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# **Real-Time Memory Management: Life and Times**

Andrew Borg, Andy Wellings, Christopher Gill, and Ron K. Cytron

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Notes: This technical report is based in part on research Andrew Borg conducted as a visiting scholar in the Center for

Type of Report: Other

# **Real-Time Memory Management: Life and Times**

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# Abstract

As high integrity real-time systems become increasingly large and complex, forcing a static model of memory usage becomes untenable. The challenge is to provide a dynamic memory model that guarantees tight and bounded time and space requirements without overburdening the developer with memory concerns. This paper provides an analysis of memory management approaches in order to characterise the tradeoffs across three semantic domains: space, time and a characterisation of memory usage information such as the lifetime of objects. A unified approach to distinguishing the merits of each memory model highlights the relationship across these three domains, thereby identifying the class of applications that benefit from targeting a particular model. Crucially, an initial investigation of this relationship identifies the direction future research must take in order to address the requirements of the next generation of complex embedded systems. Some initial suggestions are made in this regard and the memory model proposed in the Real-Time Specification for Java is evaluated in this context.

# **1** Introduction

Memory management is a major concern when developing real-time and embedded applications. The unpredictability of memory allocation and deallocation has resulted in hard realtime systems being necessarily static. Even when the analytical worst-case space and time requirements can be derived [27, 25], the high space and time overheads of traditional dynamic memory models such as fine grain explicit memory management and real-time garbage collection (GC) mean that it is hard to argue a case for these models in resource constrained environments. Alternative memory models such as that proposed in the Real-Time Specification for Java (RTSJ) can be shown to reduce these overheads but at a cost of significant development complexity. Identifying and characterising the criteria used to gauge different memory models is non-trivial. Arguing that one memory model is better than another must be qualified by a specification of the driving design forces. One important element of these driving forces is the application development cost of the chosen model. GC is the best choice in this regard as the additional development cost of using memory dynamically is small. However, the real-time community remains highly polarised as to whether real-time GC can ever reach the predictability and efficiency required by these systems. Critics of GC technologies argue that the space/time tradeoffs achievable by real-time garbage collectors are still unsuitable for the class of systems targeted by the static paradigm. In order to achieve the required performance in one semantic domain, the costs in the other are too high. On the other hand, critics of new memory models such as that proposed in the RTSJ argue that the burden of memory concerns placed on developers detracts from what is arguably the most attractive feature of developing large and complex systems in environments such as Java: freedom from memory concerns. The common ground between these two camps is that the success or failure of providing a suitable environment for developing the next generation of complex real-time and embedded systems hinges on getting the memory model right.

The motivation for this paper comes from the recent debate about which memory model the RTSJ should adopt. Indeed, this continues to be the most contentious issue of the RTSJ and more literature exists on analysis, extensions and modifications of the RTSJ memory model than any other part of the specification. Java developers could adopt a static approach in the same way as Ravenscar for Ada does [13] but even the Ravenscar profile for Java [20] recognises the need for a dynamic model by introducing a limited region-based model. The need for dynamic memory is however not a requirement only of real-time Java itself but is a requirement of the next generation of embedded real-time systems. The goals of this paper are threefold. The first goal is to identify the implications of using different dynamic memory models. The second goal is to motivate a new approach to developing dynamic memory models that goes beyond the "fine-tuning" approach of current research. The third goal is to show why the RTSJ's memory model may be a step in the right direction in this regard. However, it will also be argued that a number of changes to the model and the way it is used are necessary.

To this end, this paper provides three contributions. First, an analysis of fine grain memory management approaches from previous work is carried out in Section 2 in order to identify and quantify the space and time overheads of these approaches. Although the results described here make a strong case for an alternative memory model, current research either fails to provide models for which the overheads are sufficiently tight or proposes models which place a significant burden on the developer. Section 3 describes the second contribution of this paper. This is a novel, unified approach to distinguishing the merits of each memory model that is based on a hypothesis that highlights the relationship across three semantic domains: space, time and a characterisation of available information about the way memory is used by the application, such as the average size of object or a characterisation of their lifetime. This allows the derivation of a classification of memory models, thereby identifying the class of applications that benefit from targeting each model. The hypothesis developed in this section is used to make a case for the direction research needs to take in order to develop memory models suitable for the next generation of complex embedded systems. In particular, it is argued that rather than trying to adapt the memory models used in traditional environments to real-time ones, new memory models than directly address real-time requirements need to be developed. The Metronome collector [1, 3, 2], a state-of-the-art garbage collector, is described in order to highlight why this approach is unsuitable for the class of applications currently addressed by the static model. Section 4 describes the third contribution in this paper: an overview of a detailed investigation into the RTSJ memory model that has been carried out in order to analyse whether this model captures the shift in research strategy argued to be necessary in Section 3. Although it is argued that this is achieved to some degree, it can also be shown that the model's abstraction fails to allow an expression of common lifetime patterns or restricts the ability of this information to be expressed. These problems are often addressed by using design patterns that are orthogonal to the model. Identifying where this information is lost motivates a set of extensions to the RTSJ memory model that directly targets these failures, thereby eliminating the need for some of these design patterns and highlighting why others are required. Also, even when lifetime information can be expressed, implementations often fail to take full advantage of this information. Solutions to address this are briefly described. Finally, Section 5 identifies future work and concludes.

