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Essays on Macroeconomics and Monetary Economics

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by

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Contents

List of Figures

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Dedicated to my primary school teacher, Yılmaz Ural.

ABSTRACT OF THE DISSERTATION

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My essays that are captured in two chapters of my dissertation focus on shadow banking system, collateralized debt arrangement and monetary policy. The first chapter studies the role of shadow banking in the recent financial crisis, the relationship between shadow banking and traditional banking, and it investigates the monetary policy reaction to overcome the financial frictions associated with the scarcity of collateral or shortages of safe assets that naturally led to the liquidity constraints. On the other hand, the second chapter studies the role of housing as a collateral or as a medium of exchange and it explores how the private liquidity, in the context of home-equity loans, and public liquidity work together to overcome the limited commitment frictions.

In the first chapter, a Lagos-Wright model with costly-state verification and delegated monitoring financial intermediation, and a risk-sharing framework of banking is constructed. Lack of memory and limited commitment imply collateralized credit arrangements. In contrast to the traditional banking system, shadow banking system is not subject to the capital requirements. The relative use of shadow funded credit versus traditional bank loans entails the advantages of working outside the oversight of the bank regulations, but drawbacks of having information and transactions cost in funding entrepreneurs. I have five main findings: First, an entrepreneurial credit can help address the need for collateral. Second, the shadow funded credit shifts from risky to safer borrowers and loan creation capacity of the shadow banking sector shrinks when the economic outlook gets worse. Third, the traditional bank can fulfill the role of providing credit that shadow banks had played before the crisis, but can do it only to a certain extent. Fourth, to the extent that collateral backed

by entrepreneurial credit mitigates the limited commitment friction in the traditional banking sector, the optimal monetary policy shifts nominal interest rate towards zero lower bound. Lastly, the quantitative easing program can be welfare increasing by reinforcing the shadow funded credit versus traditional banking lending if the credit frictions in the shadow banking sector are sufficiently small.

The second chapter studies the role of home-equity loan and government debt in an environment with financial frictions. I construct a Lagos-Wright model in which private transactions must be secured under limited commitment and lack of record-keeping. Housing can be useful to support credit since it serves as collateral. It also gives direct utility as shelter and serves as a medium of exchange when the economy is inefficient. I show that when there is no efficiency loss due to exchange of housing, posting collateral is not optimal since collateralizable wealth is limited. In the state of efficiency loss, the collateral might be useful and the asset therefore bears a liquidity premium. However, once collateral becomes scarce - as it did during the financial crisis- then it amplifies the frictions and the buyer trades the asset to make up for the weak incentives associated with collateral. I show that the world is always non-Ricardian and therefore government debt implies higher welfare. As well, government debt enhances the private debt to the extent that posting collateral is always optimal. In equilibrium, full pledgeability of private collateral, in addition to government debt, completely rules out the efficiency loss arising from exchange of asset. Money and private banks are introduced. I show that as inflation imposes a tax on consumption, interest rate on cash loans imposes a tax on housing collateral. Finally, an increase in inflation raises the housing price near Friedman Rule.

1 Chapter 1: Shadow Banking, Capital Requirements and Monetary Policy

1.1 Introduction

It is important to understand the unregulated banking sector- known as *shadow banking system*- to shed light on the global financial crisis in 2008. In contrast to the Great Depression, the financial crisis in 2007-2009 did not stem from the disruption of retail payment activity in the commercial banking sector. As discussed in Gorton (2010), the financial crisis appears to have originated from disruption in the unregulated banking sector. So-called shadow banks conduct similar liquidity transformation as traditional banks; however, they are lightly regulated or not regulated. They can enhance the credit and alleviate the liquidity constraints in the financial sector. However, they are not immune from panics. In particular, shadow banking activity can be highly information sensitive and this creates incentive problems. As the liabilities of shadow banks account for assets on the balance sheet of traditional banks, the financial crisis in the *shadow banking* system has led to a decline in the transactions of traditional banks and real economic activity. After the collapse in the *shadow banking* activity, the recovery has been very slow as new loans have been originated in the *traditional banking* sector to a certain extent. However, it has been argued that traditional banks cannot fulfill the role that shadow banks had played in providing credit to the economy.

The purpose of this paper is to build a model of *shadow banking* and *traditional banking* sectors that includes the problems arising from the recent financial crisis and analyzes how monetary policy might address these problems. Under what conditions can a private loan originated from the unregulated system increase welfare? Under what conditions do traditional banks fulfill the loan creation that has departed from *shadow banking* sector? How effectively can a traditional bank perform the role of a shadow bank in terms of providing credit to economy? How do the financial frictions associated with the shadow banking sector affect real activity? How do the shadow banks interact with the private banks? Is zero nominal interest rate policy always feasible and optimal? Aside from conventional monetary policy, how does the quantitative easing program associated with the FED's purchases of private asset affect the liquidity creation capacity of shadow banks?

The model constructed in this paper has two banking sectors, intended to capture traditional banking and shadow banking. On the one hand, the traditional banking sector is modeled as a risk-sharing framework of banking in the spirit of Diamond and Dybvig (1983). On the other hand, the shadow banking sector is modeled as a financial intermediary sector subject to costly state verification and delegated monitoring as in Townsend (1979), Williamson (1987), Gale and Hellwig (1985) and Williamson (2012). While a traditional banking sector is subject to capital requirements demanded by the central bank, shadow banking sector is outside the purview of regulations. Second, the model includes features of macroeconomic credit frictions. All financial intermediaries are subject to the limited commitment problem and lack of recordkeeping as in Kiyotaki and Moore (2012), Gertler and Kiyotaki (2010), and Williamson (2014b). The model captures features of differing liquidity across assets such as money, government debt, reserves and private debts. We use some ideas in "New Monetarist Economics" as discussed in Williamson and Wright (2010c) and Williamson and Wright (2010b). Third, the model also entails unconventional purchases of private assets as discussed in Fawley and Neely (2013) and Williamson (2014a). Fourth, the shadow banking literature has been growing. In this current study, I will take the Financial Stability Board's approach on defining the shadow banking which involves entities and activities outside the oversight of the regulations. In contrast to the traditional banking, shadow banking has two distinctive properties: it has no access to the public backstops such as Federal Reserve

Discount Window and it is not subject to the capital requirements. The novelty of this paper is to theoretically assess how the capital regulations, liquidity transformation, macroeconomic credit frictions and monetary policy affect the relative use of shadow banking credit and its impact on the real economic activity. On the one hand, Duca (2014) empirically analyzes how the long-term effects of regulatory capital and financial innovation and short-run effects of financial market shocks alter the relative use of shadow funded credit. According to Adrian and Shin (2009a), Adrian and Shin (2009b), Adrian and Shin (2010) and Gorton and Metrick (2012), the shadow funded credit is pro-cyclical. On the other hand, Luck and Schempp (2014) shows that the size of the shadow banking sector affects the magnitude of the financial market shock. Overall, the contribution of this paper is to capture a unique mechanism that justifies the coexistence of the shadow banking and traditional banking sector. Not because it cannot be constructed by the Arrow-Debrue type of models quantitatively, neither because it tells anything new regarding the monetary exchange; but because it captures wide range of class of financial frictions, liquidity transformation, capital regulations and monetary policy, the framework is rich and the results are consistent with empirical findings. The scope of theoretical analysis in this paper focuses on the two roles of shadow banks, namely supplying credit and addressing the need for collateral, only for nonfinancial corporations only in short-term maturity range.

The model builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). One set of liquid assets captures currency, reserves and government debt where these are supplied by the government. The other set involves private debts: the non-contingent debt contract and individual debt contract associated with the entrepreneurial activity- like a short-term corporate debt. As well, there exist three types of financial agents in the economy: buyers, sellers and entrepreneurs. These agents have an access to frictionless Walrasian market in which debts and taxes are repaid, and new public and private debts are issued. An entrepreneur who does not possess any endowment must borrow to operate his own project. A shadow bank which is the non-bank financial intermediary ($NBFI$) of the *shadow banking system* is subject to costly state verification and might choose to finance his project. It issues non-contingent debt contract to the financial agents in the *traditional* *banking system* to finance the entrepreneurs' projects. Although this intermediation is captured in Williamson (2012), the risk-sharing banking and delegated monitoring financial intermediation are embodied in one intermediary. The value-added in our paper relative to this paper is that the *shadow banking* sector differs from the banking framework that is captured in Diamond and Dybvig (1983). Novelty here is that the central bank imposes a minimum capital requirement on the traditional banking sector while the shadow banks do not have to meet these regulations.

The interaction between a shadow bank and a traditional bank is important for how credit disruptions affect real economic activity. In fact, we show that the liabilities of a shadow bank will account for assets on the balance sheet of a traditional bank. In particular, non-contingent debt issued by a shadow bank will be in service of a private bank as collateral to secure the deposit liabilities. Therefore, availability of liquidity associated with shadow banks can alleviate the financial frictions arising from the scarcity of collateral.

The entrepreneurs who are would-be borrowers are different ex ante with respect to their verification costs. The debt contracts are the results of a solution to bilateral contracting problem as in Gale and Hellwig (1985). The monitoring decisions are made, ex post, in the case of default as in Townsend (1979). It turns out that an intermediary rejects to offer debt contract to riskier entrepreneurs. As the intermediary offers an interest rate, each entrepreneur offers so-called equilibrium debt contract. This contract and the optimal debt contract that maximizes the intermediary's expected payoff characterize the marginal contract where each entrepreneur whose verification cost exceeds the marginal entrepreneur's verification cost receives no offer. Therefore, the model exhibits an endogenous *credit rationing* where among loan applicants who appear to have different verification costs some of these receive a loan and others do not and rejected applicants would not receive a loan even if they offered to pay a higher interest rate. In contrast, the credit rationing in Stiglitz and Weiss (1981), Keeton (1979) and Williamson (1986) entail identical borrowers ex ante. In this model, an entrepreneur and a financial intermediary are asymmetrically informed, ex post, regarding the project return of the entrepreneur as in Williamson (1987). In contrast, Stiglitz and Weiss (1981) and Keeton (1979) show that equilibrium rationing arises by the moral hazard and adverse selection in credit markets. In fact, we employ a costly-state verification and delegated monitoring intermediary structure in entrepreneurial project market similar to that in Williamson (1987), Townsend (1979) and Gale and Hellwig (1985). Another novelty of the paper is that the credit rationing differs with respect to the type of banking sectors since each sector has different market interest rate. Further, the verification cost associated with the marginal contract in each sector determines its intermediary's loan creation capacity or its so-called "arm length" on entrepreneurial activity. As the marginal verification cost increases, so does the arm length of associated intermediary. A shadow bank potentially has an advantage over a private bank on reaching more projects since the shadow bank enjoys larger leverages due to lack of regulation.

In the model, lack of memory and limited commitment[1] imply that credit arrangements must be secured. The financial friction arising from collateral constraints plays an important role in how liquidity origination has an impact on the quantity of exchange, inflation, the rate of return on government debt and welfare. As well, macroeconomic credit friction entails costs of operating *shadow banking* system. These costs include additional monitoring costs and transactional costs. In particular, a fraction of shadow bank's payoff is deemed to be useless for each would-be borrower and a shadow bank while the traditional banks face none of these costs. The debt contracts originated in the shadow banking sector are information sensitive in the sense that these costs create incentive problems. In fact, these costs will not only affect the loan creation capacity of shadow banks, but also affect the equilibrium debt contract of an entrepreneur who receives an offer in the *shadow banking* sector.

The buyers are willing to have insurance against the need for liquid assets in different type of meeting and hence a bank, by allocating the resources according to the appropriate transactions, enhances the welfare, as in Diamond and Dybvig (1983). Assume, for example, the bank offers a deposit contract to mitigate the risk arises from different type of meetings - like a commercial bank. Note that all the activities of the traditional bank - also entitled as Diamond-Dybvig bank -

[1]Each entrepreneur, in contrast to other financial agents, is subject to full commitment because each lives for one period.

can be considered as the part of the *traditional banking system*. A Diamond-Dybvig bank which is subject to cash withdrawals can access to the set of assets supplied by the government. Further, this bank is within the oversight of the regulations, i.e., subject to a minimum capital requirement for each private asset held in its asset portfolio. For interesting policy analysis, we confine our attention to the stationary equilibrium in which Friedman Rule is not feasible as the private credit associated with entrepreneurial activity and the consolidated government debt are not large enough to render efficient trade. Fiscal policy is treated as given, as discussed in Williamson (2014a).

We carry out three experiments exploring the effects of financial crisis shocks on real activities. These experiments capture, first, a shift in the distribution of verification cost of entrepreneurs; second, a shift in the distribution of project returns of entrepreneurs; third, a change in the cost of monitoring technology associated with the shadow banks. We show that each creates disruption in the credit activities of traditional banking system. However, first has no impact on the payoffs of entrepreneurs whose project is funded by any intermediary. Both second and third experiments can not only change the loan creation capacity of each intermediation, but also the entrepreneurs' payoffs. According to third experiment, as cost of operating *shadow banking* system increases, we show that a private loan originated by shadow banks decreases entrepreneurs' expected payoffs. Therefore, liquidity creation in the shadow banks might decline and even disappear as traditional banks fill it by financing new projects.

A shadow bank which is subject to the limited liability posts the pool of receivables of individual debt contracts as collateral against the non-contingent debt. This asset-backed security which is purchased by a traditional bank is subject to the minimum capital requirement. The *shadow banking* activity is useful since we assume that capital requirement for receivables of debt contract directly offered by the traditional bank is at least as large as the capital requirement for asset-backed security. It turns out that the collateral associated with the loan creation in the *shadow banking* sector is cheaper. Therefore, the function associated with the incentive constraint which characterizes the equilibrium allocation exhibits a discontinuity at which a traditional bank and a shadow bank have the same loan creation capacity. This jump occurs at the level where all the loan origination departs from one intermediary to the other. The fact that the collateral originated by the shadow banks is more valuable and creates larger liquidity justifies the jump in the incentive constraint. In short, a small contraction in the shadow funded credit due to financial crisis shock may lead to large repercussions for the real economic activity by shifting the credit allocation towards traditional banking sector. Conventionally, if associated discontinuity arises near zero lower bound (ZLB), the incentive constraint will undershoot the zero nominal interest rate and ZLB will not exist in equilibrium. However, we show that if zero nominal interest rate policy is feasible, then it will be always optimal.

Finally, we confine our attention to the unconventional monetary intervention, namely quantitative easing program. The central bank's purchases of asset-backed security can increase welfare by alleviating the financial frictions associated with shadow banks if the cost of operating a shadow bank is sufficiently low. By these purchases, shadow banks offer lower interest rate and each entrepreneur can make lower equilibrium repayments if funded. If the cost of operating a shadow bank is sufficiently large, this intervention will be useless in terms of shifting the credit allocation between shadow and traditional banking as well as improvement in the real economic activity. However, if this cost is moderate, it will not only increase the welfare but also cause liquidity to move from the *traditional banking* sector to *shadow banking* sector.

We have five main findings: First, an entrepreneurial credit can help address the need for collateral. Second, the shadow funded credit shifts from risky to safer borrowers and loan creation capacity of the shadow banking sector shrinks when the economic outlook gets worse. Third, the traditional bank can fulfill the role of providing credit that shadow banks had played before the crisis, but can do it only to a certain extent. Fourth, to the extent that collateral backed by entrepreneurial credit mitigates the limited commitment friction of the depositors in the traditional banking sector, the optimal monetary policy shifts nominal interest rate towards zero lower bound. Lastly, the quantitative easing program can be welfare increasing by favoring the shadow funded credit versus traditional banking lending if the credit frictions in the shadow banking sector are tolerable.

The remainder of the paper is organized as follows. The second section captures the environment.

The shadow bank's, the traditional bank's and the entrepreneur's problems are captured in this section. The third section characterizes the equilibrium, illustrates the results of conventional monetary policy and three different experiments associated with the financial crisis. The fourth section includes the central bank's program of asset-backed security purchases and characterizes the optimal monetary policy. The fifth section concludes the paper.

1.2 Environment

We use the idea of combining decentralized trade with a periodic access to centralized market as in Lagos and Wright (2005). More precisely, time is discrete and indexed by $t = 0, 1, 2, \ldots$, and each period is divided into two subperiods, namely the centralized market (CM) and the decentralized market (DM). There are continuum of buyers and a continuum of sellers, each with unit mass. Note that the buyers can produce consumption goods only in the CM, can consume only at the DM. In contrast, the sellers can produce only at the DM, yet can consume only in the CM. Each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)] \tag{1.1}$$

where H_t is the labor supply in the CM, x_t is the consumption at the DM, and $\beta \in (0, 1)$ is the discount factor. Suppose that $u(.)$ is strictly concave, strictly increasing, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$ and $\frac{-xu''(x)}{u'(x)} < 1$ and define x^* by $u'(x^*) = 1$.

Each seller has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t] \tag{1.2}$$

where X_t is the consumption in the CM, and h_t is the labor supply at the DM. One unit of labor supply either at the DM or in the CM produces one unit of perishable consumption good.

The basic environment is also related to Rocheteau and Wright (2005) in terms of types of agent and the matching technology. At the DM, each buyer will be randomly matched with a seller with probability 1. As well, there exists no recordkeeping and therefore a seller cannot follow the buyer's past transactions. In addition, there exists a limited commitment friction, i.e., no one can be forced to work to repay debt.

In addition to buyers and sellers, there exists a continuum of entrepreneurs with mass σ. Each is born in CM, lives for only one period and then dies out in the next period of CM. This process occurs in every period. Thus, an entrepreneur who is born in the CM of period t consumes only in the CM of period $t+1$. Assume also that the entrepreneurs are risk neutral and receive no endowments. Each entrepreneur has an access to his own investment project. Each project is indivisible, requires one unit of the consumption good in the CM of period t to run, and yields a random return of ω in the CM of period $t+1$. The project return ω is distributed according to the distribution function $F(\omega)$ with associated density function $f(\omega)$. Assume that $f(\omega)$ is strictly positive and continuously differentiable on $[0,\overline{\omega}]$ where $0<\overline{\omega}$. Investment project returns are independent across entrepreneurs. The return ω is private information to the entrepreneur, but subject to costly state verification, i.e., any other intermediary can bear a fixed cost and observe ω ex post. The verification cost κ is entrepreneur-specific. As well, let $G(\kappa)$ denote the distribution of verification costs across entrepreneurs, where $\kappa \geq 0$.

At the beginning of the CM, an entrepreneur who is born in the past CM pays off, consumes the rest of her project return and then dies. Then new entrepreneurs are born with each receiving a draw from the distribution G. If a project is funded, the return ω will be drawn from the distribution F where only corresponding entrepreneur knows ω. Note that ω is independent of κ. In case of default, the lender incurs an entrepreneur-specific cost κ, learn the project return ω and eventually seize it. Default implies no consumption for the entrepreneur.

The setup for the DM directly follows from Williamson (2014b), Williamson (2014a) and Williamson (2012). A buyer will be at the *currency transactions* DM with probability ρ. That is, she will be matched with a seller who only accepts currency as a means of payment. On the other hand, a buyer will be at the *non-currency transaction* DM with probability $1-\rho$. In the latter, each seller can verify the entire portfolio held by the buyer. In fact, a buyer transfers the ownership of the claim of entire portfolio through a financial intermediary to the seller. Not only currency, but also other assets- reserves, government bonds, the non-contingent debt contract and the pool of receivables of individual loan contracts with entrepreneurs - can be verifiable in these meetings. In the currency DM meetings, the currency can be exchanged on the spot. However, the remaining assets cannot be transferred until the subsequent CM. At the beginning of the CM, a buyer does not know whether he will be in the currency or non-currency DM meeting. After the consumption and production take place and the debts are settled, each buyer will learn the type of the meeting and this is private information.

A government bond sells for z_t^b units of money in terms of CM good of period t, and pays off one unit of money in terms of CM good of period $t+1$. One unit of reserve can be acquired for z_t^m units of money in terms of CM good of period t, and pays off one unit of money in terms of CM good of period $t+1$. There is a non-contingent loan contract (backed by the receivables) between a traditional bank and shadow bank. One unit of asset-backed security sells at the price q_t in terms of CM good of period t and is a promise to pay one unit of consumption good in the CM of period $t+1$. As well, each entrepreneur is in need of one unit of consumption good to operate the project. The repayment is endogenous and depends on the distribution of the project returns, the verification cost of the entrepreneur and expected rate of return of the lender from each contract. As well, a shadow bank (traditional bank) which is perfectly diversified gains a fixed one-period return r_t^s (r_t) per unit lent to the entrepreneur. The total loan origination L_t^s (L_t) and the rate of return r_t^s (r_t) for each loan originated in the *shadow banking sector* (*traditional banking sector*) will be determined endogenously.

The consolidated government issues currency, reserves, and nominal bonds, denoted by, respectively, C_t, M_t, and B_t in nominal terms in period t. The government makes transactions only in the CM, including the lump-sum transfer τ_t to each buyer in the period t. Thus, the consolidated government budget constraints are given by

$$\psi_0(C_0 + z_0^m M_0 + z_0^b B_0) - \tau_0 = 0, \tag{1.3}$$

$$\psi_t(C_t - C_{t-1} + z_t^m M_t - M_{t-1} + z_t^b B_t - B_{t-1}) - \tau_t = 0, \quad \forall t = 1,2,3... \tag{1.4}$$

where ψ_t is the price of money in terms of CM good of period t.

All the financial arrangements in the CM are displayed in Figure 1. The following is the timing of the CM. First, all the private debts are repaid and taxes are paid. An entrepreneur who can invest in his project consumes and then dies out. Second, new entrepreneurs are born. Each receives a draw of verification cost from the distribution function G. The shadow banks and traditional banks are formed. Third, the traditional banks acquire deposits from the buyers, the government pays off bonds and reserves and it issues new government debt, reserves and currency. A bank, by using the deposits, purchases currency, reserves, government debt and asset-backed security. As well, a shadow bank issues the non-contingent debt contract and originates individual loans to fund the entrepreneurs' projects by using the proceedings of the asset-backed security. Finally, given the market interest rates offered by shadow banks and traditional banks, each entrepreneur offers a debt contract to both shadow banks and traditional banks. If both intermediaries choose to fund the project, the relevant contract is the one with smaller repayment schedule. On the other hand, if only one intermediary chooses to fund, the offer associated with the other intermediary will be irrelevant. If neither intermediaries choose to fund, this entrepreneur will die out in the next centralized market without operating her project.

Given that traditional banks are subject to the regulations, each bank must hold the fractions $\underline{\delta}_1$ and $\underline{\delta}_2$ of the asset-backed security and receivables of debt contracts, respectively, as *regulatory* capital, where $\underline{\delta}_1 \in [0,1]$ and $\underline{\delta}_2 \in [0,1]$. Note that bank capital is illiquid in the sense that it is non-collateralizable. The payoffs of the bank capital and collateralizable asset are equal. Also, we assume that $\underline{\delta}_1 \leq \underline{\delta}_2$. To be more specific, $\delta_2 - \delta_1$ is consistent with the differential in minimum capital requirements on commercial industrial loans and asset backed securities held in bank portfolios. This assumption justifies the competitive advantage of shadow banks over traditional banks on diversification and pooling of debt contracts associated with the entrepreneurial activity.[2] In

[2]According to Gorton and Metrick (2010), innovations and regulatory changes reduce the competitiveness of the traditional banks on the supply side. On the demand side, demand for collateral also justifies how the shadow

fact, it justifies the coexistence of the *shadow banking* and *traditional banking* system. Further, it implies that the loan origination in the *shadow banking* sector might yield higher welfare than the other since the liabilities of the shadow banks account for the assets on the balance sheet of the private banks. In other words, the shadow bank's loan creation capacity can be larger than that of traditional bank since lower capital requirement associated with the asset-backed security generates larger leverage and in turn lower market interest rates and smaller repayment.

1.2.1 The Shadow Bank's Problem

A non-contingent debt contract performs a useful role to finance the projects of the entrepreneurs in the CM. A shadow bank who is subject to limited commitment issues non-contingent debt contract in each period. It will collect the payoffs from individual debt contracts with each entrepreneur (who gets an offer) in the next period. In particular, the pool of these receivables backs the non-contingent debt contract. Assume that an intermediary which offers deposit claims and is subject to the withdrawal in cash is regulated. In particular, these intermediaries- Diamond-Dybvig banks - have an access to the activities by the central bank to buy or sell government debts, reserves and currency. As well, they must meet capital requirements imposed by the central bank or regulatory institutions. In contrast, the shadow banks are regulation-free intermediaries and outside the purview of the regulatory enforcements. In fact, they are not subject to the capital requirements and enjoy larger financial leverages than the regular banks.

The non-bank intermediaries are risk-neutral and are perfectly competitive profit maximizers. The shadow bank's problem can be expressed by

$$\max_{l_t^s, L_t^s} \left\{ q_t l_t^s - L_t^s - \beta l_t^s + \beta r_t^s L_t^s \right\} \tag{1.5}$$

subject to

$$-l_t^s + r_t^s L_t^s \geq 0, \tag{1.6}$$

banking sector has grown rapidly. Our assumption of differing capital requirements supports these two forces.

where (1.6) shows the collateral constraint with no capital requirements. Let L_t^s and l_t^s denote the total loan supply for entrepreneurs' projects and the quantity of non-contingent private debt, respectively. Also, let r_t^s denote the fixed rate of return from each loan contract offered by the intermediary. This rate will be determined endogenously. The intermediary's objective function (1.5) captures her expected payoff $q_t l_t^s - L_t^s$ in the CM of period t. This payoff comes from issuing l_t^s units of security and supplying L_t^s units of funds for the entrepreneurial activity. As well, the quantity $-\beta l_t^s + \beta r_t^s L_t^s$ is the discounted payoff in the CM of period $t+1$ from paying off the non-contingent private debt and collecting the project returns.

First, the intermediary issues an asset-backed security and accepts individual debt contracts of some entrepreneurs as it is committed to the verification costs. Then it diversifies the loan contracts, pools the repayments from each contract in the next CM and finally posts pool of repayments as collateral to back the non-contingent debt contract. This follows a simple form of a financial process called as *securitization*. That is, the *NBFI* turns an illiquid asset into liquid non-contingent debt contract by originating new loans and pooling the receivables of these loans. Moreover, the shadow bank specified in this model can be interpreted as *special purpose vehicle*, a shell company created by the traditional bank to get rid of minimum capital requirements.

If an intermediary defaults, then the buyer will seize all the pool of receivables. The shadow bank who is outside the purview of the regulatory institutions activates the borrowing up to full value of collateral since no capital requirements are enforced. No assets will be kept as capital and hence all the receivables of debt contracts are liquid in the sense that they will be posted as collateral to support the security. We are interested in equilibrium in which there exists a scarcity of collateral or shortages of safe assets, i.e., (1.6) binds. Therefore, the collateral is not plentiful enough to render the incentive constraint slack.

Total loan supply exhibits a perfectly elastic supply curve. That is, if the rate of return on individual debt contract is larger than the rate of return on the security, then the intermediary will make profit. Another intermediary, by marginally decreasing this rate, can increase the entrepreneurs' payoffs. On the other hand, if the first is smaller than latter, than it will optimally choose not to issue security since the costs exceed the receivables. Hence, no activities take place in the shadow banking sector. Therefore, the equilibrium rate of return on each debt contract should be equal to the rate of return on security.

1.2.2 The Traditional Bank's Problem

The traditional banks are Bertrand competitors offering deposit contracts. In equilibrium, the bank will make zero profit; otherwise, another bank would enter and offer better terms. Like all the other agents, the bank is subject to limited commitment: the deposit liabilities will be secured by its asset portfolio including reserves, government bonds, asset-backed security and pool of contract receivables. Hence, these assets will be posted as collateral.

Each buyer is willing to insure against the need of liquid assets for different type of the meetings. The traditional bank in the spirit of risk-sharing framework of banking as in Diamond and Dybvig (1983) performs a useful role since it will efficiently allocate the liquid assets to appropriate types of engagements. If the banks had never existed, the buyer who carries the currency, reserves, government debt and private assets from the CM would not have exchanged the non-currency asset for consumption at the currency DM meeting. Therefore, the banks will optimally offer a deposit contract that entails two different schemes of DM transactions according to appropriate liquid assets. Once the banks are formed, they will acquire deposits from the buyers and then purchase government debt, reserves, cash and non-contingent private debt. As well, they can offer debt contracts to the entrepreneurs as the shadow banks do. If an entrepreneur had operated a traditional bank, he would have vanished too soon in the forthcoming period without collecting the payoffs from the reserves. If a seller had operated a traditional bank, the deposit contract would have implied inefficient allocation of resources. Remember that a seller does not verify non-currency assets with probability ρ. In contrast, any buyer can run a Diamond-Dybvig bank.

It is also important to note that the deposit liabilities in the non-currency transaction are subject to the limited commitment. Hence, the asset portfolio held by the buyer must be used as collateral to secure the credit transaction. The traditional banks involve credit transactions within the oversight of the regulatory enforcements. Aside from offering a deposit claim, they must hold *regulatory capital* that involves minimum requirement for each private asset as demanded by the central bank. Otherwise, banks cannot operate in the *traditional banking* sector. Therefore, the banks cannot activate the deposit liabilities up to full capacity since the regulators impose each to hold fraction of its private asset holdings as non-collateralizable bank capital. For simplicity, we assume that capital requirements do not account for government debt, currency and reserves. In fact, Diamond-Dybvig banks can be interpreted as the commercial banks.

Note that the bank- like a shadow bank- can accept the debt contracts of some entrepreneurs, diversify the contracts and then pool them for a use of collateral. If the debt contract offered to traditional bank entails lower gross rate of return than that offered to shadow bank, corresponding entrepreneur will be better off by offering the first. As well, the traditional bank can reach larger capacity on financing the entrepreneurial activity than shadow banks and hence more projects are funded in the *traditional banking* system. Therefore, the traditional bank will fund the rest of projects that cannot be reached by the shadow banks. In other words, an entrepreneur's offer associated with the verification cost within the arm length of the traditional bank, but not the shadow banks, will be accepted only by the traditional bank. Note that lack of capital requirement in the shadow banking sector might pull shadow bank's arm longer on reaching projects and hence *shadow banking* activities might perform useful role by creating cheaper loans to the entrepreneurs.

In equilibrium, the bank offers a deposit contract that maximizes the expected utility of the buyers. The elements of the banking contract is threefold: the buyer, first, will deposit k_t units of money balance in terms of CM good of period t. Second, the bank offers c_t units of cash in terms of CM goods of period t if the depositor turns out to be in the currency DM transaction. Third, the bank offers a claim of d_t units of consumption goods in the CM of period $t+1$ and this claim will be exchanged with the seller for consumption good if the buyer is at the non-currency DM meeting.

Although the currency can be used in non-currency meeting, it will be optimal to hold currency just enough to exhaust all at the currency DM meeting as discussed in Williamson (2014b), Williamson

(2014a) and Williamson (2012). Therefore, the buyer brings currency to the subsequent DM transactions and reserves, government bonds, and securities to non-currency DM transactions. In fact, she will use the asset portfolio as collateral to engage in a collateralized credit transaction in the non-currency DM meeting. The bank can insure against the need for liquid assets in different types of transactions. Note that the currency DM meetings are subject to full commitment. We assume that there exists a strong commitment device in these meetings, like ATM.

The bank's problem is given by

$$\max_{k_t,c_t,d_t,m_t,b_t,l_t,L_t} -k_t + \rho u\left(\frac{\beta \psi_{t+1}}{\psi_t} c_t\right) + (1-\rho)u(\beta d_t) \tag{1.7}$$

subject to

$$\begin{aligned} & k_t - z_t^m m_t - z_t^b b_t - q_t l_t - L_t - \rho c_t \\ & - (1-\rho)\beta d_t + \frac{\beta \psi_{t+1}}{\psi_t}(m_t + b_t) + \beta l_t + \beta r_t L_t \geq 0, \end{aligned} \tag{1.8}$$

$$-(1-\rho)d_t + \frac{\psi_{t+1}}{\psi_t}(m_t + b_t) + l_t(1-\underline{\delta}_1) + r_t L_t(1-\underline{\delta}_2) \geq 0, \tag{1.9}$$

where (1.7), (1.8) and (1.9) denote the buyer's expected payoff, non-negative expected payoff of the bank and the bank's collateral constraint, respectively. Let $\underline{\delta}_1$ and $\underline{\delta}_2$ denote the fraction of asset-backed security and receivables of debt contracts that will be hold as required by the regulatory institutions. The deposit liabilities are backed by the government debt, reserves and liquid private assets. Note that the quantity $l_t(1-\underline{\delta}_1) + r_t L_t(1-\underline{\delta}_2)$ account for the total collateralizable asset where the bank capital forms the rest of the bank's asset holding on its balance sheet. Since $\underline{\delta}_1 \leq \underline{\delta}_2$, the security yields larger leverages than the receivables of debt contracts. By using this assumption, the receivables of debt contracts originated in the *shadow banking* sector matter since the non-contingent debt contract backed by these receivables accounts for asset in the balance sheet of the bank and it activates more liquidity than the receivables of the debt contract originated in the *traditional banking* sector.

The buyer makes a take-it-or-leave-it offer at the DM. The buyer who deposits k_t units of consumption good in the CM of period t will exchange currency worth of $\frac{\psi_{t+1}}{\psi_t}c_t$ of the consumption good in the CM of period $t+1$ in the currency DM meeting. As well, the buyer will exchange deposit liabilities worth of d_t units of good in the CM of period $t+1$ if the buyer is in the non-currency DM meeting. The constraint (1.8) binds in equilibrium. Hence, the bank's net payoff in the CM of period t is formed by the deposit acquisition from the buyers and the purchases of currency, reserve, the government bonds, non-contingent debt and loan origination for the entrepreneurs. In the subsequent CM, the bank pays off the deposit liabilities to its holder, collects the payoffs of the reserves, bonds, the asset-backed security and pool of the debt contracts.

