Report Number: WUCS-93-8

1993-01-01

Effective Loss of Multiplexed ATM Cell Streams

Seyyed M-R Mahdavian and Andreas D. Bovopoulos

Cell loss is an inherent problem of ATM networks. The magnitude of the service degeneration caused by cell loss depends on the application and loss distribution. This paper introduces a new performance criterion, called effective loss, which can quantitatively measure this degradation. Effective loss is particularly suitable for block-oriented transmissions, such as file transfer applications, but can also be applied to a broad range of other applications. In this paper the effective loss measure is applied to the study of the effectiveness of bandwidth reservation mechanisms in an ATM multiplexer. Numerical results demonstrate circumstances under which bandwidth reservation improves... **Read complete abstract on page 2.**

Effective Loss of Multiplexed ATM Cell Streams

Seyyed M-R Mahdavian and Andreas D. Bovopoulos

Complete Abstract:

Cell loss is an inherent problem of ATM networks. The magnitude of the service degeneration caused by cell loss depends on the application and loss distribution. This paper introduces a new performance criterion, called effective loss, which can quantitatively measure this degradation. Effective loss is particularly suitable for block-oriented transmissions, such as file transfer applications, but can also be applied to a broad range of other applications. In this paper the effective loss measure is applied to the study of the effectiveness of bandwidth reservation mechanisms in an ATM multiplexer. Numerical results demonstrate circumstances under which bandwidth reservation improves performance as well as circumstances in which it degrades performance.

Effective Loss of Multiplexed ATM Cell Streams

Seyyed M-R Mahdavian and Andreas D. Bovopoulos

WUCS-93-08

June 1993

Department of Computer Science
Washington University
Campus Box 1045
One Brookings Drive
St. Louis MO 63130-4899

This work was supported by the National Science Foundation, and an industrial consoritum of Ascom Timeplex, Bellcore, Bell Northern Research, Digital Equipment Corporation, Gold Star Communications, Italtel SIT, NEC America, NTT and SynOptics Communications.

Effective Loss of Multiplexed ATM Cell Streams

Seyyed M-R Mahdavian and Andreas D. Bovopoulos
Computer and Communications Research Center
Washington University, Saint Louis, USA

Abstract

Cell loss is an inherent problem of ATM networks. The magnitude of the service degradation caused by cell loss depends on the application and loss distribution. This paper introduces a new performance criterion, called effective loss, which can quantitatively measure this degradation. Effective loss is particularly suitable for block-oriented transmissions, such as file transfer applications, but can also be applied to a broad range of other applications. In this paper the effective loss measure is applied to the study of the effectiveness of bandwidth reservation mechanisms in an ATM multiplexer. Numerical results demonstrate circumstances under which bandwidth reservation improves performance as well as circumstances in which it degrades performance.

1. Introduction

The Asynchronous Transfer Mode (ATM) has been recommended by the International Telegraph and Telephone Consultative Committee (CCITT) as a suitable transport protocol for the Broadband Integrated Services Digital Network (B-ISDN). It transfers the information through fixed length packets of 48 bytes (plus 5 bytes of header information) called *cells*. Unlike the synchronous transfer mode, it is able to transfer an arbitrary bit rate (in contrast to fixed predetermined rates) and can take advantage of the statistical behavior of the sources to share communication resources effectively among users.

There are some deficiencies associated with ATM, however. Because of the statistical nature of the sources, there is a non-zero probability that congestion will occur, i.e. there will be too many cells requesting transmission at the same time. If there is no place to store excessive cells, cell loss is inevitable. To prevent cell loss, one should either demand that user sources transmit at constant rates (which is incompatible with the philosophy of

This work was supported by the Ministry of Culture and Higher Education of Iran, the National Science Foundation, and an industrial consortium of Ascom Timeplex, Bellcore, BNR, DEC, Goldstar , Italtel SIT, NEC America, NTT, SynOptics Communications.

Computer & Communications Research Center, Bryan Hall 405 – Campus Box 1115, Washington University, One Brookings Drive, St. Louis MO 63130-4899. e-mail: seyyed@wuccrc.wustl.edu, andreas@patti.wustl.edu

ATM) or take other measures, such as peak rate allocation, which usually lead to a waste of communication resources and/or unacceptable delays. Therefore *controlling* cell loss and delay has been investigated by a number of researchers. This control aims to reduce, not eliminate, cell loss and delay problems while maintaining a fair usage of communication resources. One common way of reducing cell loss is by using buffers at switch elements or concentrators. Buffers can hold excessive cells until previous ones are processed at the limited output rate. Incoming cells which find the buffers full are lost.

Cell loss has a detrimental effect on the quality of service, but the scope and extent of this effect is highly application dependent. A common measure of transmission degradation caused by cell loss is the cell loss rate or cell loss probability. However this gives only a rough estimate of the quality of service and thus is not sufficient. Depending on the distribution of cell loss and the nature of application, the perceived quality of a certain transmission can vary substantially for a fixed cell loss rate. For a review of different problems and issues regarding the performance measurement of telecommunication systems, the reader is referred to [9].

Data information is usually transmitted in the form of blocks. The information within a block is highly correlated, in the sense that any loss of information in the block results in the ruin of the information content of the whole block (which usually has to be retransmitted). For services which result in the transmission of information in blocks (bursts), such as in file transfers, the emphasis has been in trying to support multiplexing schemes which preserve bursts. Whereas these approaches aim at addressing the intuitive requirements for integrity of the burst, none so far has been able to correlate the performance requirements to the intrinsic information content of the media stream in a general and direct way. In this paper the fact that cell loss cannot describe the information loss for a particular cell stream is recognized. To cope with this, a new criterion is introduced. This criterion, called the *effective loss* , measures the effect of lost cells to the integrity of the information of the whole burst.

To show the effectiveness of this measure, a simple discrete queueing model representing the operation of a multiplexer is analyzed. Using this model, the performance of a special protocol called *bandwidth reservation* is evaluated using effective loss as the performance measure. The idea of reservation has been used by a number of researchers. Two important references are [2] and [8]. While it is a fact that reservation always increases the cell loss rate (compared to plain statistical multiplexing), it is demonstrated that under certain circumstances, the effective loss is diminished by using a reservation technique.

The rest of the paper is organized as follows. In Sections 2, 3 and 4 the general definition of effective loss is given. Section 5.1 introduces the multiplexer model. Section 5.2 examines the performance of the multiplexer when the background traffic is assumed to be non-random. The effective loss is computed for reservation and non-reservation mechanisms. In Section 5.3, the previous analysis is extended for the case in which the background traffic is assumed to be random. An important special case of random traffic, namely the Markovian traffic, is considered in Section 5.4. Finally Section 5.5 is devoted to numerical results and interpretations.

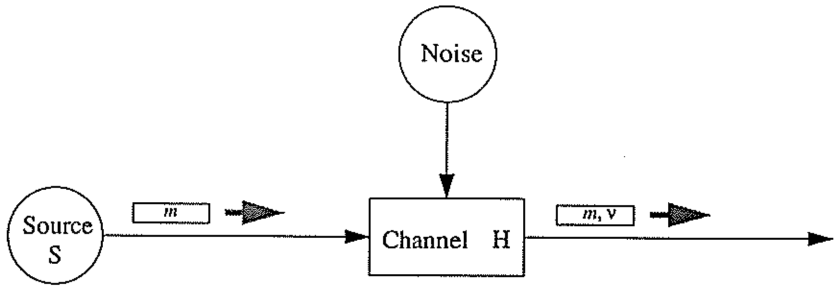


Figure 1: The loss process model in the definition of Effective Loss

2. Definition of Effective Loss

Consider an information flow which is susceptible to impairments caused by the inherent imperfection of the transmission channel. Suppose that a block of m units of information is sent by some source, of which ν units are lost during the transmission process. Although $m - \nu$ units of information are successfully transmitted, in many applications it is not a useful transmission. The cost of restoring the complete information from the impaired version may be higher than the cost of retransmitting it. Therefore in some applications, partial cell loss in a block of cells has the same effect as the loss of a much higher number of cells, possibly of the whole block. File transfer is an example of such application. While much less of a problem in voice transmission, the faulty transmission of a phoneme may result in the misinterpretation of an utterance, and the loss of a word may result in the corruption of an entire phrase. Similarly in picture transmission, a slight loss of information may result in the loss of picture details, which in turn may necessitate the retransmission of the whole picture.

In this paper a random variable is represented by a symbol with a tilde $(\tilde{})$ on top, while a particular value of a random variable is represented by the symbol alone. For example $\tilde{m}$ represents a random variable, while m represents a particular value of this random variable.

To account for the correlation between various components of an information flow, the first step is to recognize *blocks* of information within the flow. Each block is assumed to be independent from all other blocks, but within a single block, any loss of information affects the informational value of the whole block. A block of information can be thought of as a block of data in file transfers, an utterance in speech transmission, or a frame in picture transmission. To a block of m units of information, of which ν units are lost, we assign a cost of $f(m,\nu)$, where f is an arbitrary cost function. It is therefore assumed that the cost depends on the magnitude of loss, regardless of the precise location of the loss within the block. Now suppose that a source S transmits blocks of information containing $\tilde{m}$ units of information which we shall refer to as the *size* of the block. $\tilde{m}$ is assumed to be a random variable with distribution $P_{\tilde{m}}(m)$, which is the same distribution for all blocks. Suppose that each block passes through a noisy channel and loses $\tilde{\nu}$ units of information. $\tilde{\nu}$ is assumed to be a random variable with distribution $P_{\tilde{\nu}}(\nu)$. This process is depicted in Figure 1. For simplicity, the notation $P(m,\nu)$, $P(m)$ and $P(\nu)$ will be used instead of $P_{\tilde{m},\tilde{\nu}}(m,\nu)$, $P_{\tilde{m}}(m)$ and $P_{\tilde{\nu}}(\nu)$, and all three distributions will be assumed to be the same for all blocks. The effective loss of the source S passing through the channel H is then defined as:

$$\eta_{S,H} \stackrel{\text{def}}{=} \frac{\mathrm{E}f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})}$$

where E is the expectation over the probability space of $\tilde{m}$ and $\tilde{\nu}$. To simplify the notation, we shall however drop the subscript S, H. If $\tilde{m}$ and $\tilde{\nu}$ are discrete random variables, the effective loss becomes:

$$\eta = \frac{\sum_{m,\nu} f(m,\nu)\mathrm{P}(m,\nu)}{\sum_{m,\nu} f(m,m)\mathrm{P}(m,\nu)} = \frac{\sum_{m,\nu} f(m,\nu)\mathrm{P}(m,\nu)}{\sum_{m} f(m,m)\mathrm{P}(m)}$$

where $\mathrm{P}(m,\nu)$ is the probability that a block has m units of information and ν of them are lost during the transmission process, and $\mathrm{P}(m)$ is the marginal probability. Effective loss has the following properties:

1. If f is a non-decreasing function of ν (which is natural because more loss implies more cost), then $0 \leq \eta \leq 1$. The first inequality is trivial. The second is true because $f(m,\nu) \leq f(m,m)$ for all m,ν and therefore $\mathrm{E}f(\tilde{m},\tilde{\nu}) \leq \mathrm{E}f(\tilde{m},\tilde{m})$.

2. If one defines the cost function to be $f(m,\nu) = \nu$, i.e. the cost of ν losses is just ν regardless of the size of the block, then $\eta = \mathrm{E}\tilde{\nu}/\mathrm{E}\tilde{m}$ which is simply the loss rate. This is important because it shows that under the worst case condition where no natural blocks of information can be identified, an arbitrary block division together with the above cost function gives the usual loss rate measure.

3. If f represents the amount of information which is *effectively lost* in a block, e.g. the size of the block in file transfers when part of the block is lost due to channel impairments, then the effective loss represents the *average percentage* of the information flow which is *effectively lost.* This property will be presented in mathematical form in Proposition 1.