# 2 Fine Grain Models in RT Environments

The static approach to memory management is the traditional way of developing hard real-time systems. This is the approach taken in high integrity and safety critical subsets of Ada such as Ravenscar [13] and SPARK [4]. Assuming an objectoriented language, the application would allocate all necessary objects in a pre-mission phase and these would live for the duration of the mission phase. If memory constrained devices are the target of the application, developers are often forced to consider recycling objects, thereby adding another dimension of complexity to the development and maintenance processes. In some cases, this problem is aggravated as the object model may need to be broken in order to allow what would ordinarily be type-incompatible objects to replace each other. Dynamic memory allocation and deallocation can address this problem if the reduced development complexity does not come at an unacceptable space and time overhead.

Dynamic memory management is used in most development environments outside the real-time domain in the form of fine grain, user-controlled allocation and deallocation and automatic deallocation by a garbage collector. It is unsurprising therefore that recent research focuses for the most part on porting these models to the real-time domain. However, in proposing the use of these memory models in a real-time domain, the space and time overheads need to be quantified. This is investigated in the rest of this section and subsequently used to motivate research into an alternative memory model. Note that an exact evaluation of each overhead is often not possible due to the complex dependencies of the space-time tradeoff. This problem will be addressed in Section 3.

The sources of overheads in dynamic fine grain models are broadly as follows:

- 1. Both explicit and automatic memory models introduce space and time overheads due to fragmentation.
- 2. In automatic memory models, there is the additional overhead of identifying garbage.
- If memory management is made to be incremental, an additional overhead for guaranteeing the mutual integrity of the program and collector is incurred.
- 4. In order to counter (1), both explicit and automatic memory models can use defragmentation. This results in additional time but reduced space requirements.

## 2.1 The Cost of Memory Fragmentation

Memory fragmentation introduces high pessimism and unpredictability in space and time requirements that is unfavourable in real-time environments [21]. An investigation into the results from past work is carried out next in order to quantify these overheads.

#### Measuring the Cost of Fragmentation: Space

A detailed investigation into the space overhead due to fragmentation in a number of Dynamic Memory allocation algorithms (DM algorithms) can be found in previous work by Neely [23] and Johnstone [18]. These results are intriguing in that they show that the observed fragmentation is least in the simpler policies such as first and best fit as opposed to more complex policies such as buddy algorithms. These simpler algorithms exhibit fragmentation that is also very low, typically under 3%<sup>1</sup> when averaged across all applications but rises to as high as 53% in more complex algorithms such as binary buddy. When taking into consideration the implementation costs of the policy (such as the data structures maintaining free lists) and machine requirements (namely byte alignment), these overheads increase to just 34% for best-fit and first-fit policies and 74% for binary buddy. A conclusion that Johnstone draws from these results is that the fragmentation problem is solved, and has been solved for several decades. However, Johnstone's work is based on the observed rather than worst case space requirements of applications. In a series of work between 1971

<sup>&</sup>lt;sup>1</sup>Johnstone's metric, compares the memory requirements at the point of MaxLiveBytes (that is  $F = \frac{MaxHeapSizeAtMaxLiveBytes - MaxLiveBytes}{MaxHeapSizeAtMaxLiveBytes}$ ).

and 1977 [28, 29, 27], Robson derived the worst case memory requirements of the best-fit and first-fit policies whereas Knowlton [19] derived the worst case for buddy systems. Given a block size n and a maximum live memory requirement of M, the worst case memory requirements for first-fit is  $M \log_2 n$ , for binary buddy is  $2M \log_2 n$  and for best-fit is Mn. Crucially, Robson showed that there exists an optimal strategy for which the worst case memory requirements lie between  $\frac{1}{2}M \log_2 n$ and about  $0.84M \log_2 n$ . The first-fit policy therefore provides a solution that is very close to optimal.

An interesting exercise is to compare these results to Johnstone's for observed fragmentation. The work of both Johnstone and Robson would lead one to the conclusion that the first-fit policy is the best solution both in terms of observed and analytical worst case overheads. In fact, the most significant observation that can be drawn from a comparison of Johnstone's and Robson's work is that there exists a large discrepancy between the observed and analytical worst cases for all policies. For example, a program with a maximum live memory requirements of 1Mb and which allocates objects that range over a conservative size (say between 64 and 64k bytes) would still require 10Mb to guarantee against breakdown due to fragmentation when using first-fit and 20Mb when using the binary buddy algorithm. In addition to this, the memory requirements of the mechanism implementing the policy must also be considered. These total memory requirements are a significant order of magnitude higher than the 1.5Mb to 2Mb requirements one would expect to be required in the observed worst case.

#### Measuring the Cost of Fragmentation: Time

The time overhead incurred by a DM algorithm depends on the inter-arrival rate of allocation and deallocation requests and the cost of an allocation and deallocation cycle as well as the mean and variance of the request size. The most important contribution in the analysis of the time overheads incurred by DM algorithms was provided recently by Pauat in [25]. Pauat analysed the average and worst observed times of four applications using a number of different DM algorithms and compared them to the analytical worst case allocation and deallocation times of these algorithms. The average observed time overheads are similar across all DM algorithms and moreover, the worst case overheads are typically less in the simpler policies such as best-fit than in the more complex ones such as binary buddy and Fibonacci buddy [17]. The analytical worst case overheads however tell a different story. Here, the analytical worst case performance of best-fit and first-fit DM algorithms using a naïve mechanism is nearly a thousand times worse than that of the buddy systems which performs best for the analytical worst case. The significant time used by a DM algorithm in a typical program are immediately evident: the values for b-tree best fit and binary buddy would be respectively around 64% and 63% time overhead for a logic optimisation program called Espr; that is about 63% and 64% of program execution is used in servicing allocation and deallocation requests. In the worst case, these values jump to 96% and 71% respectively.

