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WASHINGTON UNIVERSITY IN ST. LOUIS

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Essays on Technology Adoption, Demographics, and Development

by

Ting-Wei Lai

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

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Dedicated to my parents.

ABSTRACT OF THE DISSERTATION

Essays on Technology Adoption, Demographics, and Development by

Ting-Wei Lai

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Professor Ping Wang, Chair

Professor B. Ravikumar, Co-Chair

This dissertation is to connect empirical findings with grounded theoretical analysis on two economic issues. One of the studies investigates industrial productivity by fitting in a theoretical model with quantitative methods. In addition, I explore how a demographic policy in China brings forth a profound impact in all aspects of the fast-growing economy.

The first chapter, “Casual Labor, Uncertainty, and Technology Adoption in Agriculture,” examines why both the technology adoption rate and labor productivity in agriculture are low in the context of developing countries. A two-stage model is built to explain how the availability of casual (non-permanent) labor ex-post, in the presence of uncertainty may affect agents’ ex-ante technology choices. A higher degree of uncertainty induces the agents to choose traditional production technology that relies heavily on the labor input instead of using any modern intermediate inputs. By calibrating the model to fit the micro data in Tanzania, I show that this proposed framework can be used to account for two targets of interest: low aggregate labor productivity and the low technology adoption rate. Counterfactual exercises suggest that the severity of uncertainty before the harvest stage and the abundance of casual labor are the potential drivers for the two targets to be explained.

The second chapter, “Growth in a Patrilocal Economy: Female Schooling, Household Savings, and China’s One-Child Policy,” is co-authored Wei-Cheng Chen. We develop a model of parental education decision to analyze how a population control policy affects saving and schooling in a patrilocal society, where sons are responsible to support aged parents more than daughters. Parent’s investment in education depends on the degree of parental altruism and the need for old-age security. A tightened population control policy makes parental altruism more important relative to the security motive and shortens gender gap in education. We also take another crucial intergenerational incentive for daughter’s education into account, since lower fertility promotes female labor market participation and increases the value of female education. Our model explains why the Chinese economy under the “One-Child Policy” exhibits a rapid growth of relative female schooling. Moreover, this chapter also articulates the relationship between household savings and demographic changes based on a general equilibrium analysis, which has been discussed extensively in recent years to explain the China’s saving puzzle.

Chapter 1

Casual Labor, Uncertainty, and Technology Adoption in Agriculture

1.1 Introduction

As documented, there is nearly 50% employment involved in agriculture sector in developing countries, and it is the sector that is characterized by much lower labor productivity and lower technology adoption rate than rich countries.[1] In this chapter, a two-stage model is built to explain how the availability of casual labor ex-post, which serves as an instrument to hedge against an aggregate productivity shock, leads to the effect of uncertainty on the agents' ex-ante technology choices. This could be an important cause leading to low aggregate labor productivity and low technology adoption rate, which are observed from micro data. In so doing, this chapter can serve to address an essential question raised by Caselli (2005) and Restuccia, Yang, and Zhu (2008): why are labor productivity gaps in the agriculture sectors between rich and poor countries so large compared to the non-agricultural sectors. Gollin, Lagakos, and Waugh (2014a) also find that substantial productivity gaps still exist even though a refined measure of inputs and outputs has been taken into account.

[1] The cross-country share of employment is referred to in Key Indicators of the Labour Market (KILM), 2013.

There are a number of reasons provided in the literature to explain why the usage of modern intermediate inputs, mainly tractors, fertilizer, high-yielding varieties and other chemicals, is so low in developing countries and how the low adoption rate influences productivity in the agricultural sector.[2] Empirical studies, mostly using survey data from a specific country, attribute the cause of the low adoption rate to the substantial fixed cost associated with poor infrastructure (e.g., Suri 2011), to the lack of insurance or other means to maintain smoothed consumption (e.g., Dercon and Christiaensen 2011), and to farmers' insufficient ability to learn from their social networks (e.g., Conley and Udry 2010). Even though evidence from field experiments by Duflo, Kremer, and Robinson (2011) shows that there exists profitability when making the investment, some reasons still impede the efficient use of intermediate inputs: credit constraints, for example. Foster and Rosenzweig (2010) identify determinants which are correlated with the probability of adopting modern intermediate inputs: education, the risks associated with a lack of insurance, the expected rate of return, and so forth.

In this chapter, I study how uncertainty will affect the agents' ex-ante decisions regarding the use of intermediate inputs if a part of the production inputs are flexible and allowed to be adjusted ex-post. I show that the availability to adjust the optimal employment ex-post will push up farmers' sunk investments in these inputs ex-ante when uncertainty rises because their expected marginal product thereby increases (the positive intensive-margin effect). Meanwhile, the rise in uncertainty will make agents reluctant to adopt modern technology (the negative extensive-margin effect), and therefore which of these competing effects dominates is therefore a quantitative issue of interest to be subsequently assessed.

This chapter is the first to study how the missing but potentially important channel, the availability of casual labor, causes the impact of uncertainty on agents' ex-ante technology choices, and to quantitatively

[2] Please refer to Alston and Pardey (2014), for example.

evaluate the effect by using a formalized model. I present stylized facts by using aggregate data from multiple countries to show how casual labor accounts for a large proportion of labor force, especially in agriculture. The motivations are primarily based on Rosenzweig (1988) and Behrman (1999), who document that casual labor has always coexisted with its counterpart, permanent labor, in rural areas of developing countries. The main difference between the two groups is that the casual labor is employed temporarily or on a daily basis, whereas its counterpart is hired for multiple periods at a fixed wage rate.[3] In order to model the feature of labor markets, a staged production process with a productivity shock is thereby introduced.[4] A recent empirical study by Rosenzweig and Udry (2014) find that a bad rainfall shock will result in a decline in wage for labor at the harvest stage in India. Their findings show that local labor demand is state-dependent and conditional on the realized rainfall outcome whereas labor supply is limited.

In this chapter, the expected costs of hiring labor ex-post is endogenized in a generalized framework in order to take the probability of matching with respect to different realized states into account. The concern is that the costs of obtaining labor on time, in addition to their regular wages, are probably low if labor supply is unlimited at any time in developing countries, as argued by Lewis (1954).[5] The household-level data to be used shows that the demand for such hired labor, in terms of a variety of production works, could be strong in the agricultural sector. Another motivating piece of evidence observed by Gollin, Lagakos, and Waugh (2014b) is that a high percentage of labor involved in part-time activities arises due to the need to smooth out seasonal fluctuations in agricultural labor demand. Hence, to account for these stylized facts regarding the fluctuations in hired labor over time and across regions, a model with planting-to-harvest

[3]The term "casual labor," based on a general definition in most official publications, refers to labor that receives wages according to daily-based or periodic contracts. It is supposed to be consistent with an alternative classification, hired labor, to be usually observed from micro data as long as the labor is paid daily wages.

[4]The reason, as argued by Newbery (1977), is that agricultural production actually involves a sequence of decisions and they are made under the effects of unforeseen events, like unexpected weather conditions.

[5]The possibility of shortages of hired labor in the U.S. farm labor market, for example, is documented by Fisher and Knutson (2013). One of the reasons that leads to a shortage is a lack of farm workers that are available to "harvest a farmer's crop when it is ready."

staged agricultural production is constructed in this chapter to explain the interactive relationship between inputs at different production stages.

The analytical approach of this chapter is related to a growing literature that examines industrial development by using micro survey data and applying quantitative analysis; e.g., Hsieh and Klenow (2009), Adamopoulos and Restuccia (2014b), among others. This chapter is also close to studies that investigate how some potential distortionary factors account for productivity differences across countries; e.g., barriers to consuming modern intermediate inputs and barriers associated with the labor market by Restuccia, Yang, and Zhu (2008), transportation cost frictions by Adamopoulos (2011), policies leading to diminishing farm size by Adamopoulos and Restuccia (2014a), and uninsurable risks owing to the incomplete market by Donovan (2014), among others. Moreover, in a way that is different from the explanation of external distortions, a self-selection effect argued by Lagakos and Waugh (2013) acts as a complement to other explanations, which amplifies productivity differences across countries.

In this chapter, an analytical framework to evaluate the role of uncertainty is built but differs from the existing literature in that I try to explore its effect on agents' production decisions. In particular, I focus on how casual labor demand comes from the need to adjust production inputs ex-post and how the availability changes farmers' ex-ante choices of optimal technology. The framework facilitates quantitative work and the results are reconciled with the main findings based on agents' decisions, including the adoption rate of modern technology, the permanent-casual labor ratio, and labor productivity. Based on the proposed framework, counterfactual exercises suggest that the severity of uncertainty before the harvest stage and the abundance of casual labor are the potential drivers of the low adoption rate and low productivity.

The rest of this chapter is organized as follows. Data sources regarding cross-country labor surveys and household-level surveys are briefly described in Section 1.2. The main descriptive statistics and stylized facts

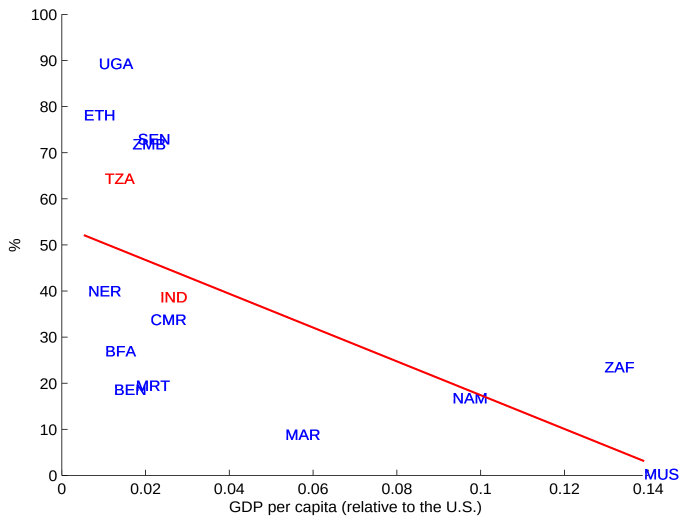


Figure 1.1: Cross-country GDP per capita vs casual/seasonal/temporary worker ratio in the rural sector around year 2010 (14 Africa countries and India)

are also provided. In Section 1.3, I build a two-stage model to analyze the factors that influences adoption rate of modern technology. Section 1.4 presents the comparative statics analysis in detail in regard to how a change in the degree of uncertainty affects agents' ex-ante decisions. Section 1.5 presents how the model is calibrated to match the micro data and how quantitative exercises proceed. Section 1.6 provides some concluding remarks.

1.2 Evidence from macro and micro survey data

The data used in this chapter are obtained from two sources: the aggregate data from the cross-country labor force survey and the micro data from the Living Standard Measurement Study - Integrated Surveys on Agriculture (LSMS-ISA). Based on aggregate data from official employment reports, it is observed that hired casual labor accounts for a large proportion of employment across countries especially in the agricultural sector. The general pattern is summarized in [Figure 1.1], in which the negative slope delivers the fact that there is substantially higher share of casual labor in the rural/agricultural sector in developing countries;

e.g., India and Tanzania.[6]

The number of hired farm workers in the U.S. is documented in various issues of quarterly reports provided by the National Agricultural Statistics Service (NASS). [Figure 1.16] illustrates the fluctuations in the numbers of such workers to be hired over time. Workers are divided into three categories in terms of the duration of time they are expected to be employed and the type of contract signed. The series, especially the number of workers to be hired on a short-term basis, exhibits a strong seasonality, and the peaks of labor demand consistently occur in the middle of each year, around the time of the harvest. In addition to the dimension of time, the demand is also contingent on regional differences, or else is dictated by local weather conditions (Fisher and Knutson, 2013). Hence, local shortages and surpluses of hired farm labor may exist nationwide at the same time.

Official and long-term statistics are limited but still available from some developing countries. For example, in India casual labor in rural areas accounts for almost twice the total employment in urban areas. [Table 1.1] is constructed based on various issues of Key Indicators of Employment and Unemployment in India and related official reports released by the National Sample Survey Office (NSSO) of India. The table summarizes the statistics regarding the labor share based on different types of employment status and the corresponding daily wage in rural and urban sectors of India in recent years. Employed persons are categorized into three broadly-defined groups: self-employed, regularly salaried, and casual labor. As shown in [Table 1.1], the share of casual labor is about 33%–39%, which is substantially higher in the rural sector than the share for those regularly salaried. By contrast, the share of labor accounted for by casual labor is much smaller in the urban sector. The evidence suggests that hiring casual labor is common in the agricultural sector since agriculture is the primary economic activity in rural areas.[7]

[6] The statistics of fourteen Africa countries is obtained from *Decent Work Indicators in Africa - A first assessment based on national sources,* which is published by International Labour Organization.

[7] It dominantly accounts for 64.1% of aggregate rural employment in India in the year 2011, for example.

The casual-labor shares in both the agricultural and non-agricultural sectors of Tanzania are reported in [Table 2], and the official data deliver a summary of the distribution in terms of the workers' employment status. It is shown that the casual-labor ratio is about 35%–74% in the agricultural sector, and that the ratio has fluctuated a lot over the years. To account for a potential miscounting problem regarding the labor actually involved in agricultural production as argued by Gollin, Lagakos, and Waugh (2014a, b), I seek micro evidence from the LSMS-ISA. The data are compiled by the development research group of the World Bank and are favorable to cross-country analysis since the questionnaires are designed on a comparable basis. The composite surveys by the LSMS-ISA project are composed of three parts, namely, household, agriculture, and community surveys, and information related to households and their agricultural production is provided. Tanzania is selected to be the country of study since it is one of the countries for which there is an available panel for multiple years. Moreover, compared to other countries investigated by the LSMS-ISA, Tanzania has more detailed information related to hired labor days devoted in both pre-harvest and post-harvest production works.

The currently available years of integrated survey data for Tanzania are the years 2008–2009 (round 1) and 2010–2011 (round 2). The total sample size of the second round is 3,924 households, of which 3,168 were visited in the first round. The basic production unit to be sampled in the agriculture survey is the plot, and a household may own multiple plots with different combinations of inputs and outputs. The number of plots, and hence the number of observations, in both years exceeds 5,000. The agriculture survey provides detailed input-output information for each basic production unit, which includes the types of crops cultivated, the consumption of a variety of intermediate inputs, and the nominal value of output to be harvested. I first present descriptive statistics and cross-correlation tables and then relevant figures, shedding light on empirical findings worthy of discussion.

[Table 3] provides descriptive statistics based on the agriculture survey in Tanzania for both of the survey years.[8] The table shows that only a moderate proportion of plots are fertilized by using chemicals; or nearly 11%–13% in both years. Over 80% of the plots are owned by households and most of them are used to cultivate subsistence crops. The use of another crucial intermediate input, pesticide/herbicide, which is considered to be complementary to chemical fertilizer is also limited. Moreover, most households choose some specific type of inorganic fertilizer based on their members' personal experiences or advice from experts and only one percent of them have obtained any seeds, fertilizers, or pesticides/herbicides on credit. Around 30% of them have recorded unexpected losses due to the area harvested being smaller than that planted during 2010-2011, and the main cause is the weather effect which is documented as unexpected drought (57.37%) and rain (10.35%). The impact of the unexpected weather conditions could be large because the region with a higher probability of suffering the unexpected losses tends to have a lower adoption rate of chemicals. [Figure 1.4] first presents the probability of having losses in years 2008–2009 and 2010–2011 on left and it suggests a consistent result across years. Then, the figure delivers a significant negative correlation between the probability of suffering losses in year 2008–2009 and the adoption rate year 2010–2011 on right.

Labor involved in production is grouped into two categories, family and hired (casual) labors, and their units are measured in days. The regular activities that they are engaged in or assigned are planting, weeding, fertilizing and harvesting, and the numbers of days on which workers engage in each activity are also observable for both groups. The hired labor is treated as casual labor in this chapter and this classification will minimize the possibility of miscounting problems because the labor is hired on a daily basis for a variety of needs in each production unit, and it is basically consistent with the definition of casual labor based on the aggregate data. It is observed that 31% and 27% of plots were previously worked on

[8] The summary statistics of the latest round in year 2012-2013 is also included in Table 3 to help understand the changes of variables over time. The table is made mainly based on information obtained by merging Tables 1A, 2A, 3A, and 4A from the agriculture survey of the LSMS-ISA. The variables and the series numbers to which they are referred are listed in the Appendix.

by hired labor in the years 2008 and 2010.[9] I divide the hired labor into two categories based on those hired ex-ante for planting, weeding and fertilizing, and ex-post for harvesting. [Figure 1.5] indicates that the reason for hiring labor at the harvest stage is motivated by the need to improve profits derived from harvested crops in the near future. Each blue (red) point in the figure represents a combination of the labor input and imputed profits based on the observation that labor was hired (not hired) at the harvest stage. To account for the scale effect, both variables are divided by the corresponding size of each plot. The figure shows that, given all predetermined and sunk inputs, the plots with hired labor at this stage tend to have higher profits per hectare with respect to different levels of the labor/land ratio.

The comparison of the last two rows of [Table 3] shows the labor productivity gains as a result of using chemical fertilizer, and the gains are measured by the value of output per labor day. The gap in terms of the mean values suggests that the labor productivity is more than doubled from not using any intermediates. To highlight the difference, the agents' discrete choices are hence denoted as using traditional and modern technology hereafter. [Figure 1.6] displays the land and labor productivity sorted from the least to the most productive production units with respect to the two technologies, and the differences indicate that the aggregate productivity level will be significantly correlated with the choice of modern technology. On the other hand, a strong correlation between the two productivity measures presented in [Figure 1.7] suggests that agents with higher labor productivity are aligned with the ones with higher land productivity, and hence the use of a different measure does not substantially change the quantitative results.

Finally, [Table 4] is constructed to show the pair-wise correlations between agents' choices of crucial intermediate inputs and how the choices are related to labor productivity in both of the survey years. It is observed that the use of one input is significantly correlated with the use of another; for example, the

[9] The hired labor with daily-paid wages is easily observed in the agricultural sector, at least in the countries surveyed by the LSMS-ISA. For example, the ratios were around 38% in Uganda (2010), 15% in Malawi (2010), and more than 25% in Nigeria (2010).

correlation coefficient between two dummies, inorganic fertilizer and pesticide/herbicide, is 0.26–0.27. In addition, agents who use these intermediate inputs in general have higher labor productivity than those who do not, confirming the significance of selecting one farming type compared to the other.

1.3 The model

In this section, a model is constructed to illustrate the staged decisions during the agricultural production cycle.[10] In contrast to the existing literature, an ex-ante technology adoption problem has resulted from the heterogeneity of agents being taken into account in a generalized model. Two technologies are available for each agent and there are no barriers or fixed costs to choose either one of them.

1.3.1 The setup of the environment

The timeline is displayed in [Figure 1.2]. Each time t is divided into two sub-periods, referred to as the planting and harvest stages. Agents in the model are risk-neutral farmers with fixed populations and are endowed with heterogenous ability to manage intermediate inputs: e.g., chemical fertilizer, among others. Workers are divided into two categories, permanent and casual, and they are assumed to be perfectly substitutable in production. An employed worker will provide a unit of labor in two successive stages if he serves as a permanent worker or only does so at one of the stages if he is hired as a casual counterpart. In order to reconcile fluctuations in hired labor ratios over time, casual labor is assumed to be hired through random matching from labor markets.

Differing from the standard competitive market assumption, the matching framework gives rise to a state-contingent matching probability in equilibrium, which is much lowered as a sharp increase in labor demand occurs. The rationale is that the additional hiring costs serve as one possible source of labor

[10] There are some possible applications based on the generalized model; an extension from Rose (2001), for example.

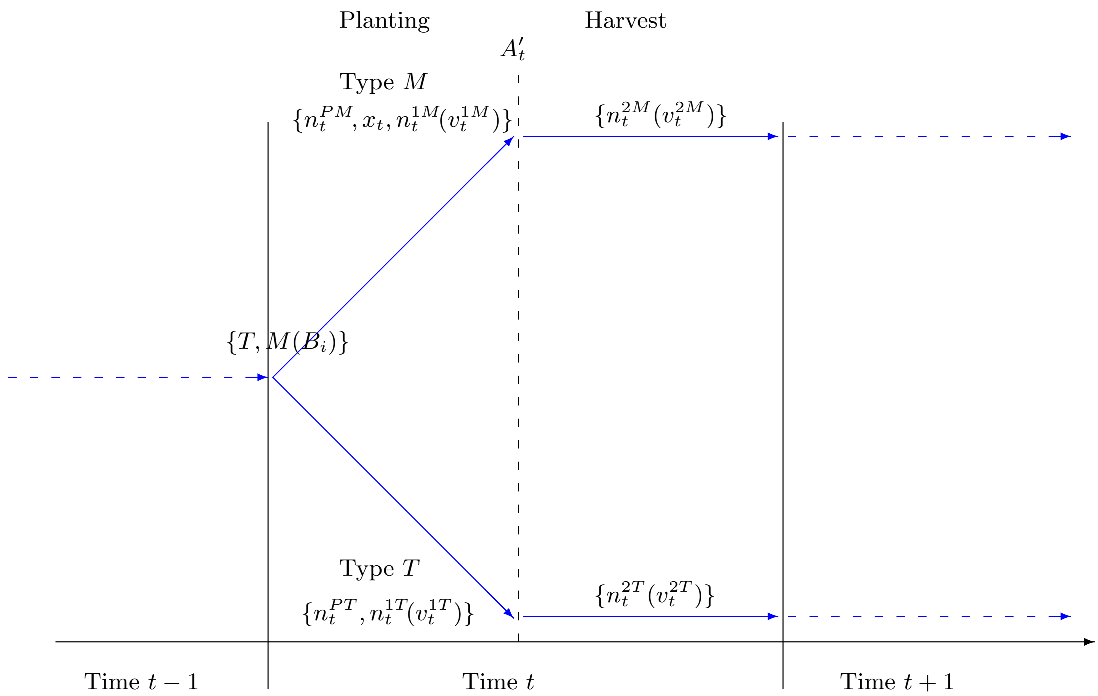


Figure 1.2: Timeline and agent i's decisions at time t

market distortions, which conceptually result from how likely labor can be obtained when it is needed at harvest time. The costs could be different across regions or over time because of labor immobility and other distortionary factors. Hence, without prices to perfectly clear the labor market, the matching probability is used to capture the costs based on the availability of casual labor at a certain time. Another implication is that labor demand is likely driven by the local weather effect and the labor market is localized, so that the matching framework could potentially better explain the large observed fluctuations in casual labor over time or across regions.

The optimization problem that agent i faces at the beginning of the planting period at time t is to determine a farming technology type, either modern M or traditional T, and then production inputs afterwards: the quantities of permanent labor n_t^{Pj}, casual labor n_t^{1j} and n_t^{2j} for a j-type farmer for $j = M$, T, as well as the amount of intermediate inputs x_t to be consumed only when he chooses to be a modern-type agent. Accordingly, the production functions of intermediate output by modern-type and traditional-type agents at the planting stage are respectively $g(B_i x_t, n_t^{PM} + n_t^{1M})$ and $h(n_t^{PT} + n_t^{1T})$, in which B_i is endowed and individual-specific knowledge or ability to use and manage intermediate inputs. The individual-specific ability follows a continuous distribution, say, $B_i \sim F(B_i; \theta)$ with support in $B_i = (0, \infty)$. The ability is augmented for intermediate inputs, and hence only takes effect when choosing modern technology.

An aggregate productivity shock A_t' occurs at the beginning/end of the harvest/planting stage. Assume that the productivity shock follows a uniform distribution, $A_t' \sim \mathcal{U}(\mu - \varepsilon/2, \mu + \varepsilon/2)$, and the parameter ε governs the variation of the realized shock around the mean value, or the degree of uncertainty.[11] Agents cannot lay off the permanent labor that was hired from the previous stage no matter how serve the realized shock is. There is no formal insurance available ex-ante, but after the shock is realized both types of agents

[11]This assumption is to facilitate the comparative statics. When dealing with computational work, other two-parameter distributions; e.g., $\ln\mathcal{N}(\mu_A, \sigma_A)$ or Beta(a, b) can be alternatives to capture the effect from an enlarged degree of uncertainty while leaving the mean value of A_t' unchanged (mean-preserving spreads).

can hire additional labor, denoted as n_t^{2M} and n_t^{2T}. On the other hand, the available short-term contracts for casual labor are assumed to be valid only at the current stage, i.e., either the planting or the harvest one, and they will expire at the end of the corresponding stage. The land input $\bar{z}$ is fixed and free to be used in production.

Based on the assumptions, the production of the agricultural good for agent i is either by

$$F^M(A_t', g(B_i x_t, n_t^{PM} + n_t^{1M}), n_t^{PM} + n_t^{2M}, \bar{z}) = A_t'[g(B_i x_t, n_t^{PM} + n_t^{1M})^\alpha (n_t^{PM} + n_t^{2M})^{1-\alpha}]^{1-\sigma} \bar{z}^\sigma$$

or by

$$F^T(A_t', h(n_t^{PT} + n_t^{1T}), n_t^{PT} + n_t^{2T}, \bar{z}) = A_t'[h(n_t^{PT} + n_t^{1T})^\alpha (n_t^{PT} + n_t^{2T})^{1-\alpha}]^{1-\sigma} \bar{z}^\sigma.$$

at the end of the harvest stage. The production functions of the intermediate output are further assumed to be $g(B_i x_t, n_t^{PM} + n_t^{1M}) = (B_i x_t)^\beta (n_t^{PM} + n_t^{1M})^{1-\beta}$ and $h(n_t^{PT} + n_t^{1T}) = n_t^{PT} + n_t^{1T}$ and remain in the CRTS form. Final outputs are hence produced by combining land, labor to be hired at each stage, and (optional) intermediate inputs.