PROPOSITION 1. *If the discrete-time process $(\tilde{m}_i, \tilde{\nu}_i)$, where i represents block number, is an ergodic process, then the following is true almost surely:*

$$\eta = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} f(m_i,\nu_i)}{\sum_{i=1}^{N} f(m_i,m_i)}$$

Proof:

$$\begin{aligned}\eta &= \frac{\mathrm{E}f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})} \stackrel{\text{a.s.}}{=} \frac{\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} f(m_i,\nu_i)}{\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} f(m_i,m_i)} \\ &= \lim_{N\to\infty} \frac{\frac{1}{N}\sum_{i=1}^{N} f(m_i,\nu_i)}{\frac{1}{N}\sum_{i=1}^{N} f(m_i,m_i)} = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} f(m_i,\nu_i)}{\sum_{i=1}^{N} f(m_i,m_i)}\end{aligned}$$

REMARK 1. $\sum_{i=1}^{N} f(m_i, \nu_i)$ *is the total sum of costs caused by losses incurred to N blocks of transmitted information.* $\sum_{i=1}^{N} f(m_i, m_i)$ *is the total cost if all the information content of the N blocks were lost during transmission. Therefore the ratio* $\sum_{i=1}^{N} f(m_i, \nu_i)/\sum_{i=1}^{N} f(m_i, m_i)$ *intuitively represents the* percentage *of the* effective loss *of information during transmission through a noisy channel.*

Example 1: Suppose that the source transmits blocks of $\tilde{m}$ cells through an ATM network, $\tilde{\nu}$ of which are lost due to buffer overflow at the nodes. Suppose that the transmission corresponds to file transfers for which the cost function is defined as follows:

$$f(m, \nu) = \begin{cases} 0 & \text{if } \nu = 0 \\ m & \text{if } \nu > 0 \end{cases}$$

The effective loss is then computed as follows:

$$\eta = \frac{\sum_m \sum_{\nu=0}^{m} \mathrm{P}(m)\mathrm{P}(\nu \mid m) f(m, \nu)}{\sum_m \mathrm{P}(m) f(m, m)} \tag{1}$$

$$= \frac{\sum_m \sum_{\nu=1}^{m} m\mathrm{P}(m)\mathrm{P}(\nu \mid m)}{\sum_m m\mathrm{P}(m)} \tag{2}$$

$$= \frac{\sum_m m\mathrm{P}(m) \sum_{\nu=1}^{m} \mathrm{P}(\nu \mid m)}{\sum_m m\mathrm{P}(m)} \tag{3}$$

$$= \frac{\sum_m m\mathrm{P}(m)[1 - \mathrm{P}(\tilde{\nu} = 0 \mid m)]}{\sum_m m\mathrm{P}(m)} \tag{4}$$

Note that if $\mathrm{P}(\tilde{\nu} = 0 \mid m)$ is independent of m, then the effective loss is simply equal to $1 - \mathrm{P}(\tilde{\nu} = 0 \mid m) = 1 - \mathrm{P}(\tilde{\nu} = 0)$, and this result is independent of the distribution of $\tilde{m}$. The same result holds if $\tilde{m}$ is equal to a fixed number M with probability one. Now suppose that $\tilde{m}$ is Poisson distributed with mean λ and that each cell in a block may be lost with equal probalility ϵ, independent of the loss of other cells. Then,

$$\mathrm{P}(\nu \mid m) = \begin{pmatrix} m \\ \nu \end{pmatrix} \epsilon^{\nu} (1 - \epsilon)^{m-\nu}$$

We wish to compute both the cell loss rate $\rho = \mathrm{E}\tilde{\nu}/\mathrm{E}\tilde{m}$ and the effective loss η for the above mentioned cost function.

$$\rho = \frac{\mathrm{E}(\tilde{\nu})}{\mathrm{E}(\tilde{m})} = \frac{\mathrm{E}_{\tilde{m}}\mathrm{E}_{\tilde{\nu}|\tilde{m}}(\tilde{\nu})}{\lambda} = \frac{\mathrm{E}(\tilde{m}\epsilon)}{\lambda} = \frac{\lambda\epsilon}{\lambda} = \epsilon$$

To compute η we use $\mathrm{P}(\tilde{\nu} = 0 \mid m) = (1 - \epsilon)^m$ in Equation 4:

$$\begin{aligned} \eta &= \frac{1}{\lambda} \sum_{m=0}^{\infty} m \frac{\lambda^m e^{-\lambda}}{m!} [1 - (1 - \epsilon)^m] \\ &= \frac{1}{\lambda} \left(\lambda - e^{-\lambda\epsilon} \sum_{m=0}^{\infty} m \frac{[\lambda(1 - \epsilon)]^m e^{-\lambda(1-\epsilon)}}{m!} \right) \\ &= \frac{1}{\lambda} \left(\lambda - e^{-\lambda\epsilon} [\lambda(1 - \epsilon)] \right) \\ &= 1 - (1 - \epsilon) e^{-\lambda\epsilon} \end{aligned}$$

If $\lambda\epsilon \ll 1$, i.e. the average number of lost cells per burst is much smaller than 1, then:

$$\eta \approx 1 - (1-\epsilon)(1-\lambda\epsilon) \approx \epsilon(1+\lambda)$$

This shows that in this case, the effective loss is higher than the loss rate by a factor of approximately $1+\lambda$, where λ is the average burst length.

3. Conditional Effective Loss

Consider the cell loss due to buffer overflow in ATM networks. The state of the buffer and the traffic condition at the time of the arrival of a block of cells directly affects the loss distribution, and thus $P(\nu \mid m)$ depends on the statistics of the traffic and the state of the buffer and cannot be defined. In general, if the description of the loss process involves random variables distinct from $\tilde{m}$ and $\tilde{\nu}$, and if these additional random variables have different distributions for different blocks, then $P(m,\nu)$ and η cannot be defined for the transmission. In cases in which additional random variables may affect the loss process, one needs to consider the *conditional effective loss*. Conditional effective loss helps in two ways: First it is an easier step in the computation of the (unconditional) effective loss when the latter is defined. Second it represents an effective loss for a subsequence of blocks for which the additional events happen, even if the unconditional effective loss is not defined for the sequence. Suppose that $\tilde{m}$ and $\tilde{\nu}$ depend on another random variable $\tilde{k}$ through the joint distribution $P(m,\nu,k)$. Let $\mathrm{E}_{k_1,k_2,\ldots} f(\tilde{m},\tilde{\nu}) = \mathrm{E}_{\tilde{m},\tilde{\nu}|\tilde{k}_1=k_1,\tilde{k}_2=k_2,\ldots} f(\tilde{m},\tilde{\nu})$. If $P(m,\nu \mid \tilde{k}=k)$ is independent of the block number, then the conditional effective loss is defined as:

$$\eta_k \stackrel{\text{def}}{=} \frac{\mathrm{E}_k f(\tilde{m},\tilde{\nu})}{\mathrm{E}_k f(\tilde{m},\tilde{m})}$$

where E_k is the expectation over the probability space of $\tilde{m}$ and $\tilde{\nu}$ conditioned on $\tilde{k}=k$. Similarly if $\tilde{k}_1, \tilde{k}_2, \ldots$ are random variables and $P(m,\nu \mid k_1,k_2,\ldots)$ is independent of the block number, then the multiply conditioned effective loss is defined as:

$$\eta_{k_1,k_2,\ldots} \stackrel{\text{def}}{=} \frac{\mathrm{E}_{k_1,k_2,\ldots} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1,k_2,\ldots} f(\tilde{m},\tilde{m})}$$

where $\mathrm{E}_{k_1,k_2,\ldots}$ is the expectation over the probability space of $\tilde{m}$ and $\tilde{\nu}$ conditioned on $\tilde{k}_1=k_1, \tilde{k}_2=k_2, \ldots$

PROPOSITION 2. *If η_k is defined, $P(k)$ is independent of the block number, and if for every block, $\tilde{k}$ and $\tilde{m}$ are independent random variables, then η exists and $\eta = \mathrm{E}_{\tilde{k}}(\eta_{\tilde{k}})$.*

Proof: If $\tilde{k}$ and $\tilde{m}$ are independent, then $P(m \mid k) = P(m)$ and therefore $\mathrm{E}_k f(\tilde{m},\tilde{m}) = \mathrm{E} f(\tilde{m},\tilde{m})$. From the definition of η_k, $P(m,\nu \mid k)$ is independent of the block number. If further $P(k)$ is independent of the block number, so are $P(m,\nu,k) = P(m,\nu \mid k)P(k)$ and $P(m,\nu) = \sum_k P(m,\nu,k)$. Therefore:

$$\eta = \frac{\mathrm{E} f(\tilde{m},\tilde{\nu})}{\mathrm{E} f(\tilde{m},\tilde{m})} = \frac{\mathrm{E}\mathrm{E}_k f(\tilde{m},\tilde{\nu})}{\mathrm{E}_k f(\tilde{m},\tilde{m})} = \mathrm{E}\left(\frac{\mathrm{E}_k f(\tilde{m},\tilde{\nu})}{\mathrm{E}_k f(\tilde{m},\tilde{m})}\right) = \mathrm{E}_{\tilde{k}}(\eta_{\tilde{k}})$$

The result of Proposition 2 can be easily extended to the multiply conditioned effective loss. The following proposition generalizes the effective loss, when conditioned on two random variables.

PROPOSITION 3. *If $\tilde{k}_1$ and $\tilde{k}_2$ are two random variables independent of $\tilde{m}$ and η_{k_1,k_2} is defined and $P(k_2 \mid k_1)$ is independent of the block number, then $\eta_{k_1} = \mathrm{E}_{\tilde{k}_2|k_1}(\eta_{k_1,\tilde{k}_2})$. If $P(k_1, k_2)$ is independent of the block number, then $\eta = \mathrm{E}_{\tilde{k}_1}(\eta_{\tilde{k}_1}) = \mathrm{E}_{\tilde{k}_2}(\eta_{\tilde{k}_2}) = \mathrm{E}_{\tilde{k}_1,\tilde{k}_2}(\eta_{\tilde{k}_1,\tilde{k}_2})$.*

Proof:

$$\begin{aligned}
\mathrm{E}_{\tilde{k}_2|k_1}\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{\nu}) &= \mathrm{E}_{\tilde{k}_2|\tilde{k}_1=k_1}\mathrm{E}_{\tilde{m},\tilde{\nu}|\tilde{k}_1=k_1,\tilde{k}_2=k_2} f(\tilde{m},\tilde{\nu}) \\
&= \mathrm{E}_{\tilde{m},\tilde{\nu}|\tilde{k}_1=k_1} f(\tilde{m},\tilde{\nu}) \\
&= \mathrm{E}_{k_1} f(\tilde{m},\tilde{\nu})
\end{aligned}$$

Since $\tilde{m}$ is independent from $\tilde{k}_1$ and $\tilde{k}_1$,

$$\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{m}) = \mathrm{E}_{k_1} f(\tilde{m},\tilde{m}) = \mathrm{E} f(\tilde{m},\tilde{m})$$

$$\begin{aligned}
\eta_{k_1} &= \frac{\mathrm{E}_{k_1} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1} f(\tilde{m},\tilde{m})} = \frac{\mathrm{E}_{\tilde{k}_2|k_1}\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{m})} \\
&= \mathrm{E}_{\tilde{k}_2|k_1}\left(\frac{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{m})}\right) = \mathrm{E}_{\tilde{k}_2|k_1}(\eta_{k_1,\tilde{k}_2})
\end{aligned}$$

$$\begin{aligned}
\eta &= \frac{\mathrm{E} f(\tilde{m},\tilde{\nu})}{\mathrm{E} f(\tilde{m},\tilde{m})} = \frac{\mathrm{E}_{\tilde{k}_1,\tilde{k}_2}\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{m})} \\
&= \mathrm{E}_{\tilde{k}_1,\tilde{k}_2}\left(\frac{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{k_1,k_2} f(\tilde{m},\tilde{m})}\right) = \mathrm{E}_{\tilde{k}_1,\tilde{k}_2}(\eta_{\tilde{k}_1,\tilde{k}_2})
\end{aligned}$$

$\eta = \mathrm{E}_{\tilde{k}_1}(\eta_{\tilde{k}_1})$ from Proposition 2. Similarly, one obtains $\eta = \mathrm{E}_{\tilde{k}_2}(\eta_{\tilde{k}_2})$ and $\eta_{k_2} = \mathrm{E}_{\tilde{k}_1|k_2}(\eta_{\tilde{k}_1,k_2})$.

REMARK 2. *A proposition similar to Proposition 1 can be written for η_k, except that one has to take the summation with respect to the subsequence of blocks for which $\tilde{k} = k$.*

Example 2: Consider the transmission model of Example 1 with the following difference: The source transmits two types of blocks, low priority blocks and high priority blocks, according to a Bernoulli random variable $\tilde{k}$ independent from $\tilde{m}$, such that $\tilde{k} = 1$ with probability p indicates a high priority block, and $\tilde{k} = 0$ with probability $1 - p$ indicates a low priority block. The channel parameter ϵ depends on the type of the transmitted block. $\epsilon = \epsilon_0$ for $\tilde{k} = 0$ and $\epsilon = \epsilon_1$ for $\tilde{k} = 1$. Applying the result of Example 1 and Proposition 2:

$$\begin{aligned}
\eta_0 &= 1 - (1-\epsilon_0)e^{-\lambda\epsilon_0} \\
\eta_1 &= 1 - (1-\epsilon_1)e^{-\lambda\epsilon_1} \\
\eta &= (1-p)\eta_0 + p\eta_1
\end{aligned}$$

One can interpret η_0 as the effective loss for low priority cells, η_1 as the effective loss for high priority cells, and η as the overall effective loss.

4. Unit Effective Loss

If different blocks in a transmission process have totally different statistics in terms of the information content of the block and the amount of lost information, then an overall effective loss for the transmission process might not make any sense. Yet the effective loss can help to study the transmission quality of a single block. Knowing the statistics of the information content and the lost information of a particular block, the effective loss can be computed as before:

$$\eta = \frac{\mathrm{E}f(\tilde{m}, \tilde{\nu})}{\mathrm{E}f(\tilde{m}, \tilde{m})}$$

Where $\tilde{m}$ and $\tilde{\nu}$ are random variables representing the information content and loss of the block under observation. We shall call this a *unit* effective loss.

5. Effective Loss of an ATM Cell Stream

In this section, a simple discrete queueing model representing the operation of a multiplexer is presented. Using this model, the performance of a special protocol called *bandwidth reservation* is evaluated using the effective loss as the performance measure. The idea of reservation has been used by a number of researchers [2] and [8]. While reservation always increases the cell loss rate (compared to plain statistical multiplexing), in the subsequent subsections it is demonstrated that under certain circumstances, the effective loss is diminished by using a reservation technique. The model used enables us to compute the joint probability distribution of the number of cell arrivals and cell losses. Cell loss probability has been computed before by many researchers for a number of models ([7, 6, 5]). By assuming independence between the loss of different cells, one can use these results to compute the cell loss distribution in a block of m arriving cells. However it is proven in [3, 4] that this is not always a good assumption, and a method is presented there to compute the exact cell loss distribution in a certain block. An exact cell loss distribution in a fixed time duration is also presented in [1]. However in these references, cell loss is computed for the aggregated input, not for individual channels. For the computation of the effective loss, the cell loss distribution for cells belonging to *one* channel under observation needs to be computed. All remaining channels are treated as background traffic (noise). The model presented in this paper makes it possible to compute the cell loss distribution on a per-channel basis, while all the other channels constitute some predetermined stochastic process with specified parameters.