# 2.2 The Cost of (Non-Incremental) GC

When considering automatic memory management, it is often implied that the garbage collector also assumes the role of the DM algorithm and defragmentor, thereby executing four tasks: servicing allocations, locating garbage through a root scan and traversal of the object graph (tracing), freeing garbage (sweeping) and defragmentation. This blurs the distinction between DM algorithms and garbage collectors and limits a direct comparison between explicit allocation and deallocation memory models and automatic memory management models. In an effort to quantify each overhead, this paper maintains the distinction between the processes of allocation, tracing, sweeping and defragmenting memory. There is an additional overhead in a garbage collected environment over an explicitly managed one serviced directly by a DM algorithm that is highlighted by this abstraction: the time overhead involved in tracing that is not present when memory in managed explicitly and the space overhead due to the delayed deallocation of memory. Given this additional overhead, it would be expected that GC would automatically imply higher total overheads than an explicit memory management, in both the space and time domain. There are several cases in the literature in which this is argued not to be the case in the time domain [8, 16]. This phenomenon occurs because the delayed deallocation of objects in a garbage collected environment results in higher space overheads but incurs lower time overheads due to infrequent vertical switching between the application and underlying memory subsystem. However, existing garbage collectors make use of strategies that are absent in existing DM algorithms but that could be readily implemented. For example, a similar technique to reduce vertical switching could be used for explicit memory management with free() calls being delayed and a single call to the DM algorithm passing the addresses of all memory to be freed. Garbage collectors will therefore always incur additional overheads over explicit fine grain models due to tracing. This is an important observation as the cost of tracing becomes the single additional overhead between explicitly managed memory models and non-incremental automatic memory management models. The results in [2] for the Metronome collector show that tracing incurs the highest cost of all collector operations, including fragmentation. Section 3.3 revisits these results in detail.

#### 2.3 The Cost of Incremental Collection

Irrespective of whether a work-based [15] or schedulebased [1, 26] approach to real-time collection is adopted, the additional time cost of an incremental approach over a stop-theworld one comes from one primary source: maintaining consistency between the mutator and collector through the execution of barriers. Quantifying this overhead is often difficult as the work done at each increment involves the execution of other tasks such as tracing and defragmentation. Since these overheads are being treated independently, the cost of a barrier here is considered only to be the cost of maintaining a suitable consistency between the mutator's view of the object graph and the actual object graph. In [31], Zorn shows that the cost of read barriers alone can incur a 20% penalty on application performance when executed in software though Cheng *et. al.* claim that their Metronome garbage collector can reduce this to 4% on average and 9% in the worst case. Considering that a read barrier based on pointer updates can be implemented with a handful of operations (an average four ALU/branch instructions in [31] and a compare, branch and load in [1]), these significant overheads are caused by the large number of times these barriers are executed.

# 2.4 The Cost of Defragmentation

One solution to reducing fragmentation is to carry out runtime *defragmentation* (or *compaction*), a process in which memory is rearranged and compacted. This approach merely shifts fragmentation overhead from the space domain to the time domain. Defragmentation is reassessed in Section 3.3 when discussing the Metronome collector.

# **3** Towards a Classification of Memory Models

Although the worst case space and time overheads for fine grain models described in the previous section are clearly too high for resource-constrained environments, making the case for an alternative model is not easy. Crucially, it is unclear what direction research needs to take in order to develop these models. Although significant research effort has been invested in this area, particularly in the field of real-time collectors, the returns have been minor. This section introduces a novel way of classifying memory models that allows a direct comparison between them to be derived and also highlights the application classes for which suitable memory models still need to be developed. This comparison is based on an evaluation metric that consists of three parameters:

- time overheads,
- space overheads and
- an expression of object information.

The relationship between space and time, although not always trivial, is in general described by a function in which an increase in overheads in one domain tends to result in a decrease in the other. The choice of whether to use a defragmentation algorithm is an example of this. The third parameter introduced in this evaluation metric captures the burden placed on the developer in describing the known information about how objects are used in the application. For example in an explicit fine grain model, this information is an expression of the lifetime of each object as specified by the malloc() and free() operations. Other models such as the Metronome collector discussed in Section 3.3 allows the expression of other information such as the average object size. Typically, a memory model is compared to another only in the space and time domains. The burden of describing the third parameter is rarely qualified in the traditional fine grain approaches introduced in Section 2, in all probability because explicit and automatic approaches at this granularity describe two extremes that are easy to characterise.

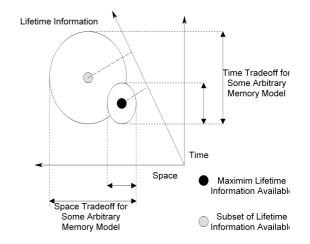


Figure 1. Space/Time in the Entropy Hypothesis

In this section, it is argued that this burden as captured by the expression of this information is not independent of the space and time dimensions; rather it defines them.

