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Filtering and Control of Traffic Volume on Arterial

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ABSTRACT

This paper studies the filtering and control of traffic volume consistently from the system-theoretic viewpoint. Based on the statistical analysis of the traffic volumes observed by detector, the time-dependent characteristics of traffic volume are formulated by the linear time-varying discrete-time system. Moreover, the time-dependent characteristics of traffic congestion length are formulated by the linear time-varying discrete-time system based on the traffic volume balance at each signalized intersection. Next, the algorithms of the Kalman filter and the MIPA Kalman filter are derived as the state estimation method for the above-mentioned dynamical system. The priority control method of the traffic congestion length with respect to the direction along the arterial is proposed by using the systematic control method of signal control parameters. Finally, the effectiveness of the MIPA Kalman filter and the priority control method of the traffic congestion length is confirmed by the simulation results of Route 2 national road in Fukuyama city.

Keywords: linear discrete-time system, MIPA Kalman filter, control of traffic congestion length.

1. INTRODUCTION

The analysis and control of the traffic congestion have been studied from the viewpoints of traffic engineering and system theory[1]~[4]. In this paper, the filtering and control methods of the traffic volume which play the most important role to the traffic congestion control on the arterial are studied consistently from the system-theoretic viewpoint.

First, the time-dependent characteristics of the traffic volume and traffic congestion length are formulated by the linear time-varying discrete-time systems, and the Kalman filter and MIPA Kalman filter algorithms are derived as the state estimation methods for these systems.

Next, the priority control method of the traffic congestion length with respect to the

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direction along the arterial is proposed by using the systematic control method of signal control parameters, and the relationship between the filtering error of traffic volume and the control error of traffic congestion length is derived. Finally, from the simulation results of Route 2 national road in Fukuyama city, it is confirmed that the MIPA Kalman filter is effective to the filtering problem of traffic volume contaminated by non-Gaussian noise, and the priority control method of the traffic congestion length on the arterial is also effective with respect to the control accuracy and the computation feasibility.

2. SYSTEM REPRESENTATION

The time-dependent characteristics of two directions of traffic volume on the arterial are assumed to be formulated for sampling period $\triangle T$ as follows.

$$x(\ell,k)=A(\ell,k-1)\,x(\ell,k-1)+u_1(\ell,k-1) \tag{1}$$
$$y_v(\ell,k)=C(k)\,[x(\ell,k)+v_1(\ell,k)] \quad k=1,2,\cdots,N \tag{2}$$

where $k=k\triangle T$, ℓ and k are a day of the week and hour respectively. $x(\ell,k)$ is the traffic volume vector, $y_v(\ell,k)$ is the observation vector, $u_1(\ell,k-1)$ is the state noise vector, $v_1(\ell,k)$ is the observation noise vector, of dimension 2*1 respectively. $A(\ell,k-1)$ is the coefficient matrix, $C(k)$ is the adjusting factor matrix of detector, of dimension 2*2 respectively. The state noise vector $u_1(\ell,k)$ and the observation noise vector $v_1(\ell,k)$ are assumed as white random sequences of the following equations

$$E\,[u_1(\ell,k)]=\overline{u}_1(\ell,k) \tag{3}$$
$$E\,[v_1(\ell,k)]=\overline{v}_1(\ell,k) \tag{4}$$
$$E\,\{[u_1(\ell,k)-\overline{u}_1(\ell,k)]\,[u_1(\ell,s)-\overline{u}_1(\ell,s)]^T\}=\delta_{ks}\,U_1(\ell,k) \tag{5}$$
$$E\,\{[v_1(\ell,k)-\overline{v}_1(\ell,k)]\,[v_1(\ell,s)-\overline{v}_1(\ell,s)]^T\}=\delta_{ks}\,V_1(\ell,k) \tag{6}$$
$$\delta_{ks}=\begin{cases}1 & k=s\\ 0 & k\neq s\end{cases} \tag{7}$$

