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WASHINGTON UNIVERSITY IN ST. LOUIS

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Essence and Ontology

by

Jan Plate

A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

December 2012

St. Louis, Missouri

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Contents

List of Figures	vi
Acknowledgments	vii
Abstract	ix
1 Introduction	1
1.1 Fine’s Asymmetry	1
1.2 Objectual and Generic Essence	5
1.3 Desiderata	7
1.4 Competing Approaches	11
1.4.1 Wiggins	11
1.4.2 Gorman	15
1.4.3 Zalta	20
1.5 Overview	23
2 Foundations I: Attributes and States of Affairs	25
2.1 Preliminaries: Classes, Functions, and Plurals	25
2.2 Variables and Constants	27
2.3 Terms	28
2.4 Instantiations of Attributes	30
2.5 Two Relations	34

2.6	Truth-Functional Connectives	35
2.7	The λ -Operator	36
2.8	Quantifiers	41
3	Foundations II: Logical Modality	44
3.1	Formulas and ‘Formal Languages’	44
3.2	Interpretation, Entailment, and Logical Modality	46
3.3	Identity and Distinctness of States of Affairs	50
3.4	Are there Modal Attributes and States of Affairs?	54
4	Individuational Ontology	59
4.1	Construction, Individuation, and Reciprocal Essence	59
4.2	Aczel’s Approach to the Ontology of Sets	64
4.3	Individuational Ontologies	67
4.4	Graphs	71
4.5	Decorations	73
4.6	Undecoratable Graphs	75
4.7	The Semantic Principle	77
4.8	Identity Criteria	80
5	Individuation, Essence, and Essentiality (Relative to an Ontology)	83
5.1	<i>O</i> -Individuation	83
5.2	<i>O</i> -Essence	84
5.3	<i>O</i> -Essentiality	89
5.4	De-Relativization	92
6	Explanatory Optimality	97
6.1	First Approximations	97
6.2	Explanatory Optimality Revised	101
6.3	The Problem of Defining ‘Comprehensive’	104

6.4	Conclusion	111
7	Systematic Optimality	112
7.1	Taking a Cue from Lewis	112
7.2	Basic Attributes	116
7.2.1	Preliminaries	116
7.2.2	An Account of Basicity	120
7.2.3	Examples of Non-Basic Attributes	124
7.3	Maximal Elegance?	127
7.4	Paying One's Dues	131
7.4.1	Informativeness and Complexity	132
7.4.2	Paying One's Dues, Relativized and <i>Simpliciter</i>	135
7.5	Parsimony	140
7.6	No Trumping	145
7.6.1	A Problem Posed by Non-Well-Founded Sets	145
7.6.2	A First Solution	150
7.6.3	Mereological Troubles	153
7.6.4	A Better Solution	157
7.7	Principality	162
7.8	Putting the Pieces together	166
8	Essence Explicated	170
8.1	Why {Socrates} Essentially Has Socrates as a Member	171
8.1.1	First Case	171
8.1.2	Second Case	177
8.2	Why Socrates Is Not Essentially a Member of {Socrates}	188
8.2.1	The Specification σ^\in	189
8.2.2	Variants of σ^\in	191
8.2.3	Specifications Using Predicates of Singleton-Membership and Parthood	196

8.2.4	An Objection from Gunk	201
8.3	Cognitively Adequate?	206
8.4	Philosophically Significant?	209
9	Metaphysical Modality	214
9.1	What Needs to Be Explained?	214
9.2	<i>De Dicto</i>	215
9.3	<i>De Re</i>	216
9.4	Iterated Modalities	219
9.5	Concluding Remark	225
A	Rebirth of the Modal Account?	227
B	Bibliographical Note on ‘Essence’ and ‘Individuation’	233
C	Improving the Semantic Principle	237
	Bibliography	242

List of Figures

4.1	A graph with edges (P, ν_1, ν_1) and (P, ν_2, ν_2) , where $P = \lambda x, \alpha (\alpha \in x)$	81
5.1	O^\exists -individuation graphs of the first four ordinals.	84
5.2	Mappings between the free variables of an O -individuation formula, the nodes of an O -individuation graph, and the entities assigned to these nodes by an O -decoration.	87
7.1	Extremely simplified sketch of a hypothetical O_ϵ -individuation graph of an urelement or well-founded set.	148
8.1	An accessible pointed graph and its circular part.	190

Acknowledgments

For discussions relevant to the topic of this dissertation, I am grateful to John Heil, Dennis Des Chene, Gillian Russell, Roy Sorensen, Kit Fine, Jonathan Schaffer, Martin Turner, Jason Gardner, John Gabriel, and Nate Adams. I would also like to thank two anonymous reviewers for *Erkenntnis* who have read a draft of a paper (currently neither published nor accepted for publication) that was based on an earlier version of §7.2.

Most of all, I am indebted to my dissertation adviser, Thomas Sattig, for valuable objections, encouragement, and numerous suggestions regarding the presentation and organization of the present material. Still, of course, I assume full responsibility for any defects that the dissertation may have.

For my parents

ABSTRACT OF THE DISSERTATION

Essence and Ontology

by

Jan Plate

Doctor of Philosophy in Philosophy

Washington University in St. Louis, 2012

Professor Thomas B. Sattig, Chair

This dissertation provides an account of essentiality that satisfies two main desiderata:

- (1) The account should offer an explanation as to why the following two intuitions are true:
 - (i) It is essential to the set {Socrates} to have Socrates as a member.
 - (ii) It is not essential to Socrates to be a member of that set.
- (2) The account should do justice to the sense of philosophical significance that has traditionally been attached to the notion of essence.

The two intuitions mentioned in (1) together form what I call ‘Fine’s asymmetry’, after Kit Fine, whose paper ‘Essence and Modality’ has persuasively undermined the traditional modal account of essentiality by pointing out (among other worries) that this account cannot plausibly accommodate both of those intuitions.

The account of essentiality proposed in this dissertation offers an alternative to the modal account. It is reductive, in the sense that it provides truth-conditions for essentialist claims without in turn relying on any fundamental notions of an entity’s ‘nature’ or ‘identity’; nor does it rely on any concepts of metaphysical modality. Instead, it is based on a framework of sets, attributes, and states of affairs, which is introduced in chapters 2 and 3. The account itself is then developed in chapters 4 to 7. The first major step in this direction is the introduction, in chapter 4, of the concept of an *individuation ontology*, which results from a generalization and modification of Peter Aczel’s approach to the theory of non-well-founded

sets. On this basis, chapter 5 introduces relativized concepts of essence and essentiality, where the relativization in question is to individuational ontologies.

The question of what conditions an individuational ontology O has to satisfy in order for essences-relative-to- O to count as essences *simpliciter* is the topic of chapters 6 and 7. Chapter 6 sets out to develop a fairly straightforward approach, but this is quickly seen to face apparently insuperable difficulties. Chapter 7 develops a fundamentally different approach, which turns out to be more successful. In chapter 8, it is shown how the resulting account of essentiality manages to accommodate Fine's asymmetry, and in the final chapter, the account is applied to an elucidation of *de re* modal discourse.

“Was ist das. – Was – ist das ...”

“Je, den Dävel ook, c’est la question, ma très chère demoiselle!”

– Thomas Mann, *Die Buddenbrooks*, beginning

L’usage commun de la raison a dès longtemps permis d’abstraire des objets singuliers d’expérience certaines notions générales qui, bien que confuses, n’en répondent pas moins à des objets.

– Etienne Gilson, *L’être et l’essence*, p. 169

Chapter 1

Introduction

1.1 Fine's Asymmetry

During much of the second half of the 20th century, it was very nearly a consensus among philosophers in the analytic tradition that the notions of essence and essentiality are to be conceived of, if at all, in terms of *de re* modality. Thus, to say that an entity x has a property P 'essentially' was usually taken to mean that x cannot exist without having P ; or, formulated in the framework of possible worlds: that x has P in every possible world in which it exists.¹

This *modal account* of essentiality has been persuasively undermined by Kit Fine in his well-known paper 'Essence and Modality' (1994). Among several other worries, Fine points out that the modal conception of essentiality fails to accommodate a certain intuitively plausible asymmetry, which I will here refer to as *Fine's asymmetry*. The example he uses is that of Socrates and the set {Socrates} (also referred to as 'Socrates' singleton'). On the intuitively plausible assumption that a set exists in every world in which all its members do, it follows that, in every world in which Socrates exists, so does his singleton.² Furthermore, it is plausible

¹E.g., see Plantinga (1974, p. 56). Some philosophers – e.g., Barcan Marcus (1971) – have proposed a variant of the modal account on which an essential property is by definition 'non-vacuous', i.e., not such that *every* entity has it in every world in which it exists.

²N.B.: Here and in the following, the expressions 'Socrates' singleton' and '{Socrates}' are not meant to refer ('non-rigidly') to whatever entity in the possible world under consideration is a set that has Socrates as its only member. Rather, these expressions are meant to refer to the set that, in the *actual* world, has

to assume that, in every possible world that contains both Socrates and his singleton, the former will be a member of the latter. So, in every world in which Socrates exists, he will be a member of his singleton. According to the modal account of essentiality, Socrates is thus *essentially* a member of {Socrates}.³

But this is not what we would suppose to be the case on an intuitive reading of ‘essential’. On an intuitive reading, it is not an essential property of Socrates’ that he should be a member of the singleton, although it is essential to the singleton to have Socrates as a member. Harking back to the Aristotelian phrase that has come to be translated as ‘essence’, Fine asks:

[C]an we not recognize a sense of nature, or of “what an object is”, according to which it lies in the nature of the singleton to have Socrates as a member even though it does not lie in the nature of Socrates to belong to the singleton? (*op. cit.*, p. 5)

In order to render the modal account compatible with this asymmetry, an adherent of the modal account would have to argue that there are some possible worlds where Socrates exists without the singleton. This might be motivated by general considerations in favor of a very liberal conception of possibility.⁴ But if we adopt such a liberal conception, it will be hard to see why we should not also allow that Socrates’ singleton exists in some worlds without having Socrates as a member. The defender of the modal account will not be in a position to argue against this possibility by appeal to intuitions about the essence of {Socrates}, for in her case, this would amount to circular reasoning. So she appears to be caught in a dilemma: if she accepts the above intuitions about what sets exist in what possible worlds, she cannot accommodate the second half of Fine’s asymmetry, viz., the thesis that it is not essential

Socrates as its only member.

³Fine also considers a different (‘categorical’) version of the modal account, on which an object has a property essentially “just in case it is necessary that the object has the property” (*op. cit.*, p. 3). To simplify the discussion, I will here ignore that version.

⁴For related discussion (though focusing on *de dicto* modal claims, rather than *de re*), see the recent literature on contingentism: e.g., Rosen (2006), Parsons (2007), Cameron (2007), and Miller (2009). For a defense of very liberal views with respect to *de re* modality, see Mackie (2006). The defense of the modal account provided by Correia (2007) also falls into this general category, and hence suffers from the weakness pointed out in the rest of this paragraph. I might add that in later chapters I will rely on an extremely liberal conception of modality myself; but this conception can likewise not be used to save the modal account. (Cf. Appendix A.)

to Socrates to be a member of {Socrates}. But if she presses for a more liberal conception of what is possible, she apparently becomes unable to motivate the thesis that Socrates' singleton cannot exist without having Socrates as a member – and hence, becomes unable to accommodate the *first* half of the asymmetry, viz., the thesis that it is essential to {Socrates} to have Socrates as a member.^{5,6}

The modal account is subject to still further worries. Fine himself identifies four additional classes of 'false positives': According to the modal account, it is essential to any entity whatsoever (i) that any given necessary truth in fact obtains, (ii) that any *other* entity should have the essential properties that it in fact has, and (iii) that the entity itself exists (*op. cit.*, p. 5f.). As he also mentions, this third point generalizes to any other entities that exist in every possible world in which the entity itself exists. Finally, (iv) for any two distinct entities x and y , it is under the modal account essential to x that it be distinct from y (*ibid.*). In all these cases, the properties in question are in Fine's view not essential to their bearers, although they are counted as such under the modal account.

In none of these four cases, however, does Fine's point seem quite as compelling as in that of the asymmetry. The second and fourth counter-examples may both be denied on the ground that in any world w , an entity x can stand in a relation to another entity y only if both x and y exist in w . So, as long as x can exist without y (which is presumably the case for most pairs of entities), it will then on the modal account not be essential to x that y should have such-and-such essential properties, nor that x be distinct from y . To be sure, if one were to hold that all entities exist necessarily (as has been maintained, e.g., by Williamson 2002), then this defense will not be available. But intuitively, it is far more plausible to think that – to use Fine's example – Socrates *can* exist without the Eiffel Tower, and similarly for most

⁵By the verb 'to accommodate', I mean something stronger than merely 'to be compatible with'. In particular, if an account is to accommodate the intuition that it is essential to Socrates' singleton to have Socrates as a member, it will not be enough for it merely to be compatible with that intuition, but rather, the account should – possibly in conjunction with other, independently plausible assumptions – allow the intuition to be *derived*.

⁶While the argument of the present paragraph does not occur in 'Essence and Modality', Fine does voice a similar worry in his (2007, p. 87f.) response to Fabrice Correia's proposal to analyze essentiality in terms of "Priorean strict implication".

other pairs of entities.

As for the other two worries, an adherent of the modal account might shrug them off as mere curiosities, or perhaps offer to accommodate them by adding further clauses to the definition of ‘essential property’.⁷ This is conceivably also true for another class of intuitively doubtful ‘essential’ properties, which Fine does not mention: viz., such *world-indexed* properties as the property of being the first person to own a motorcycle in world #521.⁸ A defender of the modal account might argue that these properties, though strictly speaking essential, are somehow less interesting or salient. This might then be used to explain away the intuition that those properties are not essential at all. But might this simple strategy not also work when applied to Fine’s asymmetry? A response of the kind just envisioned would have to dismiss the property of being a member of {Socrates} as ‘strictly speaking essential but uninteresting’ while, on the other hand, the property of containing Socrates would have to be treated as both essential *and* interesting, since this latter property should intuitively be treated as essential to the set {Socrates}. At first blush, it is not clear how this difference could be motivated. Echoing Fine, we might say that there is nothing in the “logic” of the situation that justifies an asymmetric judgment of interest or salience: “the difference lies entirely in the nature of

⁷For an example of the latter, see Barcan Marcus (1971, p. 196), or see Kripke’s laconic reaction to the problem of essential existence:

This definition [viz., that essential properties are those that an object “has to have if it exists at all”] is the usual formulation of the notion of an essential property, but an exception must be made for existence itself; on the definition given, existence would be trivially essential. We should regard existence as essential to an object only if the object necessarily exists. Perhaps there are other *recherché* properties, involving existence, for which the definition is similarly objectionable. (1971, p. 151n.)

Against this move, however, Fine has the following response. In the first place, he objects that it is *ad hoc*; for “[w]hy should the essentiality of existence consist in anything different from the essentiality of other properties?” (p. 7), and in addition, he argues that it “leads to incoherence”. But other adherents of the modal account have been much less worried than Kripke. Here, e.g., is a passage from Plantinga (1974):

Among [the ‘trivially essential’ properties] will be the property of existence—if, once more, we momentarily concede that existence is a property. Every object, clearly enough, exists in every world in which it exists; so everything has essentially the property of existing. This is not to be bogged at. (p. 61)

For a more recent discussion of trivially essential properties, see Della Rocca (1996, p. 3).

⁸Plantinga regards such properties not only as essential to whoever instantiates them, but even does them the honor of calling them ‘essences’. (Cf. his 1974, p. 72.)

the objects in question”.⁹

In addition to Fine’s worries, one might object to the modal account also on the basis of well-known Quinean doubts about the intelligibility of *de re* modal notions.¹⁰ I shall here not discuss these additional concerns. For now, it will be enough to note that, if the worries just mentioned can be made to stick, then the need for an alternative to the modal account of essentiality should seem all the more urgent. My principal aim in this dissertation is to provide such an alternative.

Before I give an overview of this dissertation in §1.5, I will list the desiderata for the to-be-presented account of essentiality in §1.3, and will briefly discuss three rival accounts in §1.4. First, however, I have to insert a preliminary note concerning my usage of the term ‘essence’.

1.2 Objectual and Generic Essence

In the following chapters, we will be concerned not only with the notion of essentiality, but also with that of *essence* – in fact, the account of essentiality to be presented below will contain an account of essence as a proper part. For this reason, it is worth distinguishing beforehand between two very general ways in which one might speak of essences. Following Correia (2006), I will mark this distinction with the labels of ‘objectual essence’ and ‘generic essence’.¹¹ Roughly, one inquires after an *objectual* essence when one asks what it is to be x , for some entity x . By contrast, when one asks what it is to be *an* F , where ‘ F ’ is replaced by a unary predicate, one inquires after a *generic* essence. And similarly, when one asks what it is to be X , where ‘ X ’ is replaced by some mass term, such as ‘water’, one again asks for a

⁹*Op. cit.*, p. 7. This first impression is called into question by Michael Gorman (2005), who claims that the property of being a member of {Socrates} is trivially essential to Socrates. However, this claim has to be rejected, as I will show in §1.4.2 below.

¹⁰For the former, see, e.g., Quine (1953; 1961; 1972, p. 492f.). A fairly recent defense of Quine’s critique can be found in Burgess (1998). Also see McMichael (1983b, p. 63).

¹¹This admittedly creates a terminological conflict with scholastic usage, where (in analogy to the Aristotelian distinction between genus and species) a distinction is drawn between ‘generic essence’ and ‘specific essence’. The term ‘sortal essence’ might have been a viable alternative, but it misleadingly connotes sortal essentialism (see below, §1.3), and so I will here stick with Correia’s labels.

generic essence.

Thus, when Kripke (1972) and Putnam (1975) talk about the essence of water or gold, they talk about generic essence; and likewise when Locke in his *Essay* talks of the ‘real essence’ of gold, or when Aristotle and the scholastic philosophers talk about the essence of man. By contrast, when these latter philosophers talk about the essence of some individual entity, such as Socrates, they talk about an *objectual* essence. We are able to make these determinations regardless of whether the respective philosophers themselves were aware of the distinction between objectual and generic essence, because what matters for applying this distinction is only what the various essences are said to be essences *of*: if of Socrates, the essence in question is objectual; if “of man”, generic.

Correia has suggested that the notion of objectual essence can be reductively explained on the basis of generic essence. For instance, the (objectual) essence of Socrates is on his view nothing else than the generic essence associated with the predicate, ‘thing that is identical with Socrates’.¹² Others have adopted the opposite approach and consider generic essence to be only a special case of objectual essence. For instance, Robertson (2008) has proposed that the (generic) essence associated with the predicate ‘cat’ should be understood in terms of the objectual essence of “the species cat”. I will here not try to take any stance on the question of whether such identifications are successful. The only notion of essence that will interest us in the following is that of objectual essence, and I shall leave aside any questions concerning generic essence. (However, we will see that, on the account to be developed here, and *contra* Correia, the essence of an entity x is in many cases *not* the same as the property of being identical with x .)

It is sometimes assumed, or even taken to be analytically true, that an entity’s essence is a certain property, or group of properties, that that entity does not share with anything else. Thus, Plantinga:

And just what sort of thing would an essence be anyhow? The initial idea is this: an essence of Socrates is a property (or a group of properties) that Socrates

¹²Correia (2006, p. 765).

has essentially and that is *unique* to him. (1974, p. 70)

The idea that essences are unique to their bearers appears to go back to Boethius' view that entities are individuated by certain special, unshareable forms.¹³ Such a view can be taken to suggest an account of essentiality that gives special consideration to *identity criteria*, as for instance the set-theoretical Axiom of Extensionality. But in contrast to Plantinga's view, the account of essentiality that I propose in this dissertation takes a very different direction. The reason for this will become clearer in the following section.

1.3 Desiderata

The account of essentiality that I will try to develop in this dissertation takes the following two desiderata as its main guidelines:

- (D1) The account should provide an answer to the question of why Fine's asymmetry holds.
- (D2) It should do justice to the sense of philosophical significance that has traditionally been attached to the notion of essence.

There are other, 'lesser' desiderata as well. In particular, the account should do justice to various intuitions (besides Fine's asymmetry) as to what sorts of properties are or aren't essential to what entities. We will encounter several of these other intuitions in chapters 6 and 7. Although none of them will be quite as central to our discussion as Fine's asymmetry, one particular class of them deserves to be mentioned already at this early stage. This includes claims to the effect that Fine's asymmetry will retain its intuitive appeal even under the assumption that such-and-such counterfactual circumstances obtain. A particularly relevant example is the following:

- (1) Fine's asymmetry will retain its intuitive appeal even if we assume that, by some lapse of the Axiom of Extensionality, Socrates has *two* singletons.

¹³Cf. King (2000, §3.1). The text in question is Boethius' *Greater Commentary on Aristotle's "De interpretatione"*.

Admittedly, this way of framing the intuition is not entirely felicitous since, if Socrates had two singletons, expressions like ‘{Socrates}’ and ‘Socrates’ singleton’, which are made use of in formulating Fine’s asymmetry, would not have a unique referent. But the general drift should be clear: Suppose that Socrates has two singletons, and let ‘{Socrates}’ denote one of these two singletons (it does not matter which). It will then still be intuitively plausible to say that it is essential to {Socrates} to have Socrates as a member.

What is interesting about this case is that the Axiom of Extensionality – which says that no two sets have all their members in common – is here assumed to be false. Given that this assumption does not seem to impair the intuitive appeal of Fine’s asymmetry, we can infer with some confidence that the answer that our account will eventually provide to the question of why it is essential to {Socrates} to have Socrates as a member had better not rely on the Axiom of Extensionality. More generally, we can infer that our account of essentiality should probably not be built around the notion of an identity criterion (nor around that of a ‘principle of individuation’), however tempting this might at first seem.

It is also worth mentioning two common views about essentiality that will *not* find consideration in the construction of our account. Those are, first, the thesis that certain objects, such as artefacts and organisms, have their respective origin essentially,¹⁴ and second, the view that many (or all) entities are essentially members of the particular ‘sorts’ to which they belong. I will here only briefly sketch my reasons for not trying to accommodate these views.

In the case of the second view, which is sometimes referred to as ‘sortal essentialism’, there are two reasons. First, the intuitive support for this view is in many cases quite weak, despite its strong historical credentials, which reach back at least to Aristotle. Thus, suppose I point at a dog and ask the Aristotelian question, ‘What is it?’. The correct answer will presumably be, ‘A dog’. But it is another view with strong historical credentials, as well as an intuitively plausible one, that if something is essentially an F , then it cannot cease to be an F unless it were destroyed. And on the face of it, it is entirely possible that a dog might

¹⁴The *locus classicus* is Kripke (1972, footnote 56).

survive transformation into a non-dog, such as a cat or a frog. The fictional literature offers an abundance of examples, the probably most prominent source being Ovid's *Metamorphoses*; and usually these examples are presented as cases of transformation rather than destruction.¹⁵

Second, sortal essentialism is subject to some deep theoretical difficulties. Penelope Mackie (1994, p. 312) points out that

[i]f sortal essentialism is true, we should expect to be able to say why it is true. In particular, we should expect to be able to answer the following two questions:

1. Why are *any* sortal properties essential, rather than accidental, properties?
2. What are the principles that determine *which* are the 'essential sortals' – the sortal concepts under which things essentially fall?

She then proceeds to examine two very different versions of sortal essentialism, viz., those offered, respectively, by Brody (1980) and Wiggins (1980), but concludes that “neither theory succeeds in giving satisfactory answers” to the two questions just quoted.¹⁶ That neither Brody's nor Wiggins's account offers satisfactory answers does of course not mean that we cannot expect such answers from *any* version of sortal essentialism. It does, however, cast some doubt on the defensibility of that view; and when this is combined with the above doubts about its intuitive plausibility, sortal essentialism does no longer seem to be a view that it would clearly be desirable to accommodate.

In this connection, it may further be worth pointing out that, when constructing an account of a given philosophical notion, it is not necessarily a fruitful strategy to try to do justice to all historically influential views that have been held on that subject. For at the beginning of such a project it is often difficult to be sure that the notion in question is in fact homogeneous: that, when one speaks of (e.g.) 'essentiality', there is really only one 'thing' that is picked out by that term. For all we can know beforehand, it may well be that the term in question is subtly ambiguous between two or more underlying concepts. In such a case, it would be natural to expect that, of the views that are commonly expressed using that term, not all are

¹⁵Also cf. Wetzel (2000).

¹⁶*Ibid.* Also cf. Mackie (2006, ch. 7–8).

true under every disambiguation. There is then, as it were, a danger that those views will ‘pull in different directions’ without there being any worthwhile compromise.

It might initially be thought that these last considerations also motivate a skeptical attitude toward an approach that sets out to satisfy (D1) and (D2) together with widely-accepted views concerning the essentiality of origin. For it may seem that those views are fueled by considerations as to what circumstances a given entity could or could not have found itself in, that is to say, by *modal* considerations. This in turn might be taken to suggest that those views hold true only under a modal conception of essentiality, while Fine’s asymmetry is best understood in terms of a different conception. Against this, however, I would maintain that a close link between essentiality and *de re* modality ought to be preserved, even if the modal account of essentiality is abandoned. (However, rather than to explicate the notion of essentiality on the basis of *de re* modality, I would instead propose to elucidate *de re* modal discourse on the basis of a concept of essentiality.¹⁷) Hence, I see no reason why modal considerations should in general be deemed irrelevant to the development of an account of essentiality.

The reason why I will here still not make any special attempt at accommodating views about the essentiality of origin lies simply in the fact that they do not seem to me to provide any guidance for an account of essentiality. As far as I can see, this is a result of the fact that (i) the thesis of the essentiality of origin is most plausibly applied to ordinary objects (as opposed to, say, sets), and (ii) the metaphysics of ordinary objects is at present a rather contentious affair. Thus, if one is told that a particular table has its origin essentially, it is not very clear what sort of constraint this will impose on one’s account of essentiality, because it is not clear *what sort of thing* that table is. This claim may sound surprising to the layman, but it will be familiar enough to anyone who has worked on the metaphysics of ordinary objects.¹⁸

¹⁷See chapter 9 below.

¹⁸Cf. the quoted passage from Korman (2011) in §8.3 below, p. 208.

1.4 Competing Approaches

To my knowledge, no account of essentiality that has so far been proposed in the literature is fully able to satisfy the two desiderata listed in the previous section. For instance, Fine's own account of essentiality (as given in Fine 1995b; 2000) does not provide any grounds on which we might be able to derive Fine's asymmetry, and so does not offer any answer to the question of why that asymmetry holds. Of course, one might propose to augment his account by adding a few additional axioms, e.g., to the effect that sets have their members essentially, and that no entity is essentially a member of any particular set. But such additions would be extremely *ad hoc*, and, for just this reason, it is questionable that the resulting account would satisfy the second desideratum.

In the rest of this section, I will briefly discuss three further accounts of essentiality, viz., those by David Wiggins (1980; 2001) and Edward Zalta (2006), as well as an account that is suggested by some remarks by Michael Gorman (2005). I do not intend this discussion to be at all complete; instead, the purpose is only to evaluate the respective accounts' ability to satisfy our two desiderata.

1.4.1 Wiggins

Although David Wiggins's account of essentiality is a version of the modal account, it still promises (at least *prima facie*) to accommodate Fine's asymmetry. The reason for this lies in the fact that his conception of *de re* possibility, or of what properties a given entity could have, is based on a notion of *conceivability*: On his account, it is possible for an entity x to have a particular property ϕ "if and only if it is possible to conceive of x 's [having] ϕ " (2001, p. 111). Thus, if it is possible to conceive of Socrates' not being a member of {Socrates}, then it is possible, according to Wiggins, that Socrates might not be a member of {Socrates}; and if so, his account will accommodate the intuition that it is not essential to Socrates to be a member of {Socrates}.