Note that I only focus on the optimization problem for the current period t, even though the framework can be readily generalized by including dynamics on n_t^{PM} or A_t' across periods, as depicted in [Figure 1.2]. Another extension is to divide t into more than two stages, and study how agents' discrete choices are observed at each stage; for example, crop choosing, types of seed planting, the use of fertilizer, and irrigation installation are sequentially affected by uninsured shocks from the future.

1.3.2 Backward solution: decisions at the harvest stage

First, consider the optimization problem for a modern-type agent with B_i and solve it backwards. The agent determines the extra labor to be hired at the harvest stage after the productivity shock is realized.

Hence, the additional value generated from hiring labor n_t^{2M} is

$$J^M(B_i,\bar{z},A_t') = \max_{v_t^{2M},\, n_t^{2M}} A_t'[g(B_i x_t, n_t^{PM}+n_t^{1M})^{\alpha}(n_t^{PM}+n_t^{2M})^{1-\alpha}]^{1-\sigma}\bar{z}^{\sigma} - \xi_2 v_t^{2M} - w_t^c n_t^{2M}$$

$$\begin{aligned} \text{s.t. } n_t^{2M} &= \eta_t\, v_t^{2M} \qquad && \text{as } A_t' \geq \tilde{A}^M \\ &= 0 && \text{as } A_t' < \tilde{A}^M \end{aligned} \tag{1.1}$$

In Eq. (1.1), the amount of intermediate inputs x_t and the quantity of permanent labor n_t^{PM} have been determined from the previous stage while n_t^{2M} is the quantity of casual labor to be hired at the wage rate w_t^c upon creating vacancies v_t^{2M}.

Assume that all agents are hiring labor in the market and that η_t stands for the probability that a vacancy is filled in, which is taken as given when the agent optimizes. ξ_2 denotes the additional costs paid at the harvest stage and hence $w_t^c + \xi_2/\eta_t$ is the mean costs of hiring a unit of labor ex-post. The agent will additionally hire labor if the realized productivity is higher than a threshold $\tilde{A}^M$, which is an endogenous cut-off value that differentiates good from bad states and is a function of inputs to be determined at the planting stage.

The optimization problem faced by traditional-type agents is analyzed under the same framework, and their decision rules are omitted here to avoid redundance. In order to simplify the following analysis, I assume that the only available short-term contract in the casual labor market is on take-it-or-leave-it basis and therefore the bargaining condition can be written as

$$J^M(B_i,\bar{z},A_t') - \bar{J}^M = J^M(B_i,\bar{z},A_t') - \bar{J}^M + (w_t^c - \lambda b)\, n_t^{2M}$$

or

$$w_t^c = \lambda b, \tag{1.2}$$

where $\bar{J}^M$ equals the value of the modern-type agent from not hiring this unit of casual labor (value from the outside option), b is disutility from providing a unit of labor in entire period t, and λ is the duration of the harvest stage if time in the whole period t is normalized to be one.

When the realized A'_t is larger than threshold values $\tilde{A}^M$ and $\tilde{A}^T$, the first-order conditions from modern-type and traditional-type agents' optimization problems imply that the ratios of labor inputs ex-post to ex-ante are

$$\frac{n_t^{PM} + n_t^{2M}}{n_t^{PM} + n_t^{1M}} = \left\{ \frac{A'_t(1-\alpha)(1-\sigma)\left[(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}})^{\alpha\beta(1-\sigma)} (\frac{\bar{z}}{n_t^{PM}+n_t^{1M}})^{\sigma} \right]}{\lambda b + \frac{\xi_2}{\eta_t(A'_t)}} \right\}^{\frac{1}{\alpha+\sigma-\alpha\sigma}}, \tag{1.3}$$

and

$$\frac{n_t^{PM} + n_t^{2T}}{n_t^{PT} + n_t^{1T}} = \left\{ \frac{A'_t(1-\alpha)(1-\sigma)\left[(\frac{\bar{z}}{n_t^{PT}+n_t^{1T}})^{\sigma} \right]}{\lambda b + \frac{\xi_2}{\eta_t(A'_t)}} \right\}^{\frac{1}{\alpha+\sigma-\alpha\sigma}}. \tag{1.4}$$

Note that the matching probability $\eta_t = \eta_t(A'_t)$ is contingent on the realized value of A'_t in equilibrium, and that both of the ratios positively depend on the realized value of A'_t given the assumption that the function of the hiring costs $\lambda b + \frac{\xi_2}{\eta_t(A'_t)}$ is concave on A'_t. This condition is set to exclude the possibility of a dramatic rise in marginal costs of hiring labor due to a higher value of A'_t, because it may lead to a case of multiple equilibria and hence make the subsequent analysis more involved.

The endogenous threshold value $\tilde{A}^M$ is solved by the following condition

$$\left.\frac{\partial F^M(A'_t, g(B_i x_t, n_t^{PM} + n_t^{1M}), n_t^{PM} + n_t^{2M}, \bar{z})}{\partial n_t^{2M}}\right|_{n_t^{2M}=0} = \lambda b + \frac{\xi_2}{\eta_t(A'_t)},$$

which implies that the agent is indifferent between hiring and not hiring an additional unit of labor around

$\tilde{A}^M$. Hence, the threshold $\tilde{A}^M$ is pinned down by

$$\tilde{A}^M = \frac{\lambda b + \frac{\xi_2}{\eta_t(\tilde{A}^M)}}{(1-\alpha)(1-\sigma)} \Big\{ \Big[\Big(\frac{B_i x_t}{n_t^{PM} + n_t^{1M}} \Big)^{\alpha\beta(1-\sigma)} \Big(\frac{\bar{z}}{n_t^{PM} + n_t^{1M}} \Big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM} + n_t^{1M}} \Big)^{-\alpha-\sigma+\alpha\sigma} \Big\}^{-1}, \tag{1.5}$$

in which the inputs x_t, n_t^{PM}, and n_t^{1M} are given while the agent is making the decision on hiring labor ex-post. In addition, the threshold value $\tilde{A}^T$ from the traditional-type agent's decision can be solved by

$$\tilde{A}^T = \frac{\lambda b + \frac{\xi_2}{\eta_t(\tilde{A}^T)}}{(1-\alpha)(1-\sigma)} \Big\{ \Big(\frac{\bar{z}}{n_t^{PT} + n_t^{1T}} \Big)^{\sigma} \Big(\frac{n_t^{PT}}{n_t^{PT} + n_t^{1T}} \Big)^{-\alpha-\sigma+\alpha\sigma} \Big\}^{-1}. \tag{1.6}$$

The discrepancy between two threshold values comes from different ex-ante input combinations of the two types of agents, and they may hence make different ex-post decisions even when facing the same aggregate shock. Given the concavity assumption on $\lambda b + \frac{\xi_2}{\eta_t(A_t')}$, Eqs. (1.5) and (1.6) pin down unique solutions of $\tilde{A}^M$ and $\tilde{A}^T$ in turn. Since the endogenous labor ratios and the threshold values rest on pre-determined input combinations and are functions of related exogenous variables, the analysis based on comparative statics and the underlying implications are postponed until a more detailed discussion is provided in Section 1.4.

1.3.3 Decisions at the planting stage

As presented in [Figure 1.2], agent i decides a farming technology, M or T, at the beginning of the planting stage at time t. He then chooses the optimal quantity of permanent and casual labor to be hired as well as the consumption of intermediate inputs if being a modern-type agent. The wage rate for hiring a unit of permanent labor is w_t^P, and the labor is hired through the competitive market without creating costly vacancies. The rationale is that permanent labor is mostly observed as family labor from the micro data: it is paid an unobservable wage rate or directly by means of agricultural output and only minimized costs for matching labor are required. On the other hand, in addition to wages for hiring a unit of casual labor implied costs ξ_1 are also introduced, making the framework consistent with the optimization problem of the harvest stage.

The optimization problem of agent i is therefore outlined as

$$V(B_i, \bar{z}) = \max_{\{M,\,T\}} \big\{ V^M(B_i, \bar{z})\,,\ V^T(\bar{z}) \big\}, \tag{1.7}$$

in which the value functions come from expected profits under two scenarios

$$\begin{aligned} V^M(B_i, \bar{z}) = \max_{\{n_t^{1M},\, n_t^{PM},\, x_t\}} \Bigg\{ & - p_t x_t - w_t^P n_t^{PM} - \Big((1-\lambda) b + \frac{\xi_1}{\bar{\eta}^c} \Big) n_t^{1M} \\ & + \mathbb{E}\Big[\, F^M(A_t', g(B_i x_t, n_t^{PM} + n_t^{1M}), n_t^{PM} + n_t^{2M}, \bar{z}) - \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')} \Big) n_t^{2M} \Big| \, A_t' > \tilde{A}^M \Big] \\ & + \mathbb{E}\Big[\, F^M(A_t', g(B_i, n_t^{PM} + n_t^{1M}), n_t^{PM}, \bar{z}) \big| \, A_t' \leq \tilde{A}^M \Big] \Bigg\}. \end{aligned} \tag{1.8}$$

and

$$\begin{aligned} V^T(\bar{z}) = \max_{\{n_t^{1T},\, n_t^{PT}\}} \Bigg\{ & - w_t^P n_t^{PT} - \Big((1-\lambda) b + \frac{\xi_1}{\bar{\eta}^c} \Big) n_t^{1T} \\ & + \mathbb{E}\Big[\, F^T(A_t', h(n_t^{PT} + n_t^{1T}), n_t^{PT} + n_t^{2T}, \bar{z}) - \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')} \Big) n_t^{2T} \Big| \, A_t' > \tilde{A}^T \Big] \\ & + \mathbb{E}\Big[\, F^T(A_t', h(n_t^{PT} + n_t^{1T}), n_t^{PT}, \bar{z}) \big| \, A_t' \leq \tilde{A}^T \Big] \Bigg\}. \end{aligned} \tag{1.9}$$

The value functions indicate that, given the two endogenous thresholds solved by Eqs. (1.5) and (1.6), the expected output is a sum of its value under the good state, which will be enlarged because of the adjustment of the labor input ex-post, and the value under the bad state.

The first-order condition for choosing x_t is

$$\begin{aligned} & p_t \\ & = \mathbb{E}\Big[\, \frac{\partial}{\partial x_t} \Big\{ F^M(A_t', g(B_i x_t, n_t^{PM} + n_t^{1M}), n_t^{PM} + n_t^{2M}, \bar{z}) - \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')} \Big) n_t^{2M} \Big\} \Big| \, A_t' > \tilde{A}^M \Big] \\ & + \mathbb{E}\Big[\, \frac{\partial}{\partial x_t} F^M(A_t', g(B_i x_t, n_t^{PM} + n_t^{1M}), n_t^{PM}, \bar{z}) \Big| \, A_t' \leq \tilde{A}^M \Big]. \end{aligned} \tag{1.10}$$

Since the optimal quantity of labor ex-post, $n_t^{PM}+n_t^{2M}$, is a function of x_t based on Eq. (1.3), the availability to hire additional labor amplifies an expected marginal product of x_t under the good state. Put differently, the argument also implies that the demand for intermediate inputs from a modern-type agent is higher when casual labor is available than the case in which casual labor is not available under an imposed condition $n_t^{2M} = 0$. In addition to the mean-value effect, a change in the uncertainty parameter will influence the optimal choice of x_t even though the agent is assumed to be risk-neutral. The essence is that $n_t^{PM} + n_t^{2M}$ is also a function of the realized state A_t'. Hence, a higher degree of uncertainty implies a higher possibility of an extreme value of A_t occurring. This also implies that using intermediate inputs is expected to be more productive because of an increase in the probability of ex-post adjustment under the good state. The result is referred to as the *intensive-margin effect*, and under the condition that casual labor is ex-post available the degree of uncertainty has a positive impact on the intensity of intermediate inputs relative to the others.

On the other hand, an agent with ability $B_i = \underline{B}$, who is previously indifferent between the two technology choices, will depart from modern to traditional technology as the uncertainty level rises. This is because a larger share of predetermined inputs will make the value of being the modern type increase less than the alternative, leading to the so-called negative *extensive-margin effect*. The two competing effects are derived from the availability of hiring additional labor ex-post, the channel emphasized by this chapter. The analytical results based on comparative statics and numerical exercises are shown in greater detail in the following sections. A simplified model by imposing a relatively restricted assumption whereby labor is fully determined ex-post is considered in the Appendix, and is used to deliver a clear sense of how the channel gives rise to the two effects. Moreover, it qualitatively shows how a complete adjustment of the labor input makes the adoption rate or modern technology lower as there is an exogenous increase in the degree of uncertainty.

1.3.4 The matching probability in equilibrium

In order to solve the state-contingent equilibrium probability that a vacancy is filled in $\eta_t(A'_t)$, I assume the matching function in the Cobb-Douglas form $\mathbb{M}(\bar{N}, V_t) = \bar{m}\bar{N}^{\rho}V_t^{1-\rho}$ with homogeneity of degree one. Since the part of the labor force that is available to be casual labor is $\bar{N}$ in both of the planting and harvest stages, then the number of matches rests on $\bar{N}$ as well as the total vacancies created by the modern and traditional type of agents (with fractions π^M and $1-\pi^M$, respectively). Both types of agents will create vacancies for hiring labor given that $A'_t \geq \tilde{A}^j$ for $j = M$ and T, whereas only traditional-type ones do so as $\tilde{A}^T < A'_t \leq \tilde{A}^M$. The equilibrium probability is determined by the matching condition

$$\eta_t = \begin{cases} \bar{m}\bar{N}^{\rho}\big[(1-\pi^M)v_t^{2T} + \int_{B_i>\underline{B}} v_t^{2M}(B_i)dF(B_i)\big]^{-\rho} & \text{if } A'_t \geq \tilde{A}^M \\ \bar{m}\bar{N}^{\rho}\big[(1-\pi^M)v_t^{2T}\big]^{-\rho} & \text{if } \tilde{A}^T \leq A'_t < \tilde{A}^M, \\ 1 & \text{if } A'_t < \tilde{A}^T \end{cases} \tag{1.11}$$

where $\bar{N}$ is the total supply of casual labor and $v_t^{2M} = n_t^{2M}/\eta_t$ and $v_t^{2T} = n_t^{2T}/\eta_t$. More precisely, each $\eta_t(A'_t)$ is pinned down by

$$\begin{aligned} {\eta_t}^{1-\rho} = \bar{m}\left(\frac{\bar{N}}{\bar{z}}\right)^{\rho}\Bigg\{(1-\pi^M)\left(\left[\frac{(1-\alpha)(1-\sigma)A'_t\big(\frac{\bar{z}}{n_t^{PT}+n_t^{1T}}\big)^{\sigma}}{\lambda b + \frac{\xi_2}{\eta_t}}\right]^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\left(\frac{n_t^{PT}+n_t^{1T}}{n_t^{PT}}\right) - 1\right)\frac{n_t^{PT}}{\bar{z}} \\ + \int_{B_i>\underline{B}}\left(\left[\frac{(1-\alpha)(1-\sigma)A'_t\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)}\big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma}}{\lambda b + \frac{\xi_2}{\eta_t}}\right]^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\left(\frac{n_t^{PM}+n_t^{1M}}{n_t^{PM}}\right) - 1\right)\frac{n_t^{PM}}{\bar{z}}dF(B_i)\Bigg\}^{-\rho}, \end{aligned} \tag{1.12}$$

as $A'_t \geq \tilde{A}^T$, and $\pi^M \equiv \int_{B_i>\underline{B}} dF(B_i)$ denotes the fraction of agents who have chosen modern technology since their individual ability is larger than the threshold value $\underline{B}$. [Figure 1.8] shows that there exists a unique solution of equilibrium probability $\eta_t = \eta_t(A'_t)$ in Eq. (1.12) since the left-hand side is an increasing and concave function of η_t, whereas the right-hand side, given all other predetermined variables, is decreasing in η_t. The comparative statics is described as

$$\eta_t = \eta_t \left(\underset{(-)}{A'_t}, \underset{(?)}{\frac{n_t^{PM}}{\bar{z}}}, \underset{(?)}{\frac{n_t^{PT}}{\bar{z}}}, \underset{(+)}{\xi_2} \right),$$

and the details are shown in Section 1.4. First, note that an increase in $n_t^{PM}/\bar{z}$ (and $n_t^{PT}/\bar{z}$) will bring undetermined effects on the equilibrium η_t. The reasons are that (i) casual labor is perfectly substituted by the permanent labor, and (ii) the complementarity between intermediate output $g(B_i x_t, n_t^{PM} + n_t^{1M})$ and total labor ex-post $(n_t^{PM} + n_t^{2M})$ is implied by the setting of the production function. The two effects will respectively lead agents to demand less and more casual labor ex-post, making the matching probability in equilibrium high and low. Second, a higher realized value of A'_t will motivate agents to hire more additional labor, given that it is larger than $\tilde{A}^M$. Last, an exogenous increase in costs ξ_2 will dampen all agents' incentives to hire labor ex-post, resulting in a higher matching probability in equilibrium.

The matching framework at the planting stage follows the same rule. The more detailed numerical analysis and comparative statics and provided in the following sections and in the Appendix.

1.4 Comparative statics analysis of the generalized model

How an exogenous change in the degree of uncertainty will affect agents' ex-ante decisions regarding input combinations is presented in this section. The main concern is that uncertainty will only affect agents' ex-post decisions indirectly through its effects on these input combinations.

In the generalized model, casual labor is available both before and after a productivity shock A'_t is realized. The technology adoption decision of an agent with ability B_i at the beginning of t is outlined as

$$V(B_i, \bar{z}) = \max_{\{M,T\}} \left\{ V^M(B_i, \bar{z}),\ V^T(\bar{z}) \right\},$$

as the two value functions described in Eqs. (1.8) and (1.9). More specifically, the value of being a modern-type agent can be expressed in detail as

$$
\begin{aligned}
V^M(B_i,\bar{z}) = & \max_{\{n_t^{1M},\, n_t^{PM},\, x_t\}} \Big\{ - p_t x_t - w_t^P n_t^{PM} - \Big((1-\lambda)b + \frac{\xi_1}{\overline{\eta}^c}\Big) n_t^{1M} \\
& + \int_{\tilde{A}^M}^{\bar{A}} \Big\{ A_t'[g(B_i x_t, n_t^{PM} + n_t^{1M})]^{\alpha(1-\sigma)} (n_t^{PM} + n_t^{2M})^{(1-\alpha)(1-\sigma)} \bar{z}^{\sigma} - \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big) n_t^{2M} \Big\} dF(A_t') \\
& + \int_{\underline{A}}^{\tilde{A}^M} \Big\{ A_t'[g(B_i x_t, n_t^{PM} + n_t^{1M}))]^{\alpha(1-\sigma)} (n_t^{PM})^{(1-\alpha)(1-\sigma)} \bar{z}^{\sigma} \Big\} dF(A_t') \Big\}.
\end{aligned} \tag{1.13}
$$

The expected costs of hiring a unit of casual labor is contingent on the realized value of A_t' whereas the expected costs ex-ante only rests on its distribution parameters in equilibrium. As mentioned, the endogenous threshold value $\tilde{A}^M$ is determined by Eq. (1.5) and it implies that the agent is indifferent between hiring and not hiring the first unit of labor as $A_t' = \tilde{A}^M$.

The quantity of labor to be hired ex-post by the agent as $A_t' \geq \tilde{A}^M$, given all other predetermined production inputs, follows the first-order condition described by Eq. (1.3). By substituting $n_t^{PM} + n_t^{2M}$ in terms of a function of $n_t^{PM} + n_t^{1M}$ into Eq. (1.13) and because of the assumption $A_t' \sim \mathcal{U}(\mu - \varepsilon/2, \mu + \varepsilon/2)$,

$$
\begin{aligned}
V^M(B_i,\bar{z}) = & \max_{\{n_t^{1M},\, n_t^{PM},\, x_t\}} \Big\{ - p_t x_t - w_t^P n_t^{PM} - ((1-\lambda)b + \frac{\xi_1}{\overline{\eta}^c}) n_t^{1M} \\
& + \int_{\tilde{A}^M}^{\mu+\varepsilon/2} \Big\{ \mathcal{C}(A_t')^{\frac{1}{\alpha+\sigma-\alpha\sigma}} (B_i x_t)^{\frac{\alpha\beta(1-\sigma)}{\alpha+\sigma-\alpha\sigma}} (n_t^{PM} + n_t^{1M})^{\frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma}} \bar{z}^{\frac{\sigma}{\alpha+\sigma-\alpha\sigma}} + \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big) n_t^{PM} \Big\} \frac{1}{\varepsilon} dA_t' \\
& + \int_{\mu-\varepsilon/2}^{\tilde{A}^M} \Big\{ A_t'(B_i x_t)^{\alpha\beta(1-\sigma)} (n_t^{PM} + n_t^{1M})^{\alpha(1-\beta)(1-\sigma)} (n_t^{PM})^{(1-\alpha)(1-\sigma)} \bar{z}^{\sigma} \Big\} \frac{1}{\varepsilon} dA_t',
\end{aligned} \tag{1.14}
$$

where the constant

$$
\mathcal{C} = (\alpha + \sigma - \alpha\sigma) \left[\frac{(1-\alpha)(1-\sigma)}{\lambda b + \frac{\xi_2}{\eta_t(A_t')}} \right]^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma-\alpha\sigma}}.
$$

The first-order conditions for n_t^{1M}, n_t^{PM}, and x_t can be derived in turn by:

$$
\begin{aligned}
&(1-\lambda)b + \frac{\xi_1}{\bar{\eta}^c} \\
&= \int_{\tilde{A}^M}^{\mu+\varepsilon/2} \Big\{ \mathcal{C} \frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma} (A_t')^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big]^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \Big\} \frac{1}{\varepsilon}\, dA_t' \\
&+ \int_{\mu-\varepsilon/2}^{\tilde{A}^M} \Big\{ \alpha(1-\beta)(1-\sigma) A_t' \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}} \Big)^{(1-\alpha)(1-\sigma)} \Big\} \frac{1}{\varepsilon}\, dA_t',
\end{aligned}
\tag{1.15}
$$

$$
\begin{aligned}
&w_t^P \\
&= \int_{\tilde{A}^M}^{\mu+\varepsilon/2} \Big\{ \mathcal{C} \frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma} (A_t')^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big]^{\frac{1}{\alpha+\sigma-\alpha\sigma}} + \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')} \Big) \Big\} \frac{1}{\varepsilon}\, dA_t' \\
&+ \int_{\mu-\varepsilon/2}^{\tilde{A}^M} \Big\{ \alpha(1-\beta)(1-\sigma) A_t' \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}} \Big)^{(1-\alpha)(1-\sigma)} \\
&\qquad + (1-\alpha)(1-\sigma) A_t' \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}} \Big)^{-\alpha-\sigma+\alpha\sigma} \Big\} \frac{1}{\varepsilon}\, dA_t',
\end{aligned}
\tag{1.16}
$$

and finally,

$$
\begin{aligned}
&p_t \\
&= \int_{\tilde{A}^M}^{\mu+\varepsilon/2} \Big\{ \mathcal{C} \frac{\alpha\beta(1-\sigma)}{\alpha+\sigma-\alpha\sigma} (A_t')^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \Big(\frac{n_t^{PM}+n_t^{1M}}{x_t} \Big) \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big]^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \Big\} \frac{1}{\varepsilon}\, dA_t' \\
&+ \int_{\mu-\varepsilon/2}^{\tilde{A}^M} \Big\{ \alpha\beta(1-\sigma) A_t' \Big(\frac{n_t^{PM}+n_t^{1M}}{x_t} \Big) \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}} \Big)^{(1-\alpha)(1-\sigma)} \Big\} \frac{1}{\varepsilon}\, dA_t'.
\end{aligned}
\tag{1.17}
$$

Note that the first-order conditions (15)–(17) and endogenous thresholds $\tilde{A}^M$ and $\tilde{A}^T$ are related to the endogenous input combinations $\Phi_1 = \Big[\big(\frac{B_i x_t}{n_t^{PM}+n_t^{1M}}\big)^{\alpha\beta(1-\sigma)} \big(\frac{\bar{z}}{n_t^{PM}+n_t^{1M}}\big)^{\sigma} \Big] \Big(\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}} \Big)^{-\alpha-\sigma+\alpha\sigma}$, $\Phi_2 = \frac{n_t^{PM}+n_t^{1M}}{x_t}$, and $\Phi_3 = \frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$. The latter two ratios are of particular interest: Φ_2 is the inverse of intensity of intermediate inputs relative to labor and Φ_3 is the permanent labor share at the planting stage. By rearrangement, conditions (3) and (5) can be rewritten as

$$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}} = \left[\frac{\lambda b + \frac{\xi_2}{\eta_t(A_t')}}{A_t'(1-\alpha)(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma-\alpha\sigma}} \left(\frac{1}{\Phi_1}\right)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}, \tag{1.18}$$

and

$$(1-\alpha)(1-\sigma)\Phi_1\tilde{A}^M = \lambda b + \frac{\xi_2}{\eta_t(\tilde{A}^M)}. \tag{1.19}$$

As shown in [Figure 1.9a] and [Figure 1.10], the two conditions determine the unique threshold value of hiring any casual labor, $\tilde{A}^M$, as well as the permanent-labor ratio ex-post. They also suggest that several factors are responsible for the higher reliance on casual labor ex-post, one of which is an indirect effect derived from uncertainty. Given its positive effect on the input combination Φ_1 (to be shown later), Eq. (1.19) and [Figure 1.9b] imply a lower threshold value $\tilde{A}^M$ to hire labor ex-post when the agent is making decisions ex-ante and taking a higher degree of uncertainty into account. On the other hand, Eq. (1.18) and [Figure 1.10] also suggest a lower permanent labor ratio or higher casual labor ratio ex-post.