Based on the traffic volume balance at jth signalized intersection on the arterial, the time-dependent characteristics of traffic congestion length are described by

$$\begin{cases}x_e^j(\ell,k)=x_e^j(\ell,k-1)+x_i^j(\ell,k)-x_o^j(\ell,k)+u_2^j(\ell,k)\\ x_e^j(\ell,k)\geqq 0\end{cases} \tag{8}$$
$$y_\ell^j(\ell,k)=D^j(\ell,k)\,x_e^j(\ell,k)+v_2^j(\ell,k) \quad j=1,2,\cdots,M \tag{9}$$

where superscript j is the location of signalized intersection, $x_e^j(\ell,k)$ is the vector of excess incoming traffic volume, $x_i^j(\ell,k)$ is the vector of incoming traffic volume, $x_o^j(\ell,k)$ is the vector of outgoing traffic volume, $y_\ell^j(\ell,k)$ is the observation vector of traffic congestion length, $u_2^j(\ell,k)$ is the state noise vector, $v_2^j(\ell,k)$ is the observation noise vector, of dimension 2*1 respectively. $D^j(\ell,k)$ is the transformation matrix of dimension 2*2, which transforms the vector of excess incoming traffic volume into the vector of traffic congestion length. $u_2^j(\ell,k)$ and $v_2^j(\ell,k)$ are assumed as white random sequences.

3. FILTERING

The observation values of the traffic volume by a detector contain the errors depending on the type, location and sensitivity of that, and so on. To remove these errors we apply the state estimation techniques such as the Kalman filter and the MIPA Kalman filter to the system represented by Equation(1) and Equation(2). The Kalman filter, which is the optimal filter in such a case that the system is linear and the noises are Gaussians, is derived as follows[5].

Kalman filter:

$$\hat{x}(\ell,k)=\tilde{x}(\ell,k)+P(\ell,k)\,[C^T(k)]^{-1}V_1^{-1}(\ell,k)\,z(\ell,k) \tag{10}$$

$$\tilde{x}(\ell,k)=A(\ell,k-1)\,\hat{x}(\ell,k-1)+\bar{u}_1(\ell,k-1) \tag{11}$$

$$P(\ell,k)=[M^{-1}(\ell,k)+V_1^{-1}(\ell,k)]^{-1} \tag{12}$$

$$M(\ell,k)=A(\ell,k-1)\,P(\ell,k-1)\,A^T(\ell,k-1)+U_1(\ell,k-1) \tag{13}$$

$$z(\ell,k)=y_v(\ell,k)-C(k)\,[\tilde{x}(\ell,k)+\bar{v}_1(\ell,k)] \tag{14}$$

where $\hat{x}(\ell,k)$ and $\tilde{x}(\ell,k)$ denote the optimal estimates and predicted value of $x(\ell,k)$ respectively.

MIPA Kalman filter:

We derive such as a robust filter that the filtering accuracy is not degraded even if either the state noise or the observation noise is non-Gaussian. In this paper, the MIPA Kalman filter for the filtering of hourly traffic volume is derived for the case where the state noise is Gaussian and the observation noise is non-Gaussian[6],[7]. Here, the MIPA is the abbreviation of m-interval polynomial approximation.

$$\hat{x}(\ell,k)=\tilde{x}(\ell,k)+P(\ell,k)\,[C^T(k)]^{-1}s\,[z(\ell,k)] \tag{15}$$

$$\tilde{x}(\ell,k)=A(\ell,k-1)\,\hat{x}(\ell,k-1)+\bar{u}_1(\ell,k-1) \tag{16}$$