The collateral constraint (1.9) binds in equilibrium. We are interested in equilibrium in which the collateral is too scarce to render the incentive constraint slack. Therefore, the quantity of exchange at the DM is inefficient, i.e., $\beta d_t < x^*$.

1.2.3 The Entrepreneur's Problem

We will permit the private production of liquidity by a way of costly-state-verification and delegated monitoring financial intermediation in the spirit of Williamson (1987) and Williamson (2012). Suppose that the entrepreneurs are subject to the full commitment and there exists no stochastic verification. As in Williamson (1986), we obtain that the optimal loan contract under incentive compatibility condition is a non-contingent debt.

In here, the non-contingent debt is associated with a specific loan contract in which an entrepreneur with verification cost κ sells for one unit of consumption good in the CM and promises to pay off (non-contingent payment) R_t units of consumption goods in the next CM. Each entrepreneur has its own indivisible project. If an intermediary chooses to accept the contract, first the entrepreneur will invest on her project and then acquire a random return ω in the next CM. If the project return turns out to be strictly lower than R_t, then the lender will incur κ, learn ω and then seize everything in the subsequent CM. In contrast to the Diamond and Dybvig (1983), monitoring decisions are taken ex-post rather than ex-ante. The gross rate of return on a loan contract maximizes the entrepreneur's expected payoff subject to the lender's incentive constraint. It turns out that the shadow banks, in contrast to the traditional banks, incur monitoring, transactional and informational costs. These costs create incentive problems. There exists larger leverage on the non-contingent debt originated by the shadow bank and hence it can have stronger pull on funding the projects. In the next subsection, we will characterize the equilibrium and "intermediary-optimal" debt contracts, respectively, originated in the *shadow banking* sector.

The Debt Contracts Originated by the Shadow Banks

The individual contract $R_t^\theta(\kappa)$ associated with the gross interest rate R_t^θ in period t and an entrepreneur whose verification cost is κ solves the following problem

$$\max_{R_t^\theta(\kappa)} \left\{ \overline{\omega} - R_t^\theta(\kappa) - \int_{R_t^\theta(\kappa)}^{\overline{\omega}} F(\omega) d\omega \right\} \tag{1.10}$$

subject to

$$\Pi_I^\theta(R_t^\theta(\kappa)) = (1-\theta)\left[R_t^\theta(\kappa) - \kappa F\left(R_t^\theta(\kappa)\right) - \int_0^{R_t^\theta(\kappa)} F(\omega) d\omega \right] \geq r_t^s, \tag{1.11}$$

where (1.10) is the expected payoff of the entrepreneur and (1.11) is the intermediary's incentive constraint. The constraint (1.11) shows that the effective expected payoff, denoted by $\Pi_I^\theta(R_t^\theta, \kappa)$, must be at least as large as the "market expected return" r_t^s. This will be treated as fixed by both entrepreneurs, and the shadow bank. We assume that a fraction $\theta \in (0,1)$ of shadow bank's expected return from each contract offered to an entrepreneur is deemed to be useless as an informational and transactional cost. Hence, the rest is the effective gain of the intermediary from funding the project. In particular, the fraction $1-\theta$ of its expected return accounts for the receivables of the debt contract. Note that θ can be interpreted as the parameter of information sensitivity for liquidity creation originated in the *shadow banking* sector. As θ increases, each contract becomes increasingly information sensitive. This not only limits the loan creation capacity of the shadow bank, but also increases the repayment of each entrepreneur and hence decreases his payoff. It turns out that large leverages associated with the *shadow banking* sector reinforce shadow bank's loan creation capacity, but monitoring costs worsen intermediary's ability to reach longer arm on financing the entrepreneurial activity.

By using (1.10), an increase in R_t^θ decreases the entrepreneur's payoff. Hence, in equilibrium (1.11) binds. Moreover, the solution R_t^θ does not depend on the project return of the entrepreneur, but the distributions F of the project returns, associated entrepreneur's verification cost κ and the information sensitivity parameter θ. Next we will define a family of equilibrium debt contracts associated with $\theta \in (0,1)$.

Definition 1 *A family $\mathbb{R}_\theta$, denoted by $\{R_\theta(\kappa)\}_{\theta\in(0,1)}$, is a set of equilibrium contracts originated in the shadow banking sector associated with an entrepreneur whose verification cost is κ and the gross rate of return $R^\theta(\kappa)$ for each $\theta \in (0,1)$ solves*

$$R_t^\theta(\kappa) - \kappa F\left(R_t^\theta(\kappa)\right) - \int_0^{R_t^\theta(\kappa)} F(\omega)d\omega = \frac{r_t^s}{1-\theta}. \tag{1.12}$$

The intermediary diversifies the loan contracts in equilibrium. Assume that $\kappa f'(R) + f(R) > 0$ for all $R \in [0, \overline{\omega}]$ and for all $\kappa \geq 0$. Then there exists a unique debt contract that maximizes the expected payoff of the intermediary. Next characterizes the optimal debt contracts for the intermediary.

Definition 2 *A family $\tilde{\mathbb{R}}^\theta$, denoted by $\{\tilde{R}^\theta(\kappa)\}_{\theta\in(0,1)}$, is a set of "intermediary-optimal" contracts originated in the shadow banking sector associated with an entrepreneur whose verification cost is κ and the gross rate of return $\tilde{R}^\theta(\kappa) < \overline{\omega}$ for each $\theta \in (0,1)$ solves*

$$0 = 1 - \kappa f(\tilde{R}_t^\theta(\kappa)) - F(\tilde{R}_t^\theta(\kappa)). \tag{1.13}$$

Note that optimal debt contracts are independent of θ since the *NBFI* incurs fixed fraction of its total payoff for each debt contract offered. Let $(R_\theta^*, \kappa_\theta^*)$ characterize the marginal contract that solves both (1.12) and (1.13) where R_θ^* shows the marginal gross rate of return associated with the marginal entrepreneur whose verification cost is κ_θ^*. The shadow bank's arm length κ_θ^* is a measurement for the loan origination capacity in the *shadow banking* sector. In other words, the shadow bank will choose not to accept any offers from those whose verification costs are larger than the marginal entrepreneur's verification cost although they can offer large rate of returns. Therefore, the intermediary can fund entrepreneur's projects whose verification cost κ is smaller than κ^*, i.e., κ_θ^*.

The Debt Contracts Originated by the Traditional Banks

We assume that there exists no informational and transactional costs associated with the traditional banks. The debt contracts are information-insensitive in the sense that the bank only incurs the verification cost of the associated entrepreneur in case of a default.

The equilibrium debt contract $R_t(\kappa)$ originated in the traditional banking sector associated with an entrepreneur whose verification cost is κ solves

$$R_t(\kappa) - \kappa F\left(R_t(\kappa)\right) - \int_0^{R_t(\kappa)} F(\omega) d\omega = r_t. \tag{1.14}$$

The Diamond-Dybvig bank diversifies the debt contracts. Note that r_t denotes the market rate of return for traditional banks. As well, "bank-optimal" contract $\tilde{R}_t(\kappa)$ of the traditional banking sector associated with an entrepreneur whose verification cost is κ solves (1.13). By assumption, it turns out that the optimal contracts for both shadow and traditional banks for each entrepreneur are equivalent. Let (R^*, κ^*) characterize the marginal contract that solves both (1.13) and (1.14) where R^* shows the marginal gross rate of return associated with the marginal entrepreneur whose verification cost is κ^*. The private bank's arm length κ^* is a measurement for the loan origination capacity in the *traditional banking* sector.

First, an entrepreneur with verification cost κ will make offers from both traditional and shadow banks if loan creation capacities for both are sufficiently large, i.e., $\kappa \leq \min\left\{\kappa_\theta^*, \kappa^*\right\}$. By using the entrepreneur's expected payoff (1.10), the relevant debt contract will be the one that yields lower gross rate of return. In fact, the traditional bank's offer is relevant if $R_t(\kappa) \leq R_t^{\theta}(\kappa)$. Otherwise, the loan will be originated by the shadow banks.

Second, an entrepreneur's offer will be accepted by only one intermediary if his verification cost lies on the funding region of the traditional bank, but not the shadow bank or vice versa. Third, an entrepreneur's offer will not be accepted by neither traditional banks nor shadow banks if his verification cost κ is sufficiently large, i.e., $\max\left\{\kappa_{\theta}^{*}, \kappa^{*}\right\} < \kappa$.

Therefore, if $\kappa_{\theta}^{*} \leq \kappa^{*}$, the total demands $L(r_t^s)$ and $L(r_t)$ of loans originated by the shadow bank and the traditional bank, respectively, can be expressed by

$$L(r_t^s) = \sigma \int_0^{\kappa_{\theta}^{*}} I[R_t^{\theta}(\kappa) \leq R_t(\kappa)] g(\kappa) d\kappa, \tag{1.15}$$

$$L(r_t) = \sigma \left[G(\kappa^{*}) - G(\kappa_{\theta}^{*}) + \int_0^{\kappa_{\theta}^{*}} I[R_t(\kappa) < R_t^{\theta}(\kappa)] g(\kappa) d\kappa \right], \tag{1.16}$$

where $I(a_1 \leq a_2) = 1$ if $a_1 \leq a_2$. Otherwise, $I(a_1 \leq a_2) = 0$. For any entrepreneur with verification cost $\kappa \leq \kappa_{\theta}^{*}$, the shadow bank funds the projects only if

$$R_t^{\theta}(\kappa) \leq R_t(\kappa). \tag{1.17}$$

That is, the repayment of the equilibrium debt contract associated with the *shadow banking* sector is smaller than or equal to the repayment of the equilibrium debt contract associated with the *traditional banking* sector. The total demand of loanable funds (1.15) originated in the *shadow banking* system is the sum of the mass of projects associated with the entrepreneurs whose debt contracts satisfy (1.17). However, if the debt contracts associated with $\kappa \leq \kappa_{\theta}^{*}$ satisfy

$$R_t(\kappa) < R_t^{\theta}(\kappa), \tag{1.18}$$

the loan will be originated in the *traditional banking* sector. However, if an entrepreneur with verification cost κ satisfies $\kappa \in (\kappa_\theta^*, \kappa^*]$, then he will receive funding only from traditional banks and accept it. Therefore, the total loanable funds (1.16) originated in the *traditional banking* sector is the sum of the mass of projects associated with the entrepreneurs whose debt contracts satisfy (1.18) for $\kappa \leq \kappa_\theta^*$ and the mass of the entrepreneurs whose verification lies in $\kappa \in (\kappa_\theta^*, \kappa^*]$. Remember that G stands for the distribution of the verification cost with the density function g. As well, σ is the mass of entrepreneurs and hence total projects.

Similarly, if $\kappa^* < \kappa_\theta^*$, the total demands $L(r_t^s)$ and $L(r_t)$ of loans originated by the *NBFI* and the private bank, respectively, can be expressed by

$$L(r_t^s) = \sigma \left[G(\kappa_\theta^*) - G(\kappa^*) + \int_0^{\kappa^*} I[R_t^\theta(\kappa) \leq R_t(\kappa)] g(\kappa) d\kappa \right], \tag{1.19}$$

$$L(r_t) = \sigma \int_0^{\kappa^*} I[R_t(\kappa) < R_t^\theta(\kappa)] g(\kappa) d\kappa. \tag{1.20}$$

1.3 Equilibrium

First, we will characterize the solutions of the shadow bank's problem. The first order conditions for the shadow bank's problem (1.5) can be expressed by

$$q_t = \beta + \lambda_t^s, \tag{1.21}$$

$$r_t^s = \frac{1}{\beta + \lambda_t^s}, \tag{1.22}$$

where λ_t^s is the Lagrange multiplier for (1.6). Also, we will assume that the shadow bank's collateral constraint (1.6) binds and later check that it binds in equilibrium. Hence, the constraint must satisfy

$$l_t^s = r_t^s L_t^s. \tag{1.23}$$

Now we will characterize the solutions of the private bank's problem (1.7). The first order conditions for (1.7) are given by

$$q_t = \beta + \lambda_t(1 - \underline{\delta}_1), \tag{1.24}$$

$$r_t = \frac{1}{\beta + \lambda_t(1 - \underline{\delta}_2)}, \tag{1.25}$$

$$z_t^m = z_t^b = \frac{\psi_{t+1}}{\psi_t}(\beta + \lambda_t), \tag{1.26}$$

$$\frac{\beta \psi_{t+1}}{\psi_t} u'\left(\frac{\beta \psi_{t+1}}{\psi_t} c_t\right) = 1, \tag{1.27}$$

$$\beta u'(\beta d_t) = \beta + \lambda_t, \tag{1.28}$$

$$\begin{aligned} & k_t - z_t^m m_t - z_t^b b_t - L_t - \rho c_t \\ & - (1-\rho)\beta d_t + \frac{\beta \psi_{t+1}}{\psi_t}(m_t + b_t) + \beta l_t + \beta r_t L_t = 0, \end{aligned} \tag{1.29}$$

where λ_t is the lagrange multiplier for binding collateral constraint (1.9). We will check that it binds in equilibrium. Hence, (1.9) must satisfy

$$(1-\rho)d_t = \frac{\psi_{t+1}}{\psi_t}(m_t + b_t) + l_t(1 - \underline{\delta}_1) + r_t L_t(1 - \underline{\delta}_2). \tag{1.30}$$

In equilibrium, asset markets must clear. Hence, the bank's demand for currency, reserves and government bonds should be equal to the supplies coming from government, respectively. Hence, we have

$$\rho c_t = \psi_t C_t, \tag{1.31}$$

$$m_t = \psi_t M_t, \tag{1.32}$$

$$b_t = \psi_t B_t. \tag{1.33}$$

The quantity of non-contingent debt issued by the *NBFI* must be equal to the quantity of security purchased by the depositors or the private bank. Thus, it must satisfy

$$l_t^s = l_t. \tag{1.34}$$

The total demand of loans for entrepreneurial activity must be equal to the quantity supplied in the *shadow banking* sector. Therefore, it must satisfy

$$L_t^s = L(r_t^s), \tag{1.35}$$

where $L(r_t^s)$ can be characterized by (1.15) and (1.19).

The total demand of loans for entrepreneurial activity must be equal to the total quantity supplied in the *traditional banking* sector. Therefore, it must satisfy

$$L_t = L(r_t), \tag{1.36}$$

where $L(r_t)$ can be characterized by (1.16) and (1.20).

We confine our attention to the stationary equilibrium, in which all the nominal quantities grow at the constant growth rate μ and the real quantities are, respectively, equal forever. That is, gross rate of return on money is given by

$$\frac{\psi_{t+1}}{\psi_t} = \frac{1}{\mu} \quad \forall\, t. \tag{1.37}$$

Note the inflation must be at least as large as the discount factor, i.e., $\mu \geq \beta$. Otherwise, a seller is better off by choosing not to consume and preserve his money holdings in the CM; however, a buyer is willing to supply labor and thus market clearing condition does not satisfy.

Using the government budget constraints (1.3) and (1.4); the market clear conditions (2.162), (2.171) and (1.33), we will obtain

$$\rho c + z^m m + z^b b - q l^g - \tau_0 = 0, \tag{1.38}$$

$$V\left(1 - \frac{1}{\mu}\right) + \frac{m}{\mu}(z^m - 1) + \frac{b}{\mu}(z^b - 1) - l^g\left[q\left(\frac{1}{\mu} - 1\right) - 1\right] = \tau. \tag{1.39}$$

Suppose that the fiscal authority fixes the real value of tax in period 0. That is, $\tau_0 = V$ and V is exogenous. Then the real value of tax τ on each buyer in each period $t = 1, 2, 3, ...$ is determined by (1.39) and hence τ is endogenous. Thus in equilibrium, we have

$$\rho c + z^m m + z^b b = V. \tag{1.40}$$

Using (1.24)-(1.34) and (1.40), the stationary equilibrium allocation can be expressed by

$$q = \beta[\underline{\delta}_1 + (1 - \underline{\delta}_1)u'(\beta d)], \tag{1.41}$$

$$z^m = z^b = \frac{u'(\beta d)}{u'(\frac{\beta c}{\mu})}, \tag{1.42}$$

$$r = \frac{1}{\beta[\underline{\delta}_2 + (1 - \underline{\delta}_2)u'(\beta d)]}, \tag{1.43}$$

$$r^s = \frac{1}{\beta[\underline{\delta}_1 + (1 - \underline{\delta}_1)u'(\beta d)]}, \tag{1.44}$$

$$\lambda^s = \lambda(1 - \underline{\delta}_1) = \beta(1 - \underline{\delta}_1)(u'(\beta d) - 1) > 0, \tag{1.45}$$

$$l = \frac{1}{[\underline{\delta}_1 + (1 - \underline{\delta}_1)u'(\beta d)]} L\left(\frac{1}{\beta[\underline{\delta}_1 + (1 - \underline{\delta}_1)u'(\beta d)]}\right), \tag{1.46}$$

where L satisfies (1.15) and (1.19).

Let x_1 and x_2 denote the consumptions in the DM of currency trade and non-currency trade, respectively. The real yields of individual debt contracts that originated in the shadow banking and traditional banking sector, respectively, are given by

$$y_i = \frac{1}{\beta[\underline{\delta}_i + (1-\underline{\delta}_i)u'(x_2)]} - 1, \tag{1.47}$$

where $i = 1$ and $i = 2$ denote the shadow and traditional banking environments, respectively. The real bond yield is given by

$$y = \frac{1}{\beta u'(x_2)} - 1. \tag{1.48}$$

There exist wedge between private asset and public debt when collateral is scarce. The differentials between rate of return on private debt and safe government debt can be expressed by

$$w_i = \frac{\underline{\delta}_i(u'(x_2) - 1)}{\beta u'(x_2)[\underline{\delta}_i + (1-\underline{\delta}_i)u'(x_2)]} > 0. \tag{1.49}$$

As $\underline{\delta}_i$ increases, so does wedge between private and public debts. As the capital requirement for the receivables of debt contract is larger, the yield of private debt originated in the traditional banking sector is larger than the yield of private debt originated in the unregulated banking sector. Therefore, the entrepreneurs will enjoy higher payoffs through shadow banks than private banks if the distribution of project returns in both sectors are equal. Next we will define the differentials of rate of return on individual debt contracts originated in the regulated and unregulated banking sector:

$$w = r - r^s = \frac{(\underline{\delta}_2 - \underline{\delta}_1)(u'(x_2) - 1)}{\beta[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)][\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} > 0. \tag{1.50}$$

Since the collateralizable asset is sufficiently scarce to the extent that collateral constraint (1.9) binds and thus $x_2 < x^*$, both private assets and government debt bear liquidity premia. Note that $\frac{1}{\beta} - 1$ expresses the "fundamental" yield for both government debt and receivables of debt contracts originated by the shadow or private banks when the collateral is plentiful enough to render the incentive constraint. In fact, the liquidity premia for private debt contracts are given by

$$p_i = \frac{(1-\underline{\delta}_i)(u'(x_2)-1)}{\beta[\underline{\delta}_i + (1-\underline{\delta}_i)u'(x_2)]}. \tag{1.51}$$

The inequality (1.50) implies that the entrepreneurs can access to cheaper individual debt contracts associated with the shadow banking sector rather than those associated with the traditional banking sector. The marginal contract depends on the rate of return, the environment that the project returns are realized and the distribution of the project returns.

1.3.1 Distributions of Project Returns and Verification Costs

For simplicity, we assume that the distribution F of the project returns follows a uniform distribution on $\omega \in [0, \overline{\omega}]$, where $0 < \overline{\omega}$, that is, $F \sim \mathcal{U}\ [0, \overline{\omega}]$. In particular, F can be expressed by

$$F(\omega) = \frac{\omega}{\overline{\omega}} \qquad \forall \omega \in [0, \overline{\omega}], \tag{1.52}$$

where the density function f associated with (1.52) can be defined by

$$f(\omega) = \frac{1}{\overline{\omega}} > 0 \qquad \forall \omega \in [0, \overline{\omega}]. \tag{1.53}$$

As well, assume that the distribution G of verification cost follows a triangle distribution[3] on $\kappa \in [0, \overline{\omega}]$, that is, $G \sim \mathcal{T}\ [0, \overline{\omega}]$. In particular, G can be expressed by

$$G(\kappa) = 1 - \left(\frac{\overline{\omega} - \kappa}{\overline{\omega}}\right)^2 \qquad \forall \kappa \in [0, \overline{\omega}]. \tag{1.54}$$

The density function g associated with the cumulative distribution function (1.54) can be expressed

[3]The choices of the distribution functions for project return and verification cost are to express the marginal contract in the most explicit form. As long as the other choices of distribution functions satisfy the existence and uniqueness of the marginal contract, new results will be in the same line with those with current choices because the focus is not on the size of the shadow banking sector, but the relative use of shadow funded credit versus traditional banking lending and its impact on real economic activity.

by

$$g(\kappa) = \frac{2(\overline{\varpi} - \kappa)}{\overline{\varpi}^2} \qquad \forall \kappa \in [0, \overline{\varpi}]. \tag{1.55}$$

Therefore, payoff of an entrepreneur whose verification cost is κ in the *traditional banking* sector and *shadow banking* sector can be expressed by

$$\pi_\theta(\kappa) = \frac{\left(\overline{\varpi} - R_\theta(\kappa)\right)^2}{2\overline{\varpi}} \qquad \forall \kappa \in [0, \kappa_\theta^*], \tag{1.56}$$

$$\pi(\kappa) = \frac{\left(\overline{\varpi} - R(\kappa)\right)^2}{2\overline{\varpi}} \quad \forall \kappa \in [0, \kappa^*], \tag{1.57}$$

where $R_\theta(\kappa)$ and $R(\kappa)$ denote the equilibrium contracts offered by the shadow bank and traditional bank, respectively.

Equilibrium Debt Contracts and "Bank-Optimal" Contracts in the Traditional Banking Sector

By using (1.14), (1.52) and (1.53), the equilibrium debt contract $R(\kappa)$ associated with an entrepreneur whose verification cost is κ can be expressed by

$$R(\kappa) = \overline{\varpi} - \kappa - \left\{ (\overline{\varpi} - \kappa)^2 - \frac{2\overline{\varpi}}{\beta[\underline{\delta}_2 + (1 - \underline{\delta}_2)u'(x_2)]} \right\}^{\frac{1}{2}}. \tag{1.58}$$

The "bank-optimal" debt contract $\tilde{R}(\kappa)$ associated with an entrepreneur whose verification cost is κ can be characterized by

$$\tilde{R}(\kappa) = \overline{\varpi} - \kappa. \tag{1.59}$$

Therefore, the marginal contract (R^*, κ^*) associated with the gross rate of return R^* and the marginal entrepreneur whose verification cost is κ^* can be characterized by

$$\kappa^* = \overline{\varpi} - \left\{ \frac{2\overline{\varpi}}{\beta[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} \right\}^{\frac{1}{2}}, \tag{1.60}$$

$$R^* = \left\{ \frac{2\overline{\varpi}}{\beta[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} \right\}^{\frac{1}{2}}. \tag{1.61}$$

Remember that an intermediary's "arm length" is defined by its loan creation capacity associated with entrepreneurial credit. Note that $G(\kappa^*)$ shows the traditional bank's arm length to finance the entrepreneurs' projects. That is, the private bank chooses not to offer those entrepreneurs whose verification costs are larger than κ^*. Next proposition shows the sufficient conditions for existence of loan creation capacity originated by the private banks.

Proposition 1 *Suppose that distribution F of projects returns follows a uniform distribution on $\omega \in [0, \overline{\omega}]$. If $\beta\overline{\varpi} > 2$, then $\kappa^* > 0$, i.e., there exists a loan origination capacity of the traditional bank.*

Proof.

By using (1.60) and the assumption, we obtain

$$\kappa^* = \overline{\varpi} - \left\{ \frac{2\overline{\varpi}}{\beta[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} \right\}^{\frac{1}{2}} > \overline{\varpi} - \left(\frac{2\overline{\varpi}}{\beta} \right)^{\frac{1}{2}} > 0. \tag{1.62}$$

■

Equilibrium Debt Contracts and "Intermediary-Optimal" Contracts in the Shadow Banking Sector

By using (1.12), (1.52) and (1.53), the equilibrium debt contract $R_\theta(\kappa) \in \mathbb{R}^\theta$ associated with an entrepreneur whose verification cost is κ can be expressed by

$$R_\theta(\kappa) = \overline{\omega} - \kappa - \left\{ (\overline{\omega} - \kappa)^2 - \frac{2\overline{\omega}}{\beta(1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]} \right\}^{\frac{1}{2}}, \tag{1.63}$$

where $\theta \in (0,1)$.

The "intermediary-optimal" debt contract $\tilde{R}_\theta(\kappa) \in \tilde{\mathbb{R}}^\theta$ associated with an entrepreneur whose verification cost is κ can be characterized by

$$\tilde{R}_\theta(\kappa) = \overline{\omega} - \kappa. \tag{1.64}$$

Therefore, the marginal contract $(R_\theta^*, \kappa_\theta^*)$ associated with the gross rate of return R_θ^* and the marginal entrepreneur whose verification cost is κ_θ^* can be characterized by

$$\kappa_\theta^* = \overline{\omega} - \left\{ \frac{2\overline{\omega}}{\beta(1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]} \right\}^{\frac{1}{2}}, \tag{1.65}$$

$$R_\theta^* = \left\{ \frac{2\overline{\omega}}{\beta(1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]} \right\}^{\frac{1}{2}}. \tag{1.66}$$

Note that $G(\kappa_\theta^*)$ shows the shadow bank's arm length to finance the entrepreneurs' projects. That is, the shadow bank chooses not to accept the offers from those entrepreneurs whose verification costs are larger than κ_θ^*. Note that if θ increases, κ_θ^* will decrease, but R_θ^* will increase. That is, if the loans originated by the shadow bank become increasingly information sensitive, the intermediary will have weaker pull on funding the entrepreneurs' projects. The following proposition specifies the sufficient and existence conditions for the loan creation capacity of the shadow banking sector.

Proposition 2 *Suppose that distribution F of projects returns follows a uniform distribution on $\omega \in [0, \overline{\omega}]$ and $\beta\overline{\omega} > 2$. There exists a loan origination capacity of the shadow banking sector, i.e., $\kappa_\theta^* > 0$ if and only if either $\theta \leq \frac{\beta\overline{\omega}-2}{\beta\overline{\omega}}$ holds or $\frac{\beta\overline{\omega}-2}{\beta\overline{\omega}} < \theta$ and $x_2 < \hat{x}_\theta$ satisfy, where $\hat{x}_\theta$ satisfies*

$$u'(\hat{x}_\theta) = \frac{2 - \beta\overline{\omega}(1-\theta)\underline{\delta}_1}{\beta\overline{\omega}(1-\theta)(1-\underline{\delta}_1)}. \tag{1.67}$$

Proof.

It follows from (1.65) and $\beta\overline{\omega} > 2$. ■

Note that we have $\frac{\partial \kappa_\theta^*}{\partial \theta} < 0$ and $\frac{\partial k_\theta^*}{\partial x_2} < 0$. Therefore, an increase in θ amplifies liquidity constraints associated with the *shadow banking* sector. As well, if the quantity of DM exchange in the non-currency DM meetings and θ are sufficiently large, then the price of non-contingent debt contract between a shadow bank and a traditional bank will be small and thus the expected rate of return on each debt contract will be large to the extent that each entrepreneur is perceived as risky borrowers by the lender. As θ decreases, it mitigates the financial frictions arising from the liquidity constraint shown by (1.12). If the non-currency DM consumption x_2 is small in equilibrium, so is interest rate r_s offered by the shadow banks. Hence, the shadow banks are willing to fund the projects as there exists at least some safe entrepreneurs whose debt contracts can be acceptable.

The Figure 1.2 displays the equilibrium conditions for existence of loan creation by shadow banks in a $(\theta, \underline{\delta}_2)$ space. When the cost of operating *shadow banking* sector is sufficiently large, as depicted by Region I in the same figure, a shadow bank loses its loan creation capacity since the financial frictions are amplified. In fact, the intermediary's effective "market expected return" exhibits dramatic increase to be able to offset the financial friction as the debt contracts offered in the unregulated banking sector become highly information sensitive. By this increase, each entrepreneur will be burdened with large repayments to the extent that the repayment in the equilibrium debt contract exceeds the repayment of the optimal contract of the intermediary and thus no offer will be made. As well, note that an increase in $\underline{\delta}_2$ has no impact on the existence of loan creation capacity since $\underline{\delta}_1$, not $\underline{\delta}_2$, is the corresponding capital requirement for the liabilities of a shadow bank.

1.3.2 Equilibrium Loan Creation by Shadow Banks and Traditional Banks

In this subsection, we focus on the interaction between shadow and private banks associated with the entrepreneurs' projects. We will concentrate on the following questions: Under which conditions can a private bank reach more entrepreneurs' projects or vice versa? How does an increase in cost of monitoring technology in the shadow banking sector affect the equilibrium debt contracts and the entrepreneurs' payoff? How do the choices of entrepreneurs whose verification costs are within the range of both shadow bank's and traditional bank's arm length shape the total demand of funds?

Proposition 3 *Suppose that distribution F of projects returns follows a uniform distribution on $\omega \in [0, \overline{\omega}]$ and $\beta\overline{\omega} > 2$. Then the following statements are equivalent:*

I. The shadow bank's loan creation capacity associated with the entrepreneurial activity is larger than the traditional bank's loan creation capacity associated with the same activity, i.e., $\kappa^* < \kappa^*_\theta$;

II. The following statement satisfies

$$\theta < \frac{\underline{\delta}_2 - \underline{\delta}_1}{1 - \underline{\delta}_1} \quad \text{and} \quad x_2 < \tilde{x}_\theta, \tag{1.68}$$

where $\tilde{x}_\theta$ satisfies

$$u'(\tilde{x}_\theta) = \frac{\underline{\delta}_2 - (1-\theta)\underline{\delta}_1}{\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)}; \tag{1.69}$$

III. All the loans associated with entrepreneurial activity are originated by the shadow banking sector.

Proof.

The proof will be constructed as follows: (I) $\implies$ (II) $\implies$ (III) $\implies$ (I).

By using $F \sim \mathcal{U}\ [0, \overline{\omega}]$, $\beta\overline{\omega} > 2$, (1.60) and (1.65), $\kappa^* < \kappa^*_\theta$ implies (1.68). Thus, (I) implies (II). Using (1.68), (1.63) and (1.58), for any entrepreneur with $\kappa \leq \kappa^*$, the set of equilibrium debt contracts $\big(R_\theta(\kappa), R(\kappa)\big)$ originated by the *NBFI* and the private bank, respectively, satisfies

$$R_\theta(\kappa) < R(\kappa). \tag{1.70}$$

An entrepreneur whose verification cost satisfies $\kappa \leq \kappa^*$ receives offer from both the shadow bank and the traditional bank. By using (1.70), he will be better off by choosing the debt contract associated with the shadow banking sector. As well, an entrepreneur whose verification cost satisfies $\kappa \in (\kappa^*, \kappa^*_\theta]$ receives offer only from the shadow bank and he optimally accepts it. Therefore, all the loan creation occurs in the shadow banking sector. Thus, (II) implies (III). Finally, if all the loans are originated by the shadow bank, then (1.70) holds for all $\kappa \in [0, \kappa^*]$ and thus (II) satisfies. Suppose that $\kappa^*_\theta \leq \kappa^*$. Then by using (1.60) and (1.65), the following statement holds

$$\text{Either} \qquad \frac{\underline{\delta}_2 - \underline{\delta}_1}{1 - \underline{\delta}_1} \leq \theta \qquad \text{or} \qquad \theta < \frac{\underline{\delta}_2 - \underline{\delta}_1}{1 - \underline{\delta}_1} \qquad \text{and} \qquad \tilde{x}_\theta \leq x_2. \tag{1.71}$$

Therefore, (II) contradicts (1.71). Therefore, (III) implies (I). ■

When the shadow banks have longer reach on the entrepreneurs' projects than the traditional banks, by using (1.19) and (1.20), it turns out that the total demand of loans associated with the entrepreneurial activity in the *shadow banking* and *traditional banking* sectors, respectively, can be expressed by

$$L(r^s_t) = \sigma G(\kappa^*_\theta), \tag{1.72}$$

$$L(r_t) = 0, \tag{1.73}$$

where κ^*_θ satisfies (1.65). Note that no loans origination takes place in the traditional banking sector.

The Figure 1.2 shows the equilibrium conditions that determine the type of banking sector whose production capacity is larger and how much loan origination takes place in each sector. In this figure, the vertical and horizontal axes represent the cost of operating a shadow bank and the capital requirement for the receivables of the debt contracts originated in the regulated banking sector, respectively. The conditions in statement II of the Proposition 3 can be displayed by the Region III, as depicted in the same figure. Note that the line that has positive slope, as depicted with cross marker in the Figure 1.2, captures the pairs of $(\theta, \underline{\delta}_2)$ in which the traditional bank's arm length is equal to the shadow bank's arm length, i.e., $\kappa^*_\theta = \kappa^*$. It turns out that if θ is sufficiently small, the financial friction arising from the liquidity constraint in the unregulated banking sector is too small to cause the shadow banks avoid accepting individual debt contracts. As well, if $\underline{\delta}_2$ is sufficiently large, an entrepreneur associated with low verification cost who potentially receives offers from both sectors will be worse off by choosing the traditional bank's offer. As the regulatory institution holds heavy capital requirements on the receivables of debt contracts, the traditional bank's expected return on each contract becomes large enough to cause the liquidity creation depart from the *traditional banking* sector to the *shadow banking* sector. In fact, as $\underline{\delta}_2$ increases, an entrepreneur knows that he needs to repay more to honor the arrangement and receives less payoff from it as long as he chooses the private bank's offer to operate his project. This is consistent with the empirical findings. For example, Basel I which increased the capital requirements on commercial and industrial loans held in portfolio from 5.5% to 8% in 1990 reinforced the relative use of shadow funded credit.