# 3.1 Memory Management as an Entropy Problem

A description of the interrelation between the parameters of the evaluation metric can be argued by the Entropy Hypothesis which we propose. The entropy hypothesis states the relationships between space, time and object information can be characterised as a form of entropy. We borrow the concept of entropy from Information Systems Theory [30] to which a parallel can be drawn. In information systems, entropy is a measure that is used to calculate the amount of information in a source or, equivalently, the *redundancy* in that source. The entropy gives a measure of the actual information in a system and dictates the maximum degree to which that system can be compressed and thereby the number of bits required to transmit that source. Every information source has a maximum entropy that sets a lower bound for the compression of that source through lossless algorithms. When a system is said to be at maximum entropy, it is implied that it exhibits maximum randomness or, equivalently, no information is known about the information source and lossless compression is impossible. However, if certain information is known about the source (i.e it is not completely random), then this can be used by a compression algorithm to reduce the number of bits required for transmission.

Our analysis of memory management techniques according to entropy is based on the hypothesis that the amount of available information about an application defines the space and time overhead domains of the application. Furthermore, just as known information of patterns in an information source can be used to reduce the cost of transmission, so information about memory usage can be used to reduce the space and time overheads of memory management. Also, using the notion of maximum entropy, maximum randomness in an information source that results in high transmission costs can be compared to high space and time requirements in application execution. For a given amount of information about an application, a solution is defined in a memory/time trade-off space for which an independent function that depends on the chosen memory model will define the tradeoff in the time and space domains. The tradeoff between these domains is a potentially unique signature for that model and can identify at a fine granularity a ranking of models for a particular application class.<sup>2</sup> However, the bounds of this space are defined by the entropy of that information: with a given amount of information, there is a bound on the solution trade-off space. The entropy hypothesis is depicted in Figure 1 for a hypothetical application and memory model.<sup>3</sup> The available information about the application defines the space of memory/time trade-offs which memory models can achieve. If less information is available, the space and time requirements could increase but are always bounded from below by the space for which maximum information is available.

The entropy hypothesis thus defines a more abstract view of the memory management problem and the function of a memory model. This allows a comparison not only between similar models that simply provide minor shifts between space and time requirements but also allows a comparison between intrinsically different memory models. It therefore provides a platform to compare models as different as real-time collection and the RTSJ scoped memory model. The entropy hypothesis motivates a more abstract definition of what a memory model is and the functions it performs. A memory model is defined as a mechanism that allows expression of knowledge of memory usage and takes advantage of this knowledge in order to reduce time and space overheads. The goals of a memory model are therefore twofold: providing a mechanism that allows this knowledge to be expressed and taking advantage of this knowledge. In promoting any memory model, the importance of making this knowledge as easy to obtain and express as possible cannot be overstated. This is particularly true of real-time environments as the guarantees the memory model can provide are directly related to the accuracy and precision of this knowledge. The ability to take advantage of this knowledge is equally important and can be used to give a comparative assessment of different implementations of the same model.

Using the Entropy Hypothesis, the investigation into the overheads of fine grain memory models described in detail in the previous section and the well-known overheads of the static approach, a spectrum of memory management technologies that describe the resultant overhead of some of these models can be defined. This spectrum is shown in Figure 2 where the space and time overheads of each solution are described in relation to the amount of information that is expressible and used by the memory model. This is based on a representative hypothetical application of non-trivial complexity and may vary for other applications. The shape of the space is likely not as well defined in practice as the figure would lead one to believe, but for illustrative purposes the tradeoff in space and time is cap-

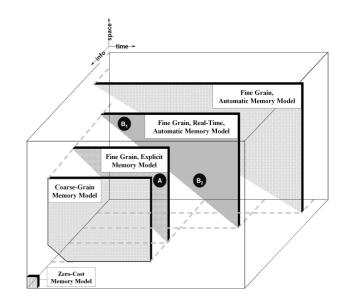


Figure 2. Classifying Memory Models

tured here by the triangle shape. In any case, an exact function describing this tradeoff is rarely available. Another simplifying assumption made here is that the general application under investigation terminates. If this were not the case then space requirements could be infinite, thereby removing the top horizontal edge.

A hypothetical zero-cost memory model that incurs zero space and time overheads is shown in Figure 2. This is tantamount to there being complete knowledge of the application's memory usage available, which is leveraged in the implementation of the adopted memory model. Most models can in practice achieve very small space or time overheads but rarely can minimise both. For example, a fine grain model can minimise space overheads by defragmenting at every deallocation but then the time overheads are high. At the other extreme, near-zero time overheads can also be achieved at high space costs by never deallocating objects.<sup>4</sup> The argument made in Section 2.2 that explicit memory management can always be made to perform at least as well as automatic memory management is captured by the entropy hypothesis; the exact lifetime of objects is unknown before runtime and therefore the space/time tradeoff must be worse. The arguments made in [8, 16] where garbage collectors are argued to perform better than explicit allocation and deallocation are then captured by the tradeoff function. For example, in Figure 2, the points A vs.  $B_1$  and  $B_2$  represent the space and time costs for an application using an explicit memory model vs. an automatic memory model respectively. Clearly, the automatic memory model is more time efficient but less space efficient, at  $B_1$ . However, to achieve the same space overheads as A at  $B_2$  the additional cost of tracing guarantees that a higher time overhead is incurred than in the explicit model. A case for the depicted location of coarse grain models in Figure 2 is made in Section 4.