5.1. Description of the Model

Consider a cell stream $\tilde{r}(n)$ passing through a buffer of size M, as depicted in Figure 2. $\tilde{r}(n)$ is a discrete-time stochastic process with values shown by letters B, R, N. If $\tilde{r}(n) = B$, a *Blue* cell enters the buffer at time n, provided that the buffer is not full, or else the cell is considered lost. A blue cell is a cell coming from the channel under observation. Similarly if $\tilde{r}(n) = R$, a *Red* cell enters the buffer provided that the buffer is not full. A red cell is

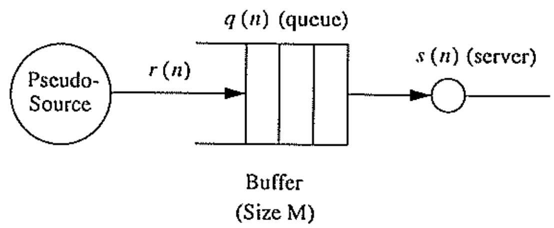


Figure 2: Illustration of the Model

a cell coming from a channel other than the channel under observation. From the point of view of this model, a red cell can be considered to be coming from the *background traffic*. If $\tilde{r}(n) = N$, no cell enters the buffer. We assume that the channel under observation is bursty, with two states active and idle. One active period is considered one block and the cell loss process is studied during an active period, beginning at time $n = 1$. The length $\tilde{L}_a$ of the active period is in general a random variable with distribution G_a. During an active period, it is assumed that $\tilde{r}(n)$ is independent from $\tilde{r}(m)$ for $m \neq n$, and can take different values according to the following table:

$\tilde{r}(n)$	Probability
B	$\alpha(n)$
R	$\tau(n)$
N	$1 - \alpha(n) - \tau(n)$

To simplify notation, we define $\rho(n) \stackrel{\text{def}}{=} \alpha(n) + \tau(n)$. If at time n, the buffer is not empty, a cell will be served (leave the buffer) with probability $\sigma(n)$ independent of other events. $\alpha(n)$, $\tau(n)$, and $\sigma(n)$ can be deterministic functions or stochastic processes. Different cases will be investigated in the sequel. $\alpha(n)$, $\tau(n)$, $\rho(n)$, and $\sigma(n)$ are time dependent probabilities representing the *rate* of the channel under observation, the rate of the background traffic, the total rate entering the buffer, and the service (departure) rate respectively.

5.2. Nonrandom Rates

Here we assume that $\alpha(n)$, $\tau(n)$ and $\sigma(n)$ are nonrandom fixed functions of time n. Since there is no stationary or cyclo-stationary assumption about the functions (or stochastic processes), there may not be an equilibrium queue length distribution. So we will assume (or condition on) the state of the queue at a time n_i and compute parameters such as the number of cells lost or the state of the queue at a later time n_f.

5.2.1. Queue Length Distribution. Of interest is the quantity $\mathcal{A}_{n_i,n_f}(q_{n_f} \mid q_{n_i})$, which is the probability that the queue length is q_{n_f} at time n_f given that the queue length is q_{n_i} at time n_i for $n_i < n_f$. Let $A_n(i,j) \stackrel{\text{def}}{=} \mathcal{A}_{n-1,n}(j \mid i)$. Then:

$$\left.\begin{array}{lcl} A_n(0,0) & = & \rho(n)\sigma(n) + (1-\rho(n)) \\ A_n(0,1) & = & \rho(n)(1-\sigma(n)) \\ A_n(0,j) & = & 0 \qquad j > 1 \end{array}\right\} \text{for } i = 0$$

$$\left.\begin{array}{lcl} A_n(i,i-1) & = & (1-\rho(n))\sigma(n) \\ A_n(i,i) & = & \rho(n)\sigma(n) + (1-\rho(n))(1-\sigma(n)) \\ A_n(i,i+1) & = & \rho(n)(1-\sigma(n)) \\ A_n(i,j) & = & 0 \qquad j > i+1 \text{ or } j < i-1 \end{array}\right\} \text{for } 0 < i < M$$

$$\left.\begin{array}{lcl} A_n(M,M) & = & \rho(n) + (1-\rho(n))(1-\sigma(n)) \\ A_n(M,M-1) & = & (1-\rho(n))\sigma(n) \\ A_n(M,j) & = & 0 \qquad j < M-1 \end{array}\right\} \text{for } i = M$$

To simplify the representation of the transition matrix, define:

$$\begin{array}{rcl} x(n) & \stackrel{\text{def}}{=} & \rho(n)\sigma(n) + (1-\rho(n))(1-\sigma(n)) \\ x_0(n) & \stackrel{\text{def}}{=} & \rho(n)\sigma(n) + (1-\rho(n)) \\ x_M(n) & \stackrel{\text{def}}{=} & \rho(n) + (1-\rho(n))(1-\sigma(n)) \\ y(n) & \stackrel{\text{def}}{=} & \rho(n)(1-\sigma(n)) \\ z(n) & \stackrel{\text{def}}{=} & (1-\rho(n))\sigma(n) \end{array}$$

Let A_n be the $(M+1) \times (M+1)$ matrix $[A_n(i,j)]$. Then:

$$A_n = \begin{pmatrix} x_0(n) & y(n) & & & & \\ z(n) & x(n) & y(n) & & 0 & \\ & z(n) & x(n) & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & 0 & & \ddots & x(n) & y(n) \\ & & & & z(n) & x_M(n) \end{pmatrix}$$

$\mathcal{A}_{n_i,n_f}(q_{n_f} \mid q_{n_i})$ in matrix form is computed using the above matrix as follows:

$$\mathcal{A}_{n_i,n_f} = \prod_{n=n_i+1}^{n_f} A_n$$

If the channel under observation produces a bursty stream with burst length (time) being a random variable $\tilde{L}_a$ with general distribution G_a, and we wish to know the queue length distribution at the end of the burst, its transition matrix can be found as follows:

$$\mathcal{A}_{0,\tilde{L}_a} = \sum_{l=1}^{\infty} G_a(l) \prod_{n=1}^{l} A_n$$

In the above formula the burst of length $\tilde{L}_a$ starts at time 1.

5.2.2. Cell Loss Distribution. Let $\mathrm{P}_{n_i,n_f}(m,\nu,q_{n_f} \mid q_{n_i})$ be the probability that from time n_i to n_f (including n_f but not n_i) the channel under observation sends m cells to the buffer, ν of which are lost, and that the number cells in the buffer at time n_f is q_{n_f}, given that at time n_i the number of cells in the buffer is q_{n_i}. This joint distribution can be computed recursively by first computing $\mathrm{P}_{n-1,n}(m,\nu,q_n \mid q_{n-1})$. The latter is found easily for all values of m, ν and q_n and is tabulated below. Any combination of m, ν and q_n which is not listed in the tables has probability zero.

for $0 < q_{n-1} < M$

m	ν	q_n	$\mathrm{P}_{n-1,n}(m,\nu,q_n \mid q_{n-1})$
0	0	q_{n-1}	$(1-\rho(n))(1-\sigma(n))+\tau(n)\sigma(n)$
0	0	$q_{n-1}+1$	$\tau(n)(1-\sigma(n))$
0	0	$q_{n-1}-1$	$(1-\rho(n))\sigma(n)$
1	0	q_{n-1}	$\alpha(n)\sigma(n)$
1	0	$q_{n-1}+1$	$\alpha(n)(1-\sigma(n))$

for $q_{n-1} = 0$

m	ν	q_n	$\mathrm{P}_{n-1,n}(m,\nu,q_n \mid q_{n-1})$
0	0	0	$(1-\rho(n))+\tau(n)\sigma(n)$
0	0	1	$\tau(n)(1-\sigma(n))$
1	0	0	$\alpha(n)\sigma(n)$
1	0	1	$\alpha(n)(1-\sigma(n))$

for $q_{n-1} = M$

m	ν	q_n	$\mathrm{P}_{n-1,n}(m,\nu,q_n \mid q_{n-1})$
0	0	M	$(1-\rho(n))(1-\sigma(n))+\tau(n)$
0	0	$M-1$	$(1-\rho(n))\sigma(n)$
1	0	M	$\alpha(n)\sigma(n)$
1	1	M	$\alpha(n)(1-\sigma(n))$

It suffices now to find the general recursion formula, $\mathrm{P}_{n_1,n_2}(\cdot,\cdot,\cdot \mid \cdot)$ in terms of $\mathrm{P}_{n_1,n_2-1}(\cdot,\cdot,\cdot \mid \cdot)$ and $\mathrm{P}_{n_2-1,n_2}(\cdot,\cdot,\cdot \mid \cdot)$. For $m,\nu \geq 1$:

$$
\begin{aligned}
&\mathrm{P}_{n_1,n_2}(m,\nu,q_{n_2} \mid q_{n_1}) = \\
&\quad \sum_q \mathrm{P}_{n_1,n_2-1}(m,\nu,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(0,0,q_{n_2} \mid q) \\
&\quad + \sum_q \mathrm{P}_{n_1,n_2-1}(m-1,\nu,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(1,0,q_{n_2} \mid q) \\
&\quad + \sum_q \mathrm{P}_{n_1,n_2-1}(m-1,\nu-1,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(1,1,q_{n_2} \mid q)
\end{aligned}
$$

For $m \geq 1, \nu = 0$:

$$
\begin{aligned}
\mathrm{P}_{n_1,n_2}(m,0,q_{n_2} \mid q_{n_1}) &= \sum_q \mathrm{P}_{n_1,n_2-1}(m,0,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(0,0,q_{n_2} \mid q) \\
&\quad + \sum_q \mathrm{P}_{n_1,n_2-1}(m-1,0,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(1,0,q_{n_2} \mid q)
\end{aligned}
$$

For $m = \nu = 0$:

$$\mathrm{P}_{n_1,n_2}(0,0,q_{n_2} \mid q_{n_1}) = \sum_q \mathrm{P}_{n_1,n_2-1}(0,0,q \mid q_{n_1})\mathrm{P}_{n_2-1,n_2}(0,0,q_{n_2} \mid q)$$

which completes the recursion formula, and therefore $\mathrm{P}_{n_1,n_2}(m,\nu,q_{n_2} \mid q_{n_1})$ can be computed for any $n_1, n_2, m, \nu, q_{n_1}, q_{n_2} \quad (n_1 < n_2;\ \nu \leq m;\ 0 \leq q_{n_1}, q_{n_2} \leq M)$.

5.2.3. Effective Loss Conditioned on the Background Traffic and the Queue State. Assume that the channel transmits blocks (or bursts) of cells of time duration $\tilde{L}_a$, where $\tilde{L}_a$ is a random variable with distribution G_a. Further, assume that the transmission period is followed by a period of 'no transmission' of duration $\tilde{L}_b$, where $\tilde{L}_b$ is a random variable with distribution G_b. The 'transmission' and 'no transmission' durations are assumed to be independent from each other and from the duration of other blocks. Suppose that the background traffic rate is a fixed function $\tau_k(n)$ during each block, where k is an index indicating that the traffic rate may not be the same function for every sample of time and every block. For each block, k can be thought of as a sample of a random variable $\tilde{k}$. To account for the dependence on this new variable k, we shall add a superscript k to the previous joint distribution which becomes: $\mathrm{P}^k_{n_i,n_f}(m,\nu,q_{n_f} \mid q_{n_i})$. The state of the queue at time zero (q_0) can also be thought of as a sample of a random variable $\tilde{q}_0$. Note that the time n is assumed to be reset to zero at the beginning of every block. Let $\tilde{m}$ be a random variable representing the number of cells in a block, and $\tilde{\nu}$ a random variable representing the number of cells in that block. Then given k and q_0, the joint distribution of $\tilde{m}$ and $\tilde{\nu}$ is clearly independent of the block number and is given by:

$$\mathrm{P}(m,\nu \mid q_0,k) = \sum_{l=1}^{\infty} G_a(l) \sum_{q=0}^{M} \mathrm{P}^k_{0,l}(m,\nu,q \mid q_0)$$

Define:

$$\begin{aligned}
\mathrm{P}_l(m,\nu,q \mid q_0) &\stackrel{\text{def}}{=} \mathrm{P}^k_{0,l}(m,\nu,q \mid q_0) \\
\mathrm{P}_l(m,\nu \mid q_0) &\stackrel{\text{def}}{=} \sum_{q=0}^{M} \mathrm{P}_l(m,\nu,q \mid q_0) \\
\mathrm{P}_l(m \mid q_0) &\stackrel{\text{def}}{=} \sum_{\nu=0}^{m} \mathrm{P}_l(m,\nu \mid q_0)
\end{aligned}$$

Then the conditional effective loss $\eta_{q_0,k}$ is computed as:

$$\eta_{q_0,k} = \frac{\sum_{l=1}^{\infty} G_a(l) \sum_{m=0}^{l} \sum_{\nu=0}^{m} \mathrm{P}_l(m,\nu \mid q_0) f(m,\nu)}{\sum_{l=1}^{\infty} G_a(l) \sum_{m=0}^{l} \mathrm{P}_l(m \mid q_0) f(m,m)}$$

5.3. Random Background Traffic Rate

The analysis of the previous section can be extended to the case where $\tau(n)$, $\alpha(n)$ and $\sigma(n)$ are not fixed functions, but are stochastic processes. However, for practical reasons, in this section we are interested only in the case where $\tau(n)$, i.e. the rate of the background traffic, is a stochastic process, and $\alpha(n)$ and $\sigma(n)$ are again fixed functions of time (just a constant in most practical applications). This case is particularly suitable for the analysis of reservation mechanisms.