Proposition 4 *Suppose that distribution F of projects returns follows a uniform distribution on $\omega \in [0, \overline{\omega}]$ and $\beta\overline{\omega} > 2$. Then the following statements are equivalent:*

I. The traditional bank's loan creation capacity associated with the entrepreneurial activity is larger than or equal to the shadow bank's loan creation capacity associated with the same activity, i.e., $\kappa^*_\theta \leq \kappa^*$;

II. Either $\frac{\delta_2-\underline{\delta}_1}{1-\underline{\delta}_1} \leq \theta$ *holds or the following statement satisfies:*

$$\theta < \frac{\delta_2 - \underline{\delta}_1}{1 - \underline{\delta}_1} \quad \textit{and} \quad \tilde{x}_\theta \leq x_2, \tag{1.74}$$

where $\tilde{x}_\theta$ *satisfies* (1.69).

III. All the loans associated with entrepreneurial activity are originated by the traditional banking sector.

Proof.

The proof will be constructed as follows: (I) $\Longrightarrow$ (II) $\Longrightarrow$ (III) $\Longrightarrow$ (I).

Using $F \sim \mathcal{U}\ [0, \overline{\omega}]$, $\beta\overline{\omega} > 2$, (1.60) and (1.65), $\kappa^*_\theta \leq \kappa^*$ implies that either $\frac{\delta_2-\underline{\delta}_1}{1-\underline{\delta}_1} \leq \theta$ holds or (1.74) satisfies. Thus, (I) implies (II). By using the debt contracts (1.58) and (1.63) associated with the traditional banks and shadow banks, respectively, implies that

$$R(\kappa) \leq R_\theta(\kappa) \qquad \forall \kappa \in [0, \kappa^*_\theta]. \tag{1.75}$$

Therefore, an entrepreneur whose verification satisfies $\kappa \leq \kappa^*_\theta$ receives offer from both the shadow bank and the traditional bank. By using (1.75), he will choose the debt contract associated with the *traditional banking* sector since it entails lower repayment in the next period. As well, an entrepreneur whose verification cost satisfies $\kappa \in (\kappa^*_\theta, \kappa^*]$ receives offer only from the regular bank and he optimally accepts it. Therefore, all the loan creation occurs in the shadow banking sector. Thus, (II) implies (III). If all the loan creation occurs in the traditional sector, $\kappa^* < \kappa^*_\theta$ contradicts (1.75). Therefore, (III) implies (I). ■

When the traditional banks have longer reach on the entrepreneurs' projects than the shadow banks, by using (1.15) and (1.16), it turns out that the total demand of loans associated with the entrepreneurial activity in the *shadow banking* and *traditional banking* sectors, respectively, can be expressed by

$$L(r_t^s) = 0, \tag{1.76}$$

$$L(r_t) = \sigma G(\kappa^*), \tag{1.77}$$

where κ^* satisfies (1.60). Note that no loans origination takes place in the *shadow banking* sector.

The conditions in statement II of the Proposition 4 can be displayed by the Region I and Region II, as depicted in the Figure 1.2. It turns out that if θ is very large, then the repayment from the equilibrium debt contract offered to the shadow bank does not make up the promise of the "intermediary-optimal" debt contract. Thus, for very large θ, the financial friction is strong to the extent that the shadow bank chooses not to accept the offers at all. The Region I displays this case in the same figure. As well, if θ is moderate, then the shadow bank will have loan creation capacity in equilibrium since the repayments in the equilibrium debt contract at least for some entrepreneurs will exceed the promise from the debt contract that maximizes the intermediary's expected payoff. However, the financial friction arising from the liquidity constraint in the *shadow banking* sector is large enough to cause entrepreneurs to make offer to traditional bank over the shadow bank. As well, if $\underline{\delta}_2$ is sufficiently small, an entrepreneur's offer to traditional bank is relevant since the rate of return from each debt contract associated with the *traditional banking* sector is sufficiently small. This yields larger payoff to the entrepreneurs whose projects are funded by the bank. The Region II, as depicted in the Figure 1.2, displays this case, namely $0 < \kappa_\theta^* \leq \kappa^*$.

By using (1.40)-(1.46), Proposition 3, (1.72) and (1.73); and Proposition 4, (1.76) and (1.77), the incentive constraint is given by

$$\begin{aligned}0 =& V - \rho x_1 u'(x_1) - (1-\rho) x_2 u'(x_2) + \frac{(1-\underline{\delta}_1)u'(x_2)}{\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)} \sigma G(\kappa_\theta^*) I(\kappa^* \leq \kappa_\theta^*) \\ &+ \frac{(1-\underline{\delta}_2)u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)} \sigma G(\kappa^*) I(\kappa_\theta^* < \kappa^*),\end{aligned} \tag{1.78}$$

where $I(\kappa_1 \leq \kappa_2) = 1$ if $\kappa_1 \leq \kappa_2$; otherwise, $I(\kappa_1 \leq \kappa_2) = 0$ for any $\kappa_1 \geq 0$ and $\kappa_2 \geq 0$. Since we assume that G is a triangular distribution that satisfies (1.54) and (1.55), by using (1.60) and (1.65), we obtain

$$G(\kappa^*) = 1 - \frac{2}{\beta\overline{\omega}[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]}, \tag{1.79}$$

$$G(\kappa_\theta^*) = 1 - \frac{2}{\beta(1-\theta)\overline{\omega}[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]}. \tag{1.80}$$

By using (1.42), the monetary policy tool z^b is given by

$$z^b = \frac{u'(x_2)}{u'(x_1)}. \tag{1.81}$$

In equilibrium, the nominal interest rate must be non-negative, i.e.,

$$z^b \leq 1. \tag{1.82}$$

The constraint (1.82) implies that zero lower bound must satisfy in equilibrium. Therefore, given the monetary policy z^b, the equilibrium allocation (x_1, x_2) satisfies (1.78), (1.81) and (1.82).

1.3.3 Conventional Monetary Policy

In this subsection, we will focus on the central bank's conventional monetary policy. More precisely, we will analyze how the equilibrium behaves near ZLB. It turns out that the function (1.78) associated with the incentive constraint exhibits a discontinuity at the level of non-currency DM consumption in which a shadow bank's loan creation capacity is equivalent to the traditional bank's capacity due to differing value of collateral with respect to the type of banking sector. We will address whether this jump causes ZLB to exist in equilibrium or not. If ZLB is feasible, is it optimal for the central bank to choose nominal interest rate to be zero? In fact, the central bank chooses the optimal z^b that yields the largest the welfare among feasible equilibrium allocations. Let us define the welfare measure by

$$W = \rho[u(x_1) - x_1] + (1-\rho)[u(x_2) - x_2]. \tag{1.83}$$

Therefore, the optimal monetary policy solves

$$\max_{x_1,x_2,z^b} \rho[u(x_1)-x_1]+(1-\rho)[u(x_2)-x_2] \tag{1.84}$$

subject to (1.78), (1.81), and (1.82).

For interesting results while doing policy analysis, we will assume

$$V+\frac{(1-\underline{\delta}_2)\sigma(\beta\overline{\omega}-2)}{\beta\overline{\omega}}<x^*. \tag{1.85}$$

The inequality (1.85) shows that the total value of consolidated government debt plus the total value of effective loans originated at the Friedman Rule does not make up the efficient quantity of DM exchange. Therefore, the Friedman Rule is not feasible in equilibrium. By using the case II of the Proposition 3, the loan creation will be originated by the *traditional banking* sector near Friedman Rule. Note that the fraction $\underline{\delta}_2$ of the receivables of debt contracts are illiquid in the sense that they are non-collateralizable. Next proposition shows the conditions under which the entrepreneurs always choose the debt contract offered by the private banks.

Let $F\sim\mathcal{U}[0,\overline{\omega}]$ and $G\sim\mathcal{T}[0,\overline{\omega}]$ denote the uniform and triangular distributions, respectively, on the same support $[0,\overline{\omega}]$, where F and G satisfy (1.52) and (1.54), respectively.

Proposition 5 *Suppose that* $F\sim\mathcal{U}[0,\overline{\omega}]$*,* $G\sim\mathcal{T}[0,\overline{\omega}]$*,* $-\frac{u''(x)x}{u'(x)}=\alpha<1$*, (1.85) and* $\beta\overline{\omega}>2$ *hold. Then if either* $\frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1}\leq\theta$ *holds or both expressions*

$$\theta<\frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1}, \tag{1.86}$$

and

$$\frac{\tilde{x}_\theta}{\underline{\delta}_2-\underline{\delta}_1-\theta(1-\underline{\delta}_1)}-\frac{(1-\underline{\delta}_2)\sigma\left\{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2-\underline{\delta}_1)-2[\underline{\delta}_2-\underline{\delta}_1-\theta(1-\underline{\delta}_1)]\right\}}{\beta\overline{\omega}(1-\theta)^2(\underline{\delta}_2-\underline{\delta}_1)^2}\leq\frac{V}{\underline{\delta}_2-(1-\theta)\underline{\delta}_1}, \tag{1.87}$$

satisfy, where $\tilde{x}_\theta$ satisfies (1.69)*, then for any feasible z^b, all the debt contracts associated with the entrepreneurs' projects will be originated by the traditional banks and $z^b = 1$ is optimal.*

Proof.

We have $\beta\overline{\omega} > 2$ and $F \sim \mathcal{U}[0,\overline{\omega}]$. Then by using Proposition 4, if either $\frac{\underline{\delta}_2 - \underline{\delta}_1}{1-\underline{\delta}_1} \leq \theta$ satisfies or both (1.86) and (1.87) satisfy, there exists no feasible monetary policy such that $x_2 < \tilde{x}_\theta$. Therefore, the traditional bank's can reach more entrepreneurs' projects. By using case (III) of Proposition 4, all the loans will be originated in the *traditional banking* sector. Moreover, by using $G \sim \mathcal{T}[0,\overline{\omega}]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$ and (1.78), the derivative of the incentive constraint is given by

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x_1)}{(1-\rho)(1-\alpha)u'(x_2) + H_1(x_2) + H_2(x_2)} < 0 \qquad \forall \ (x_1, x_2), \tag{1.88}$$

where $H_1(x)$ and $H_2(x)$ can be defined by

$$H_1(x) = \frac{-u''(x)\sigma\underline{\delta}_2(1-\underline{\delta}_2)}{[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x)]^2}\left[1 - \frac{2}{\beta\overline{\omega}[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x)]}\right] > 0, \tag{1.89}$$

$$H_2(x) = \frac{-2u''(x)u'(x)\sigma(1-\underline{\delta}_2)^2}{\beta\overline{[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x)]^3}} > 0. \tag{1.90}$$

Since the derivative of the incentive constraint is negative, as x_2 increases x_1 decreases. Given (1.87), there exists a unique allocation $x_1 = x_2 = x$ at the ZLB. We also have

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x)}{(1-\rho)(1-\alpha)u'(x) + H_1(x) + H_2(x)} > \frac{-\rho}{1-\rho}, \tag{1.91}$$

where $\frac{-\rho}{1-\rho}$ is the derivative of indifference curve associated with the welfare measure (1.83) evaluated at ZLB. Thus, $z^b = 1$ is optimal. ■

The proposition 5 states that if the cost of operating shadow banks or the consolidated government debt are sufficiently large, then zero nominal interest rate policy exists in equilibrium. Moreover, it will be optimal for the central bank to choose nominal interest rate to be zero. By using the

Proposition 4 and the sufficient conditions expressed above, it turns out that the derivative of the incentive constraint is always negative and hence there exists a unique monetary policy for any feasible z^b chosen by the central bank. Constant relative risk aversion with $\alpha < 1$ is crucial for the existence and uniqueness. For sufficiently large θ, the financial frictions arising from liquidity constraint associated with the *shadow banking* sector are amplified to the extent that the *NBFI*'s offers are too expensive. This does not only decrease the shadow bank's arm length on entrepreneur's projects but also decrease the expected payoff of the entrepreneurs who get offers. Those who can also get offers from the private banks will be better off by choosing the offers originated in the *traditional banking* sector at the expense of large capital requirements demanded by the central bank.

The Figure 1.3 is a numerical exercise for the Proposition 5 in which the economy reaches the largest welfare at the zero nominal interest rate. The incentive constraint (1.78) with $\tilde{x}_\theta \leq x_2$ describes a convex locus in (x_1, x_2) space, as depicted by the curve *IC*. The point *A*, as depicted in the Figure 1.3, shows the intersection of the ZLB and the IC. The welfare measure (1.83) describes a convex indifference curve *I* passing through *A*. Notice that the slope of the IC is flatter than the slope of the indifference curve *I*. Since no allocation is feasible in the lower triangle below the ZLB, the point *A* implies the optimal equilibrium allocation. Therefore, there exists no monetary policy away from the ZLB that accomplishes larger welfare.

Proposition 6 *Suppose that $F \sim \mathcal{U}[0, \overline{\omega}]$, $G \sim \mathcal{T}[0, \overline{\omega}]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$ and $\beta\overline{\omega} > 2$ hold. Then if* (1.86),

$$\frac{V}{\underline{\delta}_2 - (1-\theta)\underline{\delta}_1} < \frac{\tilde{x}_\theta}{\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)} - \frac{(1-\underline{\delta}_2)\sigma\left\{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2 - \underline{\delta}_1) - 2[\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)]\right\}}{\beta\overline{\omega}(1-\theta)^2(\underline{\delta}_2 - \underline{\delta}_1)^2}, \tag{1.92}$$

and

$$\frac{\tilde{x}_\theta}{\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)} - \frac{(1-\underline{\delta}_1)\sigma\left\{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2 - \underline{\delta}_1) - 2[\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)]\right\}}{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2 - \underline{\delta}_1)^2} \leq \frac{V}{\underline{\delta}_2 - (1-\theta)\underline{\delta}_1} \tag{1.93}$$

satisfy, where $\tilde{x}_\theta$ satisfies (1.69)*, then $z^b = 1$ does not exist equilibrium.*

Proof.

First, the function expressed by (1.78) exhibits a discontinuity at $x_2 = \tilde{x}_\theta$. In fact, as x_2 decreases, the economy switches from the regime in which all the loans are created by the traditional banks to the regime in which all the loans are created by the shadow banks. During this switch, for given $x_2 < \tilde{x}_\theta$, x_1 will increase dramatically. By using $F \sim \mathcal{U}[0, \overline{\omega}]$, $G \sim \mathcal{T}[0, \overline{\omega}]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$, $\beta \overline{\omega} > 2$ and the incentive constraint (1.78), for sufficiently low θ, (1.92) implies that zero nominal interest rate policy in which the loans are originated by the traditional banks occurs only if $x_2 < \tilde{x}_\theta$. This contradicts the Proposition 4. As well, (1.93) implies that ZLB binds in which the loans are originated by the shadow banks occurs in equilibrium only if $\tilde{x}_\theta \leq x_2$. This contradicts the Proposition 3. ■

Note that the function associated with the incentive constraint (1.78) is continuous everywhere except at $x_2 = \tilde{x}_\theta$. This shows the level of quantity of non-currency DM exchange in which the traditional bank's arm length equals to the one of the shadow bank. Since the liabilities of the *shadow banking* sector account for the asset in the *traditional banking* sector and capital requirement for non-contingent debt contract issued by the *NBFI* is lower than or equal to the capital requirement for the receivables of debt contracts originated by the traditional bank, it is cheaper for a buyer to post the security as collateral. Hence, this generates a jump at the point where θ is large enough to render equivalence on the production capacities between two banking sectors. As well, (1.92) implies that consolidated government debt is not large enough to satisfy non-currency DM consumption x_2 to be larger than $\tilde{x}_\theta$. As well, (1.93) implies that V is sufficiently large to the extent that it overshoots the equilibrium allocation at the ZLB.

The Figure 1.4 is a numerical exercise for the Proposition 6 in which the economy fails to satisfy ZLB. The incentive constraint (1.78) describes two convex loci in (x_1, x_2) space, as depicted by disconnected IC at $x_2 = \tilde{x}_\theta$. The point B, as depicted in the Figure 1.4, can be reached only if the central bank chooses positive nominal interest rate. As well, if x_2 goes below $\tilde{x}_\theta$, all the loan creation will depart from the Diamond-Dybvig banks to the shadow banks. This creates a jump in the IC curve from B to new allocation C. This jump displays the change in the regime where all the loan will be originated by the *NBFI* below C. However, the nominal interest rate set by the central bank undershoots the ZLB by the jump. Therefore, the point C, as depicted in the Figure 1.4, is not feasible since only negative nominal interest rate supports it. Therefore, ZLB is not feasible in equilibrium. The welfare measure (1.83) describes a convex indifference curve passing through B, as depicted by I. Notice that the slope of the IC is flatter than the slope of the indifference curve I. Therefore, monetary policy $z^b < 1$ that passes through origin and B is optimal.

Proposition 7 *Suppose that $F \sim \mathcal{U}[0,\overline{\omega}]$, $G \sim \mathcal{T}[0,\overline{\omega}]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$ and $\beta\overline{\omega} > 2$ hold. Then if* (1.86) *and*

$$\frac{V}{\underline{\delta}_2 - (1-\theta)\underline{\delta}_1} < \frac{\tilde{x}_\theta}{\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)} - \frac{(1-\underline{\delta}_1)\sigma\left\{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2 - \underline{\delta}_1) - 2[\underline{\delta}_2 - \underline{\delta}_1 - \theta(1-\underline{\delta}_1)]\right\}}{\beta\overline{\omega}(1-\theta)(\underline{\delta}_2 - \underline{\delta}_1)^2} \tag{1.94}$$

satisfy, where $\tilde{x}_\theta$ satisfies (1.69)*, then all the debt contracts associated with the entrepreneurs' projects will be originated by the shadow banks at the zero lower bound and $z^b = 1$ is optimal.*

Proof.

By using (1.78) and (1.94), if *ZLB* exists, i.e., $x_1 = x_2 = x$, then it will satisfy $x < \tilde{x}_\theta$. By using $F \sim \mathcal{U}[0,\overline{\omega}]$, $G \sim \mathcal{T}[0,\overline{\omega}]$, and $-\frac{u''(x)x}{u'(x)} = \alpha < 1$, the derivative of the incentive constraint implies

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x_1)}{(1-\rho)(1-\alpha)u'(x_2) + H_3(x_2) + H_4(x_2)} < 0 \qquad \forall\ (x_1, x_2), \tag{1.95}$$

where $H_3(x)$ and $H_4(x)$ can be defined by

$$H_3(x) = \frac{-u''(x)\sigma\underline{\delta}_1(1-\underline{\delta}_1)}{[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x)]^2}\left[1 - \frac{2}{\beta(2-\theta)\overline{\omega}[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x)]}\right] > 0, \tag{1.96}$$

$$H_4(x) = \frac{-2u''(x)u'(x)\sigma(1-\underline{\delta}_1)^2}{\beta\overline{(}1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x)]^3} > 0. \tag{1.97}$$

Therefore, x exists and is unique. By using $x < \tilde{x}_\theta$, $\beta\overline{\omega} > 2$, $F \sim \mathcal{U}[0,\overline{\omega}]$, (1.86) and Proposition 3, we obtain that all the loan creation will be originated in the shadow banking sector at $x_2 = x$. We also have

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x)}{(1-\rho)(1-\alpha)u'(x) + H_3(x) + H_4(x)} > \frac{-\rho}{1-\rho}. \tag{1.98}$$

That is, the derivative of the IC evaluated at ZLB is larger than the derivative of the indifference curve at ZLB, where $\frac{-\rho}{1-\rho}$ shows the derivative of the indifference curve at ZLB. Thus, $z^b = 1$ is optimal. ■

The proposition 7 states that if the cost of operating shadow bank and the consolidated government debt are sufficiently small, then zero nominal interest rate policy will exist in equilibrium and all the loans will be created in the shadow banking sector at the ZLB. Moreover, it is optimal for the central bank to choose nominal interest rate to be zero. By using the Proposition 3 and the sufficient conditions expressed above, it turns out that the derivative of the incentive constraint is always negative and hence there exists a unique monetary policy for any feasible z^b chosen by the central bank. Constant relative risk aversion with $\alpha < 1$ is crucial for the existence and uniqueness as in the Proposition 5. For sufficiently small θ, the financial frictions arising from liquidity constraint associated with the *shadow banking* sector are too small to cause entrepreneurs avoid accepting shadow bank's offer. Remember that the advantage of the debt contracts offered by the shadow banks is that the associated capital requirement $\underline{\delta}_1$ is lower and hence the shadow banks can have a stronger pull on financing the entrepreneurs' projects as long as θ is small enough. As θ increases, an entrepreneur with low verification cost is more willing to accept the traditional bank's offer.

The Figure 1.5 is a numerical exercise for the Proposition 7 in which the economy reaches the largest welfare at the zero nominal interest rate. The incentive constraint (1.78) with $\tilde{x}_\theta \leq x_2$ describes disconnected convex loci in (x_1, x_2) space, as depicted by the curve IC . The point D, as depicted in the Figure 1.3, shows the intersection of $x_2 = \tilde{x}_\theta$ and the IC. As the loan origination switches from one sector to the other, the IC jumps from D to E. The welfare measure (1.83) describes a convex indifference curve I passing through F, as depicted in the same figure. Notice that the slope of the IC is flatter than the slope of the indifference curve I. Since no allocation is feasible in the lower triangle below the ZLB, the point F implies the optimal equilibrium allocation. Therefore, there exists no monetary policy away from the ZLB that reaches a superior allocation.

1.3.4 Financial Crisis

After the global financial crisis, we observe decreases in the safe market rates of interest, increases in the wedge between real rate of return on risky debt and safe debt, and reductions in the credit market activity. In this model, we will concentrate on the factors that could undermine the economic activity by the disrupting lending capacity. As well, we will focus on the impact of these factors on the interest rates, inflation and consumption decisions.

The total demand of loans associated with the entrepreneurial activity can be characterized by Proposition 3, (1.72) and (1.73); and Proposition 4, (1.76) and (1.77). Potential factors that will shift the total demand curve are threefold: (a) a change in the distribution G of the verification costs of entrepreneurs; (b) a change in the distribution F of the project returns; (c) a change in the cost $\theta \in [0,1]$ of operating *shadow banking* system. These factors could be important to interpret the global crisis beginning late 2008 and the credit crunch in 2007-2009. Now suppose we will concentrate on two stages, pre-crisis and post-crisis environments. Assume that the financial crisis shock hits at the end of the first date and changes the equilibrium allocation thereafter.

Changes in the Distribution of the Verification Costs of Entrepreneurs

In this part, we carry out an experiment on the responses of a financial crisis shock that entails a change in the distribution of verification costs. The next proposition shows the impact of this experiment on the real activity.

Proposition 8 *Suppose that a financial crisis shock hits at end of the first date as a consequence of shift in the distribution G of the verification costs of entrepreneurs, i.e.,*

$$G^*(\kappa) = G(\kappa - \varepsilon) \qquad \forall \kappa \in [\varepsilon, \overline{\varpi} + \varepsilon] \tag{1.99}$$

for $\varepsilon > 0$. Then if $F \sim \mathcal{U}[0, \overline{\varpi}]$, $G^* \sim \mathcal{T}[\varepsilon, \overline{\varpi} + \varepsilon]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$, $\beta\overline{\varpi} > 2$ hold and a monetary policy z^b exists in both pre-crisis and post-crisis equilibrium, then for associated z^b, the inflation and price of the security will increase; real interest rates on government bonds, the consumptions on both currency, non-currency DM transactions, liquidity premia for both debt contracts associated with the entrepreneurs' projects and welfare will decrease in the post-crisis equilibrium. Finally, the expected payoff of the entrepreneur who gets funded after crisis does not change.

Proof. The post-crisis equilibrium satisfies

$$\begin{aligned} 0 =& V - \rho x_1 u'(x_1) - (1-\rho) x_2 u'(x_2) + \frac{(1-\underline{\delta}_1)u'(x_2)}{\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)} \sigma G^*(\kappa_\theta^*) I(\kappa^* \leq \kappa_\theta^*) \\ &+ \frac{(1-\underline{\delta}_2)u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)} \sigma G^*(\kappa^*) I(\kappa_\theta^* < \kappa^*), \end{aligned} \tag{1.100}$$

where $I(\kappa_1 \leq \kappa_2) = 1$ if $\kappa_1 \leq \kappa_2$; otherwise, $I(\kappa_1 \leq \kappa_2) = 0$ for any $\kappa_1 \geq 0$ and $\kappa_2 \geq 0$. Note that κ_θ^* and κ^* are not subject to the change since the distribution F of project returns does not change. Suppose that (x_1, x_2) and (x_1^*, x_2^*) define the equilibrium allocation in the pre-crisis and post-crisis equilibrium, respectively, for given monetary policy z^b. By definition, they exist. Moreover, the derivative of the pre-crisis and post crisis incentive constraints are given by

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x_1)}{(1-\rho)(1-\alpha)u'(x_2) - K'(x_2)} < 0, \tag{1.101}$$

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x_1)}{(1-\rho)(1-\alpha)u'(x_2) - K'^*(x_2)} < 0, \tag{1.102}$$

where $K(x)$ and $K^*(x)$ are given by

$$K(x) = \frac{(1-\underline{\delta}_1)u'(x)}{\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x)}\sigma G(\kappa_\theta^*)I(\kappa^* \le \kappa_\theta^*) + \frac{(1-\underline{\delta}_2)u'(x)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x)}\sigma G(\kappa^*)I(\kappa_\theta^* < \kappa^*),$$

$$K^*(x) = \frac{(1-\underline{\delta}_1)u'(x)}{\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x)}\sigma G^*(\kappa_\theta^*)I(\kappa^* \le \kappa_\theta^*) + \frac{(1-\underline{\delta}_2)u'(x)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x)}\sigma G^*(\kappa^*)I(\kappa_\theta^* < \kappa^*).$$

Since the derivative of incentive constraint is negative, (x_1, x_2) and (x_1^*, x_2^*) are unique in pre-crisis and post-crisis equilibrium for associated z^b. Suppose that $x_2 \le x_2^*$. Then we obtain

$$K(x_2) = \rho x_1 u'(x_1) - V \ge K(x_2^*) > K^*(x_2^*) = \rho x_1^* u'(x_1^*) - V, \tag{1.103}$$

since $G^*(\kappa) < G(\kappa)$ for all $\kappa \in [\varepsilon, \overline{\omega}]$. Using constant relative risk aversion property of utility function with $\alpha < 1$, we have $x_1^* < x_1$. Thus, we obtain

$$z^b = \frac{u'(x_2)}{u'(x_1)} > \frac{u'(x_2)}{u'(x_1^*)} \ge \frac{u'(x_2^*)}{u'(x_1^*)} = z^b, \tag{1.104}$$

which implies a contradiction. Therefore, we have $x_1^* < x_1$ and $x_2^* < x_2$. We are done. ■

The proposition 8 states that if the probability mass moves from left tail to right tail, both shadow bank and traditional bank will choose not to fund some entrepreneur whose verification costs are near marginal verification cost. Then total mass of projects that are funded by shadow banks or traditional banks will decrease. Note that the debt contracts promise receivables and these are posted as collateral to back the secured credit arrangement in non-currency DM meeting. Thus, receivables of debt contracts are highly liquid. A shift in the distribution of verification cost from G to G^* disrupts the credit by eliminating entrepreneurs' project whose verification costs are near margin. In turn, the credit disruption increases the price of collateral and hence less quantity of exchange takes place in the non-currency DM meeting. As well, a decrease in consumption decreases the rate of return on safe government debt, individual debt contract associated with entrepreneurs' projects and non-contingent debt contract issued by $NBFI$. Given the same monetary policy, as x_1

decreases, currency to consumption ratio rises and this increases the inflation. Using the welfare measure (1.83), declines in DM consumption implies a decrease in the welfare.

Changes in the Distribution of the Project Returns of Entrepreneurs

In this subsection, we are interested in exploring the impact of shift in the distribution of the project returns on the real activity. For simplicity, we will confine our attention to the class of distributions that entails a shift in the support of the project returns by preserving mean as the financial crisis shock associated with the distribution of project returns hits the economy. Suppose that at the end of the first period the distribution of project returns change from F on $[0,\overline{\omega}]$ to the new distribution F_ε on $[-\varepsilon,\overline{\omega}+\varepsilon]$ which can be expressed by

$$F_\varepsilon(\omega) = \frac{\omega+\varepsilon}{\overline{\omega}+2\varepsilon} \qquad \forall\omega \in [-\varepsilon,\overline{\omega}+\varepsilon] \qquad \text{for some} \quad \varepsilon > 0. \tag{1.105}$$

The density function f_ε associated with (1.105) is given by

$$f_\varepsilon(\omega) = \frac{1}{\overline{\omega}+2\varepsilon} \qquad \forall\omega \in [-\varepsilon,\overline{\omega}+\varepsilon] \qquad \text{for some} \quad \varepsilon > 0. \tag{1.106}$$

Note that change in ε preserves the mean which is $\frac{\overline{\omega}}{2}$. As well, the uniformity of the distribution does not change. This is a not typical shift that generates mean preserving spreads in increasing risk in the spirit of Rothschild and Stiglitz (1970) since risk is generated by extending the support to wider range. As ε rises, F_ε gets riskier on larger support in spite of preserving the mean. By using (1.105), (1.106) and (1.12), the equilibrium debt contract $R_{\theta,\varepsilon}(\kappa)$ associated with an entrepreneur whose verification cost is κ in the *shadow banking* sector can be expressed by

$$R_{\theta,\varepsilon}(\kappa) = \overline{\omega}+\varepsilon-\kappa+\left\{\kappa^2-2\kappa(\overline{\omega}+2\varepsilon)+(\overline{\omega}+2\varepsilon)(\overline{\omega}-\frac{2}{\beta(1-\theta)\overline{\omega}[\underline{\delta}_1+(1-\underline{\delta}_1)u'(x_2)]})\right\}^{\frac{1}{2}}. \tag{1.107}$$

As well, the "intermediary-optimal" debt contract $\tilde{R}_{\theta,\varepsilon}(\kappa)$ associated with an entrepreneur whose verification cost is κ can be expressed by

$$R_{\theta,\varepsilon}(\kappa) = \overline{\omega} + \varepsilon - \kappa. \tag{1.108}$$

Therefore, the marginal contract $(R^*_{\theta,\varepsilon}, \kappa^*_{\theta,\varepsilon})$ associated with gross rate of return $R^*_{\theta,\varepsilon}$ and a marginal entrepreneur whose verification cost is $\kappa^*_{\theta,\varepsilon}$ can be expressed by

$$\kappa^*_{\theta,\varepsilon} = \overline{\omega} + 2\varepsilon - \left\{ 2(\overline{\omega} + 2\varepsilon) \left(\frac{1}{\beta(1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]} + \varepsilon \right) \right\}^{\frac{1}{2}}, \tag{1.109}$$

$$R^*_{\theta,\varepsilon} = -\varepsilon + \left\{ 2(\overline{\omega} + 2\varepsilon) \left(\frac{1}{\beta(1-\theta)[\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)]} + \varepsilon \right) \right\}^{\frac{1}{2}}. \tag{1.110}$$

As well, the marginal debt contract $(R^*_{\varepsilon}, \kappa^*_{\varepsilon})$ originated by the private banks associated with gross rate of return R^*_{ε} and a marginal entrepreneur whose verification cost is κ^*_{ε} can be expressed by

$$\kappa^*_{\varepsilon} = \overline{\omega} + 2\varepsilon - \left\{ 2(\overline{\omega} + 2\varepsilon) \left(\frac{1}{\beta[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} + \varepsilon \right) \right\}^{\frac{1}{2}}, \tag{1.111}$$

$$R^*_{\varepsilon} = -\varepsilon + \left\{ 2(\overline{\omega} + 2\varepsilon) \left(\frac{1}{\beta[\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)]} + \varepsilon \right) \right\}^{\frac{1}{2}}. \tag{1.112}$$

It is important to note that as the distribution gets riskier, the arm lengths of the shadow and private banks decline, i.e., $\frac{\partial \kappa^*_{\theta,\varepsilon}}{\partial \varepsilon} < 0$ and $\frac{\partial \kappa^*_{\varepsilon}}{\partial \varepsilon} < 0$.