Using the input combinations defined above, the comparative statics will be conducted by simplifying Eqs. (1.15)–(1.17) respectively as

$$\begin{aligned}
&(1-\lambda)b + \frac{\xi_1}{\bar{\eta}^c} \\
&= \int_{\tilde{A}^M(\Phi_1,\xi_2)}^{\mu+\varepsilon/2} \left\{\mathcal{C}\frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma}(A_t'\Phi_1)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\Phi_3\right\}\frac{1}{\varepsilon}\,dA_t' + \int_{\mu-\varepsilon/2}^{\tilde{A}^M(\Phi_1,\xi_2)} \left\{\alpha(1-\beta)(1-\sigma)A_t'\Phi_1\Phi_3\right\}\frac{1}{\varepsilon}\,dA_t', \\
&\equiv f_{1C}^M(\Phi_1,\,\Phi_3;\,\varepsilon,\,\xi_2),
\end{aligned} \tag{1.20}$$

$$\begin{aligned}
w_t^P &= \int_{\tilde{A}^M(\Phi_1,\xi_2)}^{\mu+\varepsilon/2} \left\{\mathcal{C}\frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma}(A_t'\Phi_1)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\Phi_3 + \left(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\right)\right\}\frac{1}{\varepsilon}\,dA_t' \\
&+ \int_{\mu-\varepsilon/2}^{\tilde{A}^M(\Phi_1,\xi_2)} \left\{\alpha(1-\beta)(1-\sigma)A_t'\Phi_1\Phi_3 + (1-\alpha)(1-\sigma)A_t'\Phi_1\right\}\frac{1}{\varepsilon}\,dA_t' \\
&= f_{1C}^M(\Phi_1,\,\Phi_3;\,\varepsilon,\,\xi_2) + f_P^M(\Phi_1,;\,\varepsilon,\,\xi_2),
\end{aligned} \tag{1.21}$$

where

$$f_P^M(\Phi_1,;\,\varepsilon,\,\xi_2) \equiv \int_{\tilde{A}^M(\Phi_1,\xi_2)}^{\mu+\varepsilon/2} \Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big)\frac{1}{\varepsilon}\,dA_t' + \int_{\mu-\varepsilon/2}^{\tilde{A}^M(\Phi_1,\xi_2)} (1-\alpha)(1-\sigma)A_t'\Phi_1\frac{1}{\varepsilon}\,dA_t'$$

$$= w_t^P - \Big((1-\lambda)b + \frac{\xi_1}{\bar{\eta}^c}\Big), \tag{1.22}$$

and finally

$$p_t$$

$$= \int_{\tilde{A}^M(\Phi_1,\xi_2)}^{\mu+\varepsilon/2} \Big\{\mathcal{C}\frac{\alpha\beta(1-\sigma)}{\alpha+\sigma-\alpha\sigma}(A_t'\Phi_1)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\,\Phi_2\Phi_3\Big\}\frac{1}{\varepsilon}\,dA_t' + \int_{\mu-\varepsilon/2}^{\tilde{A}^M(\Phi_1,\xi_2)} \Big\{\alpha\beta(1-\sigma)A_t'\Phi_1\,\Phi_2\Phi_3\Big\}\frac{1}{\varepsilon}\,dA_t'$$

$$\equiv f_x^M(\Phi_1,\,\Phi_2,\,\Phi_3;\,\varepsilon,\,\xi_2). \tag{1.23}$$

Note that the three input combinations are jointly determined by Eqs. (1.20), (1.21), and (1.23), and hence their values will not be functions of the individual ability level. In addition, Eq. (1.22) delivers the tradeoff between hiring permanent and casual labor at the planting stage, since it means that the relative labor costs, as denoted by the right-hand side of the second equality, equal the sum of the expected marginal products under the good state $A_t' \in (\tilde{A}^M,\,\mu+\varepsilon/2)$ and the bad state $A_t' \in (\mu-\varepsilon/2,\tilde{A}^M)$. Because of the assumed perfect substitution between permanent and casual labor it can be shown that

$$w_t^P \le \Big((1-\lambda)b + \frac{\xi_1}{\bar{\eta}^c}\Big) + \mathbb{E}\Big[\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big] = \Big((1-\lambda)b + \frac{\xi_1}{\bar{\eta}^c}\Big) + \int_{\mu-\varepsilon/2}^{\mu+\varepsilon/2}\Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big)dF(A_t'), \tag{1.24}$$

where the inequality comes from the loss of flexibility under the bad state when deciding to hire one unit of the permanent labor instead of one unit of casual substitute at both stages. Secondly, by comparing the two large parentheses of Eq. (1.23), it can be shown that the marginal product of intermediate inputs (or sunk investments) is enlarged under the good state since $1/(\alpha+\sigma-\alpha\sigma) > 1$. This is because the adjustment of the labor input is available and it makes intermediate inputs become more productive.

Since uncertainty is represented by the parameter ε and it governs the amount of variation within the unform distribution, the comparative statics of uncertainty on the input combination Φ_1 is written as

$$d\Phi_1/d\varepsilon = \frac{-\partial f_P^M/\partial\varepsilon}{\partial f_P^M/\partial\Phi_1}. \tag{1.25}$$

Applying Leibniz's rule, I can derive

$$\begin{aligned}
\frac{\partial f_P^M}{\partial\varepsilon} &= \frac{1}{2}\Big[\lambda b + \frac{\xi_2}{\eta_t(\mu+\frac{\varepsilon}{2})}\Big]\frac{1}{\varepsilon} - (-\frac{1}{2})\Big[(1-\alpha)(1-\sigma)(\mu-\frac{\varepsilon}{2})\Phi_1\Big]\frac{1}{\varepsilon} \\
&\quad + \Big(-\frac{1}{\varepsilon}\Big)\Big\{\int_{\tilde{A}^M(\Phi_1,\xi_2)}^{\mu+\varepsilon/2}\Big(\lambda b + \frac{\xi_2}{\eta_t(A_t')}\Big)\frac{1}{\varepsilon}\,dA_t' + \int_{\mu-\varepsilon/2}^{\tilde{A}^M(\Phi_1,\xi_2)}(1-\alpha)(1-\sigma)A_t'\Phi_1\frac{1}{\varepsilon}\,dA_t'\Big\} \\
&= \Big(\frac{1}{\varepsilon}\Big)\Big\{\frac{1}{2}\Big[\lambda b + \frac{\xi_2}{\eta_t(\mu+\frac{\varepsilon}{2})}\Big] + \frac{1}{2}\Big[(1-\alpha)(1-\sigma)(\mu-\frac{\varepsilon}{2})\Phi_1\Big] - f_P^M\Big\} \gtreqless 0,
\end{aligned} \tag{1.26}$$

and

$$\begin{aligned}
\frac{\partial f_P^M}{\partial\Phi_1} &= \frac{\partial\tilde{A}^M}{\partial\Phi_1}\Big\{(1-\alpha)(1-\sigma)\tilde{A}^M\Phi_1 - \Big(\lambda b + \frac{\xi_2}{\eta_t(\tilde{A}^M)}\Big)\Big\}\frac{1}{\varepsilon} + \int_{\mu-\varepsilon/2}^{\tilde{A}^M}(1-\alpha)(1-\sigma)A_t'\frac{1}{\varepsilon}dA_t' \\
&= \int_{\mu-\varepsilon/2}^{\tilde{A}^M}(1-\alpha)(1-\sigma)A_t'\frac{1}{\varepsilon}dA_t' > 0.
\end{aligned} \tag{1.27}$$

The second equality of Eq. (1.27) is derived by using the determination condition of $\tilde{A}^M$ based on Eq. (1.19) and the positive, equal, or negative sign of Eq. (1.26) rests on the concavity, linearity or convexity of the function $\lambda b + \frac{\xi_2}{\eta_t(A_t')}$ on A_t', respectively. Hence, I have $d\Phi_1/d\varepsilon > 0$ if the costs of hiring, $\lambda b + \frac{\xi_2}{\eta_t(A_t')}$, are concave on A_t', as shown in [Figure 1.9a]. The assumption implies that the hiring costs do not increase too dramatically and hence excludes the possibility of multiple equilibria and less reliance on casual labor under high-productivity realizations. Because the threshold value $\tilde{A}^M$ is lowered as the degree of uncertainty rises, the result $d\tilde{A}^M/d\varepsilon < 0$ implies that the rising uncertainty will increase the extent of reliance on casual labor.

In order to understand the influence of uncertainty on the share of permanent labor at the planting stage Φ_3, I derive the components of

$$d\Phi_3/d\varepsilon = -\frac{\partial f_{1C}^M/\partial\varepsilon \, + \, (\partial f_{1C}^M/\partial\Phi_1)(d\Phi_1/d\varepsilon)}{\partial f_{1C}^M/\partial\Phi_3} \tag{1.28}$$

by using Eq. (1.20) and given $d\Phi_1/d\varepsilon > 0$ with

$$\frac{\partial f_{1C}^M}{\partial\varepsilon} = \frac{1}{\varepsilon}\left\{\frac{1}{2}\mathcal{C}\Big[\frac{\alpha(1-\beta)(1-\sigma)}{\alpha+\sigma-\alpha\sigma}\Big]\Big((\mu+\frac{\varepsilon}{2})\Phi_1\Big)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\Phi_3 + \frac{1}{2}\alpha(1-\beta)(1-\sigma)(\mu-\frac{\varepsilon}{2})\Phi_1\Phi_3 - \Big[(1-\lambda)b+\frac{\xi_1}{\overline{\eta}^c}\Big]\right\} > 0, \tag{1.29}$$

$$\frac{\partial f_{1C}^M}{\partial\Phi_3} = \Big[(1-\lambda)b+\frac{\xi_1}{\overline{\eta}^c}\Big]\frac{1}{\Phi_3} > 0, \tag{1.30}$$

and

$$\frac{\partial f_{1C}^M}{\partial\Phi_1} > \Big[(1-\lambda)b+\frac{\xi_1}{\overline{\eta}^c}\Big]\frac{1}{\Phi_1} > 0. \tag{1.31}$$

The comparative statics shows that $d\Phi_3/d\varepsilon < 0$ and hence the casual-labor ratio at the planting stage, $\frac{n_t^{1M}}{n_t^{PM}+n_t^{1M}} = 1-\Phi_3$, is increasing in the degree of uncertainty. Moreover, the casual-labor ratio at the harvest stage, $\frac{n_t^{2M}}{n_t^{PM}+n_t^{2M}}$, is also increasing in the degree of uncertainty because of Eq. (1.18) and $d\Phi_1/d\varepsilon > 0$, which hinges on the assumption that the marginal costs $\lambda b + \frac{\xi_2}{\eta_t(A_t')}$ are concave on A_t'.

Finally, the comparative statics of the ratio of labor ex-ante to intermediate inputs with respect to ε is

$$d\Phi_2/d\varepsilon = \frac{\partial f_x^M/\partial\varepsilon - (\partial f_x^M/\partial\Phi_1)(d\Phi_1/d\varepsilon) - (\partial f_x^M/\partial\Phi_3)(d\Phi_3/d\varepsilon)}{\partial f_x^M/\partial\Phi_2}, \tag{1.32}$$

which is derived by using Eq. (1.23) and its components are

$$\frac{\partial f_x^M}{\partial\varepsilon} = \frac{1}{\varepsilon}\left\{\frac{1}{2}\Big[\mathcal{C}\frac{\alpha\beta(1-\sigma)}{\alpha+\sigma-\alpha\sigma}\Big((\mu+\frac{\varepsilon}{2})\Phi_1\Big)^{\frac{1}{\alpha+\sigma-\alpha\sigma}}\Phi_2\Phi_3\Big] + \frac{1}{2}\Big[\alpha\beta(1-\sigma)(\mu-\frac{\varepsilon}{2})\Phi_1\Phi_2\Phi_3\Big] - p_t\right\} > 0 \tag{1.33}$$

and

$$\frac{\partial f_x^M}{\partial\Phi_\iota} = \frac{p_t}{\Phi_\iota} > 0 \quad \text{ for } \iota = 1,2,3. \tag{1.34}$$

Since the previous result shows that $d\Phi_1/d\varepsilon > 0$ and $d\Phi_3/d\varepsilon < 0$, the sign of $\partial\Phi_2/\partial\varepsilon$ follows

$$\frac{\partial \Phi_2}{\partial \varepsilon} \gtreqless 0 \quad \text{iff} \quad \frac{\partial f_x^M}{\partial \varepsilon} - \frac{\partial f_x^M}{\partial \Phi_3}\frac{d\Phi_3}{d\varepsilon} \gtreqless \frac{\partial f_x^M}{\partial \Phi_1}\frac{d\Phi_1}{d\varepsilon}. \tag{1.35}$$

As an interesting benchmark, consider the case in which η_t is independent of A_t', and hence $d\Phi_1/d\varepsilon = 0$, $d\Phi_2/d\varepsilon > 0$, and $d\Phi_3/d\varepsilon < 0$ as shown above. In this case, higher uncertainty not only raises the employment share of casual labor, but also lowers the intensity of intermediate inputs.

The optimization problem of the traditional-type agents can be solved by following the same rule. The details are omitted here to avoid redundance.

1.5 Quantitative analysis

In this section, I first present how the model's key parameters are calibrated to fit the micro data. Then, I consider alternative values for (i) availability of casual labor and (ii) degree of uncertainty since they represent the main channels to be highlighted in the model. Finally, related counterfactual experiments are proposed to understand how various components contribute the two targets of interest.

1.5.1 Calibration

The parameters to be calibrated are summarized in [Table 5]. The calibration rule is at first to choose the values of parameters of the production functions ($\{\alpha^M, \beta^M\}$, α^T, σ, $\{\mu_A, \sigma_A\}$, $\{\mu_B, \sigma_B\}$) primarily based on information from the data. Parameters of the labor matching function, which include ($\bar{m}$, ρ), are selected based on the standard setting. The other parameters related to hiring or searching costs are calibrated to match the main targets, the permanent-casual labor ratios (both ex-ante and ex-post) and the technology adoption rate.

First, the share of land (σ) is firstly set at 0.2 and the value follows Restuccia, Yang, and Zhu (2008), due to the lack of the rental price of land from the data. The other two parameters of modern technology (α^M, β^M) are pinned down by the following conditions: (i) the expenses on intermediate inputs account for nearly 20% of the value of output,[12] and (ii) the wage costs from hiring labor ex-post account for 14% of the value of output on average. Note that the wage costs are adjusted and remeasured by labor in effective units. An essential reason is that a large proportion of the labor days is provided by female and child workers, and they are considered on average to be less productive in agricultural production than males. Hence, the amount of labor to be employed in terms of man labor days is computed by creating estimates of the relative wage rate among the three groups. The computation work and sample selection criteria are described in more detail in the appendix. For the traditional technology, the value of the parameter α^T is selected such that the share of the wage costs from hiring labor ex-post to the value of output is around 27%–27.5%. Given the selected values for the input shares, the realized A'_{it} from traditional-type agents is constructed by

$$A'_{it} = \frac{\frac{y^T_{it}}{z_i}}{(\frac{n^{PT}_{it}+n^{1T}_{it}}{z_i})^{\alpha^T(1-\sigma)}(\frac{n^{PT}_{it}+n^{2T}_{it}}{z_i})^{(1-\alpha^T)(1-\sigma)}} \sim \ln\mathcal{N}(2.77, 1.12), \tag{1.36}$$

since individual-specific ability does not generate differences in their final output by assumption. In the same manner, the product of two random variables is expressed as

$$A'_{it}B_i^{\alpha^M\beta^M(1-\sigma)} = \frac{\frac{y^M_{it}}{z_i}}{(\frac{x_{it}}{z_i})^{\alpha^M\beta^M(1-\sigma)}(\frac{n^{PM}_{it}+n^{1M}_{it}}{z_i})^{\alpha^M(1-\beta^M)(1-\sigma)}(\frac{n^{PM}_{it}+n^{2M}_{it}}{z_i})^{(1-\alpha^M)(1-\sigma)}} \sim \ln\mathcal{N}(3.36, 1.125). \tag{1.37}$$

The above two densities are displayed in [Figure 1.11]. It is further assumed that A'_{it} from modern-type agents follows the same distribution and that A'_{it} and B_i are independent. Also note that all observed B_is are from individuals with $B_i > \underline{B}$ since only agents with a relatively high ability choose modern technology.

[12] Based on sample mean values from the data, the share of intermediate inputs is set at 0.2, which is closer to the value (0.25) predicted by Donovan (2014) for poor economies than the value (0.4) selected by Restuccia, Yang, and Zhu (2008) based on the U.S. case.

This implies that the observed B_i is drawn from the conditional distribution

$$(B_i|B_i > \underline{B}) \sim \ln\mathcal{N}(\frac{0.59}{\alpha^M\beta^M(1-\sigma)}, \frac{0.009}{(\alpha^M\beta^M(1-\sigma))^2}) = \ln\mathcal{N}(1.675, 0.225).$$

Using the properties of a truncated normal distribution and the above two moment conditions, the underlying parameters μ_B, σ_B, and $\underline{B}$ are pinned down jointly by including an additional condition to be matched, namely, the 10% technology adoption rate.

Second, the parameters of the matching function are set as $\bar{m} = 1$ and $\rho = 0.5$, and the setting follows standard assumptions since the data are not informative to the values. The value for the quantity of labor representing the potential total supply of casual labor per unit of land, which is denoted by $\bar{N}/\bar{z}$, is calibrated to meet the ex-ante casual-labor ratio, which is around 70% on average.

Finally, the parameters regarding hiring costs are selected based on the following rule. The data show that the ratio of average labor days per worker in period 2 to period 1 is 7.12/19.65 for modern-type agents and is 8.32/22.89 for traditional-type ones, and the two ratios are close to 0.36. This implies that $\lambda/(1-\lambda) \approx 0.36$, and the duration of the harvesting stage accounts for 26.5% of the time for the entire t. The labor wage of the entire t is represented by b, and the value is calibrated such that the endogenous value $\underline{B}$ to be solved by the model meets the values with a 10% technology adoption rate. Still, there is a lack of related information on the searching costs based on the data, and hence the costs at the planting stage ξ_1 are tentatively assumed to be a small value, say, 1% of the labor wage b. The last parameter represents that of the searching costs at the harvest stage ξ_2, and it is calibrated to meet the ex-post casual-labor ratio on average.

The calibration results, including the key variables of targets or non-targets, are shown in the sub-panel of [Table 5]. They predict around a 1.73–1.83-fold stage-contingent difference in the labor productivity gaps between the two technologies, which is somewhat lower than the 2.13-fold observed from the micro data.

[Figure 1.12] displays a decreasing pattern of matching probabilities with respect to different realized values of A'_t in equilibrium in the benchmark model, as argued in Section 1.3.

1.5.2 Sensitivity analysis

Based on the values of the parameters selected by the previous work, comparative statics involving exogenously changing the calibrated values is carried out by means of quantitative exercises. The results are presented in [Figure 1.14] and also listed in [Table 6] in detail. Compared with the benchmark economy, the first two exercises to some extent capture the idea of a change in the availability of casual labor. Panel A shows that exogenously doubled searching costs ex-post will have a larger positive effect on the ex-post permanent labor ratio $n_t^{PM}/(n_t^{PM} + n_t^{2M})$ than the ex-ante ratio $n_t^{PM}/(n_t^{PM} + n_t^{1M})$ since the change will directly depress the demand for casual labor ex-post, leading to an indirect effect on the labor input that is determined ex-ante. On the other hand, the increased searching costs raise the proportion of agents choosing modern technology (denoted as π^M) from 10.9% to 23.8%.[13] Meanwhile, since the change in the productivity gaps with respect to different states is moderate, switches between two technologies because of the extensive margin effect contribute most of the aggregate productivity gains.

Panel B presents an effect that results from diminishing the pool of available casual labor by 90%. Differing from the previous result, this effect is rather comprehensive since it enlarges the expected costs from hiring one unit of casual labor both ex-ante and ex-post. Hence, the quantitative results suggest that these will be a pronounced increase in the permanent labor ratio not only ex-post but also ex-ante in equilibrium. Another intuitive feature is that the fraction for choosing modern technology also increases, but the effect is relatively small when compared with solely increasing the hiring costs ex-post.

[13] The result is in line with one of the findings from Manuelli and Seshadri (2014), who provide historical evidence in the U.S. that a rise in labor costs could accelerate the adoption of less-labor-intensive technology in the long run.

The results in Panel C highlight the effects from imposing a higher degree of uncertainty through mean-preserving spreads. Note that the two related parameters of the log-normal distributions are adjusted so that their standard deviations increase while their mean values are left unchanged. The results show that, compared with the benchmark, a doubled degree of uncertainty will in general cause agents to rely more on casual labor both ex-ante and ex-post. Most importantly, it gives rise to 34% of agents who had previously chosen modern technology deciding to switch. As noted, the decrease in the adoption rate of modern technology is driven by the extensive-margin effect and the channel that makes the effect work is the availability of hiring casual labor ex-post. Panel D serves as a complementary exercise to the previous one, since it presents the effect from reducing the degree of uncertainty by half through mean-preserving contractions.

Finally, Panel E reports the results by taking a lowered degree of uncertainty combined with doubled searching costs into account. Compared with any single effect presented in Panel A and Panel D, the joint effect will motivate more agents to adopt modern technology. Meanwhile, a moderate decrease in the productivity gaps implies that the aggregate productivity gains are mostly attributed to the increased number of agents switching from traditional to modern technology.

1.5.3 Counterfactual experiments

Three related counterfactual experiments are implemented as follows. In the first exercise, I shut down the availability of casual labor while leaving the quantity of permanent labor fixed. In the second, the economy is assumed to have an abundance of casual labor at the harvest stage (with a matching probability equal to 1). This case considers whether a counterfactual effect from a fully flexible adjustment to labor will benefit one of the technologies more. In the third case, agents are characterized by perfect foresight and hence are not vulnerable to uncertainty.

The first experiment is conducted to deal with the essential question: how will agents change their ex-ante decisions if an ex-post adjustment in the labor input is not available? The quantity of permanent labor is restricted to the same level as that for the benchmark economy; more precisely

$$N_t^{CF} = \min\Big\{ \int \big[n_t^{PM} + n_t^{1M}\big] dF(B_i) + (1-\pi^M)\big[n_t^{PT} + n_t^{1T}(A_t')\big], \int \big[n_t^{PM} + n_t^{2M}\big] dF(B_i) + (1-\pi^M)\big[n_t^{PT} + n_t^{2T}(A_t')\big] \Big\},$$

in which the aggregate supply of labor N_t^{CF} is the minimum computed value of the total labor to be hired ex-ante and ex-post in the benchmark economy. This condition is imposed to confine the scale of the permanent labor market to a comparable basis with the benchmark. The quantitative results indicate that π^M will be around 19.4%–20.5%, and suggest that eliminating casual labor will motivate more agents to choose modern technology, nearly doubling the adoption rate compared to the benchmark case.

In the second experiment, the moments of interest to be predicted are (π^M, $n_t^{1M}/n_t^{2M}(A_t')$, $y_t^M(A_t')/n_t^M(A_t')$) = (0.008, 0.243, 2.015), and the state-contingent variables are evaluated by setting A_t' equal to the median value. The results show that almost *all* agents will choose traditional technology ex-ante, if labor is allowed to be fully adjusted ex-post with perfect matching. In addition, the labor ratio ex-ante to ex-post drops from 0.666 (= 0.504/0.757, in benchmark) to 0.243 because there is no need to stock up labor ex-ante to save search cost ex-post. Meanwhile, the lower adoption will trigger a decreased aggregate labor productivity due to a mild increase in the productivity gap compared to the benchmark.

The third experiment concerns the impact of eliminating uncertainty. The agents in this case are assumed to have perfect foresight, and hence their ex-ante decisions will be state-contingent and unrelated to the degree of uncertainty. Put differently, they can fully expect their future demand for casual labor with respect to different realizations and hence will behave similarly by making all their decisions ex-ante. [Figure 1.15] shows that the computed adoption rates corresponding to the realized A_t' of the 1st, 25th, 50th, 75th, and 99th percentiles are 0.4%, 5.0%, 10.2%, 23.6%, and 69.0%. A noteworthy result observed from the figure is that the adoption rate at the mean, i.e., $A_t' = \mu_A$, equals 21.6%, which is more than twice the value in the benchmark economy (10.9%, on the red line). This again validates the conjecture of how the adoption rate can be improved by mitigating the extensive-margin effect of uncertainty.

1.6 Concluding remarks

In this chapter, a channel to connect the influence of production uncertainty on agents' ex-ante technology choices is proposed, i.e., the availability of hiring casual labor ex-post, in order to explain two targets of interest: the low adoption rate of modern technology and low aggregate productivity in poor countries. I first document stylized facts about how the labor paid on a daily basis accounts for a larger proportion of the labor force in the agricultural sector based on referring to official publications from multiple countries. To account for a potential miscounting problem, I look for detailed input-output information from each basic production unit by using micro data.

A tractable two-stage model is then built and an aggregate shock is introduced to a staged production process. Differing from the existing literature, the generalized model is designed to highlight the ex-ante technology adoption problem optimized by heterogenous agents. Two competing effects derived from a rise in the degree of uncertainty are evaluated at the extensive and intensive margins under the formalized framework. Because of the availability to make adjustments to the labor input ex-post, an increase in the degree of uncertainty will lead more agents to depart from modern technology along with higher labor productivity. This contributes to the extensive-margin effect as suggested in this chapter. Meanwhile, for an agent who has chosen to be of the modern-type, the opposite intensive margin effect will be responsible for a more intensive use of intermediate inputs or more sunk ex-ante investments.