5.3.1. Generalization of the Previous Results. Define $\tilde{k}$ to be a random variable to account for the randomness of $\tau(n)$ (which will be denoted hereafter by $\tau_{\tilde{k}}(n)$), i.e. each value k of $\tilde{k}$ defines a sample path $\tau_k(n)$. We shall assume that $\tilde{k}$ is independent from all other random variables. In general $\tilde{k}$ can assume any real value but for practical purposes can be constrained to take values in a finite set. The matrix A_n defined previously will be a function of k and denoted by $A_{n,k}$. Then the transition matrix for the queue length distribution, conditioned on k, will be:

$$\mathcal{A}^k_{n_i,n_f} = \prod_{n=n_i+1}^{n_f} A_{n,k}$$
$$\mathcal{A}^k_{0,\tilde{L}_a} = \sum_{l=1}^{\infty} G_a(l) \prod_{n=1}^{l} A_{n,k}$$

We then take the expectation of the above matrices over $\tilde{k}$ on a element by element basis as follows:

$$\mathcal{A}_{n_i,n_f}(i,j) = \mathrm{E}_{\tilde{k}|\tilde{q}_{n_i}=i}[\mathcal{A}^{\tilde{k}}_{n_i,n_f}(i,j)]$$
$$\mathcal{A}_{0,\tilde{L}_a}(i,j) = \mathrm{E}_{\tilde{k}|\tilde{q}_0=i}[\mathcal{A}^{\tilde{k}}_{0,\tilde{L}_a}(i,j)]$$

which gives the transitions matrices averaged over $\tilde{k}$. The cell loss distribution is:

$$\mathrm{P}_{n_i,n_f}(m,\nu,q_{n_f} \mid q_{n_i}) = \mathrm{E}_{\tilde{k}|q_{n_i}} \mathrm{P}_{n_i,n_f}(m,\nu,q_{n_f} \mid q_{n_i},\tilde{k})$$

If the distribution of $\tilde{k} \mid q_0$ is the same for all blocks in a sequence of block transmission, the conditional effective loss can be computed as follows:

$$\eta_{q_0} = \frac{\mathrm{E}_{\tilde{k}|q_0}\mathrm{E}f(\tilde{m},\tilde{\nu})}{\mathrm{E}_{\tilde{k}|q_0}\mathrm{E}f(\tilde{m},\tilde{m})} = \frac{\mathrm{E}_{\tilde{k}|q_0}\mathrm{E}f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})} = \mathrm{E}_{\tilde{k}|q_0}(\eta_{q_0,\tilde{k}})$$

The transition matrix of the Markov chain representing the queue length at the beginning of each block, conditioned on k, is:

$$\mathcal{A}^k_{0,\tilde{L}_a+\tilde{L}_b} = \left(\sum_{l=1}^{\infty} G_a(l) \prod_{n=1}^{l} A_{n,k}\right)\left(\sum_{l=1}^{\infty} G_b(l) \prod_{n=1}^{l} B_{n,k}\right)$$

where $B_{n,k}$ is the same as $A_{n,k}$, except that $\alpha(n) = 0$ is used in its computation (because there is no transmission during L_b). $\mathcal{A}_{0,\tilde{L}_a+\tilde{L}_b}$, i.e. the transition matrix averaged over $\tilde{k}$ is found by taking the expectation over the elements of $\mathcal{A}^k_{0,L_a+L_b}$ as follows:

$$\mathcal{A}_{0,\tilde{L}_a+\tilde{L}_b}(i,j) = \mathrm{E}_{\tilde{k}|\tilde{q}_0=i}[\mathcal{A}^{\tilde{k}}_{0,\tilde{L}_a+\tilde{L}_b}(i,j)]$$

The equilibrium queue length distribution is then the solution of the equation:

$$\pi = \pi.\mathcal{A}_{0,\tilde{L}_a+\tilde{L}_b}$$

where π is the row vector containing the equilibrium probabilities. The unconditional effective loss can then be computed using the indicated equilibrium distribution for the queue length, and taking the expectation of the conditional effective loss using that equilibrium distribution:

$$\eta = \mathrm{E}_{\tilde{q}_0}(\eta_{\tilde{q}_0}) = \sum_i \pi_i \eta_i$$

5.3.2. Bandwidth Reservation Mechanism. In this section a bandwidth reservation mechanism is described. The channel under observation generates a burst of cells that lasts for $\tilde{L}_a$ units of time, where $\tilde{L}_a$ is a random variable with distribution G_a. During the burst the channel generates cells with fixed cell rate $\alpha(n) = \alpha$. With the bandwidth reservation scheme,

1. The total rate $\rho(n)$ entering the queue should not exceed certain fixed level ρ_t which is called the *rate threshold*, with $0 < \rho(n) \leq \rho_t < 1$.
2. If, at the beginning of the bursts of the observed channel, the total background rate is less than $\rho_t - \alpha$, the burst is accepted, and bandwidth equal to α is *reserved* for its service for as long as the burst lasts. On the other hand, if, at the beginning of the burst, the total background rate exceeds $\rho_t - \alpha$, then all the cells in the burst are rejected.

The above definition of bandwidth reservation can be formulated as follows. Let:

$$\tau_k^R(n) \stackrel{\text{def}}{=} \begin{cases} \tau_k(n) & \text{if } \tau_k(n) < \rho_t - \alpha \\ \rho_t - \alpha & \text{if } \tau_k(n) \geq \rho_t - \alpha \end{cases}$$

Further, let:

$$A \stackrel{\text{def}}{=} \{k : \tau_k(0) \leq \rho_t - \alpha\}$$

and

$$B \stackrel{\text{def}}{=} \{k : \tau_k(0) > \rho_t - \alpha\}$$

The burst is accepted if $k \in A$. In this case the incoming burst will enter the queue, and the traffic $\tau_k(n)$ will be changed to $\tau_k^R(n) < \tau_k(n)$ to account for the bandwidth *reserved* for the incoming burst. The burst is rejected if $k \in B$. In this case the incoming burst will not enter the queue and all its cells will be lost at a cost $f(\tilde{m}, \tilde{m})$. $\tau_k(n)$ will remain unchanged. With the above reservation model, the following questions arise: How efficient is this technique? Under what circumstances does it work and when does it fail? Here the performance of the reservation scheme is evaluated by computing the conditional effective loss:

$$\eta_{q_0} = \frac{[\mathrm{E}_{\tilde{k}|A}\mathrm{E}_R f(\tilde{m}, \tilde{\nu})]\mathrm{P}(\tilde{k} \in A) + [\mathrm{E}_{\tilde{k}|B}\mathrm{E} f(\tilde{m}, \tilde{m})]\mathrm{P}(\tilde{k} \in B)}{\mathrm{E} f(\tilde{m}, \tilde{m})}$$

where $\mathrm{E}f(\tilde{m},\tilde{m})$ is the expectation of the cost $f(\tilde{m},\tilde{m})$, $\mathrm{E}_R f(\tilde{m},\tilde{\nu})$ is the expectation of the cost $f(\tilde{m},\tilde{\nu})$ when the traffic is $\tau_k^R(n)$ (given k), $\mathrm{E}_{\tilde{k}|A}$ is the expectation over those k's belonging to A and $\mathrm{E}_{\tilde{k}|B}$ is the expectation over k's belonging to B. Since the arrival process is independent of $\tilde{k}$, so is $f(\tilde{m},\tilde{m})$ and the above formula can be simplified to yield:

$$\begin{aligned}\eta_{q0} &= \frac{[\mathrm{E}_{\tilde{k}|A}\mathrm{E}_R f(\tilde{m},\tilde{\nu})]\mathrm{P}(\tilde{k}\in A)+[\mathrm{E}f(\tilde{m},\tilde{m})]\mathrm{P}(\tilde{k}\in B)}{\mathrm{E}f(\tilde{m},\tilde{m})}\\ &= \frac{\mathrm{P}(\tilde{k}\in A)\mathrm{E}_{\tilde{k}|A}\mathrm{E}_R f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})}+\mathrm{P}(\tilde{k}\in B)\\ &\le \frac{\mathrm{E}_{\tilde{k}|A}\mathrm{E}_R f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})}+\mathrm{P}(\tilde{k}\in B)\end{aligned}$$

Usually $\mathrm{P}(\tilde{k}\in B)$ is small so that the above upper bound is a good approximation. The effect on η_{q0} of the reservation scheme is now examined. First there is an additional $\mathrm{P}(\tilde{k}\in B)$ term added to the effective loss. This comes from the fact that some bursts are denied entrance to the queue and therefore are lost. On the other hand, $\mathrm{E}_{\tilde{k}|A}$ replaces $\mathrm{E}_{\tilde{k}}$, and $\mathrm{E}_R f(\tilde{m},\tilde{\nu})$ replaces $\mathrm{E}f(\tilde{m},\tilde{\nu})$, both of which tend to decrease η_{q0}. The reason is that given that $\tilde{k}\in A$, because of correlation in the traffic $\tau_k(n)$, it is likely that the traffic is low for $n>1$. Also in the calculation of $\mathrm{E}_R f(\tilde{m},\tilde{\nu})$ the traffic is $\tau_k^R(n)$ which is less than the $\tau_k(n)$ used to compute $\mathrm{E}f(\tilde{m},\tilde{\nu})$. With reservation introducing terms that tend both to increase and to decrease the effective loss, it is not clear whether the overall effective loss will increase or decrease by using a reservation technique. The result depends on $\mathrm{P}(\tilde{k}\in B)$, $\tau_k(n)$, $\tau_k^R(n)$ and also on $f(\tilde{m},\tilde{\nu})$; although a small decrease in $\tau_k(n)$ does not decrease dramatically the number of lost cells in a burst, it may or may not decrease drastically $\mathrm{E}f(\tilde{m},\tilde{\nu})$, depending on the cost function. In fact the gain obtained from reservation (if any) comes from the fact that $\mathrm{E}_R f(\tilde{m},\tilde{\nu})$ can be much smaller than $\mathrm{E}f(\tilde{m},\tilde{\nu})$ for an appropriate cost function and a suitable traffic. If $\mathrm{E}_R f(\tilde{m},\tilde{\nu})\approx \mathrm{E}f(\tilde{m},\tilde{\nu})$, then from the first equation for η_{q0}:

$$\begin{aligned}\eta_{q0} &\approx \frac{[\mathrm{E}_{\tilde{k}|A}\mathrm{E}f(\tilde{m},\tilde{\nu})]\mathrm{P}(\tilde{k}\in A)+[\mathrm{E}_{\tilde{k}|B}\mathrm{E}f(\tilde{m},\tilde{m})]\mathrm{P}(\tilde{k}\in B)}{\mathrm{E}f(\tilde{m},\tilde{m})}\\ &\ge \frac{[\mathrm{E}_{\tilde{k}|A}\mathrm{E}f(\tilde{m},\tilde{\nu})]\mathrm{P}(\tilde{k}\in A)+[\mathrm{E}_{\tilde{k}|B}\mathrm{E}f(\tilde{m},\tilde{\nu})]\mathrm{P}(\tilde{k}\in B)}{\mathrm{E}f(\tilde{m},\tilde{m})}\\ &= \frac{\mathrm{E}_{\tilde{k}}\mathrm{E}f(\tilde{m},\tilde{\nu})}{\mathrm{E}f(\tilde{m},\tilde{m})} = \text{effective loss without reservation}\end{aligned}$$

Thus in this case, reservation increases η_{q0} and therefore is not desirable. Intuitively, if the frequency of variation of $\tau_k(n)$ is low, $\mathrm{E}_{\tilde{k}}f(\tilde{m},\tilde{\nu})$ is close to $\mathrm{E}f(\tilde{m},\tilde{\nu})$, because given that $\tilde{k}\in A$, the traffic is low at $n=0$ and will remain low most likely during the whole burst length; thus $\tau_k^R(n)\approx\tau_k(n)$ and $\mathrm{E}_R f(\tilde{m},\tilde{\nu})\approx \mathrm{E}f(\tilde{m},\tilde{\nu})$. Therefore reservation is desirable only if the frequency of variation of the traffic is sufficiently high. In applications such as file transfers, the cost function f is a sharp (discontinuous) function of ν. $f(m,\nu)=m\mathrm{I}\{\nu>\nu_0\}$ is such an example ('I' is the indicator function). In such cases, if ρ_t is properly chosen, $\mathrm{E}_R f(\tilde{m},\tilde{\nu})$ can be much smaller than $\mathrm{E}f(\tilde{m},\tilde{\nu})$ and reservation can be quite effective.

5.4. Markovian Background Traffic Rate

This section describes the case in which the channel generates bursts of cells at a constant rate α, the server has a fixed rate σ, and the background traffic generates cells at a rate $\tilde{\tau}(n)$ which is a Markov chain. Let S be the set of all possible values for $\tilde{\tau}(n)$, and T its transition matrix. It is assumed that this Markov chain is homogeneous. Let:

$$T_{\tau_1,\tau_2} = \Pr(\tilde{\tau}(n) = \tau_2 \mid \tilde{\tau}(n-1) = \tau_1) \stackrel{\text{def}}{=} \mathrm{P}_1(\tau_2 \mid \tau_1)$$

be the one step transition probability. Let e be the vector representing the equilibrium probabilities for the Markov chain $\tilde{\tau}(n)$, i.e. $e = e \cdot T$. It is assumed that the channel is bursty with burst length $\tilde{L}_a$, a random variable with distribution G_a.

