

According to the preceding discussion, a selective survey of computational imaging was presented. Attention has been focused on the discontinuity of streamlines generated by image processing techniques as implemented by the semi-analytical method compared to the D8 algorithm. It began with the review of streamline model creation processes that consisted of a sequence of three phases: digital elevation model (DEM), interpolation model, and streamline model. The first two phases: DEM and interpolation model were briefly detailed in concepts of each model and focused on the various techniques to increase resolution of the elevation data as the input data for the streamline model in raster and vector forms. For the raster form, the D8 algorithm was exploited whereas the analytical solution of the boundary integral equation was used for the vector form. The numerical results of both methods were carried out and compared to investigate the advantages and drawbacks of each method. The results from the semi-analytical method were very much more detailed drainage links over the hill region and more continuous streamlines than those of the D8 algorithm. Furthermore, the drawbacks of the D8 algorithm compared with the semi-analytical method were consuming lots of memory in process of flat terrain case, difficult to adjust the different degree of elevation to lower complexity, and providing exaggerated flowing direction along the axes of grid. Consequently, the semi-analytical method is a most promising candidate for further development in enhancing the smoothness and continuity of streamlines models, for the estimation of the size of the catchment area, and for determining the time-varying rate of flow.

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