Quantitative analysis is conducted based on a benchmark economy which is calibrated to match the main features from the micro-data. The numerical results of comparative statics are presented to access the effects of uncertainty; especially, how they are amplified through the proposed channel. Three counter-factual experiments are in addition provided to ascertain the significance of the channel, and are conducted by cases of (i) an unavailability of a labor adjustment ex-post, (ii) a wholly flexible labor adjustment, and (iii) agents with perfect foresight of future shocks. The quantitative results suggest that the severity of the uncertainty and an abundance of casual labor at the harvest stage are important drivers for low adoption and low productivity in a developing country. These factors also help to explain why agricultural productivity in poor countries is low relative to productivity in the non-agricultural sectors.

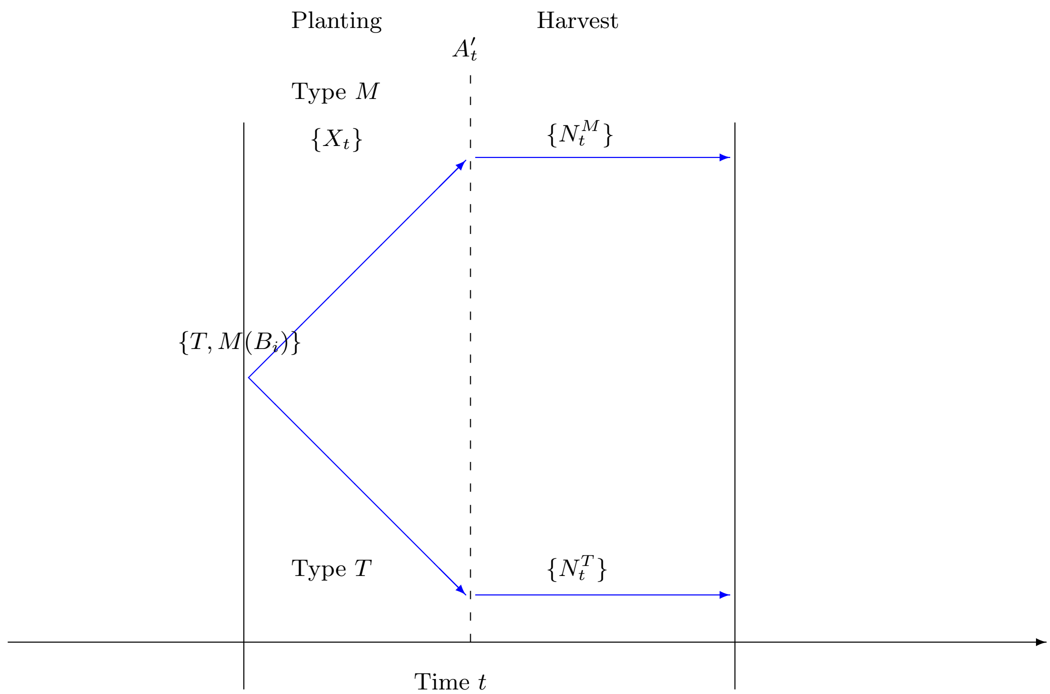


Figure 1.3: Timeline and agent i's decisions at time t (the simplified model)

1.7 Appendix

1.7.1 Comparative statics from the simplified model

A heterogeneous agent with ability $B_i = B\zeta_i$ is considering the following optimization problem

$$V(B_i, \bar{Z}) = \max_{\{M,\,T\}} \big\{V^M(B_i, \bar{Z})\,,\; V^T(\bar{Z})\big\},$$

in which the variable of individual-specific ability B_i follows all assumptions that have been made. The setting of dual production technology mainly follows the Cobb-Douglas form by Restuccia, Yang, and Zhu (2008) and Yang and Zhu (2013), i.e.,

$$Y_t^M = A'_t\big((B_i X_t)^{\alpha} N_t^{1-\alpha}\big)^{\gamma} \bar{Z}^{1-\gamma},$$

$$Y_t^T = A_t' N_t^{\gamma} \bar{Z}^{1-\gamma}.$$

The timeline of this simplified case is displayed in [Figure 1.3] and an additional assumption is made under the baseline framework: labor is fully hired ex-post but intermediate inputs are determined ex-ante. The backward solution shows that for a modern-type agent the use of inputs follows

$$n_t^M = \frac{N_t^M}{\bar{Z}} = \Big[\frac{(1-\alpha)\gamma A_t'(B_i x_t)^{\alpha\gamma}}{w_t}\Big]^{\frac{1}{1-(1-\alpha)\gamma}}, \tag{A.1.1}$$

$$x_t = \frac{X_t}{\bar{Z}} = (B_i)^{\frac{\alpha\gamma}{1-\gamma}} \Big(\frac{\alpha\gamma}{p_t}\Big)^{\frac{1-(1-\alpha)\gamma}{1-\gamma}} \Big[\frac{(1-\alpha)\gamma}{w_t}\Big]^{\frac{(1-\alpha)\gamma}{1-\gamma}} \Big[\frac{g(\mu,\varepsilon)}{1-(1-\alpha)\gamma}\Big]^{\frac{1-(1-\alpha)\gamma}{1-\gamma}}, \tag{A.1.2}$$

in which $g(\mu,\varepsilon) = \mathbb{E}(A_t'^{\frac{1}{1-(1-\alpha)\gamma}})$ and g is a positive function of μ and ε. Accordingly, the profit functions of both types are increasing in realized A_t'

$$\Omega_t^M(B_i) = \frac{V^M(B_i,\bar{Z})}{\bar{Z}} = \big[(1-\alpha)\gamma A_t'\big]^{\frac{1}{1-(1-\alpha)\gamma}} w_t^{\frac{-(1-\alpha)\gamma}{1-(1-\alpha)\gamma}} \left[\frac{1-(1-\alpha)\gamma}{(1-\alpha)\gamma}\right] (B_i x_t)^{\frac{\alpha\gamma}{1-(1-\alpha)\gamma}} - p_t x_t$$

$$\Omega_t^T = \frac{V^T(\bar{Z})}{\bar{Z}} = (\gamma A_t')^{\frac{1}{1-\gamma}} w_t^{\frac{-\gamma}{1-\gamma}} \big(\frac{1-\gamma}{\gamma}\big).$$

The expected profits (values) from choosing modern and traditional technologies are

$$\mathbb{E}(\Omega_t^M) = \mathcal{C}^M(B_i)\, p_t^{-\frac{\alpha\gamma}{1-\gamma}} w_t^{-\frac{(1-\alpha)\gamma}{1-\gamma}} \mathbb{E}(A_t'^{\frac{1}{1-(1-\alpha)\gamma}})^{\frac{1-(1-\alpha)\gamma}{1-\gamma}} \tag{A.1.3}$$

$$\mathbb{E}(\Omega_t^T) = \mathcal{C}^T w_t^{\frac{-\gamma}{1-\gamma}} \mathbb{E}(A_t'^{\frac{1}{1-\gamma}}), \tag{A.1.4}$$

in which

$$\mathcal{C}^T = \gamma^{\frac{1}{1-\gamma}} \big(\frac{1-\gamma}{\gamma}\big)$$

and

$$\mathcal{C}^M = (B_i)^{\frac{\alpha\gamma}{1-\gamma}}\Big[\big(1-(1-\alpha)\gamma\big)^{1-\frac{\alpha\gamma}{1-\gamma}} - \alpha\gamma\big(1-(1-\alpha)\gamma\big)^{-1-\frac{\alpha\gamma}{1-\gamma}}\Big](\alpha\gamma)^{\frac{\alpha\gamma}{1-\gamma}}[(1-\alpha)\gamma]^{\frac{(1-\alpha)\gamma}{1-\gamma}}$$

is a positive function of B_i.

Note that for an agent with idiosyncratic ability B_i the expected values in choosing both types are equalized so that $\mathbb{E}(\Omega_t^M) = \mathbb{E}(\Omega_t^T)$ if $\alpha = 0$ or $\varepsilon = 0$. The two conditions imply that either the share of intermediate inputs is zero or there is no uncertainty during the production process. Secondly, it can be shown that $\mathbb{E}(A_t'^{\frac{1}{1-\gamma}}) \geq \mathbb{E}(A_t'^{\frac{1}{1-(1-\alpha)\gamma}})^{\frac{1-\gamma+\alpha\gamma}{1-\gamma}}$ by Jensen's inequality. Hence, for an agent with $B_i = \underline{B}$ such that $\mathbb{E}(\Omega_t^M) = \mathbb{E}(\Omega_t^T)$, an exogenous rise in uncertainty will relatively increase the value of choosing the traditional type more if $\alpha \neq 0$ and $\varepsilon \neq 0$. This contributes the extensive-margin effect and motivates the agent, who was indifferent between choosing the two alternatives before, to depart from modern technology. Thirdly, uncertainty does not have any effect if no inputs are allowed to be adjusted ex-post. Finally, the setting of the production functions implies labor productivity in an economy without market distortions:

$$\frac{y_t^M}{n_t^M} = \frac{w_t}{(1-\alpha)\gamma} \geq \frac{w_t}{\gamma} = \frac{y_t^T}{n_t^T},$$

and the productivity gap captures what is observed from the micro data if the value of α is sufficiently large.

1.7.2 Structural estimation under the simplified model

Since the extent to which uncertainty influences agents' decisions rests on the parameters of the two production functions, an inaccurately measured value may under- or overstate the results from the counter-factual analysis. Based on available information regarding the input-output combination for each production unit, the structural estimation proceeds in the following context.

Assume $A_i = A\nu_i$ where $\nu_i \overset{i.i.d}{\sim} \ln\mathcal{N}(\mu_\nu, \sigma_\nu)$ and $\zeta_i \overset{i.i.d}{\sim} \ln\mathcal{N}(\mu_\zeta, \sigma_\zeta)$. Denote the structural parameters to be estimated by $\Theta = (\alpha^M, \gamma^M, \alpha^T, \mu_\nu, \sigma_\nu, \mu_\zeta, \sigma_\zeta)'$. An observation from agent i includes $\omega_i = (y_i, d_i, x_i, l_i, z_i)'$, in which i's discrete technology choice $d_i \in \{M, T\}$ and y_i, x_i, l_i, and z_i respectively represent agent i's observable decisions on output, and the quantity of intermediate inputs, labor and land. Individual ability ζ_i is a latent variable and known by agent i. The likelihood function is built upon

$$L(\Theta; \omega_1, .., \omega_N) = \prod_{i=1}^{N} \mathcal{L}_i(\Theta | d_i, y_i, x_i, l_i, z_i) = \sum_{i=1}^{N} \ln \mathcal{L}_i(\Theta | d_i, y_i, x_i, l_i, z_i),$$

in which each

$$\begin{aligned}
&\mathcal{L}_i(\Theta | y_i, d_i, x_i, l_i, z_i) = \Pr(y_i, d_i, x_i, l_i | z_i) = \int \Pr(y_i, d_i, x_i, l_i | z_i, \zeta_i) f(\zeta_i) d\zeta_i \\
&= \int\int f(y_i | d_i, x_i, l_i, z_i, \zeta_i, \nu_i) f(x_i, l_i | d_i, z_i, \zeta_i) \Pr(d_i | z_i, \zeta_i) f(\nu_i) f(\zeta_i) d\nu_i d\zeta_i \\
&= \int\int f(y_i | d_i, x_i, l_i, z_i, \zeta_i, \nu_i) f(l_i | d_i, x_i, z_i, \zeta_i) f(x_i | d_i, z_i, \zeta_i) \Pr(d_i | z_i, \zeta_i) f(\nu_i) f(\zeta_i) d\nu_i d\zeta_i \\
&= \int\int f(y_i | d_i, x_i, l_i, z_i, \zeta_i, \nu_i) \Big[\int f(l_i | d_i, x_i, z_i, \zeta_i, \nu_i) f(\nu_i) d\nu_i \Big] f(x_i | d_i, z_i, \zeta_i) \Pr(d_i | z_i, \zeta_i) f(\nu_i) f(\zeta_i) d\nu_i d\zeta_i
\end{aligned} \tag{A.1.5}$$

in which the second and third equalities of Eq. (A.1.5) hold because both the unobservable ability ζ_i and the productivity shock ν_i are assumed to be independent and identically distributed random variables. Since

$$\ln(y_i^M) = \ln(A\nu_i) + \alpha^M \gamma^M \left[\ln(B\zeta_i) + \ln(x_i)\right] + (1 - \alpha^M)\gamma^M \ln(l_i) + (1 - \gamma^M)\ln(z_i),$$

I can derive that

$$f\big(\ln(y_i) | d_i = M, x_i, l_i, z_i, \zeta_i, \nu_i; \Theta\big) \sim \mathcal{N}(\mu_y^M, \sigma_y^M),$$

where

$$\mu_y^M = C^M + \alpha^M \gamma^M \ln x_i + (1 - \alpha^M)\gamma^M \ln l_i + (1 - \gamma^M) \ln z_i + \mu_\nu + \alpha^M \gamma^M \mu_\zeta,$$

$$\sigma_y^M = \sigma_\nu + (\alpha^M \gamma^M)^2 \sigma_\zeta.$$

Because of conditions (A.1.1) and (A.1.2), the conditional densities $f\big(\ln(l_i)|d_i = M, x_i, z_i, \zeta_i, \nu_i\big)$ and $f\big(\ln(x_i)|d_i = M, z_i, \zeta_i\big)$ are also Gaussian. The conditional probability of discrete choice d_i is

$$\begin{aligned}\Pr(d_i|z_i, \zeta_i) &= \Pr(d_i = M|z_i, \zeta_i)^{\mathbb{I}(d_i = M)} \Big[1 - \Pr(d_i = M|z_i, \zeta_i)\Big]^{1-\mathbb{I}(d_i = M)} \\ &= \Pr\Big(\mathbb{E}[\Omega_i^M|z_i, \zeta_i] - \mathbb{E}[\Omega_i^T|z_i, \zeta_i] > 0\Big)^{\mathbb{I}(d_i = M)} \Pr\Big(\mathbb{E}[\Omega_i^M|z_i, \zeta_i] - \mathbb{E}[\Omega_i^T|z_i, \zeta_i] < 0\Big)^{1-\mathbb{I}(d_i = M)},\end{aligned}$$

where the indicator function $\mathbb{I}(d_i = M) = 1$ while $d_i = M$. Hence, $\Pr(d_i|z_i, \zeta_i)$ is a function of structural parameters as stated.

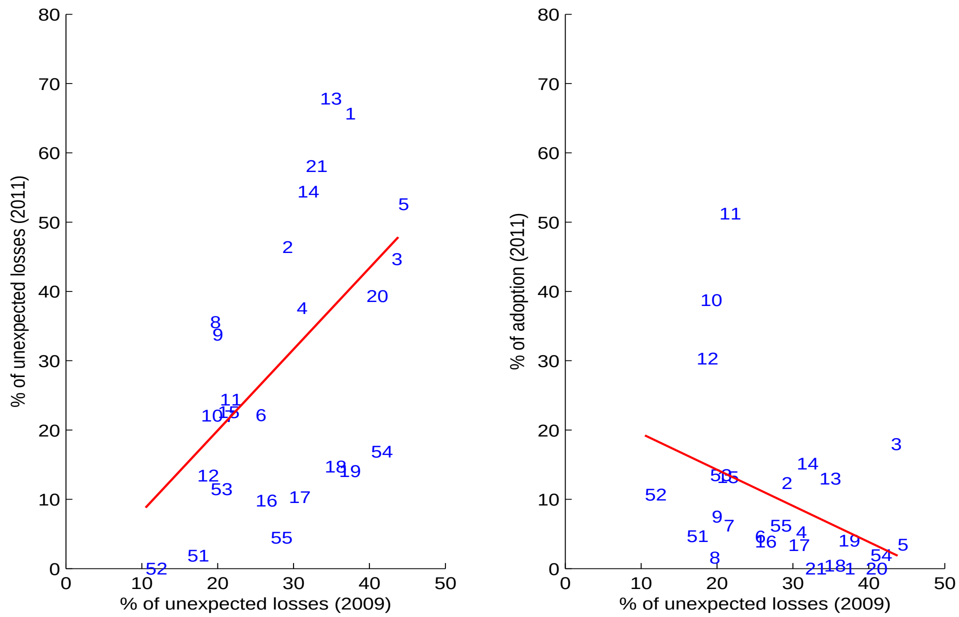


Figure 1.4: Probability of suffering losses and cross-regional adoption rate of Tanzania

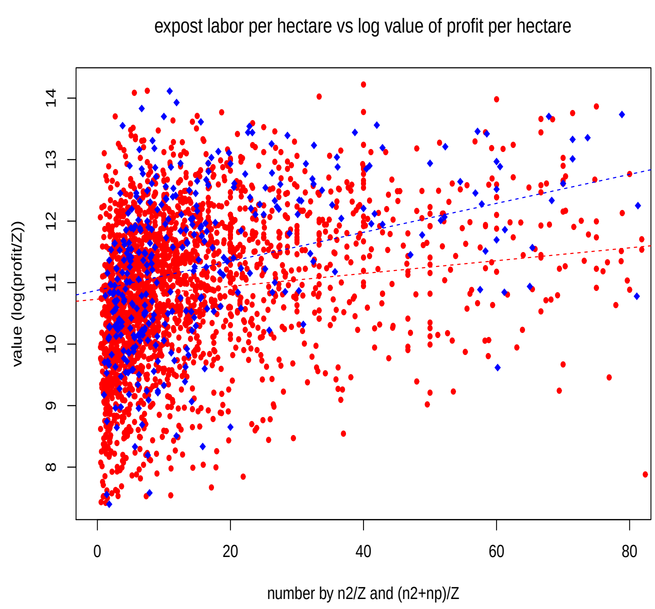


Figure 1.5: Expost labor per hectare vs log value of profit per hectare: $(n^2 + n^P)/\bar{Z}$ in blue dots and $n^P/\bar{Z}$ in red dots

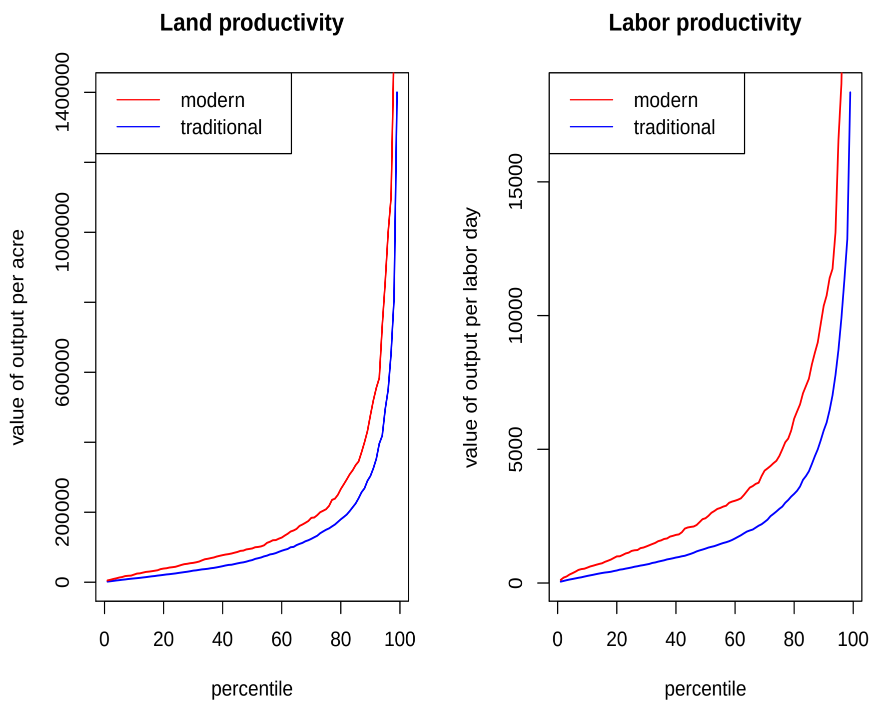


Figure 1.6: Land and labor productivity by percentile

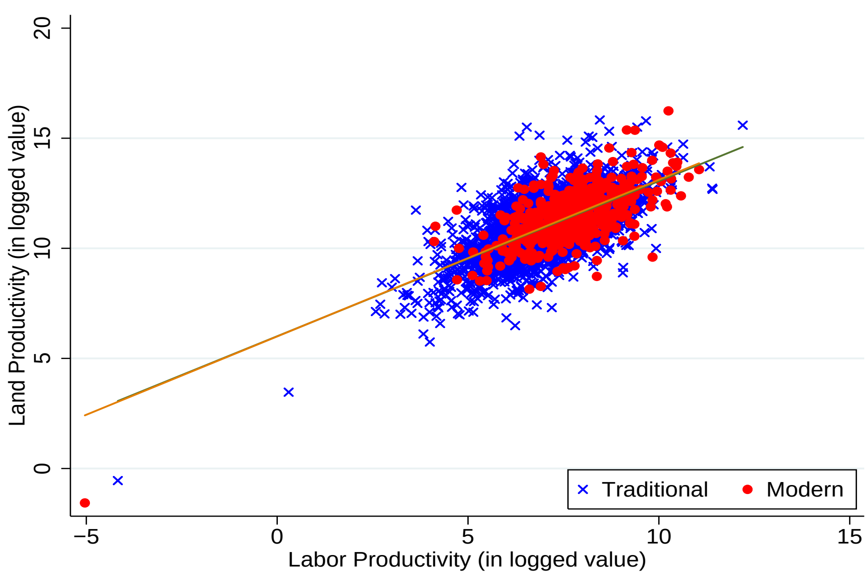


Figure 1.7: Land and labor productivity by technology type

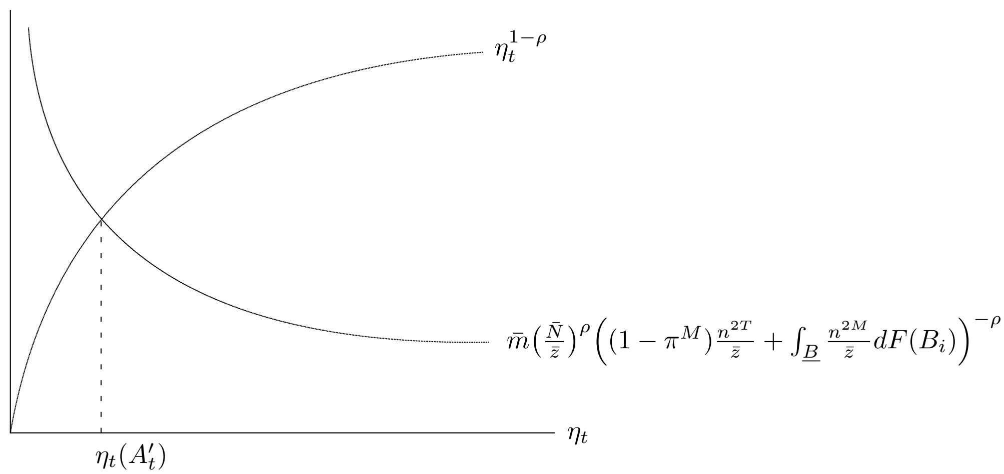


Figure 1.8: Determine the equilibrium probability that a vacancy is filled in

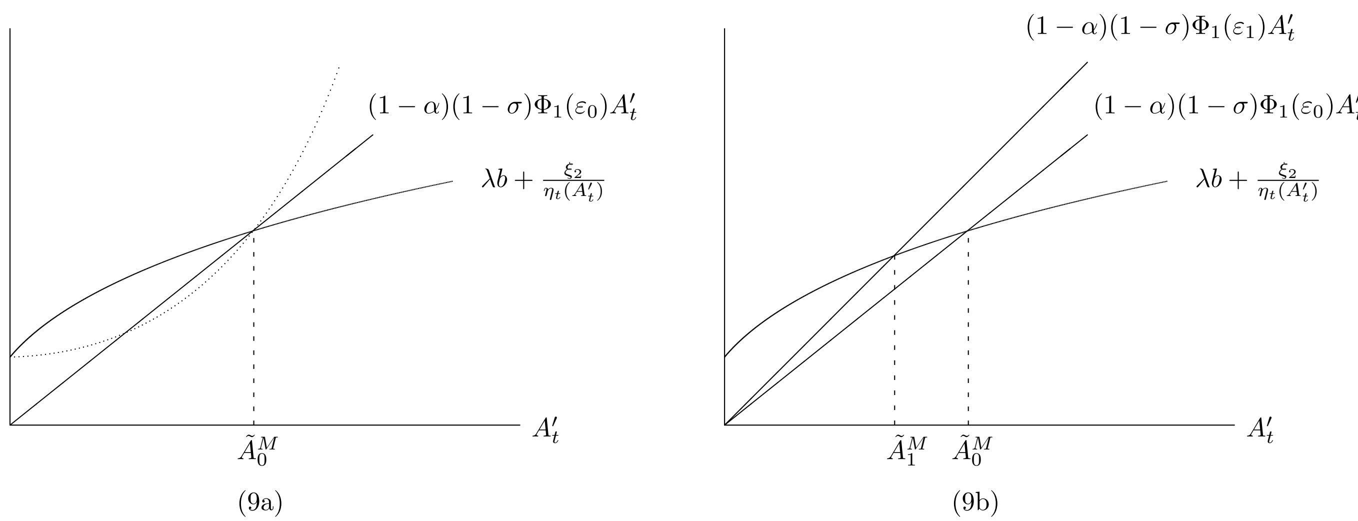


Figure 1.9: Concavity of hiring costs (on left) and the negative effect of uncertainty on the threshold value $\tilde{A}^M$ by $\varepsilon_0<\varepsilon_1$ (on right)