5.4.1. Cell Loss Distribution.

Assuming that an incoming burst of length l at time n, is able to observe the state of the queue at time $n-1$ and the state of the traffic at time n, it is interesting to know what is the probability that ν out of m cells will be lost. So we are interested in computing:

$\mathrm{P}_l(m, \nu \mid q_1, \tau_1) \stackrel{\text{def}}{=}$ The probability that given that $\tilde{q}(n-1) = q_1$ and $\tilde{\tau}(n) = \tau_1$ (and the channel is active, transmitting cells at rate α), m cells come during l discrete times (from n to $n+l-1$) and ν of them are lost.

Similar notation will be used for other probabilities. $\mathrm{P}_l(m, \nu, q_2, \tau_2 \mid q_1, \tau_1)$ is the probability that given that the queue length and traffic are q_1 and τ_1 respectively, l units of time later they will be q_2 and τ_2, m cells from the channel will come and ν of them will be lost. To find $\mathrm{P}_l(m, \nu \mid q_1, \tau_1)$, we use again a recursive approach.

$$\begin{aligned} \mathrm{P}_1(m, \nu, q_2, \tau_2 \mid q_1, \tau_1) &= \mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)\mathrm{P}_1(\tau_2 \mid \tau_1, q_1, q_2, m, \nu) \\ &= \mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)\mathrm{P}_1(\tau_2 \mid \tau_1) \\ &= \mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)T_{\tau_1,\tau_2} \end{aligned}$$

$\mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)$ is found by elementary analysis of the queueing process and is tabulated below:

for $0 < q_1 < M$

m	ν	q_2	$\mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)$
0	0	q_1	$(1-\alpha-\tau_1)(1-\sigma)+\tau_1\sigma$
0	0	$q_1 - 1$	$(1-\alpha-\tau_1)\sigma$
0	0	$q_1 + 1$	$\tau_1(1-\sigma)$
1	0	q_1	$\alpha\sigma$
1	0	$q_1 + 1$	$\alpha(1-\sigma)$

for $q_1 = 0$

m	ν	q_2	$P_1(m, \nu, q_2 \mid q_1, \tau_1)$
0	0	0	$(1-\alpha-\tau_1)+\tau_1\sigma$
0	0	1	$\tau_1(1-\sigma)$
1	0	0	$\alpha\sigma$
1	0	1	$\alpha(1-\sigma)$

for $q_1 = M$

m	ν	q_2	$P_1(m, \nu, q_2 \mid q_1, \tau_1)$
0	0	M	$(1-\alpha-\tau_1)(1-\sigma)+\tau_1$
0	0	$M-1$	$(1-\alpha-\tau_1)\sigma$
1	0	M	$\alpha\sigma$
1	1	M	$\alpha(1-\sigma)$

$P_1(m, \nu \mid q_1, \tau_1)$ is computed as:

$$P_1(m, \nu \mid q_1, \tau_1) = \sum_q P_1(m, \nu, q \mid q_1, \tau_1)$$

and the general recursion formula is computed as follows:

$$\begin{aligned} P_l(m, \nu \mid q_1, \tau_1) &= \sum_{q,\tau} P_1(0, 0, q, \tau \mid q_1, \tau_1) P_{l-1}(m, \nu \mid q, \tau) \\ &+ \sum_{q,\tau} P_1(1, 0, q, \tau \mid q_1, \tau_1) P_{l-1}(m-1, \nu \mid q, \tau) \\ &+ \sum_{q,\tau} P_1(1, 1, q, \tau \mid q_1, \tau_1) P_{l-1}(m-1, \nu-1 \mid q, \tau) \end{aligned}$$

with the convention that whenever ϵ or δ are negative, $P_{l-1}(\epsilon, \delta \mid q, \tau) = 0$. Therefore $P_l(m, \nu \mid q, \tau)$ can be computed for all m, ν, q, τ with $0 \leq m \leq l$, $0 \leq \nu \leq m$, $0 \leq q \leq M$, $\tau \in S$.

5.4.2. Effective Loss and Equilibrium Probabilities. It is assumed that at time 1 the under observation channel generates a burst for a time period of $\tilde{L}_a$. To find the equilibrium joint distribution of the queue size and the rate of the background traffic, the distribution of the active period $\tilde{L}_a$ and idle period $\tilde{L}_b$ of the under observation channel must be utilized. If the under observation channel cell stream constitutes only a small part of the whole cell stream entering the main buffer queue, it is a good approximation to assume that the equilibrium distribution of $\tilde{q}$ and $\tilde{\tau}$ is only a function of the traffic $\tilde{\tau}(n)$. This is true specially if $L_b \gg L_a$, because under this condition, between two activity periods of the under observation channel, the channel remains in the idle time for an amount of time sufficiently long to allow the queue to return to its equilibrium state as if only the background traffic were present. Under this assumption, the couple $(\tilde{q}(n-1), \tilde{\tau}(n))$ is a Markov chain over the time n. The transition probability matrix is defined by $P_1(q_2, \tau_2 \mid q_1, \tau_1)$ for all possible values of q_1, τ_1, q_2, τ_2.

$$\begin{aligned} \mathrm{P}_1(q_2, \tau_2 \mid q_1, \tau_1) &= \mathrm{P}_1(q_2 \mid q_1, \tau_1)\mathrm{P}_1(\tau_2 \mid q_1, q_2, \tau_1) \\ &= \mathrm{P}_1(q_2 \mid q_1, \tau_1)\mathrm{P}_1(\tau_2 \mid \tau_1) \\ &= \mathrm{P}_1(q_2 \mid q_1, \tau_1)T_{\tau_1,\tau_2} \\ &= \left(\sum_{m,\nu} \mathrm{P}_1(m, \nu, q_2 \mid q_1, \tau_1)\right) T_{\tau_1,\tau_2} \end{aligned}$$

Let $\pi(q, \tau)$ be the equilibrium probability that the Markov chain is in state (q, τ). Then,

$$\eta = \frac{\mathrm{E}f(\tilde{m}, \tilde{\nu})}{\mathrm{E}f(\tilde{m}, \tilde{m})} = \frac{\sum_{l=1}^{\infty} G_a(l) \sum_{m=0}^{l} \sum_{\nu=0}^{m} \mathrm{P}_l(m, \nu) f(m, \nu)}{\sum_{l=1}^{\infty} G_a(l) \sum_{m=0}^{l} \mathrm{P}_l(m) f(m, m)}$$

where

$$\mathrm{P}_l(m, \nu) = \sum_{q,\tau} \mathrm{P}_l(m, \nu \mid q, \tau)\pi(q, \tau)$$

and $\mathrm{P}_l(m) = \sum_{\nu=0}^{m} \mathrm{P}_l(m, \nu)$.

5.4.3. Bandwidth Reservation. We model bandwidth reservation in the presence of a Markovian background traffic as follows:

1. The total rate $\rho(n)$ entering the queue should not exceed a fixed value ρ_t.
2. A burst is accepted if $\tau(1) \leq \rho_t - \alpha$ and rejected otherwise. If a burst is accepted, then $\tilde{\tau}(n)$ changes to $\tilde{\tau}^R(n)$ with transition matrix T^R which is a lower dimension matrix obtained from T as follows:

$$T^R_{\tau_1,\tau_2} \stackrel{\text{def}}{=} \begin{cases} T_{\tau_1,\tau_2} & \text{for } \tau_1 \neq \tau_2 \\ T_{\tau_1,\tau_1} + \sum_{\forall j > \rho_t - \alpha} T_{\tau_1,j} & \text{for } \tau_1 = \tau_2 \end{cases}$$

   where $0 \leq \tau_1, \tau_2 \leq \rho_t - \alpha$.
3. Define $A = \{\omega : \tau(1) \leq \rho_t - \alpha\}$ be the set of outcomes that a burst is accepted and $B = \{\omega : \tau(1) > \rho_t - \alpha\}$ the event that it is rejected.

Now the effective loss is:

$$\begin{aligned} \eta &= \frac{\mathrm{P}(A)\mathrm{E}_R f(\tilde{m}, \tilde{\nu}) + \mathrm{P}(B)\mathrm{E}f(\tilde{m}, \tilde{m})}{\mathrm{E}f(\tilde{m}, \tilde{m})} \\ &= \frac{\mathrm{P}(A)\mathrm{E}_R f(\tilde{m}, \tilde{\nu})}{\mathrm{E}f(\tilde{m}, \tilde{m})} + \mathrm{P}(B) \\ &\approx \frac{\mathrm{E}_R f(\tilde{m}, \tilde{\nu})}{\mathrm{E}f(\tilde{m}, \tilde{m})} + \mathrm{P}(B) \end{aligned}$$

In these formulas, E refers to the expectation assuming background traffic $\tilde{\tau}(n)$, and E_R means expectation assuming background traffic $\tilde{\tau}^R(n)$.

Again note that for the reservation to be effective, $\mathrm{E}_R f(\tilde{m}, \tilde{\nu})$ must be considerably smaller than $\mathrm{E}f(\tilde{m}, \tilde{\nu})$. In the case where α is small compared to ρ_t, this can happen only if the loss of a few cells is quite expensive (as expressed by the function $f(m, \nu)$).

5.5. Numerical Results

A few computer programs have been run to compute the conditional effective loss η_{q_0} for a random shifted phase sinewave as the background traffic rate, and the unconditional effective loss η for a couple case of Markovian background traffic rate. The sine wave and the Markovian background traffic rate will each be discussed in a seperate section.

5.5.1. Random Phase Sinewave Background Traffic Rate. In this example, it is assumed that the background traffic rate is a sine wave which is randomly shifted in time. More precisely, the traffic rate is the following process:

$$r_{\tilde{k}}(n) = \frac{A}{2}\left(1 + \sin\left(\frac{2\pi nF}{N} + 2\pi\tilde{k}\right)\right)$$

where $\tilde{k}$ is a uniform random variable over the interval $[0, 1)$. This case is interesting because it represents variation in the amount of traffic (sometimes high, sometimes low), with the frequency of the wave representing how fast the traffic changes. The amplitude of the traffic rate can also be varied to see the effect on the effective loss. To simplify the computations, the following assumptions are made:

- The time duration of a burst is a deterministic constant N.
- $\alpha(n)$ and $\sigma(n)$ are constant functions of time.

While it has not been proven, it is reasonable to assume that these assumptions do not have a significant effect on the conclusions implied by the numerical results. The effective loss is a function of traffic rate amplitude, traffic rate frequency, and rate threshold. Two different cost functions are used:

index	cost function
$C = 0$	$f(m,\nu) = m \cdot u(\nu - 1)$
$C = 1$	$f(m,\nu) = m \cdot u(10 \cdot \nu - m)$

where $u(n)$ is the step function defined as follows:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{otherwise} \end{cases}$$

The first cost function could be useful for data transmissions and the second cost function could work in the case of forward error correction, or the transmission of information such as a picture, which can tolerate up to a certain degree of cell loss. The first set of results are shown in Figures 3 through 5. These are obtained for $q_0 = M/2$. The second set of results are obtained for larger buffer sizes, but only with the first cost function ($C = 0$). These are shown in Figures 6 through 9. Paremeters not shown on the second set of figures are the same as those of the first set of figures.

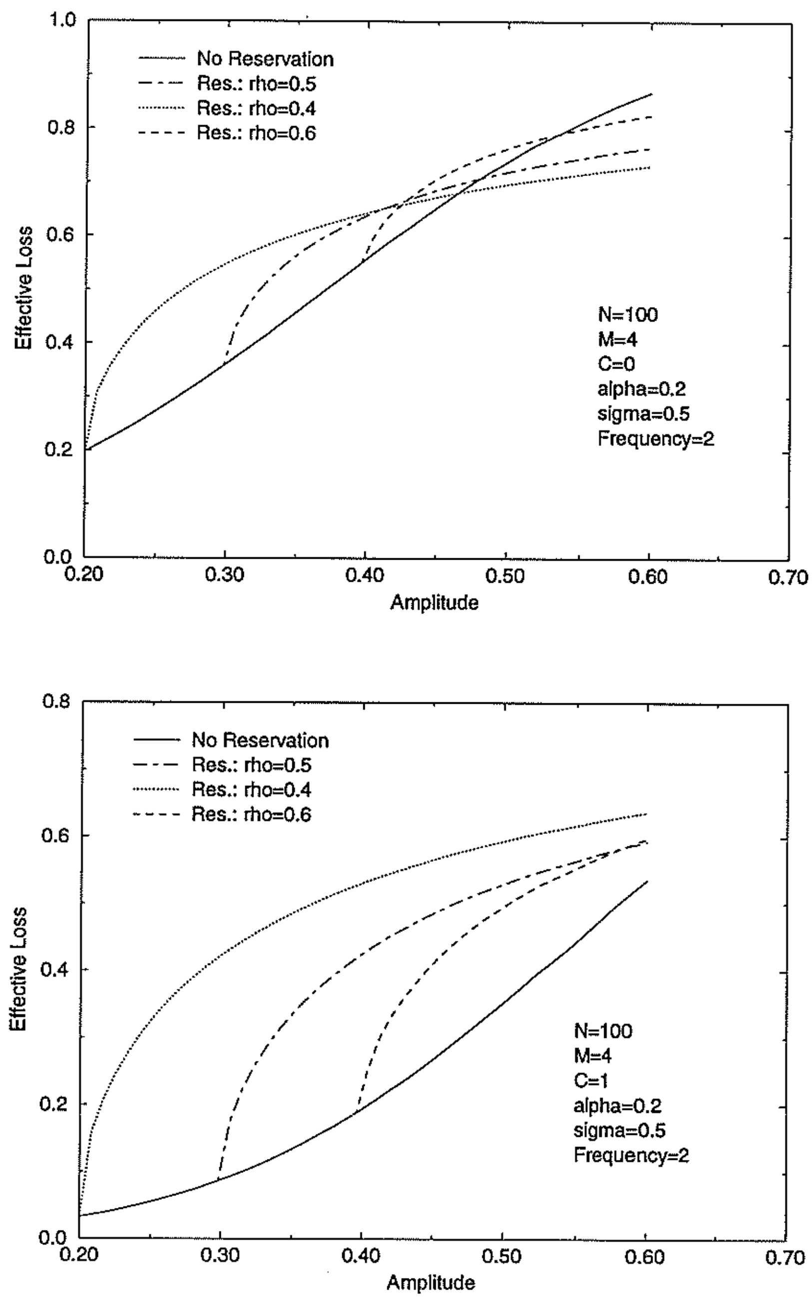


Figure 3: Effective loss as a function of traffic rate amplitude, for different cost functions.