$$P(\ell,k)=M(\ell,k)-M(\ell,k)\,\Lambda(\ell,k)M(\ell,k) \tag{17}$$

$$M(\ell,k)=A(\ell,k-1)\,P(\ell,k-1)\,A^T(\ell,k-1)+U_1(\ell,k-1) \tag{18}$$

$$\Lambda(\ell,k)=\sum_{j=1}^{m}[b_j(\ell,k)P_{1j}+c_j(\ell,k)P_{0j}] \tag{19}$$

$$s[z(\ell,k)]=\sum_{j=1}^{m}\{[b_j(\ell,k)\,z(\ell,k)+c_j(\ell,k)]\cdot I_{Aj}[z(\ell,k)]\} \tag{20}$$

$$z(\ell,k)=y_v(\ell,k)-C(k)\,[\tilde{x}(\ell,k)+\bar{v}_1(\ell,k)] \tag{21}$$

where the interval $A_j=(a_{j-1},a_j]$, $a_0=-\infty$, $a_m=+\infty$, and satisfies the following equation for the probability density function $f[z(\ell,k)]$

$$\int_{A_j} f[z(\ell,k)]\,dz(\ell,k)=1/m \tag{22}$$

$I_{Aj}[z(\ell,k)]$ denotes the indicator function of A_j and is defined by

$$I_{Aj}[z(\ell,k)]=\begin{cases}1 & z(\ell,k)\in A_j\\ 0 & z(\ell,k)\notin A_j\end{cases} \tag{23}$$

The coefficients $b_j(\ell,k)$ and $c_j(\ell,k)$ are calculated by

$$b_j(\ell,k) = \frac{U_{0j}P_{1j} - U_{1j}P_{0j}}{U_{2j}U_{0j} - U_{1j}^2} \tag{24}$$

$$c_j(\ell,k) = \frac{U_{2j}P_{0j} - U_{1j}P_{1j}}{U_{2j}U_{0j} - U_{1j}^2} \tag{25}$$

with

$$U_{ij} = \int_{A_j} z^i[\ell,k]\, f[z(\ell,k)]\, dz(\ell,k) \qquad i=0,1,2 \tag{26}$$

$$P_{0j} = f(a_{j-1}) - f(a_j) \tag{27}$$

$$P_{1j} = a_{j-1} f(a_{j-1}) - a_j f(a_j) + U_{0j} \tag{28}$$

The important differences of the MIPA Kalman filter from the Kalman filter are as follows. The probability density function of residual process $f[z(\ell,k)]$ is approximated based on the m-interval polynomial approximation method, and both the function vector of residual process $s[z(1,k)]$ and the variance matrix of filtering error $P(\ell,k)$ are also computed by using the same approximation method.

4. CONTROL

The priority control method of the traffic congestion length with respect to the direction along the arterial is proposed by using the systematic control method of signal control parameters. The control error vector is defined by

$$e_c(\ell,k) = \ell_r(\ell,k) - \hat{y}_c(\ell,k) \tag{29}$$

where $\ell_r(\ell,k)$ denotes the vector of permitted queueing length, $y_\ell(\ell,k)$ denotes the vector of traffic congestion length, of dimension 2*1 respectively. The cycle length vector $c_y(\ell,k)$ and the green split vector $r_g(\ell,k)$ are controlled so as to satisfy the condition $e_c(\ell,k) \geqq 0$. The offset vector $t_{off}(\ell,k)$ is controlled according to

$$t_{off}(\ell,k) = V_s^{-1}(\ell,k)\,\ell_d - S_f^{-1}(\ell,k)\,q(\ell,k) \tag{30}$$

such that the motor-cars flow smoothly between 2-adjacent signalized intersections[8], where $V_s(\ell,k)$ is the average speed matrix, ℓ_d is the road length vector between jth- and (j+1)th-signalized intersection, $S_f(\ell,k)$ is the saturation flow matrix on the approach at the downstream signalized intersection, $q(\ell,k)$ is the queueing number vector of motor-cars while the signal has been red at the downstream signalized intersection.

By using the systematic control method of signal control parameters, the priority control algorithm of the traffic congestion length with respect to the direction along the arterial is outlined as follows.