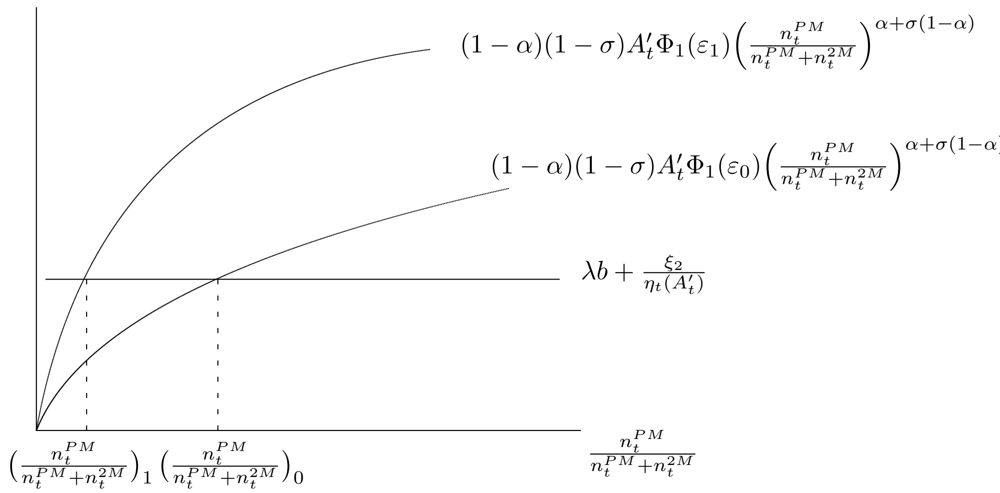


Figure 1.10: Effect of uncertainty on permanent labor ratio ex-post by $\varepsilon_0 < \varepsilon_1$

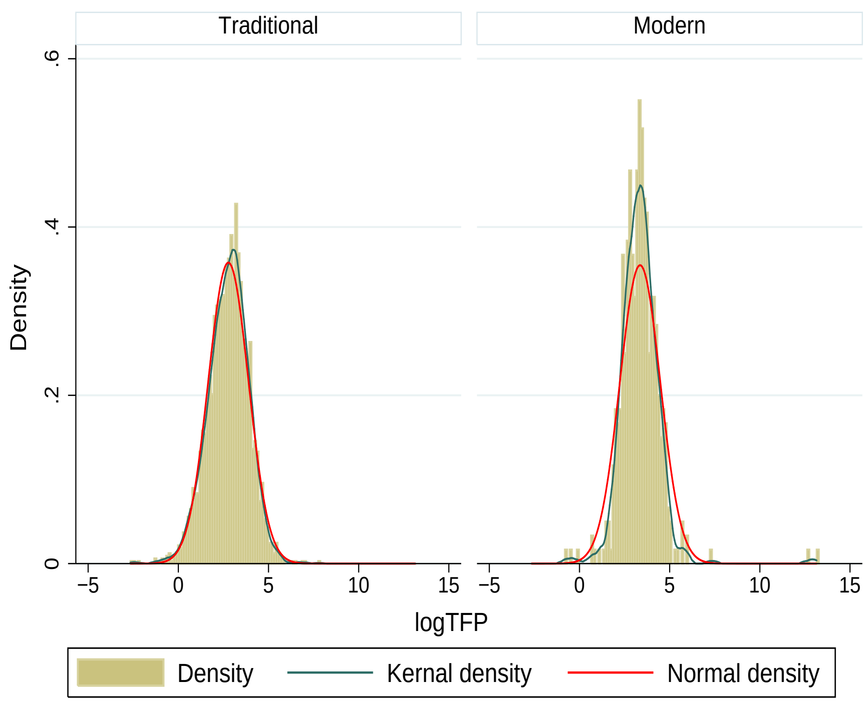


Figure 1.11: Densities of $\ln(A_{it}')$ from traditional-type agents (left) and $\ln(A_{it}'B_i^{\alpha\beta(1-\sigma)})$ from modern-type ones (right)

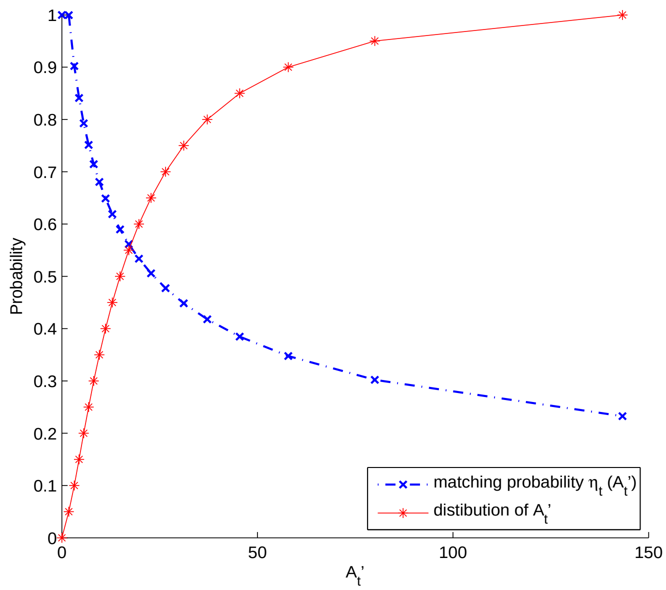


Figure 1.12: Equilibrium matching probability w.r.t. realized value of A_t' (benchmark)

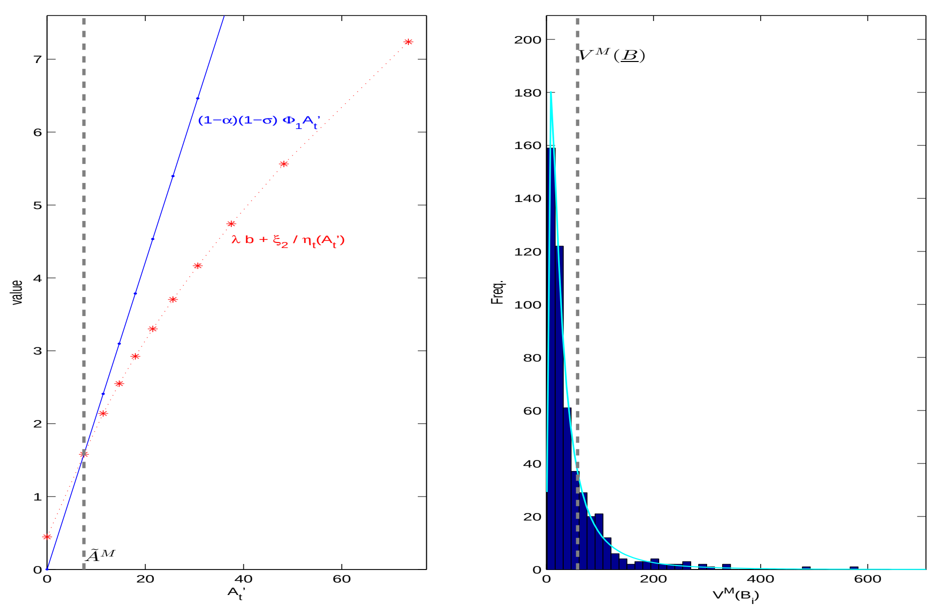
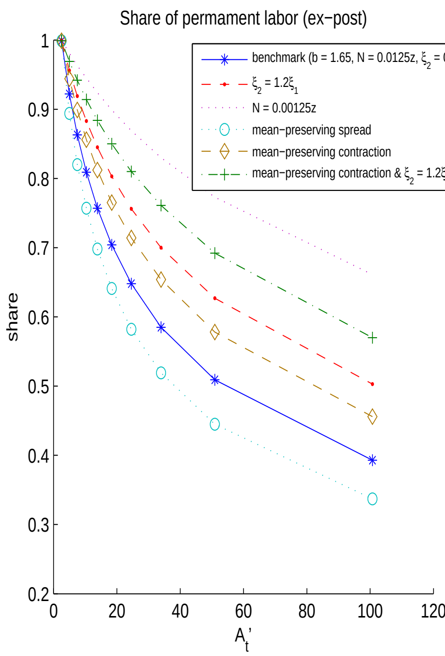
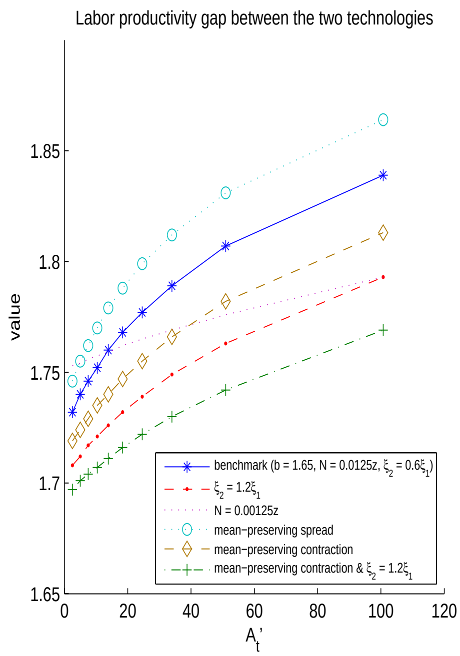


Figure 1.13: Technology adoption rate (10%) and values corresponding to B_i (benchmark)





[htb]

Figure 1.14: Sensitivity analysis on permanent labor share (left) and labor productivity gap (right)

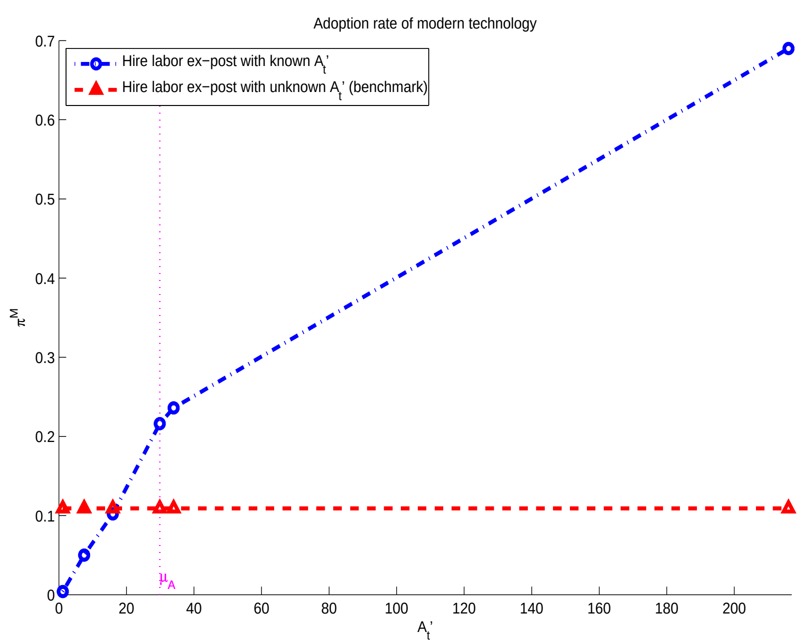


Figure 1.15: Counter-factual experiment: adoption rate of modern technology as agents have perfect foresight

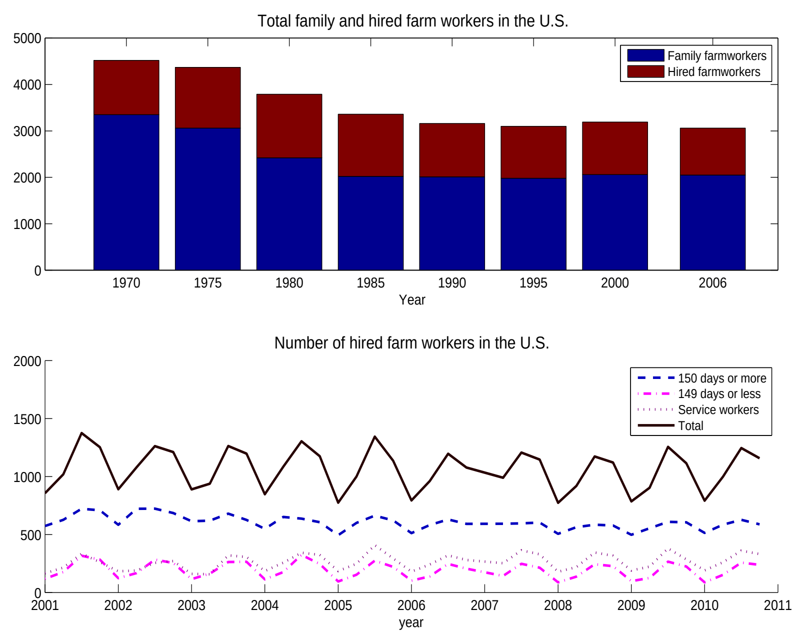


Figure 1.16: Composition of workers in agriculture in the U.S.

Table 1.1: Percentage of casual and regular waged labor by gender and by sector from different rounds of labor survey of India*

Casual labor	male	female	total	male	female	total	wage (daily)	
Survey period (round)	rural			urban			rural	urban
Jul.2011–Jun.12 (68nd)	35.5%	35.1%	35.4%	14.9%	14.3%	14.8%	138.62	170.10
Jul.2009–Jun.10 (66nd)	38.0%	39.9%	38.6%	17.0%	19.6%	17.5%	93.06	121.83
Jul.2007–Jun.08 (64th)	35.5%	37.6%	36.2%	15.4%	19.9%	16.2%	60.33	72.24
Jul.2005–Jun.06 (62nd)	33.3%	33.9%	.	15.7%	16.5%	.		
Jul.2004–Jun.05 (61st)	32.9%	32.6%	32.8%	14.6%	16.7%	15.0%	48.89	68.68
Jan.–Jun.2004 (60th)	33.5%	34.7%	.	15.3%	19.2%	.		
Jan.–Dec.2003 (59th)	33.5%	35.1%	.	15.6%	20.7%	.		
Jul.–Dec.2002 (58th)	34.4%	40.6%	.	15.0%	23.3%	.		
Jul.2001–Jun.02 (57th)	33.9%	38.2%	.	15.4%	26.1%	.		
Jul.2000–Jun.01 (56th)	31.6%	37.5%	.	17.5%	24.1%	.		
Jul.1999–Jun.00 (55th)	36.2%	39.6%	37.4%	16.8%	21.4%	17.7%	40.23	57.98
Jan.–Jun.1998 (54th)	37.7%	44.2%	.	18.1%	28.8%	.		
Jan.–Dec.1997 (53rd)	33.3%	40.9%	.	18.5%	29.0%	.		
Jul.1995–Jun.96 (52nd)	33.3%	41.2%	.	16.5%	26.8%	.		
Jul.1994–Jun.95 (51st)	32.8%	40.8%	.	16.5%	27.3%	.		
Jul.1993–Jun.94 (50th)	33.8%	38.7%	35.6%	16.3%	26.1%	18.3%	20.54	28.77

* The data are obtained from (i) Employment and Unemployment Situation in India 2007-08, pp. 39–41, 50 and 52, (ii) Key Indicators of Employment and Unemployment in India 2009–2010, pp. 16, 19 and 20, and (iii) Key Indicators of Employment and Unemployment in India 2011–2012, pp. 18, 15 and 23. All of the reports are published by the National Sample Survey Office (NSSO) of India.

Table 1.2: Total employment by industry and employment status in Tanzania (persons)

Employees	2001	2002	2005	2006	2007	2010	2011	2012
Agriculture								
Regular	45946	65256	37459	47051	47051	55063	44884	49932
Casual	31327	34887	42973	96832	56084	83011	128370	45923
Share (casual)	0.41	0.35	0.53	0.67	0.54	0.60	0.74	0.48
Non-Agriculture								
Regular	649241	611407	770537	764926	851992	964441	1057589	1273801
Casual	92346	214063	173371	176433	205007	174467	131716	180362
Share (casual)	0.12	0.26	0.18	0.19	0.19	0.15	0.11	0.12

Notes: The data are obtained from the Employment and Earnings Survey, Analytical Reports 2001, 2002, 2007, 2011, and 2012, which are published by the National Bureau of Statistics of the United Republic of Tanzania. The employees are defined as all wage earners and salaried employees whether full-time, part-time, or casually for a full working day.

Table 1.3: Descriptive statistics by household-level survey (LSMS-ISA) in Tanzania

	TZA (08-09)	TZA (10-11)	TZA (12-13)
-Number of effective samples (unit: plot)	5126	6038	7447
-Share of subsistence crops cultivated	75.33%	77.14%	71.68%
-Ownership of this plot (owned)	83.26%	84.27%	84.25%
-Did you use any organic fertilizer (yes)	10.66%	10.68%	11.76%
-Did you use any inorganic fertilizer (yes)	10.90%	12.80%	11.20%
-Did you use pesticide/herbicide (yes)	10.75%	9.01%	9.86%
-Average values of inorganic fertilizer (TSH)	62060.46	60312.7	88189.25
-(If used), why choose this type of fertilizer			
(own experience)	67.98%	49.43%	56.15%
(advice from agricultural officer or neighbor)	31.34%	30.90%	31.69%
-Did you receive any seeds, fertilizer, pesticide/herbicide on credit (yes)	1.05%	0.96%	1.59%
-Was area harvested less than planted (yes)	26.53%	31.31%	29.74%
-If yes, the main reason?			
(drought + rain)	51.85%	67.72%	63.13%
(lack of casual labor)	1.23%	0.96%	0.77%
-Did you hire any labor to work on this plot in the long rainy season[1] (yes)	31.14%	27.03%	32.00%
-Casual labor share (by days) if employed any	25.33%	30.75%	29.84%
-Labor days hired for planting[2]	11.6671	11.9608	13.8659
-Labor days hired for weeding	13.4630	13.6967	15.5407
-Labor days hired for fertilizing	NA	5.7641	6.8601
-Labor days hired for harvesting	11.4565	12.4678	13.9121
-Average daily wage (TSH) paid for planting	2405.55	3094.37	3819.35
-Average daily wage (TSH) paid for harvesting	2041.37	2152.74	3000.76
-Labor prod. if using inorganic fertilizer (TSH/day)	4116.25	4320.42	
-Labor prod. if not using inorganic fertilizer (TSH/day)	1930.63	2035.93	

[1] The long rainy season in Tanzania lasts during March to May.
[2] The labor was hired on a daily basis.

Table 1.4: Description of the data by pairwise correlation, years 2010–11 (upper) and 2008–09 (lower)

2010--11									
Variables	s.crop	o.fert	i.fert	p.cide	ir.gate	c.labor	c.ratio	land	l.prodt
s.crop	1.000								
o.fert	-0.010	1.000							
i.fert	0.061**	0.089**	1.000						
p.cide	-0.146**	0.164**	0.267**	1.000					
ir.gate	-0.045**	0.083**	0.102**	0.098**	1.000				
c.labor	0.006	0.000	0.064**	0.124**	0.060**	1.000			
c.ratio	-0.023	-0.016	0.039**	0.099**	0.102**	0.690**	1.000		
land	-0.018	-0.002	0.040**	0.078**	-0.006	0.114**	0.069**	1.000	
l.prodt	-0.074**	0.036*	0.145**	0.156**	0.151**	0.158**	0.230**	0.097**	1.000
2008--09									
Variables	s.crop	o.fert	i.fert	p.cide	ir.gate	c.labor	c.ratio	land	l.prodt
s.crop	1.000								
o.fert	-0.012	1.000							
i.fert	0.033*	0.138**	1.000						
p.cide	-0.148**	0.140**	0.257**	1.000					
ir.gate	-0.056**	0.128**	0.160**	0.181**	1.000				
c.labor	0.009	0.036*	0.066**	0.095**	0.060**	1.000			
c.ratio	0.008	0.056**	0.084**	0.089**	0.107**	0.654**	1.000		
land	-0.007	-0.013	-0.010	0.024	-0.011	0.080**	0.102**	1.000	
l.prodt	-0.075**	0.081**	0.182**	0.201**	0.138**	0.148**	0.259**	0.120**	1.000

1. The data are obtained from the LSMS-ISA of the World Bank. Observations are from the agriculture survey on Tanzania in the year 2010–2011 by merging Tables 2A, 3A, and 4A. The land size of each plot is obtained from Table 2A, the value of output harvested is from Table 4A, and all the other information is from Table 3A.

2. Variables: s.crop = 1 if subsistence crop cultivated (dummy); o.fert = use of organic fertilizer (dummy); i.fert = use of inorganic fertilizer (dummy); p.cide = use of pesticide/herbicide (dummy); ir.gate = irrigation (dummy); c.labor = hired any casual labor during the past production season (dummy); c.ratio = ratio of hours worked by casual labor to hours of total labor; land = size of plot based on GPS measurement; l.prodt = labor productivity measured by value of output harvested divided by total hours worked by labor

Table 1.5: Summary of parameters to be calibrated

Parameter	**Values**	**Target/source**
Production functions		
(α^M, β^M)	(0.83, 0.30)	Based on intermediate input share and labor share from the data (modern-type only)
α^T	0.65	Based on labor shares from the data (traditional-type only)
σ	0.20	Restuccia, Yang, and Zhu (2008)
$(\log\mu_A, \log\sigma_A)$	(2.77, 1.12)	Computed from the data
$(\log\mu_B, \log\sigma_B)$	(-0.35, 1.15)	Computed from the data
Costs of hiring labor		
b	1.65	Calibrated to meet 10% technology adoption rate
λ	0.265	By average labor days provided by each worker (ratio of ex-ante to ex-post)
ξ_1	1%*b	Assumed to be 1% of labor wages
ξ_2	$0.6^*\xi_1$	Calibrated to meet casual labor ratio (ex-post)
Matching function		
$\bar{m}$	1	Standard
ρ	0.5	Standard
$\bar{N}/\bar{z}$	0.0125	Calibrated to meet casual labor ratio (ex-ante)

Calibrated parameters in benchmark: $b = 1.65$, $\bar{N}/\bar{z} = 0.0125$, $\xi_2 = 0.6\xi_1$

Calibration targets: (i) technology type, (ii) permanent labor ratio ex-ante, and (iii) permanent labor ratio ex-post

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A_t')}$									
0.109	0.504	0.815	0.999	0.922	0.863	0.809	0.757	0.704	0.648	0.585	0.509	0.393

Non-calibration targets: labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} / \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.732	1.740	1.746	1.752	1.760	1.768	1.777	1.789	1.807	1.839

Table 1.6: Quantitative exercises

[Panel A] comparative statics 1: $b = 1.65$, $\bar{N}/\bar{z} = 0.0125$, $\xi_2 = 1.2\xi_1$, $A'_t \sim \ln\mathcal{N}(2.77, 1.12)$

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A'_t)}$									
0.238	0.494	0.829	1.000	0.956	0.919	0.883	0.845	0.803	0.756	0.700	0.627	0.503

Labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} \Big/ \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.708	1.712	1.717	1.721	1.726	1.732	1.739	1.749	1.763	1.793

[Panel B] comparative statics 2: $b = 1.65$, $\bar{N}/\bar{z} = 0.00125$, $\xi_2 = 0.6\xi_1$, $A'_t \sim \ln\mathcal{N}(2.77, 1.12)$

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A'_t)}$									
0.155	0.691	1.206	1.000	0.981	0.963	0.945	0.924	0.899	0.869	0.830	0.773	0.661

Labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} \Big/ \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.757	1.759	1.761	1.764	1.766	1.770	1.774	1.780	1.789	1.809

[Panel C] comparative statics 3: $b = 1.65$, $\bar{N}/\bar{z} = 0.0125$, $\xi_2 = 0.6\xi_1$, $A'_t \sim \ln\mathcal{N}(2.42, 1.40)$

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A'_t)}$									
0.072	0.507	0.799	0.999	0.894	0.820	0.757	0.698	0.641	0.582	0.519	0.445	0.337

Labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} \Big/ \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.746	1.755	1.762	1.770	1.779	1.788	1.799	1.812	1.831	1.864

[Panel D] comparative statics 4: $b = 1.65$, $\bar{N}/\bar{z} = 0.0125$, $\xi_2 = 0.6\xi_1$, $A'_t \sim \ln\mathcal{N}(3.15, 0.70)$

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A'_t)}$									
0.168	0.500	0.826	0.999	0.944	0.899	0.856	0.812	0.765	0.714	0.654	0.578	0.456

Labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} \Big/ \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.719	1.724	1.729	1.735	1.740	1.747	1.755	1.766	1.782	1.813

[Panel E] comparative statics 5: $b = 1.65$, $\bar{N}/\bar{z} = 0.0125$, $\xi_2 = 1.2\xi_1$, $A'_t \sim \ln\mathcal{N}(3.15, 0.70)$

π^M	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{1M}}$	$\frac{n_t^{PT}}{n_t^{PT}+n_t^{1T}}$	$\frac{n_t^{PM}}{n_t^{PM}+n_t^{2M}(A'_t)}$									
0.332	0.491	0.839	0.999	0.969	0.942	0.914	0.884	0.850	0.810	0.761	0.692	0.570

Labor productivity gaps between modern and traditional technologies

$\frac{y_t^M}{\lambda(n_t^{PM}+n_t^{1M})+(1-\lambda)(n_t^{PM}+n_t^{2M})} \Big/ \frac{y_t^T}{\lambda(n_t^{PT}+n_t^{1T})+(1-\lambda)(n_t^{PT}+n_t^{2T})}$									
1.697	1.701	1.704	1.707	1.711	1.716	1.722	1.730	1.742	1.769

Note: All computations are executed 500 times and each time 500 samples are drawn from the distribution $B_i \sim \ln\mathcal{N}(-0.35, 1.15)$

1.8 References

1. **Adamopoulos, Tasso** (2011). "Transportation Costs, Agricultural Productivity, and Cross-Country Income Differences." *International Economic Review*, 52 (2), 489–521.

2. **Adamopoulos, Tasso and Diego Restuccia** (2014a). "The Size Distribution of Farms and International Productivity Differences." *American Economic Review*, 104 (6), 1667–97.