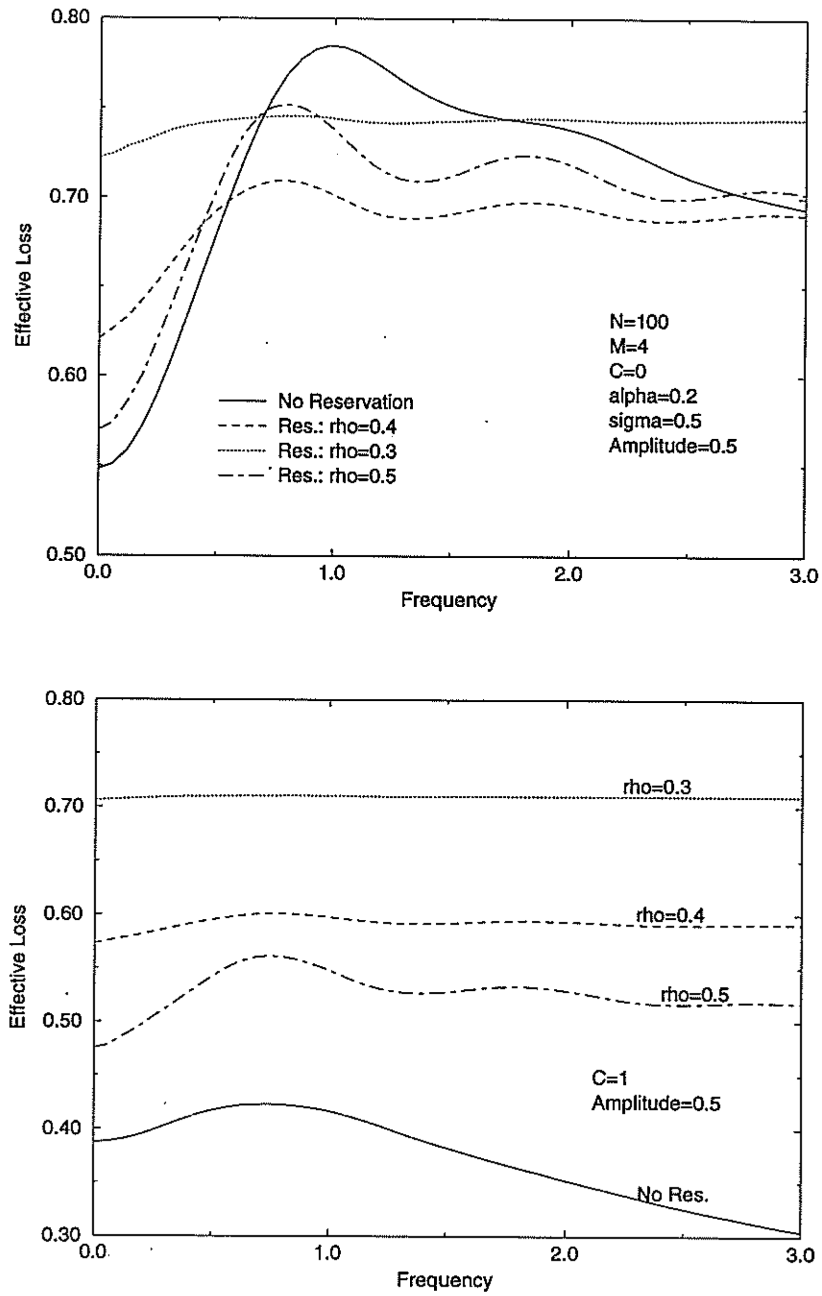


Figure 4: Effective loss as a function of traffic rate frequency, for different cost functions.

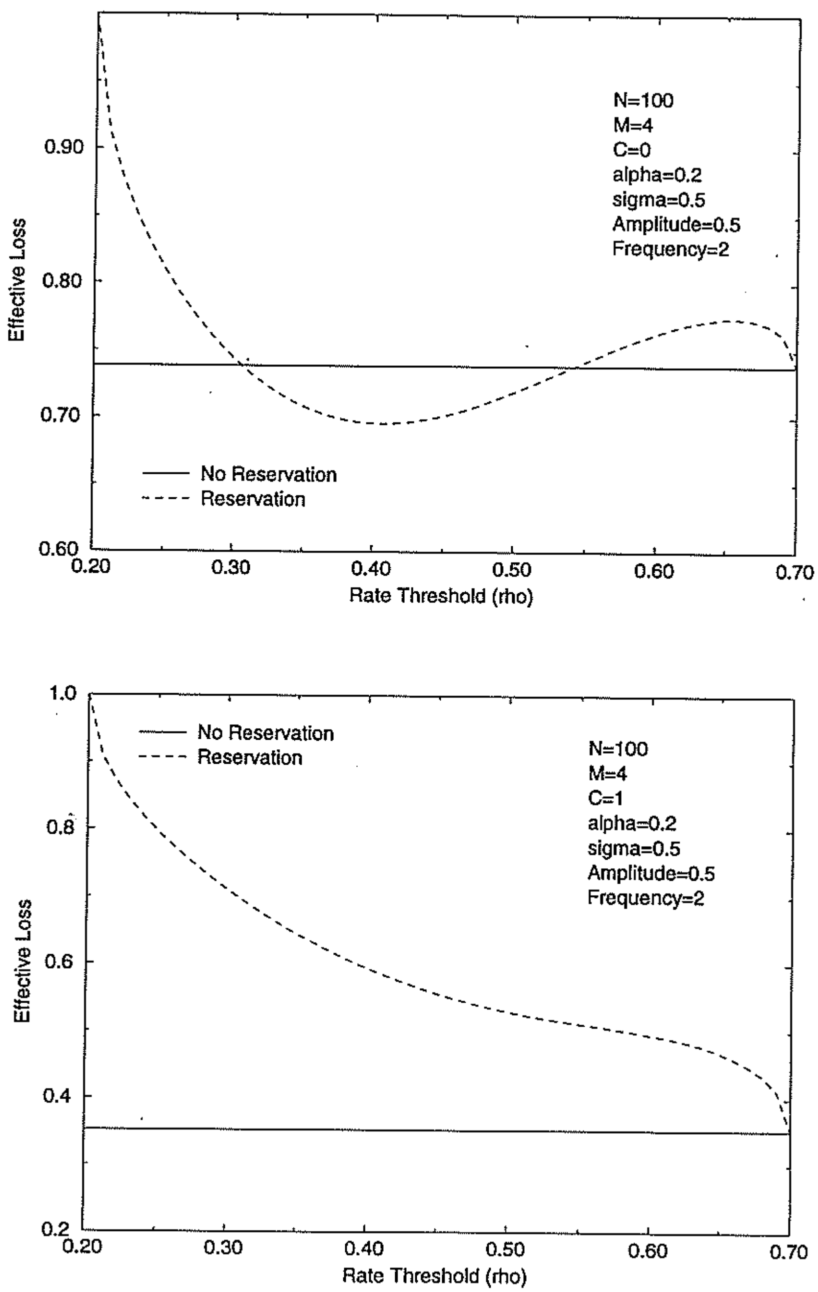
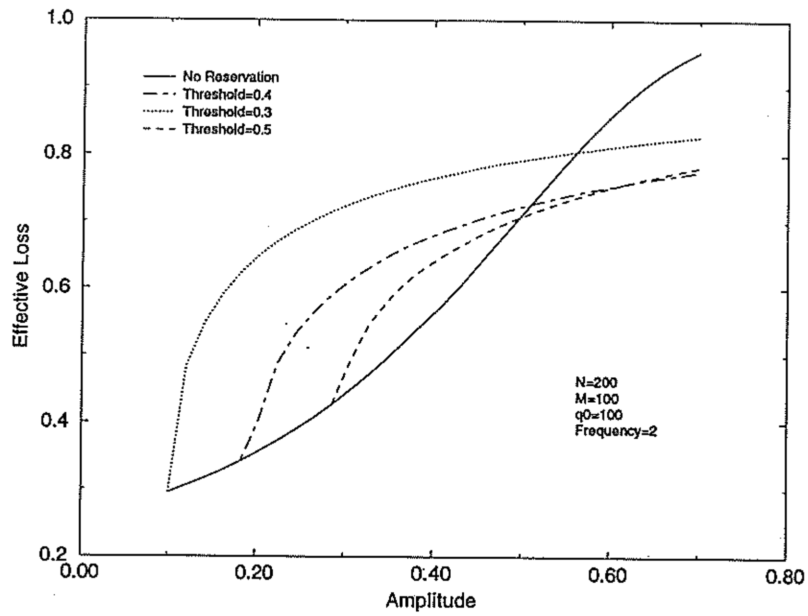


Figure 5: Effective loss as a function of rate threshold, for different cost functions.



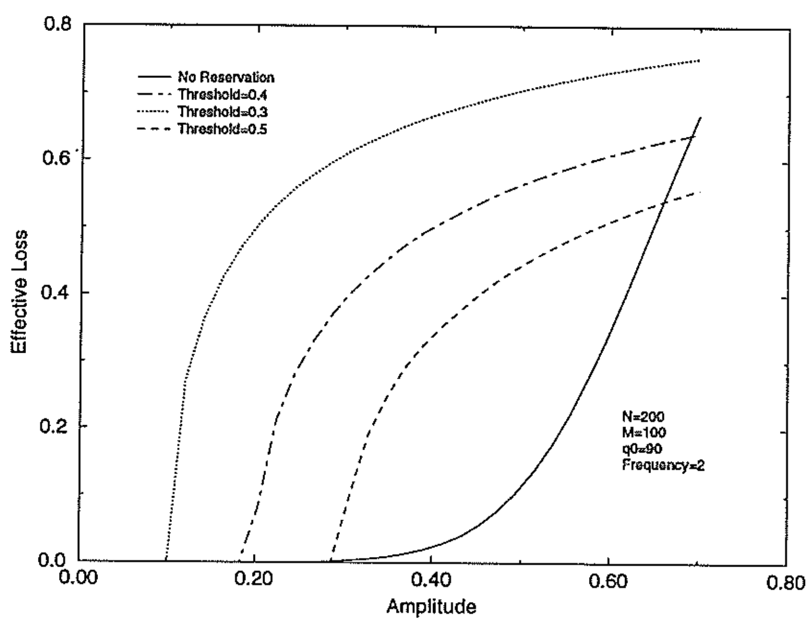
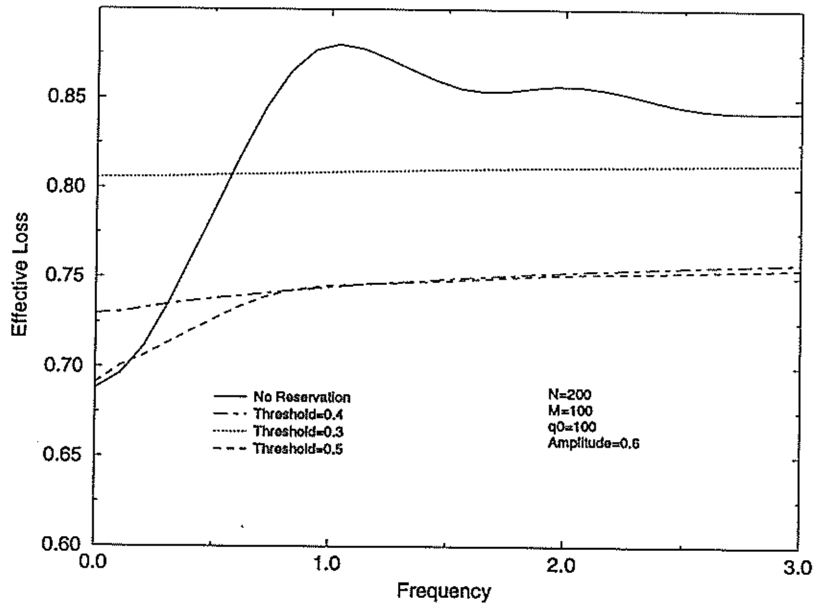


Figure 6: Effective loss as a function of traffic rate amplitude, for different initial buffer occupancies.