<sup>&</sup>lt;sup>2</sup>The assumption here is that the tradeoff is defined for the worst-case values of the respective domains. There is a significant complexity in defining this function that is not addressed here.

 $<sup>^{3}</sup>$ Note that the oval shape is not necessarily indicative of the true shape of this space.

<sup>&</sup>lt;sup>4</sup>This is never exactly zero due to the time required to update the pointer to free memory.

A caveat of the Entropy Hypothesis is that which memory model is to be used for an application must be decided before implementation begins. For example, if an explicit coarse grain model such as a memory pool model is to be used, then the developer must identify appropriate clusterings to place inside each pool. In time-critical applications, consideration must also be given to the timing requirements of tasks. In quantifying the development costs of a chosen model and the corresponding expected time and space overheads, an important assumption is made: the developer must target that memory model. Targeting a memory model plays an important part in leveraging the advantages of that model. For example, if a memory-pool model is provided, then programming with a fine grain approach by placing one object in each pool will fail to achieve the reduced time overheads of the coarse grain approach. The importance of providing the right abstractions to capture this information is crucial.<sup>5</sup> The success of bringing dynamic memory models to constrained real-time environments must therefore lie in providing the right abstractions to capture information about memory usage in the application.

The entropy hypothesis therefore hints at the direction future research must take in order to fill the gap between the static approach and fine grain models. In the absence of a proven lowerbound for the space-time tradeoff of existing memory models, there are two possible research directions that can be taken to improve on the static approach: invest further in refining existing fine grain models models or derive alternative models that target regions of the trade-off space that fall between the static and dynamic approaches. In either case, the entropy hypothesis makes a case for memory models to allow more information to be expressed in order to reduce space and time overheads. Three questions that future research must therefore address are:

- 1. What types of information can be captured?
- 2. Which combination of these types of information best allows the shift towards the hypothetical zero cost model?
- 3. What is the best way to capture this information so as to place the least possible burden on the developer?

The first of these questions is addressed next. The overheads seen for the Metronome collector are describe in Section 3.3 and, together with the analysis carried out in Section 2, is used to motivate a move towards allowing the expression of new types of information. In Section 4, coarse grain memory models and the RTSJ memory model are evaluated in order to show how this approach can achieve lower overheads. The third question is then answered in relation to the RTSJ memory model.

#### 3.2 Information in Object-Based Systems

Information about objects in an application can be expressed either for a specific application or a more general application class. The difference between general and application-specific knowledge is mainly one of tuning in the adopted strategy; general information is used to define a memory model's basic strategy whereas application-specific information is used as a parameter to refine this strategy. For example, generational garbage collectors [14] define a basic global strategy based on the lifetime of objects but may also provide variable generational parameters that can be specialised based on knowledge of the lifetime patterns of objects for a particular application. In some cases there is no default for this type of information. For instance, real-time garbage collectors require user-defined parameters such as the maximum allocation rate. In this case, this information is by definition application-specific rather than general. General and application-specific information can be further decomposed into local and global information. In this case, the difference is the granularity for which the information is specified. At one extreme, fine grain allocation and deallocation using malloc() and free() operators is local information. At the other extreme, the information in the parameters described for real-time collectors are global to the whole program. Coarse grain models such as memory pools lie between these two extremes with aggregates being specified to define the lifetime of objects. The case for application-specific being preferred over general information is clear when the worst case has to be considered; an application that does not fit a general model is often easy to develop and such an application will perform poorly. The case for local as opposed to global information is more difficult to argue as this implies a significant development burden.

#### 3.3 The Metronome Collector

The Metronome collector is a time-based collector that uses a best-fit policy implemented with a segregated free list mechanism in its DM algorithm and an incremental mark-sweep collector that defragments when required. The segregated policy implies that internal fragmentation is observed rather than external fragmentation. A read barrier is implemented to ensure moved objects are properly referenced by the application. In motivating the time-based approach of the Metronome collector [1, 3, 2], Bacon et. al. derive an analysis that allows a guarantee of the minimum mutator time. For a given time period in the application, the mutator and collector have two properties specified: for the mutator, the allocation rate over a time interval and the maximum live memory usage and; for the collector, the rate at which memory can be traced. By defining the frequency of invocation of the collector, the memory required for a given utilisation requirement is derived. Alternatively, the maximum available memory is specified and the minimum guaranteed utilisation is derived. Briefly, given the mutator quantum  $Q_T$  and the collector quantum  $C_T$  the mutator utilisation is trivially  $\frac{Q_T}{Q_T+C_T}$ . If the allocated memory between  $t_1$  and  $t_2$  is given by  $\alpha(t_1, t_2)$ , the rate of collection is R and the amount of live data at time t is given by  $m_t$ , then it can be shown that excess space of  $\alpha(t, t + \frac{m(t)}{R}, \frac{Q_T}{C_T})$  is required.