3. **Adamopoulos, Tasso and Diego Restuccia** (2014b). "Land Reform and Productivity: A Quantitative Analysis with Micro Data." Unpublished Manuscript, University of Toronto.

4. **Alston, Julian M. and Philip G. Pardey** (2014). "Agriculture in the Global Economy." *Journal of Economic Perspectives*, 28 (1), 121–46.

5. **Behrman, Jere R.** (1999). "Labor Markets in Developing Countries." In O. Ashenfelter, and D. Card (eds.) *Handbook of Labor Economics*, Vol. 3B. Elsevier Science B. V., Amsterdam, 2859–2939.

6. **Caselli, Francesco** (2005). "Accounting for Cross-Country Income Differences." In Aghion P. and S. Durlauf (eds.) *Handbook of Economic Growth*, Vol. 1A. North Holland, New York, 679–741.

7. **Conley, Timothy G. and Christopher R. Udry** (2010). "Learning about a New Technology: Pineapple in Ghana." *American Economic Review*, 100 (1), 35–69.

8. **Dercon, Stefan and Luc Christiaensen** (2011). "Consumption Risk, Technology Adoption and Poverty Traps: Evidence from Ethiopia." *Journal of Development Economics*, 96 (2), 159–73.

9. **Donovan, Kevin** (2014). "Agricultural Risk, Intermediate Inputs, and Cross-Country Productivity Differences." Unpublished Manuscript, University of Notre Dame.

10. **Duflo, Esther, Michael Kremer, and Jonathan Robinson** (2011). "Nudging Farmers To Use Fertilizer: Theory and Experimental Evidence from Kenya." *American Economic Review*, 101 (6), 2350–90.

11. **Feder, Gershon** (1980). "Farm Size, Risk Aversion and the Adoption of New Technology under Uncertainty." *Oxford Economic Papers*, 32 (2), 263–83.

12. **Fisher, Dennis U. and Ronald D. Knutson** (2013). "Uniqueness of Agricultural Labor Markets." *American Journal of Agricultural Economics*, 95 (2), 463–69.

13. **Foster, Andrew D. and Mark R. Rosenzweig** (2010). "Microeconomics of Technology Adoption." *Annual Review of Economics*, 2, 395–424.

14. **Gollin, Douglas, David Lagakos, and Michael Waugh** (2014a). "The Agricultural Productivity Gap." *Quarterly Journal of Economics*, 129 (2), 939–93.

15. **Gollin, Douglas, David Lagakos, and Michael Waugh** (2014b). "Agricultural Productivity Differences across Countries." *American Economic Review: Papers & Proceedings*, 104 (5), 165–70.

16. **Hsieh, Chang-Tai and Peter J. Klenow** (2009). "Misallocation and Manufacturing TFP in China and India." *Quarterly Journal of Economics*, 124 (4), 1403–48.

17. **ILO** (2012). *Decent Work Indicators in Africa: A first assessment based on national sources*, Geneva, ILO.

18. **ILO** (2013). *Key Indicators of the Labour Market (KILM)*, Seventh Edition, 2013.

19. **Lagakos, David and Michael E. Waugh** (2013). "Selection, Agriculture, and Cross-Country Productivity Differences." *American Economic Review*, 103 (2), 948–80.

20. **Lewis, W. Arthur** (1954). "Economic Development with Unlimited Supplies of Labour." *Manchester School*, 22 (2), 139–91.

21. **Manuelli, Rodolfo E. and Ananth Seshadri** (2014). "Frictionless Technology Diffusion: The Case of Tractors." *American Economic Review*, 104 (4), 1368–91.

22. **Newbery, David M. G.** (1977). "Risk Sharing, Sharecropping and Uncertain Labour Markets." *Review of Economic Studies*, 44 (3), 585–94.

23. **Restuccia, Diego, Dennis T. Yang, and Xiaodong Zhu** (2008). "Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis." *Journal of Monetary Economics*, 55 (2), 234–50.

24. **Rose, Elaina** (2001). "Ex Ante and Ex Post Labor Supply Response to Risk in a Low-income Area." *Journal of Development Economics*, 64 (2), 371–88.

25. **Rosenzweig, Mark R.** (1988). "Labor Markets in Low-income Countries." In Chenery, H. and T. N. Srinivasan, (eds.) *Handbook of Development Economics*, Vol. 1. North Holland, New York, 713–62.

26. **Rosenzweig, Mark R. and Christopher R. Udry** (2014). "Rainfall Forecasts, Weather, and Wages over the Agricultural Production Cycle." *American Economic Review: Papers & Proceedings*, 104 (5), 278–83.

27. **Suri, Tavneet** (2011). "Selection and Comparative Advantage in Technology Adoption." *Econometrica*, 79 (1), 159–209.

28. **Yang, Dennis T. and Xiaodong Zhu** (2013). "Modernization of Agriculture and Long-term Growth." *Journal of Monetary Economics*, 60 (3), 367–82.

Chapter 2

Growth in a Patrilocal Economy: Female Schooling, Household Savings, and China's One-Child Policy

2.1 Introduction

The rise of education attainment of females relative to males is widely observed in both developed and emerging countries.[1] The shrinking gender gap in education opportunity is prominent in the case of China. In this chapter, we develop an overlapping generation (OLG) model in order to understand the interactions between fertility, savings, and gender difference in education. An important application of the model is to illustrate the effects of a population regulation, i.e., the China's "One-Child Policy" (OCP), on female-to-male schooling ratio as well as the propensity for households to save for retirement. To our knowledge, this chapter is the first to connect the rise of China's savings with the gender difference on relative changes of human capital under the OCP. The motivation is that raising children (and hence investing their education) is taken as an important means of savings, in addition to other possible savings instruments.

[1] Please refer to Becker et al. (2010), Rosenzweig and Zhang (2013), Orazem and King (2007), among others. In Section 2.1, we also refer to the gender parity index (GPI) provided by UNESCO as a measure of the gender gap.

This chapter is motivated by four stylized facts in China after the implementation of the OCP in late 1970s. Firstly, the cohorts who were born under the OCP have a rising trend of female-to-male enrollment in high school. Such a trend is even more dramatic when we take into account a growing imbalance of sex ratio in these cohorts. Secondly, when we break down the data of school enrollment into family sizes, there is an inverse relationship between the number of children a family has and the relative chances of a young woman attending schools compared with a young man. Thirdly, during the same period, the rates of return to women's schooling are persistently higher than men's (Zhang et al., 2005), suggesting a differential of parents' incentive to invest in their daughters' and sons' education, or a "son preference." Finally, we draw conclusions from a household survey on their reasons to save to show the origin of the son preference: the more sons a household has, the less need for the household to save for retirement.

To highlight our channels, we model parental transfers as parents' investment in children's human capital through schooling, and we model filial transfers as financial support provided by sons after retirement. We assume that sons are more likely to provide their parents old-age support (Banerjee et al., 2014; Zhou, 2014). It is consistent with a patrilocal social norm which requires a married couple to live with or close to the husband's parents. Therefore, sons' support is a substitute for the parent's savings, and a household might choose to invest more in sons' education so that sons could provide more support when their parents are old. Parental investment in education of their children, therefore, depends on the degree of parental altruism and the need for old-age security.

The degree of parental altruism is affected by the household's fertility decisions via the quantity-quality trade-off *à la* Barro and Becker (1989). Lower fertility induces higher parental altruistic behavior toward children and hence benefits both sons and daughters. Nevertheless, diminishing marginal returns in schooling implies that sons' education becomes a less efficient means of providing old-age support. Thus the son preference is weakened. In addition, if there is a sex division of labor, women's human capital depreciates more than men's after reproduction. Therefore, expecting that daughters will also have fewer offspring, altruistic parents will invest more in daughters' education. Both effects tend to increase the relative female schooling. Nevertheless, the gap in the rate of return to schooling persists due to the additional benefits for parents to increase sons' earnings.

Several authors have documented gender difference of schooling in China. For example, the gender-based difference in rate of return to schooling is examined by Zhang et al. (2005) and Chen and Hamori (2009) by respectively using data from China Urban Household Surveys (UHS) and China Health and Nutrition Surveys (CHNS).[2] Their estimation results based on Mincerian wage regressions reconcile the findings from Psacharopoulos (1994) in most of developing countries' cases. The simultaneous rise in both female relative schooling years and returns to female schooling is also noted by Pitt, Rosenzweig, and Hassan (2012). In order to incorporate the estimated marginal effect of schooling on wage income based on micro evidence and thereby model the gender difference, we adopt the setting of individual human capital by Bils and Klenow (2000) into the production function.

Our model could also explain the change of households' demand on savings. There are two opposing forces that affect household savings. Firstly, parents need to save more for old age security when they have fewer sons. Secondly, higher parental altruism puts more weight on children's welfare over old age consumption, and therefore reduces the need for saving for retirement. Assuming that the elasticity of parental altruism with respect to number of children is not too large, we see household savings increase with lower fertility as observed in the data.[3]

[2]Underlying forces that drive the prominent difference, argued by Rosenzweig and Zhang (2013) and Zhang et al. (2005), are females' comparative advantage on skilled work and a self-selection effect.

[3]To keep our model tractable, we do not consider other reasons to save, such as a bequest motive. Incorporating this motive will further increase the saving demand and make it easier to match the data.

The ascending savings in China has been documented by Yang, Zhang, and Zhou (2011), which is characterized by persistently high saving rate and its strikingly upward pattern since economic reforms began in 1978. This dramatic increase in domestic savings is distinct to other developed and emerging economies especially for years 2000 to 2008. Possible explanations put forward by the literature include precautionary motives, habit formation, and reasons related to demographic changes resulted from the OCP. However, when it comes to the most reliable old-age income support, the answer is mostly children instead of household savings or pensions.[4]

This chapter is therefore closely related to a growing literature that connects the relation between household savings (or capital accumulation) and demographic changes.[5] Modigliani and Cao (2004) suggest that changes in China's demographic structure started since the implement of population-related policies contributes to the rising saving rate in China. Among these studies, the one closest to ours is done by Banerjee et al. (2014), which also analyzes the general equilibrium effect of China's OCP on household savings. They consider a counterfactual removal of the OCP and show that the partial equilibrium effect estimated from micro data is offset by the increase in interest rates. That is, relaxation of population control policy may not lead to substantially descending of household savings while taking the rise of population and hence the rise of interest rate into account. In contrast, our model suggests that the rise of human capital with population control could be substantial, and that the general-equilibrium effect may amplify instead of dampening the saving motive under the OCP.

The rest of this chapter proceeds as follows. Section 2.2 presents empirical findings from aggregate data as well as household surveys. Section 2.3 presents an OLG general equilibrium model and points to an

[4]The argument is based on the survey question from China Health and Retirement Longitudinal Study (CHARLS) 2011-12 baseline survey, in which 66% and 21% of cross-region participants answer children and pensions.

[5]Please refer to Boldrin and Jones (2002), Banerjee, Meng, and Qian (2010), Choukhmane, Coeurdacier, and Jin (2013), Curtis, Lugauer, and Mark (2015), and Ge, Yang, and Zhang (2012), for example.

additional channel for forward looking parents. Section 2.5 concludes and outlines future work.

2.2 Motivating facts

The data used in this chapter are obtained mainly from two sources. We first summarize the stylized facts regarding female secondary education based on the statistics from China Education Statistical Yearbooks (CESY, 1973-2012). In order to highlight our points, micro evidence regarding households decisions on education is mostly derived from the Chinese House Income Project of 2002 (CHIP-2002, hereafter).

Why the case of China is prominent? The case is noteworthy not just because of its influential demographic policy but also because its equality of education opportunity is improved rapidly in the context of a cross-country comparison. In order to measure the equality of education opportunity, we apply the GPI provided by UNESCO and the index is constructed by using gross enrollment ratio (GER) of females relative to males. The most advantage of this indicator is that it not only delivers the female-to-male enrollment ratio in a static sense but also shows how the dynamics is taken place over time; see Subrahmanian (2005).

In Table 2.1, we focus on the main East Asia and the Pacific countries (with population 2 million and above) and compare the changes of GPI across countries over the same period; that is around year 1995–2010. The GPI of China increases dramatically compared to other countries/regions in the tertiary level. In order to take different initial values into account, we also compute the relative change rates of GER of females relative to males across countries. The second indicator again confirms that females gained access to higher education more than other countries/regions.

2.2.1 Motivations from the aggregate data

[Figure 2.1] shows the series of the percentage of female students enrolled in lower- and uppe - secondary education in the upper panel, while the series of percentage of female new-entrants in the lower panel. Three points of time are marked for noteworthy episodes. The year 1979, which is marked by OCP, is regarded as the starting year of the rigorous implement of the OCP. The year 1986 is the time when China enforced 9-year compulsory education on both men and women and it is denoted by CE9. Finally, the point marked by Y15 represents the year 1994 when the first cohort of teenagers who were born under the OCP reached 15 years old.

The point Y15 is marked in order to underscore the impact of the OCP on secondary education, because it is also the time when the cohort is expected to enter senior high school. The enrollment ratio of female students in lower-secondary education is selected as a benchmark for comparison because after CE9 both boys and girls are equally required to attend junior high schools, suggesting a comprehensive policy effect. As shown in the figure, this ratio increases after CE9 and reaches a long-run rate of 47%, and meanwhile the entrance ratio of female students exhibits a consistent pattern. On the other hand, the enrollment ratio of female students in upper-secondary education remains stable around 40% before Y15. As suggested by the upper panel of [Figure 2.2], it starts to increase and eventually overtakes males' the enrollment ratio in upper-secondary level around year 2007, implying that females are more likely to enter high schools than males.

In order to present this change, we construct the following index to measure the relative opportunity of accessing higher education between female and male graduates from junior high school in year t; namely,

$$GPI_t = \frac{\text{female new entrants of SHS/female graduates from JHS}}{\text{male new entrants of SHS/male graduates from JHS}},$$

in which SHS and JHS are abbreviations of upper and lower secondary schools, respectively. This index is informative because compared to the previous ratios it additionally reduces the influence of the changing sex ratio over time. It is comparable to the GPI though a mild difference is that our index uses the number of current-year graduates from junior high schools as a proxy to the number of *potential* students.[6] As shown in the lower panel of [Figure 2.2], the index attains the minimum value around 1997 and after that it grows. That is, the rising trend of women' secondary education becomes more striking when taking the imbalanced sex ratio into account.

One might suspect that this trend is due to a change in the rate of return to schooling between men and women. However, as argued by Psacharopoulos and Patrinos (2004) and Pitt, Rosenzweig, and Hassan (2012), women typically have a dominating rate of return to schooling in developing countries. In addition, [Figure 2.3] presents the estimated returns to years of schooling by gender in China based on Zhang et al. (2005) and it shows females have persistently higher returns than males during the period 1988–2001.[7] Since women always have a persistently higher return to schooling in China, here a question arises due to the change of relative education opportunity. We argue that it could be a result of the change in long-standing "son preference" in China's society, and the OCP is one of the key drivers.

Another noteworthy feature that is related to the change in female schooling is the rise in household saving rates in recent decades. The reason is that Chinese parents are used to relying heavily on children, especially sons, for providing old-age care of themselves. The convention contributes a close connection between the rate of return on physical capital and on human capital. [Figure 2.4] presents the gross household saving rates from Flow of Funds Accounts in the China Statistical Yearbook (CSY, 2012) and from World Development Indicators (WDI) in the upper panel as well as household saving rates by using the data from China Urban and Rural Household Survey in the lower panel. Note that, though there exist some discrepancies between different measures, the household saving rates are continuously increasing after 1990. [8] Moreover, the series

[6] See the definition by `http://www.unicef.org/infobycountry/stats_popup5.html`

[7] Chen and Hamori (2009) have similar findings by using CHNS data of years 2004-2006 after controlling potential selection bias.

[8] The discrepancies occur perhaps due to differences on the definition of saving rates to be used. For example, household saving rates in urban and rural sectors are computed by (1 - Annual Per Capita Consumption Expenditure/Annual Per Capita Disposable Income)*100% and (1 - Annual Per Capita Net Income/Annual Per Capita Living Expenditure)*100%, respectively.

of saving rates of urban households displays a significantly upward pattern whereas rural-household savings fluctuate a lot over time.

2.2.2 Implications and evidence from CHIP household surveys

[Table 2.2] summarizes the most and second-most important purposes of saving for rural households from a survey data in the CHIP-2002. This survey question has been studied by Wei and Zhang (2011), but the motive to save that we put emphasis here is *preparing for elderly life after retirement*. We begin by comparing three-person households, and they are mostly have a single daughter or son. There are 46 percent of the households with a single daughter that choose preparing for retirement as an important incentive to save, in contrast to 37% of the households with the only son. A similar result is also observed in samples of four-person households since the percentage of saving for retirement is decreasing in the number of sons, from 46% to 23%. In addition, households with one or two daughters are more likely to rank preparing for sickness as their main concern, because the ratio is 6-7% relatively higher than the counterparts with the same number of sons. The results confirm our primary conjecture that gender of the child should be considered regarding the motives to save. The statistics also suggests that sons are expected to provide financial supports to retired parents more than daughters.[9]

In addition to the information regarding the motives to save, the CHIP-2002 survey also provides suggestive evidence to validate our argument that the fertility constraint due to the OCP has influence on household's gender-based education decisions. [Table 2.3] outlines the composition of households in urban and rural areas. Nearly 70% of Households in urban area are classified into the 3-persons category, and they are mostly composed of two parents and one child. Another 7.9% of households have multiple children, and

[9] One of possible conjectures to explain this significant difference is existence of *patrilocality*, or the custom of residence with husband's relatives. [Figure 2.5] depicts main reasons to migration by gender from the China Population Census 2005, and it is observed that females are more likely to migrate for marriage and the ratio is about 10% higher than males.

the number of children for most of households is not greater than two. Meanwhile, it is observed that fertility constraint is less restrictive in rural areas since 54.5% of households have 2 children or more. The comparison between households' decisions in urban and rural area is informative because it is used to understand the changes of households' concern under different tightness of fertility constraint in some sense.

In [Table 2.4], all of the individuals in the CHIP-2002 with identity "child" and ages between 16 to 22 are sorted in terms of divisions by region, gender, and the size of households that they were born. We select the samples only from children at age 16–22 because they are supposed to study senior high school (or above) at that time. Ratios of non-schooling with respect to each division are correspondingly tabulated. The ratio is defined by the fraction of children who are not currently full-time students and it is used to capture the aggregate enrollment/non-enrollment ratio of female to male students on a comparable basis. Note that we abstract from the potential issue of sibling composition effect and focus on the effect of number of children on schooling decisions, which will be highlighted in our model.

The table shows that the rate of non-schooling for both male and female children is increasing in the number of children that each household has. For example, there are 60.7% female and 60.4% male children at age 16–22 under non-schooling status from households with a single child in rural area, and the rates respectively increase by 14.3% and 9.3% for those who were born in households with more than 3 children. In contrast, the non-schooling rate of female children in urban area does not have a relative increase with the rise of number of children compared to the rate of males. The change of values may justify our suspect that females are likely to obtain educational resources under tighter fertility constraint.[10]

[10] Note that Li, Zhang, and Zhu (2008) also document the similar findings by using 1% sample of China Population Census 1999. Similar to our observations based on CHIP-2002, their study shows that female children, in terms of different family sizes, consistently have lower rate of receiving secondary education and above than males in rural area, but the differences are not observed simultaneously in urban area.

2.3 The Model

The dynamics of population Time is discrete and goes to infinite, $t = 0, 1, 2, \cdots$. The economy is populated by agents who live for 3 periods. In addition, each age cohort is evenly distributed by male and female agents. We denote the state of age structure at the beginning of time t as (L_{t-2}, L_{t-1}), where L_{t-k} is the number of male (female) agents who were born at the beginning of period $t-k$, $k = 1, 2$. For convenience, we call the people who were born at period $t-1$ the young generation at time t, and the people who were born at period $t-2$ the old generation at time t. In addition, individuals who were born during period t are called the newborn at time t.

In each period, each male agent in the young generation marries a female agent in the same generation. Couples married at time t become time-t representative households. We model the fertility decision of the representative household as a reproduction rate $n_t \in [0, \bar{n}]$, $\bar{n} < \infty$. Note that $\bar{n}$ represents a population control policy imposed by the government. If the government does not impose any restrictions on reproduction, we set $\bar{n} = n_b$, where n_b is the biological limit of reproduction. At the end of each period, the newborn turns into the young generation, the young turn into old, and the old generation dies. Hence, the number of the young-generation agents evolves according to

$$L_t = n_t L_{t-1}. \tag{2.1}$$

We say that the population is along the balanced growth path if $n_t = n$ is constant over time. In such case,

$$L_t = L_0 n^t, \tag{2.2}$$

for some $n \geq 0$.

Production technologies There is a final output Y_t that requires physical and human capital as inputs and can be consumed by households or turned into investment. We denote K_t to be the stock of physical capital at time t, and $h_{i,t}$ to be the stock of human capital for male and female agents at time t, where $i = m$ for male agents and $i = f$ for females.

We make an usual assumption about the law of motion for physical capital:

$$K_{t+1} = (1 - \delta)K_t + I_t, \tag{2.3}$$

where δ is the depreciation rate for physical capital, and I_t investment at time t.

For human capital, we assume that a young agent's human capital is measured by the amount of schooling. A male agent provides ${h_{m,t}}^{\gamma}$ efficient units of labor when they are young, and a female agent provides $(1 - b(n_t))z{h_{f,t}}^{\gamma}$ efficient units of labor, where $z > 0$ measures the gender productivity difference between a woman with no children and a man given that they have the same level of education. We also assume that reproduction reduces a female's human capital, therefore $b'(n_t) > 0$. This assumption is to capture that wives specialize in reproduction activities. We also assume that $b(n_b) < 1$, so that female productivity is non-negative within their fertility limit.

The production of final goods is constant return to scale, with a constant labor-augmented technology growth rate g. That is,

$$Y_t = F(K_t, g^t H_t), \tag{2.4}$$

where the supply of human capital into production of consumption goods is

$$H_t = L_{t-1}\left({h_{m,t}}^{\gamma} + (1 - b(n_t))z{h_{f,t}}^{\gamma}\right) - \phi L_t\left(h_{m,t+1} + h_{f,t+1}\right). \tag{2.5}$$

That is, the total amount of schooling demand is $L_t\left(h_{m,t+1}+h_{f,t+1}\right)$, each unit of schooling requires ϕ units of efficient labor, and the rest of labor will be put into production of final goods.

Feasible allocations and aggregate state transitions The aggregate state at the beginning of time t is $(L_{t-2}, L_{t-1}, K_t, h_{m,t}, h_{f,t})$. A resource allocation at time t is a consumption plan for the young generation and the old, (c_t^y, c_t^o), investment I_t, reproduction n_t, and education for the next generation $(h_{m,t+1}, h_{f,t+1})$.

For a closed economy, given initial state $(L_{-2}, L_{-1}, K_0, h_{m,0}, h_{f,0})$, we say a sequence of allocations $\{c_t^y, c_t^o, I_t, n_t, h_{m,t+1}, h_{f,t+1}\}_{t=0}^{\infty}$ is *feasible* if given the allocations, the transition of aggregate state

$$(L_{t-2}, L_{t-1}, K_t, h_{m,t}, h_{f,t}) \xrightarrow{c_t^y, c_t^o, I_t,, n_t, h_{m,t+1}, h_{f,t+1}} (L_{t-1}, L_t, K_{t+1}, h_{m,t+1}, h_{f,t+1}), \tag{2.6}$$

evolves according to equations (2.1), (2.3), and (2.5), and the allocations satisfy the feasibility constraint

$$Y_t \geq c_t^o L_{t-2} + c_t^y L_{t-1} + I_t \tag{2.7}$$

for $t = 0, 1, \cdots$.

Preferences and parental altruism We assume that the preference of a time-T household is defined over allocations $\{c_t^y, c_t^o, n_t, h_{m,t+1}, h_{f,t+1}\}_{t=T}^{\infty}$. Note that the history of allocations and the physical capital investment in the current period do not directly affect household preference. We denote $\mathbf{c^T} = (c_T^y, c_{T+1}^o)$ be the consumption path over the life cycle of the time-T household, and $\mathbf{h^T} = (h_{m,T+1}, h_{f,T+1})$ be the education level that the time-T household chooses for their children. We say that the household is *non-altruistic* if their preference only depends on $(\mathbf{c^T}, \mathbf{h^T})$. We say that the household exhibits *parental altruism* if their preference also depends on $(\mathbf{c^t}, \mathbf{h^t})$ for $t > T$. As we shall see clear in the following, what really distinguishes these two types of preferences is that in a decentralized economy, a household who has parental altruism cares about the actions taken by future generations, while a non-altruistic household concerns only actions taken by themselves.

Since one of the main object of the dynamic setting is to explore parental altruistic behavior, we simplify the analysis by assuming that households care about their children's education to the extent which it affects future generations' welfare. That is, a time-T household's utility function is defined over $\{\mathbf{c^t}\}_{t=T}^{\infty}$. We further assume that $U^*(\{\mathbf{c^t}\}_{t=T}^{\infty})$ is homogeneous of degree $1-\rho$ and additively separable, and $\psi(n_t)$ is the weight of parental altruism toward each child.[11]

Assumption 1.

$$U^*(\{\mathbf{c^t}\}_{t=T}^{\infty}) = u^*(\mathbf{c^T}) + \sum_{t=T}^{\infty}\left(\prod_{t'=T}^{t} n_{t'}\psi(n_{t'})\right)u^*(\mathbf{c^{t+1}}), \tag{2.8}$$

where $\psi(n)$ is a decreasing function of n, $n\psi(n) < 1$ for all $n \leq n_b$, and

$$u^*(\mathbf{c^t}) = \frac{(c_t^y)^{1-\rho}}{1-\rho} + \beta\frac{(c_{t+1}^o)^{1-\rho}}{1-\rho}, \tag{2.9}$$

for $0 < \rho < 1$ and $\beta > 0$.