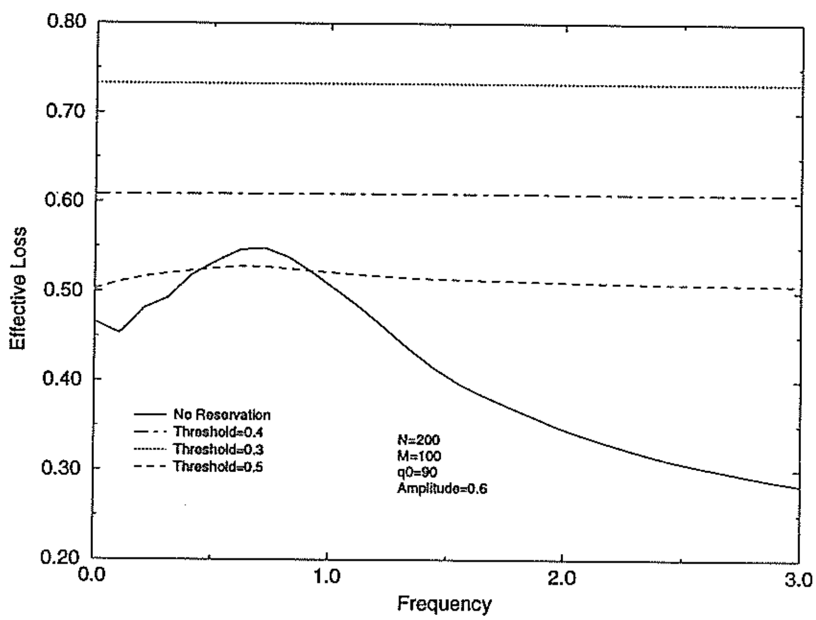
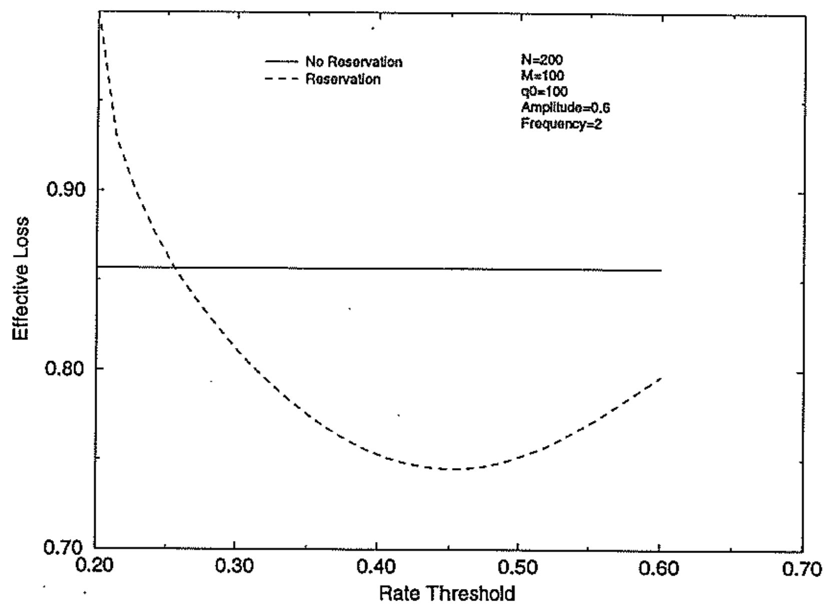


Figure 7: Effective loss as a function of traffic rate frequency, for different initial buffer occupancies.



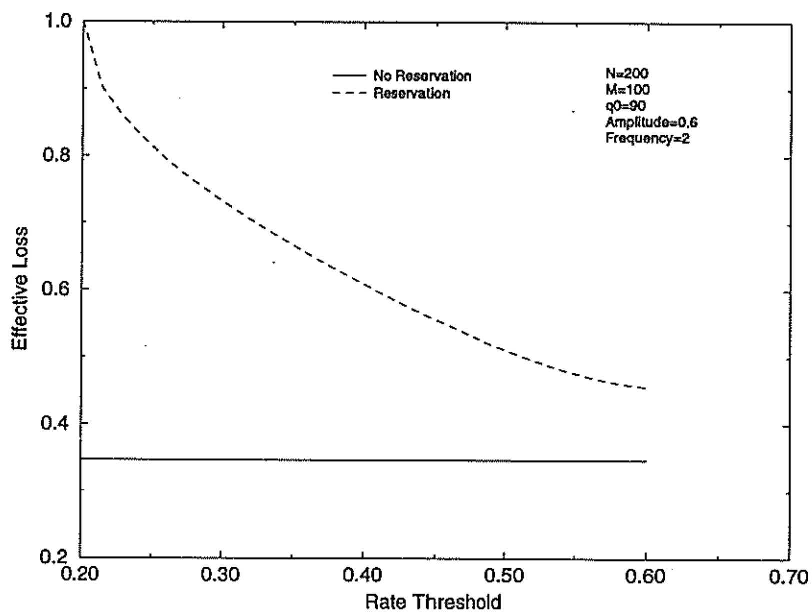


Figure 8: Effective loss as a function of rate threshold, for different initial buffer occupancies.

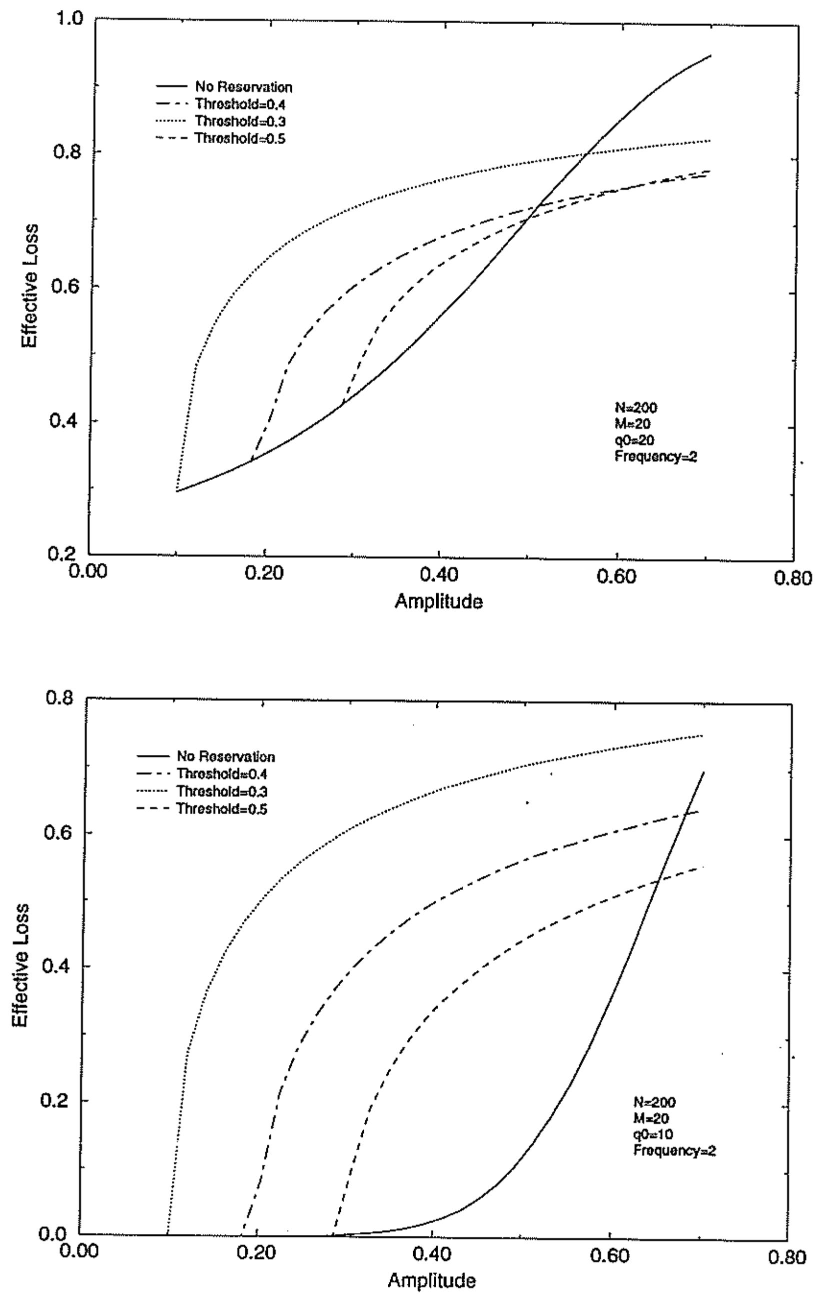


Figure 9: Effective loss as a function of traffic rate amplitude, for different initial buffer occupancies and $M = 20$.

As can be seen from Figure 3, the effective loss always increases when the traffic rate amplitude increases. However, in the case in which reservation is used, the effective loss is less than the effective loss in ordinary statistical multiplexing at high traffic intensity only. Note that when the traffic intensity is very low, the total bandwidth never crosses the reservation threshold ρ_t, and thus the reservation request is always successful. In this case, as shown in the figure, both non-reservation and reservation result in the same effective loss.

As the frequency of variation of the traffic rate is changed, the reservation mechanism results in a lower effective loss in a certain frequency range, namely the range for which the period of variation is comparable to the burst length (see Figure 4). Note that the unit for the frequency of the traffic rate is cycles per burst duration. At low frequencies, there is no need for reservation, because if, at the time the channel burst is generated, the background traffic rate is low (implying the acceptance of the burst), it will most probably remain low during the whole duration of the burst. On the other hand if the frequency is sufficiently high, the rate variation will be easily absorbed by the buffer and the loss will be low. Recall that in both the above cases the reservation mechanism contributes a fixed amount of effective loss which is related to the probability of rejecting a burst. Only when the traffic variation is comparable to the burst length is reservation useful, because when a burst is accepted, it is protected for the duration of the burst.

When the rate threshold ρ_t is varied, the effective loss reaches a minimum value (see Figure 5). For the particular example for which the computations were carried out, the optimum value for the rate threshold was 20% less than the output rate of the multiplexer. If the threshold is very low, almost all bursts are rejected and the effective loss is almost '1'. If the rate threshold is too high, the reservation request is always successful, and the effective loss is the same as the non-reservation case.

The final important result of these numerical computations is the effect of the cost function on effective loss. If the cost function is such that the loss of one cell is equal to the cost of losing the entire burst, reservation incurs a lower effective loss. But if cell loss can be tolerated up to a certain level (in our example, 10% of the total number of cells), then the performance of the reservation method deteriorates. In fact in our example, it was always worse than plain statistical multiplexing. Therefore in applications in which some cell loss can be tolerated, or if other measures such as forward error correction coding are used, reservation may not be beneficial.

5.5.2. Markovian Background Traffic Rate. In this example, it is assumed that the background traffic rate is a Markov chain as explained in Section 5.4. As in the previous example, it is assumed that the time duration of a burst is a deterministic constant N, and that $\alpha(n)$ and $\sigma(n)$ are constant functions of time with the same values as in the previous example.

Before computations can be performed, a transition matrix for the Markov chain representing the rate of the background traffic must be specied. Consider a 2-state (idle, active)

Markov chain $\tilde{\tau}_1(n)$ with transistion matrix:

$$T_1 = \begin{pmatrix} 1-\epsilon_u & \epsilon_u \\ \epsilon_d & 1-\epsilon_d \end{pmatrix}$$

$\tilde{\tau}_1$ can be thought of as a background traffic consisting of a single on-off source. While in active state, the source generates traffic with rate δ. Now suppose that

$$\tilde{\tau}(n) \stackrel{\text{def}}{=} \sum_{i=1}^{K} \tilde{\tau}_i(n)$$

is a background traffic consisting of the sum of K i.i.d. processes, all with the same transition matrix T_1. $\tilde{\tau}(n)$ is a $(K+1)$-state Markov chain with transition matrix T. Let $Q(i,j \mid k_1)$ be the probability that from time $n-1$ to time n, i sources become active and j sources become idle, given that k_1 sources are active at time $n-1$. Then:

$$Q(i,j \mid k_1) = \begin{pmatrix} K-k_1 \\ i \end{pmatrix} {\epsilon_u}^i (1-\epsilon_u)^{K-k_1-i} \cdot \begin{pmatrix} k_1 \\ j \end{pmatrix} {\epsilon_d}^j (1-\epsilon_d)^{k_1-j}$$

and

$$T(k_2 \mid k_1) = \sum_{i=\max(0,k_2-k_1)}^{\min(K-k_1,k_2)} Q(i, i+k_1-k_2 \mid k_1)$$

The results in Figure 10 are obtained for $K=5$ and $\epsilon_u = \epsilon_d = 0.1$. The effective loss is computed as a function of the amplitude $A \stackrel{\text{def}}{=} K \cdot \delta$ which is the maximun possible rate of the background traffic.

One can see again that the effective loss of the non-reservation scheme is lower than the reservation scheme, except at high traffic amplitude in which case the effective loss is prohibitively high (more than 50%).