Proposition 9 *Suppose that a financial crisis shock hits at end of the first date as a consequence of shift in the distribution F of the project returns of entrepreneurs, i.e., it changes from F on $[0, \overline{\omega}]$ to F_ε on $[-\varepsilon, \overline{\omega} + \varepsilon]$ for $\varepsilon > 0$. Then if $F_\varepsilon \sim \mathcal{U}[-\varepsilon, \overline{\omega} + \varepsilon]$, $G \sim \mathcal{T}[-\varepsilon, \overline{\omega} + \varepsilon]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$, $\beta\overline{\omega} > 2$ hold and a monetary policy z^b exists in both pre-crisis and post-crisis equilibrium, then for associated z^b, the inflation and price of the security will increase; real interest rates on government bonds, the consumptions on both currency, non-currency DM transactions, liquidity premia for both debt contracts associated with the entrepreneurs' projects and welfare will decrease in the post-crisis equilibrium. Finally, expected payoff of an entrepreneur who gets funded after crisis* *decreases.*

Proof. The post-crisis equilibrium satisfies

$$
\begin{aligned}
0 =& V - \rho x_1 u'(x_1) - (1-\rho)x_2 u'(x_2) + \frac{(1-\underline{\delta}_1)u'(x_2)}{\underline{\delta}_1 + (1-\underline{\delta}_1)u'(x_2)} \sigma G(\kappa^*_{\varepsilon,\theta}) I(\kappa^*_{\varepsilon} \leq \kappa^*_{\varepsilon,\theta}) \\
&+ \frac{(1-\underline{\delta}_2)u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)} \sigma G(\kappa^*_{\varepsilon}) I(\kappa^*_{\varepsilon,\theta} < \kappa^*_{\varepsilon}),
\end{aligned} \tag{1.113}
$$

where $I(\kappa_1 \leq \kappa_2) = 1$ if $\kappa_1 \leq \kappa_2$; otherwise, $I(\kappa_1 \leq \kappa_2) = 0$ for any $\kappa_1 \geq 0$ and $\kappa_2 \geq 0$. Note that for $\varepsilon > 0$, the loan creation capacity of both banking sector associated with the entrepreneurs' projects decrease. Therefore, the effect of decreases in $\kappa^*_{\varepsilon,\theta}$ and κ^*_{ε} is parallel with the shift in the distribution of verification cost shown in Proposition 8. The rest follows from the proof of Proposition 8. ■

The Proposition 9 states that if the distribution of project returns get riskier, in the spirit of Proposition 8, we observe credit disruptions in both banking sectors. In this experiment, the risk is defined by enlarging the support, yet preserving the mean and the nature of distribution. It turns out both private and shadow banks can reach less projects after the crisis. Therefore, the verification cost of the marginal entrepreneur whose project is the last to be offered by the lender decreases since the contract that maximizes the lender's expected payoff increases with ε. Therefore, as ε rises, the range of verification costs in which the repayment in the equilibrium contract exceeds the one in the optimal contract shrinks. Thus, a lender's pull to capture the entrepreneurs' projects becomes weaker. In addition, each entrepreneur who is in the funding range of the lender must repay more since he needs to repay a risk premium. Therefore, in contrast to the results of Proposition 8, if an entrepreneur receives an offer after the financial crisis shock associated with the shift in F, his expected payoff will decrease. All the real activities follow the pattern as described in the Proposition 8.

Changes in the Cost of Operating a Shadow Bank

In contrast to the last two subsections, an increase in the cost of operating a shadow bank does not only disrupts the credit associated with the entrepreneurial activity, but also affects the location from which the loan origination takes place. In other words, by dramatic changes in θ, the debt contracts initially originated in the *shadow banking* sector might depart the scene and *traditional banking* sector might fill it by funding the projects returns or vice versa.

Proposition 10 *Suppose that* $F \sim \mathcal{U}[0,\overline{\omega}]$*,* $G \sim \mathcal{T}[0,\overline{\omega}]$*,* $-\frac{u''(x)x}{u'(x)} = \alpha < 1$*,* $\beta\overline{\omega} > 2$*,* $\theta_1 < \frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1}$ *and*

$$\frac{V}{\underline{\delta}_2-(1-\theta_1)\underline{\delta}_1} \leq \frac{\tilde{x}_{\theta_1}}{\underline{\delta}_2-\underline{\delta}_1-\theta_1(1-\underline{\delta}_1)} - \frac{(1-\underline{\delta}_1)\sigma\left\{\beta\overline{\omega}(1-\theta_1)(\underline{\delta}_2-\underline{\delta}_1)-2[\underline{\delta}_2-\underline{\delta}_1-\theta_1(1-\underline{\delta}_1)]\right\}}{\beta\overline{\omega}(1-\theta_1)(\underline{\delta}_2-\underline{\delta}_1)^2} \tag{1.114}$$

hold where $\tilde{x}_{\theta_1}$ *satisfies* (1.69)*. Then if the cost of operating a shadow bank changes from* $\theta = \theta_1$ *to* $\theta = \theta_2$ *such that* $\frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1} \leq \theta_2$ *after a financial crisis shock that hits at the end of the first date, then all the loan contracts associated with the entrepreneurs' projects that are initially originated by the shadow bank departs from the shadow banking sector to the traditional banking sector at the ZLB.*

Proof. Initially, by using $\theta_1 < \frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1}$, (1.114) and Proposition 3, all the loans are originated in the *shadow banking* sector near ZLB. However, by using Proposition 4, $\frac{\underline{\delta}_2-\underline{\delta}_1}{1-\underline{\delta}_1} \leq \theta_2$ implies that all the loans will now be originated in the *traditional banking* sector near ZLB. Therefore, by using (1.78) and $\kappa^*_\theta \leq \kappa^*$, the derivative of the incentive constraint is negative and hence there exists a unique monetary policy at ZLB. Moreover, the derivative of the incentive constraint is flatter than the derivative of the indifference curve at ZLB. Thus, $z^b = 1$ is optimal. ■

The Proposition 10 states that for sufficiently small θ_1 and consolidated government debt V in the pre-crisis environment, we observe that the *NBFI* does not only have a better reach on the entrepreneurs' projects, but also gross rate of return associated with shadow banking sector is more favorable by the entrepreneurs. However, when there is a change on the cost of monitoring technology to the extent that new θ_2 is sufficiently large, the financial frictions get amplified and entrepreneurs become better off by choosing private bank's contract offer if they get offers from both. As well, large θ_2 implies low capacity of loan origination by the shadow banks to the extent that shadow bank's arm length becomes shorter than that of private bank. Therefore, as θ increases, the debt contracts associated with shadow banking sector become increasingly information sensitive- as it occurred during the recent global financial crisis. In turn, the loan origination in the shadow banking sector vanishes and moves to the traditional banking sector.

The Figure 1.6 is a numerical exercise for the Proposition 10. The incentive constraint (1.78) describes discontinuous two convex loci in (x_1, x_2) space, as depicted by the curve IC_1 with bold line. The point H, as depicted in the Figure 1.6, shows the intersection of ZLB and the IC_1. As the regime switched from $\theta = \theta_1$ to $\theta = \theta_2$, the incentive constraint shifts to the curve IC_2 for $x_2 < \tilde{x}_{\theta_1}$, as depicted in the same figure. Note that if the central bank preserves optimal zero nominal interest rate policy, the equilibrium jumps from H to G and hence there exists a decline in credit activity in the traditional banking sector. It turns out that in the new regime all the loans will be originated from the private banks.

1.4 Central Bank's Unconventional Monetary Policy

The central bank's unconventional monetary policy captures the purchases of asset-backed securities issued by the $NBFI$. We will concentrate on the effects of this intervention on shadow banking sector's loan creation capacity over the traditional banking sector and the real activities such as quantity of DM exchange, rate of return on safe government debt and private debts associated with entrepreneurs' projects and welfare. t is important to note that the central bank sets a monetary policy tool $\tilde{q}$, that is how much the central bank will pay for a unit of security. The first type of intervention involves that the central bank purchases at the market price q. Second, the central bank accounts for all the security, i.e., $\tilde{q} > q$. In other words, the $NBFI$ chooses to sell everything to the central bank and hence the depositors and Diamond-Dybvig bank are phased out. Now we will reorganize the consolidated government budget constraint as follows

$$\rho c + z^m m + z^b b - \tilde{q} l^g - V = 0. \tag{1.115}$$

The equilibrium does not exist if $\tilde{q}$ goes to the infinity. To eliminate the existence problem, we will assume that whatever the central purchases as a means of unconventional intervention, the rate of return on this asset cannot be smaller than the rate of return on safe government debts. That is,

$$\frac{1}{\beta u'(x_2)} \leq \frac{1}{\tilde{q}}, \tag{1.116}$$

where $\frac{1}{\beta u'(x_2)}$ and $\frac{1}{\tilde{q}}$ denotes the rate of return of government debt and non-contingent private debt, respectively.

1.4.1 Central Bank and Private Market are Both Active

In this type of intervention, the central bank purchases the asset-backed security at the market price, respectively. Therefore, the central bank sets

$$\tilde{q} = q, \tag{1.117}$$

where the market price q is given by (1.41). Let l and l^g denote the supply of private loans from the buyers and the central bank, respectively. Then in equilibrium, the marker clears, i.e.,

$$l^s = l + l^g, \tag{1.118}$$

where l^s is the total demand. Then by using (1.41)-(1.46), (1.117), (1.115), Proposition 3, (1.72) and (1.73); and Proposition 4, (1.76) and (1.77), the incentive constraint is given by

$$
\begin{aligned}
0 =& V - \rho x_1 u'(x_1) - (1-\rho) x_2 u'(x_2) + \left(\beta \underline{\delta}_1 l^g + \frac{(1-\underline{\delta}_1) u'(x_2)}{\underline{\delta}_1 + (1-\underline{\delta}_1) u'(x_2)} \sigma G(\kappa_\theta^*) \right) I(\kappa^* \leq \kappa_\theta^*) \\
&+ \frac{(1-\underline{\delta}_2) u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2) u'(x_2)} \sigma G(\kappa^*) I(\kappa_\theta^* < \kappa^*),
\end{aligned}
\tag{1.119}
$$

where $I(\kappa_1 \leq \kappa_2) = 1$ if $\kappa_1 \leq \kappa_2$; otherwise, $I(\kappa_1 \leq \kappa_2) = 0$ for any $\kappa_1 \geq 0$ and $\kappa_2 \geq 0$. Note that neither private bank's nor $NBFI$'s production capacities exhibit a change after the purchases since the central bank competes at the market prices. Therefore, the rate of returns on each debt contract associated with the entrepreneurs' projects are not subject to the change. If the private bank has a longer reach on funding the entrepreneur's projects, i.e., $\kappa_\theta^* < \kappa^*$, then no debt contracts will be issued by the shadow banks and the central bank's purchases are irrelevant. However, if the $NBFI$'s arm length is larger, i.e., $\kappa^* \leq \kappa_\theta^*$, then an increase in l^g will increase the welfare and hence it will be optimal for the central bank to account for all the securities. That is, $l^s = l^g$ is optimal. Therefore, the incentive constraint can be rewritten by

$$
\begin{aligned}
0 =& V - \rho x_1 u'(x_1) - (1-\rho) x_2 u'(x_2) + \sigma G(\kappa_\theta^*) I(\kappa^* \leq \kappa_\theta^*) \\
&+ \frac{(1-\underline{\delta}_2) u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2) u'(x_2)} \sigma G(\kappa^*) I(\kappa_\theta^* < \kappa^*),
\end{aligned}
\tag{1.120}
$$

It turns out that central bank's purchase at the market rate increases welfare when the loans are originated by the $NBFI$ as it shifts the IC curve towards more superior equilibrium allocations. As well, the IC exhibits a larger jump at $x_2 = \tilde{x}_\theta$ with the purchases where the private bank's production capacity equals to the shadow bank's production capacity, $\kappa^* = \kappa_\theta^*$. Therefore, there might be feasible allocations at the zero nominal interest rate policy where ZLB fails to exist in equilibrium after the central bank's intervention due to large discontinuity.

1.4.2 Central Bank Accounts for All Securities

In this subsection, the central bank will purchase the asset-backed security at a higher price than what competitive asset market offers. Therefore, it must satisfy

$$\tilde{q} > \beta\left(\underline{\delta_1} + (1-\underline{\delta_1})u'(x_2)\right). \tag{1.121}$$

Then by using (1.41)-(1.46), (1.117), (1.115), Proposition 3, (1.72) and (1.73); and Proposition 4, (1.76) and (1.77), the incentive constraint is given by

$$\begin{aligned} 0 =& V - \rho x_1 u'(x_1) - (1-\rho)x_2 u'(x_2) + \sigma G(\kappa^*_{\theta,\tilde{q}})I(\kappa^* \leq \kappa^*_{\theta,\tilde{q}}) \\ &+ \frac{(1-\underline{\delta}_2)u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)}\sigma G(\kappa^*)I(\kappa^*_{\theta,\tilde{q}} < \kappa^*), \end{aligned} \tag{1.122}$$

where $\kappa^*_{\theta,\tilde{q}}$ can be expressed by

$$\kappa^*_{\theta,\tilde{q}} = \overline{\omega} - \left\{\frac{2\overline{\omega}}{(1-\theta)\tilde{q}}\right\}^{\frac{1}{2}}. \tag{1.123}$$

Note that as $\tilde{q}$ increases, so does $\kappa^*_{\theta,\tilde{q}}$. Therefore, it is optimal for the central bank to choose the largest feasible $\tilde{q}$. Hence, we have $\tilde{q} = \beta u'(x_2)$. Then the shadow bank's arm length κ^{**}_θ at optimum is given by

$$\kappa^{**}_\theta = \overline{\omega} - \left\{\frac{2\overline{\omega}}{(1-\theta)\beta u'(x_2)}\right\}^{\frac{1}{2}}. \tag{1.124}$$

Therefore, (1.122) can be rewritten as

$$\begin{aligned} 0 =& V - \rho x_1 u'(x_1) - (1-\rho)x_2 u'(x_2) + \sigma G(\kappa^{**}_\theta)I(\kappa^* \leq \kappa^{**}_\theta) \\ &+ \frac{(1-\underline{\delta}_2)u'(x_2)}{\underline{\delta}_2 + (1-\underline{\delta}_2)u'(x_2)}\sigma G(\kappa^*)I(\kappa^{**}_\theta < \kappa^*). \end{aligned} \tag{1.125}$$

Suppose again $\beta\overline{\omega} > 2$. Then the *shadow banking* sector's loan creation capacity as the central bank conducts optimal purchases is positive if and only if θ satisfies

$$\theta < \frac{\beta\overline{\omega}u'(x_2) - 2}{\beta\overline{\omega}u'(x_2)}. \tag{1.126}$$

Moreover, if θ satisfies

$$\underline{\delta}_2\left(\frac{u'(x_2) - 1}{u'(x_2)}\right) < \theta, \tag{1.127}$$

then the private bank's loan creation capacity associated with the entrepreneurs' projects will be at least as large as the *NBFI*'s loan creation capacity as the central bank conducts optimal purchases of asset-backed security at the price of government debt. That is, expressions $\theta < \underline{\delta}_2$ and $x_2 \leq \bar{x}_\theta$ satisfy, where $\bar{x}_\theta$ satisfies

$$u'(\bar{x}_\theta) = \frac{\underline{\delta}_2}{\underline{\delta}_2 - \theta}, \tag{1.128}$$

The Figure 1.7 displays the equilibrium conditions for existence of loan creation by the *NBFI* in a $(\theta, \underline{\delta}_2)$ space with and without central bank's unconventional intervention. When the cost of monitoring technology associated with the *shadow banking* sector is sufficiently large, as depicted by region I in the same figure, the *NBFI* has no loan capacity and hence purchases are irrelevant. In the region *II* and *III*, the central bank's purchases can only work for increasing shadow bank's capacity, but the private bank's arm length is larger than the *NBFI* even after the purchases because the financial frictions arising from large θ is too strong. The region *IV* captures the pair of $(\theta, \underline{\delta}_2)$ in which central bank purchase matters. Initially, all the loans are originated by the private banks; however, with the optimal purchases, the loans will be now created in the shadow banking sector. Therefore, the central bank's purchase matters in this region. In other words, with the optimal purchases the loan creation flows from the private banks to shadow banks. In the section V, the loans are already originated in the shadow banking sector and it gives an extra liquidity by decreasing the rate of return on the debt contract associated with the *NBFI*.

Proposition 11 *Suppose that* $F \sim \mathcal{U}[0, \overline{\omega}]$, $G \sim \mathcal{T}[0, \overline{\omega}]$, $-\frac{u''(x)x}{u'(x)} = \alpha < 1$ *and* $\beta\overline{\omega} > 2$ *hold. Then* *if* $\frac{\delta_2-\delta_1}{1-\underline{\delta}_1} \leq \theta < \underline{\delta}_2$ *and*

$$\frac{\bar{x}_\theta \underline{\delta}_2}{\underline{\delta}_2-\theta} - \frac{\sigma\{\beta\overline{\omega}(1-\theta)\underline{\delta}_2 - 2(\underline{\delta}_2-\theta)\}}{\beta\overline{\omega}(1-\theta)\underline{\delta}_2} \leq V \tag{1.129}$$

satisfy, where $\bar{x}_\theta$ *satisfies* (1.128)*, all the debt contracts associated with the entrepreneurs' projects that are initially originated by the traditional banks will be created by the shadow banking sector at the ZLB as the central bank conducts the optimal monetary policy. Moreover,* $z^b = 1$ *is optimal.*

Proof.

By $F \sim \mathcal{U}[0,\overline{\omega}]$, $G \sim \mathcal{T}[0,\overline{\omega}]$, $\beta\overline{\omega} > 2$, $\frac{\delta_2-\delta_1}{1-\underline{\delta}_1} \leq \theta$, (1.78) and Proposition 4, all the loans associated with entrepreneurial activity are originated by the traditional banking sector for any monetary policy. When the central bank conducts the optimal monetary policy, (1.125) and (1.129) imply that $x < \bar{x}_\theta$ where $x_1 = x_2 = x$ at ZLB. By using Proposition 3 and $\theta < \underline{\delta}_2$, all the loans will be originated by the shadow banks at ZLB. The derivative of (1.125) is given by

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x_1)}{(1-\rho)(1-\alpha)u'(x_2) - \frac{2\sigma u''(x_2)}{\beta(1-\theta)u'(x_2)^2}} < 0. \tag{1.130}$$

Hence, ZLB exists in equilibrium and x is unique. Moreover, the derivative of incentive constraint evaluated at ZLB is given by

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1-\alpha)u'(x)}{(1-\rho)(1-\alpha)u'(x) - \frac{2\sigma u''(x)}{\beta(1-\theta)u'(x)^2}} > \frac{-\rho}{1-\rho}, \tag{1.131}$$

where $\frac{-\rho}{1-\rho}$ is derivative of indifference curve at ZLB. Hence, $z^b = 1$ is optimal.

■

The Figure 1.8 is a numerical exercise for the Proposition 11. The incentive constraint (1.78) describes convex locus in (x_1, x_2) space, as depicted by the curve *IC* with bold line. The point *M*, as depicted in the Figure 1.8, is the intersection of the ZLB and IC. Note that all the loans are initially originated in the traditional banking sector. As the central bank conducts private asset purchases, the incentive constraint curve below the threshold value shifts from IC to $\breve{IC}$. New IC curve shows discontinuity. However, the purchase is welfare improving, shifting the curve towards northeast direction. As well, around ZLB the new incentive constraint implies that all the loans will be originated from the shadow banking sector. Thus, al the loans initially originated by the private bank will be now move to the shadow banking sector in a welfare increasing fashion. Finally, the point N, as depicted in the same figure, is the intersection of the ZLB and IC. Note that N exists and unique. The welfare measure (1.83) describes a convex indifference curve passing through N, as depicted by I. Notice that the slope of the IC is flatter than the slope of the indifference curve I. Therefore, N characterizes the optimal equilibrium allocation, i.e., there exists no monetary policy away from the ZLB that yields larger welfare.

1.5 Conclusion

A Lagos-Wright model with costly-state verification and delegated monitoring financial intermediation, and a risk-sharing framework of banking is constructed. First, lack of memory and limited commitment imply collateralized credit arrangements. Second, it is costly to operate *shadow banking system* and shadow bank is subject to the financial frictions arising from incentive problems. An intermediary who is subject to the minimum capital requirements and the cash withdrawal operates in the *traditional banking system.* In contrast, shadow banks are outside the purview of the regulatory limitations. By lack of regulation, a shadow bank has a comparative advantage over a traditional bank on loan creation capacity associated with the entrepreneurs' projects.

We carry out three different experiments exploring the effects of financial crisis shocks on real activities. First experiment captures a shift in the distribution of verification cost of entrepreneurs. In this experiment, we show that inflation and the price of asset-backed security increase; the consumption in both DM meetings, the rate of return on safe government debt and welfare decrease. If an entrepreneur still gets an offer from the lender after the shift, his expected payoff does not change. Second captures a shift in the distribution of project returns of entrepreneurs. In fact, the distribution gets riskier by enlarging the support, but preserving the nature of the distribution and the mean. This has the same impact on the real activities as the first shift does. However, it also decreases the loan creation capacities of the intermediations. It turns out that if an entrepreneur is still within arm length of the intermediation after this shift, he needs to repay the risk premium because increasing risk imposes larger payments on equilibrium contracts to be able to receive the fixed rate of return from each contract. Third shows a change in the cost of operating *shadow banking system*. Similar to first and second shift, the real activities are hampered by the change. Similar to second shift, it decreases each entrepreneur's expected payoff that are originated in the shadow banking sector. More importantly, we show that this shift can change the source of loan origination. In fact, liquidity creation in the shadow banks might depart the scene and private banks fill the void by financing new projects if costs are sufficiently large.

When a traditional bank's loan creation capacity equals to shadow bank's loan creation capacity, the incentive constraint exhibits a jump since the liabilities of the *shadow banking* sector account for the asset in the *traditional banking* sector and capital requirement for non-contingent debt contract issued by the shadow bank is weaker than the receivables of debt contracts originated by the private bank. Although both sectors generate equal amount of production capacity, the debt contracts originated by the shadow bank create larger liquidity. As a value of collateral is different according to which banking sector it is originated from, the function associated with incentive constraints might exhibit discontinuity near zero lower bound. However, if zero nominal interest rate is feasible, then it is always optimal.

Finally, we work on unconventional monetary policy. First, the central bank purchases the securities at the market rate. These purchases do not change the shadow bank's loan creation ability. If it is optimal to create loans in the shadow banking sector, the purchases are welfare increasing. However, if central bank purchases private asset at larger prices, this program will increase welfare by alleviating the financial frictions associated with shadow banks. These purchases, in turn, might bring the liquidity creation back to the *shadow banking system* from the *traditional banking* sector.

In other words, the central bank's purchases mitigate the financial friction arising from liquidity constraint associated with operating costs of shadow banks. We also show that if the cost of operating *shadow banking system* is sufficiently large, central bank's unconventional purchases will have no impact on the shadow bank's loan creation capacity.

Figure 1.1: The Transactions in the Centralized Market

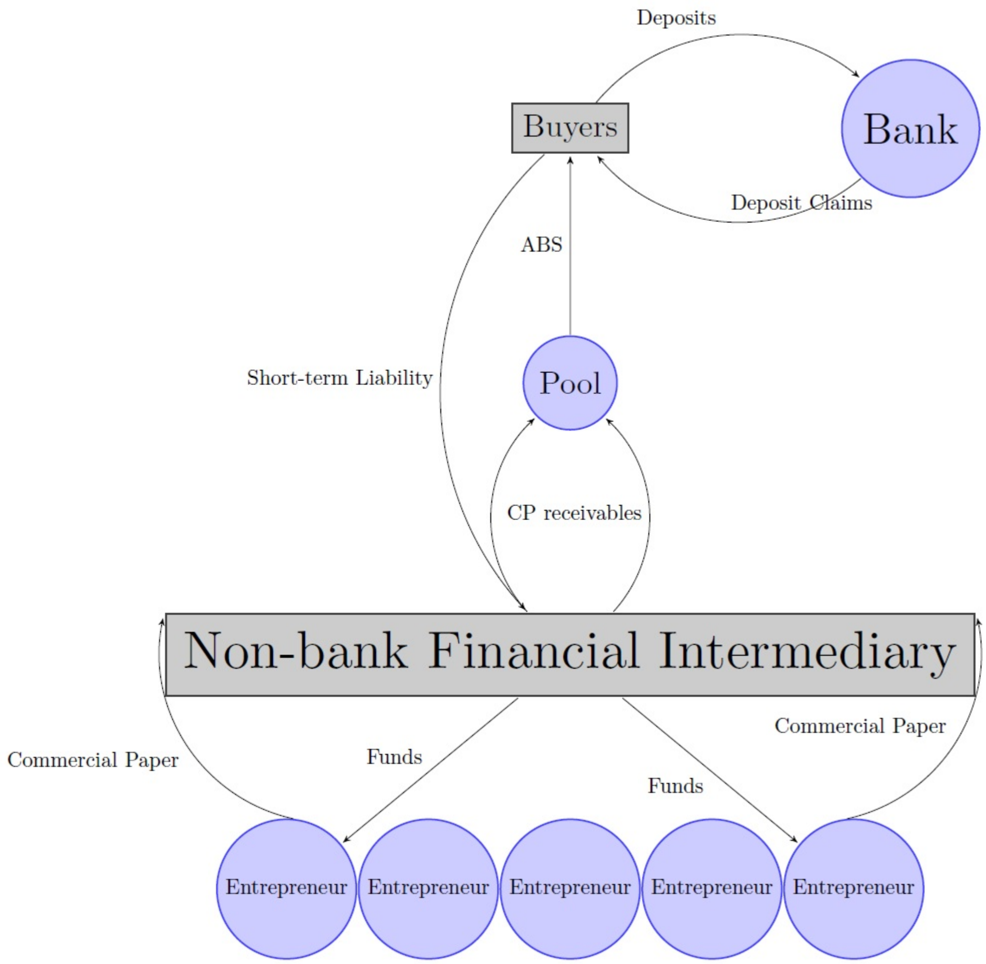


Figure 1.2: Equilibrium Conditions for Loan Origination in Different Banking Sectors

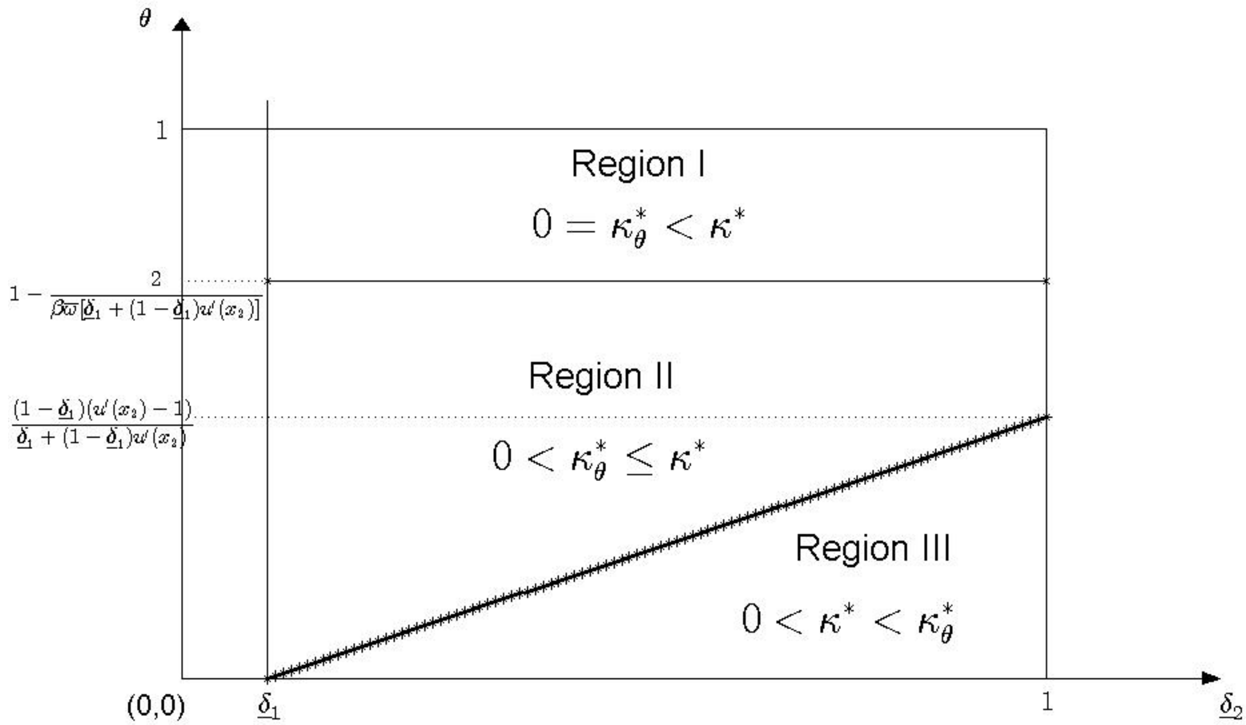


Figure 1.3: Conventional Monetary Policy I: All Loans Originated by Diamond-Dybvig Banks

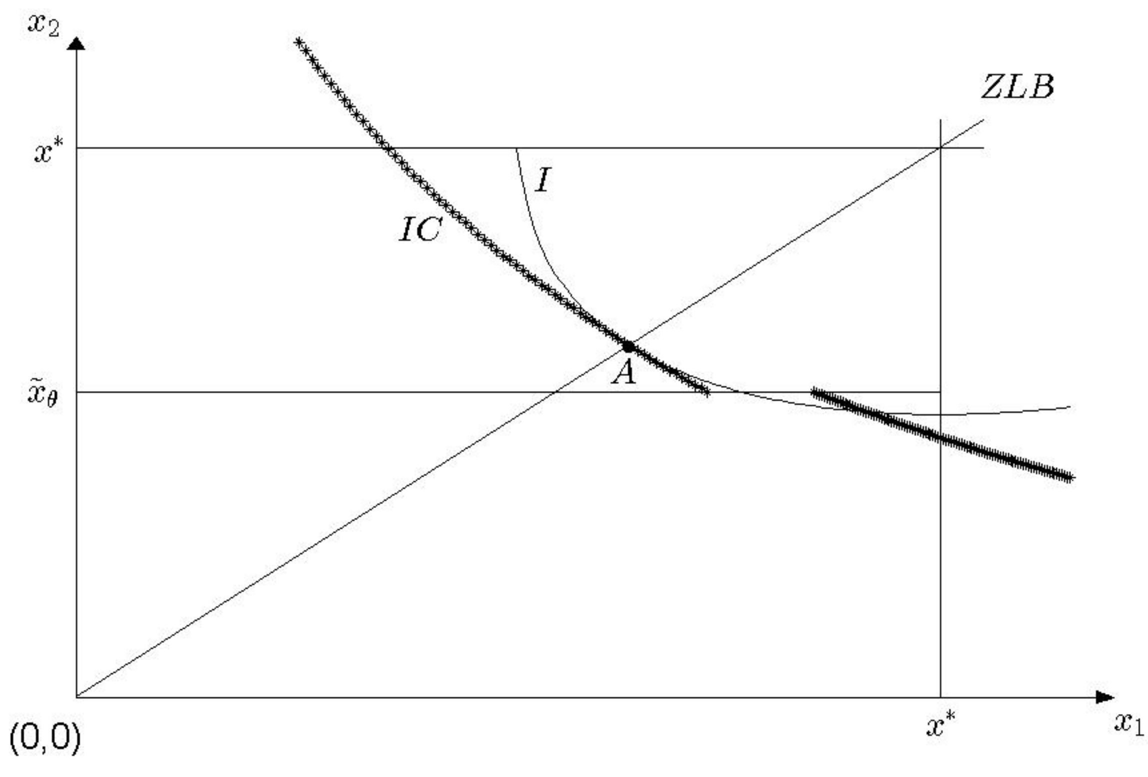


Figure 1.4: Conventional Monetary Policy II: ZLB not Feasible

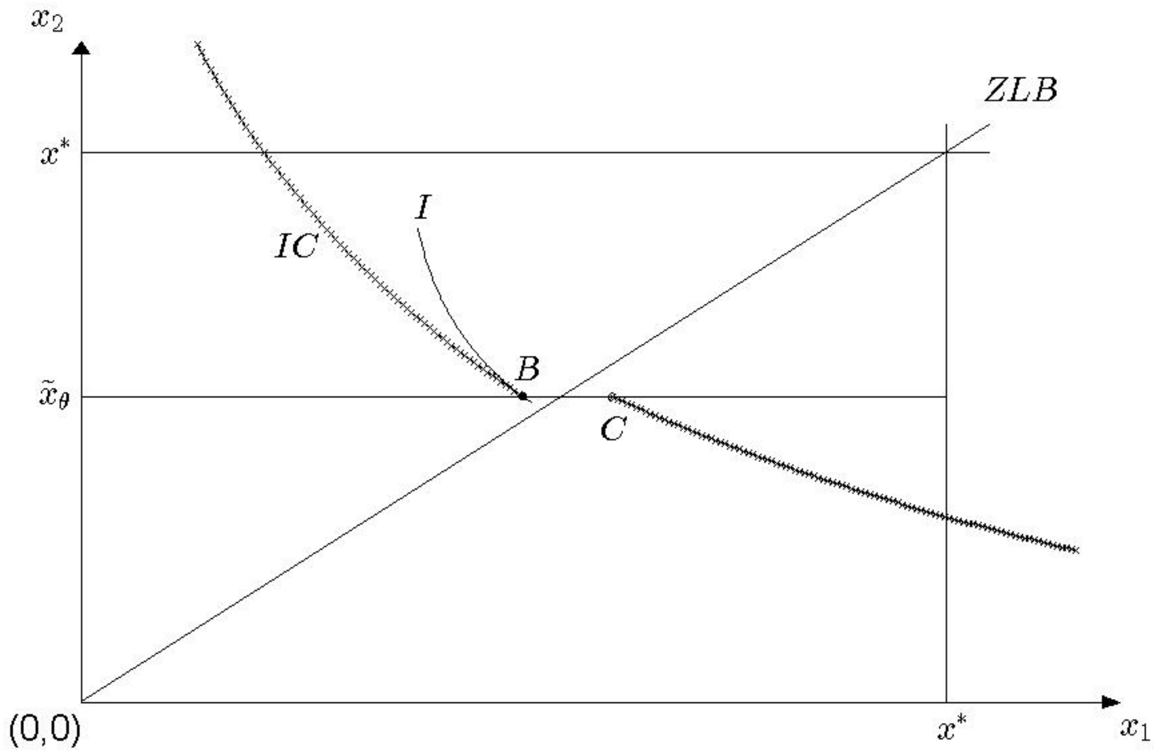


Figure 1.5: Conventional Monetary Policy III: All loans Originated by Shadow Banks at the ZLB