Decentralized markets We want to solve the allocation over time in a decentralized economy. Assume that agents before adulthood have no access to capital markets and need their parents to provide their education expenses. Production technologies are run by competitive firms, who hire labor and rent capital from households. Denote r_t to be the interest rate and $g^t w_t$ to be the wage rate per efficient unit of labor. Assume that the goods market is fully competitive. Define $k_t = \frac{K_t}{H_t}$ to be the physical-human capital ratio. The interest rate at time-t is pinned down by the marginal product of physical capital at time-$t+1$ less

[11] See the setting by Becker et al. (1990).

depreciation,

$$r_t = F_1(g^{-t-1}k_{t+1}, 1) - \delta, \tag{2.10}$$

and the wage rate at time-t is pinned down by the marginal product of human capital at time-t; therefore,

$$w_t = F_2(g^{-t}k_t, 1). \tag{2.11}$$

Patrilocality and savings for old-age security We assume that postmarital residence to be *patrilocal*, which means that a married couple lives with or near to the husband's parents. Effectively, we assume that female adults cannot support her parents, while male adults are obliged by social norms.We also assume that the economy is dictated by a social norm that male adults should support their parents' consumption after retirement with a fraction τ of the male's earnings. Therefore, the consumption of a young generation household at time-t is determined by household income subtracting financial transfers, education expenses, and savings

$$c_t^y = g^t w_t \left[(1-\tau){h_{m,t}}^{\gamma} + (1-b(n_t))z{h_{f,t}}^{\gamma} - \phi n_t(h_{m,t+1} + h_{f,t+1})\right] - a_t. \tag{2.12}$$

Household saves for their old-age security, which is partially supported by their adult sons. Therefore,

$$c_{t+1}^o = (1+r_t)a_t + \tau n_t g^{t+1} w_{t+1}{h_{m,t+1}}^{\gamma}. \tag{2.13}$$

We can combine the above expressions to derive the life-time budget constraint of a time-t young household. That is,

$$c_t^y + \frac{c_{t+1}^o}{1+r_t} = g^t w_t \left[(1-\tau){h_{m,t}}^{\gamma} + (1-b(n_t))z{h_{f,t}}^{\gamma} - \phi n_t(h_{m,t+1} + h_{f,t+1})\right] + \frac{\tau n_t g^{t+1} w_{t+1} {h_{m,t+1}}^{\gamma}}{1+r_t}. \tag{2.14}$$

Also note that all physical capital is held by the old generation, and therefore we have

$$K_t = a_{t-1}L_{t-2}, \tag{2.15}$$

for $t = 0, 1, \cdots$.

Household decision problem Under Assumption 1, the household cares about the welfare of future generations, and it could affect the decisions of their offspring by providing education to their children. Anticipating that their offspring will allocate resources optimally by their preferences, the household's decision problem can be formulated recursively as follows.

$$V_t(h_{m,t}, h_{f,t}) = \max_{c_t^y, c_{t+1}^o, h_{m,t+1}, h_{f,t+1}, n_t} \frac{(c_t^y)^{1-\rho}}{1-\rho} + \beta \frac{(c_{t+1}^o)^{1-\rho}}{1-\rho} + n_t\psi(n_t)V_{t+1}(h_{m,t+1}, h_{f,t+1}) \tag{2.16}$$

subject to the life-time budget constraint

$$c_t^y + \frac{c_{t+1}^o}{1+r_t} = g^t w_t \left[(1-\tau){h_{m,t}}^{\gamma} + (1-b(n_t))z{h_{f,t}}^{\gamma} - \phi n_t(h_{m,t+1} + h_{f,t+1})\right] + \frac{\tau n_t g^{t+1} w_{t+1} {h_{m,t+1}}^{\gamma}}{1+r_t} \tag{2.17}$$

given that $n_t \leq \bar{n}$. Note that

$$g^t w_t \left[(1-\tau){h_{m,t}}^{\gamma} + (1-b(n_t))z{h_{f,t}}^{\gamma}\right] \tag{2.18}$$

is the household wage income minus the transfer to the parents,

$$g^t w_t \phi n_t (h_{m,t+1} + h_{f,t+1}) \tag{2.19}$$

is the education expenditure on their children, and

$$\frac{\tau n_t g^{t+1} w_{t+1} {h_{m,t+1}}^{\gamma}}{1 + r_t} \tag{2.20}$$

is the discounted value of old-age financial support from their male children.

Based on the first order conditions, we have

$$n_t \psi(n_t) \frac{g^{t+1} w_{t+1} (1-\tau) \gamma {h_{m,t+1}}^{\gamma-1}}{(c^y_{t+1})^{\rho}} + \frac{n_t}{(c^y_t)^{\rho}} \frac{\tau g^{t+1} w_{t+1} \gamma {h_{m,t+1}}^{\gamma-1}}{1 + r_t} = \frac{n_t}{(c^y_t)^{\rho}} g^t w_t \phi, \tag{2.21}$$

$$n_t \psi(n_t) \frac{g^{t+1} w_{t+1} (1 - b(n_{t+1})) z \gamma {h_{f,t+1}}^{\gamma-1}}{(c^y_{t+1})^{\rho}} = \frac{n_t}{(c^y_t)^{\rho}} g^t w_t \phi, \tag{2.22}$$

and

$$V_{t+1}(h_{m,t+1}, h_{f,t+1}) \geq \frac{g^t w_t \left[\phi(h_{m,t+1} + h_{f,t+1}) + b'(n_t) z {h_{f,t}}^{\gamma}\right] - \frac{\tau {g^{t+1}} w_{t+1} {h_{m,t+1}}^{\gamma}}{1+r_t}}{(\psi(n_t) + n_t \psi'(n_t)) \, (c^y_t)^{\rho}}, \tag{2.23}$$

with equality holds if $n_t < \bar{n}$.

The first-order conditions of consumption imply

$$c^y_t = \frac{1}{1 + \beta^{\frac{1}{\rho}} (1 + r_t)^{\frac{1}{\rho} - 1}} C_t, \tag{2.24}$$

and

$$c_{t+1}^o = \frac{\beta^{\frac{1}{\rho}}(1+r_t)^{\frac{1}{\rho}-1}}{1+\beta^{\frac{1}{\rho}}(1+r_t)^{\frac{1}{\rho}-1}}(1+r_t)C_t, \tag{2.25}$$

where C_t is the present value of the expenditures on consumption; namely,

$$C_t = g^t w_t \left[(1-\tau){h_{m,t}}^\gamma + (1-b(n_t))z{h_{f,t}}^\gamma - \phi n_t(h_{m,t+1}+h_{f,t+1})\right] + \frac{\tau n_t g^{t+1} w_{t+1}{h_{m,t+1}}^\gamma}{1+r_t}. \tag{2.26}$$

The household asset holdings can be derived based on equation (2.13)

$$a_t = \frac{c_{t+1}^o}{1+r_t} - \frac{\tau n_t g^{t+1} w_{t+1}{h_{m,t+1}}^\gamma}{1+r_t}. \tag{2.27}$$

Definition of competitive equilibrium Given an initial state $(L_{-2}, L_{-1}, a_{-1}, h_{m,0}, h_{f,0})$, a competitive equilibrium is a sequence of allocations $\{c_t^y, c_t^o, a_t, n_t, h_{m,t+1}, h_{f,t+1}\}_{t=0}^\infty$ and prices $\{g^t w_t, r_{t-1}\}_{t=0}^\infty$ such that

i) Given $\{g^t w_t, r_{t-1}\}_{t=0}^\infty$, $\{c_t^y, c_{t+1}^o, a_t, n_t, h_{m,t+1}, h_{f,t+1}\}_{t=0}^\infty$ satisfies equations (2.21)-(2.25) and (2.27), given

$$c_0^o = (1+r_{-1})a_{-1} + \tau\frac{L_{-1}}{L_{-2}}w_0{h_{m,0}}^\gamma; \tag{2.28}$$

ii) Population L_t evolves according to equation (2.1), aggregate physical capital K_t evolves according to equation (2.3), and aggregate human capital in production H_t evolves according to equation (2.5);

iii) Prices $\{g^t w_t, r_{t-1}\}_{t=0}^\infty$ evolves according to the conditions (2.10) and (2.11).

Education levels on the balanced growth path Assume that the economy is on its balanced growth path and that $r_t = r$, $w_t = w$. Assume that household's fertility and education choices are constant over time. That is,

$$h_{m,t+1} = h_m$$

$$h_{f,t+1} = h_f$$

$$n_t = n.$$

The following lemma can be proven by standard arguments.

Lemma 1. $V_t(h_m, h_f) = g^{(1-\rho)t} V_0(h_m, h_f)$.

Notice that from equation (2.23) on the balanced growth path the fertility decision satisfies

$$(\psi(n) + n\psi'(n))\, g^{(1-\rho)} \frac{\left[\frac{(c_0^y)^{1-\rho}}{1-\rho} + \beta \frac{(c_1^o)^{1-\rho}}{1-\rho}\right]}{1 - n\psi(n)g^{1-\rho}} \geq \frac{w}{(c_0^y)^{\rho}} \left[\phi(h_m + h_f) + b'(n)z{h_f}^{\gamma} - \frac{\tau g {h_m}^{\gamma}}{1+r}\right], \tag{2.29}$$

and a population control policy is strictly binding if the above condition holds with strictly inequality with $n = \bar{n}$. Based on equations (2.21) and (2.22), we know that on the balanced growth path,

$$\phi = \left[(1-\tau)\psi(n)g^{1-\rho} + \tau \frac{g}{1+r}\right] \gamma {h_m}^{\gamma-1}, \tag{2.30}$$

and

$$\phi = (1 - b(n))z\psi(n)g^{1-\rho}\gamma {h_f}^{\gamma-1}. \tag{2.31}$$

Note that both of the $(1 - b(n))$ (females' labor supply) and the $\frac{\psi(n)}{n}$ (parental altruism) decrease as n increases. This implies a larger h_f as the binding population control policy restricted n a smaller number. Moreover, from equation (2.30) we also see that h_m increases as n decreases. Moreover, we know that the ratio of rate of return on education

$$\frac{(1-\tau)\gamma {h_m}^{\gamma-1}}{(1-b(n))z\gamma {h_f}^{\gamma-1}} = 1 - \frac{\tau g \gamma {h_{m,t}}^{\gamma-1}}{(1+r)\phi} \tag{2.32}$$

becomes closer to 1 as n decreases. Also,

$$\left(\frac{h_f}{h_m}\right)^{1-\gamma} = \left(1 - \frac{\tau g \gamma {h_m}^{\gamma-1}}{(1+r)\phi}\right) \frac{(1-b(n))z}{1-\tau}, \tag{2.33}$$

which implies that $\frac{h_f}{h_m}$ increases as n decreases, and $h_f > h_m$ if

$$\left(1 - \frac{\tau g \gamma {h_m}^{\gamma-1}}{(1+r)\phi}\right) \frac{(1-b(n))z}{1-\tau} > 1. \tag{2.34}$$

Equation (2.33) delivers another intergenerational incentive for altruistic parents since an additional channel takes effect via the term $\frac{(1-b(n))z}{1-\tau}$. The underlying reason is that education level of daughters is weighed more as they are expected to have fewer children since parents care about the discounted value of their children. Put differently, females will spend more time participating in the labor market when they have fewer children, making daughters' human capital more valuable and their parents' investments worthwhile. As a result, altruistic parents will endogenously adjust toward female schooling. Note also there is no wealth effect for n on the balanced growth path because the marginal rate of substitution between generations remains constant.

Saving rates on the balanced growth path Since $\frac{c_0^y}{c_0^o}$ is pinned down by $\frac{u_{1,t}}{u_{2,t}} = (1+r)$, we have

$$c_t^y = \eta(r) C_0, \tag{2.35}$$

$$c_t^o = (1+r)(1-\eta(r)) C_0, \tag{2.36}$$

where C_0 is the present value of lifetime consumption expenditure,

$$C_0 = w_0 \left[(1-\tau){h_m}^{\gamma} + (1-b(n))z{h_f}^{\gamma} - \phi n(h_m + h_f)\right] + \frac{\tau n g w_0 {h_m}^{\gamma}}{1+r},$$

and

$$\eta(r) = \frac{1}{1 + \beta^{\frac{1}{\rho}} (1+r)^{\frac{1}{\rho}-1}}. \tag{2.37}$$

The asset holdings of the young generation are

$$a_t = (1 - \eta(r))\, C_0 - \frac{\tau n w_{t+1} {h_m}^{\gamma}}{1+r}, \tag{2.38}$$

and the corresponding saving rate is

$$s_t^y = \frac{a_t}{W_t}, \tag{2.39}$$

where $W_t = w_t[(1-\tau){h_m}^{\gamma} + (1-b(n))z{h_f}^{\gamma}]$ is the wage income of the young generation.

The old generation has dividend income ra_{t-1}, and sells all financial assets. Therefore the saving rate of the old is

$$s_t^o = \frac{-(1+r)a_{t-1}}{ra_{t-1}} = \frac{-(1+r)}{r}. \tag{2.40}$$

The aggregate savings is a weighted average of the age-specific saving rates,

$$s_t = \mu_t s_t^y + (1-\mu_t) s_t^o, \tag{2.41}$$

where

$$\mu_t = \frac{L_{t-1}W_t}{L_{t-1}W_t + L_{t-2} r a_{t-1}}, \tag{2.42}$$

and is constant over time on the BGP. In particular, on the BGP $s_t^y = s^y$, and

$$s_t = s = \mu s^y + (1-\mu)\frac{-(1+r)}{r}, \tag{2.43}$$

where

$$\mu = \frac{g}{g + \frac{r}{n}s^y}. \tag{2.44}$$

We summarize the effects of the population control policy on aggregate saving rate via the following channels

- (Income effect) $\bar{n}$ affects s^y since fewer children implies

  - fewer children to provide education;

  - higher household wage income;

  - fewer children to support old-age security. These factors tend to increase s^y;

  but

  - more education for each children. This factor reduces s^y.

- (Ageing population effect) $\bar{n}$ affects μ since

  - lower n produces an aging population, and the old generation dissaves;

  - lower n changes s^y, therefore change the relative wealth between the old and the young generation.

  When $\frac{ds^y}{\bar{n}} < 0$, both effects tend to reduce μ when $\bar{n}$ is smaller.

We calibrate our model to evaluate the net effect of OCP on saving rate and on the children's education.

Physical-human capital ratio and the general equilibrium effects To find the general equilibrium effect, we next solve the equilibrium interest rate and wage as functions of physical-human capital ratio k, and solve k via the market clearing condition. Assume that the aggregate production function is $F(K_t, H_t) = A_0 K_t^{\alpha} H_t^{1-\alpha}$. Based on the equations (2.10) and (2.11), we have

$$\frac{K_t}{H_t} = kg^t, \tag{2.45}$$

$$w = (1-\alpha) A_0 k^{\alpha}, \tag{2.46}$$

$$r = \alpha A_0 k^{\alpha-1} - \delta, \tag{2.47}$$

where $k = \frac{K_0}{H_0}$. Normalize $L_{-2} = 1$. Then $K_0 = a_{-1} = a_0/g$. Market clearing on the balanced growth path implies that $Y_0 = c_0^o L_{-2} + c_0^y L_{-1} + K_1 - (1-\delta)K_0$. Therefore, k is pinned down by

$$A_0 \frac{a_0}{g} k^{\alpha-1} = \frac{c_1^o}{g} + c_0^y n + a_0 n - (1-\delta)\frac{a_0}{g}, \tag{2.48}$$

where n, a_0, c_1^o, c_0^y are functions of k.

2.4 Quantitative Analysis

We calibrate our model to fit the main features of China's economy and quantify the effects of the implementation of the "One-Child" policy on main variables of interest by comparing their changes under the two policy regimes.

2.4.1 Calibration

Our calibration strategy is firstly to characterize the features of China's economy at two steady states (balanced growth paths): one with endogenous fertility rate and the other with exogenously constrained fertility rate under the population control policy. We set the model period to be 20 years and adjust related variables consistent with the setting. Parameters in the model are hence calibrated to fit the moments: (i) unconstrained fertility rate, (ii) inequality of years of schooling, (iii) household saving rate, and (iv) share of household's education expenditure. Information regarding the parameters to be calibrated and the source of reference is summarized in [Table 2.5]. The policy effect is then evaluated through the release of the fertility constraint. Secondly, the dynamics of population, physical, and human capital is included to generate the transitions of variables.

The parameters are selected or calibrated based on the following source of reference. The parameters related to the production function are $(\alpha, \delta)'$, in which capital share and depreciation rate are set to $\alpha = 0.5$ and $\delta = 0.88$ to be consistent with values documented by Bai, Hsieh, and Qian (2006). The depreciation rate in the 20-year horizon is computed based on the annual rate at 10%; namely, $0.88 = 1 - (1 - 0.1)^{20}$. Annual growth rate of TFP equal to 2.3% at constant national price level is calculated by using data of Penn World Table and the number gives us $g = (1 + 0.023)^{20} - 1 = 0.58$.

The male's rate of return on schooling around year 1995, $\gamma = 0.057$, is chosen based on the estimates provided by Zheng et al. (2005). The gender gap on the rate of return to education, $z = 1.18$, is calibrated to match the computed index $h_f/f_m = 0.87$ at the initial steady state. Following Bar and Leukhina (2010), we set the cost of raising children as a deduction from female's hours of working in the form of $b(n) = b_0 n$, and the value of $b_0 = 0.129$ is calibrated so that it matches the estimated effect on women's labor force participation by Maurer-Fazio et al. (2011). Proportion of financial transfer to pre-transfer income ($\tau = 0.15$) is obtained from the statistics sorted by Lei et al. (2012, pp.214).[12] Finally, the unit price of education relative to consumption goods, $\phi = 0.01$, is calibrated to match the share of rural household education expenditure on 7.35% in year 1995 documented by CSY.

In addition, the parental altruistic preference is represented by an increasing function of n, i.e., $\psi(n) = \lambda n^{1-\psi}$, and this setting conforms our assumption $\psi(n) > \psi'(n)n$. The parameter governing the elasticity of parental altruism, $\psi = 0.5$, follows the value chosen by Liao (2013) while the other parameter $\lambda = 0.383$ is calibrated to match the fertility rate 2.71 in year 1980. The objective discounted rate $\beta = 0.404$ is computed by using the annual rate 0.95 since $0.404 = (0.95)^{20}$. The remaining preference parameters is the inverse of the elasticity substitution, $\rho = 0.91$, is calibrated so that the initial household saving rate equals 26%, which is the same with the value provided by Banerjee et al. (2014).

It shows that the variables subject to the second steady-state value of fertility $\bar{n} = 1.60$: saving rate equal to 26.8%, and relative female ratio h_f/h_m equal to 1.05. This result, to some extent, delivers the simultaneously increases of household saving rate and female student ratio under OCP, as depicted in [Figure 2.1] and [Figure 2.3].

2.4.2 The role of the fertility constraint on the transition

2.5 Conclusion

One of the striking structural changes of developing economies is the reduction of population's birth rates, and many countries have tried various population control policies as an instrument to promote economic development. In this chapter, we propose an overlapping generation model to analyze the effect of a binding population control policy on physical and human capital accumulation via household's saving and schooling

[12] They use pilot of CHARLS of year 2008 and focus on provinces of Zhejiang and Gansu of China.

decisions. The model will be calibrated to explain the rapid growth of saving rate and schooling years in China after its implementation of the "One-Child Policy."

We emphasize gender difference in filial support and childcare for two reasons. Firstly, the role of women in economic activities, in particular their education level and labor supply, is crucial to promote economic development in low-income and developing countries. It is therefore important to understand how gender difference within family affects women's productivity. Secondly, vast empirical evidence suggests that in a patrilocal society such as China, there exists gender difference in filial support and childcare; namely, men have more responsibility in supporting their parents and women in childcare. Therefore, incorporating gender difference helps explain household saving decision and parental investment in education.

In addition, this chapter is among a few studies to consider the general equilibrium effects of a population control policy. Unlike previous studies, however, we argue that the need for a household to save does not offset by the slowdown of population growth. In our model, higher levels of human capital compensate lower fertility, and hence the interest rate does not drop under population control. Therefore, household savings increase with a tighter population control policy.

So far, we focus on the effects of a population control policy on the balanced growth path. For the case of China under the "One-Child Policy," we are able to explain the co-movement of household saving and the gender difference regarding schooling years. Our future work is to see how our model can explain the transition of savings and schooling after the implementation of China's population control policy.

2.6 References

1. **Bai, Chong-En, Chang-Tai Hsieh, and Yingyi Qian** (2006). "The Return to Capital in China." *Brookings Papers on Economic Activity*, **2006 (2)**, 61–88.

2. **Bar, Michael and Oksana Leukhina** (2010). "Demographic Transition and Industrial Revolution: A Macroeconomic Investigation." *Review of Economic Dynamics*, **13 (2)**, 421–51.

3. **Banerjee, Abhijit, Xin Meng, and Nancy Qian** (2010). "The Life Cycle Model and Household Savings: Micro Evidence from Urban China." Mimeo.

4. **Banerjee, Abhijit, Xin Meng, Tommaso Porzio, and Nancy Qian** (2014). "Aggregate Fertility and Household Savings: A General Equilibrium Analysis with Micro Data." NBER Working Paper.

5. **Barro, Robert J. and Gary S. Becker** (1989). "Fertility Choice in a Model of Economic Growth." *Econometrica*, **57 (2)**, 481–501.

6. **Becker, Gary S., William H.J. Hubbard, and Kevin M. Murphy** (2010). "Explaining theWorldwide Boom in Higher Education ofWomen." *Journal of Human Capital*, **4 (3)**, 203–41.

7. **Becker, Gary S., Keven M. Murthy, and Robert Tamura** (1990). "Human Capital, Fertility, and Economic Growth." *Journal of Political Economy*, **98 (5)**, S12–37.

8. **Bils, Mark and Peter J. Klennow** (2000). "Does Schooling Cause Growth?" *American Economic Review*, **90 (5)**, 1160–83.

9. **Boldrin, Michele and Larry E. Jones** (2002). "Mortality, Fertility, and Saving in a Malthusian Economy." *Review of Economic Dynamics*, **5**, 775–814.

10. **Chen, Guifu and Shigeyuki Hamori** (2009). "Economic Returns to Schooling in Urban China: OLS and the Instrumental Variables Approach." *China Economic Review*, **20**, 143–52.

11. **Chamon, Marcos D. and Eswar S. Prasad** (2010). "Why Are Saving Rates of Urban Households in China Rising?" *American Economic Journal: Macroeconomics*, **2 (1)**, 93–130.

12. **Choukhmane, Taha, Nicolas Coeurdacier, and Keyu Jin** (2013). "The One-Child Policy and Household Savings." Miemo, London School of Economics.

13. **Curtis, Chadwick C., Steven Lugauer, and Nelson C. Mark** (2015). "Demographic Patterns and Household Saving in China." *American Economic Journal: Macroeconomics*, **7 (2)**, 58–94.

14. **Ge, Suqin, Dennis T. Yang, and Junsen Zhang** (2012). "Population Policies, Demographic Structural Changes, and the Chinese Household Saving Puzzle." IZA Discussion Paper No. 7026.

15. **Lei, Xiaoyan, John Giles, Yuqing Hu, Albert Park, John Strauss, and Yaohui Zhao** (2012). "Patterns and Correlates of Intergenerational Nontime Transfers: Evidence from CHARLS." In Smith, James P. and Malay Majmundar (eds.) *Aging in Asia: Findings from New and Emerging Data Initiatives*, The National Academies Press, Wasington, D. C., 207–28.

16. **Li, Hongbin, Junsen Zhang, and Yi Zhu** (2008). "The Quantity-quality Trade-off of Children in Developing Country: Identification Using Chinese Twins." *Demography*, **45 (1)**, 223–43.

17. **Liao, Pei-Ju** (2013). "The One-child Policy: A Macroeconomic Analysis." *Journal of Development Economics*, **101**, 49–62.

18. **Maurer-Fazio, Margaret, Rachel Connelly, Lan Chen, and Lixin Tang** (2011). "Childcare, Eldercare, and Labor Force Participation of Married Women in Urban China, 1982–2000." *Journal of Human Resources*, **46 (2)**, 291–94.

19. **Modigliani, Franco and Shi L. Cao** (2004). "The Chinese Saving Puzzle and the Life-Cycle Hypothesis." *Journal of Economic Literature*, **42**, 145–70.

20. **Orazem, Peter F. and Elizabeth M. King** (2007). "Schooling in Developing Countries: The Roles of Supply, Demand and Government Policy." in T.P. Schultz and J. Strauss (eds), *Handbook of Development Economics*, vol. 4, North-Holland, Amsterdam, 3475-3559.

21. **Psacharopoulos, George** (1994). "Returns to Investment in Education: A Global Update." *World Development*, **22**, 1325–43.

22. **Pitt, Mark M., Mark R. Rosenzweig, and Mohammad N. Hassan** (2012). "Human Capital Investment and the Gender Division of Labor in a Brawn-Based Economy." *American Economic Review*, **102 (7)**, 3531–60.

23. **Rosenzweig, Mark and Junsen Zhang** (2013). "Economic Growth, Comparative Advantage, and Gender Differences in Schooling Outcomes: Evidence from the Birthweight Differences of Chinese Twins." *Journal of Development Economics*, **104**, 245–60.