Granting that Wiggins's account can accommodate this second half of Fine's asymmetry,

the next question is whether it is also able to accommodate the first half, viz., the thesis that it is essential to the set $\{\text{Socrates}\}$ to have Socrates as a member. In order for this to be the case, it has to be *impossible* to conceive of that set as not containing Socrates. Wiggins here faces the task of identifying a clear sense of ‘to conceive’ on which such conceiving is determinately impossible. At first, Wiggins tries to appeal to the Axiom of Extensionality, and the fact that theorists have typically taken “the identity of a set” to be determined by its members. But that axiom is not a modal principle, and so he has to come back to what needs to be proved, viz., “the *de re* modal thought that, if α is a class containing x and y , then α could not have lacked x ” (p. 119).¹⁹ To support this thought, he invokes the authority of mathematicians: “Set theorists who say that it is a peculiarity of sets to be determined by their members, or who distinguish sets from attributes in Quine’s way, are surely saying at least this” (p. 119f.). However, supposing that this is correct and that set theorists do assert the *de re* modal thought just mentioned, why should it be true? In a footnote, Wiggins finally appeals directly to considerations as to what it is possible to conceive:

Against those not convinced, here is a final or Parthian shot: try conceiving the unit set of x , namely $\{x\}$, as the unit set of y where $y \neq x$. Now try conceiving the pair set $\{x, y\}$ as lacking either x or y (p. 119n.)

Does the shot find its mark? Suppose that, at this moment, I try to think of the set $\{\text{Socrates}\}$ and imagine how, in some other possible world, it has *no members at all*. Wiggins might say that I am only deceiving myself, and am not truly conceiving of $\{\text{Socrates}\}$ when I am thinking of that empty set in that other possible world. But it is far from obvious on what grounds he can make this assessment.²⁰

Clearly, what is needed here is a theory of conceiving that would tell us what properties a given entity can be conceived of as having. Wiggins formulates such a theory in a single paragraph immediately after his discussion of the essentiality of set-membership:

¹⁹Concerning attempts to derive this modal thought from the Axiom of Extensionality, also cf. Appendix B.

²⁰For a thorough recent critique of conceivability-based accounts of essentialist knowledge, see Roca-Royes (2011). In connection with Wiggins’s account, iterated possibility raises a further interesting question: Can I at least conceive of *myself* as conceiving of $\{\text{Socrates}\}$ as having no members? If yes, then on Wiggins’s account it is possible that I am conceiving of that set in the specified way, which should mean, if his theory of possibility is correct, that it’s *possibly possible* that $\{\text{Socrates}\}$ should have no members.

[A] thinker genuinely conceives of x as having ϕ only if there is *some* sortal concept f such that: (i) f adequately answers the Aristotelian question of what x is, and commits anyone who singles a thing out as an instantiation of f to an identity-cum-persistence condition for x ; and (ii) f and ϕ are cosatisfiable by x , and if x had the property ϕ that would not preclude x 's being singled out as this very instantiation of f . (p. 121)

The important part of this theory for the present purposes is condition (ii). A problematic feature of this condition is that it makes use of the *de re* modal notion of being “cosatisfiable by x ”, which renders Wiggins’s account of *de re* possibility – and hence, of essentiality – circular. (He notes this circularity himself.) In addition, it is not clear just what it means for an entity to be singled out “as this very instantiation of f ”. If someone were to single out the set {Socrates} in some possible world where it has no members at all, she would presumably single it out as ‘the empty set’; or at least this seems to be most natural description under which she might take herself to have singled it out. But at the same time, that set is of course still {Socrates}, i.e., the very same entity that it is in the actual world. So, why exactly should it then not be the case that our subject has singled it out as *that very thing*, only under a different description?

In his §4.10, Wiggins suggests a different reading of condition (ii). In spite of his use of the subjunctive (“if x had the property ϕ ”), he there elaborates his theory in a way that suggests that the singling-out he has in mind should make reference to a property that is had by the entity in question in the *actual* world. More specifically: when he says that x 's having the property ϕ should “not preclude x 's being singled out as this very instantiation of f ”, what he seems to mean (given what he says in §4.10, specifically on p. 131) is that there should be some property χ that (1) is actually instantiated by x , (2) serves to single out x (again, in the actual world), and (3) is “co-conceivable” with ϕ . One consequence of this is that the disjunction of all the properties that single out x in the actual world is a property that x could not have failed to have.

Thus, to elaborate Wiggins’s own example, Julius Caesar cannot have failed to have the disjunctive property of being either the man killed by Brutus in 44 BCE, or the general who led his legion across the Rubicon on January 10th in 49 BCE, or But evidently such

properties come very cheap. For instance, let χ be the property of being the first human being who crossed the Rubicon on the morning of January 10th, 49 BCE. Suppose Caesar actually instantiates χ . Then, on Wiggins’s account, all the properties co-satisfiable with χ are properties that Caesar might have had. Thus, it turns out that he might have been a Scottish schoolboy, a Russian housemaid, an Assyrian princess, etc. (One wonders whether he could also have been Aristotle or Brutus, but presumably this is ruled out by the necessity of distinctness, which constrains what properties ϕ are such that, for some sortal f , f and ϕ are co-satisfiable by Caesar.) Other than the properties of being human (besides possibly other sortal properties), of being Caesar, of being distinct from Aristotle, Brutus, etc., and other than the long disjunctive property mentioned before, as well as any property entailed by the conjunction of these, Caesar will thus not have any essential properties at all.

More important, Wiggins’s account, as elaborated in §4.10, does not succeed in accommodating Fine’s asymmetry. For consider again the set {Socrates}, and let χ' be the property of *being a set that has a snub-nosed, hemlock-drinking philosopher as its only member*. This may well single out {Socrates} in the actual world, but it is co-satisfiable with many other properties. For example, let ϕ' be the property of *being a set that has not Socrates but Heraclitus as its only member*. Even if Heraclitus was not actually both snub-nosed and hemlock-drinking, he surely could have been (at least on Wiggins’s account), and so ϕ' should count as co-conceivable with χ' . As a result, it is on this account not essential to {Socrates} to have Socrates as a member, contradicting Fine’s asymmetry.

For this reason, as well as because of its circular nature, Wiggins’s account cannot easily be regarded as satisfying our desideratum (D1). In addition, it is doubtful that it will fully satisfy (D2), either, for the philosophical significance of the concept of essence has virtually always been taken to lie in its *metaphysical* import.²¹ This last fact makes it seem unlikely that an adequate account of essentiality will rely on a notion of conceivability as heavily as Wiggins’s account does. For, even if that notion is augmented by some auxiliary account

²¹As MacIntyre notes: “Aristotle made the concept [of essence] express a timeless and necessary element in the nature of the object itself” (1967, p. 59).

of what it would take to conceive of some entity as having such-and-such properties, it still remains a psychological rather than a metaphysical notion.

1.4.2 Gorman

Michael Gorman (2005) has proposed an account of essentiality according to which an entity's essential properties are those "characteristics" of it that are not "explained" by any of its other characteristics.²² As an example, he offers a hydrogen atom's characteristic of containing a proton: the latter is not explained by any other characteristics of the atom and is therefore essential to it. By contrast, the same atom's characteristic of "being prone to bond" is explained by its having a proton, and is therefore merely accidental.²³

By distinguishing in this way the essential from the accidental, Gorman takes himself to have "identifie[d] what is truly at the core of what [a] thing is" (*ibid.*). Unfortunately, his account rests on the largely unexplained notion of a *characteristic*. He says (p. 279) that, while he uses "the word 'feature' to indicate anything that can be said about a thing", he reserves the word 'characteristic' for the use of "indicat[ing] features that really characterize" the entity in question. But he does not say what it means to "really characterize" something. As a result, his account offers no guidance at all to anyone who wishes to determine whether, on Gorman's understanding of essentiality, it is essential to {Socrates} to have Socrates as a member. For he does not specify any grounds on which one might determine whether the "feature" of having Socrates as a member is a characteristic of Socrates' singleton, much less whether it is an *unexplained* characteristic.²⁴

²²*Op. cit.*, p. 284.

²³*Ibid.*, p. 286.

²⁴A similar criticism can be found in Nathan Wildman's recent Ph.D. thesis (2010, p. 81). Wildman proposes a variant of the modal account of essentiality, according to which a property is essential to a given entity just in case (i) the entity has the property in every world in which it exists, and (ii) the property is "sparse" (p. 88). Although he in effect concedes that this account cannot easily accommodate Fine's asymmetry (pp. 95–102), his discussion of the various moves by which the account might be defended is extremely interesting. Wildman arrives at his account chiefly by a process of elimination, through which he hopes to rule out all alternatives to (variants of) the modal account. However, this argument of elimination does not seem to me entirely successful. For example, at one crucial point he argues that a reduction of modal notions to a concept of

For this reason, it is clear that Gorman’s account fails to satisfy our desideratum (D1). What I want to consider in the following is therefore not Gorman’s account itself, but rather an alternative account that is suggested by some remarks that he makes in an earlier passage of the same paper. He there draws on Della Rocca’s (1996) identification of two kinds of “trivial necessary” property, where a “necessary property” of a given thing is one that the thing has in every world in which it exists. Following Della Rocca, he characterizes these two kinds of property (or “feature”) as follows:

Some features, such as being male if a bachelor, are trivial because they belong to everything. Others belong only to some things but are still trivial because they follow logically from the first type of trivial feature. Socrates, for example, has the feature of being identical with Socrates. He does not share this feature with everything, but his having it follows from his having a feature that he does share with everything, namely, being self-identical. (p. 278)

He goes on to claim that “[s]omeone who is willing to follow Della Rocca on this can evade” the counter-example that Fine’s asymmetry poses to the modal account “by arguing that being a member of {Socrates} is trivial in the second sense” (*ibid.*). In other words, what Gorman suggests here is that Socrates’ being a member of {Socrates} “follows from” Socrates’ having some feature F that he shares with every entity whatsoever. If this suggestion can be made to work, it would seem that Fine’s asymmetry can be accommodated by a near relative of the modal account, according to which the essential properties of an entity x are exactly those properties P such that (i) x has P in every world in which it exists, and (ii) P is not a “trivial necessary” property of x , in the sense specified by Della Rocca.

Let us consider Gorman’s suggestion: Is there really some property (or “feature”) F that is shared by every entity whatsoever and is such that, from Socrates’ having F , it follows that Socrates is a member of {Socrates}? The first property that comes to mind as a possible

essence (or essentiality) will fail if it also relies on a concept of consistency (p. 48). His basic thought here is that consistency is itself a modal concept and as such not available for use in a reduction of modal notions. But this seems somewhat rash, considering that consistency alone does not yet give us a satisfactory grasp of *de re* modality. Even if it is granted that consistency is a modal notion, there is certainly still work to be done in order to elucidate *de re* modality, and as far as I can see, there is no reason why, in doing so, one should not be able to make use of other, better-understood modal notions, such as that of consistency. (For a detailed sketch of how one might construct an account of *de re* modality based on concepts of essentiality and logical entailment, see §9.3 below.)

candidate is the property of being a member of one's own singleton. Another possible candidate is the property of being a member of both one's singleton and some other set. Of course there are many further candidates. But each of them, as far as I can see, is such that, from an entity's possession of it, we can infer that the entity in question is a member of its own singleton. This is, after all, what makes them plausible candidates in the first place: because the hope is that, if we can infer from Socrates' possession of the property in question that he is a member of his own singleton, then this will mean (given that the property in question is shared by every entity whatsoever) that the property of being a member of {Socrates} is *trivially* essential to Socrates.

However, it can be seen that the hope just mentioned is in fact unfounded. This becomes evident once we carefully distinguish between two properties that may both be referred to, somewhat loosely, as the *property of being a member of Socrates' singleton*. First, there is the property that we are concerned with in connection with the second half of Fine's asymmetry, i.e., what I have been calling "the property of being a member of {Socrates}". Crucially, the expression '{Socrates}' should here *not* be read as a definite description along the lines of 'the set that has Socrates as its only member', but is rather intended as referring to the particular entity that is Socrates' singleton *in the actual world*.²⁵ Another way to describe this property is as follows: Let x be Socrates' singleton. Then the property in question is simply the property of being a member of x .

The second property, by contrast, cannot be referred to in anything like this way. Instead, one would have to refer to it, for instance, as the property of being a member of the set that has Socrates as its only member. The relevant difference between the two properties can be made clearer if we refer to them by means of λ -expressions. Thus, the first property may be denoted by $\ulcorner \lambda x (x \in t) \urcorner$, where t is a constant denoting Socrates' singleton. (Note that the λ -expression itself does not contain anything to indicate that the referent of t is a set that has Socrates as a member.) By contrast, the second property may be denoted by the considerably

²⁵Cf. above, footnote 2.

more complicated expression

$$\lambda x \exists y (x \in y \wedge \forall z (z = y \leftrightarrow \forall w (w \in z \leftrightarrow w = \text{Socrates})))$$

Being a member of the only entity that has Socrates as its only member.

Consider now the universally shared property of being a member of one's singleton. This, too, can be denoted by a λ -expression, namely:

$$\lambda x \exists y (x \in y \wedge \forall z (z = y \leftrightarrow \forall w (w \in z \leftrightarrow w = x)))$$

Being an entity x that is a member of the only entity that has x as its only member.

From the fact that Socrates has this property, it follows that he is a member of the only entity that has Socrates as its only member. But does it also follow that Socrates is a member of x , where x is some (arbitrarily chosen) entity? On a strict conception of logical consequence, this will only follow once we adopt some additional premise concerning the properties of x . In particular, even if x should happen to be Socrates' singleton, it will not follow that Socrates is a member of x , unless we adopt the additional premise that x has Socrates as a member.

From this consideration, it can be seen that Socrates' being a member of $\{\text{Socrates}\}$ does not strictly follow from his having the property of being a member of one's singleton; and it is hard to see what other universally shared property there could be such that Socrates' being a member of $\{\text{Socrates}\}$ follows from his possession of that property. But if there isn't, then the property in question – i.e., of being a member of $\{\text{Socrates}\}$ – is *not* a “trivial necessary” property of Socrates, after all.²⁶

In defense of the account of essentiality that we are here considering, it might now be urged that we should relax our conception of logical consequence. In particular, it might be thought that we should relax that conception in such a way that, in determining whether p follows from q , certain further premises (such as that x has Socrates as a member) may be

²⁶The mistake of focusing on the property of being a member of one's singleton, rather than on the property of being a member of $\{\text{Socrates}\}$, also seems to underlie Oderberg's discussion of Fine's asymmetry in his (2007, p. 7f.).

added *ad libitum*. This of course raises the question of how we are to restrict the class of premises that may be added, without producing the unwelcome result that everything follows from everything. If we are to preserve the desired consequence that, from Socrates' being a member of his singleton, it follows that Socrates is a member of {Socrates} (where '{Socrates}' is, as above, treated as a constant rather than as a description), then we will *at least* have to admit premises of the form,

X has Socrates as a member,

where ' X ' may be replaced by any constant. However, this in turn creates trouble with respect to the *first* half of Fine's asymmetry, according to which it is essential to {Socrates} to have Socrates as a member. For, if we can just help ourselves to the premise that {Socrates} has Socrates as a member, then, on the accordingly relaxed conception of logical consequence, it 'follows' from the mere fact that {Socrates} is self-identical that {Socrates} has Socrates as a member. Given that the property of being self-identical is shared by every entity whatsoever, it thus turns out that the property of having Socrates as a member is a "trivial necessary" property of Socrates' singleton. On the account under consideration, such properties are disqualified from counting as essential; and so, the property of having Socrates as a member is on this account not essential to {Socrates}, contrary to the first half of Fine's asymmetry.

In this way, we see that the proposed account of essentiality faces a dilemma similar to the one we already encountered in §1.1. For when it is combined with a strict conception of logical consequence, the account is unable to accommodate the second half of Fine's asymmetry; and when it is combined with a conception of logical consequence that is relaxed just enough to enable it to accommodate that second half, the account instead becomes unable to accommodate the first half. In either case, it fails to accommodate Fine's asymmetry, and consequently also fails to satisfy our desideratum (D1).

1.4.3 Zalta

Edward Zalta has developed an account of essentiality on the basis of his neo-Meinongian theory of abstract objects.²⁷ A crucial feature of this theory is its fundamentally different treatment of abstract objects on the one hand and ‘ordinary’ objects on the other. Thus, whereas abstract objects may on this theory both *exemplify* and *encode* properties, ordinary objects never encode properties. Another characteristic of Zalta’s theory lies in the fact that it does not straightforwardly allow for {Socrates} to have Socrates as a member. Rather, that the one should have the other as a member is supposed to hold only *according* to a certain mathematical theory M . It is for this reason that, speaking without qualification, Socrates is not at all a member of the set {Socrates}, and hence does not exemplify the property of being a member of that set.

On Zalta’s account of essence, this also means that it is not *essential* to Socrates to be a member of {Socrates}. For, if x is an ordinary object (such as Socrates), then a property P is taken to be essential to x if and only if x exemplifies P in every possible world in which x is ‘concrete’.²⁸ (In Zalta’s scheme, all the entities that exist in other possible worlds also exist in the actual world. However, he allows that other possible worlds contain entities that are ‘concrete’ in those worlds yet ‘abstract’ in the actual one. He thereby seeks to accommodate the intuition that it is possible for there to be entities that are distinct from all those that actually exist.) On the other hand, if x is an abstract object, then P is taken to be essential to x just in case x *encodes* P . For instance, the abstract object {Socrates} encodes the property of having Socrates as a member, and consequently, that property is on Zalta’s account essential to {Socrates}.

As this example illustrates, Zalta’s account of essentiality treats ordinary and abstract objects very differently: one might say that he offers a *modal* account of essentiality for

²⁷Cf., e.g., Zalta (1983).

²⁸More precisely, P is in such a case at least *weakly* essential to x . To be *strongly* essential to x , P must in addition be such that it is not necessarily the case that x exemplifies P . For the sake of simplicity, I will here ignore this complication and use the term ‘essential’ to express what Zalta (in the case of ordinary objects) would express by ‘weakly essential’.

ordinary objects and a separate, *non*-modal account for abstract objects. He defends this bifurcation by appeal to the fundamental difference between ordinary and abstract objects:

Whereas ordinary objects exemplify their properties in the classical way, abstract objects are the kind of object which can both encode and exemplify properties. Such a basic distinction in kinds of objects merits a distinction in the notion of ‘essential property’ that applies to each kind. (p. 678)

Whether that “basic distinction in kinds of objects” in fact merits (or at least excuses) the bifurcation in Zalta’s account of essentiality, it seems to me that this division renders the account’s ability to satisfy (D2) at least somewhat questionable. For, intuitively, one would not have thought that a philosophically significant notion such as that of essentiality should have such a strongly bisected account. Ought there not to be something that essentiality-for-abstract-objects has in common with essentiality-for-ordinary-objects, by virtue of which both of them merit the name ‘essentiality’?

Moreover, although Zalta’s account apparently manages to accommodate Fine’s asymmetry very easily, there is also reason to doubt that it fully satisfies the desideratum (D1). For, on the face of it, the plausibility of Fine’s asymmetry does not depend on our acceptance of Zalta’s theory of abstract objects and its crucial claim that Socrates is not in fact a member of {Socrates}. On the face of it, Fine’s asymmetry loses nothing of its appeal even if we adopt a staunchly realist view of singletons and take Socrates to be a member of {Socrates} in every world in which he exists. For this reason, Zalta’s account does not quite seem to satisfy (D1). It tells us why Fine’s asymmetry holds *within the framework of Zalta’s theory of abstract objects*; but for those who stand outside of that framework, it provides no such illumination.²⁹

²⁹In view of the fact that the account that I am presenting in the following chapters also presupposes a somewhat controversial framework – namely, of attributes and states of affairs –, an advocate of Zalta’s approach might here press an analogous objection against my own account. However, the objection does arguably not have much force. Although my account, as formulated below, presupposes an ontology of attributes and states of affairs, this does not mean that those attributes and states of affairs might not be substituted by other kinds of entities. For instance, if some nominalistically-inclined philosopher were to favor an ontology of linguistic entities instead, then much of the work that is here done by attributes and states of affairs might in the nominalist’s framework be done by linguistic entities (though *how* much, will depend, among other things, on how abundant her ontology of linguistic entities is). By contrast, Zalta’s account of essentiality is not only based on his theory of abstract objects as a source of expressive resources, but rather

There is also another, related argument that can be levelled against Zalta's account. For, if his account were accepted as correct, then the following two theses would have to be regarded as incompatible:

- (1) It is not essential to Socrates to be a member of {Socrates}.
- (2) Socrates is a member of {Socrates} in every possible world in which he exists.

The reason why these two theses would have to be regarded as incompatible is that, on Zalta's account, and given that Socrates is uncontroversially an 'ordinary' object, (2) just amounts to the denial of (1). However, judging from the reception of Fine (1994), many philosophers regard Fine's asymmetry as a genuine counter-example to the modal account of essentiality, and presumably this is precisely because they hold both (1) and (2) to be true, or at least hold them to be compatible with each other. (Of course, if they do regard both theses as true, this will commit them to the view that they are compatible.)

Hence, Zalta's account appears to stand in conflict with essentialist intuition: Given that, on his account, the two theses are incompatible, the philosophers just alluded to must either (a) have committed the logical error of taking two incompatible theses to be compatible, or (b) have been confused about their own understanding of the word 'essential', or (c) have understood the word 'essential' in some sense that is different from the one proposed by Zalta. The first two disjuncts are rather unattractive, because they entail either widespread logical error or confusion. But if the third disjunct is true, then the result will be that many philosophers – at least all those who take Fine's asymmetry to be a genuine counter-example to the modal account – have been making use of a notion of essentiality that is not adequately captured by Zalta's account.³⁰

* * *

requires, if it is to make Fine's asymmetry come out true, a tenet of that theory that does not merely stem from a particular choice of resources: viz., it requires the thesis that Socrates is not in fact a member of {Socrates}.

³⁰Additional discussion of Zalta's account can be found in Malec (2009, ch. 6).

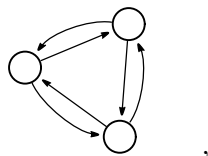
In this section, we have seen that the accounts of essentiality proposed by Wiggins and Zalta, and also the account that is suggested by certain remarks of Michael Gorman’s, do not meet both of the two main desiderata listed in §1.3 above. As I hope to show in the rest of this dissertation, it is possible to construct an account that does meet those desiderata.

1.5 Overview

The next two chapters lay the groundwork for the account of essentiality that I aim to develop in this dissertation: a rudimentary ontology of attributes and states of affairs, which is outlined in chapter 2, and a conception of ‘logical’ modality, which is introduced in chapter 3. The task of developing the account itself is then carried out in chapters 4–7. In order to describe these later chapters, and in particular chapter 4, it is necessary to go into a little more detail.

In §4.1, I formulate the basic idea that underlies the to-be-presented account of essentiality, viz., that an entity’s essential properties are determined by the way in which the entity is ‘constructed’. With a view to turning this idea into an account of essentiality, one of the first questions to arise is what exactly ‘construction’ amounts to. A crucial guiding thought here is that it should be possible for two or more entities to be constructed *from each other*. Some authors who work on the theory of non-well-founded sets have already shown how such mutual construction can be formally described; the classic text on this topic is Peter Aczel’s *Non-Well-Founded Sets* (1988).

Aczel’s main representational tools are directed graphs in which the nodes represent sets and the edges represent set-membership.³¹ An example would be



which represents a trio of sets that have each other (and nothing else) as members. With

³¹My talk of representation is here admittedly quite loose.

the use of such graphs, Aczel develops what essentially amounts to an ontology of non-well-founded sets. This can be considered an example of a type of ontology that I will refer to as *individuational*. The main task of chapter 4 is to characterize this type of ontology in a systematic and general fashion that, in particular, abstracts away from Aczel's set-theoretic context.

Based on the concept of an individuational ontology, chapter 5 introduces concepts of essence and essentiality that are 'relativized' to individuational ontologies. This in turn gives rise to the task of 'de-relativizing' those concepts. That is, I will have to identify conditions that an individuational ontology O has to satisfy in order for properties that are essential to a given entity *relative to* O to deserve being called simply essential properties of that entity. In chapter 6, I describe a fairly straightforward approach to this problem, which, however, runs into apparently insuperable difficulties. An alternative, more complex approach is then developed in chapter 7, which turns out to be more successful. At the end of chapter 7, the account of essentiality is complete.

The tasks that remain for the final two chapters are, first, to show that the account satisfies the two main desiderata laid down in §1.3; and second, to apply the account to an elucidation of *de re* modal discourse. The first of these tasks is taken up in chapter 8, the second in chapter 9.

Chapter 2

Foundations I: Attributes and States of Affairs

The aim of this chapter is to formulate some basic assumptions of an ontology of attributes and states of affairs, and to introduce a formal system of expressions that can be used to refer to these entities.¹ This ontology can be aptly characterized as ‘abundant’: it includes, for instance, conjunctive, negative, disjunctive, and quantificational attributes as well as states of affairs, and even attributes with infinitely many argument-places.

2.1 Preliminaries: Classes, Functions, and Plurals

For the purpose of constructing a rudimentary ontology of attributes and states of affairs, it will be necessary to adopt some background theory. More specifically, I shall accept Zermelo–Fraenkel set theory, with Choice axiom, ur-elements, and an open list of large cardinal axioms. The abundance of the metaphysics of attributes and states of affairs that I will sketch in the following will directly depend on the strength of the underlying set-theoretic assumptions. However, it will not be necessary to be more specific as to *which* large-cardinal axioms are

¹By ‘attribute’, I mean anything that is either a property or a relation: a property is thus a unary attribute, whereas relations are attributes that are at least binary.

supposed to hold.

Unlike attributes, *functions* will be treated in the traditional way as extensional entities: if f and f' are functions that have the same domain and map each element of their domain to the same entity, then f and f' will be regarded as identical. More specifically, functions will here be treated as classes of tuples, or, in the infinitary case, as classes of sequences.

Talk of functions, as well as of classes, should be taken in a perfectly literal sense only where this talk can be understood as referring to *sets*. Talk of proper classes, on the other hand, should here always be understood as an informal way of speaking about properties with the intention of ‘abstracting away’ from all non-extensional differences. That is to say, when in the following I appear to speak of classes, this should be understood as indicating that it does not matter for the purpose at hand which of any two properties I am referring to, as long as they are instantiated by exactly the same entities. Similarly for functions: unless it is clear that the domains of the functions in question are sets (so that the functions themselves may also be thought of as sets), talk that is outwardly about functions is to be understood as talk about properties of tuples or sequences.

It might be thought that talk of classes would be better understood in terms of *irreducibly plural quantification*.² The reason why I have here not opted for this route is that it seems to lead to difficulties in connection with the metaphysics of attributes and states of affairs. For example, suppose that c is a plural constant denoting the ordinals. Then if σ is a sentence containing c at subject-position, what should the state of affairs look like that is expressed by σ ? Either c will correspond to a single constituent of that state of affairs, or to a plurality of constituents (viz., the ordinals). In the first case, that single constituent would have to be, if not a set, some other ‘container object’ that contains all the ordinals. In the second case, it will be the state of affairs itself that functions as a container object, which would again contain all the ordinals. The second option has the advantage that it does not impose on users of plural terms an ontological commitment to sets or other container objects.

²Cf. Lewis (1986a, p. 50n.). The *locus classicus* is Boolos (1984).

However, both options court trouble in roughly the same way as naïve set theory does. For suppose we choose the second option. Then let c' be a plural constant denoting ‘the states of affairs that don’t have themselves as constituents’, and let σ' be a sentence containing that constant at subject-position. Suppose further that σ' expresses a state of affairs s' , and consider whether s' is among the states of affairs denoted by c' . If it is, then s' will have itself as a constituent, and hence cannot be among the things denoted by c' . But in that case, s' would not have itself as a constituent (unless σ' contains other self-referential devices besides c' , but let us assume it doesn’t); and so, it would be among the states of affairs denoted by c' , after all. So we have a contradiction.

To avoid this outcome, one option would be to bar σ' from expressing any state of affairs at all, but of course this would have to be done in some systematic fashion. An alternative solution would be to begin by interpreting all plural quantification in terms of classes, and, in the second step, to interpret all talk of classes in terms of sets and properties, in the way indicated above.

2.2 Variables and Constants

In order to formulate a system of expressions by which to refer to attributes and states of affairs, let us first of all impose a strict syntactic distinction between two kinds of atomic terms, viz., *variables* and *constants*. In particular, variables will be either Greek or Roman letters (either italicized or calligraphic, and possibly with sub- or superscripts and diacritics), whereas constants will include non-italicized English words, Arabic numerals, etc. Occasionally, a constant can also be composed of more than one English word. For example, ‘teacher-of’ will be taken as a constant denoting the teacher–student relation. In addition to constants and variables, I will also be using the familiar logical connectives and quantifiers, as well as the λ -operator for intensional abstraction and delimiters such as commas and parentheses.