Step 1. Setting of the initial values of the parameters and variables for two directions (see Step 2) at each signalized intersection.

Step 2. Filtering of the incoming traffic volume by using such filters as Kalman filter and MIPA Kalman filter.

$y_v^m(\ell,k) \rightarrow \hat{x}_i^m(\ell,k)$, $m=1,2$

where m=1 denotes the direction which is along the arterial, and m=2 denotes the direction which is cross the arterial.

Step 3. Estimation of the green time $g^m(\ell,k)$ needed to flow the incoming traffic volume.

Step 4. Estimation of the cycle length by

$$\hat{C}_y^m(\ell,k) = [R_g^m(\ell,k)]^{-1}\,\hat{g}^m(\ell,k) \tag{31}$$

where $R_g^m(\ell,k)$ denotes the green split matrix.

Step 5. Estimation of the capacity[4].

$$\hat{C}_x^m(\ell,k)=R_{g\ell}^m(\ell,k)\,\hat{C}_{x\ell}^m(\ell,k)+R_{gs}^m(\ell,k)\,\hat{C}_{xs}^m(\ell,k)+R_{gr}^m(\ell,k)\,\hat{C}_{xr}^m(\ell,k) \tag{32}$$

where $R_{g\ell}^m(\ell,k)$, $R_{gs}^m(\ell,k)$ and $R_{gr}^m(\ell,k)$ denote the green split matrix for left-turn-, straightforward- and right-turn-traffic lanes respectively. $C_{x\ell}^m(\ell,k)$, $C_{xs}^m(\ell,k)$ and $C_{xr}^m(\ell,k)$ denote the capacity vector for each traffic lane respectively.

Step 6. Estimation of the excess incoming traffic volume from Equation(8), and that of the traffic congestion length from the following equation

$$\hat{y}_\ell^m(\ell,k)=D^m(\ell,k)\,\hat{x}_e^m(\ell,k) \tag{33}$$

Step 7. When $\hat{e}_c^m(\ell,k)\geqq 0$, or $r_g^m(\ell,k)=r_{g,\max}^m(\ell,k)$, we apply $\hat{c}_y^m(\ell,k)$ and $r_g^m(\ell,k)$ at the time to the optimal values. Then proceed to step 8. When $\hat{e}_c^m(\ell,k)<0$, correct $r_g^m(\ell,k)$ according to the following equation

$$r_g^{m\,(n+1)}(\ell,k)=r_g^{m\,(n)}(\ell,k)+\triangle r_g^m \tag{34}$$

where superscript(n) denotes the repetition times of computation, and $\triangle r_g^m$ denotes the small increment vector. Then return to step 4.

Step 8. Estimation of the optimal relative offset from Equation (30). We execute this control algorithm in sequence from m=1 to m=2, and from k=1 to k=N. The cycle lengths and the green splits in the case of m=2 are determined definitely restricted by those in the case of m=1 [9]. Therefore, step 3, step 4 and step 7 are not executed in the case of m=2.

5. FILTERING AND CONTROL

The blockdiagram of the feedback control system of traffic congestion length is shown in Fig.1 based on the filtering and control of traffic volume at each signalized intersection on the arterial. The filtering error $e_f(\ell, k)$ is defined by