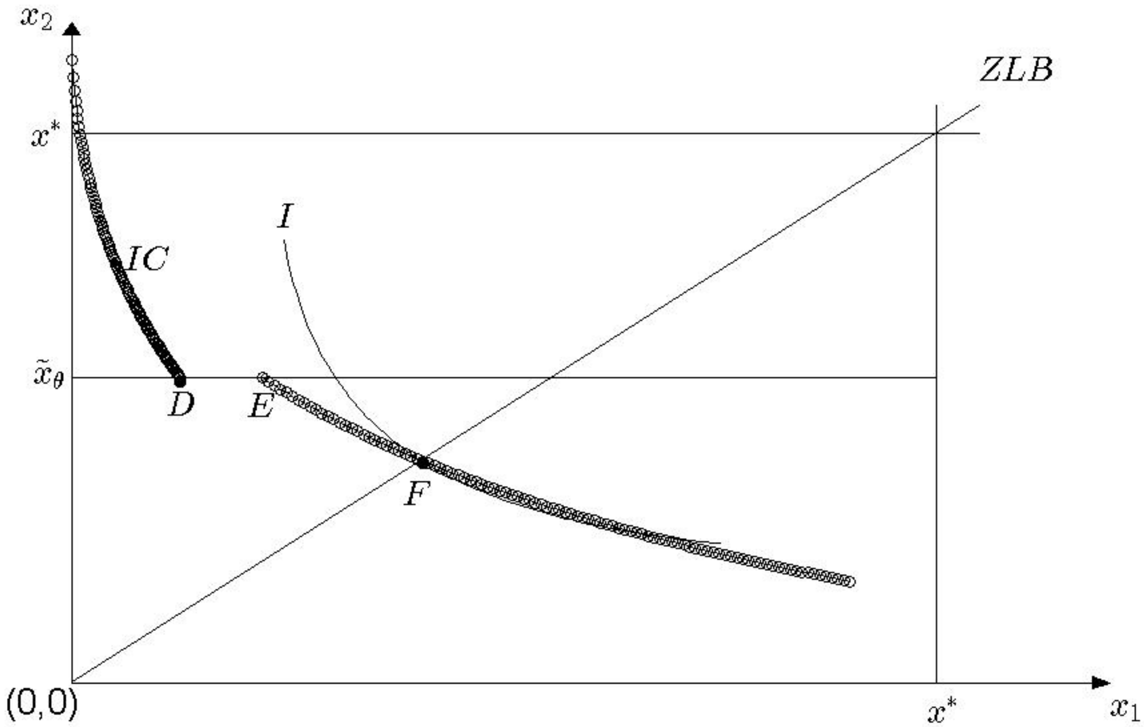


Figure 1.6: Increase in Cost of Operating Shadow Banks

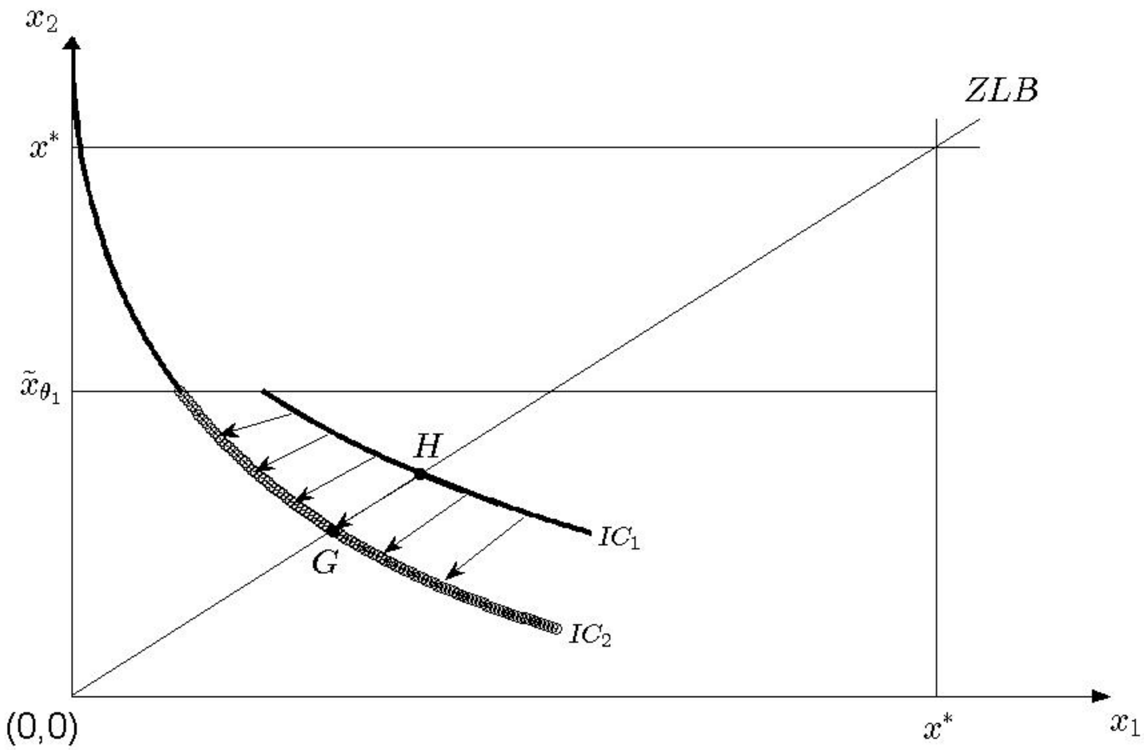


Figure 1.7: Equilibrium Conditions for Loan Origination in Different Banking Sectors with Central Bank Purchases

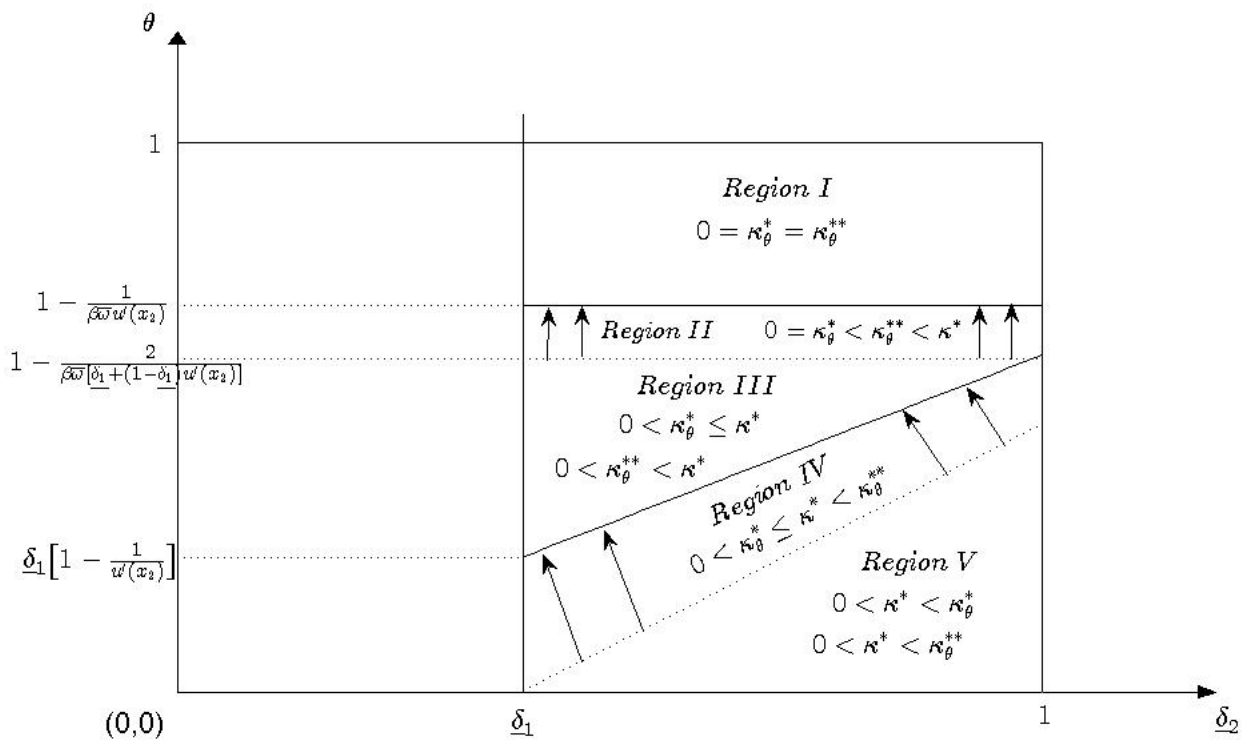
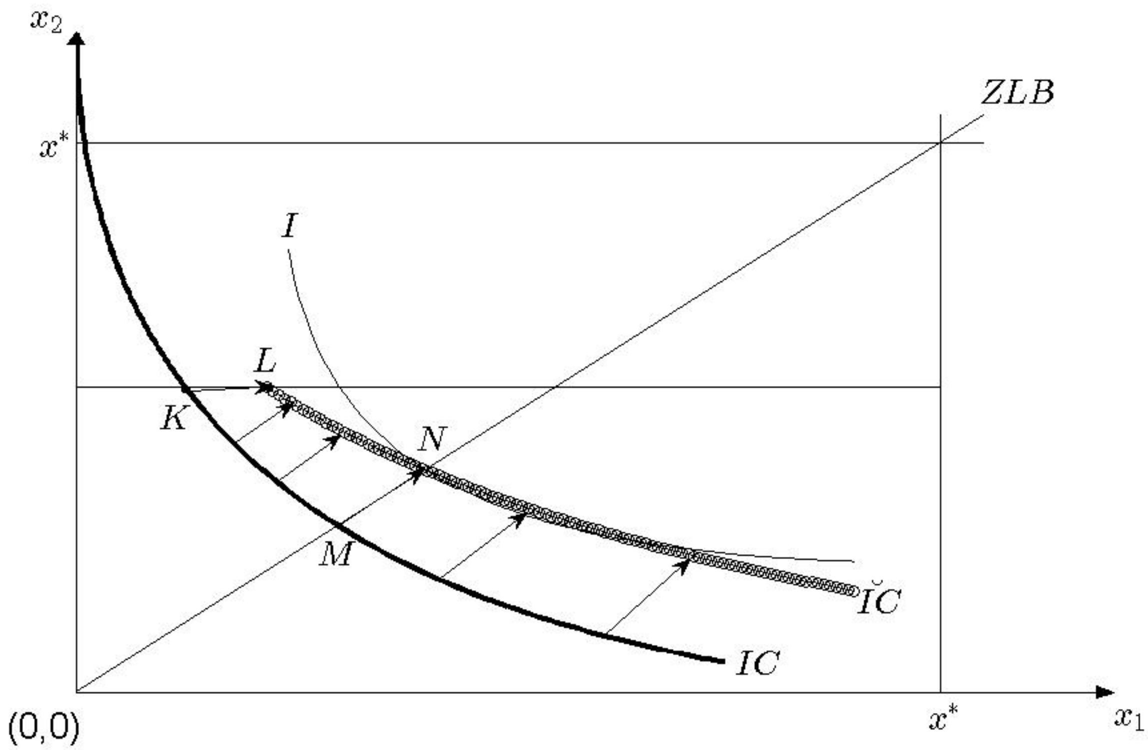


Figure 1.8: Unconventional Monetary Policy



2 Chapter 2: Collateralized Debt, Government Debt and Liquidity

2.1 Introduction

Housing provides direct utility- like a shelter. It also serves as a medium of exchange when the economy is inefficient. As well, housing can be useful to support credit. In fact, it facilitates intertemporal transactions when the credit markets are imperfect. Lack of commitment weakens the usage of unsecured lending and hence this creates a role for a use of housing as collateral to back the private credit arrangement- like a home-equity loan. A wide range of literature shows the empirical results that US households withdrew large volume of home-equity for consumption. These findings, as discussed in He, Wright, and Zhu (2015), Ferguson (2008), Disney and Gathergood (2011), Greenspan and Kennedy (2008) and Mian and Sufi (2011), are strong evidences that the housing assets are widely used to generate cash implying the existence of a liquidity premium. However, once home-equity becomes scarce- as it did in the recent financial crisis, and is now- then it amplifies the financial frictions. This paper explores macroeconomic credit frictions that can arise in the context of limited commitment and explores the potential role of housing in overcoming these inefficiencies. As in Williamson and Carapella (2014), Woodford (1990) and Kiyotaki and Moore (2012), we also study how the government debt enhances economic welfare. Moreover, we show that as liquidity constraints tighten up and the individuals start trading houses to make up for the weakness in collateral, safe government debt supports the private debt backed by housing and this mitigates the financial frictions associated with the liquidity constraints of the private agents.

The purpose of this paper is to build a model of housing that analyzes the role of housing as collateral and how public debt interacts with private debt in alleviating financial frictions associated with limited commitment and limited accessibility to public records. In this paper, we will address the following questions: Under what conditions does housing perform a useful role for collateral? Under what circumstances does a household prefer trading her house over posting it as collateral in exchange for consumption? How do supplies of private and public debt interact under limited commitment? Does government debt always enhance welfare when the credit markets are imperfect? Can a supply of government debt reinforce weak incentives on housing collateral as it becomes scarce in the state of economic downturn? How does an increase in nominal interest rate affect housing prices, collateralizable wealth and interest rate on home-equity loan near Friedman Rule?

The model that we build in this paper captures two different economic environments and focuses on the one after the other. On the one hand, we study the environment in which there exists no efficiency loss from the exchange of houses as all the individuals are potential buyers of houses. This environment can justify the behavior of the individuals and policy makers in the small economies with "shallow" home-equity loan market. It turns out that a use of collateral is costly and inefficient to the extent that the liquidity creation by a home-equity loan does not make up for the liquidity creation by a direct sale of housing. On the other hand, we analyze the environment in which asymmetry between agents on their willingness to consume housing services creates welfare loss from the exchange of houses. This environment captures the economies with significantly large home-equity loan market where this asymmetry might justify the efficient use of housing as collateral. Second, everybody faces limited commitment problem in the private engagements as in Kiyotaki and Moore (2012), Gertler and Kiyotaki (2010) and Williamson (2014a). Third, an individual posts his or her house as collateral to create liquidity and therefore we use home-equity to directly collateralize the consumption goods in the baseline model as in Kiyotaki and Moore (1997) and Kiyotaki and Moore (2005). We use some ideas in He, Wright, and Zhu (2015) by adding money- instead of government debt- and an intermediation to the model and analyzing the impacts of inflation and nominal interest rates on housing markets. Fourth, as in Kehoe and Levine (1993), Kocherlakota (1996) and Sanches and Williamson (2010), the default never occurs in equilibrium, but its threat can change the equilibrium allocation. Finally, we study the implications of safe government debt on welfare as in Williamson and Carapella (2014), Woodford (1990) and Kiyotaki and Moore (2012).

The model builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). One set of liquid assets captures safe government debt or currency where these are supplied by the government. The other set involves private assets: housing and non-contingent debt contract associated with housing collateral- like a home-equity loan. As well, there exist two types of financial agents in the economy: buyers and sellers. These agents have an access to frictionless Walrasian market in which debts and taxes are repaid, new government debts are issued and houses are traded under perfect competition. The non-contingent debts are originated in the frictional market in which the technology implies a buyer with an access to a seller and makes it possible for her to deal directly with associated seller rather than a given prices in centralized location.

First, there exists no efficiency loss from the house trade between buyers and sellers when both agents are willing to obtain housing for its services. This environment represents the economy in which both buyer and seller take the housing services into account as making consumption decisions at the frictional market. In particular, we show that it is never optimal for a buyer to use housing as collateral because limited collateralizable wealth prevents the buyer to activate liquidity creation up to full capacity. In contrast, housing trade entails largest quantity of exchange and hence welfare. Thus, the buyer who acquires the house in the centralized location will trade it again in every subsequent frictional market when the asset trade does not imply a welfare loss.

Second, the government issues short-term government bonds that are supported by taxation. We study how the government debt mitigates the financial frictions arising from liquidity constraints. Also, we study the interaction between public and private supplies of liquidity. In our model, one set of seller is subject to limited information on corresponding buyer's past tax records. On the other hand, the other set of seller is subject to full information on accessing the buyer's past tax records as in Williamson and Carapella (2014). The value-added in our paper relative to this paper is that no private engagements are recorded while there exists recordkeeping on tax payments. Moreover, the government's ability to collect taxes differs from the private agent's ability to collect collateralized loan repayments. In particular, the tax non-compliance implies full confiscation of the defaulter's bond holdings by the government. In contrast, each seller will seize a fraction of corresponding buyer's collateralizable wealth in case of default.

We will analyze the equilibrium allocations both with and without provisions of government debt as they are financed by tax payments of buyers. We will characterize the equilibria according to two different schemes of off-the-equilibrium punishment as government debt is supplied. On the one hand, *global punishment* implies autarky in case of a default. On the other hand, *individual punishment* implies that a defaulter can potentially pool with non-defaulters in the off-the-equilibrium path and avoid from the punishment in state of limited accessibility to public records on tax payments. In the spirit of Kehoe and Levine (1993), Kocherlakota (1996) and Sanches and Williamson (2010), there is only threat of a default. Indeed, the default never occurs in the equilibrium.

We show that the world is non-Ricardian, i.e., the economy always accomplishes larger welfare by the supply of government debt. In fact, it implies a larger set of information for private agents since a seller can reach the tax records of associated buyer in contrast to non-contingent private debt. This can make government debt relevant differing from the Ricardian economy under global punishment as in Williamson and Carapella (2014). That is, the Ricardian equivalence never holds independent of the punishment scheme. Therefore, the equilibrium allocation with the government debt is superior than the allocation without government debt unless the trade is efficient.

We also concentrate on the economy when only buyers can have the housing services. Suppose, for example, that there exists abundant supply of houses in the economy and each seller- like a retailer- can use the housing only for investment purposes. As in Williamson (2014a), this assumption implies the environment in which there exists an asymmetry on willingness to consume housing services between buyers and the sellers. The housing trade at the frictional market implies a welfare loss since a seller who acquires a house in exchange for consumption freezes the housing services. That is, by acquiring the house at the trade, a seller prevents buyers to benefit from its services in current period.

We show that when there exists an efficiency loss due to the asset trade, if the housing is sufficiently plentiful- the dividends are sufficiently rich, the pledgeability factor and the discount factor are sufficiently large- then it will be optimal for a buyer to post it as collateral. In other words, the buyer purchases the house at the first date and keeps it forever since collateral is always useful to support the trade. Housing bears liquidity premium, that is, its price is larger than the present discounted value of future stream of housing dividends. In contrast, if the housing is sufficiently scarce in quality, then it will serve as a medium of exchange. That is, once collateral becomes scarce then the financial frictions mount in the housing sector. Incentive constraint tightens up and a buyer optimally starts trading houses to make up for the weakness in collateral at the expense of future stream of housing dividends. Then a buyer will trade the asset in the frictional market and purchase it back in the subsequent centralized market. In spite of the welfare loss from the trade, a buyer avoids using her house as collateral because the cost of collateralized debt increases and immediate liquidation of the asset takes place as housing services, discount factor and pledgeability factor decrease. As well, we characterize the ”buffer” equilibrium in which the buyer is indifferent between trading the house and using it as collateral. In fact, it turns out that if the quality of collateral is moderate, then we will obtain the mixed equilibrium in which the buyer will trade some of her asset holdings and use the remaining as collateral.

Another key contribution of this paper is that when there exists a welfare loss in exchange for housing, the government debt alleviates the frictions by supporting housing collateral. In fact, by decreasing marginal utility of consumption, the government debt mitigates the financial frictions to the extent that posting housing as collateral is always optimal. In fact, the optimal purchases of government debt rule out the bad equilibrium allocation where a buyer exchanges housing for consumption. Similar to the first state with no surplus loss from the trade, the government debt accomplishes larger welfare and therefore the economy is always non-Ricardian. Further, we show that optimal purchases of the government debt and the full pledgeability completely eliminate the efficiency loss arising from the house trade as collateral activates the full capacity of liquidity creation.

We also concentrate on the framework of money and bank in which buyers can withdraw cash from the private banks and the cash loans are collateralized by the houses in the extension of the baseline model, as in Berentsen, Camera, and Waller (2007). The traditional bank is modeled as a risk-sharing framework of banking in the spirit of Diamond and Dybvig (1983). The buyers face preference shock about whether they are willing to consume or not in frictional market. Therefore, they are willing to have insurance against the need for cash in some meetings and hence a bank, by allocating the resources according to the appropriate transactions, enhances the welfare. We show that the inflation limits the consumption while the interest rate on cash loans limits the housing collateral. It turns out that when the financial frictions amplify- like an increase in haircut and a decrease in the collateralizable wealth- an increase in the nominal interest rate might increase the housing prices and hence collateralizable wealth. For low inflation, the bank deposits are not exhausted; an increase in the inflation rises the marginal utility of consumption and therefore liquidity premium. However, if the frictions are moderate, then an increase in the nominal interest rate will decrease the housing prices since an increase in the inflation puts more restriction on consumption than interest rate of home-equity loan puts on collateral. If the financial frictions are irrelevant, then the inflation has no impact on asset price since increase in inflation increases the interest rate on cash loan one-for-one.

For related papers, the effect of inflation on housing demand is captured in Aruoba, Davis, and Wright (2012), Kearl (1979), Poterba (1992) and Follain (1982). The effect of public debt on intergenerational transfers and optimal capital accumulation is covered in Diamond (1965).

The remainder of the paper is organized as follows. Second section captures the environment. The third section characterizes the equilibrium allocations with and without government debt when there exists no efficiency loss due to the housing trade. The fourth section characterizes the equilibrium allocations with and without government debt when there exists an efficiency loss due to the asset trade. The fifth section extends the baseline model with adding money- instead of government debt- and the private bank and displays the effects of nominal interest rate on the housing price. The sixth section concludes the paper.

2.2 Model

Time is indexed by $t = 1, 2, 3, ...$, and each period consists of two subperiods in which trade occurs. Each period begins with the centralized market (CM) and ends with the decentralized market (DM). Housing is one of the asset in the economy and it is in fixed supply with continuum of houses with unit mass. There are two types of agents with a unit mass: buyers and sellers. If a buyer holds a_t units of housing at the DM of period t, she will receive $a_t y$ units of housing services in the CM. Each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [F_t^b - H_t + u(x_t)] \tag{2.1}$$

where H_t is the labor supply during the CM, x_t is the consumption at the DM, and F_t^b is the dividend (housing service) that the buyers enjoy in the CM. As well, $\beta \in (0,1]$ is the discount factor. Notice that a buyer consumes only at the DM; however, she supplies labor only in the CM. Suppose that $u(.)$ satisfies certain properties: $u(.)$ is strictly concave and strictly increasing with $u(0) = 0$, $u'(0) = \infty$ and define x^* as the solution to $u'(x^*) = 1$. Each seller has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [F_t^s + X_t - h_t] \tag{2.2}$$

where X_t is the consumption in the CM, h_t is the labor supply at the DM and F_t^s is the dividend (housing service) that the sellers consume in the CM. Notice that a seller consumes only in the CM; however, she produces only at the DM. Each agent has a constant returns to scale technology when productive: A unit of labor yields one unit of perishable consumption good.

It is important to note that we will use two different assumptions on the given preferences of the sellers, respectively. The preference expressed by (2.2) entails the first assumption:

Assumption 1 *Every seller consumes the housing services.*

Assumption 1 entails the state of economy in which there exists no efficiency loss from the exchange of houses. Both buyers and sellers are potential buyers of houses. This environment captures the economy in which the home-equity loan market is shallow since housing collateral is inefficient. This economy can be interpreted as open economies with significantly small housing market.

A house is traded in the perfectly competitive market at a price ϕ_t in every CM of period t. Second, there is no money in this benchmark model although we will include money and the banking in the following section to make it more realistic. Third, a buyer is willing to make credit arrangements with a seller at the DM since she can not produce at this subperiod; she promises to make the repayment in the next CM unless she trades all the asset in exchange for consumption at the DM. Therefore, all the debts, if any, are settled in the CM.

We assume that there exists no recordkeeping for the private credit. As well, a credit arrangement involves a limited commitment friction: no buyer can be forced to repay a loan. Therefore, if a buyer acquires unsecured credit, she will default at the optimum. Hence, the private loan must be secured. That is, a buyer should post a collateral against the loan taken out at the DM and she can run away with a fraction $1-\theta$ of the collateral in the next CM in case of a default.

We interpret the DM as the frictional market as opposed to the frictionless CM. First, each buyer matches randomly with a seller at the DM. Holding a_t units of asset, a buyer makes a take-it-or-leave-it offer. The non-contingent debt contract can be summarized as follows: she borrows x_t consumption good at the DM of period t, she pledges a''_t units of housing as collateral and she trades a'_t units of housing at the DM. As well, she must repay D_{t+1} consumption good at the CM of period $t+1$. Posting collateral is costless and hence $a''_t + a'_t = a_t$. More specifically, a buyer who owns a_t units of assets takes out a private loan and the bargaining determines the optimal outcome

$(x_t, D_{t+1}, a'_t, a''_t)$. In case of default, the seller seizes fraction θ of collateral in the CM. We can interpret $\theta \in (0,1]$ as the pledgeability factor.

If the government issues short-term government bond, then the government finances it by acquiring tax returns τ_t from the buyers in the CM of each period t. Note that the buyers' tax payments will be recorded in each period. As well, we assume that the seller does not have an access to the buyer's past records at the DM with probability σ in each period. These meetings are called *limited information meetings*. On the other hand, the seller has an access to the buyer's past records at the DM with probability $1-\sigma$ in each period. As well, these meetings are called *full information meetings*. It is important to note that whether a buyer defaults on the tax return or not, her action will be recorded in the public record and therefore the public record is complete. In contrast, the private arrangements are never kept under the records.

A unit of bond sells at q_t units of consumption good in the CM of period t and it is a promise to return a unit of consumption good in the CM of period $t+1$. The bond market is perfectly competitive. If the buyer defaults on tax repayment, then the government will confiscate defaulter's bond holdings. The confiscation will take place only at the CM. When government issues B_t units of government debt in period t, government's budget constraint is given by

$$\tau_0 = -q_0 B_0, \tag{2.3}$$

$$\tau_t = B_{t-1} - q_t B_t, \qquad \forall\, t = 1,2,3,... \tag{2.4}$$

Consider the case when only buyers can benefit from the housing services. This creates an inefficiency on social surplus. Given posting collateral is costless, if the asset is more beneficial to a buyer than a seller per se, why would a household choose to liquidate her asset in exchange for the DM consumption? The household's action whether to pledge the asset as collateral or sell it depends on the limited commitment friction arising from the collateral constraint. It turns out that if the quantity of collateral increases when the financial constraint is binding, it will increase the social welfare by taming the inefficiency arising from the asset trade and hence makes the buyer better off. In contrast, this causes an extra disutility by tightening up the collateral constraint and hence decreases the quantity of exchange and welfare. The tradeoff between these opposing forces will determine the bargaining outcome at the DM. The collateral constraint, the main source of financial friction, captures the collateralizable wealth which depends on the discount factor, the pledgeability factor and the dividends. These fundamentals will determine the buyers' optimal action at the DM. For low pledgeability and poor dividends, posting collateral will be too expensive to take out. In contrast, if the asset is sufficiently plentiful, the housing collateral performs a useful role. Further, the bargaining rule entails a take-it-or-leave-it offer.

Consider the case when both buyers and sellers can consume the housing dividends. This eliminates the surplus inefficiencies and hence an increase in collateral only tightens up the collateral constraint; thus, this decreases the consumption and the welfare. Under Assumption 1, the buyer uses no collateral at the optimum. To specify these implications, we will obtain an explicit map that elucidates a buyer's optimal action as a function of fundamentals. In the next section, we will characterize the equilibrium under Assumption 1.

2.3 Equilibrium with Symmetric Intrinsic Values of Housing across Agents

We will concentrate on stationary equilibrium in which all the real quantities stay unchanged forever, respectively. In this section, we will characterize the equilibrium when the Assumption 1 satisfies.

2.3.1 The Buyer's Problem Without Government Debt

By Assumption 1, a seller consumes the dividends. Then if a seller obtains a' units of asset at the DM, he will acquire $a'(\phi + y)$ units of consumption good at the next CM. As well, the buyer makes a take-it-or-leave-it offer. Also, there exists a limited commitment friction and hence the buyer faces the collateral constraint. To be more specific, every buyer solves

$$S = \max_{a,D,x,a''} \left\{ -\phi a + u(x) - \beta D + \beta(\phi + y)a'' \right\} \tag{2.5}$$

s.t

$$x = \beta D - \beta(\phi + y)(a'' - a), \tag{2.6}$$

$$-D + (\phi + y)a''\theta \geq 0, \tag{2.7}$$

where (2.6) and (2.7) stand for the bargaining rule equation and collateral constraint, respectively. Remember that (x, D, a, a'') characterizes the contract where x and D denote loan at the DM and repayment at the CM in terms of consumption good, respectively. Also, a and a'' denote the buyer's asset purchase at the CM and collateral posted at the DM, respectively. Solving (2.5) implies that the buyers trades all the asset at the DM at the optimum and this statement is captured in the next proposition.

Proposition 12 *If the Assumption 1 holds, then no collateral will be posted at the optimum.*

Proof. If (2.7) does not bind, then posting collateral is irrelevant. If (2.7) binds, using (2.6), we obtain $x = \beta(\phi + y)a - \beta(\phi + y)a''(1 - \theta)$. Taking the first order condition with respect to a'' implies $\frac{\partial S}{\partial a''} = -\beta(\phi + y)(1 - \theta)(u'(x) - 1) < 0$ and hence $a'' = 0$. ■

This proposition states that a buyer who already purchases house in the CM sells her asset in exchange for consumption at the DM and then purchases again in the next CM. In fact, the buyer never carries her asset holdings to the CM since posting collateral effects the social welfare in a decreasing fashion. A buyer who takes out loan by posting all the asset as collateral can not create as much consumption as the one who trades all the asset since the pledgeability factor limits the buyer's ability to take out large enough loans for any quantity of collateral posted. Therefore, in this part housing serves as a medium of exchange. The buyers exchange houses for consumption goods at the DM.

Then by using Proposition 12, the buyer optimally trades all the asset and hence the problem (2.5) can be rewritten as

$$S = \max_{a,x} \left\{ -\phi a + u(x) - x + \beta(\phi + y)a \right\} \tag{2.8}$$

s.t

$$-x + \beta(\phi + y)a \geq 0, \tag{2.9}$$

For simplicity, suppose that $u(x) = 2x^{\frac{1}{2}}$ throughout the paper.

The Incentive Constraint (2.9) **Does not Bind.**

The first order conditions imply

$$\phi = \frac{\beta y}{1-\beta}, \tag{2.10}$$

$$x = x^* = 1. \tag{2.11}$$

The market clearing condition implies $a = 1$. This equilibrium holds if and only if

$$\frac{1}{1+y} \leq \beta. \tag{2.12}$$

The Incentive Constraint (2.9) **Binds.**

The first order conditions imply

$$\phi = \beta(\phi + y)u'(\beta(\phi + y)), \tag{2.13}$$

that is,

$$\phi = \frac{\beta + (\beta^2 + 4\beta y)^{\frac{1}{2}}}{2}, \tag{2.14}$$

$$x = \frac{\beta^2 + 2\beta y + \beta(\beta^2 + 4\beta y)^{\frac{1}{2}}}{2}. \tag{2.15}$$

This equilibrium holds if and only if

$$\beta < \frac{1}{1+y}. \tag{2.16}$$

Define β^* by

$$\beta^* = \frac{1}{1+y}. \tag{2.17}$$

If the discount factor is larger than the cut-off discount factor β^*, then the equilibrium is efficient; however, if the buyer's patience is sufficiently low, then the equilibrium does not support the efficient trade. We show this result in a numerical exercise, as depicted in Figure 2.1. This figure shows the types of equilibrium in (β, σ) space where vertical axis and horizontal axis represent the discount factor and the probability of limited information meetings at the DM, respectively. The upper rectangular area, above the horizontal line at $\beta = 1/2$, exhibits (2.12) and hence supports the unconstrained optimum. On the other hand, the lower rectangular area, below the horizontal line at $\beta = 1/2$, displays (2.16) and thus supports the constrained optimum where the trade is inefficient. Without the government debt, there exists no recordkeeping and hence the individual punishment takes place. The frequency of limited information meetings are therefore irrelevant. By using (2.14) and (2.15), an increase on the dividend will rise the asset price, the DM consumption and hence the welfare if the trade is inefficient.

2.3.2 The Buyer's Problem With the Government Debt

In this section, the government issues B units of government debt and acquires τ from the tax return in the CM in terms of consumption good. Similar to the collateralized debt, a buyer faces limited commitment problem: she cannot be forced to work to make the tax returns. If a buyer defaults, the government will track down the buyer's bond holdings and confiscate them. Remember that the confiscation only takes places at the CM. By using the Assumption 1 and Proposition 12, it is clear that a buyer is worse off by pledging collateral and hence no collateral will be posted at the optimum as in the last section. Then an optimizing buyer who can also purchase government debt solves

$$V = \max_{a,b,b'} \Big\{ -\phi a - qb + u\big(\beta(\phi + y)a + \beta b - \beta b'\big) - \beta\tau + \beta b' + \beta V \Big\} \tag{2.18}$$

s.t

$$-\beta\tau + \beta b' + \beta V \geq \beta\hat{V}, \tag{2.19}$$

where V, b and b' denote the buyer's value function on the equilibrium path, the buyer's bond purchase at the CM and the buyer's bond holding that are not traded at the DM. As well, $\hat{V}$ denotes the value function on the off-the-equilibrium path.