In the following, talk of denotation should always be understood as relativized to an *interpretation* and a *variable-assignment*. The concept of an interpretation will be introduced

more fully below (§3.2). For now, it will be sufficient to say that an interpretation is a partial function whose domain comprises the constants and existential quantifiers of the language in question. A variable-assignment, on the other hand, is a partial function whose domain comprises only the variables of the language. Since these domains do not overlap, the union of an interpretation and a variable-assignment will itself be a function. Thus, suppose that I is an interpretation, g a variable-assignment, and e some expression that is either a constant or a variable. Then e will have a denotation relative to I and g if and only if the function that is the union of I and g (in symbols: $I \cup g$) maps e to some entity; in which case that entity will be what e denotes relative to I and g .

Since it would be quite cumbersome to speak always only of denotation ‘relative to’ an interpretation and a variable-assignment, this relativization will be suppressed wherever possible. And also *as far* as possible: e.g., if in the following I say that some expression e denotes an entity x relative to some variable-assignment g , this should be understood as saying that e denotes x relative to g *and* relative to some interpretation supplied by the context.

2.3 Terms

The next step is to define what sorts of expressions should count as terms:

- (T) A *term* is any expression that is either a variable, a constant, a formula, or a λ -expression.³

Because individual constants are in the present framework not syntactically distinguished from atomic predicates, we will have to draw a distinction between the positions that a term occupies when it occurs either ‘as a predicate’ or ‘as a subject’. (The need for this distinction will become clearer below, p. 38.) To this end, let us first introduce a notion of containment and say that, for any expressions e and e' , e *contains* a certain occurrence of e' if and only if

³It may be worth noting that definite descriptions are deliberately excluded from this list. I will give the reason for this in footnote 11 below.

that occurrence corresponds to some node in the parse tree of e .⁴ If e contains some occurrence of e' , I will also simply say that e contains e' . In this sense, then, every expression trivially contains itself, because every expression corresponds to the root node of its own parse tree. By contrast, the result of enclosing some expression e in quotation marks will, at least on a typical treatment of quotation, not contain e ; for the semantic value of e will (typically) be quite irrelevant to the semantic value of the quotation.

With the notion of containment in hand, we can now formulate the following definitions:

- (PP) An expression e contains a term t at *predicate-position* if and only if e contains an occurrence of t that precedes a (non-empty) list of terms enclosed in angled brackets.⁵
- (SP) An expression e contains a term t at *subject-position* if and only if e contains an occurrence of t as an element in a (non-empty) list of terms that is enclosed by angled brackets and preceded by a term.

⁴By a ‘parse tree’, I mean what is also often called a ‘phrase structure tree’ by linguists. It is important to note that, on this conception, what makes a tree a *parse* tree has to do, on the one hand, with the way in which it is constructed (so that it makes sense to speak of a ‘correspondence’ between its nodes and portions of the to-be-evaluated expression), and also with the role it plays as the primary input to semantic interpretation. It is here not possible to describe this role in any detail, but the basic picture is given in Heim and Kratzer (1998, ch. 3).

What kinds of things should occurrences of expressions be taken to be? This question requires that we first settle how we want to conceive of expressions. For our purposes, we may think of the latter as *sequences of symbols*, where the specification of what is to count as a ‘symbol’ of a given language is simply part of the specification of that language. Such sequences may in turn be thought of as functions from ordinals to symbols, each such function having as its domain some initial segment of the ordinals. To define the concept of an expression’s occurrence, we can then simply weaken this last requirement by leaving out the word ‘initial’. In particular, we may say that an *occurrence* of an expression e is a function f such that, for some ordinal α , the domain of f is the set $\{\alpha + \beta : \beta \in \text{dom } e\}$, and for each $\beta \in \text{dom } e$, $f(\alpha + \beta) = e(\beta)$. Further, let us say that an expression e' *occurs within* an expression e if and only if, for some occurrence f of e' , f is a (not necessarily proper) subset of e .

For example, if ‘a’ and ‘b’ are two symbols, let e be the expression ‘ababa’, and let e' be the expression ‘aba’. As we can see, e' occurs twice within e : its two occurrences are the functions $\{(0, \text{‘a’}), (1, \text{‘b’}), (2, \text{‘a’})\}$ and $\{(2, \text{‘a’}), (3, \text{‘b’}), (4, \text{‘a’})\}$. Or for another example, if it is asked what the occurrence of the expression ‘ababa’ in ‘ababa’ itself is, the answer will be that it is ‘ababa’ itself. I think this is what one would intuitively expect, and I consider it an attractive consequence of the present conception of occurrences. (For a brief discussion of other approaches, see Wetzel 1993.)

⁵Of course it would be more convenient to use parentheses instead of angled brackets. But I will also continue to use expressions like ‘ $f(x)$ ’, where the ‘ f ’ is replaced by some function symbol; and it is important that these expressions not be confused with formulas. Since we do not distinguish between function-symbols and predicates, we thus have to introduce a syntactic distinction between functional and predicate ‘application’.

For example, the formula ‘ $G\langle x, y \rangle$ ’ contains the variable ‘ G ’ at predicate-position and the variables ‘ x ’ and ‘ y ’ at subject-position. Further concepts of this sort (such as that of ‘sentence-position’) might also be defined, but these two will suffice for our purposes.

2.4 Instantiations of Attributes

The notion of *obtainment*, as it applies to states of affairs, will here be taken as primitive. To a first approximation, one might explicate this notion in terms of truth: a state of affairs obtains just in case it is expressed by a true sentence. However, this is assuming that every state of affairs is denoted by some sentence of the language in question. There may be languages for which this is the case (e.g., Lagadonian ones⁶), but I will here not assume that we have such a language at our disposal.

At least provisionally, I shall also take as primitive the notion of an attribute’s *instantiation* by a given sequence of entities, where the sequence in question may have any set-sized length greater than zero. In the special case where the sequence contains only a single entity x , I will simply speak of the attribute’s instantiation ‘by x ’, rather than by the sequence that contains only x ; and similarly, I will speak of an attribute’s instantiation ‘by entities’ or more specifically ‘by entities x_1, x_2, \dots ’, where x_1, x_2, \dots are not necessarily pairwise distinct. Every instantiation of an attribute by (a sequence of) entities will be taken to be a state of affairs. Further, to say that some entity or entities x_1, x_2, \dots (jointly) *instantiate* a given attribute means that the instantiation of that attribute by x_1, x_2, \dots obtains.

While I am prepared to take almost every meaningful English predicate to denote an attribute (at least if vagueness is ignored), I will *not* assume that the predicates ‘obtains’ and ‘instantiates’, though meaningful, denote attributes, or any other entities. The reason for this stems from the Liar paradox: if there were a property of obtainment, there would – given our other assumptions, and given that we allow names of states of affairs to occupy subject-position – also be a state of affairs to the effect that all states of affairs that have a certain

⁶Cf. Lewis (1986a, p. 145).

property P do not obtain. And if here P were chosen in such a way that it is instantiated only by that state of affairs, we would have a situation where the latter obtains if and only if it doesn't obtain. As far as I can see, a similar paradox does not arise if we assume that there exists an instantiation relation, as for instance a ternary relation \mathcal{I} whose instantiation by entities x, y, z (in this order) is the state of affairs that x is the instantiation of y by z .⁷ But even so, I will here assume the existence of neither an obtainment property nor of an instantiation relation.

The notion of an attribute with a given arity can be defined on the basis of the notion of instantiation, as follows: if κ is a set-sized cardinality greater than zero, I will say that an entity x is a κ -ary attribute just in case, for some entities x_1, x_2, \dots (imagine here κ -many variables written; the entities themselves need not be pairwise distinct), there is a state of affairs that is the instantiation of x by x_1, x_2, \dots .

The following principle governs the syntax and semantics of basic formulas:

(I1) For any terms a and a_1, a_2, \dots (κ -many variables; $\kappa > 0$), the expression

$$a\langle a_1, a_2, \dots \rangle$$

is a formula, and has a denotation if and only if a denotes an attribute A and a_1, a_2, \dots denote entities x_1, x_2, \dots such that there exists a state of affairs s that is the instantiation of A by x_1, x_2, \dots ; in which case the expression will denote s .

Note the immediate consequence: if G is a κ -ary predicate and a_1, a_2, \dots are terms, then the formula $\ulcorner G\langle a_1, a_2, \dots \rangle \urcorner$ will not denote anything *unless* G and all the terms a_1, a_2, \dots have a

⁷If one does accept the existence of such a relation, one will arguably also have to admit instantiation relations of higher arity, beginning with a relation whose instantiation by entities x, y, z, w is the state of affairs that x is the instantiation of y by z and w (in this order). My remarks in the text are intended to hold also for these additional relations.

denotation.^{8,9}

In what cases is there “a state of affairs s that is the instantiation of A by x_1, x_2, \dots ”? My general assumption here will be that there is *always* such a state of affairs, except where considerations about semantic or other paradoxes call for an exception. For instance, if a denotes the property of non-self-instantiation, supposing that there exists such a property, then it might be held that there is no such state of affairs as the instantiation of this property by itself.¹⁰ The assumption just alluded to can thus be stated as follows:

⁸The present treatment of formulas is thus similar to the evaluation of sentences in a neutral free logic (cf. Priest 2008, §21.7). However, Priest claims that “one would have to make exceptions for the existence predicate itself” (*ibid.*, p. 466); and supervaluationists like van Fraassen (1966) or Skyrms (1968) might wish to introduce further exceptions for the identity predicate or for predicates like $\ulcorner \lambda x(G\langle x \rangle \vee \neg G\langle x \rangle) \urcorner$. By contrast, I intend to make no such exceptions. To be sure, this raises the question of what one should say about negative existentials, such as, ‘Pegasus does not exist’. Tentatively, I would respond to this along the lines of a ‘historical chain view’, as expressed, e.g., in Donnellan (1974). However, the issue may well be more difficult to settle, as one might hold that, in order for a sentence of the form ‘ X does not exist’ to be meaningful, the name that takes the place of the ‘ X ’ will need to have some sort of descriptive meaning and not merely a history. (Consider, e.g., whether ‘The Jabberwock does not exist’ would have had any meaning if Lewis Carroll had never written the poem ‘Jabberwocky’ but instead only begun a poem entitled ‘The Jabberwock’, and had in fact never gotten beyond that title. Similar remarks might be made with respect to *nomina nuda* in zoological and botanical taxonomy.)

⁹ In this and the following chapters, quotations that make use of corner-quotes should be evaluated according to a scheme that consists of two rules. The first and main rule is as follows:

- (1) All variables (except for sub- and superscripts) should be replaced by their respective referents, *provided* that they have a referent in the context in which the quotation is evaluated.

For example, in evaluating $\ulcorner \forall x(x \in S \rightarrow G\langle x \rangle) \urcorner$ in a given context, ‘ x ’, ‘ S ’, and ‘ G ’ should all be replaced by their respective referents, provided that they are assigned any in the context in question. On the other hand, for sub- and superscripts, the relevant rule is the following:

- (2) If the quotation is evaluated in a context in which the variable ξ is bound to some entity x , then all occurrences of ξ within the quoted expression should be replaced by occurrences of some constant denoting x . (Within any given context, the same constant should be used for replacing all occurrences of ξ , even if these occur in different quotations.)

When evaluating corner-quotations, this second rule should be applied before the first. For example, suppose the expression $\ulcorner G\langle x_1, \dots, x_m \rangle \urcorner$ is to be evaluated in a context where the variable ‘ G ’ is assigned a finitary predicate G and the variable ‘ m ’ is assigned the arity of G . The first step would be to apply rule (2) and to replace ‘ m ’ by a numeral denoting the arity of G . Suppose the chosen numeral is ‘17’. In the second step, one would then apply rule (1) and replace the variables ‘ G ’ and ‘ x_1 ’, \dots , ‘ x_{17} ’ by their respective referents.

My use of single quotes varies between scare quotes and the straightforward sort of quotation that simply yields a name of the quoted expression. In order to mark quotations of material that has already appeared elsewhere, I will continue to use double quotes. Finally, I will for the most part follow the custom of omitting all quotation marks around indented expressions. The respective context will hopefully be sufficient in such cases to disambiguate whether the omitted quotes are double, single, or corner-quotes – or whether no quotes have been omitted at all.

¹⁰Note that I am here not committing myself to any such claims, but only wish to make room for them.

- (I2) For any κ -ary attribute A and for any entities x_1, x_2, \dots (κ -many variables), there exists, barring paradoxes, an instantiation of A by x_1, x_2, \dots .

Of course, this is less of a principle than a policy, since it does not say how exactly the paradoxes are supposed to be avoided. I shall leave this question to be settled by a more specific metaphysics.

Semantic paradoxes constitute only *one* reason as to why a formula may fail to denote a state of affairs. Another reason are denotationless terms. For example, suppose we introduce a constant c as denoting whatever attribute is picked out by the predicate ‘consists of phlogiston’, or we introduce c as denoting whatever entity is referred to by ‘the present king of France’. This will plausibly leave c without a denotation, and so one might equally plausibly conclude that no basic formula that contains c will denote a state of affairs, or anything at all.¹¹

The above principle (I1) speaks of *the* instantiation of a given attribute by certain entities. This is not an accident, for I will assume that

- (I3) For any attribute A and any entities x_1, x_2, \dots (κ -many variables), there exists at most one instantiation of A by x_1, x_2, \dots .¹²

I will in addition assume that the notions of attribute and state of affairs are even more closely linked, viz., by the following principle:

- (I4) For any attributes A and A' , A will be identical with A' if, for every cardinality κ and all entities x_1, x_2, \dots (κ -many variables): whenever there is an instantiation of A by x_1, x_2, \dots , there also exists an instantiation of A' by x_1, x_2, \dots , and the latter is identical with the former.

¹¹At this point it may be worth noting that, given the role that the concept of a term plays in the current framework, this concept should plausibly not be extended to definite descriptions. The reason is that it would not be plausible to say that an expression like $\lceil G\langle d \rangle \rceil$, where G denotes a property P and d is a definite description denoting some entity x , simply denotes x 's instantiation of P . For example, suppose that P is the property of being greater than 7 and d is the description, ‘the number of planets in the solar system’. Even though d denotes the number 8, it would not be plausible to regard $\lceil G\langle d \rangle \rceil$ as denoting the fact that 8 is greater than 7. Failure to heed this tends to open the way (via a ‘slingshot’ argument) to the unwelcome conclusion that all facts are one. (For a discussion of the philosophical significance of slingshot arguments, see Neale (1995) and Oppy (1997).)

¹²In the following, the specification ‘in this order’ will usually be suppressed.

In other words: if A and A' are two distinct attributes, then there will be some cardinality κ and entities x_1, x_2, \dots (κ -many variables) such that there is an instantiation of A by x_1, x_2, \dots , but either no instantiation of A' by x_1, x_2, \dots , or, if there is, then the two instantiations will be two distinct states of affairs.¹³

2.5 Two Relations

I will assume the existence of two particular attributes, viz., of the identity relation and of set-membership:

(A1) There exists a binary relation of *identity*, denoted by the constant ‘I’. For any entities x and y , the state of affairs $I\langle x, y \rangle$ obtains if and only if x is identical with y .

(A2) There exists a binary relation of *set-membership*, denoted by the constant ‘E’. For any entities x and y , the state of affairs $E\langle x, y \rangle$ obtains if and only if x is a member of y .

Following common usage, for any terms a and b , the formula $\ulcorner I\langle a, b \rangle \urcorner$ will usually be abbreviated as $\ulcorner (a = b) \urcorner$, and the formula $\ulcorner E\langle a, b \rangle \urcorner$ will usually be abbreviated as $\ulcorner (a \in b) \urcorner$.

We have now reached a stage where it can be said to follow from our above assumptions that there exists at least one state of affairs. Namely, if no paradoxes are in the way (as seems reasonable to assume), there exists, according to (I1), (I2), and (A1), the state of affairs that is the self-identity of the identity relation, i.e., $I = I$; and similarly, by (I1), (I2), and (A2), there exists a state of affairs that is the self-identity of set-membership. Note, however, that

¹³It might here be wondered whether I should not instead adopt an alternative principle, according to which any attributes A and A' are identical whenever there are *any* entities x_1, x_2, \dots such that the instantiation of A by x_1, x_2, \dots is identical with the instantiation of A' by x_1, x_2, \dots . But this proposal would be inconsistent with the other principles that I have adopted, or will adopt, in this chapter. Suppose, for instance, that R is a binary relation, and that y and z are two distinct entities such that $R\langle y, z \rangle \neq R\langle z, y \rangle$. Consider now the following two properties: (i) the property of being R -related to z , and (ii) the property of being an entity x such that z is R -related to x . Anticipating notation that will be introduced below, these two properties can be denoted, respectively, by ‘ $\lambda x R\langle x, z \rangle$ ’ and ‘ $\lambda x R\langle z, x \rangle$ ’. Intuitively and also by the principles of this chapter (including one that will be introduced below, but excluding (I4) for the sake of argument), these are two distinct properties: because the instantiation of $\lambda x R\langle x, z \rangle$ by y is the same as $R\langle y, z \rangle$ and the instantiation of $\lambda x R\langle z, x \rangle$ by y the same as $R\langle z, y \rangle$, and because, by hypothesis, $R\langle y, z \rangle$ is distinct from $R\langle z, y \rangle$. However, if we consider the instantiation of either of these two properties by z , we get in each case the same state of affairs, viz., $R\langle z, z \rangle$, and this violates the alternative principle.

nothing that we have assumed so far entails that $\mathbf{I} = \mathbf{I}$ and $\mathbf{E} = \mathbf{E}$ are two distinct states of affairs.

2.6 Truth-Functional Connectives

The following four principles guarantee the existence of negative and conjunctive states of affairs, as well as – once further principles are in place – of negative and conjunctive *attributes*:

(N1) For any state of affairs s , there is exactly one state of affairs that is the negation of s .

The negation of s obtains if and only if s does not obtain.

(N2) For any variable, constant, or formula φ , the expression $\ulcorner \neg\varphi \urcorner$ is a formula, and has a denotation if and only if φ denotes a state of affairs s , in which case the expression will denote the negation of s .

(C1) For any states of affairs s_1, s_2, \dots (κ -many variables), there is exactly one state of affairs that is the conjunction of s_1, s_2, \dots .¹⁴ The conjunction of s_1, s_2, \dots obtains if and only if each of s_1, s_2, \dots obtains.

(C2) For any variables, constants, or formulas $\varphi_1, \varphi_2, \dots$ (κ -many variables), the expression $\ulcorner (\varphi_1 \wedge \varphi_2 \wedge \dots) \urcorner$ is a formula, and has a denotation if and only if $\varphi_1, \varphi_2, \dots$ respectively denote states of affairs s_1, s_2, \dots , in which case the expression will denote the conjunction of s_1, s_2, \dots .

In addition to ‘ \neg ’ and ‘ \wedge ’, I shall also make use of the connectives ‘ \vee ’, ‘ \rightarrow ’, and ‘ \leftrightarrow ’. These may be defined in the usual way on the basis of conjunction and negation. Thus, any expression $\ulcorner (\varphi_1 \vee \varphi_2 \vee \dots) \urcorner$ should be read as an abbreviation of the formula $\ulcorner \neg(\neg\varphi_1 \wedge \neg\varphi_2 \wedge \dots) \urcorner$, and similarly for ‘ \rightarrow ’ and ‘ \leftrightarrow ’: i.e., any expression $\ulcorner (\varphi \rightarrow \psi) \urcorner$ should be read as abbreviating $\ulcorner \neg(\varphi \wedge \neg\psi) \urcorner$, and any expression $\ulcorner (\varphi \leftrightarrow \psi) \urcorner$ should be read as an abbreviation of $\ulcorner (\neg(\varphi \wedge \neg\psi) \wedge \neg(\psi \wedge \neg\varphi)) \urcorner$. In addition, for any terms a and b , the formulas $\ulcorner \neg\mathbf{I}\langle a, b \rangle \urcorner$

¹⁴As usual, the order of the conjuncts does not matter.

and $\lceil \neg E\langle a, b \rangle \rceil$ will usually be abbreviated as $\lceil a \neq b \rceil$ and $\lceil a \notin b \rceil$, respectively. Finally, parentheses may everywhere be omitted if they are not needed for disambiguation.

If the self-identity of the identity relation is the first state of affairs whose existence follows from the above assumptions, we now have a *second* such state of affairs in the negation of the former, which may be written as ‘ $I \neq I$ ’. Since a state of affairs cannot both obtain and fail to obtain, it is clear that the two must be distinct. Note, however, that the present commitment to the existence of *conjunctive* states of affairs does not similarly entail a commitment to a greater number of states of affairs. For instance, nothing that has been said so far rules out that $(E = E) \wedge (I \neq I)$ is identical with $(I \neq I)$.

2.7 The λ -Operator

Besides variables, constants, and formulas, the fourth class of expressions that count as terms consists of λ -expressions:¹⁵

(L) An expression is a λ -*expression* if and only if it is identical with

$$\lambda v_1, v_2, \dots \varphi,$$

where v_1, v_2, \dots are κ -many pairwise distinct variables (with $\kappa > 0$), and φ is a formula that contains each of the v_1, v_2, \dots , but contains each of them only at *subject*-position.

The syntax of λ -expressions, as thus specified, differs in several ways from the traditional treatment.¹⁶ First of all, φ is required to be a formula, rather than just any well-formed expression. This restriction is mainly motivated by the particular purpose for which λ -expressions are intended in the present framework, which is to serve as names of attributes.

Second, an expression $\lceil \lambda v_1, v_2 \varphi \rceil$ is here not to be read as an abbreviation of $\lceil \lambda v_1 (\lambda v_2 \varphi) \rceil$. The reason for this deviation from standard practice stems from the fact that we are in

¹⁵Cf. the definition of ‘term’ in §2.3 above.

¹⁶As specified in Barendregt (1984).

general allowing for attributes with infinitely many argument-places. Consequently we should not impose on λ -expressions any limits with respect to the number of argument-places; but if $\ulcorner \lambda v_1, v_2 \varphi \urcorner$ were to be read as an abbreviation of $\ulcorner \lambda v_1 (\lambda v_2 \varphi) \urcorner$, this would automatically mean that the list of variables v_1, v_2, \dots has to be finite.

Third, the variables that follow the ' λ ' (also known as λ -variables) are required to be pairwise distinct, and each of them has to occur in the formula φ – which is also referred to as the *matrix* of the λ -expression – at least once. An expression like ' $\lambda x \text{human}\langle \text{Socrates} \rangle$ ' will thus not count as a λ -expression, and neither will ' $\lambda x, x \text{human}\langle x \rangle$ '. The reasons for this presuppose material that will be introduced only further below, but to anticipate: If ' $\lambda x, x \text{human}\langle x \rangle$ ' were to count as a λ -expression, then, by (L1) below, it would denote a binary attribute. The expression

$$\lambda x, x \text{human}\langle x \rangle \langle \text{Socrates}, \text{Plato} \rangle \quad (*)$$

would then, by (I1), have a denotation only if the term 'Plato' does. But, given what will be said in later sections (in particular, §2.8 and §3.2), this would mean that the state of affairs denoted by (*) *necessitates* Plato's existence. On the other hand, by (I1) and the principle (L2) (to be adopted shortly), (*) would denote just the same as ' $\text{human}\langle \text{Socrates} \rangle$ ', i.e., the state of affairs that Socrates is human.¹⁷ Yet it is surely an unwelcome consequence that Socrates' being human should necessitate Plato's existence. Very similar considerations lead me to reject expressions like ' $\lambda x \text{human}\langle \text{Socrates} \rangle$ ', in which the would-be λ -variable (viz., ' x ') fails to occur in the would-be matrix.

Fourth, the present treatment of λ -expressions differs from the usual practice in that the matrix of a λ -expression should contain each one of the λ -variables only at subject-position. The motivation for this requirement lies in the thought that we should not court paradox unnecessarily. If it were not for the requirement just stated, nothing would prevent expressions like ' $\lambda x \neg x$ ' and ' $\lambda x, y x \langle y \rangle$ ' from counting as λ -expressions. By semantic principles to be

¹⁷Cf. the argument at the end of this section.

adopted below (viz., (L1) and (L2)), the first of these would then denote a property that a state of affairs instantiates just in case it fails to obtain, and the second would denote a relation that an entity x bears to another entity y just in case y instantiates x . Moreover, given what will be said in §2.8, I would be committed to the thesis that, for every given property P , there exists a state of affairs to the effect that no state of affairs that instantiates P obtains, or in symbols:

$$\neg\exists x (P\langle x \rangle \wedge x).$$

If P were chosen in such a way that it is instantiated only by this particular state of affairs, we would here have a version of the Liar paradox. Whether P *can* be chosen in such a way – i.e., whether there exists a suitable property – is not particularly obvious;¹⁸ but in any event, the safer route seems to be not to commit ourselves unnecessarily to the existence of attributes and states of affairs of the sort just considered.

With the syntax of λ -expressions settled, the next step is to specify under what conditions a λ -expression has a denotation, and also (in very general terms) what that denotation will be:

- (L1) An expression of the form $\ulcorner \lambda \underbrace{v_1, v_2, \dots}_{\kappa\text{-many}} \varphi \urcorner$ has a denotation if and only if it is a λ -expression in the sense of (L), and there exists an assignment of entities to variables relative to which φ denotes a state of affairs; in which case the expression will denote a κ -ary attribute.

Evidently, this principle does not yet tell us *what* attribute any given λ -expression denotes, nor even when two λ -expressions denote the same property. To remedy this, we will have to adopt a further principle that specifies how to evaluate a formula that is composed of a λ -expression and a list of terms.

The principle just mentioned will be easier to formulate if we first introduce a piece of notation for manipulating functions. Thus, let f be some function, and let x_1, x_2, \dots and

¹⁸Cf. chapter 3, footnote 6.

y_1, y_2, \dots (both lists should be thought of as containing κ -many variables) be some entities. Then $f[y_1/x_1, y_2/x_2, \dots]$ will be the function that is just like f , except that it maps x_1 to y_1 , x_2 to y_2 , and so on, with the further exception that ‘the least index counts’ if any of the x_i should occur more than once in the list.¹⁹ For example, if g is an assignment of entities to variables, then $g[x/v]$ will be the assignment that is just like g , except that it assigns to v the entity x .

The principle itself can now be stated as follows:

- (L2) For any variable-assignment g , any κ -ary attribute A , and any entities x_1, x_2, \dots (κ -many variables), if A is denoted (relative to g) by a λ -expression $\lceil \lambda v_1, v_2, \dots \varphi \rceil$, then there will be an instantiation of A by x_1, x_2, \dots if and only if, relative to $g[x_1/v_1, x_2/v_2, \dots]$, φ denotes a state of affairs; in which case the instantiation of A by x_1, x_2, \dots will be identical with that state of affairs.

This principle does still not tell us what property a given λ -expression denotes, but at least it can be combined with our other principles – notably, (I1) and (I4) – to yield information as to what λ -expressions denote the same attribute. An example may help to see how.

Consider the λ -expression ‘ λx human $\langle x \rangle$ ’. Intuitively, it may be quite clear that this should denote the same property as the constant ‘human’, and that its instantiations should therefore be instantiations of the property of being human. It may, however, be less clear that this is also borne out by the above principles.²⁰

We first have to verify that ‘ λx human $\langle x \rangle$ ’ denotes anything at all. That is, given (L1), we will need to find some variable-assignment f relative to which the formula ‘human $\langle x \rangle$ ’

¹⁹More formally: $f[y_1/x_1, y_2/x_2, \dots]$ is the function whose domain is $(\text{dom } f \cup \{x_1, x_2, \dots\})$ and which maps each element x in its domain to the unique entity y that satisfies the following two conditions:

- (i) If $x \in \{x_1, x_2, \dots\}$, then $y = y_k$, where k is the least index i such that $x_i = x$.
- (ii) Otherwise, $y = f(x)$.