$$e_f(\ell, k) = \hat{x}_i(\ell, k) - x_i(\ell, k) \tag{35}$$

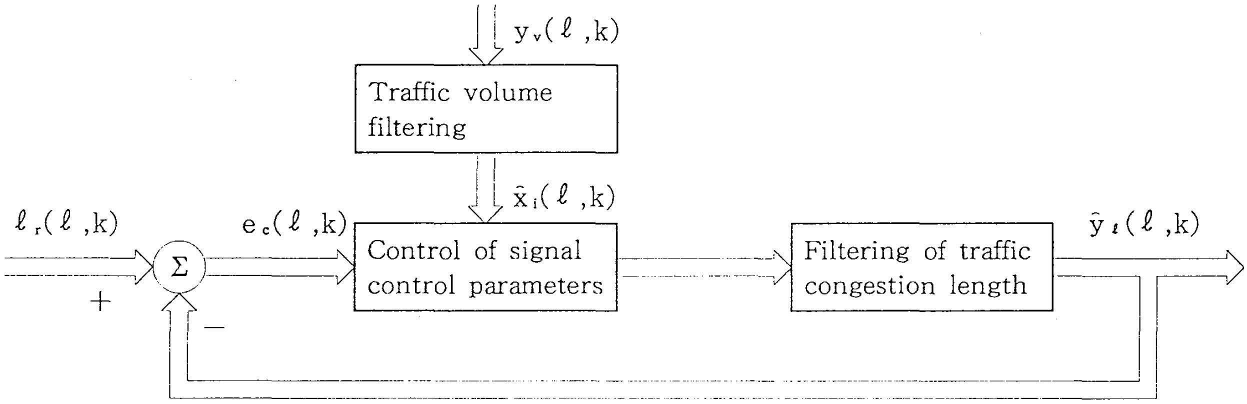

Fig.1 Blockdiagram of feedback control system for traffic congestion length based on filtering and control of traffic volume

By substituting the filtering values of the incoming traffic volume $\hat{x}_i(\ell, k)$ for the priority control algorithm mentioned above, the relationship between the filtering error and the control error is obtained. In non-congested case the filtering error never gives effect to the control error, but in other case

$$e_c^e(\ell, k) = -D(\ell, k)\,[I - R(\ell, k)]\;e_f(\ell, k) \tag{36}$$

where $e_c^e(\ell, k)$ denotes the control error generated by the filtering error, and $R(\ell, k)$ denotes the outgoing ratio matrix defined by

$$\hat{x}_0(\ell, k) \triangleq R(\ell, k)\,\hat{x}_i(\ell, k) \tag{37}$$

6. SIMULATION RESULTS AND DISCUSSIONS

In congested case, the deterioration of the filtering accuracy of incoming traffic volume cause that of the control accuracy of traffic congestion length as described by Equation(36). Therefore, the filtering errors are compared the Kalman filter with the MIPA Kalman filter by using the observation data of Route 2 national road in Fukuyama city in April 1979 to July 1980. From the comparison shown in Table 1, it is confirmed that the MIPA Kalman filter is actually effective to the filtering of the hourly traffic volume,

Table 1 Comparison of filtering errors and observation error on up line

	Kalman Filter	MIPA Kalman Filter	Observation Value
Absolute Mean (%)	13.58	7.71	13.79
Mean (%)	0.14	−0.05	4.31
Variance ($\%^2$)	290.5	114.3	364.0

whose observation values are contaminated by non-Gaussian noise.

Next, we consider the computer simulation results of traffic congestion length in two directions, that is the eastward-(m=1) and southward-(m=2) directions. In the eastward direction, the green splits are controlled widely reacting sensitively to the incoming traffic volumes as shown in Fig.2 and Fig.3. The controlled values of cycle length are smaller than the real data by 18 seconds on the average as shown in Fig.4. As the results, the peak value of traffic congestion length in the morning are reduced from 400 meters to 227 meters, and the continuative time of traffic congestion is reduced remarkably as shown in Fig.5. Moreover, at the downstream signalized intersection the capacities are always controlled so as to exceed the incoming traffic volumes and no traffic congestion occurs during the day. Therefore, the offsets are controlled such that their values are always constant as shown in Fig.6. On the other hand, in the southward direction the capacities are always controlled so as to exceed the incoming traffic volumes, and no traffic congestion occurs during the day.