24. **Subrahmanian, Ramya** (2005). "Gender Equality in Education: Definitions and Measurements." *International Journal of Educational Development*, **25**, 395–407.

25. **Wei, Shang-Jin and Xiaobo Zhang** (2011). "The Competitive Saving Motive: Evidence from Rising Sex Ratios and Savings Rates in China." *Journal of Political Economy*, **119 (3)**, 511–64.

26. **Yang, Dennis T., Junsen Zhang, and Shaojie Zhou** (2011). "Why Are Saving Rates so High in China?" NBER Working Paper No. 16771.

27. **Zhang, Junsen, Yaohui Zhao, Albert Park, and Xiaoqing Song** (2005). "Economic Returns to Schooling in Urban China, 1988 to 2001." *Journal of Comparative Economics*, **33**, 730–52.

28. **Zhou, Weina** (2014). "Brothers, household financial markets and savings rate in China." *Journal of Development Economics*, **111**, 34–47.

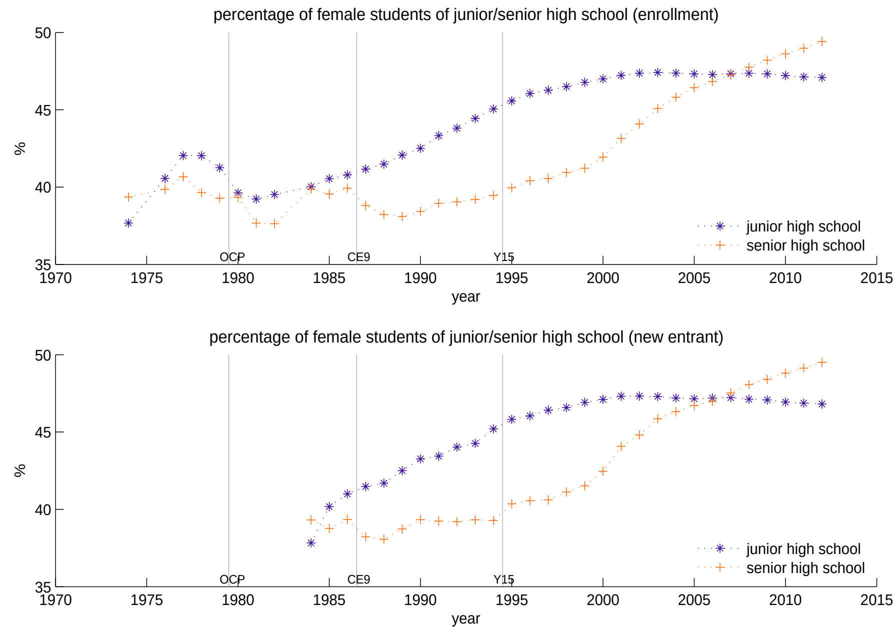


Figure 2.1: Percentage of female student in secondary education (measured by total enrolled and new entrant)

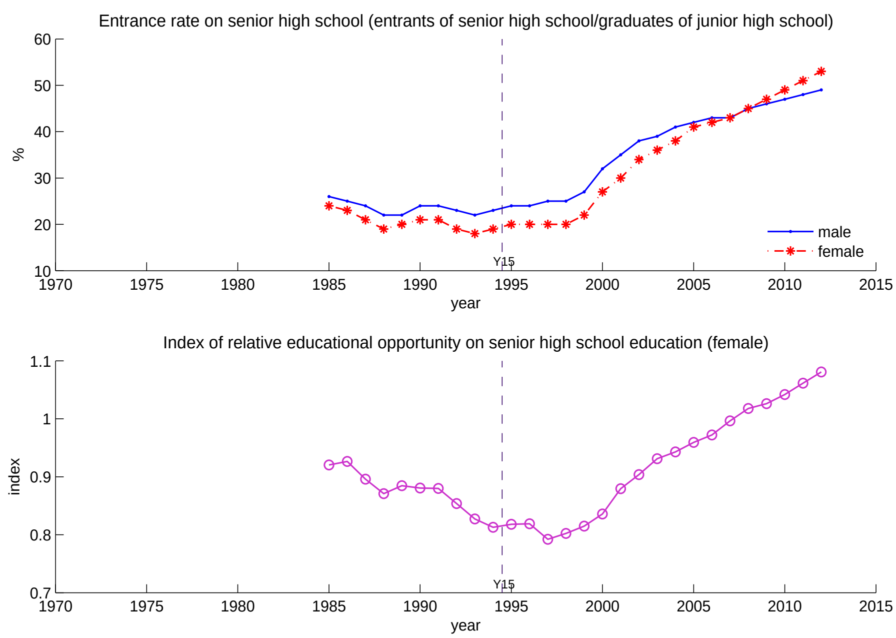


Figure 2.2: Entrance rate of male and female students and the GPI

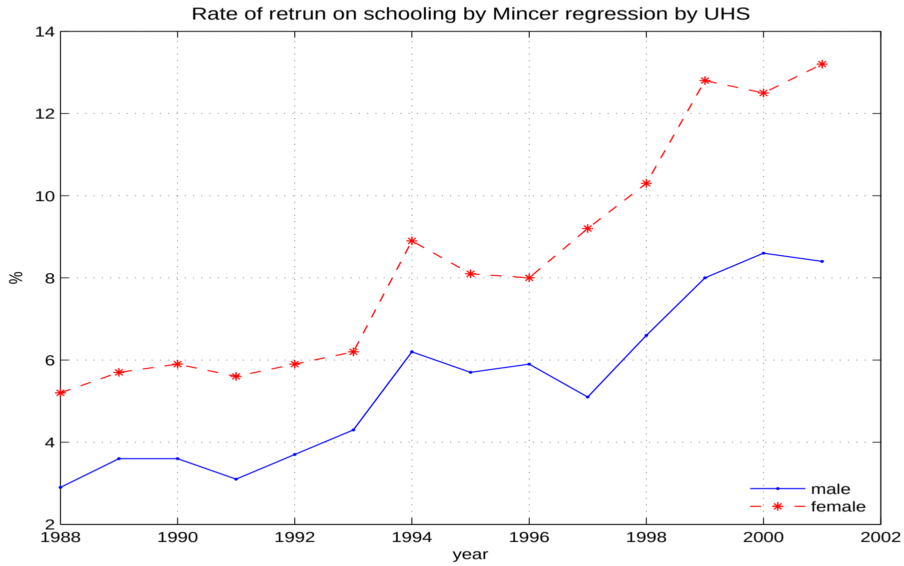


Figure 2.3: The rate of return on schooling estimated by Zhang et al. (2005)

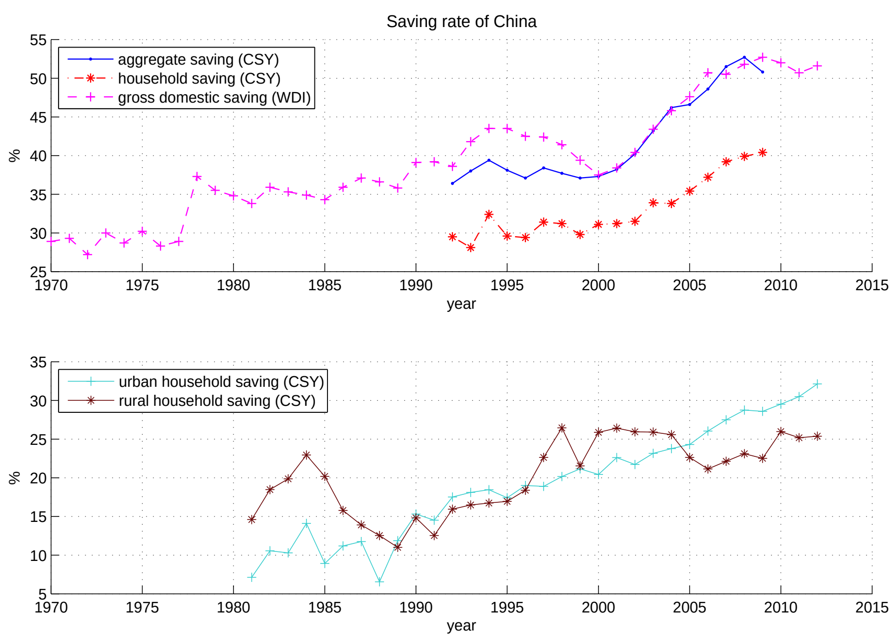


Figure 2.4: Household saving rate of China by different measure

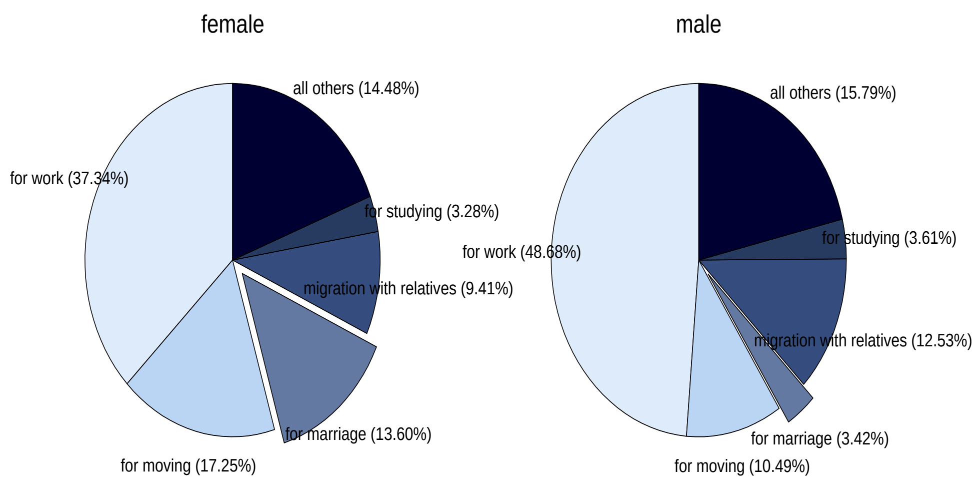


Figure 2.5: Reasons to migration by China Population Census 2005 (1% samples)

Table 2.1: Change of gender parity across countries and regions in tertiary education*

	Country/Region	Change of GPI**	Change rate of gross enrollment ratio (female over male)	Sample year
Asia Countries	China	0.56	2.35	1994–2010
	Cambodia	0.41	3.57	1995–2010
	India	0.15	1.40	1995–2010
	Indonesia	0.24	1.80	1995–2010
	Japan	0.07	1.31	1995–2010
	Korea (The Repulic of)	0.16	1.60	1995–2010
	Lao	0.36	2.02	1995–2010
	Malaysia	0.16	1.40	1998–2010
	Mongolia	-0.74	0.58	1995–2010
	Myanmar	-0.23	0.78	1995–2010
	Philippines	-0.14	-0.20	1995–2009
	Thailand	0.14	1.21	1993–2010
	East Asia and the Pacific countries	0	1.00	1995–2010
	Low income countries	0.13	1.46	1995–2010
	Middle income countries	0.20	1.43	1995–2010
	High income countries	0.16	1.59	1995–2010
	All developing countries	0.24	1.58	1995–2010
	World	0.13	1.30	1995–2010

*Source: UNESCO

**The GPI is measured by GER of females relative to males, i.e., FGER/MGER, and hence the changes of GPI are computed by GPI_{2010} - GPI_{1995}. On the other hand, the relative change rates of GER are computed by $\frac{\mathrm{FGER}_{2010}-\mathrm{FGER}_{1995}}{\mathrm{FGER}_{1995}} / \frac{\mathrm{MGER}_{2010}-\mathrm{MGER}_{1995}}{\mathrm{MGER}_{1995}}$.

Table 2.2: The main motives to saving from survey of rural households by CHIP-2002*

	Family type						
1st (2nd) motive of saving	0	1	2	3	4	5	Total
for children's education	18.5 (8.6)	47.9 (10.6)	41.2 (7.7)	52.5 (12.5)	54.7 (8.4)	50.5 (9.5)	47.0 (8.9)
for the children's wedding	6.8 (9.5)	6.6 (15.6)	17.3 (19.6)	10.1 (17.3)	14.6 (26.8)	19.4 (28.8)	14.2 (21.6)
for parental bequest	3.6 (12.3)	3.2 (9.5)	3.9 (12.2)	1.8 (9.3)	2.0 (9.2)	1.8 (6.3)	2.9 (10.0)
for elderly life	50.0 (17.3)	28.0 (25.6)	23.7 (26.8)	23.3 (30.1)	15.4 (20.8)	13.6 (19.9)	21.9 (23.9)
for future sickness	9.5 (25.9)	4.1 (14.5)	3.9 (11.2)	3.9 (12.5)	2.3 (9.1)	3.6 (8.7)	3.7 (11.5)
for building a house	5.0 (5.9)	6.6 (10.0)	6.7 (10.5)	6.9 (9.9)	8.2 (13.5)	8.4 (13.5)	7.3 (11.4)
for other reasons	8.2 (20.5)	3.5 (14.1)	3.2 (11.9)	1.5 (8.4)	2.8 (12.2)	2.6 (13.3)	3.2 (12.6)

* There are 9200 rural households under investigation by CHIP 2002. We only consider 3-persons and 4-persons, and hence totally 5286 households. They are then divided into 6 groups, in which type 1 to type 5 respectively represent "3-persons family with only one daughter," "3-persons family with only one son," "4-persons family with two daughters," "4-persons family with one son and one daughter," "4-persons family with two sons." Additionally, households with other combinations of members are denoted by type 0.

Table 2.3: Rate of non-schooling in age 16–22 children by region, gender, and family type

ratio (number of obs.)	Urban		Rural	
	female	male	female	male
1-child	22.6% (730)	22.5% (746)	60.7% (270)	60.4% (568)
2-child	33.1% (163)	33.6% (149)	62.7% (1170)	60.8% (1383)
3-child	-	-	71.2% (764)	63.6% (698)
> 3-child	-	-	75.0% (452)	69.7% (323)
total	25.0% (908)	24.6% (904)	67.1% (2656)	62.3% (2972)

The data is from the same source as stated in [Table 2.3]. Individuals whose relation with household heads are "child" and ages between 16–22 are selected. Their current education status is analyzed by gender and by area. The ratios and numbers in each block represent the rate of non-schooling and number of observations from individuals in each division. The statistics of urban part is limited on individuals from 1-child and 2-child households since the remaining observations only account for a small and negligible proportion.

Table 2.4: Summary of parameters to be calibrated

Parameter	Value	Source	Category / Target
$\bar{n}$	1.610	WDI	constrained fertility rate under OCP (fertility rate in 1980)
g	0.576	PWT 8.0	growth rate of TFP of China (2.3% annual rate)
β	0.404	standard	subjective discount factor (0.95 annually)
ψ	0.5	Liao (2013)	parameter of parental altruism
λ	0.38	calibrated	parameter of parental altruism
τ	0.15	Lei et al. (2012)	financial transfers from sons to parents
ρ	0.916	calibrated	inverse of the intertemporal elasticity of substitution
ϕ	0.010	calibrated	price of each unit of education good
γ	0.057	Zhang et al. (2005)	male's rate of return on years of schooling
z	1.178	calibrated	female's premium on the rate of retun to education
b_0	0.129	calibrated	parental time spent on each child
α	0.50	Bai, Hsieh, and Qian (2006)	capital share of the production function
δ	0.88	Bai, Hsieh, and Qian (2006)	depreciation rate of capital (10% annual rate)

2.7 Appendix: numerical solutions

In this appendix, we first describe how variables along the two steady-state paths are solved. Then, a computation scheme to solving transition periods is provided by using a simple case.

2.7.1 Solving the steady state

The steady state at $t = 0$ is solved by

$$\phi = \left[(1-\tau)\psi(n)g^{1-\rho} + \tau\frac{g}{1+r}\right]\gamma {h_m}^{\gamma-1}, \tag{A.2.1}$$

$$\phi = (1-b(n))z\psi(n)g^{1-\rho}\gamma {h_f}^{\gamma-1}, \tag{A.2.2}$$

$$\frac{g^{1-\rho}u(c_0^y, c_1^o)}{w_0 u_{1,0}}\frac{\psi'(n)}{(1-g^{1-\rho}\psi(n))} \geq \phi(h_m + h_f) + b'(n)z{h_f}^{\gamma} - \frac{\tau g {h_m}^{\gamma}}{1+r}, \tag{A.2.3}$$

and the strick inequality of eq. (3) holds as n = $\bar{n}$. The first order conditions on consumption give

$$c_0^y = \frac{1}{1+\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}C_0, \tag{A.2.4}$$

$$c_1^o = \frac{\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}{1+\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}(1+r)C_0, \tag{A.2.5}$$

in which

$$C_0 = w_0\left\{[(1-\tau){h_m}^{\gamma} + (1-b(n))z{h_f}^{\gamma} - \phi n(h_m+h_f)] + \frac{\tau n g {h_m}^{\gamma}}{1+r}\right\},$$

is growing at a constant rate g. The prices of physical and human capitals are determined by

$$r = \alpha A_0 k^{\alpha-1} - \delta, \tag{A.2.6}$$

$$w = (1-\alpha)A_0 k^{\alpha}, \tag{A.2.7}$$

The market clear conditions are written as

$$A_0 \frac{a_0}{g} k^{\alpha-1} = \frac{c_1^o}{g} + c_0^y n + a_0 n - (1-\delta)\frac{a_0}{g}, \tag{A.2.8}$$

where the initial value of capital is given by

$$K_0 = a_0/g \tag{A.2.9}$$

Set the functional form of $b(n) = b_0 n$ and $\psi(n) = \lambda n^{1-\psi}$, in which $0 < \psi \leq 1$. The asset holdings of the young cohort

$$a_t = \left[\frac{\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}{1+\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}\right] C_t - \frac{\tau n w_{t+1} {h_m}^{\gamma}}{1+r},$$

which implies at the initial steady state

$$a_0 = \left[\frac{\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}{1+\beta^{\frac{1}{\rho}}(1+r)^{\frac{1}{\rho}-1}}\right] C_0 - \frac{\tau n (g w_0) {h_m}^{\gamma}}{1+r}. \tag{A.2.10}$$

Hence, the saving rate of the young cohort at $t = 0$ is defined as the ratio of the asset holdings to the wage income

$$s_0 = \frac{a_0}{w_0[(1-\tau){h_m}^{\gamma} + (1-b(n))z{h_f}^{\gamma}]}. \tag{A.2.11}$$

Table 2.5: Settings of the timeline (OCP imposed at t)

$(h_m,\, h_f,\, n)$	-1	...	t	...	$T-1$	T
Parents	$({h_m}^*,\, {h_f}^*)$		$({h_m}^*,\, {h_f}^*)$		$(h_{m,T-1},\, h_{f,T-1})$	$(\bar{h}_m,\, \bar{h}_f)$
Children	$({h_m}^*,\, {h_f}^*)$		$(h_{m,t},\, h_{f,t})$		$(\bar{h}_m,\, \bar{h}_f)$	$(\bar{h}_m,\, \bar{h}_f)$
Fertility	n^*		$\bar{n}$		$\bar{n}$	$\bar{n}$

2.7.2 Computation scheme at the transition economy

Refer to the timeline displayed in [Table 2.5] and assume that the economy is on its balanced growth path at $t = -1$ with its initial steady-state values of $(L_{-2}, L_{-1}, K_0, h_{m,0}, h_{f,0}) = (1, n^* L_{-2}, K_0, h_m^*, h_f^*)$ and now the policy that restricted $n_t = \bar{n}$ is imposed at $t = 0$.

We now consider a simple 4-period case by setting $T = 2$ and list all of equations that characterize the transition from $t = 0$ to $t = 2$. We solve the sequence of allocations $\{c_t^y, c_t^o, a_t, n_t, h_{m,t+1}, h_{f,t+1}\}_{t=0}^1$ and prices $\{w_t, r_{t-1}\}_{t=0}^1$ as well as the aggregates $\{K_1, K_2\}$ and $\{H_0,\, H_1\}$ in order to pin down the prices and the dynamics of population L given the initial value $L_{-2} = 1$.

First, we use the equations in Section 2.6.1 to determine variables at the two steady states (balanced growth); namely, $(1, n^*, K^*, h_m^*, h_f^*)$ for $n_t = n^*$ at $t = 0$ and $(1, \bar{n}, \bar{K}, \bar{h}_m, \bar{h}_f)$ for $n_t = \bar{n}$ at $t = 2$.

The firm's optimization conditions at $t = 0$ are

$$w_0 = (1-\alpha) A_0 (\frac{K_0}{H_0})^\alpha, \tag{A.2.12}$$

$$r_{-1} = \alpha A_0 (\frac{K_0}{H_0})^{\alpha - 1} - \delta \tag{A.2.13}$$

where the initial K_0 is given. The aggregate human capital is

$$H_0 = L_{-1} \Big\{ ({h_m}^*)^\gamma + (1 - b(n_0)) z ({h_f}^*)^\gamma - \phi n_0 (h_{m,1} + h_{f,1}) \Big\}, \tag{A.2.14}$$

in which the population of middle-aged is

$$L_{-1} = n^* \tag{A.2.15}$$

and

$$L_{-2} = 1. \tag{A.2.16}$$

Old generation's consumption at $t = 0$ is determined by

$$c_0^o = (1 + r_{-1})\frac{K_0}{L_{-2}} + \tau n^* w_0 ({h_m}^*)^\gamma. \tag{A.2.17}$$

Young generation's consumption and asset holdings at $t = 0$ are

$$\begin{aligned} c_0^y = \frac{1}{1 + \beta^{\frac{1}{\rho}}(1 + r_0)^{\frac{1}{\rho}-1}} \Bigg\{ & w_0 \left[(1-\tau)({h_m}^*)^\gamma + (1 - b(n_0))z({h_f}^*)^\gamma - \phi n_0 (h_{m,1} + h_{f,1})\right] \\ & + \frac{\tau n_0 g w_1 (h_{m,1})^\gamma}{1 + r_0} \Bigg\}, \end{aligned} \tag{A.2.18}$$

and

$$a_0 = w_0 \left[(1-\tau)({h_m}^*)^\gamma + (1 - b(n_0))z({h_f}^*)^\gamma - \phi n_0 (h_{m,1} + h_{f,1})\right] - c_0^y. \tag{A.2.19}$$

Under the population control policy, the fertility rate is fixed at

$$n_0 = \bar{n}. \tag{A.2.20}$$

The allocation of $(h_{m,1}, h_{f,1})$ is derived by

$${n_0}^{1-\psi}\left\{\frac{g w_1 (1-\tau)\gamma (h_{m,1})^{\gamma-1}}{(c_1^y)^\rho}\right\} + \frac{n_0}{(c_0^y)^\rho}\left\{\frac{\tau g w_1 \gamma (h_{m,1})^{\gamma-1}}{1 + r_0}\right\} = \frac{n_0}{(c_0^y)^\rho} w_0 \phi, \tag{A.2.21}$$

$${n_0}^{1-\psi}\left\{\frac{g w_1 (1 - b(n_1))z\gamma (h_{f,1})^{\gamma-1}}{(c_1^y)^\rho}\right\} = \frac{n_0}{(c_0^y)^\rho} w_0 \phi, \tag{A.2.22}$$

Note that the equations (A.2.17)-(A.2.22) also depend on the variables to be determined at $t = 1$; they are w_1, r_0, c_1^y, n_1 to be solved jointly by the following equations.

At $t = 1$, these variables are chosen such that the new balanced growth path is reached at $t = 2$. The firm's optimization conditions given that $T = 2$ are

$$w_{T-1} = (1-\alpha)A_0(g^{T-1})^{-\alpha}(\frac{K_{T-1}}{H_{T-1}})^{\alpha}, \tag{A.2.23}$$

$$r_{T-2} = \alpha A_0(g^{T-1})^{1-\alpha}(\frac{K_{T-1}}{H_{T-1}})^{\alpha-1} - \delta. \tag{A.2.24}$$

The supply of physical capital is from the assets held by the old-aged cohort at $T-1$, i.e.,

$$K_{T-1} = L_{T-3}a_{T-2} = L_{T-3}\Big\{g^{T-2}w_{T-2}\left[(1-\tau)(h_m{}^*)^{\gamma} + (1-b(n_{T-2}))z(h_f{}^*)^{\gamma} - \phi n_{T-2}(h_{m,T-1} + h_{f,T-1})\right] - c^y_{T-2}\Big\}. \tag{A.2.25}$$

The aggregate human capital is

$$H_{T-1} = L_{T-2}\Big\{(h_{m,T-1})^{\gamma} + (1-b(n_{T-1}))z(h_{f,T-1})^{\gamma} - \phi n_{T-1}(h_{m,T} + h_{f,T})\Big\}, \tag{A.2.26}$$

where

$$L_{T-2} = n^*\bar{n}. \tag{A.2.27}$$

Old generation's consumption at $t = T-1$ is determined by

$$c^o_{T-1} = (1 + r_{T-2})a_{T-2} + \tau n_{T-2}g^{T-1}w_{T-1}(h_{m,T-1})^{\gamma}. \tag{A.2.28}$$

Young generation's consumption at $t = T-1$ is

$$c^y_{T-1} = g^{T-1}w_{T-1}[(1-\tau)(h_{m,T-1})^{\gamma} + (1-b(n_{T-1}))z(h_{f,T-1})^{\gamma} - \phi n_{T-1}(h_{m,\,T} + h_{f,\,T})] - a_{T-1} \tag{A.2.29}$$

so that savings and education can match the state at $t = T$

$$a_{T-1} = \frac{K_T}{L_{T-2}}. \tag{A.2.30}$$

The constrained fertility rate, and the choices on human capital are

$$n_{T-1} = \bar{n}, \tag{A.2.31}$$

$$h_{m,T} = \bar{h}_m, \tag{A.2.32}$$

and

$$h_{f,T} = \bar{h}_f. \tag{A.2.33}$$