The first order conditions imply

$$\phi = \beta(\phi + y)u'(x), \tag{2.20}$$

$$q = \beta u'(x), \tag{2.21}$$

where

$$x = \beta(\phi + y)a + \beta b - \beta b'. \tag{2.22}$$

As well, the market clearing conditions for both asset market and bond market are given by

$$a = 1, \quad b = B, \quad \text{and} \quad \tau = B(1 - q). \tag{2.23}$$

2.3.3 The Government's Problem

In this section, we will express the government's problem. The government will issue the quantity of bond B that yields the highest the social welfare given the incentive constraint (2.19). Therefore, the government solves

$$\max_{B} \quad u(\beta(\phi+y)a+\beta B\big)-\beta(\phi+y)a-\beta B \tag{2.24}$$

s.t

$$-\beta B(1-q)+\beta V \geq \beta \hat{V}, \tag{2.25}$$

where V, q, a and ϕ satisfy (2.18), (2.21), (2.23) and (2.20), respectively, where x in (2.22) satisfies

$$x=\beta(\phi+y)a+\beta B. \tag{2.26}$$

As well, we will express $\hat{V}$ in detail in the following section.

2.3.4 Government Debt With the Global Punishment

In this section, we assume that if a buyer defaults on tax payments, then both housing and bond markets will entirely shut down. This entails the most severe punishment. In the next section, we will work on individual punishment which is more realistic.

In here, the punishment implies that if any buyer defaults, everybody gets nothing forever, i.e.,

$$\hat{V}=0. \tag{2.27}$$

In other words, a tax non-compliance triggers perpetual ostracism and thus both private market and bond market entirely shut down forever.

The Incentive Constraint (2.25) **Does not Bind.**

By using (2.27), (2.20), (2.21), and (2.22) and solving (2.24), then the DM trade is efficient as the tax constraint (2.25) does not bind. Hence x satisfies (2.11). As well, ϕ is at its fundamental value, i.e., the present discounted value of the future dividends. Thus, the asset price satisfies (2.10). As well, the bond price is at its fundamental value, i.e.,

$$q = \beta. \tag{2.28}$$

The incentive constraint (2.25) must satisfy. Then the necessary and sufficient conditions for this equilibrium to exist is given by

$$\frac{1}{y+2} \leq \beta. \tag{2.29}$$

The Incentive Constraint (2.25) **Binds.**

By using (2.27), (2.20), (2.21), and (2.22) and solving (2.24), as the liquidity constraint binds, the buyer's value function on the equilibrium path can be expressed by

$$V = \frac{-\phi a - qB + u\big(\beta(\phi + y)a + \beta B\big) - \beta B(1-q)}{1-\beta}. \tag{2.30}$$

The binding constraint (2.25) implies

$$B(1-q) = V. \tag{2.31}$$

Using (2.30) and (2.31), the asset price ϕ, the bond price q, the optimal quantity of bond B and the DM consumption x are given by

$$\phi = \frac{\beta^{\frac{1}{2}} y}{(\beta + y)^{\frac{1}{2}}}, \tag{2.32}$$

$$q = \frac{\beta}{\beta + \beta^{\frac{1}{2}}(\beta + y)^{\frac{1}{2}}}, \tag{2.33}$$

$$B = \frac{\left[\beta + \beta^{\frac{1}{2}}(\beta + y)^{\frac{1}{2}}\right]^2}{\beta^{\frac{1}{2}}(\beta + y)^{\frac{1}{2}}}, \tag{2.34}$$

$$x = \left[\beta + \beta^{\frac{1}{2}}(\beta + y)^{\frac{1}{2}}\right]^2 \tag{2.35}$$

Then this equilibrium exists if and only if

$$\beta < \frac{1}{y+2}. \tag{2.36}$$

Define β^{**} by

$$\beta^{**} = \frac{1}{2+y}. \tag{2.37}$$

The discount factor is at least as large as the new cut-off discount factor β^{**} if and only if the equilibrium is efficient. Otherwise, the equilibrium does not support the efficient trade. These results are exhibited as a numerical exercise in the Figure 2.2. The vertical axis and horizontal axis stand for the discount factor and the probability of limited information meetings at the DM, respectively. Figure 2.2 shows the sufficient and necessary conditions for each type of equilibrium. In fact, the upper rectangular area, above the horizontal line at $\beta = 1/3$, exhibits (2.29) and hence supports the unconstrained optimum. On the other hand, the lower rectangular area, below the horizontal line at $\beta = 1/3$, displays (2.36) and thus supports the constrained optimum where the trade is inefficient. The frequency of limited information meetings is irrelevant because a default on tax repayments implies perpetual ostracism. As well, by using (2.32), (2.33), (2.34) and (2.35), we obtain

$$\frac{\partial \phi}{\partial y} > 0, \quad \text{and} \quad \frac{\partial q}{\partial y} < 0; \tag{2.38}$$

$$\frac{\partial B}{\partial y} > 0, \quad \text{and} \quad \frac{\partial x}{\partial y} > 0. \tag{2.39}$$

By using (2.38) and (2.39), if the trade is inefficient, an increase in the dividends will increase the housing price, DM consumption and optimal quantity of public debt issued by the government;

however, it will decrease the bond price. That is, an increase in dividend make each house more expensive and then it creates more consumption goods at the DM and therefore rises the welfare. Since the private debt becomes more valuable against the public debt, the price of a bond will decrease. Note that the pledgeability factor has no impact on the equilibrium allocation since posting collateral is never optimal under Assumption 1.

2.3.5 Government Debt With the Individual Punishment

If a buyer has defaulted on tax payments and the matched seller is in the full information meeting, then latter will choose not to trade with the first. The seller knows that the defaulter's bond holdings used at the DM exchange will be confiscated in the subsequent CM by the government. We also assume that if she learns that the buyer is a defaulter, then neither bonds nor collateralized debt will be used as means of exchange for consumption and hence no trade takes place at the DM. This happens in the full information DM meetings where the seller can access to the public records. However, when the seller does not have an access to the public records, a defaulter will behave as if she has not defaulted. In another words, a defaulter optimally chooses to pool with the non-defaulters. Therefore, a defaulter chooses an allocation (a, B, a'', x, D) that is identical to the non-defaulter. If the defaulter deviates from the non-defaulter's strategy, then the seller will immediately distinguish the defaulter and choose not to trade with her. Thus, the off-the-equilibrium punishment can be expressed by

$$\hat{V} = \max\left[0, \frac{-\phi a - qB + \sigma u\big(\beta(\phi+y)a + \beta B\big) + (1-\sigma)\big(\beta(\phi+y)a + \beta B\big)}{1-\beta}\right] \tag{2.40}$$

The Incentive Constraint (2.25) **Does not Bind.**

By using (2.40), (2.20), (2.21), and (2.22) and solving (2.24), as the tax constraint is unbinding, the DM trade is efficient and hence x satisfies (2.11). As well, ϕ is at its fundamental value and thus it satisfies (2.10). Also, the bond price satisfies (2.28). A non-defaulter's value function V, a defaulter's value function $\hat{V}$ and the optimal quantity of bonds B, respectively, can be expressed by

$$V = \frac{u(x^*) - x^* - \beta^2(1-\beta)}{1-\beta}, \tag{2.41}$$

$$\hat{V} = \frac{\sigma\big(u(x^*) - x^*\big)}{1-\beta}, \tag{2.42}$$

$$B = \begin{cases} \frac{1-\beta-\beta y}{\beta(1-\beta)} & \text{if } \beta \in \big[\frac{1}{y+2-\sigma}, \frac{1}{1+y}\big) \\ 0 & \text{if } \frac{1}{1+y} \leq \beta. \end{cases}, \tag{2.43}$$

The incentive constraint (2.25) must satisfy in equilibrium. Hence, the necessary and sufficient condition for this equilibrium is given by

$$\frac{1}{y+2-\sigma} \leq \beta. \tag{2.44}$$

The Incentive Constraint (2.25) Binds and $\hat{V} > 0$.

The buyer's value function V can be expressed by (2.30). In this case, the off-the-equilibrium punishment $\hat{V}$ is positive, i.e., a defaulter optimally pools with the non-defaulters and the asset prices are cheap enough to satisfy pooling in the off-the-equilibrium path in contrast to autarky. By using (2.40), (2.20), (2.21), (2.22) and the binding constraint (2.25), the asset price ϕ, the bond price q, optimal quantity of bonds B and the DM consumption x, respectively, are given by

$$\phi = \frac{2\beta y\big(1+\beta(1-\sigma)\big)}{\beta(3-2\sigma) - 2\beta\big(1+\beta(1-\sigma)\big) + \big[\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\big]^{\frac{1}{2}}}, \tag{2.45}$$

$$q = \frac{2\beta\big(1+\beta(1-\sigma)\big)}{\beta(3-2\sigma) + \big[\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\big]^{\frac{1}{2}}}, \tag{2.46}$$

$$B = \frac{\beta(3-2\sigma)^2 + 2y\big(1+\beta(1-\sigma)\big) + (3-2\sigma)\left[\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\right]^{\frac{1}{2}}}{2\big(1+\beta(1-\sigma)\big)^2}$$

$$- \frac{\beta(3-2\sigma)y + \left[\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\right]^{\frac{1}{2}} y}{\beta(3-2\sigma) - 2\beta\big(1+\beta(1-\sigma)\big) + \left[\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\right]^{\frac{1}{2}}}, \tag{2.47}$$

$$x = \frac{\left[\beta(3-2\sigma) + \left(\beta^2(3-2\sigma)^2 + 4\beta y\big(1+\beta(1-\sigma)\big)\right)^{\frac{1}{2}}\right]^2}{4\big(1+\beta(1-\sigma)\big)^2}. \tag{2.48}$$

The binding constraint (2.25) must satisfy in equilibrium. Hence, we obtain

$$\beta < \frac{1}{y+2-\sigma}; \tag{2.49}$$

In equilibrium, $\hat{V} > 0$ holds, i.e.,

$$\frac{1}{2} \leq \sigma, \quad \text{or} \quad \sigma < \frac{1}{2} \quad \text{and} \quad \frac{(1-2\sigma)^2}{y(1-\sigma)^2 + 2(1-\sigma)(1-2\sigma)} < \beta. \tag{2.50}$$

The Incentive Constraint (2.25) Binds and $\hat{V} = 0$.

The defaulter's value function $\hat{V}$ satisfies $\hat{V} = 0$. The financial frictions exacerbate and the incentive constraint is severely binding. Then large liquidity premia cause defaulters to abandon the asset markets and hence off-the equilibrium path is autarky. By using (2.40), (2.20), (2.21), (2.22) and the binding constraint (2.25), the asset price ϕ, the bond price q, optimal quantity of bonds B and the DM consumption x, respectively, are expressed by (2.32), (2.33), (2.34) and (2.35). In other words, the equilibrium allocation with individual punishment coincides the one with the global punishment in here.

The necessary and sufficient conditions for this equilibrium to exist are given by

$$\sigma < \frac{1}{2} \quad \text{and} \quad \beta \leq \frac{(1-2\sigma)^2}{y(1-\sigma)^2 + 2(1-\sigma)(1-2\sigma)}. \tag{2.51}$$

The subsections above describe the necessary and sufficient conditions that characterize the type of equilibrium allocations. These types can be exhibited as the result of a numerical exercise, as depicted in the Figure 2.3. Note that we assume the government issues public debt, individual punishment takes place and Assumption 1 satisfies. The Figure 2.3 displays a (β, σ) space where the vertical axis and horizontal axis represent the discount factor and the probability of limited information meetings- like in the Figure 2.1 and Figure 2.2.

If the discount factor is sufficiently small, then the incentive constraint will bind. If σ decreases, then the financial frictions will amplify and liquidity premia of the assets will increase. Therefore, it will become suboptimal for a defaulter to pool with the non-defaulters at the DM because it will be too expensive for a defaulter to take out government debt. However, if σ increases, a defaulter would be more willing to pool with the non-defaulter and hence off-the-equilibrium punishment will become weaker as depicted in Figure 2.3.

Define $\tilde{\beta}$ and $\tilde{\tilde{\beta}}$ by

$$\tilde{\beta} = \frac{1}{2+y-\sigma}, \tag{2.52}$$

$$\tilde{\tilde{\beta}} = \frac{(1-2\sigma)^2}{y(1-\sigma)^2 + 2(1-\sigma)(1-2\sigma)}. \tag{2.53}$$

It is important to note that $\tilde{\tilde{\beta}} < \beta^{**} < \tilde{\beta} < \beta^*$, where β^* and β^{**} are expressed by (2.17) and (2.37). As well, x^{nd}, x_G^d and x_I^d denote the quantity of DM exchanges without government debt, with government debt and global punishment, and with government debt and individual punishment, respectively. Also, the welfare measure under Assumption 1 is given by

$$W = u(x) - x. \tag{2.54}$$

The efficient trade implies $x = 1$. If the incentive constraint binds, then the higher the consumption the higher the welfare is. Hence, we can characterize the social welfare by using the quantity of exchange at the DM. Therefore, by using the equilibrium allocations that are characterized in the previous subsections, we can suggest a general proposition on the welfare:

Proposition 13 *Suppose that the Assumption 1 satisfies. Then*

1. *If $\beta^* \le \beta$, then*
$$x^{nd} = x_G^d = x_I^d = 1. \tag{2.55}$$

2. *If $\tilde{\beta} \le \beta < \beta^*$, then*
$$x^{nd} < x_G^d = x_I^d = 1, \tag{2.56}$$
where x^{nd} satisfies (2.15)*.*

3. *If $\beta^{**} \le \beta < \tilde{\beta}$, then*
$$x^{nd} < x_I^d < x_G^d = 1, \tag{2.57}$$
where x^{nd} and x_I^d satisfy (2.15) *and* (2.48)*, respectively.*

4. *If $\frac{1}{2} \le \sigma$ and $\beta < \beta^{**}$ or $\sigma < \frac{1}{2}$ and $\tilde{\tilde{\beta}} < \beta < \beta^{**}$, then*
$$x^{nd} < x_I^d < x_G^d < 1 \tag{2.58}$$
where x^{nd}, x_I^d and x_G^d satisfy (2.15)*,* (2.48) *and* (2.35)*, respectively.*

5. *If $\sigma < \frac{1}{2}$ and $\beta \le \tilde{\tilde{\beta}}$, then*
$$x^{nd} < x_I^d = x_G^d < 1. \tag{2.59}$$
where x^{nd} and x_I^d satisfy (2.15) *and* (2.35)*, respectively.*

Proof. Directly follows from the necessary and sufficient conditions that are characterized in the previous subsections. ■

The most important result above is that the world is non-Ricardian, i.e., the government debt matters. In contrast to Williamson and Carapella (2014) in which the private credit is unsecured, in here the government debt always accomplishes larger quantity of exchange and therefore larger welfare. Under the individual punishment, the government debt alleviates the liquidity needs in spite of the financial constraints and thus implies a larger quantity of exchange. Since a defaulter can mimic the non-defaulters in the limited information meetings with the individual punishment, the off-the-equilibrium punishment is weaker than the global punishment since the latter implies a perpetual ostracism in case of any default. That is, global punishment, although unrealistic, implies larger welfare than the one with the individual punishment.

When the financial frictions are amplified- low discount factor and small probability of limited information meetings- the DM trade will be inefficient and a defaulter would not choose to pool with the non-defaulters because both assets bear large premia. As the liquidity constraint are severely binding, a defaulter would optimally choose autarky and therefore the equilibrium allocation with the individual punishment becomes identical with the one with the global punishment. This state is described by the case 5 of the Proposition 13 and is exhibited by the area 5 in the Figure 2.4. However, if the frictions are moderate, then the liquidity constraint (2.25) will be still binding, but this time it would be optimal for a defaulter- who behaves as if she is a non-defaulter- to trade in the limited information meeting. This equilibrium is expressed by case 4 of Proposition 13. In fact, as σ increases, a defaulter's incentive to pool with the non-defaulters will increase since the seller's ability to verify her history of tax return will decrease. Hence, off-the-equilibrium punishment becomes weaker and therefore welfare decreases. As well, the equilibrium with individual punishment differs from the one with the global punishment although both equilibria imply inefficient trade. This is depicted by the area 4 in Figure 2.4.

As $\beta \in [\beta^{**}, \tilde{\beta})$ holds, the optimal public debt with the global punishment supports the efficient trade; however, the optimal public debt with the individual punishment is not large enough to support the efficient trade at the DM. As well, the public debt mitigates the liquidity constraints and therefore the welfare without the government debt is the lowest. This equilibrium is expressed by the case 3 of Proposition 13 and it is depicted by the area ③ in the Figure 2.4. If $\beta \in [\tilde{\beta}, \beta^*)$ holds, then the optimal supply of public debt is large enough to support the efficient trade regardless of the punishment types. However, without the government debt the private credit is too small to support the efficient outcome. This equilibrium is characterized by the case 2 of Proposition 13 and the conditions that satisfies this equilibrium are displayed by the area ② in the Figure 2.4.

Finally, the differences in the quantity of exchange between the states with government debt and without government debt vanish when the discount factor is high enough. In particular, all the trades are efficient if $\beta^* \leq \beta$. In fact, the asset prices are at their fundamental values. This state is expressed by the case 1 of Proposition 13 as depicted by the area ① in (β, σ)-space in the Figure 2.4.

In the next section, we confine our attention to the environment in which there exits an asymmetry between a buyer and a seller on willingness to consume housing services. In fact, only buyer can consume the housing dividends. This environment involves following assumption.

Assumption 2 *No seller can consume the housing services.*

The novelty here is that no seller derives direct utility from the housing. This environment can be interpreted as the economies with abundant supply of housing and significantly large home-equity loan market. In fact, each seller uses housing only for investment purposes. Whether the seller consumes the dividend or not will help us to understand the role of collateral and asset trade during the DM transactions. We analyze the environment in which asymmetry between agents on their willingness to consume housing services creates welfare loss from the exchange of houses. This asymmetry might justify the efficient use of housing as collateral.

2.4 Equilibrium with Asymmetric Intrinsic Values of Housing across Agents

In this section, we confine our attention to the stationary equilibrium with and without government debt when Assumption 2 satisfies. The significance of this assumption lies on the state of asymmetry between a buyer's and the seller's surplus. In contrast to the equilibrium under Assumption 1, in here the welfare measure is given by

$$W = u(x) - x + \beta y(a'' - a). \tag{2.60}$$

This welfare measure implies that if a buyer chooses to trade housing at any quantity, it implies a welfare loss. This loss comes from the fact that the seller can derive no utility from the house per se. He does not use the house as a shelter, but only for medium of exchange. He can exchange the consumption good for the asset with the intention of selling it in the next CM. We can interpret the seller as a retailer. If a buyer collateralizes all the asset, then there will be no welfare loss.

2.4.1 The Buyer's Problem Without Government Debt

By Assumption 2, a seller consumes the dividends. Then if a seller obtains a' units of asset at the DM, he will acquire $a'\phi$ units of consumption good at the next CM. Remember that the buyer makes a take-it-or-leave-it offer. Then each buyer solves

$$R = \max_{a,D,x,a''} \left\{ -\phi a + u(x) - \beta D + \beta(\phi + y)a'' \right\} \tag{2.61}$$

s.t

$$x = \beta D - \beta\phi(a'' - a), \tag{2.62}$$

$$-D + (\phi + y)a''\theta \geq 0, \tag{2.63}$$

where (2.62) and (2.63) stand for the bargaining rule equation and collateral constraint, respectively. Notice that the fact that the buyer makes a take-it-or-leave-it offer implies (2.62) and this bargaining rule is different than (2.6) under Assumption 1. Solving (2.61) implies that a buyer can post all the asset holdings as collateral in contrast to the Proposition 12 when Assumption 1 holds.

The Incentive Constraint (2.63) **Does not Bind.**

The first order conditions imply

$$a'' = a, \quad x = x^* = 1, \quad \text{and} \quad D = \frac{1}{\beta}, \tag{2.64}$$

and the asset price is at its fundamental value, i.e., ϕ satisfies (2.10). The necessary and sufficient conditions for this equilibrium to exist is given by

$$\frac{1-\beta}{\beta\theta} \leq y. \tag{2.65}$$

The Incentive Constraint (2.63) **Binds and All the Houses are Collateralized.**

In this case, all the assets are collateralized, i.e., $a'' = a$. This holds if the marginal increase in collateral increases the welfare, i.e., it satisfies

$$-\beta\big(\phi(1-\theta) - y\theta\big)\big(u'(x) - 1\big) + \beta y \geq 0. \tag{2.66}$$

As well, the bargaining rule (2.62) and the binding constraint (2.63) imply

$$D = \frac{x}{\beta}, \quad \text{and} \quad x = \beta(\phi + y)a\theta. \tag{2.67}$$

The first order condition implies

$$\phi = \beta(\phi + y)\big(1 - \theta + \theta u'(x)\big). \tag{2.68}$$

Using (2.67) and (2.68), the asset price ϕ and DM consumption x are given by

$$\phi = \frac{\beta\theta + 2y\beta(1-\theta)\big(1-\beta(1-\theta)\big) + \big[\beta^2\theta^2 + 4\beta\theta y\big(1-\beta(1-\theta)\big)\big]^{\frac{1}{2}}}{2\big(1-\beta(1-\theta)\big)^2}. \tag{2.69}$$

$$x = \left(\frac{\beta\theta + \big[\beta^2\theta^2 + 4\beta\theta y\big(1-\beta(1-\theta)\big)\big]^{\frac{1}{2}}}{2\big(1-\beta(1-\theta)\big)}\right)^2. \tag{2.70}$$

By using $x < 1$ and (2.66), the necessary and sufficient conditions for this equilibrium to exist are given by

$$y < \frac{1-\beta}{\beta\theta}, \quad \text{and} \quad \frac{\beta(1-\beta)(1-\theta)}{\theta} \le y. \tag{2.71}$$

The Incentive Constraint (2.63) Binds and the Buyers are Indifferent Between Trading Asset and Posting it as Collateral.

In this case, a buyer is indifferent between trading the asset and pledging it as collateral. That is, $a'' = \hat{a} \in (0, a)$. Therefore, the first order condition with respect to a'' implies

$$-\beta\big(\phi(1-\theta) - y\theta\big)\big(u'(x) - 1\big) + \beta y = 0. \tag{2.72}$$

By using the bargaining rule (2.62) and the binding constraint (2.63), the DM consumption x and the debt payment D are given by

$$x = \beta\phi a - \beta a''\big(\phi(1-\theta) - y\theta\big), \quad \text{and} \quad D = (\phi + y)\hat{a}\theta. \tag{2.73}$$

The first order condition with respect to a implies

$$x = \beta^2 \tag{2.74}$$

Using (2.72) and (2.74), the asset price ϕ and quantity of collateral a'' are given by

$$\phi = \frac{y\big(\theta + (1-\theta)\beta\big)}{(1-\beta)(1-\theta)}, \tag{2.75}$$

$$\hat{a} = \frac{y\big(\theta + (1-\theta)\beta\big) - \beta(1-\beta)(1-\theta)}{(1-\theta)\beta y}. \tag{2.76}$$

The necessary and sufficient condition for this equilibrium to exist is $\hat{a} \in (0, a)$, that is,

$$\frac{\beta(1-\beta)(1-\theta)}{\theta + (1-\theta)\beta} < y, \quad \text{and} \quad y < \frac{\beta(1-\beta)(1-\theta)}{\theta}. \tag{2.77}$$

The Incentive Constraint (2.63) **Binds and All the Houses are Traded.**

In this case, no collateral is posted at the optimum. That is, $a'' = 0$. The first order condition with respect to a'' implies

$$-\beta\big(\phi(1-\theta) - y\theta\big)\big(u'(x) - 1\big) + \beta y \leq 0. \tag{2.78}$$

By using the bargaining rule (2.62) and the binding constraint (2.63), the DM consumption x and the debt payment D are given by

$$x = \beta\phi a, \quad \text{and} \quad D = 0. \tag{2.79}$$

The first order condition with respect to a implies

$$x = \beta^2 \tag{2.80}$$

Using (2.80), this equilibrium holds if and only if

$$y \leq \frac{\beta(1-\beta)(1-\theta)}{\theta + (1-\theta)\beta}. \tag{2.81}$$

Define cut-off values of dividends y^*, y^{**} and y^{***} by

$$y^* = \frac{1-\beta}{\beta\theta}, \tag{2.82}$$

$$y^{**} = \frac{\beta(1-\beta)(1-\theta)}{\theta}, \tag{2.83}$$

$$y^{***} = \frac{\beta(1-\beta)(1-\theta)}{\theta+(1-\theta)\beta}. \tag{2.84}$$

When Assumption 2 satisfies, the types of equilibrium without the public debt can be shown by a numerical exercise, as depicted in Figure 2.5. In this figure, the vertical axis and horizontal axis represent the the quantity of dividends and the pledgeability factor, respectively.

The efficient equilibrium involves rich dividends, large pledgeability and high discount factor. In fact, the larger the dividend is, the more plentiful assets are; the greater the pledgeability of the asset is, the cheaper the private debt is; the larger the discount factor is, the weaker the financial friction is. The private asset is more than enough to support the efficient trade at the DM when the assets are used as collateral. Hence, a buyer acquires the housing in the initial period and then keeps it forever. The assets are never traded and the asset price is at its fundamental value, i.e., the present discounted value of the future dividends. The sufficient and necessary condition for an efficient equilibrium is expressed by (2.65) which is exhibited as upper area of the convex locus on the extreme north-east in the Figure 2.5.

The asset can be plentiful to the extent that collateral is useful to support the credit arrangement; however, the value of collateralizable wealth is not large enough to render the collateral constraint (2.63) slack. A buyer purchases housing at the initial period and keeps it forever. As well, collateral creates liquidity and hence ameliorates the financial friction. Hence, it bears liquidity premium. Also, (2.50) defines the necessary and sufficient condition for constrained optimum with all the assets are collateralized. Hence, the area in between two convex loci towards north-east direction, as depicted in the Figure 2.5, displays the conditions (2.71) for this equilibrium.

The collateral can be also sufficiently plentiful to the extent that the buyer cannot completely tolerate the surplus loss from the asset trade; at the same time, it can be sufficiently scarce to the extent that the private credit cannot be completely supported. Hence, this is a ”buffer” state of mixed equilibrium in which the buyer trades some assets and collateralize the rest. As well, (2.77) states the necessary and sufficient conditions for inefficient equilibrium in which the buyer is indifferent between trading the asset and posting it as collateral. The area of (y,θ)-pairs in between two convex loci that are located along southwest, as depicted in the Figure **??**, represents the conditions (2.77) for this equilibrium.

Finally, we will focus on the equilibrium in which the buyer optimally trades all the asset. This equilibrium involves poor dividends, small pledgeability and low discount factor. Moreover, the smaller the dividend is, the scarcer the asset is; the smaller the pledgeability of assets is, the weaker the collateral incentive is; the more impatient the household is, the more amplified the financial friction becomes. What is working against the asset trade is asymmetry in payoffs: a buyer receives payoff from keeping the asset during the DM, whereas the seller does not. Therefore, as the financial friction tightens up, the buyer trades the asset to make up for the weak incentives in collateral although asset trade causes net surplus loss. It is useful to think of the global crisis in 2008 as decreasing pledgeability θ or decreasing dividends y. In this equilibrium, housing serves as a medium of exchange; the buyer will sell the asset at the DM and purchase it back at the next CM to create liquidity at the forthcoming DM. As well, (2.81) expresses the necessary and sufficient condition for inefficient equilibrium with no collateral posted. (2.81) can be shown by the lower area of the convex locus that is located on the extreme southwest and upper area of the horizontal and vertical axes, as depicted in the Figure 2.5.

2.4.2 The Buyer’s Problem With the Government Debt

The government issues B units of government debt and acquires τ from the tax return in the CM in terms of consumption good. Remember that Assumption 2 holds. Then a buyer who engages in purchasing government bond solves

$$V = \max_{a,x,D,a'',b,b'} \left\{ -\phi a - qb + u(x + \beta b - \beta b') - \beta D + \beta(\phi + y)a'' - \beta\tau + \beta b' + \beta V \right\} \quad (2.85)$$

s.t

$$x = \beta D + \beta\phi(a - a''), \quad (2.86)$$

$$-\beta D + \beta(\phi + y)a'' - \beta\tau + \beta b' + \beta V \geq \beta(\phi + y)a''(1 - \theta) + \beta\hat{V}, \quad (2.87)$$

where $\hat{V}$ denotes the defaulter's continuation value.

We will assume that the collateralized debt is not large enough to support the efficient trade, i.e., $y < y^*$. As well, we will confine our attention to the equilibrium in which the incentive constraint (2.87) can be separated into two constraints: The collateral constraint involves the seller's seizure the of the collateral in case of default and can be described as

$$-\beta D + \beta(\phi + y)a''\theta \geq 0, \quad (2.88)$$

and the tax constraint involves the government seizure of the defaulter's bond holdings in case of the tax non-compliance and can be expressed as

$$-\beta\tau + \beta b' + \beta V \geq \beta\hat{V} \quad (2.89)$$

Since we assume $y < y^*$, then the collateral constraint has to bind at the optimum. Therefore, using the bargaining rule (2.86) and the binding collateral constraint (2.88), we have

$$x = \beta\phi a - \beta a''\big(\phi(1 - \theta) - y\theta\big), \quad (2.90)$$

Then the buyer's problem can be rewritten by

$$V = \max_{a,x,a'',b,b'} \left\{ -\phi a - qb + u(x + \beta b - \beta b') - x + \beta \phi a + \beta y a'' - \beta \tau + \beta b' + \beta V \right\} \tag{2.91}$$

subject to (2.90) and (2.89).

The first order condition with respect to b is given by

$$q = \beta u'(x + \beta b - \beta b'), \tag{2.92}$$

where x satisfies (2.90). For simplicity, we will define the other first order conditions under case-by-case analysis.

The Incentive Constraint (2.89) **Does not Bind.**

The first order conditions imply that

$$a'' = a, \quad x = \beta(\phi + y)a\theta, \quad x + \beta b - \beta b' = x^* = 1, \quad \text{and,} \quad D = \frac{1}{\beta}, \tag{2.93}$$

and the housing price ϕ satisfies (2.10) and the bond price q satisfies (2.28). The continuation value can be expressed by

$$V = \frac{u(x^*) - x^* - \beta \tau}{1 - \beta}. \tag{2.94}$$

The incentive constraint (2.89) must meet

$$-\beta \tau + \beta b + \beta(\phi + y)a\theta - x^* + \beta V \geq \beta \hat{V}. \tag{2.95}$$

The Incentive Constraint (2.89) **Binds and All the Houses are Collateralized.**

In this case, all the assets are collateralized, i.e., $a'' = a$. This holds if

$$-\beta\big(\phi(1-\theta)-y\theta\big)\big(u'(x+\beta b-\beta b')-1\big)+\beta y \geq 0. \tag{2.96}$$

By using the FOCs, the allocation (x, D, ϕ, q, V) is given by

$$x = \beta(\phi+y)a\theta, \quad \text{and} \quad D = \frac{1}{\beta}, \tag{2.97}$$

$$\phi = \beta(\phi+y)\big[1-\theta+\theta u'\big(\beta(\phi+y)a\theta+\beta b-\beta b'\big)\big], \tag{2.98}$$

$$q = \beta u'(\beta(\phi+y)a\theta+\beta b-\beta b'). \tag{2.99}$$

$$V = \frac{-\phi a - qb + u\big(\beta(\phi+y)a\theta+\beta b-\beta b'\big)+\beta(\phi+y)a(1-\theta)-\beta\tau+\beta b'}{1-\beta}. \tag{2.100}$$

The incentive constraint (2.89) binds, i.e.,

$$-\tau + b' + V = \hat{V}. \tag{2.101}$$

The Incentive Constraint (2.89) **Binds and the Buyers are Indifferent Between Trading Asset and Posting it as Collateral.**

In this case, a buyer is indifferent between trading the asset and pledging it as collateral. That is, $a'' = \hat{a} \in (0, a)$. Therefore, the first order condition with respect to a'' implies

$$-\beta\big(\phi(1-\theta)-y\theta\big)\big(u'(x+\beta b-\beta b')-1\big)+\beta y = 0, \tag{2.102}$$

where x and D satisfy (2.73). The first order condition with respect to a implies

$$x+\beta b-\beta b' = \beta^2. \tag{2.103}$$

Using (2.103) and (2.92), we have

$$q = 1. \tag{2.104}$$

Using (2.102), the housing price ϕ satisfies (2.75). As well, the continuation value is given by

$$V = \frac{-\phi a - qb + u(\beta^2) - \beta^2 + \beta\phi a + \beta y\hat{a} - \beta\tau + \beta b}{1-\beta}. \tag{2.105}$$

The incentive constraint (2.89) binds, i.e.,

$$-\beta\tau + \beta\phi a - \beta\hat{a}\big(\phi(1-\theta) - y\theta\big) + \beta b - \beta^2 + \beta V = \beta\hat{V}. \tag{2.106}$$

The Incentive Constraint (2.89) **Binds and All the Houses are Traded.**

In this case, the buyer trades all the asset at the optimum. That is, $a'' = 0$. The first order condition with respect to a'' implies

$$-\beta\big(\phi(1-\theta) - y\theta\big)\big(u'(x+\beta b-\beta b') - 1\big) + \beta y \leq 0, \tag{2.107}$$

where x and D satisfy (2.79). The first order condition with respect to a implies (2.103). Hence, by using (2.103) and (2.92), q satisfies (2.104).

As well, the continuation value is given by

$$V = \frac{-\phi a - qb + u\big(\beta^2\big) - \beta^2 + \beta\phi a - \beta\tau + \beta b}{1-\beta}. \tag{2.108}$$

The incentive constraint (2.89) binds, i.e.,

$$-\beta\tau+\beta\phi a+\beta b-\beta^2+\beta V=\beta\hat{V}. \tag{2.109}$$

The market clearing conditions for both asset market and bond market satisfy (2.23).