²⁰To be sure, those principles do not even tell us that the constant ‘human’ has any denotation at all; but then, they are not supposed to. Rather, they are meant to be combined with further assumptions as to what constants denote what attributes, as for instance, in the present example: the assumption that ‘human’ denotes the property of being human. Assumptions of this sort will here usually be left implicit.

denotes a state of affairs. By way of an educated guess, let f be any variable-assignment that maps the variable ‘ x ’ to Socrates. By (I1), the formula in question will then, relative to f , denote the instantiation by Socrates of the property of being human, provided that such an instantiation exists. But by (I2), we may assume that such an instantiation does exist, since no semantical paradoxes seem to arise from this assumption. It then follows from (L1) that our λ -expression $\lceil \lambda x \text{ human}\langle x \rangle \rceil$ denotes some property, as desired. The next step will be to prove that this property, call it ‘ P ’, is identical with the property of being human, which I will for now refer to as ‘ H ’.

To do this in a reasonably concise and perspicuous way, let us introduce a little more notation. First, in order to abbreviate the phrase, ‘the denotation of $\lceil \dots \rceil$ relative to g ’, I shall simply write $\llbracket \dots \rrbracket^g$. This notation is common in the literature on formal semantics.²¹ Second, I will use the operator ‘ \doteq ’ to construct equations that are true if and only if both sides are ‘either undefined or equal’, which is to say: either the expressions flanking the operator both denote the same thing, or neither of them denotes anything.

Let now z be some entity, ξ some variable, and g some assignment that maps ξ to z . To begin with, we can establish the following equation:

$$P\langle z \rangle \doteq \llbracket \lambda x \text{ human}\langle x \rangle \langle \xi \rangle \rrbracket^g,$$

or in other words: $\lceil \lambda x \text{ human}\langle x \rangle \langle \xi \rangle \rceil$ has a denotation relative to g if and only if there is such a state of affairs as z ’s instantiation of P ; and if there is, then that will be the denotation (relative to g) of the former expression. This much follows from (I1), given that, relative to g , ‘ $\lambda x \text{ human}\langle x \rangle$ ’ denotes P and ξ denotes z . Further, we have:

$$\begin{aligned} \llbracket \lambda x \text{ human}\langle x \rangle \langle \xi \rangle \rrbracket^g &\doteq \llbracket \text{human}\langle x \rangle \rrbracket^{g[z/'x']} && \text{by (L2),} \\ &\doteq H\langle z \rangle && \text{by (I1).} \end{aligned}$$

²¹Cf., e.g., Heim and Kratzer (1998, ch. 2).

Putting this together with the previous equation, it follows that $P\langle z \rangle \doteq H\langle z \rangle$. But we have concluded this for *any* entity z , variable ξ and assignment g mapping ξ to z , and so the result holds for every such z , ξ , and g . And since we can safely enough assume that there will be such an assignment g and variable ξ for *every* entity z , the condition that g should map ξ to z does not constitute a limitation. In other words, the result simply holds for every entity z . Hence, for every entity z , if there is such a state of affairs as $H\langle z \rangle$ (i.e., the state of affairs that z is human), there will also be such a state of affairs as $P\langle z \rangle$, and moreover, $P\langle z \rangle$ will be identical with $H\langle z \rangle$. With (I4), it then follows that H and P are the same property. So ‘ λx human $\langle x \rangle$ ’ denotes the property of being human, which is what we wanted to show.

2.8 Quantifiers

Finally, it will be necessary to allow for states of affairs that involve quantification.²² In analytic metaphysics, it has for a long time been the dominant view that ‘existence’ is univocal.²³ But there has also been some opposition to this doctrine: on the one hand from those who think that there may well be multiple equally legitimate ways of carving the world into individual entities,²⁴ and on the other hand from those who think that there may well be multiple kinds, or ‘modes’, of existence.²⁵

Although it will not be necessary for the purposes of this dissertation to take a stance on these issues, I do at least want to leave room for the possibility that there may be more than

²²As far as I am aware, quantification-involving states of affairs have first been recognized by Russell (1985, p. 103), who admitted both ‘existence facts’ and ‘generality facts’. The latter have more recently also been admitted by Armstrong (1997) under the label of (obtaining) ‘totality states of affairs’.

²³Cf., e.g., Quine (1948; 1960, pp. 131, 242), and van Inwagen (1998).

²⁴The most prominent recent proponent of this form of opposition is Eli Hirsch (2002; 2005), whose view has important predecessors in the work of Hilary Putnam (e.g., 1987; 1994) and Rudolf Carnap (1950). For a discussion of the differences between Hirsch and Carnap, see Hirsch (2008). A related line of thought, which goes back to Frege (1884), has to do with the possibility of giving deflationary truth-conditions for sentences that *prima facie* carry ontological commitment to certain sorts of entity. On this issue, see in particular the work of Agustín Rayo (e.g., 2008; MSa).

²⁵See, e.g., McDaniel (2009; 2010), Turner (2010). Both McDaniel and Turner point out that their view has a long historical pedigree. For an overview of the recent literature, see Spencer (2012).

one philosophically respectable interpretation of the existential quantifier. For this reason, I shall take the unusual step of treating the existential quantifier ‘ \exists ’ as a *denoting expression* that denotes whatever sense of ‘existence’ is assigned to it by the relevant interpretation. Just what sort of thing these senses of ‘existence’ are will here be left open. On the metaphysically most conservative view, they are simply (equivalence classes of) concepts. In this case, it would perhaps be more appropriate to say that the existential quantifier *expresses*, rather than denotes, a sense of ‘existence’; but nothing important depends on this choice.

As for *universal* quantifiers, these will be treated not as denoting expressions, but rather as abbreviatory devices. Thus, if \mathcal{E} is an existential quantifier denoting a certain sense of ‘existence’, then a corresponding universal quantifier \mathcal{A} may be introduced by stipulating that for every formula φ and κ -many pairwise distinct variables v_1, v_2, \dots , the formula $\lceil \mathcal{A}v_1, v_2, \dots \varphi \rceil$ abbreviates $\lceil \neg \mathcal{E}v_1, v_2, \dots \neg \varphi \rceil$. The universal quantifier that corresponds to ‘ \exists ’ will here, of course, be the familiar symbol ‘ \forall ’.

In order to ensure that the present framework allows for quantification-involving states of affairs, I will adopt the following two principles:

- (Q1) For any sense of ‘existence’ S and any κ -ary attribute A , there is exactly one state of affairs to the effect that there exist, in the sense S , some entity or entities that (jointly) instantiate A .²⁶ This state of affairs obtains if and only if there are such entities.
- (Q2) For any existential quantifier \mathcal{E} , the expression $\lceil \mathcal{E} \underbrace{v_1, v_2, \dots}_{\kappa\text{-many}} \varphi \rceil$ will be a formula if and only if φ is a formula and v_1, v_2, \dots are pairwise distinct variables. In that case, the expression will have a denotation if and only if \mathcal{E} denotes a sense of ‘existence’ E and $\lceil \lambda v_1, v_2, \dots \varphi \rceil$ is a λ -expression that denotes an attribute A ; in which case the expression will denote the state of affairs that there exist, in the sense E , some entity or entities that (jointly) instantiate the attribute A .

With (Q1) and (Q2), the list of metaphysical assumptions and semantic stipulations that constitute the present framework is almost complete. It only remains to add two further

²⁶For the sake of brevity, the words ‘to the effect’ will often be omitted.

assumptions concerning the *identity conditions* of states of affairs and thus, indirectly, also of attributes. Their formulation, however, requires some additional groundwork, which will occupy us in the first two sections of the next chapter. The assumptions themselves will then be formulated in §3.3.

Chapter 3

Foundations II: Logical Modality

In the previous chapter, we have introduced a system of symbolic expressions for the purpose of referring to attributes and states of affairs. In addition, we have listed some metaphysical assumptions as to what attributes and states of affairs there are. However, it still remains to specify necessary and sufficient conditions for the identity of states of affairs. This lacuna will be filled in §3.3. In preparation for this, I shall need to introduce the concept of *necessitation*, which will in turn be based on that of *entailment*. Moreover, I will introduce concepts of ‘logical’ possibility and necessity, as applied to states of affairs. This work constitutes one part of the basis for the elucidation of *de re* modal discourse that I will propose in §9.3. The other part will be supplied by the concept of essentiality that will be developed in chapters 4 to 7.

3.1 Formulas and ‘Formal Languages’

Several of the principles introduced in the previous chapter (in particular: (I1), (N2), (C2), and (Q2)) state sufficient conditions under which a symbolic expression counts as a formula, and thereby provide part of a definition of what it means to be a formula in the present framework. To complete the definition, I now only have to add a stipulation to the effect that no *further* expressions are to count as formulas:

(F) An expression is a formula *only if* it is a formula according to the above principles, and is ‘built up’ in a finite number of steps from atomic constituents.

Atomic constituents include, in particular: all variables, constants, existential quantifiers, logical connectives, and delimiters (i.e., commas and parentheses). For the purposes of the following, *abbreviations* of formulas will also be treated as formulas, even though they do not strictly count as such according to the present definition.

Just what expressions are to count as variables, constants, or existential quantifiers has so far been left relatively vague. This has made for welcome flexibility, as it has here allowed us – and will continue to allow – to introduce new constants and variables more or less *ad libitum*. For the purposes of the following, however, it will be convenient to work with a conception of language under which every given language has fixed classes of (respectively) variables, constants, and existential quantifiers. Within the context of the present framework, I shall refer to these languages simply as *formal languages*. Each formal language \mathcal{L} is fully determined by three pairwise disjoint classes of atomic symbols:

- $\text{Var } \mathcal{L}$ is the class of \mathcal{L} ’s variables. It is a *proper class*, i.e., it has as many elements as there are ordinals.
- $\text{Const } \mathcal{L}$ is the class of \mathcal{L} ’s constants. It contains at least one symbol, viz., ‘I’.
- $\text{Quant } \mathcal{L}$ is the class of \mathcal{L} ’s existential quantifiers. It contains at least one symbol, viz., ‘E’.

Further, every formal language \mathcal{L} is itself a proper class of expressions – viz., precisely of those expressions that count as *terms* under the definitions of the previous chapter, and under the assumption that all and only the members of $\text{Var } \mathcal{L}$ are variables, all and only the members of $\text{Const } \mathcal{L}$ are constants, and all and only the members of $\text{Quant } \mathcal{L}$ are existential quantifiers.

3.2 Interpretation, Entailment, and Logical Modality

With the concept of a formal language in place, the corresponding notion of interpretation can be defined as follows:

- (I) An *interpretation* of a formal language \mathcal{L} is a partial function I whose domain comprises all the existential quantifiers and constants of \mathcal{L} and which satisfies the following conditions:
 - (i) To the constant ‘ \mathbf{I} ’, I assigns the identity relation;
 - (ii) To the quantifier ‘ \exists ’, I assigns a sense of ‘existence’ such that for every term t of \mathcal{L} and every entity x : if t denotes x relative to I , then x exists in that sense.

This definition is admittedly very liberal, since it does not require that any constant other than ‘ \mathbf{I} ’ be assigned a denotation. However, *if* a given interpretation I assigns a denotation to some constants, then the second clause, in conjunction with the principles of the previous chapter, requires that not only the entities denoted by those constants, but also any entities that are (relative to I) denoted by terms that can be built up from those constants, exist in the sense that I assigns to the quantifier ‘ \exists ’. Hence, while I may assign very specialized senses of ‘existence’ to other quantifiers, the sense that it assigns to ‘ \exists ’ will nearly always – i.e., as long as I assigns a denotation to any constant at all – have to be a fairly broad one.

Next, we can define the concepts of validity, entailment, and equivalence:

- (V) A constant or formula φ in a formal language \mathcal{L} is *valid* if and only if, for every interpretation I and every variable-assignment g , φ denotes, relative to I and g , an obtaining state of affairs.
- (En1) If Γ is a class of variables, constants, or formulas in a formal language \mathcal{L} , and if ψ is a variable, constant, or formula of that same language \mathcal{L} , then Γ *entails* ψ if and only if, for every interpretation I of \mathcal{L} and for every variable-assignment g : if each member of Γ denotes, relative to I and g , a state of affairs, then ψ will likewise denote, relative

to I and g , a state of affairs; and if each member of Γ denotes, relative to I and g , an obtaining state of affairs, then the state of affairs denoted by ψ will also obtain.

(En2) If φ and ψ are variables, constants, or formulas in a formal language \mathcal{L} , then φ *entails* ψ if and only if the singleton $\{\varphi\}$ entails ψ .

(Eq) If φ and ψ are variables, constants, or formulas in a formal language \mathcal{L} , then φ and ψ are *equivalent* if and only if they entail each other.

The general form of these last definitions should not be surprising, and, as far as entailment and equivalence are concerned, the usual results of reflexivity and transitivity are easily reproduced.¹

The concept of a (logically) necessary state of affairs can now be very naturally defined as follows:

(N) A state of affairs is *necessary* if and only if it is denoted, relative to some interpretation and variable-assignment, by a valid constant or formula.

The adverb ‘logically’ seems appropriate, given the way in which this definition relies on the concepts of validity and interpretation, and given how these latter concepts have here been defined. For it can be seen from the relevant definitions that a formula is valid only if it contains neither free variables nor any constants other than ‘I’; and, at least from the assumptions we have made so far, it does not follow that any ‘mathematical fact’ – i.e., any fact that would naturally be expressed by a formula using mathematical vocabulary such as ‘ \in ’ – is denoted by any such formula. Consequently, it does not follow that any mathematical fact is denoted by a valid formula, and so it likewise does not follow that any such fact is (logically) necessary.

Given this definition of ‘necessary’, how should one define the concept of a logically *possible* state of affairs? It would be tempting to follow the tradition of taking possibility to be the

¹The present concept of entailment is somewhat related to Parry’s (1933) notion of analytic implication, as well as to Lewy’s (1976) notion of analytic entailment. (Thanks here to Kit Fine.)

dual of necessity, and to say that a state of affairs is possible just in case its negation fails to be necessary. But this would arguably be a mistake. For instance, it seems evident that the state of affairs (Socrates \neq Socrates), i.e., Socrates' non-self-identity, should count as impossible. The proposal in question, however, does not bear this out. What it *does* bear out is rather only that Socrates' non-self-identity is the negation of a state of affairs denoted by 'Socrates = Socrates'. Given that this latter formula is not valid, it is therefore not clear – i.e., it does not follow from our assumptions – that Socrates' non-self-identity is the negation of a necessary state of affairs, and so it does not follow that this state of affairs is impossible. To accommodate the intuition that any state of affairs that is denoted by a formula of the form ' $\alpha \neq \alpha$ ' is impossible, we will therefore have to adopt a different definition.

To see how to construct such a definition, it will be helpful first to consider, in the light of the above definition of 'valid', what it means for a state of affairs to be necessary. By combining (N) and (V), we find that a state of affairs is necessary just in case it is denoted, relative to some interpretation and variable-assignment, by a constant or formula φ such that, "for every interpretation I and every variable-assignment g , φ denotes, relative to I and g , an obtaining state of affairs". It should then seem natural to define the concept of a (logically) impossible state of affairs as follows:

- (Ip) A state of affairs is *impossible* if and only if it is denoted, relative to some interpretation and variable-assignment, by a constant or formula φ such that, for every interpretation I and every variable-assignment g , φ does *not* denote, relative to I and g , an obtaining state of affairs.

Given that the formula 'Socrates \neq Socrates' does not denote an obtaining state of affairs relative to any interpretation and variable-assignment, it is now clear, as it should be, that the state of affairs denoted by that formula is impossible. Further, I will say that a state of affairs is (logically) *possible* if and only if it is not impossible, and that a state of affairs is (logically) *contingent* if and only if it is not necessary.

The present way of defining modal concepts is admittedly non-standard in at least three respects. First, it takes the ‘primary bearers’ of necessity and possibility to be states of affairs rather than statements. Second, it does not rely on a notion of possible world (whether abstract or concrete). And third, it does not take possibility to be the dual of necessity, since for a state of affairs to be possible does under the present definition not mean that the negation of that state of affairs fails to be necessary. However, none of these features is unprecedented. Notably, the third feature is also present in Arthur Prior’s (1957) “system Q”. Prior saw it as a virtue of his system that, due to the fact that it does not treat possibility as the dual of necessity, it manages to avoid the implausible conclusion that everything exists necessarily: a conclusion that, in his words, would make “gods of us all” (p. 48).² The present framework can claim the same virtue – once it is suitably extended. In particular, it will be necessary to formulate a distinctness condition for states of affairs, which will be done in the next section.

For the purpose of formulating such a condition, it will be convenient to have at hand the notion of (logical) *necessitation*, which can be regarded as a ‘metaphysical analogue’ of the notion of entailment:

(Nc) If s and t are states of affairs, then s *necessitates* t if and only if there are variables, constants, or formulas φ and ψ such that φ entails ψ and, relative to some interpretation and variable-assignment, φ denotes s and ψ denotes t .

Note the trivial consequence that every state of affairs necessitates itself.³

²The thesis that everything exists necessarily has in recent years been defended by Williamson (2002), but for most others, it seems to have retained its implausibility. See Efrid (2010) for a recent proposal to avoid the conclusion in the manner of Prior; also see Fine (1985). Further, Correia (2007) employs an avowedly ‘Priorean’ treatment of modality in constructing an account of essence. (In contrast to Prior, however, Correia does treat necessity and possibility as duals.)

³In *The Nature of Necessity*, Alvin Plantinga has introduced a rather similar notion, namely, that of *inclusion*: “Let us say that a state of affairs S *includes* a state of affairs S' if it is not possible (in the broadly logical sense) that S obtain and S' fail to obtain” (p. 44f.). I would normally have been happy to adopt Plantinga’s terminology, but the term ‘necessitation’ seems to suggest itself more naturally, given its usage in English. A notion that is related to the present concept of necessitation, and which has drawn a great deal of attention from contemporary metaphysicians, is the notion of *grounding*. It would be an interesting question to investigate whether, and if so to what extent, the former notion might help to elucidate the latter. (For an excellent introduction to the notion of grounding, see Correia and Schnieder (2012). A seminal paper is Fine (2001).)

3.3 Identity and Distinctness of States of Affairs

Using the concept of necessitation, we can formulate the following simple condition for the distinctness of states of affairs:

(S1) No two states of affairs necessitate each other.

Or in other words, if s and t are states of affairs, then s will be identical with t if (and only if) s and t necessitate each other. I take this principle to be mainly motivated by considerations of ideological parsimony: there will be no theoretical need, at least not for the purposes of this dissertation, to distinguish between states of affairs that necessitate each other. Hence, it appears admissible to treat any states of affairs s and t that necessitate each other as one and the same.⁴

Together with the above principles, (S1) has some far-reaching consequences. For example, let A and B be attributes, respectively denoted by λ -expressions $\ulcorner \lambda x_1, x_2, \dots \varphi \urcorner$ and $\ulcorner \lambda x_1, x_2, \dots \psi \urcorner$, such that φ and ψ entail each other. It then follows from (S1) and our earlier assumptions that A and B are one and the same attribute. For, given that φ and ψ entail each other, it will be the case that, relative to any interpretation I and any variable-assignment g , φ and ψ both denote either nothing at all, or they denote states of affairs that necessitate each other, which by (S1) means that they denote the *same* state of affairs. The rest of the

⁴A similar principle can already be found in Wittgenstein's *Tractatus*: "If p follows from q and q from p , they are one and the same proposition" (5.141). More recent authors who have explicitly endorsed similar principles are Taylor (1976, p. 264), Pollock (1984, p. 54), Olson (1987, p. 91), and Stalnaker (2012, p. 25). As Pollock remarks, this principle

is a safe assumption even if there is another sense of 'state of affairs' in which equivalent states of affairs [for present purposes, this may be read as: states of affairs that necessitate each other] need not be identical, because we can always regard states of affairs in the present sense as being equivalence classes of the more finely individuated kind of states of affairs. (*ibid.*)

It may also be worth noting that Plantinga appears tacitly to assume this principle – which, in his terminology, would have to be stated as, 'No two states of affairs include each other' – when he argues (1974, §4.1) that there is at most one actual world. On the other hand, some critical discussion of this principle can be found in Fine (1982, p. 55–62).

argument proceeds in a fashion similar to the one given at the end of §2.7 above.⁵

The principle (S1) has moreover the consequence that there exists only one necessary state of affairs. For, by definition, every necessary state of affairs is denoted by a valid formula, and all valid formulas entail each other, so that all necessary states of affairs necessitate each other. On the other hand, our assumptions so far do still not rule out that there may exist only two states of affairs, viz., the necessary state of affairs (which I shall refer to as ‘ \top ’) and its negation (‘ \perp ’). Thus, for instance, nothing that has been said up to this point rules out that the state of affairs that Socrates exists is in fact the same as \top . Of course, since the latter is necessary, this would turn Socrates into a necessary existent, contradicting the intuitively attractive thought that Socrates’ existence is only contingent.⁶

⁵Thus, let g be some variable-assignment that maps the variables ξ_1, ξ_2, \dots respectively to the entities z_1, z_2, \dots . We then have:

$$\begin{aligned}
 A\langle z_1, z_2, \dots \rangle &\doteq \llbracket \lambda x_1, x_2, \dots \varphi\langle \xi_1, \xi_2, \dots \rangle \rrbracket^g && \text{by (I1),} \\
 &\doteq \llbracket \varphi \rrbracket^{g[z_1/'x_1', z_2/'x_2', \dots]} && \text{by (L2),} \\
 &\doteq \llbracket \psi \rrbracket^{g[z_1/'x_1', z_2/'x_2', \dots]} && \text{by (S1),} \\
 &\doteq \llbracket \lambda x_1, x_2, \dots \psi\langle \xi_1, \xi_2, \dots \rangle \rrbracket^g && \text{by (L2),} \\
 &\doteq B\langle z_1, z_2, \dots \rangle && \text{by (I1).}
 \end{aligned}$$

So, for any entities z_1, z_2, \dots , if there exists an instantiation of A by z_1, z_2, \dots , then there likewise exists an instantiation of B by z_1, z_2, \dots , and the latter is identical with the former. By (I4), A and B are therefore one and the same attribute.

⁶In the literature on states of affairs and attributes, one sometimes finds a line of reasoning that seems to start from the assumption that a state of affairs (or ‘fact’, or ‘proposition’) cannot, as it were, be constituted in more than one way. E.g., Mellor argues, essentially following Ramsey (1925), that the fact Ua , where $U = \sim P$ (to use his notation), cannot be the same as $\sim Pa$, because “they have different constituents” (Mellor 1991, p. 264). Transposed into the present framework, the argument has the conclusion that the instantiation of a property $\lambda x \sim P\langle x \rangle$ by an entity a cannot be the same as the state of affairs $\sim P\langle a \rangle$, on the ground that these states of affairs have different constituents; in particular, the former has as a constituent the property $\lambda x \sim P\langle x \rangle$, which the latter lacks. This reasoning appears to derive its plausibility from a conception of states of affairs according to which, for each state of affairs, there is only one way in which it is ‘composed’ from its constituents. The present framework, by contrast, does not involve any such conception. The mentioned line of reasoning should therefore not be seen as having any force in the present context.

There is also, however, another line of reasoning that should be mentioned here, which goes back not to Ramsey, but to Tarski and Gödel. If there are only two states of affairs, say, \top and \perp , of which only the former obtains, then, with ‘ $\lambda x (x = \top)$ ’, we seem to have at our hands something like a *truth-predicate*. More precisely, we could call $\lambda x (x = \top)$ an ‘obtainment property’, in the sense that it is instantiated by a given state of affairs if and only if that state of affairs obtains. Under the impression of Tarski’s (1936) undefinability result, one might think that the assumption of there being such a property leads to inconsistency, because there will then exist a state of affairs that ‘says’ of itself that it does not obtain.

In fact, however, it is far from clear that this follows, because a proof for the existence of such a state of affairs requires certain assumptions that have here not been made. In particular, we have not assumed that

In order to honor this intuition, we will have to adopt a principle that imposes a necessary condition on the identity of states of affairs. I shall do so here by postulating that no two formulas denote the same state of affairs unless they meet a certain requirement.

In formulating such a requirement, special care has to be taken to avoid contradicting the above assumptions. For there is a wide range of cases where those principles entail that a state of affairs denoted by some formula φ is also denoted by another formula ψ . To give only one example: if G and H are two terms denoting the same property P , and if a and b are two terms denoting the same entity x , then it follows from (I1) and (I3) that the formulas $\ulcorner G\langle a \rangle \urcorner$ and $\ulcorner H\langle b \rangle \urcorner$ denote, if anything at all, the same state of affairs (viz., the instantiation of P by x). And of course, further such identities follow from our latest principle (S1). The arguably easiest way to take all of these principles into account is to make explicit reference

there is such a thing as an instantiation relation (i.e., a relation that holds between a state of affairs, an attribute, and one or more further entities x_1, x_2, \dots if and only if the state of affairs is an instantiation of that attribute by x_1, x_2, \dots). And even if we *did* assume that there is such a relation, it seems that the ensuing difficulty would not be fatal. Thus, suppose we assume that there is a relation \mathcal{I} that holds between entities x, y , and z just in case x is the instantiation of y by z , or in other words: $\mathcal{I}\langle x, y, z \rangle$ iff $x = y\langle z \rangle$. It will then follow from our above assumptions that the λ -expression

$$\lambda x \neg \exists y (\mathcal{I}\langle y, x, x \rangle \wedge y = \top)$$

denotes a certain property Q . (I am here adapting the relevant part of Tarski's proof as presented in Enderton 2001, p. 235.) Suppose further that there exists a state of affairs that is the instantiation of Q by itself, i.e., the state of affairs $Q\langle Q \rangle$. As we can see from the displayed λ -expression, $Q\langle Q \rangle$ obtains if and only if there is *no* entity y such that, first, $\mathcal{I}\langle y, Q, Q \rangle$ and, second, $y = \top$. But of course, $Q\langle Q \rangle$ *itself* already fulfills the first condition, and, by (I3), there will be nothing else that does. So $Q\langle Q \rangle$ will obtain if and only if it is not identical with \top . But since \top is the sole obtaining state of affairs, that in turn means that $Q\langle Q \rangle$ obtains if and only if it doesn't obtain: contradiction.

To avoid this outcome, a proponent of the view that \top and \perp are the only two states of affairs may choose between two possibilities. According to the first, there exists – at least in the sense of ‘existence’ denoted by ‘ \exists ’ – no such thing as the instantiation relation. The second alternative would be to say that, even though \mathcal{I} , and then also Q , exists, there is still no such state of affairs as $Q\langle Q \rangle$. This latter alternative may seem *ad hoc*, but perhaps it can be defended, after all. At any rate, there is no need for us to settle on either alternative.

Finally, I should also note that, to encounter a similar difficulty, we need not go so far as to claim that there is only one obtaining state of affairs. It will be enough if we merely hold that there exists only one *necessary* state of affairs – a view to which we are committed by (S1). For consider again the property Q . It is of course still the case that $Q\langle Q \rangle$ obtains if and only if it is not identical with \top , which now means: if and only if it is not necessary. So it has to obtain, for otherwise it would be necessary, and, if necessary, it could not but obtain. But if so, we have just shown that $Q\langle Q \rangle$ obtains, *and* that it is not necessary. And there is something decidedly odd about a state of affairs that can be shown to obtain but is nevertheless not necessary. (We would here have a new form of the contingent *a priori*.) To avoid this presumably unwelcome consequence, we can again choose between the two alternatives of banning either \mathcal{I} (and hence the property Q itself) or only the state of affairs $Q\langle Q \rangle$. But, as above, there is no need for us, at least for present purposes, to settle on either of these alternatives.

to them. That is, one might simply say that two formulas φ and ψ of a formal language \mathcal{L} denote the same state of affairs *only if* it follows from the above principles – together with some information as to ‘what denotes the same as what’ (more specifically: what constants, variables, or quantifiers denote the same entities as what other terms or quantifiers) – that φ and ψ denote the same state of affairs.

The ‘information’ just alluded to may here be thought of as a set of true sentences formulated in plain English. In particular, we may think of it as a set of sentences that satisfies the following two conditions:

- (i) Each member of the set is a true sentence of the form ‘ a denotes the same as b ’, where ‘ a ’ is replaced by a name of a constant, variable, or existential quantifier, and where ‘ b ’ is replaced by a name of a term (not necessarily atomic) or existential quantifier of \mathcal{L} .⁷
- (ii) If c is any constant, variable, or existential quantifier, then the set contains only *at most* one sentence of the form ‘ a denotes the same as b ’ where ‘ a ’ is replaced by a name of c .