The results from Metronome are of particular use to this investigation for two reasons: firstly, the cost of each of the four processes of GC described in the previous section are bro-

<sup>&</sup>lt;sup>5</sup>We note in passing that the abstraction problem is one also faced by realtime developers in the area of scheduling. The traditional cost/deadline model fails to capture more complex abstractions of timing requirements with the result that mapping this requirements to the traditional model is often complex and inefficient.

Bench	m	S	$T_{GC}$	$T_I$	$T_R$	$T_M$	$T_S$	$T_D$
javac	86	172	2.21	0.001	0.061	1.973	0.137	0.124
db	82	137	2.63	0.001	0.043	2.408	0.148	0.163
jack	82	146	1.73	0.001	0.042	1.076	0.094	0.047
mtrt	80	122	1.59	0.001	0.046	1.386	0.115	0.078
jess	73	126	0.63	0.001	0.186	0.554	0.046	0.031
fragger	72	151	3.20	0.001	0.147	1.700	0.175	1.295

# Table 1. Time and Space Overheads at 50% Util-isation in Metronome (Taken from [3])

ken down and quantified; secondly, the extra parameters that must be specified by the developer capture global, applicationspecific information that can be used to evaluate the potential of this type of information. Ignoring allocation costs, collection overheads are broken down into the costs of initialisation and termination of the collector  $(T_I)$ , root scanning  $(T_R)$ , marking  $(T_M)$ , sweeping  $(T_S)$  and defragmentation  $(T_D)$ . For a 50% utilisation across the selection of applications from the SPECjvm98 benchmarking suite, the time overheads (in seconds) are reproduced in Table 1 where the amount of live memory (m) and the maximum heap size (s) are also shown. It is interesting to note that the majority of the collection cycle is spent tracing, with the time taken for defragmentation being significant only in the fragger application.

The results from the research for Metronome initially appear promising. By requiring the user to specify information about the pattern of object usage, the space and time overheads are significantly smaller than those shown to hold in the worst case for an explicit model using a DM algorithm. The information that needs to be specified includes the average object size and locality in the size. These parameters are used to reduce the pessimism in the worst case overheads incurred during tracing and defragmentation. Tools for automatically calculating global parameters for fine-tuning garbage collectors are also common [22]. The magnitude of the overheads of the DM algorithm due to fragmentation described in the last section in comparison to the results achieved here require further investigation in relation to the entropy hypothesis. The space overheads are as little as two and half times the amount of live memory. By the entropy hypothesis, this reduction in overheads from what was shown to hold for a DM algorithm can only be achieved by an increase in time overheads and/or a source of extra information about the application's use and lifetime of objects. Although this tradeoff is partly captured by the use of the defragmentation algorithm in Metronome the time overheads due to fragmentation are relatively small. This is achieved by capturing the pessimism of the worst-case fragmentation through a factor  $\lambda$  that specifies the locality of size of objects and thereby the amount of possible fragmentation that can occur.

Although the cost of tracing and using an incremental approach are still significant, Metronome's  $\lambda$  factor appears to address the fragmentation problem for its DM algorithm. This research therefore provides not only a real-time collector with tighter space and time overheads but, more importantly, a solu-

tion that can be applied to explicit fine grain models for use in more resource constrained environments. This could therefore fill the gap between the environments real-time collectors address and the those addressed by the the static approach. The  $\lambda$  factor allows the developer to capture Johnstone's thesis that fragmentation in real-world systems is negligible. However, there are two problems with this approach: firstly, identifying the  $\lambda$  factor for an application is non trivial; secondly, the analysis does not capture the possible variance of this value during the application's lifetime. Therefore, the chosen  $\lambda$  factor will always always be the smallest value during the entire lifetime of the application. This leads to a number of assumptions in the derivation of the worst-case space requirements for a given minimum mutator utilisation that inhibit a true calculation of the worst case space requirements. Crucially, the space-time relationship between heap size and utilisation is not well defined due to the interdependence of parameters leading to undesirable recursive functions. In particular, the amount of extra space required depends on the time needed for a collection cycle which in turn depends on the amount of heap space. The chosen or derived heap size is based on an "expansion factor" of the maximum amount of live memory that is chosen to be around 2.5 based on experimentation. In the absence of the pessimistic values that would be obtained from a recursive relationship of space and time overhead, the results given in [1, 3] are essentially observed rather than analytical quantities. If the heap size is given at a large order of magnitude relative to this, then the factor of 2.5 would probably suffice. If this is not the case, then the choice of heap size could be made a function of the maximum allocation rate and  $T_{GC}$  rather than the maximum live memory. This could result in the heap size being several orders of magnitude larger than the maximum live memory.