2.4.3 The Government's Problem

In this section, we will express the government's problem. The government will choose the optimal bond supply B that yields the highest the social welfare given the incentive constraint (2.89). Therefore, the government solves

$$\max_{B} \quad u(x+\beta B)-x-\beta B+\beta y(a''-a) \tag{2.110}$$

s.t

$$-\beta B(1-q)+\beta V\geq\beta\hat{V}, \tag{2.111}$$

where the allocation (V,q,ϕ,x,a'') is expressed in the subsections above. As well, we will express $\hat{V}$ in detail in the next sections.

2.4.4 Government Debt With the Global Punishment

In this section, we assume that if a buyer defaults on tax payments, then both housing and bond markets entirely shut down. This is the most severe punishment since any default implies ostracism for everybody. In here, the punishment implies that if a buyer defaults, everybody gets nothing, i.e., $\hat{V}$ satisfies (2.27).

The Incentive Constraint (2.111) **Does not Bind.**

By using (2.27), (2.93) and solving (2.110), we reach that the DM trade is efficient and hence x satisfies (2.11). As well, the asset prices ϕ and q satisfies (2.10) and (2.28), respectively. As well, x and B can be expressed by

$$x = \frac{\beta y \theta}{1-\beta}, \tag{2.112}$$

$$B = \frac{1-\beta-\beta y \theta}{\beta(1-\beta)}. \tag{2.113}$$

Then by using (2.94), (2.23), (2.10), and (2.28), the continuation value for the non-defaulter is given by

$$V = \frac{u(x^*) - x^* - \beta^2(1-\beta)}{1-\beta}. \tag{2.114}$$

This equilibrium exists if and only if (2.111) satisfies, i.e.,

$$\frac{1-2\beta}{\beta\theta} \leq y. \tag{2.115}$$

The sufficient and necessary condition for this equilibrium to exist can be shown as a numerical exercise. This is depicted in the Figure 2.6 which lies on (y,σ) space. Upper rectangular area, above the horizontal line at $y = 8/3$, exhibits (2.115) and supports the efficient equilibrium. The collateralizable wealth is not large enough to support the efficient trade; however, the supply of public debt renders tax constraint slack and the trade reaches efficiency. As well, in this equilibrium it is optimal for a buyer to use all the asset holdings as collateral, i.e., $a'' = 1$.

The Incentive Constraint (2.111) **Binds and All the Houses are Collateralized.**

In here, the buyer optimally chooses to post all the housing as collateral. Hence, we obtain $a'' = a$. Denote the DM consumption by x_1 which can be expressed by

$$x_1 = \beta(\phi + y)a\theta + \beta B. \tag{2.116}$$

As well, the house price, the bond price and the continuation value for the non-defaulters are given by

$$\phi = \beta(\phi + y)\left[1 - \theta + \theta u'(x_1)\right], \tag{2.117}$$

$$q = \beta u'(x_1), \tag{2.118}$$

$$V = \frac{-\phi a - qB + u(x_1) - x_1 + \beta(\phi + y)a + \beta qB}{1 - \beta}. \tag{2.119}$$

Then all the assets are used as collateral if a marginal increase in collateral relaxes the binding constraint. That is, by using (2.96), (2.116) and (2.117), in equilibrium it must satisfy

$$\beta^2 \leq x_1. \tag{2.120}$$

By using (2.27), (2.116)-(2.118) and (2.119), and the binding constraint (2.111), we can write x_1 in implicit form as follows:

$$\frac{\beta y\theta}{(1 - \beta + \beta\theta)x_1^{\frac{1}{2}} - \beta\theta} = \frac{x_1 - 2\beta x_1^{\frac{1}{2}}}{x_1^{\frac{1}{2}} - \beta}. \tag{2.121}$$

Notice that by Inverse Function Theorem[1], there exists a unique real solution $\tilde{x}_1$ that satisfies (2.120) and (2.121). The unique solution supports this equilibrium if the trade is inefficient, i.e., $\tilde{x}_1 < 1$. This satisfies

$$y < \frac{1-2\beta}{\beta\theta}. \tag{2.122}$$

The sufficient and necessary condition for this equilibrium to exist is depicted in the Figure 2.6. The lower rectangular area, below the horizontal line at $y = 8/3$, exhibits (2.122) and supports the inefficient equilibrium in which a buyer uses all her houses as collateral. The housing is not plentiful enough and hence the private credit is not large enough to support the efficient trade. In addition, the supply of public debt mitigates the liquidity constraint; however, the public debt is not large enough to render the tax constraint slack. The buyer purchases her house at the first date and keeps it forever at the optimum. In spite of the inefficient trade at the DM, the collateral performs a useful role to support the credit transactions.

The Incentive Constraint (2.111) **Binds and the Buyers are Indifferent Between Trading Asset and Posting it as Collateral.**

In this case, a buyer is indifferent between trading the asset and pledging it as collateral. That is, $a'' = \hat{a} \in (0, a)$. Denote the DM consumption by x_2 and it can be defined by (2.103). Hence, the first order condition with respect to a implies

$$x_2 = \beta^2. \tag{2.123}$$

Thus, the house price and the bond price are given by (2.75) and (2.104), respectively. The continuation value in (2.105), the prices (2.75) and (2.104) and the DM consumption (2.123) contradict

[1]Define functions f_1 and f_2 by $f_1(x) = \frac{\beta y\theta}{(1-\beta+\beta\theta)x_1^{1/2}-\beta\theta}$ and $f_2(x) = \frac{x_1-2\beta x_1^{1/2}}{x_1^{1/2}-\beta}$, respectively. We have $f_1'(x) < 0 < f_2'(x)$ for all $x \geq \beta^2$. As well, we obtain $\lim_{x\to\beta^2} f_2(x) = -\infty < 0 < f_1(\beta^2) = \frac{y\theta}{(1-\beta)(1-\theta)}$ and $\lim_{x\to\infty} f_1(x) = 0 < \lim_{x\to\infty} f_2(x) = \infty$. Therefore, IFT applies; existence and uniqueness satisfy.

the binding the collateral constraint (2.106). Therefore, this equilibrium does not exist.

The Incentive Constraint (2.111) Binds and All the Houses are Traded.

In this case, $a'' = 0$. Denote the DM consumption by x_3 and it can be defined by (2.79). Hence, the first order condition with respect to a implies

$$x_3 = \beta^2. \tag{2.124}$$

Thus, the bond price is given by (2.104). The continuation value (2.105), the bond price (2.104) and DM consumption (2.124) contradict the binding collateral constraint (2.109). Therefore, as in the previous subsection, this equilibrium never holds.

2.4.5 Government Debt With the Individual Punishment

As in the first section, if a buyer has defaulted on tax payments and the seller learns the default, then she will choose not to trade with her. The seller knows that the bonds he acquire from the defaulter will be confiscated at the CM by the government. We also assume that if she learns that the buyer has defaulted, then no trade will take place at the DM. However, when the seller does not have an access to the public records, a defaulter optimally behaves as if she has not defaulted. That is, a defaulter optimally chooses to pool with the non-defaulters. If the defaulter deviates from the non-defaulter's strategy, then the seller will optimally reject trading with her. Thus, the off-the-equilibrium value function is given by

$$\hat{V} = \max\left[0, \frac{-\phi a - qB + \sigma\big(u(x+\beta B) - \beta D + \beta(\phi+y)a''\big) + (1-\sigma)\big(\beta(\phi+y)a + \beta B\big)}{1-\beta}\right], \tag{2.125}$$

where x, D and a'' are expressed in subsections 4.2.1, 4.2.2, 4.2.3 and 4.2.4, respectively.

The Incentive Constraint (2.111) **Does not Bind.**

By using (2.125), (2.93), (2.94), and solving (2.110), then DM trade is efficient. As well, ϕ is at its fundamental value and the house and bonds prices satisfy (2.10) and (2.28), respectively. As well, a non-defaulter's value function and optimal quantity of bond issued by the government can be expressed by

$$\hat{V} = \frac{\sigma\big(u(x^*) - x^*\big)}{1-\beta}, \tag{2.126}$$

$$B = \frac{1-\beta-\beta y\theta}{\beta(1-\beta)}, \tag{2.127}$$

The incentive constraint (2.111) must satisfy. Hence, this equilibrium exists if and only if

$$\frac{1-2\beta+\beta\sigma}{\beta\theta} \leq y. \tag{2.128}$$

The sufficient condition (2.128) for this equilibrium to exist can be shown as a numerical exercise, as depicted in Figure 2.7. In this figure, the vertical axis and horizontal axis represent the quantity of dividends for each house and the probability of limited information meeting, respectively. Upper triangular area, above the horizontal line with positive slope, exhibits (2.128) and supports the efficient equilibrium with the individual punishment and the supply of government debt.

The Incentive Constraint (2.111) **Binds, All the Houses are Collateralized and** $\hat{V} > 0$**.**

The off-the-equilibrium punishment $\hat{V}$ is positive, i.e., a defaulter optimally pools with the non-defaulters. By using (2.125), (2.97)-(2.100) and the binding constraint (2.101), the asset price ϕ, the bond price q, the non-defaulter's continuation value V and the DM consumption x_1, respectively, are given by (2.117), (2.118), (2.119), and (2.121). The buyer uses all the houses as collateral, i.e., $a'' = a$ if x_1 satisfies (2.120).

By using (2.125) and (2.117)-(2.101), then x_1 satisfies:

$$\frac{\beta y \theta}{(1-\beta+\beta\theta)x_1^{\frac{1}{2}} - \beta\theta} = \frac{x_1(1+\beta-\beta\sigma) - \beta(3-2\sigma)x_1^{\frac{1}{2}}}{x_1^{\frac{1}{2}} - \beta}. \tag{2.129}$$

Notice that by Inverse Function Theorem[2], there exists a unique real solution $\hat{x}_1(y,\rho)$ that satisfies (2.129) and (2.120). This solution supports this equilibrium if $\hat{x}_1(y,\sigma) < 1$, that is,

$$y < \frac{1-2\beta+\beta\sigma}{\beta\theta}. \tag{2.130}$$

In equilibrium, it must also satisfy $\hat{V} > 0$, that is,

$$\begin{aligned} &\frac{1}{2} \leq \sigma, \quad \text{or} \\ &\sigma < \frac{1}{2}, \quad \text{and} \quad x_1(y,\sigma) > \frac{(1-2\sigma)^2}{(1-\sigma)^2}. \end{aligned} \tag{2.131}$$

The sufficient conditions (2.130) and (2.131) for this equilibrium to exist can be shown as a numerical exercise, as depicted in Figure 2.7. The area in between the horizontal line with positive slope and vertical line with negative slope exhibits (2.130) and (2.131) and supports the inefficient equilibrium in which the buyer collateralizes all the asset and a defaulter optimally would pool with the non-defaulters in the limited information meetings.

The Incentive Constraint (2.111) Binds, All the Assets are Collateralized, and $\hat{V} = 0$.

The defaulter's value function $\hat{V}$ satisfies $\hat{V} = 0$ because a defaulter abandons pooling with non-defaulters due to large asset prices. Hence, off-the equilibrium path is autarky. Define the DM

[2] Define functions g_1 and g_2 by $g_1(x) = \frac{\beta y\theta}{(1-\beta+\beta\theta)x_1^{1/2} - \beta\theta}$ and $g_2(x) = \frac{x_1(1+\beta-\beta\sigma)-\beta(3-2\sigma)x_1^{1/2}}{x_1^{1/2}-\beta}$, respectively. We have $g_1'(x) < 0 < g_2'(x)$ for all $x > \beta^2$. As well, we obtain $\lim_{x\to\beta^2} g_2(x) = -\infty < 0 < g_1(\beta^2) = \frac{y\theta}{(1-\beta)(1-\theta)}$ and $\lim_{x\to\infty} g_1(x) = 0 < \lim_{x\to\infty} g_2(x) = \infty$. Therefore, IFT applies; existence and uniqueness satisfy.

consumption for this case by

$$x_2 = \beta(\phi + y)a\theta + \beta B. \tag{2.132}$$

The buyer posts all the asset as collateral, i.e., $a'' = a$. This satisfies if

$$\beta^2 \leq x_2. \tag{2.133}$$

By using (2.125), (2.97)-(2.100) and the biding constraint (2.101), the asset price ϕ, the bond price q, the non-defaulter's continuation value V, respectively, are given by

$$\phi = \beta(\phi + y)\big(1 - \theta + \theta u'(x_2)\big), \tag{2.134}$$

$$q = \beta u'(x_2), \tag{2.135}$$

$$V = \frac{-\phi a - qB + u(x_2) - x_2 + \beta(\phi + y)a - \beta qB}{1 - \beta}. \tag{2.136}$$

By using (2.111), (2.132), and (2.134)-(2.136), then x_2 satisfies:

$$\frac{\beta y \theta}{(1 - \beta + \beta\theta)x_2^{\frac{1}{2}} - \beta\theta} = \frac{x_2 - 2\beta x_2^{\frac{1}{2}}}{x_2^{\frac{1}{2}} - \beta}. \tag{2.137}$$

Therefore, (2.137) coincides (2.121). That is, a defaulter would choose autarky in equilibrium because of the large liquidity premia. Hence, the individual punishment coincides the global punishment in this state.

We will concentrate on the implicit solutions since they are enough to make the welfare comparison with other equilibrium allocations and determine the cut-off dividends that set the conditions for characterization. Notice that by Inverse function theorem, there exists a unique real solution

$\hat{x_2}(y,\sigma)$ that satisfies (2.133) and (2.137). The proof follows from the section 4.4.2. The unique solution supports this equilibrium if $\hat{x_2}(y,\sigma) < 1$, that is,

$$y < \frac{1-2\beta}{\beta\theta}. \tag{2.138}$$

In equilibrium, it must also satisfy $\hat{V} = 0$, that is,

$$\sigma < \frac{1}{2} \quad \text{and} \quad x_2(y,\sigma) < \frac{(1-2\sigma)^2}{(1-\sigma)^2}. \tag{2.139}$$

The implicit solutions $x_1(y,\sigma)$ and $x_2(y,\sigma)$ are given by (2.129) and (2.137), respectively. Define $\tilde{y}$ by

$$\tilde{y} = \frac{\left[(1-2\sigma)^2 - 2\beta(1-\sigma)(1-2\sigma)\right]\left[(1-\beta+\beta\theta)(1-2\sigma) - \beta\theta(1-\sigma)\right]}{\beta\theta(1-\sigma)^2\left[(1-2\sigma)-\beta(1-\sigma)\right]}. \tag{2.140}$$

By confining our attention to positive real solutions, $x_1(y,\sigma) = x_2(y,\sigma) \equiv x$ and $\sigma < \frac{1}{2}$ imply that

$$x = \frac{1-2\sigma}{1-\sigma}, \quad \text{and} \quad y(\sigma) = \tilde{y}. \tag{2.141}$$

By using (2.129) and (2.137), we have

$$\frac{\partial x_1}{\partial y} > 0, \quad \text{and} \quad \frac{\partial x_2}{\partial y} > 0. \tag{2.142}$$

By using (2.131), (2.141) and (2.142), the equilibrium in which (2.111) binds and all the assets are collateralized with $\hat{V} > 0$ if it satisfies

$$\begin{aligned} &\frac{1}{2} \le \sigma, \quad \text{and} \quad y < \frac{1-2\beta+\beta\sigma}{\beta\theta}, \quad \text{or,} \\ &\sigma < \frac{1}{2}, \quad \text{and} \quad \tilde{y} < y < \frac{1-2\beta+\beta\sigma}{\beta\theta}. \end{aligned} \tag{2.143}$$

As well, by using (2.128), (2.141) and (2.142), the equilibrium in which (2.111) binds and all the assets are collateralized with $\hat{V} = 0$ if it satisfies

$$\sigma < \frac{1}{2}, \quad \text{and} \quad y \leq \tilde{y}. \tag{2.144}$$

The Incentive Constraint (2.111) Binds, the Buyers are Indifferent Between Trading Asset and Posting it as Collateral and $\hat{V} > 0$.

In here, a buyer is indifferent between trading the asset and pledging it as collateral. That is, $a'' = \hat{a} \in (0, a)$. By using (2.102)-(2.106), and (2.125), we reach the contradiction. Hence, for $\hat{V} > 0$, this equilibrium never satisfies. When $\hat{V} = 0$, the individual punishment coincides the global punishment. By using the subsection 4.4.3, as well, the equilibrium in which the buyers are indifferent between trading asset and using it as collateral with $\hat{V} = 0$ never satisfies.

The Incentive Constraint (2.111) Binds, All the Houses are Traded and $\hat{V} > 0$.

In this case, the buyer trades all the asset and hence $a'' = 0$. By using (2.108) and (2.109), then we reach the contradiction. Hence, the equilibrium in which all the assets are traded with $\hat{V} > 0$ does not exist. The individual punishment coincides the global punishment when $\hat{V} = 0$. Thus, by using the subsection 4.4.4, the equilibrium in which all the assets are traded with $\hat{V} = 0$ does not exist.

For the following propositions, we suppose that the incentive constraint (2.87) can be separated into the collateral constraint (2.88) and the tax constraint (2.89). As well, we assume that the collateralized debt is too scarce to satisfy the efficient trade by itself, i.e., $y < y^*$.

Proposition 14 *Suppose that the Assumption 2 satisfies, $y < y^*$ holds and the collateral constraint* (2.89) *and the tax constraint* (2.90) *are separable. Then if the government issues public debt, then no houses will be traded at the optimum.*

Proof.

It follows from both subsections 4.4.3 and 4.4.4 when there is a global punishment. As well, it follows from both subsections 4.5.4 and 4.5.5 when there is an individual punishment.

■

We show that the collateral is not useful when there exists no surplus loss due to the private asset trade, as in the Proposition 12. In contrast, if Assumption 2 satisfies, then a buyer will optimally post her house as collateral if it is plentiful enough. However, if the dividends and/or pledgeability factor are low, then the collateral constraint will tighten up and the buyer will trade the house to make up the trade loss. Remember that we interpret this state as the global crisis in 2008. Further, this proposition states that the government debt supports the collateral. In particular, with the government debt a buyer never trades her house in exchange for consumption goods at the optimum. She therefore acquires the house at the first date and keeps it forever since it is useful to serve as collateral to support the DM transactions.

First, an increase in collateral increases the potential loans that cannot be activated due to the default risk and this tightens up the collateral constraint. More precisely, a unit of collateral increase will decrease the welfare by $\beta\lambda[\phi(1-\theta)-y\theta]$, where λ is the lagrange multiplier of the collateral constraint (2.63). Second, a collateral increase tames the surplus loss arising from the private asset trade since it decreases the house trade. By using the welfare measure (2.60), a unit of increase in collateral increases welfare by βy. These are the two opposite forces that determine the equilibrium outcome. If the first dominates the latter, it will be optimal for a buyer to trade all the houses at the DM. However, if the latter overwhelms the first, then all the asset will be posted as collateral. With the government debt, the marginal utility of consumption decreases so does the shadow value λ. In particular, optimal supply of government debt decreases λ to the extent that a marginal increase in collateral rises the total surplus, i.e., the government debt implies that no house trade will take place at the DM at the optimum.

For simplicity, define $\hat{y}$ and $\hat{\hat{y}}$ by

$$\hat{y} = \frac{1-2\beta+\beta\sigma}{\beta\theta}, \tag{2.145}$$

$$\hat{\hat{y}} = \frac{1-2\beta}{\beta\theta}. \tag{2.146}$$

If the Assumption 2 holds, the welfare measure is given by (2.60). The efficient trade implies $x = 1$ and $a'' = a$. The next proposition combines the welfare measures under the Assumption 2 and makes comparison on welfare states by using the DM consumptions. Note that $\bar{x}^{nd}$, $\bar{x}_G^d$ and $\bar{x}_I^d$ denote the DM consumptions without government debt, with the government debt and global punishment, and with the government debt and the individual punishment, respectively. As well, a''^{nd}, $a_I''^d$ and $a_G''^d$ denote the quantity of collateral in equilibrium without government debt, with government debt and the individual punishment, and with the government debt and global punishment, respectively. Further, remember that $\hat{y}$, $\hat{\hat{y}}$, y^* and $\tilde{y}$ are expressed by (2.145), (2.146), (2.82) and (2.140), respectively. Notice that we have

$$\tilde{y} \leq \hat{\hat{y}} \leq \hat{y} \leq y^* \quad \forall(\sigma, \theta) \in [0,1]. \tag{2.147}$$

Proposition 15 *Suppose that the Assumption 2 satisfies, $y < y^*$ holds and the collateral constraint* (2.89) *and the tax constraint* (2.90) *are separable. Then*

1. *If $\hat{y} \leq y < y^*$, then*
$$\bar{x}^{nd} < \bar{x}_G^d = \bar{x}_I^d = 1, \tag{2.148}$$
where $\bar{x}^{nd}$ satisfies (2.70) *if* (2.71) *holds, satisfies* (2.74) *if* (2.77) *holds and satisfies* (2.80) *if* (2.81) *holds;*

2. *If $\hat{\hat{y}} \leq y < \hat{y}$, then*
$$\bar{x}^{nd} < \bar{x}_I^d < \bar{x}_G^d = 1, \tag{2.149}$$
where $\bar{x}_I^d$ is the unique real solution associated with (2.129) *in equilibrium;*

3. *If $\frac{1}{2} \leq \sigma$ and $y < \hat{\hat{y}}$ or $\sigma < \frac{1}{2}$ and $\tilde{y} < y < \hat{\hat{y}}$, then*
$$\bar{x}^{nd} < \bar{x}_I^d < \bar{x}_G^d < 1, \tag{2.150}$$

where $\bar{x}_G^d$ is the unique real solution associated with (2.121) *in equilibrium;*

4. *If $\sigma < \frac{1}{2}$ and $y \le \tilde{y}$, then*

$$\bar{x}^{nd} < \bar{x}_I^d = \bar{x}_G^d < 1. \tag{2.151}$$

where $\bar{x}_G^d$ and $\bar{x}_I^d$ are equivalent to unique real solution that satisfy (2.121) *in equilibrium.*

Further, we have

$$a''^{nd} \le a_I''^{d} = a_G''^{d} = 1, \tag{2.152}$$

where

$$a''^{nd} = \begin{cases} 0 & \textit{if } y \in [0, y^{***}], \\ \in (0,1) & \textit{if } y \in (y^{***}, y^{**}), \\ 1 & \textit{if } y \in [y^{**}, y^{*}]. \end{cases}$$

Proof.

Note that the solutions $\bar{x}_I^d$ and $\bar{x}_G^d$ are expressed in implicit forms. Among solutions, we show that there exists unique solution that supports the equilibrium in subsections 4.4.2, 4.5.2 and 4.5.3, respectively. Therefore, we confine our attention to these solutions and eliminate those which cannot satisfy the equilibrium. In spite of implicit solutions, we can still compare the states of welfare between the punishment types. Finally, the proof directly follows from the sufficient conditions that are characterized in the previous subsections. ■

As in the Proposition 13, in here the world is non-Ricardian. With the government debt, the economy accomplishes larger welfare. Although the punishment imposed after the tax non-compliance limits not only supply of government debt, but also private credit, the government debt- by creating additional liquidity- ameliorates the financial frictions that arise from liquidity constraints. Since a defaulter's pooling with the non-defaulters in the limited information meetings amplifies the financial frictions, the individual punishment equilibrium implies smaller welfare than the one with the global punishment, as in the Proposition 13. Further, by using Proposition 14, remember that the government debt always improves collateral and therefore with the government debt the private asset trade at the DM is never optimal. In contrast, if the financial frictions are strong enough in the economy without government debt, then the buyer can optimally trade her house to make up for the weak collateral.

Second, if the houses are sufficiently plentiful, i.e., the dividends and/or the pledgeability factor are large enough, then both global and individual punishments imply the efficient trade. The private credit is not plentiful enough to support the efficient trade by assumption. However, with the government debt, the additional liquidity is large enough to support the efficient equilibrium. This state is described in the case 1 of Proposition 15. Further, the conditions that support this case are displayed in a numerical exercise, as depicted by the area ① in (y, σ)-space in the Figure 2.8.

Consider the case in which the housing is too scarce to the extent that government with individual punishment does not support the efficient trade; however, it is sufficiently plentiful to the extent that the government debt with global punishment supports the efficiency. In fact, this state can be characterized by the case 2 of the Proposition 15 and the conditions to support this equilibrium can be exhibited by the area ② in the Figure 2.8. Further, when the private credit is scarce enough to the extent that the government debt even under the global punishment does not imply an efficient outcome, then a defaulter would optimally choose to pool with the non-defaulters in the off-the-equilibrium path if the tax constraint is sufficiently relaxed. When the assets are too scarce to support the efficient trade and too plentiful to trigger the equivalence between global and individual punishments, the defaulters optimally choose to pool with non-defaulters and this rises the off-the-equilibrium punishment. Therefore, the state with global punishment accomplishes higher welfare than the one with individual punishment. This state is characterized by the case 3 of the Proposition 15. As well, the sufficient conditions for this equilibrium to exist are exhibited by the area ③ in the Figure 2.8. When the dividends and/or pledgeability factor are very low and the frequency of the full information meetings are very high, then both house and bond prices bear large liquidity premia. Therefore, large prices trigger the defaulters not to pool with the non-defaulters and imply autarky. Under this state, the global punishment is equivalent to the individual punishment. This state is expressed by the case 4 of the Proposition 15 and the sufficient conditions for this equilibrium to exist are shown by the area ④ in the Figure 2.8.

One way to get rid of implicit solutions in (2.121) and (2.129) is to concentrate on the equilibrium with full pledgeability. What does the equilibrium look like when $\theta = 1$? The next proposition shows a link between the equilibrium allocation when Assumption 1 satisfies and the one when Assumption 2 satisfies.

Proposition 16 *The equilibrium allocation $(x, \phi, q, V, \hat{V})$ with the government debt under the Assumption 1 is equivalent to the one with government debt under the Assumption 2 and $\theta = 1$.*

Proof.

Plug $\theta = 1$ to the equations (2.121) and (2.129) in the subsections 4.4.2, 4.5.2 and 4.5.3. When $\theta = 1$, the implicit function can be written explicitly and the results are equivalent to the equilibria characterized in the subsections 3.4.2, 3.5.2 and 3.5.3.

■

First, remember that if Assumption 2 holds, the welfare measure (2.60) will produce surplus loss from the house trades at the DM. The source of surplus loss originates from the extra component expressed by $\beta y(a'' - a)$. If a buyer trades the asset, this will imply a welfare loss since the sellers do not enjoy the housing services. However, it turns out that when government issues debt, the Proposition 14 states that collateral is always useful and the buyer optimally purchases the house at the first date and keeps it forever in order to post it as collateral. The equilibrium outcome eliminates the welfare loss because the buyers keep all the houses. Therefore, by the Proposition 14, the welfare function (2.54) under the Assumption 1 and (2.60) under the Assumption 2 are equivalent in equilibrium since the surplus loss due to the house trade is optimally eliminated.

Second, the collateral constraint (2.63) under Assumption 2- in contrast to liquidity constraint (2.9) under Assumption 1- includes additional source of financial friction: the pledgeability factor. If the pledgeability factor gets smaller, the collateral constraint will tighten up and hence it will amplify the financial friction. If Assumption 2 holds, then full pledgeability will trigger the buyers to activate the funds up to full capacity by reaching the highest amount of collateralizable wealth. This amount $\beta(\phi+y)a$ reaches the value of the house trade at the DM under Assumption 1. Remember that when Assumption 1 holds, there exists no efficiency loss since everybody acquires direct utility from housing. Therefore, by using Proposition 16 above, in equilibrium the provision of government debt and the full pledgeability eliminate the efficiency losses arising from both the house trades at the DM and lack of private credit creation from the collateralization.

2.5 Private Bank, Government, and Money

In this section, we will introduce money- instead of government bonds- to the model. This allows us to analyze the effects of inflation and nominal interest rates on housing markets, as well as the interaction between public and private supplies of liquidity.

In here, we suppose that a buyer prefers consumption at the DM with probability α and she does not consume with probability $1-\alpha$. This requires a private bank that mitigates the liquidity needs arising from different types of DM meetings. In the spirit of Diamond and Dybvig (1983), the bank efficiently allocates the resources. A buyer first deposits cash to the bank in the CM to insure against the preference shock. Then if she prefers to make consumption, she will take out home-equity loan (HEL) and repay in the following CM. HEL requires housing as collateral and HEL is the only instrument for those who prefers to consume at the DM since we assume that a seller gives no loans and only accepts cash in the bargaining. Denote the price of money in terms of CM good in period t by ψ_t. In equilibrium, it must satisfy $\frac{\psi_t}{\beta\psi_{t+1}} \geq 1$ for all $t = 1, 2, \ldots$, i.e., the money growth rate in each period must be at least as large as the discount factor; otherwise, the buyer optimally chooses not to consume today, but the sellers are willing to produce all and hence this violates the market clearing condition. Further, suppose that only buyers can consume the housing services, i.e., Assumption 2 holds. In equilibrium, it will be optimal for a buyer to post all the assets as collateral. Therefore, a buyer will purchase her house at the first date and keep it forever since it is useful to take out HEL.

We introduce a new subperiod, namely the Home-Equity Loan Market (HELM), in the spirit of He, Wright, and Zhu (2015). The buyers will deposit cash at the end of the CM and then the cash will be reallocated in the forthcoming HELM. In the HELM, a buyer not only privately learns whether she will trade or not, but also has an access to the bank to borrow cash against the collateral. Hence, she can use the cash at the next DM in exchange for consumption goods. A buyer who is willing to consume withdraws cash, but those who are unwilling to consume leave their deposits alone. The debts are settled in the CM of the next period. That is,a buyer who withdraws cash will make repayment and that who does not withdraw in the HELM will withdraw in the CM. We assume that the HELM is competitive under a concrete friction: limited commitment. The limited penalties will be imposed if a buyer reneges on her promise. The fraction γ_h of the home-equity will be seized if she absconds with her borrowing. In here, we can interpret $1-\gamma_h$ as the haircut for the home-equity loan. The home-equity loan interest rate in the period t is denoted by ρ_{t+1}. For one unit of money withdrawal in the HELM at period t, a household repays $1+\rho_{t+1}$ in the next CM. The price of the loan is determined in a competitive market. Further, the central bank makes a lump-sum transfer τ_t to each buyer in the CM of period t.

2.5.1 The CM Problem

In the CM, a buyer chooses the optimal labor supply H_t, private asset purchase a_t, and the deposit M_t'. A trader in t incurs a real obligation of D_{t+1} owed to the bank; otherwise, she will be a non-trader and leave deposits alone during DM. As well, she will withdraw $1+\rho_{t+1}$ units of cash at the CM of period $t+1$ for every unit of deposit in the CM of period t . For simplicity, we will confine our attention to stationary equilibrium and use the real variables in the representation: A buyer has m' units of real demand deposits in the bank at the CM in each period. She incurs D consumption goods in the CM and holds a units of housing. As well, she determines the optimal quantity of asset to purchase a', the labor supply H and the real deposits m' at the CM. As well, money grows at the same rate, i.e., $\frac{\psi_t}{\psi_{t+1}} = \mu$. Then the CM value function can be expressed as

$$W(D,a) = \max_{H,a',m'} \left\{ -H + ay + V(a',m') \right\} \tag{2.153}$$

subject to

$$H + \phi a + \tau = D + \phi a' + m', \tag{2.154}$$

where V denotes the value function of the buyer at the beginning of the HELM and (2.154) shows the buyer's budget constraint.

2.5.2 The HELM Problem

At the HELM, each buyer learns her type, whether she is willing to consume at the DM or not. It is important to remember that this is a private information. A buyer who will trade at the DM withdraws m'' units of real money balance from the bank and uses the withdrawals to purchase consumption good at the DM. Therefore, she incurs $\frac{1+\rho}{\mu}(m'' - m')$ repayment in terms of consumption good in the next CM. Due to the limited commitment friction, there exists personal collateral constraint that enforces a debt limit. If a buyer breaks the promise, then the bank seizes the fraction γ_h of the buyer's asset holding. On the other hand, a non-trader leaves the deposits in the bank and withdraws $\frac{1+\rho}{\mu} m'$ units of real money balance in the next CM. We assume that each trader makes a take-it-or-leave-it offer. That is, the trader who withdraws m'' units of cash consumes $\frac{\beta}{\mu} m''$ units of good.