If a set of sentences satisfies these two conditions, let us call it an \mathcal{L} -set. The envisaged principle can then be stated as follows:

- (S2) Two formulas φ and ψ of a formal language \mathcal{L} denote the same state of affairs *only if*, for some \mathcal{L} -set S , it follows from the previously adopted principles, together with the sentences in S , that φ and ψ denote the same state of affairs.

With its explicit reference to the above principles, as well as to sets of metalinguistic sentences, this formulation is admittedly somewhat unusual. However, it does have the advantage of being considerably more concise than any equivalent formulation from which all reference to those principles is eliminated.

To see how (S2) ‘works’, it may help to consider an example of two formulas for which (S2) does *not* rule out that they denote the same state of affairs. In particular, consider the two

⁷For the sake of brevity, I am here suppressing the usual relativization to interpretations and variable-assignments. Note that, since constants, variables, and existential quantifiers have their respective denotations only relative to an interpretation or variable-assignment, and since interpretations and variable-assignments are functions, it follows that every constant, variable, and quantifier will only denote at most one entity.

formulas $\ulcorner c_1 \wedge c_2 \urcorner$ and $\ulcorner c_1 \wedge (c_2 \vee \neg c_3) \urcorner$, where c_1 , c_2 , and c_3 are distinct constants. Suppose also that c_1 and c_3 both denote a certain state of affairs s_1 , while c_2 denotes a different state of affairs s_2 . It now follows from the principles of §2.6, together with the information that c_3 denotes the same as c_1 , that $\ulcorner c_1 \wedge (c_2 \vee \neg c_3) \urcorner$ denotes the same as $\ulcorner c_1 \wedge (c_2 \vee \neg c_1) \urcorner$. But this latter formula obviously entails $\ulcorner c_1 \wedge c_2 \urcorner$, and vice versa; so the states of affairs denoted by these two formulas necessitate each other, and are thus identical, by (S1). Consequently, $\ulcorner c_1 \wedge c_2 \urcorner$ denotes the same state of affairs as $\ulcorner c_1 \wedge (c_2 \vee \neg c_3) \urcorner$. From the way this has just been concluded (viz., from the previously adopted principles, together with information as to what constants denote the same entities as what other terms), it is clear that this result will not be contradicted by (S2).

As a corollary of (S2), it may be worth noting that the only necessarily existing entities are attributes and states of affairs: in particular, attributes and states of affairs that can be denoted by terms that contain no constants other than ‘I’ and no existential quantifiers other than ‘ \exists ’.

With (S2), the list of assumptions that constitute our rudimentary background ontology of attributes and states of affairs is finally complete. In the next section, I will now briefly address a question that arises in connection with the present framework and the conception of logical modality developed in this chapter. In particular, the question is whether there are such things as ‘modal’ states of affairs, which include states of affairs to the effect that a certain other state of affairs is possible or necessary. This issue will become relevant when I discuss the problem of iterated modalities in §9.4 below.⁸

3.4 Are there Modal Attributes and States of Affairs?

Let us say, very informally, that a *canonical description* of a state of affairs is a description that we use when we refer to a given state of affairs as ‘the state of affairs that ...’. Similarly,

⁸Some readers may at this point worry that the above account of necessary and possible states of affairs gives rise to a modal account of essentiality that (unlike traditional versions of the modal account) promises to accommodate Fine’s asymmetry. I address worries of this sort in Appendix A.

let us say that a canonical description of a particular property is a description that we use when we speak of the latter as ‘the property of being ...’.⁹ In the following, I shall use the label ‘modal’ to pick out, very roughly, any attributes or states of affairs whose canonical description makes use of any of the notions of possibility, necessity, and necessitation that have been introduced in §3.2 above, or of any notion that can be defined on the basis of these. For example, if there should exist a state of affairs that can be characterized as ‘the state of affairs that it’s possible that grass is green’, then that will count as a modal state of affairs. And if there should exist such a thing as the property of being a possible state of affairs, then that will count as a modal property.

On an account that treats states of affairs as the primary bearers of necessity and possibility, the question of whether there are modal states of affairs has considerable significance for the interpretation of sentences that make use of iterated modal operators, such as ‘Possibly, it’s possible that grass is green’, or ‘It is necessarily possible that pigs can fly’. For, if there *are* modal states of affairs (and if they are suitably abundant), one could interpret such sentences in the same general way as those that do not contain any iterated modal operators. For instance, the sentence ‘Possibly, it’s possible that grass is green’ may then be interpreted as saying that the state of affairs *s* is possible, where *s* is the state of affairs that it’s possible that grass is green.

If there are to be modal states of affairs, there will arguably also have to be modal properties. In particular, there will have to be such things as the property of being possible and the property of being necessary. For if there is to be a state of affairs that may appropriately be described as, say, ‘the state of affairs that it’s possible that grass is green’, then, on an account of the sort in question, it will be hard to see what this state of affairs could be other than the instantiation of the property of being possible by the state of affairs that grass is green. But what does it take for there to be such modal properties?

Here we naturally turn to the way in which the *concepts* of possibility and necessity have

⁹Attributes of higher arity will have to be described in a different format, but for now they can be safely ignored.

been defined. As can be seen from the relevant definition in §3.2, the concept of impossibility, and hence that of possibility, rests on the concept of an obtaining state of affairs. Similarly, the concept of necessity is defined on the basis of that of a valid formula – which again rests on the concept of an obtaining state of affairs. Those definitions are of course not intended as accounts of modal *properties*; instead, they only aim to specify how certain predicates should be understood. However, if we are going to assume that those predicates do denote (or express) modal properties, then those definitions can of course not simply be ignored when it comes to the question of what those properties are, and hence, of what is required for the existence of any such properties.

For the purpose of a first approximation, let us suppose that the above definition of ‘impossible’ can at the same time be regarded as a characterization of the property of being an impossible state of affairs. If we take this quite literally, we will then be identifying that property with the property of being “denoted, relative to some interpretation and variable-assignment, by a constant or formula φ such that, for every interpretation I and every variable-assignment g , φ does *not* denote, relative to I and g , an obtaining state of affairs”.¹⁰ Now this may admittedly seem very suspicious. Who would have thought that the property of being an impossible state of affairs should have *this* much to do with interpretations, formulas, and variable-assignments? But there are good reasons why that definition of ‘impossible’ is framed in this way. In effect, the definition describes states of affairs by way of describing formulas by which those states of affairs are denoted. The main advantage of this indirect route lies in the fact that it avoids any commitment as to how states of affairs are structured: i.e., in exactly what sorts of relations they stand to their various constituents.

On the other hand, if we had instead adopted an account as to how (i.e., by what relations) states of affairs are themselves constructed out of their various constituents, then all the above talk about denotation, formulas, interpretations, and variable-assignments could have been replaced by direct descriptions of states of affairs. For this reason, an account of what is

¹⁰See above, p. 48.

required for there to be such a thing as the property of being an (im)possible state of affairs need not itself make reference to formulas and variable-assignments.

What might a definition of ‘impossible’ look like that is purged of all reference to linguistic entities? Very roughly, we might say that a state of affairs is impossible just in case, for some ‘level’ of its structure, no matter how the constituents below that level (except for the identity relation¹¹) are replaced by other entities, the resulting state of affairs will always fail to obtain.

Could we not also get rid of the reference to the notion of obtainment? In particular, it might be wondered whether we could not draw up some purely formal description as to how a state of affairs would need to be structured in order to be impossible. If such a description can be given, it is clear that it would have to be extremely complicated. But more important, such a purely structural description would no longer seem to do justice to the present conception of impossibility, for the link between impossibility and obtainment appears to be central for what impossibility *is*. For this reason, an adequate account of that property (if it exists) should retain some reference to the notion of obtainment. So, again very roughly, the property of being an impossible state of affairs would have to be identified with the property of being a state of affairs that fails to obtain and that is such that any state of affairs that results from it by such-and-such replacements of its constituents also fails to obtain. But it is hard to see how there could be such a property without there being a property of obtainment. So we seem forced to conclude that there will be no property of being an impossible state of affairs unless there is a property of obtainment; and similarly for other modal attributes. It then further follows that, without a property of obtainment, there will be no modal states of affairs.

Of course, if there are no modal states of affairs, then sentences containing iterated modal operators cannot be understood in the natural way sketched above, viz., as saying of such-and-such modal states of affairs that these are (im)possible, necessary, or contingent. This consideration might be taken to militate in favor of accepting that a property of obtainment exists, but this move would be far from unproblematic. For if we accept the idea that there

¹¹Cf. the above definition of ‘interpretation’ (p. 46).

exists a property of obtainment, it will not be clear how we can in a principled way avoid the resulting version of the Liar paradox.¹² This difficulty does certainly not rule out the existence of modal attributes and states of affairs. However, the issue of how best to avoid the semantic paradoxes is extremely delicate, and I will here not try to take a stance on it. My aim in this section was only to give an indication of what sorts of problems we might be led into if we assume that modal attributes and states of affairs exist.

¹²See above, p. 30.

Chapter 4

Individuational Ontology

In this chapter, I develop the concept of an individuational ontology, which will provide the basis for the ‘relativized’ concepts of essence and essentiality that will be introduced in chapter 5.

4.1 Construction, Individuation, and Reciprocal Essence

The intuitive kernel of the account of essentiality that I will develop in this dissertation is the following, very rough explanation or diagnosis as to why Fine’s asymmetry holds:

- (E) It is essential to the set {Socrates} to have Socrates as a member, because the former is in some sense *constructed* from Socrates. On the other hand, it is *not* essential to Socrates to be a member of that set, because Socrates is not constructed from it.

It is worth noting that the link that is here drawn between essence and construction can already be found in Fine’s own work from the early 1980s. Consider, e.g., the following passage, in which he aims to explicate his notion of ‘ontological genesis’:

Say that certain objects generate another object if they are used to explain its identity. The canonical description of an object then displays its ontological genesis, the objects from which it is generated and the manner of generation. (1982, p. 52)

It would not be far-fetched to read the phrase ‘canonical description’, as it occurs here, in something like the traditional sense of ‘real definition’, which is precisely how Fine, a decade later, has proposed to understand the concept of essence.¹ If this reading is on the right track, then the passage suggests that we can find an entity’s essential properties by asking for its canonical description, i.e, by asking how and from what objects it is generated.²

In order to turn the rough-and-ready diagnosis given in (E) into an account of essentiality, we will have to explain what it means for something to be ‘constructed from’ something else. And in doing so, it seems advisable not to rule out the possibility that an entity may be constructed from something else that is in its turn constructed from the former; or, more generally, the possibility that an entity may be constructed, directly or indirectly, from itself. The reason for this is that one should arguably allow for cases in which two things x and y are ‘essentially related’ to each other, in such a way that it is essential to x to stand in a certain relation to y , and essential to y to stand in a certain relation to x . Fine himself mentions two potential cases of this sort in his (1995a):

Consider the case of two interrelated fictional objects, for example. Could it be maintained that it was of the essence of Holmes to be admired by Watson (or, at least, to be admired by him in the Conan Doyle stories) and also of the essence of Watson to admire Holmes. Or again, given two points x and y in Euclidean space, could it be maintained that x is essentially distinct from y and y essentially distinct from x . (p. 65)

Fine expresses himself very tentatively here, and the examples he mentions – Holmes and Watson, and points in Euclidean space – are certainly not uncontroversial cases of entities that are essentially related to each other. But the real question is whether an account of essence

¹Quoting from ‘Essence and Modality’:

I shall also argue that the traditional assimilation of essence to definition is better suited to the task of explaining what essence is. It may not provide us with an analysis of the concept, but it does provide us with a good model of how the concept works. Thus my overall position is the reverse of the usual one. It sees real definition rather than de re modality as central to our understanding of the concept. (p. 3)

The account of essence and essentiality that I will develop in the following is completely in harmony with this position.

²For a fuller account of this sort of ontological generation or ‘construction’, see Fine (1991). Similar ideas are also present in Fine’s later papers, such as his (2005; 2010; 2012).

should allow for such reciprocity, or whether it ought to rule it out.

This issue is a subtle one, and merits some more detailed discussion. A little further below from the passage just quoted, Fine says:

It seems very reasonable that reciprocal essences should be forbidden under the constitutive conception; for how else could a noncircular account of the essence of things be given? But this leaves it open whether reciprocal essence should be allowed under a consequential conception. (p. 65f.)

The “consequential conception” of essence is broader than the “constitutive conception”, in that – very roughly – it counts all those properties as essential that in a certain sense follow from those that are ‘constitutive’ of a given entity’s nature. Fine himself points out, however, that it is not very clear where the line between constitutive and consequential essence should be drawn (p. 58).

Although Fine prohibits reciprocal constitutive essence, he does allow that it may be “part of the constitutive essence of x and y (in that order) that they stand in the relation R ” (p. 66). He allows, in other words, for essences that are somehow associated with two entities in a particular order. In this way, it would seem that he can, after all, allow for the possibility that two entities are in some sense essentially related to each other: viz., in the sense that x ’s being R -related to y is part of something that might be called the ‘joint’ essence of x and y . However, it is conceptually very odd, at least to my mind, to have an essence associated with two entities in a certain order, where this essence is (at least to some extent) independent of those essences that are associated with the two entities individually. By contrast, it strikes me as much more natural to treat all essences (at least where these are not generic ones, such as ‘the essence of man’³) as associated with individual entities, rather than with pluralities thereof, whether ordered or not. At least, this is the conception of essence that I shall adopt for the purposes of the following.

Suppose, then, that all essences are associated with individual entities. Does this preclude that there are pairs of entities that are essentially related to each other? According to Fine, the existence of such pairs is indeed ruled out, at least on the constitutive conception of essence.

³For the purposes of this dissertation, generic essences have already been set aside in §1.2 above.

His argument for this conclusion appears to be that, if it were part of x 's constitutive essence to be R -related to y , and vice versa, then no “noncircular account of the essence” of x and y could be given. But it is not at all obvious, as far as I can see, that an account of an entity's essence must always be noncircular, and Fine does not offer any argument for this thesis. So it is unclear why constitutive essences should be prohibited from being reciprocal.

Of course, these considerations do not yet give us a reason to think that constitutive essences should *not* be prohibited from being reciprocal; the above examples of potential cases where things are essentially related to each other are, as I said, not uncontroversial. But there is still another class of examples that seem to be more compelling, viz., non-well-founded sets. These can be conceived of as starting-points of infinitely long descending chains of set-membership. In other words, a set x is non-well-founded just in case there are entities x_1, x_2, \dots (not necessarily pairwise distinct) such that $x \ni x_1, x_1 \ni x_2, x_2 \ni x_3$, and so on *ad infinitum*. Fine himself makes use of one such set to illustrate the idea of “reflexive essence”: viz., the set that has itself as its only member.⁴ The theory of non-well-founded sets clearly also affords examples of *prima facie* reciprocal essence. Thus, consider two non-well-founded sets x and y , where x contains y as a member, and where y contains x as a member.

Examples taken from the theory of non-well-founded sets may admittedly seem artificial (though this might also be said of examples taken from classical set theory), and some philosophers may regard non-well-founded set theory to be incoherent, on the grounds that set-membership can only be properly conceived of as well-founded. However, the naturalness or coherence of non-well-founded set theory is not at issue here. For the present purposes, the relevant question is only this: If we accepted the existence of non-well-founded sets, would we still say that for any entities x and y such that x has y as a member, it is *essential* for x to have y as a member? I think that the intuitively most plausible answer to this question is clearly ‘yes’. Even if x and y happen to have *each other* as members, we should say that it is essential to x to have y as a member, and essential to y to have x as a member.

⁴*Op. cit.*, p. 63.

Further, based on what has been said above, a case of reciprocal essence should arguably not be analyzed as one in which it is part of some ‘joint essence’ of x and y (in which order?) that x has y as a member and vice versa. An adherent of Fine’s views on this issue might insist that it is only part of the *consequential* essences of x and y to have the respective other entity as a member. But what is here the relevant constitutive essence, from which the membership of x in y , or that of y in x , would follow? There does not seem to be any plausible candidate, and so we are apparently forced to conclude that it is part of x ’s constitutive essence to have y as a member, and also part of y ’s constitutive essence to have x as a member. That is, we should say this if we wanted to continue to distinguish between constitutive and consequential essence. However, since it is not very clear how to draw this distinction, I shall in the following leave it aside.

Considerations of non-well-founded sets, then, suggest quite strongly that our account of essentiality should make room for the possibility, not only of entities that are essentially related to themselves, but also of pairs of entities that are essentially related to each other. A terminological side-effect of this is that the metaphor of *construction* does no longer appear to be altogether apt. After all, it does not make much sense to say that an entity is constructed from itself, or that two entities are constructed from each other; yet this is what we would have to say if we applied the above diagnosis (E) to two entities x and y that have each other as members. For this reason, I will in the following prefer to speak of *individuation* rather than construction: instead of saying that x is constructed from y , I will say that x is *individuated by* y . Naturally, the term ‘individuation’ should in this context not be understood simply in the sense of ‘singling out’, but rather in a more ambitiously metaphysical sense, whose clarification is part of the present project, or at least closely related to it.⁵

As I said above, in order to turn (E) into an account of essentiality, we will have to explain what it means for something to be ‘constructed from’ something else. In accordance with the terminological shift proposed in the previous paragraph, this goal has now to be

⁵For a brief bibliographical note on different senses of ‘individuation’ and their (ir)relevance for considerations of essentiality, see Appendix B.

reformulated as follows: We will have to explain what it means for something to be *individuated* by something else. And as can be seen from the above discussion, in constructing this account of individuation, we should leave room for the possibility that an entity may (directly or indirectly) individuate itself.

This last constraint, which at first seems to make the mentioned task more difficult, in fact makes it easier, because it points us, as it were, in the direction of the desired account of individuation. In the first place, it directs us toward formal treatments of non-well-founded sets, the standard reference for which is Peter Aczel’s *Non-Well-Founded Sets* (1988). Aczel’s account can effectively be regarded as an *ontology* of both well-founded and non-well-founded sets. By suitable generalizations, it is possible to abstract from it a certain approach to the construction of ontologies that I will refer to as the *individuational* approach. The main task of this chapter will be to explain this approach, and the central concept for that purpose is that of an individuational ontology, which will be introduced in §4.3.

4.2 Aczel’s Approach to the Ontology of Sets

In his classic study of non-well-founded set theory, Aczel (1988) adopts the standard axioms and axiom schemata of Zermelo–Fraenkel set theory, except for the Foundation Axiom, which he replaces with the so-called Anti-Foundation Axiom (or *AFA*), and the Schema of Replacement, which he replaces with the Collection Schema.⁶ The Anti-Foundation Axiom belongs to a family of related axioms, some of which (including Aczel’s own) had already been studied by other authors.⁷

⁶The Collection Schema can be written as follows:

$$\forall a \exists z (\forall x \exists y (x \in a \rightarrow \varphi) \rightarrow \forall x \exists y (x \in a \rightarrow y \in z \wedge \varphi)),$$

where ‘ φ ’ may be replaced by any formula in which the variable ‘ z ’ does not occur free (cf. Aczel 1988, p. 117f.). As Aczel notes, every instance of the Replacement Schema “can easily be proved from” the Collection Schema (*ibid.*).

⁷In particular, Aczel’s axiom had previously been studied by Forti and Honsell (1983). He himself acknowledges this fact, and offers an extensive discussion of several alternative anti-foundation axioms in later chapters of his book.

A remarkable fact about *AFA* is that, in a certain sense, it amounts to a complete ontology of both well-founded and non-well-founded sets. Aczel himself states the axiom as follows:

Every graph has a unique decoration. (p. 6)

In order to see what this means, one has to consult Aczel's definitions of 'graph' and 'decoration'. The definition of 'graph' is still quite simple:

Here a graph consists of a set of NODES and a set of EDGES, each edge being an ordered pair (n, n') of nodes. (p. 3f.)

We may interpret this as saying that a graph is an ordered pair (M, E) , where M is a set of nodes and E a set of edges. Aczel tells us that edges are pairs of nodes; but he does not say what sort of thing a *node* is. For most purposes, this does not matter, because nodes are only needed as the coordinates (or 'end points') of edges, so that their various intrinsic properties are largely irrelevant. Hence, anything whatsoever can be a node – including sets. It is here important to keep in mind that Aczel retains, except for the Axiom of Foundation and the Axiom Schema of Replacement, all the axioms and axiom schemata of classical ZF set theory. He has thus available to himself all the sets that exist already according to ZF set theory, and he can draw on these for his nodes as well as for his graphs (which, as we have just seen, are set-theoretic entities constructed on the basis of nodes).⁸

To be able to understand Aczel's *AFA*, it only remains to see what 'decoration' means:

A DECORATION of a graph is an assignment of a set to each node of the graph in such a way that the elements of the set assigned to the node are the sets assigned to the children of the node. (p. 4)

In order to understand this, one first has to ask what the 'children' of a node are.

If (n, n') is an edge then I will write $n \longrightarrow n'$ and say that n' is a CHILD of n . (*ibid.*)

This formulation is somewhat loose. According to the Axiom of Pairing, there will be a pair (n, n') , and hence an edge, for *any* two nodes n and n' ; but it is plainly not supposed to be the case that every node is a child of every other. What Aczel seems to have in mind is a

⁸Aczel explicitly uses sets as nodes of graphs – see *op. cit.*, p. 14 and p. 30.

notion of child that is relativized to graphs: Relative to a graph (M, E) , a node n' is a *child* of another node n if and only if E contains an edge (n, n') . The above definition of ‘decoration’ must then be understood accordingly: A *decoration* of a graph G is an assignment of a set to each node n of G in such a way that the elements of the set assigned to n are the sets assigned to those nodes that are, relative to G , the children of n . In the following, however, this relativization to graphs will usually be left implicit.

Let us now apply *AFA* to an example. The following is a depiction of a graph with two nodes and two edges:⁹



What does it mean to say that this graph has a ‘unique decoration’? For ease of reference, let us first give names to the two nodes, calling the one on the left ‘ ν_1 ’ and the one on the right ‘ ν_2 ’. Then the two edges will be (ν_1, ν_2) and (ν_2, ν_2) : both ν_1 and ν_2 have ν_2 as their only child. A *decoration* of this graph is accordingly an assignment of sets to these two nodes, such that the set assigned to ν_1 has only the set assigned to ν_2 as a member, and the set assigned to ν_2 has only itself as a member. Notice that this does not rule out that the two nodes are assigned one and the same set. So it is at first blush possible that the graph should have at least *two* decorations: one in which the two nodes are assigned the same set, and another, in which they are assigned distinct sets. But according to *AFA*, the graph has a *unique* (that is to say, exactly one) decoration. So, which one of the two candidate decorations just mentioned is the actual decoration of the above graph? To answer this, one has to note that



is a graph as well, and must therefore, by *AFA*, have exactly one decoration. Of course, this simply means – setting aside ontological commitment to the decoration itself – that there is

⁹The same graph appears in Aczel’s book, at p. 47. Of course he gives many more examples. Still further examples can be found online in Moss (2009), whose introduction to non-well-founded set theory is much less condensed than the present exposition.

exactly one entity that has only itself as a member. Following Aczel, we may call this set ‘ Ω ’. But now, nothing more is needed for there to be a decoration that assigns Ω to both nodes of the above graph; which is to say that such a decoration exists. According to *AFA*, then, the latter must remain the *only* decoration of that graph, and there cannot be any additional entity, distinct from Ω , that some other decoration of that graph would assign to the left-hand node.¹⁰

This example illustrates how *AFA* fulfills two functions, corresponding to its two ‘halves’.¹¹ For, on the one hand, it says that every graph has *at least* one decoration, and this gives rise to ontological commitment: in saying that every graph has at least one decoration, the axiom in effect asserts, for every single graph, that there exist entities that can be assigned to the nodes of that graph in such a way that the assignment amounts to a decoration. Provided only that there are enough graphs, *AFA* will thus ‘generate’ all the well-founded sets of classical set theory, and in addition a host of non-well-founded ones. But on the other hand, because the axiom states that every graph has *at most* one decoration, it also imposes a certain restriction on what (pluralities of) sets there are, as we have seen in the above example. These two functions or roles might be referred to as those of ‘generation’ and ‘restriction’, respectively. When in the following we generalize Aczel’s approach to ontologies of entities other than sets, these functions will be assigned to two disjoint classes of statements.

4.3 Individuational Ontologies

The individuational approach to the construction of ontologies that I shall now develop is in several ways a generalization and modification of Aczel’s approach to the ontology of sets. The following is a list of some of the most important changes:

¹⁰It may be worth noting that the same result follows already from the Axiom of Extensionality. For if there were a decoration that assigned two distinct entities to the two nodes, then both of these entities would have as their only member the entity assigned to the right-hand node. So they would have all their members in common and yet be distinct, contradicting Extensionality.

¹¹Cf. Aczel (1988, p. 19), where he refers to these two halves as ‘*AFA*₁’ and ‘*AFA*₂’, respectively.

1. I will sharply distinguish between two levels of theory: on the one hand the *meta-level*, which will be home to an ontology of nodes and graphs, and on the other hand the *object-level*, on which individuational ontologies are formulated. In Aczel's approach, these two levels are not distinguished from each other, since *AFA* is presented as only one among a number of other axioms and axiom schemata, which provide the graphs and decorations that are quantified over by *AFA*.
2. The approach will be generalized so that it becomes applicable to entities other than sets. As a result, we will deal with graphs that may have more than one kind of edge (including unary ones), and these edges will no longer be exclusively interpreted as representing set-membership.
3. The mentioned roles of 'generation' and 'restriction' that are played by the two halves of *AFA* will be given to two disjoint classes of statements from which individuational ontologies are formed.
4. The role that in Aczel's approach is played by decorations *tout court* will be assigned to *injective* decorations, i.e., decorations that assign distinct entities to distinct nodes.

All this is admittedly easier said than done. In particular, the generalization to entities other than sets requires that we generalize all the central notions of Aczel's approach, i.e., the notions of graph, edge, child, and decoration. As a first step in this direction, I will now introduce the notion of an *individuational ontology*.

An individuational ontology is a set of statements (possibly empty), where each of these statements is either an *individuating specification* or an *ontological restriction*. An individuating specification is a statement of the following form:

Properties of the form ' $\lambda x \varphi$ ' are individuating,

where ' φ ' is to be replaced by some first-order formula φ that is free in x and possibly further variables.¹² These other variables will in the context of an individuating specification be

¹²As will emerge from the following, the only 'active ingredient' in an individuating specification is the predicate-schema that takes the place of ' $\lambda x \varphi$ ', the rest being essentially boilerplate. Consequently, one might

referred to as *meta-variables*, and will here be written as lower-case Greek letters. (Except for ‘ x ’ and these meta-variables, φ should not contain any other free variables.) For example, an individuating specification might state that

Properties of the form ‘ $\lambda x (\alpha \in x)$ ’ are individuating.

A *form* of properties is simply a schematic λ -expression, such as ‘ $\lambda x (\alpha \in x)$ ’ in the specification just shown. Meta-variables, like ‘ α ’, are here replaceable by individual constants: for instance, in order to obtain a λ -expression that denotes a property of the form ‘ $\lambda x (\alpha \in x)$ ’, one would have to replace all occurrences of ‘ α ’ by occurrences of some individual constant (using the same constant for all occurrences). More generally, if $\ulcorner \lambda x \varphi \urcorner$ is a form of properties for some formula φ that possibly contains meta-variables, then a given property is ‘of’ that form just in case it is denoted by an expression $\ulcorner \lambda x \psi \urcorner$, where ψ results from φ by way of a uniform substitution of individual constants for meta-variables. Thus, in the case of the above example, one such property would be the one denoted by ‘ $\lambda x (\text{Socrates} \in x)$ ’, i.e., the property of having Socrates as a member.