# 4 The Case for Coarse Grain Memory Models

Whereas it could be argued that the analysis provided by both work and schedule based approaches of fine grain models fulfil real-time predictability requirements, the overheads of these approaches may make this prohibitive in resourceconstrained environments. From the evaluation of fine grain approaches in Sections 2 and 3.3, it is immediately apparent that the most urgent information required is that which addresses fragmentation and the cost of tracing. Therefore, the solution to the memory management problem for resource-constrained real-time systems could lie in an explicit model (thereby eliminating the need for tracing)<sup>6</sup> and directing research at a more local characterisation of the application's information, particularly fragmentation. The entropy hypothesis argues the case for more information to be expressible and for this information to then be used by that model. The key problem is identifying what this information is and how it can be captured. Rather than arguing for similar global, application specific information to be used in these models, a case for more localised information can be made. For example, global parameters could be made more localised in Metronome by being sensitive to the

<sup>&</sup>lt;sup>6</sup>A model that uses static escape analysis [7] could help the collector by identifying objects that are believe to have become unreachable.

program's flow. Therefore, rather than there being just one integer  $\lambda$  factor, a number of values could be assigned that depend on the current execution trace. This could then dynamically alter the collection rate or where defragmentation is triggered.<sup>7</sup> The feasibility of such a characterisation is unclear, both in terms of identifying this function as well as how this fits into the scheduling model. Until such research is carried out, an alternative is available in the form of coarse grain memory models.

# 4.1 Object Lifetime in Coarse Grain Models

Coarse grain models take advantage of the phenomenon that it is often possible to aggregate objects according to their lifetime. For example, objects that are created close together typically have similar lifetimes due to spatial locality of reference [5]. This phenomenon is used implicitly by generational garbage collectors [14] as a general and global piece of information. For a real-time environment, a more application specific and local approach is required as accuracy and precision is necessary in order to provide the required guarantees with low pessimism. Coarse grain models involve specifying the boundaries of aggregates appropriately and then specifying the lifetime of each aggregate. The advantage of a coarse grain approach is that the extra information that comes from the relative ordering of objects means that a significant reduction in fragmentation can be achieved. The extra space cost of retaining objects for a slightly longer period than required is still significantly less than the fragmentation overheads in fine grain models. This is because the variance between the size of allocated blocks can be made smaller, thereby minimising the worst case space cost by Robson's equations. Therefore, an explicit coarse grain model targets the two largest sources of overhead identified above: tracing and fragmentation.

The importance of the relative order of object lifetimes is rarely taken advantage of in most memory models. Theoretically, knowing the exact lifetime of objects can be used to completely eliminate fragmentation. This can be achieved by using a bin-packing algorithm that guarantees that at the point of maximum live data, no fragmentation occurs and any fragmentation that occurs at other times results in memory requirements that are less than the size of this live data. The indeterminability of knowing the exact creation and deallocation of all objects in a dynamic environment coupled with the probable intractability of such a bin-packing algorithm make this approach impractical. However, a coarse grain model allows this type of information to be captured in a less-exact way. By the entropy hypothesis, this automatically implies greater overheads than a zero-cost model but potentially smaller overheads than an explicit fine grain approach. The space occupied by coarse grain models is shown in Figure 2 to be equivalent to fine grain models in the worse case. This occurs when no aggregation information is available and therefore allocation and deallocation in the coarse grain model is done at an equivalent granularity as in the fine grain one. However, there exists a space within the coarse grain model that can outperform the fine grain model in both time and space. As an example, consider the case where a fine grain first-fit model is used in an application that creates objects of sizes 64, 128 and 64k bytes. By Robson's analysis, this would require about ten times the amount of live memory. However, if it were known that all 64 byte objects are deallocated at the same time in pairs, then they could be allocated next to each other every time. In this way, Robson's analysis shows that only 5 times the amount of live memory is required.

#### 4.2 The RTSJ Memory Model: Criticism

The RTSJ adopts a novel approach to memory management with the introduction of scoped regions. This model is essentially a coarse grain model that aggregates object lifetime based on program flow. The main criticisms of this model are broadly as follows:

#### The model is complex to use:

The complexity of the RTSJ model could be partially argued to be a failure of developers to target the model. As argued in Section 3.1, developers must target a memory model rather than apply an orthogonal abstraction to the chosen model. Since the RTSJ defines object aggregates based on locality in the program flow, developers must express lifetime information around this abstraction. However, it is sometimes the case that the real-world pattern of memory usage does not follow this approach. For example, it is a well known problem that applications that employ a producer/consumer pattern of memory usage are hard to describe in the RTSJ as the implicit information in this application is not well captured by the scoped memory abstraction.

#### Reference rules inhibit the expression of object lifetime:

A second source of the complexity in using the RTSJ's memory model comes from the model's reference rules. Despite an object's lifetime clearly belonging to some aggregate, these rules require a change in the lifetime of objects based on the reference graph. Restricting the flexibility of how aggregates are defined is an example of a memory model unnecessarily restricting the expression of known lifetime information.<sup>8</sup>

#### The possibility of reusing code is limited:

The reuse problem of RTSJ code is caused by the embedding of memory concerns within application code. The absence of an interface that captures how the memory model is used in existing classes means that there is no way to export the lifetime of objects created in this code.

#### Lifetime information is poorly utilised in implementations:

Although the RTSJ specifies when the backing store of regions are allocated and freed in the runtime, no constraints on

<sup>&</sup>lt;sup>7</sup>These values therefore become points on the flow graph of the application and new analysis would be required to identify how the transition between these points changes the behaviour of the collector.

<sup>&</sup>lt;sup>8</sup>Reference objects are inspired by the reference objects used in GC environments. In particular, the semantics described here are similar to those for weak references.

the underlying DM algorithm is specified. In particular most implementations do not take advantage of the scoping structure to reduce fragmentation. Also developers are often unaware of the implications of where aggregate boundaries are applied.