The second traffic behaviour we want to evaluate is the one in which the background traffic is low most of the time and very high during some limited time periods. To simulate this type of background traffic, the following transition matrix is used:

$$T = \begin{pmatrix} 1-\epsilon_a & \epsilon_a & 0 & 0 & \cdots \\ \epsilon_i & p_{11}-\epsilon_i & p_{12} & p_{13} & \cdots \\ 0 & p_{21} & p_{22} & p_{23} & \cdots \\ 0 & p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

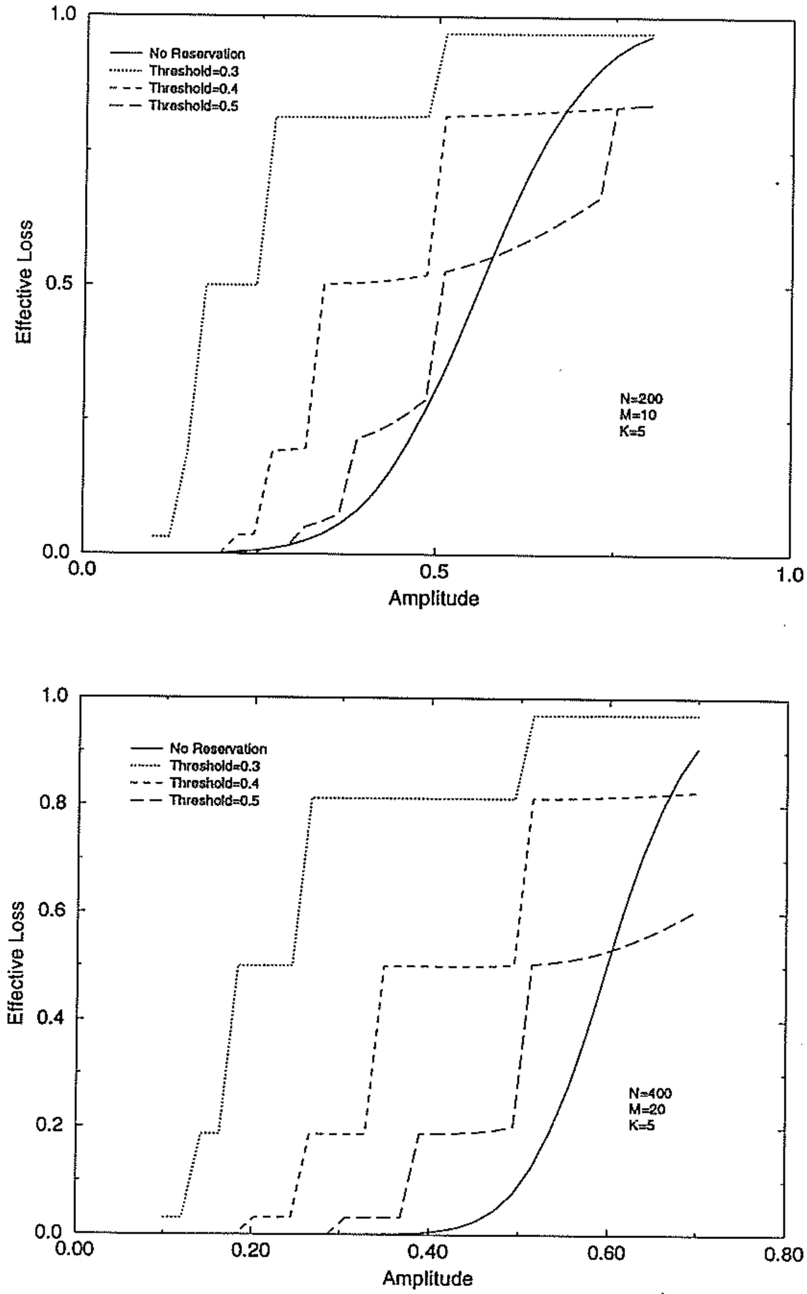


Figure 10: Variation of effective loss with the amplitude of a Markovian background traffic.

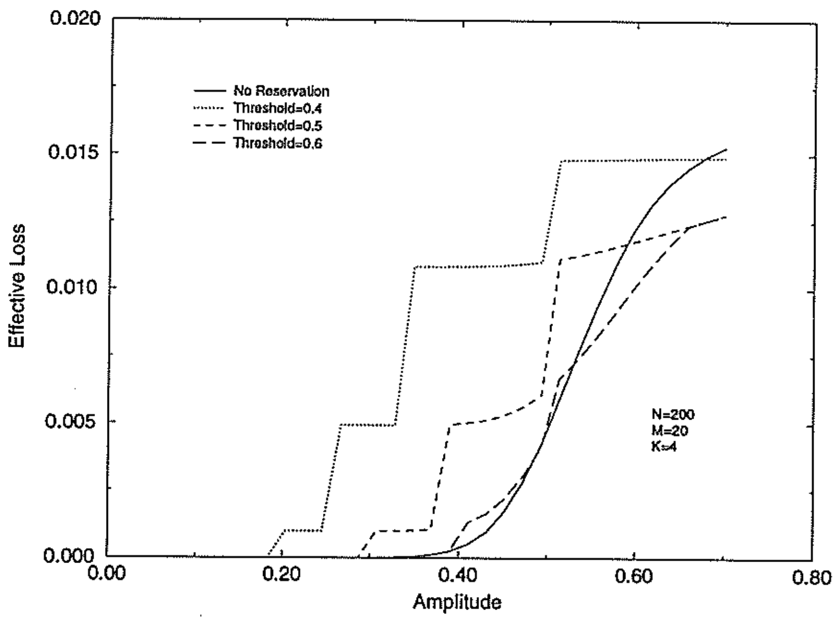


Figure 11: Variation of effective loss with the amplitude of a bursty Markovian background traffic.

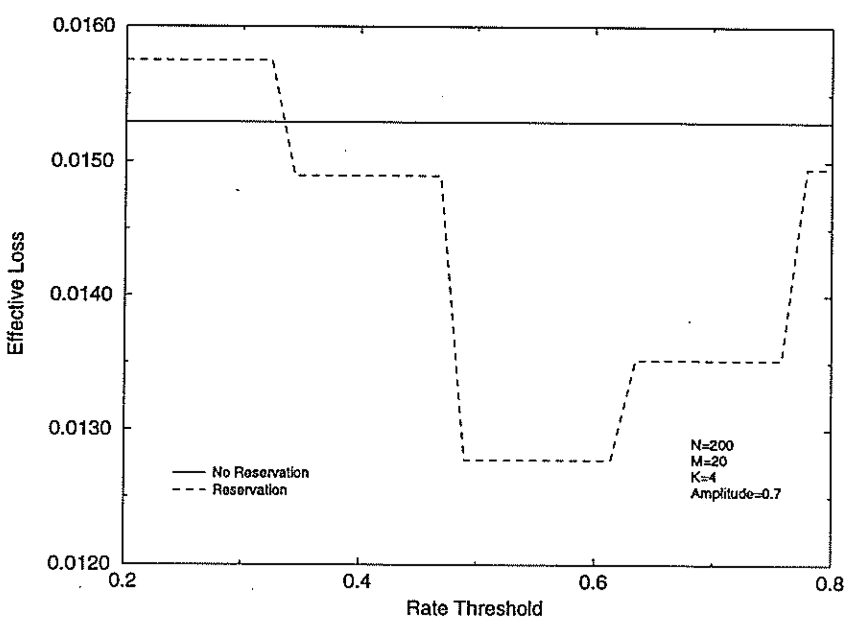


Figure 12: Variation of effective loss with the rate threshold in the presence of a bursty Markovian background traffic.

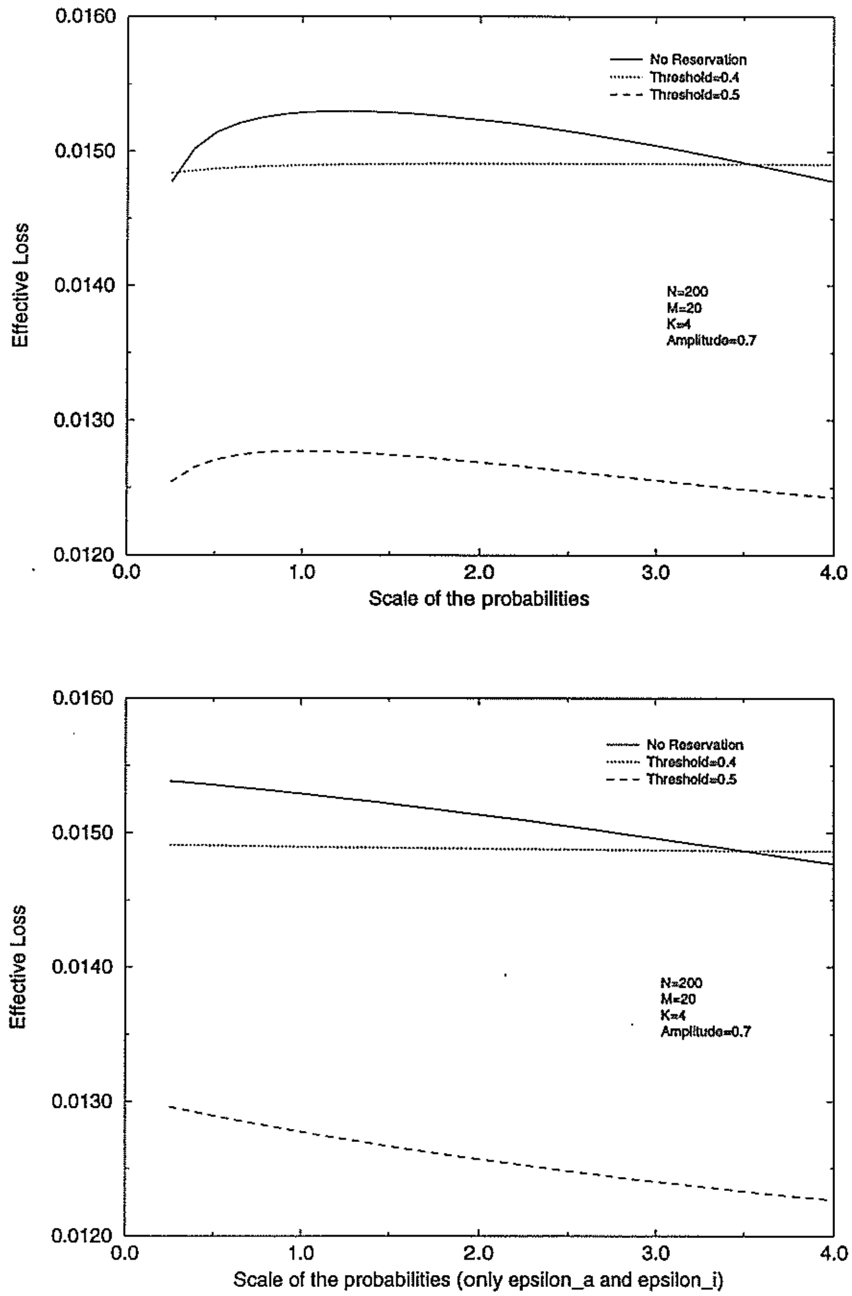


Figure 13: Variation of effective loss with the 'frequency' of a bursty Markovian background traffic.

is the transition matrix of a Markov chain consisting of K on-off i.i.d. sources, each with transition matrix T_1.

Figures 11, 12 and 13 show the results corresponding to this type of background traffic. These numerical results have been obtained for $\epsilon_a = 10^{-5}$ and $\epsilon_i = 10^{-2}$ and $\epsilon_u = \epsilon_d = 0.1$. Figure 11 again shows the effective loss as a function of the traffic amplitude, which is the maximum possible rate of the background traffic. Figure 12 shows the variation of the effective loss as the rate threshold ρ_t is varied. Again it is observed that there is an optimum value for ρ_t which minimizes the loss when the reservation scheme is used. Finally Figure 13 shows the variation of the effective loss as the ϵ's are varied by multiplying them by a scale factor. This simulates the change of frequency of variation of the background traffic. In the top figure, the scales of $\epsilon_a, \epsilon_i, \epsilon_u$ and ϵ_d are varied all at the same time according to $\epsilon_a \rightarrow \epsilon_a \times \text{scale}, \epsilon_i \rightarrow \epsilon_i \times \text{scale}, \epsilon_u \rightarrow \epsilon_u \times \text{scale}, \epsilon_d \rightarrow \epsilon_d \times \text{scale}$. In the bottom figure, the scales of ϵ_a and ϵ_i are varied according to $\epsilon_a \rightarrow \epsilon_a \times \text{scale}, \epsilon_i \rightarrow \epsilon_i \times \text{scale}$, and $\epsilon_u = \epsilon_d = 0.1$.

To summarize, these numerical results show that reservation is indeed a useful method to protect the channel and improve the transmission efficiency, but only under certain conditions. These conditions include:

- The traffic (and therefore the loss) at the multiplexer being high.
- The frequency of variation of the background traffic being comparable to that of the channel under consideration.
- In the case of bursty Markovian background traffic, the occurrence rate of the burst being low.
- The rate threshold being properly chosen.

It is suggested that in an adaptive algorithm, these conditions be evaluated before reservation is used.

References

[1] A.D. Bovopoulos. Performance evaluation of a traffic control module for ATM networks. Technical Report WUCCRC-91-22, Washington University in St. Louis, 1991.

[2] P. Boyer. A congestion control for the ATM. In *Proceedings 7th ITC Seminar*, October 1990.

[3] I. Cidon, A. Khamisy, and M. Sidi. On packet loss processes in high-speed networks. In *Proceedings INFOCOM 1992*, volume 2, pages 0242–0251, 1992.

[4] I. Cidon, A. Khamisy, and M. Sidi. Analysis of packet loss processes in high-speed networks. *IEEE Transactions on Information Theory*, 39(1):98–108, January 1993.

[5] M. Hirano and N. Watanabe. Characteristics of a cell multiplexer for bursty ATM traffic. In *Conference Record - International Conference on Communications*, volume 1, pages 399–403, June 1989.

[6] T-C Hou and A.K. Wong. Queueing analysis for ATM switching of mixed continuous-bit-rate and bursty traffic. In *Proceedings - IEEE INFOCOM*, volume 2, pages 660–667, June 1990.

[7] H. Saito, M. Kawarasaki, and H. Yamada. An analysis of statistical multiplexing in an ATM transport network. *IEEE Journal on Selected Areas in Communications*, 9(3), April 1991.

[8] J. Turner. A proposed bandwidth management and congestion control scheme for multicast ATM networks. Technical Report WUCCRC-91-1, Washington University in St. Louis, 1991.

[9] S. Wolf et al. How will we rate telecommunications system performance? *IEEE Communications Magazine*, 29(10):23–29, October 1991.