The *ontological restrictions* of an individuating ontology, on the other hand, do not have a fixed format, but are simply statements, made in a possibly higher-order language, whose sole purpose is to rule out the existence of certain pluralities of entities (more precisely: the existence of pluralities of entities that collectively satisfy certain descriptions). An example of such a statement would be the Axiom of Extensionality, according to which there are no two sets that have all their members in common. The basic idea is that an ontological restriction should not make any existence claims of its own, not even where the existence claim is embedded in a conditional, as for instance in, ‘If everything is F , then something is G ’. This requirement can be more formally stated as follows:

- (R) If an ontological restriction is put into prenex form (i.e., with all the quantifiers out front), then it will not contain any existential quantifiers.

say that a more appropriate format for individuating specifications would be something like ‘The predicate-schema ... is individuating’. However, nothing important turns on this issue, and the present format seems to me the most natural.

But this is only a first approximation; it still has to be refined in two respects.

First, an exception has to be made for those existential quantifiers that range over objects of the background ontology of sets, attributes, and states of affairs that has been sketched in the previous two chapters. This step is motivated by the thought that it should be admissible to have an ontological restriction according to which there are (in the ontology's domain) no more than countably many entities. But to express such a restriction in a perspicuous way requires, at least in the present framework, existential quantification over sets: 'There is a set S such that S contains all entities, and for every infinite subset of S , there is a bijection from that subset onto S .'

Second, a precaution has to be added against the use of atomic predicates that 'hide' existential quantifiers. For example, if someone defined a unary predicate F to denote the property of *being someone's ancestor*, then a statement like \lceil Everything is F \rceil would look like an admissible ontological restriction; but of course it should not count as one, since it implicitly entails the existence of ancestors. In order to guard against statements such as this, we have to require that an ontological restriction should not contain any existential quantifiers – with the exception noted in the previous paragraph – when put into prenex form *and* formulated in such a way that all its atomic predicates denote *basic* attributes. To a first approximation, basic attributes may be thought of as the sort of properties and relations that David Lewis (1983) has referred to as 'perfectly natural'. However, I will propose an alternative conception in §7.2 below.

Individuational ontologies need not always be constructed for 'all of existence', but may also be associated with smaller domains. Those things that fall into the domain associated with a given ontology O will in the following also be referred to as ' O -entities'. In order to represent the association of ontologies with domains in a formal way, one may think of individuational ontologies not as sets of statements, but rather, e.g., as ordered pairs (S, P) , where S is a set of statements and P a predicate that is true of all and only those entities that belong to the ontology's domain. For the sake of convenience, however, I shall continue to speak of ontologies as sets of statements 'associated' with domains.

In order to signify that a given quantifier is intended to range only over O -entities, I will use a name of O as a subscript. Thus, if P is the predicate that is used to characterize the entities of O 's domain, then the expression ‘ $\exists_O x \text{ horse}\langle x \rangle$ ’ will be intended as an abbreviation of the formula $\lceil \exists x (P\langle x \rangle \wedge \text{horse}\langle x \rangle) \rceil$; and analogously, ‘ $\forall_O x \text{ horse}\langle x \rangle$ ’ will abbreviate the formula $\lceil \forall x (P\langle x \rangle \rightarrow \text{horse}\langle x \rangle) \rceil$.

In this section, I have only stated what sorts of statement individuation ontologies consist of, but have not yet addressed the important question of how such an ontology, and in particular its individuating specifications, are to be *interpreted*. This question will be answered in §4.7 by way of a semantic principle, or ‘meaning postulate’. In a certain sense, this principle will be a generalization of the first half of Aczel’s Anti-Foundation Axiom, which states that every graph has at least one decoration. It is for this reason that, in order to formulate the principle, we will first have to generalize Aczel’s notions of graph, edge, child, and decoration.

4.4 Graphs

Of the two “levels of theory” mentioned above (p. 67), individuation ontologies belong to the *object*-level. The latter has to be sharply distinguished from the *meta*-level, which “will be home to an ontology of nodes and graphs”. Such an ontology will be needed because we will have to quantify over graphs, or suitable other entities, in order to interpret individuation ontologies. The work of nodes and graphs can admittedly also be done by formulas (as described in Appendix C); but for the time being, I shall continue to work with nodes and graphs, if only because they are relatively familiar from Aczel’s treatment.

What should an ontology of such entities look like? As we have seen, Aczel’s approach was to use the axioms of classical set theory to ‘generate’ the usual set-theoretic hierarchy, the members of which could then be used as nodes. An alternative to this, less austere though perhaps more intuitive, would be to conceive of nodes as *sui generis*. Nothing important for our purposes will depend on this choice. The simplest course of action would probably be to follow Aczel in relying on the axioms and axiom schemata of classical set theory. Unlike

Aczel, however, I will here also be able to rely on the commonly accepted Foundation axiom and the Replacement schema, for doing so on the meta-level will not preclude consideration of ontologies (on the object-level) that do admit the existence of non-well-founded sets. In addition, as already mentioned in §2.1 above, I propose to accept a (necessarily incomplete) list of large-cardinal axioms. This will allow us to draw on large infinite numbers of pure sets, which will in the following play the role of nodes.

The fact that the present approach has to be applicable to entities other than sets gives rise to a certain complication: for Aczel's notion of graph will have to be generalized in such a way that graphs can have different 'kinds' of edges, representing different attributes. To this end, let us say that an edge is an ordered tuple (P, ν_1, \dots, ν_n) , where ν_1, \dots, ν_n are nodes and P is an n -ary predicate ($n \geq 1$).¹³ Here, the number n – intuitively, the number of the edge's 'end points' – will also be called the *arity* of the edge in question. Further, the node ν_1 will be referred to as the edge's *parent-node*, and the nodes ν_2, \dots, ν_n , if present, will be referred to as its *child-nodes*.

As can be seen from these last definitions, the present meta-level ontology will have to be an ontology not only of nodes, but also of predicates. This aspect can for now be left implicit, however.

Having so far sketched an ontology of nodes and edges, we next have to introduce the concept of an *O-graph*, where O is an individuational ontology. As a first step in this direction, it will be convenient to define the notion of a predicate's being *derived* from a given individuational specification. Thus, suppose that σ is an individuational specification, i.e., a statement of the form 'Properties of the form ... are individuating', where the dots are replaced, as shown in the previous section, by some λ -expression L enclosed in single quotes. L will contain exactly one λ -variable (i.e., the variable immediately following the ' λ ') and zero or more meta-variables. A predicate P will then count as *derived* from σ just in case P is a λ -expression that is obtained from L by writing all the meta-variables behind the λ -variable,

¹³Why not use attributes instead of predicates? The answer is that we have no guarantee that all the predicates used by a particular ontology actually denote something.

separated by commas and in the order in which they occur in the original expression. For example, from ‘ $\lambda x (\alpha \in x)$ ’ one would obtain ‘ $\lambda x, \alpha (\alpha \in x)$ ’; from ‘ $\lambda x R(x, \alpha_1, \alpha_2)$ ’ one would obtain ‘ $\lambda x, \alpha_1, \alpha_2 R(x, \alpha_1, \alpha_2)$ ’, and so on. In this way, the meta-variables are turned into ordinary λ -variables, and the resulting expressions will be $(n + 1)$ -ary predicates, where n is the respective number of meta-variables.¹⁴

Now, for any individuational ontology O , an O -graph may be thought of as a graph whose edges have as their first coordinates predicates derived from O ’s individuational specifications. More formally, an O -graph is a pair (M, E) , where M is a set of nodes (i.e., pure sets) and E a set of tuples (P, ν_1, \dots, ν_n) as described above, but with the additional requirement that the predicate P be derived from one of O ’s individuational specifications.

A useful convention for naming individuational specifications is to write ‘ σ^P ’ as a name for the specification in question, where ‘ P ’ is to be replaced by (an abbreviation of) the predicate that is derived from that specification. For example, consider the individuational specification to the effect that properties of the form ‘ $\lambda x (\alpha \in x)$ ’ are individuating. The predicate derived from this is ‘ $\lambda x, \alpha (\alpha \in x)$ ’, which, for the purposes of naming said specification, may be abbreviated to ‘ \exists ’. So, by the proposed convention, we may refer to that specification as ‘ σ^\exists ’. (I will make repeated use of σ^\exists as an example further below, and it will become even more important in chapters 7 and 8.)

4.5 Decorations

To generalize Aczel’s concept of a node’s child, we now have to introduce the concept of a node’s P -children-tuple, where P may be any predicate. Thus, if G is a graph that contains the node ν , let us say that any tuple of nodes (ν_1, \dots, ν_n) is a P -children-tuple of ν relative to G just in case G ’s set of edges contains the edge $(P, \nu, \nu_1, \dots, \nu_n)$. Further, let us call the coordinates of ν ’s P -children-tuples relative to G the P -children of ν relative to G . For

¹⁴The fact that, e.g., ‘ $\lambda x, \alpha (\alpha \in x)$ ’ is here treated as a well-formed λ -expression should not seem objectionable, since there is no general rule according to which Greek letters are always to be read as meta-variables.

instance, if ν has, relative to G , the two P -children-tuples (ν_1, ν_2) and (ν_1, ν_3) , then the P -children of ν relative to G will be ν_1 , ν_2 , and ν_3 . Finally, let us say that a node's P -edges relative to G are the edges of G that have the predicate P as their first coordinate and the node itself as their second coordinate. In other words, the P -edges of a node ν relative to G are all the edges of G that are of the form (P, ν, \dots) .

Before we can generalize Aczel's concept of decoration, we will need to introduce two more auxiliary concepts. First, for any entity x , let us say that a σ -indivuator-tuple of x is any tuple of entities (x_1, \dots, x_n) such that x and x_1, \dots, x_n , in this order, jointly satisfy the predicate derived from σ . And second, in analogy to the way in which we have in the previous paragraph introduced the concept of a node's P -children, let us call the coordinates of x 's σ -indivuator-tuples the σ -individuator-tuples of x . Thus, for instance, the σ^\exists -individuator-tuples of a set will be exactly the set's members, because its σ^\exists -indivuator-tuples are precisely the 1-tuples of its members. The set {Socrates, Plato}, for example, has the σ^\exists -indivuator-tuples (Socrates) and (Plato), and its σ^\exists -individuator-tuples are therefore Socrates and Plato.

We are now in a position to generalize Aczel's concept of decoration. As before, let O be an individuational ontology, and let G be an O -graph. Then an O -decoration of G is any assignment s of O -entities to the nodes of G in such a way that, for each node ν of G , each O -entity x , each individutive specification σ of O , and each predicate P : if P is the predicate derived from σ , and if x is the entity that s assigns to ν , then one of the following two conditions is satisfied:

- (i) The arity of P is 1, and x satisfies P if and only if (P, ν) is one of G 's edges; or
- (ii) The arity of P is greater than 1, and the σ -individuator-tuples of x are *tuple-wise matched* with the P -children (relative to G) of ν .

This last locution ('tuple-wise matched') is here to be understood as follows: If $n + 1$ is the arity of P , then both of the following two conditions are satisfied:

- (i) For each P -children-tuple (ν_1, \dots, ν_n) of ν (relative to G), there is a σ -indivuator-tuple (x_1, \dots, x_n) of x such that s assigns x_i to ν_i for each i with $1 \leq i \leq n$.

- (ii) For each σ -individuator-tuple (x_1, \dots, x_n) of x , there is a P -children-tuple (ν_1, \dots, ν_n) of ν (relative to G) such that s assigns x_i to ν_i for each i with $1 \leq i \leq n$.

To construct a simple example of an O -decoration, consider an ontology that contains as its only statement the individuating specification σ^\exists , according to which properties of the form ‘ $\lambda x (\alpha \in x)$ ’ are individuating. For future reference, let us call this ontology ‘ O^\exists ’. The domain of O^\exists may be taken to be simply ‘all of existence’; in other words, the predicate associated with O^\exists will be ‘ $\lambda x \exists y (y = x)$ ’.¹⁵

Further, consider an O^\exists -graph G , as depicted on p. 66 above, with two nodes ν_1 and ν_2 and two edges (P, ν_1, ν_2) and (P, ν_2, ν_2) , where P is the predicate derived from σ^\exists , i.e., ‘ $\lambda x, \alpha (\alpha \in x)$ ’, and let s be some assignment that assigns the entities x_1 and x_2 , respectively, to the nodes ν_1 and ν_2 . In order for s to count as an O^\exists -decoration of G , the σ^\exists -individuators of x_1 have to be tuple-wise matched with the P -children of ν_1 , and the σ^\exists -individuators of x_2 have to be tuple-wise matched with the P -children of ν_2 . Now, relative to G , the only P -children-tuple of ν_1 is (ν_2) . Hence, in order for the σ^\exists -individuators of x_1 to be tuple-wise matched with ν_1 ’s P -children, x_1 has to have precisely one σ^\exists -individuator-tuple, viz., (x_2) , for x_2 is the entity that s assigns to the only P -child of ν_1 . In other words, x_1 has to have exactly one member, namely, x_2 . By analogous reasoning, we find that, in order for the σ^\exists -individuators of x_2 to be tuple-wise matched with the P -children of ν_2 , x_2 will have to have only itself as a member.

4.6 Undecoratable Graphs

By way of further illustration, let us now consider some cases where, for a given individualational ontology O , some O -graphs do *not* have an O -decoration, either for semantic or for logical reasons. In the first three cases below, the graphs in question may be said to be (relative to O) *semantically* undecoratable, in the sense that their lack of a decoration results, at least in

¹⁵It is worth noting that O^\exists -decorations are simply decorations in Aczel’s sense, except for the fact that Aczel’s notion of graph must here be replaced by that of an O^\exists -graph.

part, from facts about the semantics of the predicates that occur as first coordinates of the respective graphs' edges:

- Suppose that O contains an individuating specification σ from which a semantically defective predicate P can be derived. (Typical examples of such a predicate would be ' λx consists-of-phlogiston $\langle x \rangle$ ' and ' λx is-a-bandersnatch $\langle x \rangle$ '.) In such a case, if the graph in question contains any P -edges, then it will have no O -decoration, because there are no entities that jointly satisfy P .
- Suppose that, from O 's individuating specifications, two distinct predicates P_1 and P_2 can be derived that denote the *same* attribute, though this cannot be read off the predicates themselves: e.g., P_1 may be ' λx consists-of-water $\langle x \rangle$ ' while P_2 is ' λx consists-of-H₂O $\langle x \rangle$ '. If a particular node ν of the graph in question has (relative to that graph) a P_1 -edge but no P_2 -edge, then that graph cannot have an O -decoration, for the entity assigned to ν would – possibly together with some other entities – have to satisfy P_1 but not P_2 , which cannot be if the two predicates denote the same attribute.
- For a similar case, suppose that O contains two individuating specifications from which two binary predicates Q_1 and Q_2 can be derived, where the relation denoted by Q_2 is the *converse* of the relation denoted by Q_1 (though, again, let us suppose that this cannot be read off the predicates themselves). Then, if the graph in question contains an edge (Q_1, ν_1, ν_2) but no edge (Q_2, ν_2, ν_1) , the graph cannot have an O -decoration.

These cases of semantic undecoratability can be distinguished from what I will call *logical* undecoratability, an example of which is the following:

- Suppose that, from one of the ontology's individuating specifications, the predicate P can be derived, where $P = \lceil \lambda x, \alpha (R\langle x, \alpha \rangle \wedge R\langle \alpha, x \rangle) \rceil$ for some binary predicate R , and suppose also that the graph in question contains an edge (P, ν_1, ν_2) , but no edge (P, ν_2, \dots) . Then, by purely logical reasoning – i.e., independently of the semantics of whatever non-logical vocabulary is used in the graph's predicates –, one can see that the

graph has no decoration. For let x_1 and x_2 be the entities that are assigned, respectively, to ν_1 and ν_2 . If the assignment were a decoration, then it would, on the one hand, have to be the case that $R\langle x_1, x_2 \rangle \wedge R\langle x_2, x_1 \rangle$, since ν_2 is ν_1 's P -child; and so it would have to be both the case that $R\langle x_1, x_2 \rangle$ and $R\langle x_2, x_1 \rangle$. But on the other hand, it would have to be *not* the case that $R\langle x_2, x_1 \rangle \wedge R\langle x_1, x_2 \rangle$, since ν_2 has no P -children of its own. Consequently, it could not both be the case that $R\langle x_1, x_2 \rangle$ and $R\langle x_2, x_1 \rangle$, which contradicts what we have said two sentences ago.

Generally speaking, we have a case of logical undecoratability whenever it is logically impossible that the graph in question should have a decoration, regardless of how one interprets the non-logical vocabulary that is used in its predicates.

If a graph is neither semantically nor logically undecoratable, it may of course still fail to have a decoration, simply because there may not be any entities that could be assigned to the graph's nodes in such a way that the assignment amounts to a decoration. In such a case, I will say that the graph in question is (relative to the ontology in question) *ontologically undecoratable*.

4.7 The Semantic Principle

With the concept of O -decoration in place, we can at last specify how individuational ontologies should be interpreted. Thus, if O is an individuational ontology, then the following principle indicates – with a qualification to which I will turn immediately below – what claims O should be interpreted as making:

- (S) For every O -graph G , O claims, unless this is ruled out by O 's ontological restrictions, that G has an injective O -decoration.¹⁶

In order to forestall misunderstanding, I should note that the part after 'that' is here not meant to express ontological commitment to decorations, but rather only to the existence of

¹⁶Recall that an O -decoration is *injective* just in case it assigns distinct entities to distinct nodes. (Cf. p. 68 above.)

entities x_1, x_2, \dots that jointly satisfy a certain formula, which itself need normally not mention any decorations at all. In Appendix C (p. 237ff.), I describe an algorithm by which such a formula can be constructed on the basis of a given individuation ontology O and an O -graph G .

The concept of ‘ruling out’ that is employed in (S) also has to be clarified. Let $\chi_{O,G}$ be the formula just alluded to, which is jointly satisfied by entities x_1, x_2, \dots just in case there exists an injective O -decoration of the graph G that assigns these entities to the graph’s nodes. Further, let $\exists\chi_{O,G}$ be the result of prefixing $\chi_{O,G}$ with the quantifier ‘ \exists_O ’ and a list of all the free variables in $\chi_{O,G}$.¹⁷ What $\exists\chi_{O,G}$ in effect ‘says’ is that there are entities in the domain of O that jointly satisfy $\chi_{O,G}$. Thus $\exists\chi_{O,G}$ expresses the ontological commitment that is in (S) expressed, with the noted imperfections, by saying that G has an injective O -decoration.

Now, to say that O ’s ontological restrictions *rule out* that G has an injective O -decoration should be understood as saying that the conjunction of those restrictions is logically incompatible with $\exists\chi_{O,G}$. In the special case in which G is logically undecoratable, so that $\exists\chi_{O,G}$ is logically incompatible with itself, we will say that O ’s ontological restrictions rule out that G has an injective O -decoration, even if it should happen that O contains no ontological restrictions at all. Thus understood, our principle (S) will not saddle any individuation ontology with inconsistent existence claims, such as ‘ $\exists_O x, y (R\langle x, y \rangle \wedge \neg R\langle x, y \rangle)$ ’. But on the other hand, since the notion of ‘ruling out’ is here taken in a strictly logical sense, (S) does not prevent any individuation ontologies from making *semantically* impossible existence claims, such as ‘ $\exists_O x \text{ is-a-bandersnatch}\langle x \rangle$ ’ or ‘ $\exists_O x (\text{consists-of-water}\langle x \rangle \wedge \neg \text{consists-of-H}_2\text{O}\langle x \rangle)$ ’.

It is worth emphasizing that the semantics of individuation specifications is meant to be exhaustively described by (S). In other words, nothing more is to be read into them than the ‘impact’ they have on the semantics of the individuation ontologies containing them, as determined by that principle.

¹⁷We may here assume that each free variable in $\chi_{O,G}$ corresponds, one-to-one, to a node of G . (It may also be worth emphasizing that the symbol ‘ $\exists\chi_{O,G}$ ’ is still only a *name* of a formula, and not itself a formula.)

In the remainder of this section, I shall now explain why (S) talks of *injective* decorations, rather than of decorations *tout court*. In a nutshell, the reason for this slight complication lies in the fact that the omission of the word ‘injective’ would have made it unnecessarily difficult to construct ontologies that make certain sorts of existence claims. Suppose, for example, that we want to construct an individuation ontology O according to which there exist at least two entities that instantiate a certain property P . Under the present principle, this requires of O nothing more than that it should contain an individuating specification σ to the effect that ‘Properties of the form ‘ $\lambda x P\langle x \rangle$ ’ are individuating, where ‘ P ’ is replaced by a predicate denoting P . For let F be the predicate derived from σ , which will accordingly denote that same property. Then there will be an O -graph with two nodes ν_1 and ν_2 and unary edges (F, ν_1) and (F, ν_2) , and according to (S), O will consequently claim (using the predicate F) that there exist at least two *distinct* entities that instantiate P .

By contrast, consider the alternative principle (S⁻) that results from (S) by deleting the word ‘injective’. To achieve the goal mentioned in the previous paragraph, an ontology O would then have to be made more complex. An initially promising approach might be to include an additional individuating specification σ^\neq , to the effect that properties of the form ‘ $\lambda x (x \neq \alpha)$ ’ are individuating. Then there would exist an O -graph with two nodes ν_1 and ν_2 and edges (F, ν_1) , (F, ν_2) , (Z, ν_1, ν_2) , and (Z, ν_2, ν_1) , where Z is the predicate derived from σ^\neq , viz., ‘ $\lambda x, \alpha (x \neq \alpha)$ ’. The existence claim that corresponds to this graph would indeed entail that there are at least two entities that both satisfy P . However, it would also entail that there are *no further* entities than just those two.¹⁸ In fact, for every set-sized cardinality $\kappa > 0$, there will be graphs that correspond to claims entailing that there are exactly κ -many entities. The ontology would consequently be inconsistent, unless we add an ontological restriction to the effect that there exists no more than a single entity. (We could not add a restriction to the effect that there are exactly *two* entities, because this would go against the definition of

¹⁸For let x and y be the two entities that a decoration of that graph assigns, respectively, to ν_1 and ν_2 . Then, since ν_2 is the only Z -child of ν_1 , y must be the only σ^\neq -individuator of x . (Analogously, it can be seen that x must be the only σ^\neq -individuator of y .) But this just means that y is the only entity that is distinct from x .

‘ontological restriction’; see above, p. 69.) But then we would still not have achieved the goal of constructing an ontology according to which at least two entities instantiate P .

The only way in which that goal can be achieved under the mutilated principle (S^-) is to exploit differences between the entities that satisfy P . For example, suppose that one of those entities has a certain property Q while another doesn’t. (This need not be a qualitative difference; e.g., Q might simply be the property of being identical with x , where x is one of the entities that instantiate P .) We might then add to O a further individuating specification, to the effect that ‘Properties of the form ‘ $\lambda x Q\langle x \rangle$ ’ are individuating’, where ‘ Q ’ is replaced by a name of Q . If H is the predicate derived from that specification, there would then exist an O -graph with two nodes ν_1 and ν_2 and three edges (F, ν_1) , (H, ν_2) , and (F, ν_2) , but without an edge (H, ν_1) . To say that this graph has an O -decoration would thus amount to the claim that there exist entities x and y such that x and y instantiate P while x but not y instantiates Q . Since this entails that x and y are distinct, O would accordingly entail that there are at least two entities that both have P , which is what we wanted to achieve. It does, however, seem unusually cumbersome that we should have to add another individuating specification merely in order to claim that there exist at least two entities that both have P . I think that this should count as a strong reason for preferring the original principle (S).

4.8 Identity Criteria

A class of ontological restrictions that deserves special mention is that of *identity criteria*, which in general state that no two (distinct) entities stand in a certain relation R to each other, where R is some equivalence relation. According as R can be defined by first-order or only by second-order means, we can speak of the identity criterion in question as a first-order or second-order criterion. A typical example for a first-order criterion is the familiar Axiom of Extensionality (henceforth AE), according to which no two sets have all their members in common. An example for a second-order criterion would be the second half (referred to as ‘ AF_2 ’) of Aczel’s Anti-Foundation Axiom, which says that no graph has more than one



Figure 4.1: A graph with edges (P, ν_1, ν_1) and (P, ν_2, ν_2) , where $P = \lambda x, \alpha (\alpha \in x)$.

decoration. To put AFA_2 into the form of an identity criterion within the present framework, one could rephrase it as follows: No two distinct entities x and y are such that for some node ν of some O^\exists -graph G , there are O^\exists -decorations d_1 and d_2 of G such that d_1 assigns x to ν and d_2 assigns y to ν .¹⁹ Due to the quantification over O^\exists -graphs, this criterion has to be regarded as second-order.

AFA_2 is a strengthening of AE : whereas AE allows the graph shown in figure 4.1 to have an injective O^\exists -decoration, it is easy to see that AFA_2 doesn't. On the other hand, both of them rule out that the two-node graph depicted on p. 66 has an injective O^\exists -decoration.

In the relatively narrow context of non-well-founded set theory, AFA_2 may seem the most natural choice for an identity criterion, considering that it is the obvious counterpart to the attractively simple existence claim that every graph has at least one decoration. But in the present framework, this first half of the Anti-Foundation Axiom has been replaced by the semantic principle (S), which yields, for any ontology O , existence claims to the effect that certain O -graphs have *injective* O -decorations. Accordingly, the most natural identity criterion in the context of the present framework would seem to be an ontological restriction to the effect that every O -graph has at most one injective O -decoration, where O is the ontology in question.

In contrast to both AFA_2 and AE , this alternative criterion, if added to the ontology O^\exists , allows for the two-node graph on p. 66 to have an injective O^\exists -decoration. In further contrast to AE (though not to AFA_2), it does *not* allow the graph depicted in Figure 4.1 to have an injective O^\exists -decoration. For if there were an O^\exists -decoration that assigned distinct entities x

¹⁹Recall that O^\exists is the individuation ontology that has σ^\exists as its only individuation specification (and which contains no ontological restrictions). O^\exists -decorations are therefore a close equivalent to Aczel's decorations. Now, to say – à la AFA_2 – that every O^\exists -graph has at most one O^\exists -decoration means that no O^\exists -graph G is such that, for any two decorations d_1 and d_2 and any node ν of G , d_1 and d_2 assign different entities to ν . This is equivalent to the statement given in the text.

and y to, respectively, the left-hand node and the right-hand node, there would then also be another injective O^\exists -decoration that, instead, would assign x to the right-hand node and y to the left-hand node. The mentioned criterion therefore rules out that the graph has any injective O^\exists -decoration at all. Hence, as a statement within an individuational ontology O that also contains σ^\exists , the mentioned criterion neither implies nor is implied by AE .²⁰

However, regardless of what kind of identity criterion we may take to be the most natural, in general there is no reason to suppose that an individuational ontology always has to contain some identity criterion (or, for that matter, any other ontological restriction) in order to be true.²¹

²⁰Related discussion can be found in Rieger (2000), who argues for a criterion that is intermediate in strength between AE and FA_2 .

²¹For valuable discussion of the alleged need for identity criteria, see Carrara and Giarretta (2004). Also see Horsten (2010) and the references therein.

Chapter 5

Individuation, Essence, and Essentiality (Relative to an Ontology)

In the previous chapter, I have introduced the concept of an individuational ontology, as well as several related concepts, notably those of O -graph, σ -individuator, and O -decoration. On the basis of these concepts, I will now introduce concepts of individuation, essence, and essentiality that are relativized to individuational ontologies. In the final section, I will then sketch what role the relativized concept of essentiality will play in the eventual account of essentiality *simpliciter* that will be proposed in §7.8.

5.1 O -Individuation

Suppose that O is some individuational ontology, and that x and y are two entities. We may then say that x *O -individuates* y if and only if there exist entities x_1, \dots, x_n (for some $n \geq 1$) such that the following three conditions are satisfied:

- (i) $x_1 = x$;
- (ii) $x_n = y$;
- (iii) For each i with $1 \leq i < n$, there is an individuating specification σ of O such that x_i is

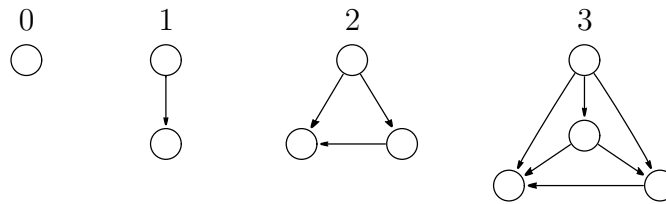


Figure 5.1: O^\exists -individuation graphs of the first four ordinals. (The nodes that are here depicted at the top are intended to be the ‘points’ of the respective graphs.)

a σ -individuator of x_{i+1} .