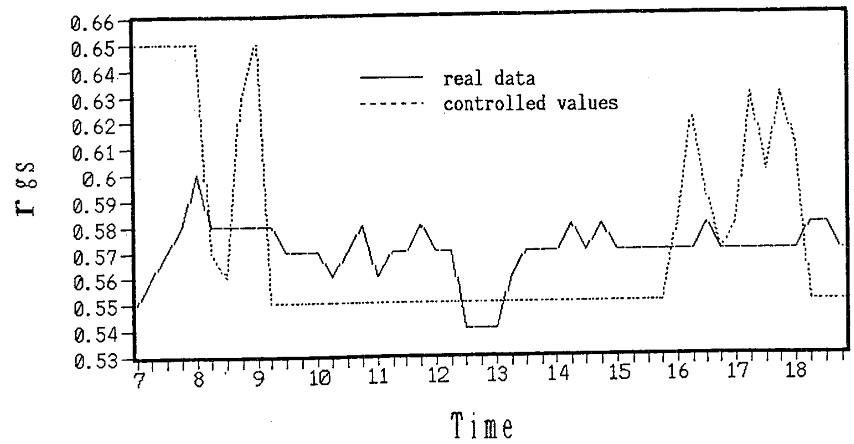

Fig. 2 Comparison between real data and controlled values of green split of straightforward- traffic lanes at upstream signalized intersection

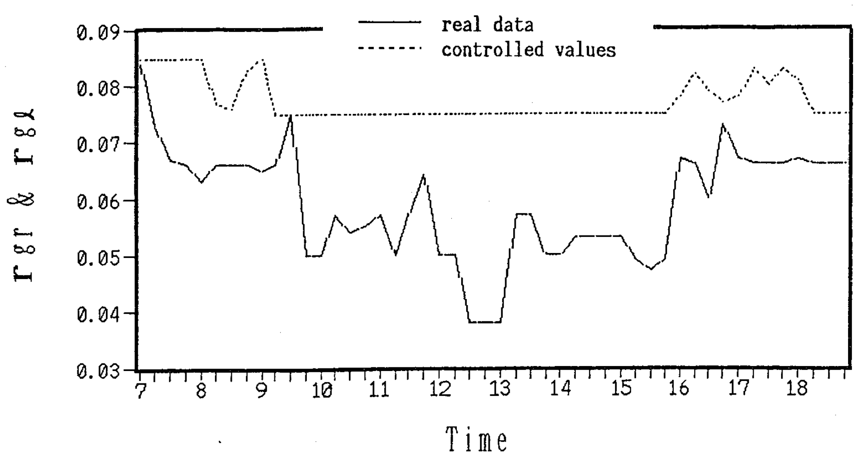

Fig. 3 Comparison between real data and controlled values of green split about left- and right- turn- traffic lanes at upstream signalized intersection

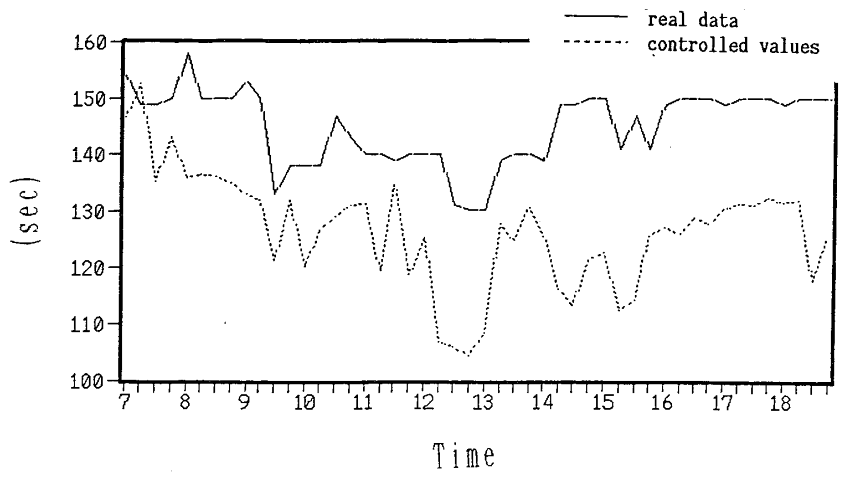

Fig. 4 Comparison between real data and controlled values of cycle length at upstream signalized intersection