The HELM problem can be expressed by

$$\begin{aligned} V(a',m') = {} & (1-\alpha)\beta W\left(-\frac{(1+\rho)m'}{\mu}, a' \right) + \\ & + \ \alpha \left\{ \max_{m''} u\left(\frac{\beta m''}{\mu}\right) + \beta W\left(\frac{(1+\rho)(m''-m')}{\mu}, a' \right) \right\}, \end{aligned} \tag{2.155}$$

subject to

$$-\frac{\beta}{\mu}(1+\rho)(m''-m')+\beta(\phi+y)a'\gamma_h \geq 0, \tag{2.156}$$

where ρ is the interest rate on the bank loan. With probability α, a buyer's real cash holding increases from m' to m'' in the HELM, and she consumes at the DM and incurs a real obligation of $\frac{1+\rho}{\mu}(m''-m')$. The limit on the loan owed to the bank is $(\phi+y)a'\gamma_h$. With probability $1-\alpha$, the buyer cannot consume at the DM and hence she has no debt payment. She leaves the deposit alone and the bank pays off a real return of $\frac{1+\rho}{\mu}m'$ in the CM. The inequality (2.156) shows the collateral constraint for the home-equity loan. It turns out that (2.156) is not the only constraint. The total cash withdrawals at the DM cannot be larger than the bank's total deposit reserves. Hence, the deposit constraint is given by

$$\alpha M_t'' \leq M_t'. \tag{2.157}$$

The government issues currency M_t' in nominal terms in every CM of period $t=1,2,3,...$ The government stimulates money into the economy through lump-sum transfer τ_t to each household in the CM in period t: Thus, given M_0', the monetary policy rule $\left\{\frac{\psi_t}{\psi_{t+1}}, \tau_{t+1}\right\}_{t=0}^{t=\infty}$ follows

$$M_t' + \frac{\tau_{t+1}}{\psi_{t+1}} = M_{t+1}' \quad \forall t=0,1,2,..., \tag{2.158}$$

$$\tau_0 = M_0' \tag{2.159}$$

2.5.3 Equilibrium

We will first characterize the solutions to the CM problem and HELM problem, and then solve for stationary equilibrium. From the CM problem (2.153), the first order conditions of optima are given by

$$-\phi+\frac{\partial V}{\partial a'}=0, \tag{2.160}$$

$$-1+\frac{\partial V}{\partial m'}=0, \tag{2.161}$$

where

$$m' = M_t' \psi_t. \tag{2.162}$$

Then the followings express the envelope conditions for (2.153):

$$\frac{\partial W}{\partial D} = -1, \tag{2.163}$$

$$\frac{\partial W}{\partial a} = \phi + y, \tag{2.164}$$

Next is the HELM problem (2.155). Suppose that the collateral constraint is slack. Then the first order condition implies

$$\frac{\beta}{\mu} u'(\frac{\beta}{\mu} m'') + \frac{\beta}{\mu}(1+\rho)\frac{\partial W}{\partial D} = 0, \tag{2.165}$$

Then by using (2.163) and (2.164) the envelope conditions for (2.155) are given by

$$\frac{\partial V}{\partial a'} = (1-\alpha)\beta \frac{\partial W}{\partial a'} + \alpha\beta \frac{\partial W}{\partial a'} = \beta(\phi + y), \tag{2.166}$$

$$\frac{\partial V}{\partial m'} = \frac{\beta}{\mu}(1+\rho) \tag{2.167}$$

Define the gross nominal interest rate R and gross HEL interest rate $\hat{\rho}$ by $R = 1 + i = \frac{\mu}{\beta}$ and $\hat{\rho} = 1 + \rho$, respectively. Now suppose that the collateral constraint binds. Then the HELM value function can be written as

$$\begin{aligned} V(a', m') = {} & (1-\alpha)\beta W\left(-\frac{1}{\mu}(1+\rho)m', a' \right) \; + \\ + \;\; & \alpha \left\{ u\left(\frac{\beta}{\mu} m' + \frac{\beta(\phi + y)a'\gamma_h}{1+\rho} \right) + \beta W\big((\phi + y)a'\gamma_h, a' \big) \right\}, \end{aligned} \tag{2.168}$$

Then the envelope conditions by using (2.168) are given by

$$\frac{\partial V}{\partial m'} = \frac{\beta}{\mu}\big(1 - \alpha + \alpha u'(x) \big) \tag{2.169}$$

$$\frac{\partial V}{\partial a'} = \beta(\phi + y)\left(1 + \frac{\alpha\gamma_h(u'(x) - 1)}{1+\rho}\right) \tag{2.170}$$

As (2.156) binds, there are two cases: First, it is the case where deposit constraint is slack. In another words, total cash demand does not exhaust the deposits.

$$\rho(m' - \alpha m'') = 0. \tag{2.171}$$

By using the market clearing condition for HEL defined above, the bank's reserve superiority over the loan demand implies that the HEL interest rate is zero, i.e., $\rho = 0$. Second is the case where the loan demand exhausts all the reserves. Then by using (2.171), the HEL interest rate is positive, i.e., $\rho \in (0, i)$, where i is the nominal interest rate.

Collateral Constraint (2.156) **does not Bind.**

By using (2.160), (2.161), (2.163) and (2.164), then in equilibrium the asset reaches its fundamental price and the HEL interest rate equals to the nominal interest rate. In particular,

$$\phi = \frac{\beta y}{1-\beta}, \quad \text{and} \quad \rho = i, \tag{2.172}$$

$$x = \frac{1}{R^2}, \quad \text{and} \quad m'' = \frac{1}{R}. \tag{2.173}$$

In equilibrium, (2.172) and (2.173) have to satisfy (2.156), i.e.,

$$\frac{(1-\alpha)(1-\beta)}{\beta y R} \leq \gamma_h. \tag{2.174}$$

Collateral Constraint (2.156) **Binds, Deposit Constraint** (2.157) **does not Bind.**

By using (2.168), (2.169), (2.170) and (2.171), then in equilibrium we have

$$\phi = \frac{\beta y(1-\gamma_h+\gamma_h R)}{1-\beta(1-\gamma_h+\gamma_h R)}, \quad \text{and} \quad \rho = 0, \tag{2.175}$$

$$x = \frac{\alpha^2}{(R-(1-\alpha))^2}, \quad \text{and} \quad m'' = \frac{R\alpha^2}{(R-(1-\alpha))^2}. \tag{2.176}$$

Moreover, in equilibrium, (2.175) and (2.176) have to satisfy the deposit constraint (2.157), i.e.,

$$\gamma_h \leq \frac{\alpha^2(1-\alpha)(1-\beta)}{\alpha^2(1-\alpha)\beta(R-1)+\beta y(R-(1-\alpha))^2} \tag{2.177}$$

Both Collateral Constraint (2.156) **and Deposit Constraint** (2.157) **Bind.**

By using (2.168), (2.169), (2.170), (2.171) and binding deposit constraint (2.157), then in equilibrium we have $\rho \in (0,i)$. In particular, the gross HEL interest rate is given by

$$\hat{\rho} = \frac{2\beta y\gamma_h R + \alpha^2(1-\beta+\beta\gamma_h) - \left(4\beta y\gamma_h\alpha^2 R(1-\beta+\beta\gamma_h\alpha)+\alpha^4(1-\beta+\beta\gamma_h)^2\right)^{\frac{1}{2}}}{2\beta y\gamma_h(1-\alpha)}, \tag{2.178}$$

and the asset price as a function of gross HEL interest rate $\hat{\rho}$ and gross nominal interest rate R is given by

$$\phi = \frac{\beta y\left((1-\gamma_h)\hat{\rho}+\gamma_h R\right)}{(1-\beta+\beta\gamma_h)\hat{\rho}-\beta\gamma_h R}. \tag{2.179}$$

The DM consumption x and cash withdrawal m'' are given by

$$x = \frac{\alpha^2}{(R-\hat{\rho}(1-\alpha))^2} \quad m'' = \frac{R\alpha^2}{(R-\hat{\rho}(1-\alpha))^2} \tag{2.180}$$

Moreover, in equilibrium, (2.178) and (2.179) have to satisfy $\rho \in (0,i)$, i.e.,

$$\frac{\alpha^2(1-\alpha)(1-\beta)}{\alpha^2(1-\alpha)\beta(R-1)+\beta y(R-(1-\alpha))^2} < \gamma_h < \frac{(1-\alpha)(1-\beta)}{\beta y R} \tag{2.181}$$

By using the sufficient conditions (2.174), (2.177) and (2.181), we characterize the steady state equilibrium in the next theorem. In particular, we will show the effect of the nominal interest rate on the asset prices and HEL interest rate.

Proposition 17 *There exists a unique stationary equilibrium which implies*

1. *If* $\frac{(1-\beta)(1-\alpha)}{\beta y R} \leq \gamma_h$, *then*

$$\rho = i, \quad \phi = \frac{\beta y}{1-\beta}, \quad \textit{and} \quad \frac{\partial \phi}{\partial R} = 0; \tag{2.182}$$

2. *If* $\frac{(1-\beta)\alpha^2(1-\alpha)}{\beta\alpha^2(1-\alpha)(R-1)+\beta y(R-(1-\alpha))^2} < \gamma_h < \frac{(1-\beta)(1-\alpha)}{\beta y R}$, *then*

$$\rho \in (0,i), \quad \frac{\partial \phi}{\partial R} < 0, \quad \textit{and} \quad \frac{\partial \hat{\rho}}{\partial R} > 0, \tag{2.183}$$

   where asset price ϕ *and gross HEL interest rate* $\hat{\rho}$ *satisfy* (2.179) *and* (2.178), *respectively;*

3. *If* $\gamma_h \leq \frac{(1-\beta)\alpha^2(1-\alpha)}{\beta\alpha^2(1-\alpha)(R-1)+\beta y[R-(1-\alpha)]^2}$, *then*

$$\rho = 0, \quad \textit{and} \quad \frac{\partial \phi}{\partial R} > 0, \tag{2.184}$$

   where asset price ϕ *satisfies* (2.175).

The Figure 2.9 shows the sufficient conditions for each equilibrium in the (γ_h, R) space where the vertical axis and horizontal axis represent the pledgeability factor and gross nominal interest rate, respectively. Further, CC and DC denote the collateral constraint (2.156) and deposit constraint (2.157), respectively. Therefore, if the γ_h is sufficiently small and there is enough liquidity in the system, the binding collateral constraint becomes severe in spite of unbinding deposit constraint. Therefore, an increase in the inflation relaxes the collateral constraint and hence increases the housing prices. However, if γ_h is moderate, then an increase in the nominal interest rate will have an overwhelming increasing tax effect on the buyers and thus lower the housing price. It is important to note that under this case, not only collateral constraint binds, but also deposit constraint binds. Thus, an increase in nominal interest rate increases the interest rate on HEL. When the pledgeability for housing collateral is large enough to support efficient trade, then housing reaches its fundamental value. Thus, an increase in nominal interest rate has no impact on housing prices and an increase in nominal interest rate increases the HEL interest rate one-for-one.

The Figure 2.10 shows a numerical exercise and captures three graphs: first shows the HEL interest rate; second shows the housing price; and third shows the DM consumption as a function of the gross nominal interest rate in equilibrium. Notice that the inflation and the interest rate on HEL effectively function as the tax on consumption and the tax on collateral, respectively. Near Friedman Rule, the deposit reserves are not exhausted and hence an increase in inflation has no impact on the HEL interest rate. Therefore, an increase in inflation increases the marginal utility of consumption so does the Lagrange multiplier or the shadow value of the collateral constraint (2.156). As the constraint tightens up, the asset bears larger liquidity premium and therefore house price increases. The quantity of exchange will decrease sharply not only because of the inflationary effect per se but also because rising house price increases buyer's housing expenditure and the cost of HEL. Given all the deposits are exhausted for moderate inflation, the deposit constraint will bind and hence an increase in the inflation will increase the HEL nominal interest rate, but not one-for-one. It turns out that for moderate values of inflation an increase on inflation causes larger effect of taxation on housing then the consumption and therefore the house price starts to decline. As a result, the DM exchange will decrease, but not as sharp as the one near the Friedman Rule. Finally, for sufficiently large inflation, the collateral constraint does not bind and house price reaches its fundamental value. An increase in inflation increases the interest rate on HEL one-for-one and hence its effect as taxation on consumption is equivalent to the one on housing. Therefore, the house price does not change.

2.6 Conclusion

Housing offers many services. This paper captures some services that housing provides. First, when there exists no surplus loss from the house trade at the DM, collateralized debt is suboptimal since the risk of default limits the private loan taken out by the buyer. Since the buyer can create the maximum liquidity by selling the asset in this state, it will be optimal for a buyer to trade it in exchange for consumption. Hence, the housing serves as a medium of exchange. We show that government debt, in addition to asset trade, alleviates the liquidity constraints and hence accomplishes larger welfare for any type of punishment. In particular, the Ricardian equivalence never holds even under the harshest punishment.

When there exists an asymmetry on willingness to consume dividends between buyers and sellers, the inefficiency from the asset trade occurs. Under this state, the housing collateral can perform a useful role only if the asset is sufficiently plentiful. Then the buyer purchases the house at the first date and keeps it forever. Since it supports the collateralized debt, it bears liquidity premium. When the households are sufficiently impatient and the asset is sufficiently scarce, the buyer optimally trades the asset to make up for the weak incentives in collateral. When the financial frictions amplify as they did during the financial crisis in 2008 and during the credit crunch in 2007-2009, collateralized debt becomes too expensive to take out. This happens as fire sales become prevalent during the crisis when overwhelming desire for liquidity exceeds the efficiency loss associated with the asset trade. We show that the optimal purchases of government debt- like central bank's *open market operations*- supports the housing collateral and in fact eliminates the fire sale equilibrium in which houses are traded at the DM. Therefore, with the government debt it is always optimal for a buyer to post her house as collateral and the economy enjoys larger welfare. Further, the full pledgeability in the private engagement, in addition to the government debt, completely cures the efficiency loss arising from the asset trade in equilibrium.

We also add money, instead of the government debt, to the benchmark model. The interesting result is that when the financial frictions amplify such as an increase haircuts and a decrease in the collateralizable wealth, an increase in the nominal interest rate might increase the housing prices and hence collateralizable wealth. However, if the frictions are moderate, then an increase in the interest rate decreases the housing prices, but increases the interest rates on HEL. When the financial frictions are irrelevant, then an increase in the nominal interest rate has no impact on the asset prices. It is important to note that the inflation and interest rates on HEL can be interpreted as taxation on consumption and the housing collateral, respectively. Near Friedman Rule, there is enough liquidity in the system in spite of the binding collateral constraint and an increase in inflation rises the marginal utility of consumption so does the shadow value of the incentive constraint. Therefore, house price increases. For moderate inflation economy, an increase in inflation creates larger effect of taxation on housing collateral than the one on consumption and hence the asset price starts to decline. For large inflation economy, the financial friction arising from liquidity constraint is irrelevant.

Figure 2.1: Individual Punishment, No Government Debt and Assumption 1

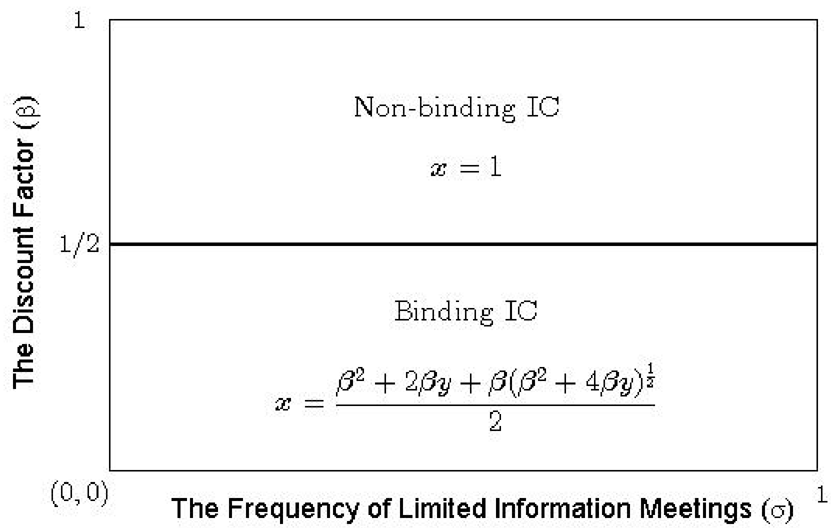


Figure 2.2: Global Punishment, Government Debt and Assumption 1

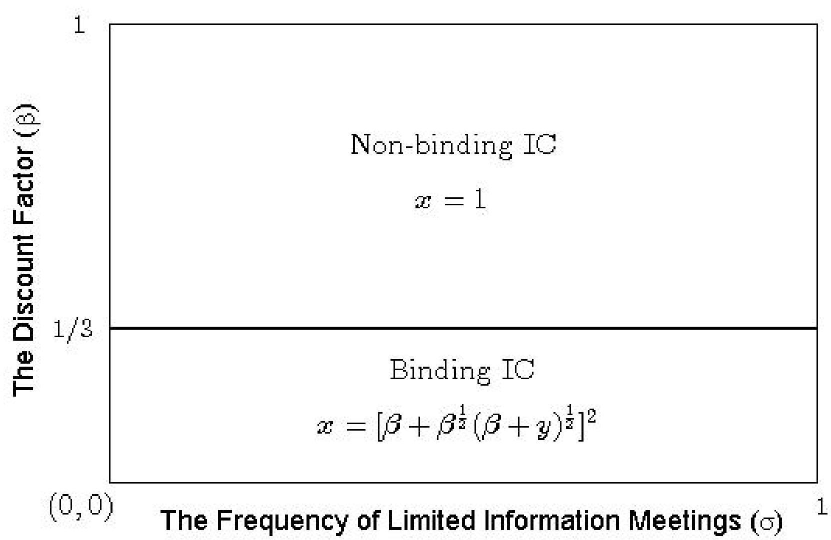


Figure 2.3: Individual Punishment, Government Debt and Assumption 1: Exercise 2

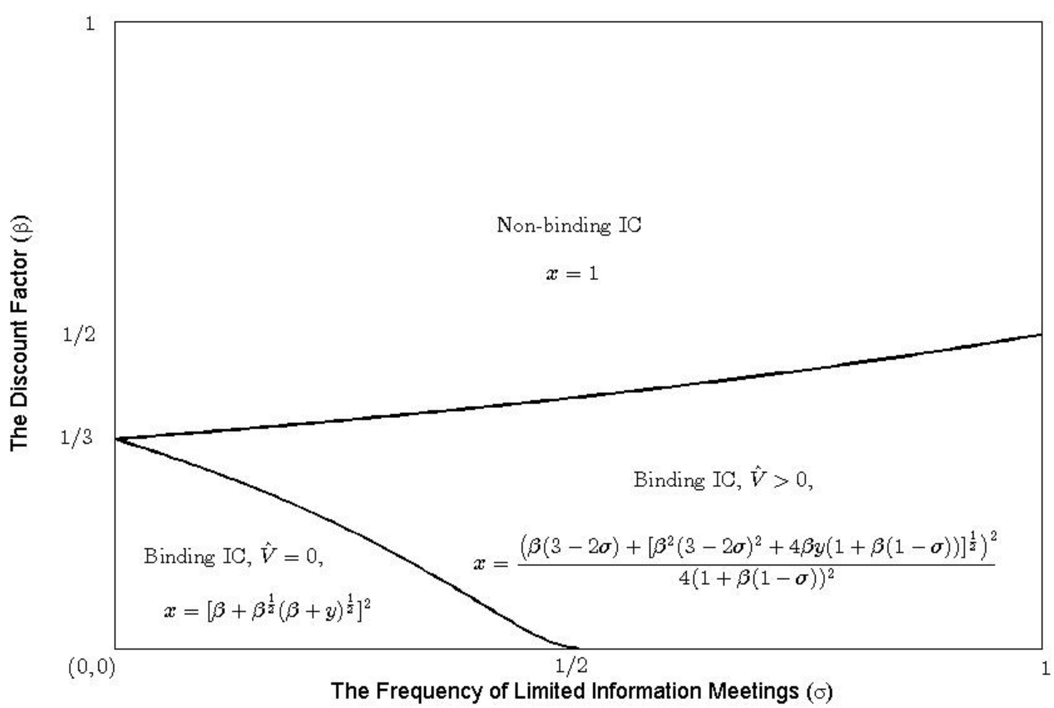


Figure 2.4: Proposition 13: Combination of Exercise 1 and Exercise 2

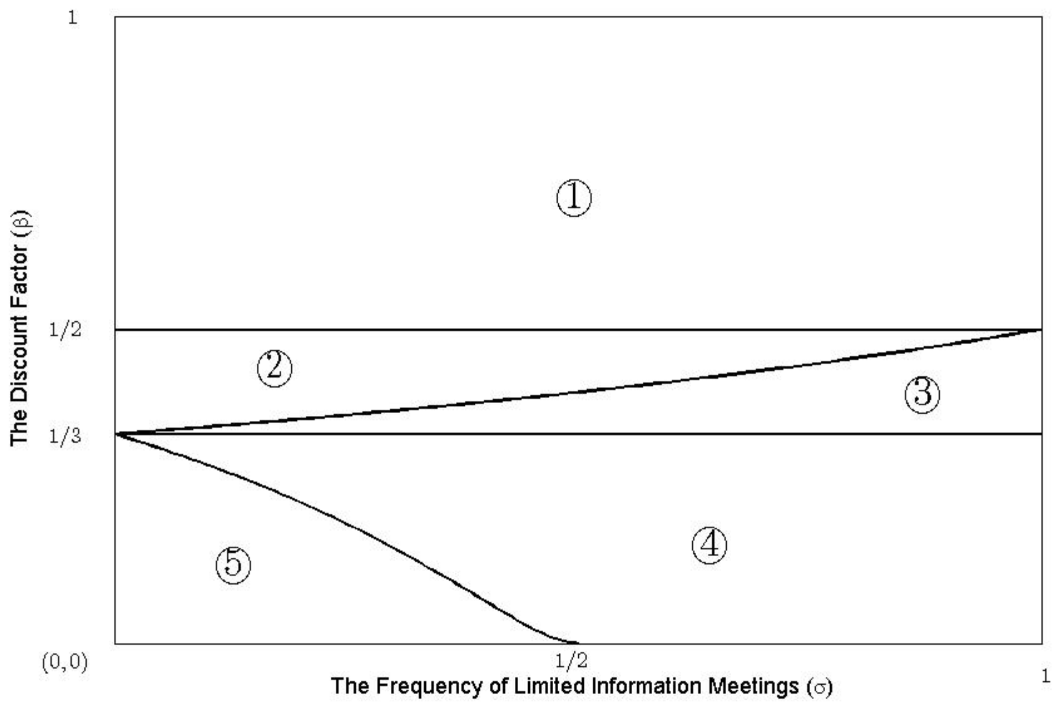


Figure 2.5: Individual Punishment, No Government Debt and Assumption 2

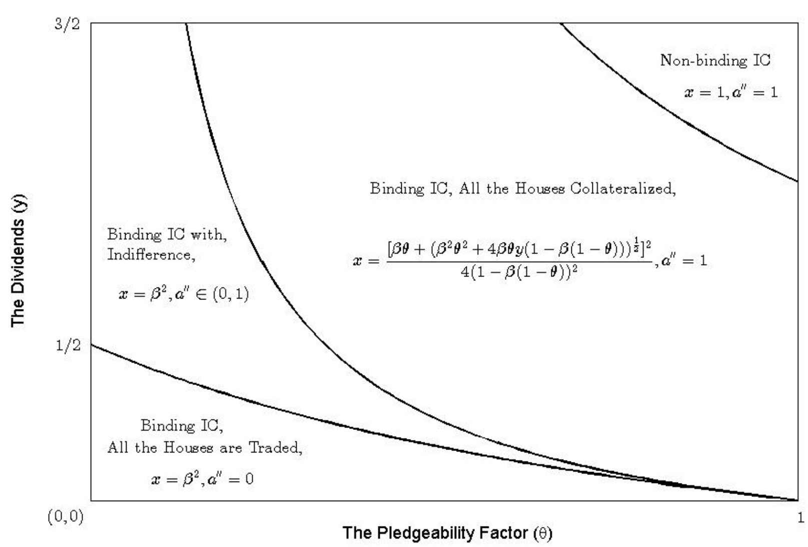


Figure 2.6: Global Punishment, Government Debt and Assumption 2

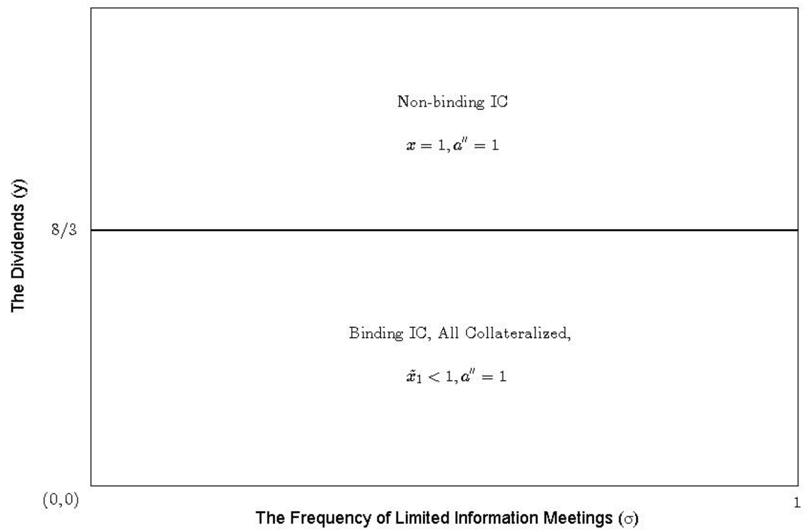


Figure 2.7: Individual Punishment, Government Debt and Assumption 2: Exercise 4

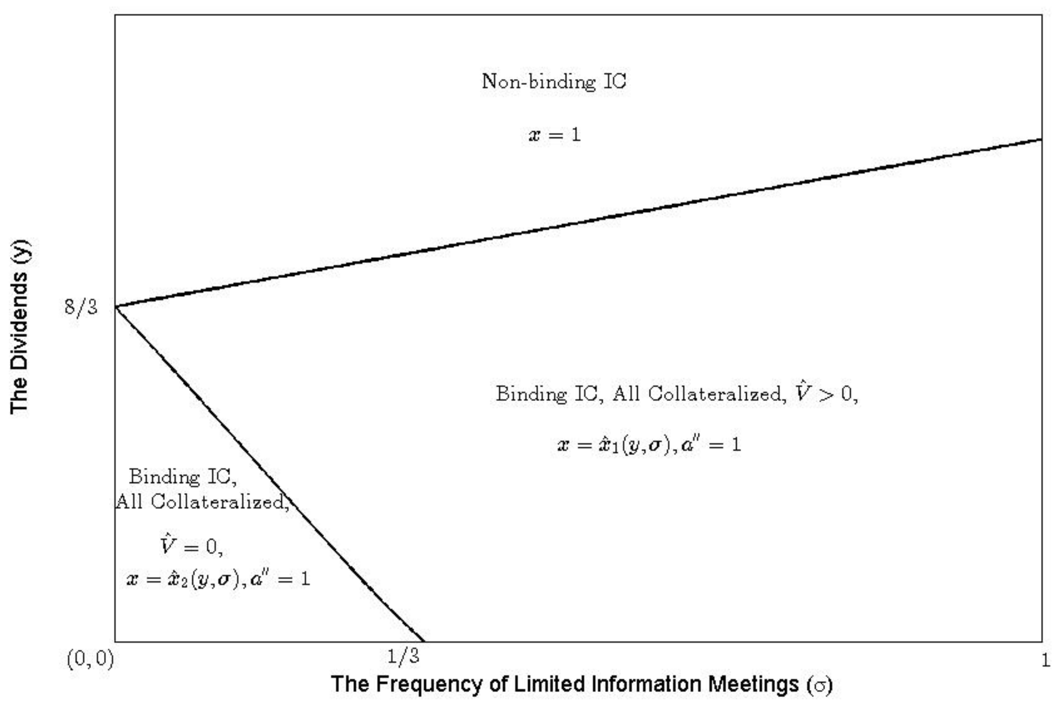


Figure 2.8: Proposition 15: Combination of Exercise 3 and Exercise 4

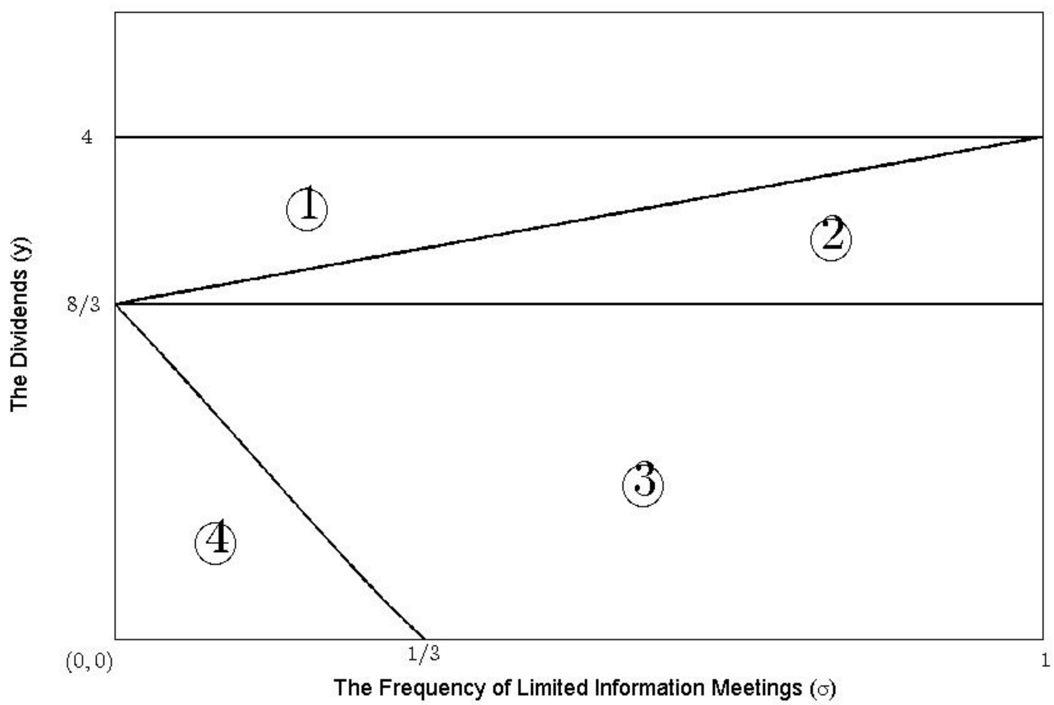


Figure 2.9: Equilibrium with Money and Bank

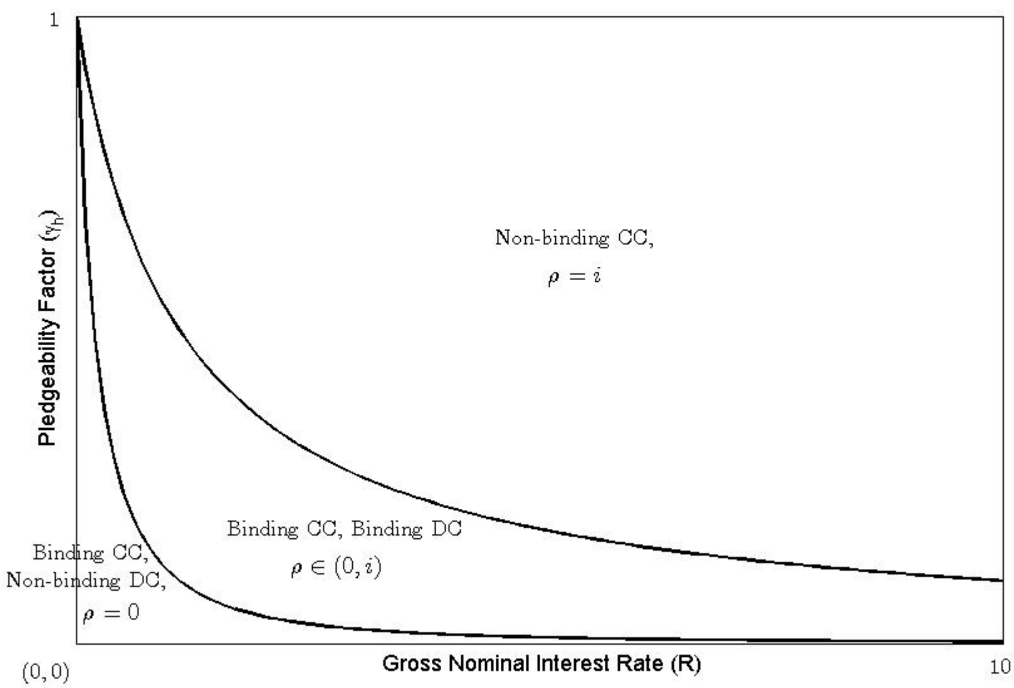
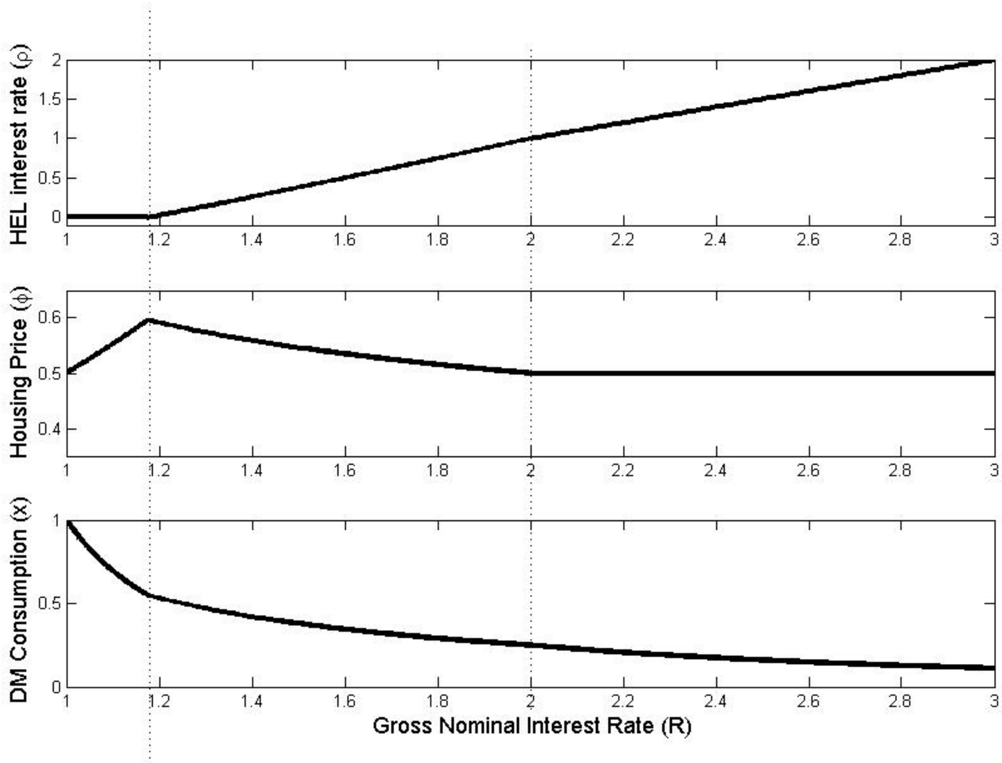


Figure 2.10: HEL Interest Rate, House Price and Quantity of DM Exchange



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