Note the trivial consequence that, for every individuational ontology O , every entity is O -individuated by itself. (To see this, consider the case in which $n = 1$.)

5.2 O -Essence

It is somewhat more complicated to define the concept of O -essence. The first step here is to define the notion of an O -individuation graph. Adapting once more the terminology of Aczel (1988), let us say that a *pointed O -graph* is an O -graph with a designated node that is referred to as the graph’s ‘point’; and let us say that a pointed O -graph G is *accessible* just in case every node of G can be reached from G ’s point by following the edges in the parent–child direction.¹ To put this more formally: If ν_0 is the point of G , then for each node ν of G that is distinct from ν_0 , G contains nodes ν_1, \dots, ν_n (for some $n > 0$) such that the following two conditions are satisfied: $\nu_n = \nu$; and for each i with $0 \leq i < n$, there is an edge of G of which ν_i is the parent-node and of which ν_{i+1} is a child-node.² The O -individuation graph of an entity x is then an accessible pointed O -graph that has an injective O -decoration assigning x to the graph’s point. For example, figure 5.1 shows the O^\exists -individuation graphs of the first four von-Neumann ordinals.³

Suppose now that G is a pointed O -graph. Further, as in §4.7 above, let $\chi_{O,G}$ be a formula

¹See *op. cit.*, p. 4.

²For the definitions of ‘parent-node’ and ‘child-node’, see §4.4 above.

³Cf. the diagrams in Aczel (1988, p. 3).

that is satisfied by some plurality of O -entities if and only if G has an injective O -decoration (*modulo* the commitment to graphs and decorations); this formula may be called the O -*individuation formula* that corresponds to G . In constructing this latter formula, one has to make use of some injective function from the nodes of G to variables, which then end up as free variables in $\chi_{O,G}$.⁴ Supposing that f is the function that has been used in constructing $\chi_{O,G}$, let V be the set of those variables to which f maps the nodes of G . Given that G is a pointed graph, one of the variables in V will be distinguished by the fact that it is the one to which f maps the point of G . Let v be this variable, and let w be a further variable that does not occur free in $\chi_{O,G}$. We can then construct the following λ -expression:

$$\lambda w \exists_O v_1, v_2, \dots (w = v \wedge \chi_{O,G}),$$

where v_1, v_2, \dots are the variables in V . Assuming that neither the predicate that is used in characterizing the domain of O (see above, p. 71), nor any of the predicates in $\chi_{O,G}$ is semantically defective, this λ -expression will denote a certain property, of which we will say that it is (relative to O) *expressed* by G . For example, the O^\exists -individuation graph of the number 1, as depicted in figure 5.1, corresponds to the (simplified) O^\exists -individuation formula

$$y \in x \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = y) \wedge \forall y_1 \neg(y_1 \in y) \wedge x \neq y,$$

and the property that is (relative to O^\exists) expressed by that graph is accordingly

$$\lambda z \exists x, y (z = x \wedge y \in x \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = y) \wedge \forall y_1 \neg(y_1 \in y) \wedge x \neq y)$$

Being an entity x that has exactly one member, which itself has no members and is distinct from x .⁵

⁴See Appendix C for details. By construction, $\chi_{O,G}$ will not contain any other free variables than those just mentioned.

⁵Strictly speaking, the quantifier ' \exists ' in the λ -expression should be ' \exists_{O^\exists} ', but this can be simplified to ' \exists ' because the domain of O^\exists is 'all of existence' (see above, p. 75), which means that all entities are O^\exists -entities. Further, where the original, unsimplified O^\exists -individuation formula contains sub-formulas $\ulcorner \lambda x, \alpha (\alpha \in x)(a, b) \urcorner$ (for any terms a and b), these have here been replaced by $\ulcorner b \in a \urcorner$.

Based on the concept of an entity's O -individuation graph, we may now introduce the concept of an 'abstract' O -essence:

(AEs) For any entity x and property P : if x has an O -individuation graph that (relative to O) expresses P , then P is the *abstract O -essence* of x .

If an entity has an O -individuation graph, it may still lack an O -essence. This will be the case if some of O 's individuating specifications make use of predicates that, although meaningful, do not denote any attributes. (The predicate 'obtains' may be an example of such a predicate.) For most practical purposes, however, individuation graphs of this sort may be safely ignored.

For the purpose of constructing an account of essentiality, we now still have to introduce (at least) one further concept of relativized essence. The need for this arises from the role that (relativized) essences are going to play in the account of relativized essentiality that will be provided in the next section. Roughly, an entity's *O -essential* properties are exactly those properties that 'follow' from its O -essence, or more precisely: those properties P such that the entity's instantiation of P is necessitated by its instantiation of its own O -essence. However, if the relevant essence were of the abstract sort introduced above, then an entity's instantiation of its own O -essence would generally fail to necessitate that the entity stands in such-and-such relations to any *particular* other entities. For instance, the property of having 0 as a member would not be O -essential to the von-Neumann ordinal 1, unless that property were explicitly mentioned in the ontology in question. And analogously, it would not be O -essential to {Socrates} to have Socrates as a member unless O contained an explicit reference to Socrates.

Consider, in addition, the role that the concept of relativized essentiality is going to play in the to-be-proposed account of essentiality *simpliciter*. According to this account, a property is essential to a given entity just in case it is O -essential to that entity, for some ontology O that meets a certain optimality condition. In order to accommodate the intuition that it is essential to {Socrates} to have Socrates as a member, the relevant account of optimality would – if all

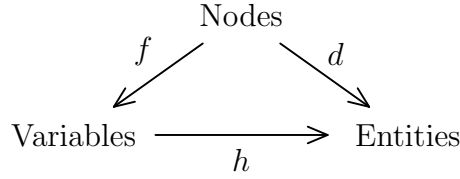


Figure 5.2: Mappings between the free variables of an O -individuation formula, the nodes of an O -individuation graph, and the entities assigned to these nodes by an O -decoration.

relativized essences were abstract – have to observe the constraint that there exists at least one optimal ontology that makes explicit reference to Socrates. Certainly, this *could* be done, but it would threaten to render our account of optimality exceedingly complicated. Moreover, it seems *prima facie* natural to think of an optimal ontology as one that is particularly elegant; but an ontology can hardly be regarded as elegant if it makes explicit reference to all sorts of individual entities. For this reason, I shall here take the alternative route of using a more complicated concept of O -essence, viz., that of *concretized* O -essence.

If x is some entity, let G be the O -individuation graph of x , let $\chi_{O,G}$ be the O -individuation formula that corresponds to G , and let f be a mapping of the nodes of G onto the free variables of $\chi_{O,G}$, as specified above. In addition, let d be an injective O -decoration of G that assigns x to the point of G (there has to be such a decoration, since otherwise G would not be an O -individuation graph of x), and let h be an assignment of entities to the free variables in $\chi_{O,G}$, such that a variable v is under h assigned an entity z just in case d assigns z to the particular node that f maps to v . (For illustration, see Figure 5.2.) By way of example, suppose that f maps the point of the O -individuation graph of the ordinal 1 to the variable v . In that case, the entity that h assigns to v will be the ordinal 1 itself, because the latter is what d assigns to the graph's point.

Given an entity x and an assignment h as just specified, we may say that a property P is a *concretized* O -essence of x just in case P is, relative to h , denoted by

$$\lambda w \exists_{O} u_1, u_2, \dots (w = v \wedge \chi_{O,G}),$$

where w is a variable that does not occur free in $\chi_{O,G}$, v is the variable to which f maps the point of G , and where the u_1, u_2, \dots form only a *proper subset* of the variables to which f maps the nodes of G . In the special case in which this subset is empty, we have a *fully concretized* O -essence, which is denoted, relative to h , by the expression $\ulcorner \lambda w (w = v \wedge \chi_{O,G}) \urcorner$, with v and w as before.

For example, the fully concretized O^\exists -essence of the ordinal 1 is the property that is denoted by

$$\lambda z (z = x \wedge y \in x \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = y) \wedge \forall y_1 \neg(y_1 \in y) \wedge x \neq y)$$

relative to an assignment h that assigns 1 to the variable ‘ x ’ and 0 to the variable ‘ y ’. In other words, if we take ‘0’ and ‘1’ to be constants denoting respectively 0 and 1, then we may say that the fully concretized O^\exists -essence of 1 is the following property:

$$\lambda z (z = 1 \wedge 0 \in 1 \wedge \forall y_1 (y_1 \in 1 \rightarrow y_1 = 0) \wedge \forall y_1 \neg(y_1 \in 0) \wedge 1 \neq 0)$$

Being identical with 1 and such that 0 is the only member of 1, nothing is a member of 0, and 0 is distinct from 1.

By contrast, the only two *partially* (i.e., not fully) concretized O^\exists -essences of 1 are

$$\lambda z \exists x (z = x \wedge 0 \in x \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = 0) \wedge \forall y_1 \neg(y_1 \in 0) \wedge x \neq 0)$$

Being an entity x such that 0 is the only member of x , nothing is a member of 0, and 0 is distinct from x .⁶

and

$$\lambda z \exists y (z = 1 \wedge y \in 1 \wedge \forall y_1 (y_1 \in 1 \rightarrow y_1 = y) \wedge \forall y_1 \neg(y_1 \in y) \wedge 1 \neq y)$$

Being identical with 1 and such that 1 has exactly one member, which itself has no members and is distinct from 1.

⁶For the use of ‘ \exists ’ instead of ‘ \exists_{O^\exists} ’, see the previous footnote.

If the term ‘*O*-essence’ is used without qualification, it should in the following be understood in an inclusive sense, covering an entity’s abstract *O*-essence as well as its (partially or fully) concretized *O*-essences. In other words, a property is an *O*-essence of an entity x just in case it is an abstract or concretized *O*-essence of x .

5.3 *O*-Essentiality

Finally, on the basis of the concept of *O*-essence, we can now introduce that of *O*-essentiality. Thus, let E be the fully concretized *O*-essence of some entity x . Then P will be said to be *O*-essential to x just in case $E\langle x \rangle$ (i.e., the instantiation of E by x) necessitates $P\langle x \rangle$.⁷ Given that an entity’s fully concretized *O*-essence is the logically strongest of all its *O*-essences, an

⁷For the relevant notion of necessitation, see §3.2 above. The question of how an entity’s essential properties are related to its essence(s) has recently also been dealt with by Oderberg (2011) and Correia (2012). On Oderberg’s account, the essential properties (which he would prefer to call simply ‘properties’) of an object of a kind K are exactly those that “are caused by and originate with the form of a K ”, where the form at the same time “provides the essence” (p. 101). Hewing closely to Aristotle, Oderberg here links essential properties to kinds. However, his proposal does not easily lend itself to an attempt at accommodating Fine’s asymmetry. If we ask to what kind the set {Socrates} belongs, the perhaps most plausible answer will be that it belongs to the kind *set*. But it is not very plausible to say that the property of having Socrates as a member (which we intuitively regard as essential to {Socrates}) is “caused by and originates with” some “form” associated with that very general kind; for if it did, it would be hard to see why that property should not be essential to any set whatsoever.

Correia’s account is somewhat closer to the present one. The main difference here lies in the fact that his account is *rule-based*; i.e., where the present account is based on the semantic notion of entailment, Correia’s account is based on the notion of provability in a particular system, where the admissible inference rules are determined by the entity (or plurality of entities) in question. This is motivated by the desire to accommodate certain intuitions, as e.g. the intuition that the property of being such that “if Sam is a philosopher, then Sam is a philosopher” is not essential to every entity whatsoever. In the present framework, these intuitions are accommodated by the use of neutral free logic (see below). In Correia’s system, by contrast, the reason why the mentioned property is not essential to (e.g.) Socrates lies in the fact that Socrates is not a logical concept. To illustrate his approach a little further, consider, instead of Socrates, the *plurality* that consists of Socrates as well as the logical concept of the material conditional: Although the mentioned property – i.e., of being such that, if Sam is a philosopher, then Sam is a philosopher – is on Correia’s account not essential to Socrates alone, it *is* essential to this plurality, because the introduction rule for the material conditional will allow us to prove, from any premise whatsoever, that if Sam is a philosopher, then Sam is a philosopher (cf. *op. cit.*, p. 649).

My principal worry about Correia’s account is that it puts all-too narrow limits on the class of an entity’s essential properties. In particular, suppose that Socrates’ “basic” (i.e., non-derivative) essence comprises exactly the following two propositions: that he is human, and that he has sprung from a certain zygote z . Then it will not be even derivatively essential to him that he is both human and sprung from z , because the derivation of this would require the use of a rule associated with the logical concept of conjunction. And nor would it be essential to him that he is sprung from *some* zygote, because this would require the use of a rule associated with existential quantification. Both of these results seem to me strongly counter-intuitive.

equivalent definition can be stated as follows:

(Es) A property P is *O-essential* to an entity x just in case, for some *O-essence* E of x , $E\langle x \rangle$ necessitates $P\langle x \rangle$.

For example, the following properties are all O^\exists -essential to the ordinal 1:

- $\lambda x (x = 1)$ (being identical with 1),
- $\lambda x \exists y (y = x)$ (existence),
- $\lambda x (0 \in x)$ (having 0 as a member),
- $\lambda x \exists y (y \in x \wedge \forall z \neg(z \in y))$ (having an entity as a member that has no members itself).

By contrast, the following three properties are *not* O^\exists -essential to 1:

- $\lambda x (x = 1 \vee x = 2)$ (being either 1 or 2),
- $\lambda x (x \in 2)$ (being a member of 2),
- $\lambda x (x = x \wedge \text{Socrates} = \text{Socrates})$ (being self-identical and such that Socrates is Socrates).

The reason why none of these three properties is O^\exists -essential to 1 is in each case the same: For each of the properties, its instantiation by 1 is denoted by some formula that contains a variable or constant denoting either the ordinal 2 or Socrates. But these variables or constants will denote different things under different variable-assignments and interpretations of the respective formal language, and will therefore, relative to some such assignments and interpretations, fail to denote anything at all.⁸ Further, they are not needed to construct a formula that denotes the instantiation by 1 of any of the latter's O^\exists -essences. As a result, if φ is such a formula, and if ψ is a formula that denotes the instantiation by 1 of one of the three properties in question, there will be an interpretation and variable-assignment relative

⁸See §3.2 above.

to which φ denotes a state of affairs while ψ does not denote anything at all. Hence, φ does not entail ψ ; and so the instantiation by 1 of any of 1's O^\exists -essences does not necessitate the instantiation by 1 of any of those three properties, which in turn means that none of those properties is O^\exists -essential to 1.

One of Fine's criticisms of the modal account of essentiality rests on the observation that, according to the modal account, Socrates essentially exists. In Fine's view, this consequence should be avoided, for "we do not want to say that [Socrates] essentially exists" (1994, p. 6). The same consequence, however, arises also from the present account, since the property of existence, i.e., $\lambda x \exists y (y = x)$, can easily be seen to be O -essential to every entity, for every ontology O . Should we then not try to avoid this consequence?

Against Fine, I would argue that we shouldn't. If we can accept that the property of having 0 as a member is essential to the ordinal 1, then we should presumably also be ready to accept that the property of having *something* as a member is essential to 1. By analogy, if we have no qualms about accepting that the property of being identical with 1, or $\lambda x (x = 1)$, is essential to 1, then we should likewise have no qualms about accepting that the property of being identical with *something*, i.e., $\lambda x \exists y (y = x)$, is essential to 1; and that is just the property of existing. But surely, if anything is essential to a given entity, that will be the property of being *that very entity*. (For – to invoke the independently plausible link between essence and *de re* modality – how could something fail to be the entity that it is?) So it seems extremely plausible to say that the property of being identical with 1 is indeed essential to 1. By the above reasoning, we should therefore also accept that 1 essentially exists. This point generalizes to all entities whatsoever, for in the case of any entity x , we would want to say that it is essential to x that it should be identical with x . All this is perfectly consonant with the present account, given that, for every entity x and ontology O , the property $\lambda y (y = x)$ is O -essential to x .

It may finally be worth emphasizing that, as it is here understood, O -essentiality should *not* be thought of as a feature that properties have independently of their bearers. This may be somewhat surprising, because it may be thought that, if a given property is essential to one

particular entity, it will also be essential to every other entity that instantiates it; and that the same should therefore hold for *O*-essentiality. But this is not always the case. For instance, if the *O*-individuation graph of some particular entity has, say, fifty nodes, then the property of being one of at least fifty entities will be an *O*-essential property of that entity. Similarly, the property of being one of at least a *hundred* entities may be *O*-essential to some other entity (namely, of one whose *O*-individuation graph has a hundred nodes). And of course, this latter property may in some cases also be instantiated by the first entity, even if it is not in fact *O*-essential to it.⁹ So we cannot in general assume that a property that is *O*-essential to a particular entity will be *O*-essential to every entity that instantiates it.

5.4 De-Relativization

As has already been mentioned, the concepts of *O*-essence and *O*-essentiality are central components of the account of essence and essentiality that I here aim to develop. The task ahead is to ‘de-relativize’ those concepts, i.e., we will have to identify the conditions that an ontology *O* has to satisfy in order (i) for *O*-essences to deserve being referred to as *essences* and (ii) for an entity’s *O*-essential properties to deserve being referred to as *essential* properties of that entity. That there should be such conditions is of course far from obvious, but is rather one of the central claims implicit in the account. Supposing that there are such conditions, let us call an ontology ‘optimal’ just in case it satisfies them. Once this notion of optimality has been adequately fleshed out, the account can be succinctly stated as follows:

(E1) A property is an (abstract or concretized) *essence* just in case it is an (abstract or

⁹Plantinga has further counter-examples:

Are there properties that some things have essentially and others have, but have accidentally? Indeed there are; *being non-green* is a property 7 has essentially and the Taj Mahal accidentally. *Being prime or prim* is essential to 7; it is accidental to Miss Prudence Allworthy, Headmistress of the Queen Victoria School for Girls. (1974, p. 61)

Both of these examples are instructive, but debatable. If it isn’t essential to 7 that the property of *being green* exists (and why should it be?), then it should also not be essential to 7 that the property of *being non-green* exists; but if so, then – at least on the present account – the property of being non-green cannot be essential to 7. Analogous considerations suggest that the property of *being prime or prim* is not essential to 7, either.

concretized) *O*-essence, for some optimal ontology *O*.

(E2) For any entity *x*, a property is *essential* to *x* just in case it is *O*-essential to *x*, for some optimal ontology *O*.

In view of what has been said at the beginning of the previous chapter, it might be wondered where this leaves the concept of individuation. To recapitulate, I there offered a rough-and-ready diagnosis as to why Fine's asymmetry holds, based on the intuitive notion of construction: I suggested that it is essential to the set {Socrates} to have Socrates as a member because the former is in some sense 'constructed' from Socrates, whereas it is not essential to Socrates to be a member of that set because he is not constructed from it (p. 59). Taking this as my starting-point, I further suggested that, in order to arrive at an account of essentiality, one would first need an account of construction; but with a view to avoiding the potentially misleading connotations of 'building from the ground up', I abandoned the term 'construction' in favor of 'individuation'. Now, in order to arrive at (E1) and (E2), we have in fact not needed to develop an account of individuation, after all. Nor will we need to develop such an account in order to spell out what it means for an ontology to be optimal.

However, the to-be-developed account of essentiality *is* of course based on the concept of an individuational ontology, and on this same basis, as we have seen in §5.1, a concept of individuation relativized to ontologies can be very naturally defined. Further, on the basis of this latter concept and the still-to-be-fleshed-out concept of optimality, it is very natural to explicate the *unrelativized* concept of individuation – in analogy to those of essence and essentiality – as follows:

(In) For any two entities *x* and *y*, *x* is *individuated* by *y* just in case *x* is *O*-individuated by *y*, for some optimal ontology *O*.

Although it was not necessary to develop this account of individuation in order to arrive at the account of essence and essentiality sketched above, the question of what it means for an entity to be individuated by another was what initially put us on the path toward the concept of an individuational ontology. Moreover, while it is true that (E1) and (E2) are not based on (In),

this does not mean that the initial diagnosis of Fine’s asymmetry was incorrect. For if it is O -essential to $\{\text{Socrates}\}$ to have Socrates as a member (for some optimal ontology O), then this will be due to the same characteristics of the O -individuation graph of $\{\text{Socrates}\}$ that make it the case that this set is O -individuated by Socrates. And likewise, if it is not O -essential to Socrates to be a member of $\{\text{Socrates}\}$, then this will be due to the same characteristics of *his* O -individuation graph that make it the case that he is not O -individuated by his singleton.

Before we move on to the task of developing an account of optimality, I should like to address a further concern that might arise in connection with (E2). In particular, one might worry that nothing that has so far been said rules out the possibility that there might exist an entity x , two properties P_1 and P_2 , and two optimal ontologies O_1 and O_2 such that

- (i) P_1 is O_1 -essential but not O_2 -essential to x ,
- (ii) P_2 is O_2 -essential but not O_1 -essential to x , and
- (iii) there is no optimal ontology O such that the property $\lambda x (P_1\langle x \rangle \wedge P_2\langle x \rangle)$ – i.e., the ‘conjunction’ of P_1 and P_2 – is O -essential to x .

In this case, the conjunction of P_1 and P_2 would, according to (E2), *not* be essential to x . But this seems counter-intuitive, given that (again according to (E2)) both P_1 and P_2 *are* essential to x .

It might at first be thought that this outcome can be easily avoided if we adopt a different account of essentiality, such as the following:

(E2’) For any entity x , a property P is *essential* to x just in case the state of affairs $P\langle x \rangle$ is necessitated by some non-empty conjunction

$$E_1\langle x \rangle \wedge E_2\langle x \rangle \wedge \dots,$$

where E_1, E_2, \dots are essences of x .

This proposal is in effect a variant of the above account of O -essentiality (p. 90), in which the references to O -essences have been replaced by references to essences *simpliciter*. But this

solution to the problem is unsatisfactory, because it addresses the problem only, as it were, at the topmost level. To get to the root of the problem, it seems that we have to descend all the way to the level of ontologies.

To see this, it will be helpful first of all to consider what makes the above scenario so counter-intuitive. The idea that there should be an entity to which two properties P_1 and P_2 are both essential while their conjunction isn't appears to contradict not merely intuitions about essentiality, but indirectly also intuitions about essence. For if we consider an entity's essential properties to be those that 'follow' from its essences (where this notion of 'following' may be understood in terms of necessitation, as seen in the above definition of O -essentiality), then the case in question must be one in which x has at least two essences E_1 and E_2 such that: (i) $E_1\langle x \rangle$ necessitates $P_1\langle x \rangle$, (ii) $E_2\langle x \rangle$ necessitates $P_2\langle x \rangle$, but (iii) x does not have any essence E such that $E\langle x \rangle$ necessitates both $E_1\langle x \rangle$ and $E_2\langle x \rangle$. I think that it is the implied absence of such an 'overarching' essence E that accounts for the counter-intuitive character of the case in question.

Admittedly, it is already somewhat counter-intuitive to allow that an entity may have multiple essences in the first place: for an entity's essence is naturally thought of as a *full* answer to the question of what it is to be that entity. But it seems nevertheless tolerable to adopt a conception that extends the usage of the word 'essence' in such a way that certain 'merely partial' answers to that question also count as essences; this is in effect what we have here been doing when we allowed an entity to have multiple (e.g., more or less concretized) essences. However, for every set of partial answers to the question of what it is to be x , there will plausibly exist a fuller answer that implies each of those partial ones. An analogous thesis should then also hold for essences. In other words, for every set S of essences of x , there should exist a 'more complete' essence E such that the state of affairs $E\langle x \rangle$ necessitates $E_i\langle x \rangle$ for each E_i in S . In particular, if E_1 and E_2 are essences of x , there should exist an essence E of x such that $E\langle x \rangle$ necessitates both $E_1\langle x \rangle$ and $E_2\langle x \rangle$.

It would of course be easy to accommodate these considerations by adding a clause to (E1) to the effect that, for every set of essences of a given entity x , their conjunction is itself

an essence of x . But this would be almost as unsatisfactory as the above proposal to replace (E2) with (E2'), because it would address the problem only at the relatively superficial level of essence. The arguably most satisfactory solution would address the problem at the deepest level possible, which in this case is the level of ontologies. Thus, I propose to retain (E2) and to impose the following constraint on the account of optimality that I will try to develop in the next two chapters:

- (C) For every set S of optimal ontologies, there exists an optimal ontology O that contains every individuating specification that is contained in any given member of S .

This constraint guarantees that the scenario described above cannot arise. In particular, whenever a property P_1 is O_1 -essential to x and another property P_2 is O_2 -essential to x , where O_1 and O_2 are both optimal, there must exist an optimal ontology O that contains the individuating specifications of both O_1 and O_2 . This ontology will then be such that both P_1 and P_2 , and hence their conjunction, are O -essential to x . (This latter claim can be easily verified by reflecting on the way in which, for any ontology O and entity x , the O -individuation graph of x is constructed on the basis of O 's individuating specifications.)

In the following two chapters, we will now turn to the question of what, exactly, optimality amounts to. According to the approach that I set out to develop in chapter 6, an optimal ontology is one that, roughly, forms part of a best explanation of what there is. Unfortunately, however, this approach runs into some grave difficulties. For this reason, I will in chapter 7 develop an alternative approach, which identifies optimality with *systematic* optimality. This latter name is motivated by the affinity that the approach bears to David Lewis's best-system account of lawhood: *systematically optimal* ontologies are individuating ontologies that, in a certain sense, are analogous to the 'best systems' of Lewis's account.

Chapter 6

Explanatory Optimality

6.1 First Approximations

What account should we give of optimality? The first approach to this problem that I would like to explore is based on the general idea that, in order for *O*-essences to deserve being called ‘essences’, an ontology *O* should in some sense provide a best explanation of what there is, or in a word, that it should be *explanatorily optimal*. This is motivated by the – as I think, intuitively attractive – idea that an ontology that can best explain what there is will also best be able to tell us, for every single entity, *what that entity is*. Accordingly, the task for the following will be to set out in more precise terms what one might reasonably mean by a “best explanation of what there is”.¹

A useful way to start on this task is to consider what would count as a ‘best explanation’ when the explanandum is not *what there is*, but rather only some particular phenomenon. In such cases, it seems natural to think of a best explanation as one that is as simple or elegant as possible, within the bounds set by certain constraints. The perhaps most obvious constraint

¹The idea that the essentiality of a property might be accounted for in terms of explanatory power has been well-entrenched, in contemporary metaphysics as well as historically. For example, in a passage already quoted above (p. 59), Fine suggests that an entity’s “canonical description” (which may plausibly be read in the sense of ‘real definition’) “displays [...] the objects from which it is generated”, where for the entity to be generated from those objects means that they “are used to explain its identity”. Talk of explaining an object’s ‘nature’ or ‘identity’ recurs in more recent writings of Fine’s (in particular, his (2010, p. 582)), as well as of other Fineans, such as Koslicki (2012):

is the requirement that the explanation should not entail anything that is known to be false.