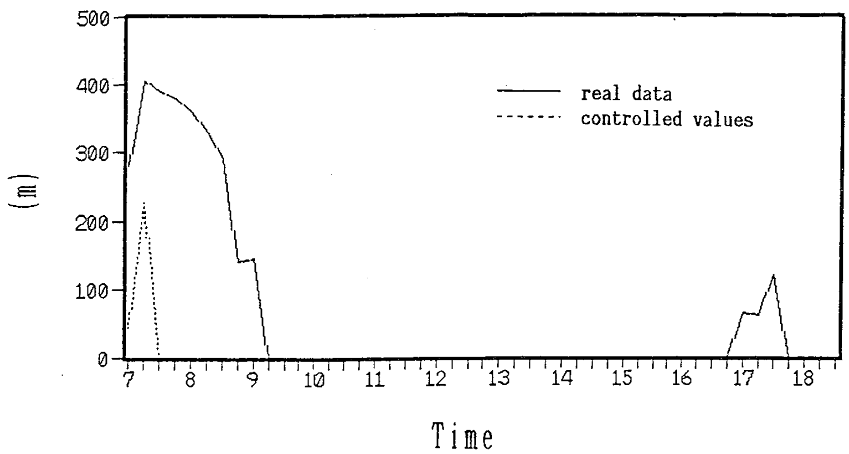

Fig. 5 Comparison between real data and controlled values of traffic congestion length at upstream signalized intersection

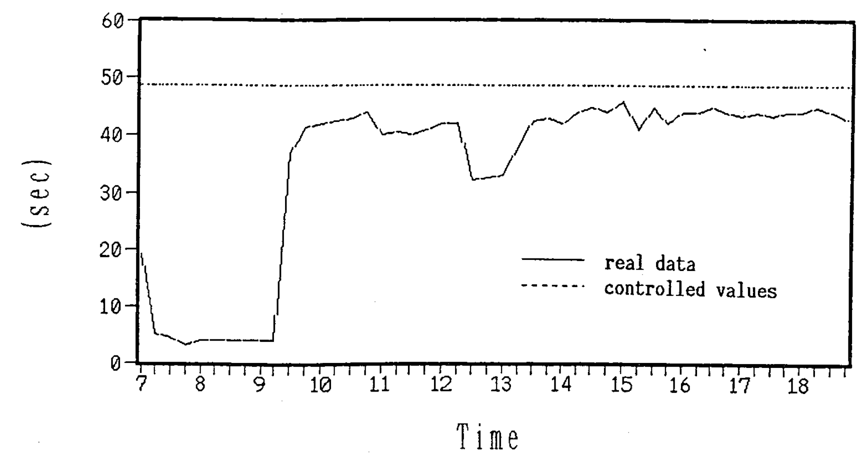

Fig. 6 Comparison between real data and controlled values of offset between 2-adjacent signalized intersections

7. CONCLUSIONS

In this paper, the time-dependent characteristics of traffic volume are formulated by the linear time-varying discrete-time system, and the state estimation algorithms of this system are derived.

The priority control algorithm of traffic congestion length with respect to the direction along the arterial is derived for two directions at each signalized intersection. The relationship between the filtering error and the control error is derived for the feedback control system of traffic congestion length. From the computation results of Route 2 national road in Fukuyama city, the following are confirmed.

1) The MIPA Kalman filter is actually effective to the filtering problem of hourly traffic volume, whose observation values are contaminated by non-Gaussian noise.
2) By controlling the three signal control parameters systematically and controlling the traffic congestion length on the arterial prior to other roads, the cycle lengths, the peak value of traffic congestion length and the continuative time of traffic congestion are reduced remarkably.

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