# 4.3 The RTSJ Memory Model: Solutions

#### The model is complex to use:

In order to address this problem, researchers have provided solutions that are orthogonal to the RTSJ scoped memory model only because the model fails to allow these patterns to be expressed. For example, Pizlo [24] proposes "wedge threads" that are used to keep a scoped region alive when its reference count would have otherwise dropped to zero. Although the introduction of these patterns highlights the complexity of using the scoped memory model, forcing a solution on top of the existing RTSJ model has earned them the term "anti-patterns". There are two possible conclusions that can be drawn from this: either the RTSJ needs to be extended to allow these patterns to be expressed as an integral part of the model or the way the model should be targeted is still not understood. The second possibility is improbable as the model's rationale is intuitive. Earlier work we have carried out [12, 11, 9] shows that in translating the same information present in an explicit coarse grain model to a scoped model results in a tradeoff space that can reduce space and time overheads is some cases but can lead to potentially unbounded space requirements in others.<sup>9</sup> This conclusion is important as it makes a strong case for the RTSJ to provide alternative memory models in addition to scoped memory that allow these patterns of object lifetime to be expressed.<sup>10</sup> The RTSJ scoped memory model can express some patterns of object lifetime better than other models and is therefore useful when these patterns are manifested in the real-world. When this is not the case, the RTSJ must provide other approaches that allow the expression of object lifetimes that the patterns such as those described in [6, 24] address. Describing these patterns as an abstraction on top of the RTSJ scoped model is a poor approach. In conclusion therefore, the entropy hypothesis can be used to argue that the RTSJ model is complex only when it is used to express information that is poorly captured by its abstraction. The solution is therefore not an alternative model but an extended model that allow a wider range of patterns of memory usage to be expressed.

#### **Reference rules inhibit the expression of object lifetime:**

The problem of expressing some aggregate lifetime patterns in the RTSJ as described above is compounded by the RTSJ making it harder to define these aggregates. We have developed reference objects [10] in order to address this. Using a reference object rather than a normal reference achieves a compromise between maintaining the safety of objects and allowing this lifetime information to be expressed. Reference objects carry lifetime information that is looser than that specified by a regular reference. In the RTSJ, if an object A holds a reference to an object B then B *must* live as long as A. However, if A holds a reference object to an object B then B *can* live as long as A but the reference object must throw a caught exception if this is shown not to be the case at runtime. Reference objects are an example of allowing the developer to specify lifetime information in order to reduce space overheads.

#### The possibility of reusing code is limited:

This issue comes back to the question of: "What is the best way of allowing known information to be expressed without placing unnecessary burden on the developer?" If a memory model can allow an equivalent expression of this information externally to application code then code reuse is again possible. We have developed a solution to this problem as part of our work [12, 11, 9] that is based on extracting the cross-cutting memory management concern as a separate aspect. This is achieved by defining the boundaries and lifetime of aggregates on the program's control flow graph. An automatic algorithm then fines the optimal scoping structure and annotates the application to enter and exit regions when specified.

#### Lifetime information is poorly utilised in implementations:

The second role of the memory model, taking advantage of expressed memory usage, is also under-specified in the RTSJ. The underlying DM algorithm allocates and frees regions in a similar way to explicit allocation and deallocation, thereby resulting in similar fragmentation problems. In particular, the advantage of reduced fragmentation due to scoping is lost in multithreaded environments as the lifetimes of aggregates across different threads of control are unspecified. A solution to this problem is to have separate partitions for each branch of the scope stack when this can be statically determined to be possible. When this is not possible, the model experiences similar fragmentation to fine grain models if the variance in the sizes of regions is large. This therefore partly eliminates the rationale for a scoped approach. Again, the inability to define separate partitions for scope stacks is an example of how, unavailable information leads to higher overheads. In this case, a simple analysis of the variance of region sizes can be used to merge regions of similar lifetime so that fragmentation can be reduced.

#### 5 Conclusion

The search for suitable memory models that address the requirements of complex embedded real-time systems continues to gain momentum. The choice of a suitable memory model for the RTSJ is viewed as a contentious issue by many, particularly where a choice between real-time GC and scoped memory must be made. The entropy hypothesis shows that an argument for a some memory model is not absolute to a particular domain,

<sup>&</sup>lt;sup>9</sup>Using the entropy hypothesis it can be argued that this occurs due to a loss of information in the translation process. Understanding what constitutes this lost information is an important goal that is still being investigated. It appears at this point that this lost information is due to the scoped model being only able to capture the relative ordering of a subset of regions at any time, rather than the complete ordering available in an unscoped model.

<sup>&</sup>lt;sup>10</sup>Interestingly, Benowitz [6] proposes solutions that map alternative memory models on top of the RTSJ. Mapping alternative memory models to the RTSJ is of little advantage in terms of space and time performance as the disadvantages or these models is only supplemented by the extra cost of maintaining this abstraction.

whether that domain is defined in terms of allowable space and time overheads or development costs. Rather, a memory model is suitable for a given application only in the degree to which it can capture information of memory usage in the application. The goal of future research must therefore lie in identifying this information and providing ways of allowing this information to be expressed in order for the underlying memory subsystem to make use of it. This is a significant shift from current research directions that deliver only marginal improvements due to the implicit assumption that expressing lifetime information implies unnecessary burdens on application developers.

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