A second potential constraint might be aptly termed ‘holism’: in seeking an explanation for some phenomenon, we should take into account not only that phenomenon itself, but also any other phenomena that might need explaining. For instance, if a certain attractively simple explanation of a particular phenomenon were to rule out any and all even remotely satisfactory explanations of numerous other phenomena, then this should certainly count against that explanation. But how should this constraint be implemented? Presumably the best way of making sure that one does not accept a simple explanation of a given phenomenon at the expense of many others is to conceive of all the to-be-explained phenomena together as of one ‘great phenomenon’, and to try to find the simplest explanation of *that*; though, again, under the constraint of avoiding known falsehoods. The resulting explanation of all to-be-explained phenomena will then also (in a certain sense) contain an explanation of the original phenomenon, and it is this latter explanation that we will be able justifiably to regard as the ‘best explanation’ of that phenomenon. However, if this is a reasonable way of implementing the holism constraint, then it appears to be not so much a constraint *under*

[T]he conditions, being the successor of the successor of the number 0 and being the predecessor of the number 3, both do the job we expect necessary and sufficient conditions to do, i.e., they single out the number 2 and differentiate it from a certain contrast-class [...]. However, one might reasonably claim that the first condition [...] is *more explanatory of the essential nature* of the number 2 than the second condition [...], since the first mirrors more closely than the second does the method by which the number 2 is constructed from a basic entity, the number 0, together with a relation that is taken as primitive, viz., the successor relation. (p. 199, emphasis added)

Earlier in the twentieth century, Irving Copi (1954) has proposed to analyze essentiality in terms of ‘causal derivability’. This proposal in turn goes back directly to Locke, who characterized the ‘real essence’ that is associated with a given species as “that real constitution of any Thing, which is the foundation of all those Properties, that are combined in, and are constantly found to co-exist with the *nominal Essence*” (*Essay*, III.6.6). But of course, Locke did not think that it made sense to speak of an essence that a thing might have in itself, independently of any manner of its description:

That which is *essential*, belongs to [a material object] as a Condition, whereby it is of this or that Sort: But take away the consideration of its being ranked under the name of some abstract *Idea*, and then there is nothing necessary to it, nothing inseparable from it. (*ibid.*)

For a more realist account, we can instead turn to Francisco Suárez (*Disp. met.* II, §4.6–7), or, going further back, to Thomas Aquinas, who paraphrased the Aristotelian expression that has come to be translated as ‘essence’ with the words, “that on account of which something is what it is (*hoc per quod aliquid habet esse quid*)” (2007, p. 228f.). Finally, the connection between explanation and (generic) essence is present already in Aristotle’s *Posterior Analytics* and *Metaphysics* (Charles 2010), and can arguably also be found in Plato’s *Phaedo* (Politis 2010).

which the simplicity of an explanation is to be maximized, as rather a constraint on *what* is to be maximized: viz., not the simplicity of the explanation of the particular phenomenon in question, but rather the simplicity of the more encompassing explanation of all to-be-explained phenomena, of which the former explanation is supposed to be a part.

These remarks are, I think, by and large correct with regard to what would count as a best explanation of some phenomenon, as considered from the point of view of an epistemically finite agent. An *explanatorily optimal* ontology, on the other hand, will here not be understood as a best explanation in quite the same sense, though there are also some important similarities. Roughly, I will say that an ontology is explanatorily optimal just in case it forms a part of an ontology that is (i) universal, in the sense that its domain comprises ‘everything whatsoever’, and (ii) as simple as possible, within the bounds set by two constraints.² The first of these two constraints is to the effect that the ontology in question should be *true*. So, rather than not to entail any *known* falsehoods, the ontology is required not to entail any falsehoods at all. The reason for this deviation from the above (and possibly more common) conception of best explanation lies in the desideratum that an account of essentiality should do justice to the sense of philosophical significance that has traditionally been attached to the concepts of essence and essentiality.³ This desideratum *prima facie* militates against an overly mind-dependent account of essentiality, and so it seems desirable that the question of what properties are essential to what entities should, on our account, not depend on what is or isn’t known to some epistemic agent. The same will thus have to be true for the conditions under which an ontology is explanatorily optimal.

The *second* constraint might be roughly characterized as requiring that the ontology in question should be ‘sufficiently informative’. Such a constraint is evidently needed, because it is all-too easy to construct a true and simple ontology if the latter need not actually contain any

²As noted in §2.8, it is not obvious that we can in an unambiguous sense speak of everything whatsoever (hence the scare quotes around that phrase), and so it is also not obvious that we can in an unambiguous sense speak of universal ontologies. Nevertheless, I shall in the following leave this complication aside, since the ambiguities in question do not seem to affect our account in any material way.

³Cf. §1.3 above.

amount of information at all. To have a more technical term for what I have here referred to as ‘sufficiently informative’, I will in the following speak of *comprehensiveness*. The following gives a first, very rough approximation of what comprehensiveness amounts to:

(Cp₀) An ontology is *comprehensive* if and only if it entails all the facts (i.e., all the obtaining states of affairs).

I will try to revise this explication in §6.3 below. Using the concept of comprehensiveness, the notion of explanatory optimality may, again to a first approximation, be defined as follows:

(EO₀) An ontology O is *explanatorily optimal* just in case it forms a part of an individuational ontology that is as simple as possible while still being both true and comprehensive.

This explication is also in need of revision. But before we come to that, it is necessary to add two clarificatory remarks.

First, it has to be specified what it means for an ontology to entail a given state of affairs. This task is unfortunately complicated by the fact that the ontologies that we will be dealing with – which include, but are not limited to, individuational ontologies – do not all consist of sentences of a formal language. We can therefore not immediately bring to bear the concept of entailment defined in §3.2, but instead have to presuppose that the respective ontologies can be translated into classes of formulas (in the sense of ‘formula’ given in §3.1). In the case of an individuational ontology O , this translation can of course not be performed sentence-by-sentence. Instead, the class of formulas that appropriately translates O will be the union of two classes C_1 and C_2 , where C_1 is the class of existence-claims that are ‘generated’ from the (possibly empty) set of O ’s individuational specifications in accordance with the semantic principle given in §4.7, and where C_2 is the class of formulas that translate O ’s ontological restrictions. Further, the existence-claims in the former class should here not be given in the form, ‘Such-and-such a graph has an injective O -decoration’, but rather as a formula that does not mention either graphs or decorations. (Appendix C describes how such formulas may be constructed.) Once we have a class of formulas C that translates the ontology in question, we

may say that the ontology *entails* a given state of affairs s just in case C entails some formula that denotes s , in the sense of ‘entails’ specified in §3.2.

Second, it has to be clarified what it means for an ontology to ‘form a part of another’. On a very straightforward reading, an ontology is a part of another just in case the former, taken as a set of statements, is a subset of the latter. This conception goes roughly in the right direction, but in the end it is all-too literal and restrictive. Instead, I would propose to understand parthood among ontologies along the following lines: If O and O' are two ontologies, then O' is a *part* of O just in case every statement of O' is either identical with some statement of O , or results from some statement of O by restricting the quantifiers in the latter to subclasses of their original domains. In addition, if this latter restriction of quantifiers enables certain simplifications, then those should also be allowed. Thus, suppose that O deals with sets as well as non-sets, while O' deals only with sets, and that the former contains the following statement: ‘No two entities are such that they are both sets and have all their members in common’. Since O' deals only with sets, it may instead contain the following, simplified version of that statement: ‘No two entities have all their members in common’. This simplification should not prevent O' from counting as a part of O .

In the following section, I shall now first revise the present definition of ‘explanatorily optimal’, as given by (EO₀). Then, in §6.3, I will try to revise the definition of ‘comprehensive’ that is given by (Cp₀). Unfortunately, we will see that this revision runs into some considerable difficulties, which call for a fundamentally different approach. I will develop such an alternative approach in the next chapter.

6.2 Explanatory Optimality Revised

Consider a hypothetical universe in which a particular property – say, the property of being orange – is distributed fairly haphazardly. We would not usually want to say that such a property is essential to all its bearers. But if an ‘optimal’ ontology is taken to be an *explanatorily* optimal ontology, and if explanatory optimality is conceived of in accordance with (EO₀),

then, on the account of essentiality sketched in §5.4 (p. 93), exactly that will be the result. For, according to (EO₀), an ontology is explanatorily optimal only if it is an individuatinal ontology that is both true and comprehensive. In order to be comprehensive, the ontology will have to entail all the facts about the distribution of the mentioned property. However, since it is an individuatinal ontology, it cannot entail those facts simply by stating them: after all, each one of its statements is either an individuating specification or an ontological restriction. Individuating specifications all have the fixed format specified in §4.3, and ontological restrictions are prohibited from containing existential quantifiers (with the exception of those quantifiers that range over objects of the ‘background ontology’; see p. 70 above). But it is clearly impossible to state how a particular property is distributed without making an existence claim, either to the effect that there are some things that have that property, or to the effect that there are some things that lack it. Consequently, the distribution of that property cannot be stated using ontological restrictions alone.

In order to be comprehensive, then, the ontology will have to contain individuating specifications that enable the ontology to entail either that some entities have, or that some entities do not have, the property in question. The most straightforward way to do so is of course to make direct reference to that property. Thus, if the property is that of being orange (say), then the relevant individuating specification might look as follows:

(*) Properties of the form ‘ λx orange $\langle x \rangle$ ’ are individuating.

An ontology O that contains this specification will thereby entail *both* that there are orange entities and that there are non-orange entities; for if P is the predicate derived from this specification (viz., the unary predicate ‘ λx orange $\langle x \rangle$ ’, since the specification does not contain any meta-variables), then there will be O -graphs with nodes that have P -edges, as well as O -graphs with nodes that do *not* have P -edges. O will accordingly entail, unless this is prevented by its ontological restrictions, that some entities are orange, and also that some entities are not orange. However, as a result of the fact that (*) is one of O ’s individuating specifications, the property of being orange will be O -essential to all orange entities, and the property of being

non-orange will be O -essential to all non-orange entities. Hence, if O is explanatorily optimal (and if optimality is identified with *explanatory* optimality), then, on the account currently under consideration, all orange entities will be essentially orange, and all non-orange entities will be essentially non-orange. Since this point generalizes to all kinds of other properties, the account thus appears to be excessively liberal.

The most natural way to avoid this consequence seems to be to revise the present conception of explanatory optimality by widening the field of competing ontologies. In particular, let us say that the true and comprehensive ontologies among which the simplest are selected as ‘explanatorily optimal’ include both individuational and *hybrid* ontologies. A hybrid ontology can here be thought of as consisting of (i) an individuational ontology and (ii) at least one further statement, in a first- or higher-order language, that is neither an individuational specification nor an ontological restriction.⁴ For any hybrid ontology, the class of those statements that fall under (i) may be called the ontology’s ‘individuational part’, while the others may be said to constitute its ‘non-individuational part’. To avoid giving an unfair advantage to hybrid ontologies, the statements that make up the latter part must be required to be formulated in a certain austere vocabulary; for otherwise a very simple statement would be enough to entail all the facts that are entailed by an arbitrarily complex individuational ontology.⁵ The question of how exactly this notion of an austere vocabulary should be spelled out can for now be left aside, but I will return to it in §7.2 below.

With the concept of a hybrid ontology in place, the previous chapter’s definitions of O -individuation, O -essence, and O -essentiality can be straightforwardly generalized so as to allow the respective ontology O to be either individuational or hybrid. It is in this more general sense that those notions will be understood in the rest of this chapter. The above conception of explanatory optimality can then be revised as follows:

⁴More particularly, the language mentioned under (ii) should be such that its sentences admit of being interpreted in isolation from each other, rather than having to be interpreted ‘holistically’, as in the case of individuational specifications.

⁵The difficulty here is basically the same as that which has been noted by Lewis (1983) with regard to his original account of lawhood.

(EO) An ontology O is *explanatorily optimal* just in case it forms a part of a universal (and possibly hybrid) ontology that is as simple as possible while still being both true and comprehensive.⁶

As a result of this revision, an ontology that contains certain individuating specifications will not count as explanatorily optimal as long as there is still another (hybrid) ontology that, while not containing those specifications, is both true and comprehensive, and simpler than the former. Or in other words: if the constraints of truth and comprehensiveness can be more simply satisfied by an ontology that does not count certain sorts of properties as individuating, then an explanatorily optimal ontology will not count them as such. This obviously has consequences for the question of what properties will count as O -essences for some explanatorily optimal ontology O . Very roughly, there will tend to be fewer such properties, and the resulting account of essentiality will be less liberal, as desired.

It is important to note in this connection that the revision that turns (EO₀) into (EO) is not a merely technical maneuver designed to avoid a certain unwelcome consequence, for (EO) is certainly capable of being given a philosophical rationale. That rationale takes the form of a view as to the conditions under which entities have essences. In a nutshell, the view is to the effect that entities have essences only if the individuating approach yields a simpler ontology than the non-individuating alternative. When we apply the individuating approach, we are trying to find a certain sort of pattern in the world. According to the view in question, such a pattern deserves to be regarded as ‘real’ (and an entity’s O -essential properties deserve to be regarded as essential to it) only in such cases where the approach offers an advantage in terms of simplicity.

6.3 The Problem of Defining ‘Comprehensive’

Now, however, we have to turn to a revision of the above account of comprehensiveness. According to (Cp₀), an ontology is comprehensive just in case it entails all the facts. This raises

⁶For the relevant sense of ‘universal’, see p. 99 above.

an obvious problem in connection with ‘particular’ facts (i.e., facts that are instantiations of properties by individual entities) and the size of comprehensive ontologies. For if a comprehensive ontology has to entail *all* the facts, then it will have to entail, for every given entity x , the fact that x exists, the fact that x is identical with x , etc. But it could not entail these latter facts unless it contains a name for x . For example, if x is the empty set, then, in order for an ontology to entail that x exists, it will not be enough for the ontology to entail merely that there exists a (unique) empty set. Rather, the ontology will have to contain some name for the empty set, such as ‘ \emptyset ’, and then entail, e.g., the formula ‘ $\exists x (x = \emptyset)$ ’. And similarly for all other entities. So, for each entity, a comprehensive ontology would have to contain a statement that asserts that that particular entity exists or has such-and-such other properties. This would have the unwelcome consequence that (on the assumption that classical set-theory is correct) there are no comprehensive ontologies; for, counting only the pure sets, there are more entities than any set-sized cardinality, and ontologies, as well as the statements they contain, are only set-sized.

In order to avoid this problem, it is clear that we have to widen the concept of a comprehensive ontology. A natural proposal in this direction might be stated as follows:

(Cp₁) An ontology is *comprehensive* if and only if it entails all purely qualitative facts.

A state of affairs may here be called *purely qualitative* just in case it is not an instantiation of a property by an entity whose existence is logically contingent, or more formally: just in case there exists no property P and no entity x such that (i) the existence of x is logically contingent and (ii) the state of affairs in question is identical with $P\langle x \rangle$.⁷ For an example of a state of affairs that is *not* purely qualitative, consider the state of affairs that John loves Mary. This is identical with, e.g., the instantiation of the property $\lambda x \text{ loves}\langle x, \text{Mary} \rangle$ by John, and John is a contingent (i.e., not necessarily existing) entity. The state of affairs in question is thus not purely qualitative. By contrast, the state of affairs that there is at least one horse, $\exists x \text{ horse}\langle x \rangle$, is purely qualitative, because there is no property P and no contingent entity x

⁷By contrast, if there *is* such a property P and entity x , we may say that the state of affairs in question *involves* x .

such that this state of affairs is identical with $P\langle x \rangle$. There are, however, properties P such that $\exists x \text{ horse}\langle x \rangle$ is identical with $P\langle \text{I} \rangle$, where I is the (necessarily existing) identity relation: one such property is $\lambda x (x = x \wedge \exists y \text{ horse}\langle y \rangle)$, i.e., the property of being self-identical and such that there is at least one horse. For, by the principle (S1) (p. 50), the state of affairs $\exists x \text{ horse}\langle x \rangle$ is identical with $(\text{I} = \text{I} \wedge \exists y \text{ horse}\langle y \rangle)$. Hence the need for the term ‘contingent’ in the above definition.

Unfortunately, the conception of comprehensiveness formulated in (Cp_1) is still not adequate to the role that this notion plays in the present account of essentiality. Consider again the basic idea underlying the account of explanatory optimality at which we have arrived in the previous subsection: entities have essences only if the individuation approach yields a simpler ontology than its non-individuation rivals. ‘A simpler ontology’ means here in particular: a simpler ontology that is both true and comprehensive. But now, if the term ‘comprehensive’ is understood in the sense of (Cp_1) , then individuation ontologies, in order to be true and comprehensive, will inevitably have to contain statements that also form part of a comprehensive *non-individuation* ontology. As a result, the question of whether a set has its members essentially will on the present account turn on comparisons that, intuitively, seem irrelevant to that question.

To see the problem, consider what statements an individuation ontology O would have to contain in order to be both true and comprehensive; and for the sake of the example, let us ignore all those facts that don’t concern the instantiation of set-membership. In order to entail the existence of various kinds of sets, the ontology would need to contain an individuation specification. A very natural choice would be σ^\exists , i.e., the specification according to which properties of the form ‘ $\lambda x (\alpha \in x)$ ’ are individuating. If O contains this specification, it will entail (unless its ontological restrictions prevent this) a plethora of existence claims: e.g., it will entail that there are memberless entities, that there are entities that have exactly one member, that there are entities with infinitely many members, etc. In effect, O will entail the existence of the entire set-theoretic hierarchy, including the non-well-founded sets. Of course, if there should happen to be no non-well-founded sets, then, in order for O not to be false,

it will have to contain some version of the Foundation Axiom as an ontological restriction. Moreover, if the universe does not in fact contain a proper class of memberless entities, then O will have to contain an ontological restriction to the effect that there are at most so-and-so many memberless entities, for otherwise O will entail that there are κ -many memberless entities for every set-sized cardinality κ . And finally, O will have to contain some restriction to the effect that, at least for non-empty well-founded sets, no two of them have exactly the same members. To summarize: if O contains σ^\exists , then, in order not lapse into falsehood, it will also have to contain up to three ontological restrictions, which we may respectively label ‘Foundation’, ‘Limitation’, and ‘Extensionality’.⁸

Now, if O is to be not only true but also *comprehensive*, then it will have to contain even further statements. For instance, it is presumably a purely qualitative fact that every entity has a singleton, and also that for any two entities, there exists a set that contains only them. Although, as a result of containing σ^\exists , O entails the existence of singletons and pair sets, it does not entail the two facts just mentioned. The reason is that those facts are so-to-speak universally quantified: they are states of affairs to the effect that *every* entity (or every two entities, etc.) meets such-and-such a condition. It is clear that O cannot entail such facts merely by virtue of containing σ^\exists . Rather, some additional statement is called for. For example, in order to entail that for every two entities x and y there is a set $\{x, y\}$, O would have to contain something like the classical Axiom of Pairing. The same point could now be repeated for all the other axioms and axiom schemata of ZFC that start with a universal quantifier, and so O will also have to contain some version of, respectively, Separation, Union, Powerset, Replacement, and Choice, or some equivalent set of restrictions (Foundation and Extensionality have already been mentioned).⁹ Since many of these fail to

⁸I say “up to three” because two of the three restrictions just mentioned are needed only under certain conditions, and it is not clear whether those conditions are met. (To wit: Foundation is needed only if there are no non-well-founded sets, and Limitation is needed only if the universe does not contain a proper class of memberless entities.) But O may certainly have to incorporate still further ontological restrictions if it contains any individuating specifications besides σ^\exists .

⁹I should note, however, that all these restrictions are needed only in the presence of urelements. For if we were instead assuming that all there is are pure sets, then the Axiom of Extensionality (or some variant

satisfy the requirements for being an ontological restriction, it follows that O will have to be (contrary to our initial supposition) not an individuational but rather a *hybrid* ontology.

The only classical axioms that do not have to be included in O are those that do not start with a universal quantifier, such as the Axiom of Infinity (“There exists an infinite set”) and large-cardinal axioms. The Axiom of Infinity does not have to be included because, thanks to σ^\exists , O already entails that there is an infinite set (assuming that this is not ruled out by its ontological restrictions), and the same holds for large-cardinal axioms.

In order to be comprehensive, then, O will have to contain nearly all the same statements that would have to be included in an ontology that contains no individuating specifications at all, and the only difference is that the latter would have to contain, instead of σ^\exists , the Axiom of Infinity or some stronger large-cardinal axiom. Now it is commonly thought that no single large-cardinal axiom is strong enough (even when combined with the other axioms) to capture the full richness of the set-theoretic universe, and in that case, no finite ontology without individuating specifications can hope to be comprehensive. This point may seem to favor quite strongly the individuational approach, because an individuational ontology is not subject to the same limitations. The main reason for this lies in the difference between the conceptions of entailment that respectively apply to individuational ontologies and mere sets of axioms. A set of axioms is understood to entail only what holds in each one of its models, however small. By contrast, an individuational ontology O entails, within the bounds set by its ontological restrictions, every existence claim that corresponds to an O -graph, and the question of what O -graphs there are depends on the *actual* richness of the set-theoretic universe. Due to this difference, the mentioned advantage of the individuational approach might be considered ‘unfair’, but that advantage is arguably not relevant for our present

of it, if non-well-founded sets are admitted) would guarantee that no two sets have the same set-theoretic ‘structure’. As a result, all the axioms just listed would, if combined with (e.g.) the Axiom of Infinity, have only the effect of entailing the existence of a plethora of different set-theoretic ‘structures’, and just the same effect is achieved by σ^\exists . To be sure, σ^\exists would not achieve this effect if it were not for the richness of the universe of sets, on which the present framework relies for its supply of graphs and formulas. But the present point is only that the individuational ontology O need not contain any other statements than σ^\exists in order to entail the existence of that plethora of structures.

discussion. For even if the set-theoretic universe were so small that the Axiom of Infinity were already sufficient, with no need for any large-cardinal axioms, our intuitions about essentiality would presumably be the same.

If no large-cardinal axioms were needed, would the individuation approach still have an advantage in terms of simplicity? As we have seen, an ontology O that contains σ^\exists (or any other individuating specifications, for that matter) will, in order to be both true and comprehensive, have to contain several further statements as well. As a result, an ontology O' that is also true and comprehensive, but devoid of individuating specifications, could be constructed from O by simply replacing σ^\exists by the Axiom of Infinity (again assuming that the universe is small enough, and ignoring the other individuating specifications that O may contain). The question of whether O is simpler than O' – and hence the question of whether O is explanatorily optimal – would thus boil down to the question of whether σ^\exists is simpler than the Axiom of Infinity. But intuitively, this is not at all the sort of question on which the essentiality of membership should depend.

The broad idea that questions of essentiality should have something to do with questions of simplicity does seem to have some intuitive appeal – at least if essentiality is a matter of real definition, real definition a matter of canonical description, and the canonicity of descriptions a matter of their simplicity. (Or again, if essentiality is a matter of best explanation, and explanations are *ceteris paribus* better if they are simpler.) The central question is what exactly those descriptions are whose simplicity is supposed to be maximized. According to the account being developed here, the descriptions in question are individuation or hybrid ontologies. There is indeed reason to hope that, among such ontologies, some criterion of simplicity might single out those that yield intuitively acceptable answers to the question of what properties are essential to a given entity. For the ontology that contains only σ^\exists (plus a small number of ontological restrictions to keep it from falsehood) is evidently quite simple, and nevertheless achieves rather a lot, given that it entails existence claims for all kinds of different sets. And moreover, it does justice to the intuition that sets have their members essentially, since the property of having a certain entity as a member is, relative to such an

ontology, essential to any set that contains that entity.

In the light of this consideration, it might seem that the present approach is still largely on the right track, and that we have so far erred only in adopting an all-too demanding concept of comprehensiveness, namely one that requires of ontologies that they entail existentially and universally quantified facts alike. As we have seen above, this has had the effect that the ease with which individuation ontologies manage to entail a wealth of existence facts did not really come to bear in the relevant comparisons of simplicity.

The seemingly obvious way to address this problem is to adopt a new concept of comprehensiveness that requires of an ontology only that it should entail all purely qualitative facts that are, in a certain sense, ‘pure existence facts’. Unfortunately, however, it is not at all clear how to make this latter notion more precise. What we need is a concept that, on the one hand, does not apply to such universally quantified states of affairs as the fact that everything is a member of some set:

$$(1) \forall x \exists y (x \in y).$$

On the other hand, we need the concept to apply to at least some of the states of affairs that an individuation ontology can entail merely by virtue of its individuating specifications. A typical example would be the state of affairs that there is at least one memberless entity. But this latter state of affairs evidently necessitates the universally quantified fact that for every entity x , there is an entity of which x is not a member:

$$(2) \forall x \exists y (x \notin y).$$

This raises the following question: How can a state of affairs that necessitates the fact denoted by (2) count as a ‘pure existence fact’, when the state of affairs denoted by (1) – and thus also, presumably, any state of affairs that necessitates it – doesn’t? The difference between the two formulas is, after all, only that (2) contains ‘ \notin ’ where (1) contains ‘ \in ’. What can this difference have to do with the question of whether a given state of affairs is a ‘pure existence fact’?

I have to admit that I do not know how this difficulty might be overcome – i.e., how one might frame an acceptable account of what should count as a ‘pure existence fact’, in such a way that (at least some) facts that necessitate the state of affairs denoted by (2) are counted as pure existence facts while those that necessitate the state of affairs denoted by (1) are excluded. If such an account is not forthcoming, the currently envisioned approach has to be abandoned.

6.4 Conclusion

In this chapter, I have considered a fairly straightforward approach to the problem of fleshing out the notion of optimality, according to which an ontology is ‘optimal’ just in case it is as simple as possible while being both true and comprehensive. This approach is attractive in its simplicity. However, in the previous section we have found that it is not obviously possible to explicate the notion of comprehensiveness in such a way that the resulting account of essentiality remains capable of accommodating the intuition that sets have their members essentially. So I shall now develop a fundamentally different approach, for which this sort of difficulty does not arise.

Chapter 7

Systematic Optimality

7.1 Taking a Cue from Lewis

In his *Counterfactuals*, David Lewis has proposed a conception of lawhood according to which

a contingent generalization is a *law of nature* if and only if it appears as a theorem (or axiom) in each of the true deductive systems that achieves a best combination of simplicity and strength. (p. 73)

Strength (or informativeness) of course tends to conflict with simplicity: greater simplicity can often be achieved at the loss of strength, and vice versa. Lewis deliberately leaves open what exactly a “best combination of simplicity and strength” amounts to. To have a convenient term for the goodness of the “combination of simplicity and strength” that a given system achieves, let us refer to this quality as *elegance*. So we may say that, according to Lewis’s proposal, a contingent generalization is a law of nature just in case it appears as a theorem (or axiom) in each deductive system that is maximally elegant among those that are true.¹

Though we may not be able to say much about elegance, the following basic principle seems intuitively hard to deny:

¹Very similar accounts of lawhood can also be attributed to Mill (1882, bk. 3, ch. 4) and Ramsey (1928), but Lewis’s proposal is arguably the most detailed of the three. Also cf. Lewis (1986b, p. 122–4) and (1994). The many critics of Lewis’s approach include Tooley (1977), Armstrong (1983), Carroll (1990; 1994), Maudlin (2007), Bird (2007), and Roberts (2008). I am here not able to defend Lewis against these critics, but perhaps, even if his account of lawhood should fail, a roughly analogous account of essence can claim some plausibility.

(El) For any two deductive systems S and S' : if S entails more facts than S' , but is no more complex than the latter, then S is more elegant than S' .

In an important later paper (1983), Lewis observed that this fact leads to a problem for his proposal. For we can axiomatize any system S by the simple statement ‘ $\forall xFx$ ’ (in Lewis’s notation), where ‘ F ’ is an atomic predicate “that applies to all and only things at worlds where S holds” (p. 367). If such predicates are allowed, any amount of strength can be achieved at extremely modest cost in terms of complexity. So the strongest true system, which omits no truths whatsoever, will at the same time be the most elegant. But in that case, every truth whatsoever will on this proposal count as a law.

Lewis’s solution to this problem was to introduce an additional constraint (besides truth) under which elegance is to be maximized. In particular, he proposed that we “let the primitive vocabulary that appears in the axioms refer only to perfectly natural properties” (p. 367f.). In Lewis’s system, properties are simply sets of possible entities, most of which exist in non-actual possible worlds. Some of these properties differ from all others in being ‘perfectly natural’, and this special status is thought to render them suitable for various philosophically important roles, such as that of the referents of the primitive vocabulary in which natural laws should be formulated. Lewis did not take a stance on the question of what exactly it means for a property to be perfectly natural, and was prepared to accept this notion as primitive.²

Now, Lewis’s best-system account of lawhood may at first not seem very relevant to the project of identifying the conditions that an ontology O has to satisfy in order for O -essences to deserve being called essences. However, I think that it is, in fact, a promising alternative to the approach taken in the previous chapter to construct an analogue of his account. The basic motivating idea here is as follows: Just as physical possibility *de dicto* may be analyzed as conformity with the laws of nature, so metaphysical possibility *de dicto* may be thought of as conformity with the ‘laws of metaphysics’ (whatever these may be). But in order to have

²He tended, however, (e.g., in his 1986a, p. 64) to offer two ways in which naturalness may be reduced to other notions: one being based on Armstrong’s (1978) theory of sparse universals, the other on a theory of tropes à la Williams (1966).

a full account of metaphysical possibility, we would still have to account for metaphysical possibility *de re*; and this is where one may naturally expect that the notion of essence will prove helpful. Essences and laws may thus be envisioned as playing complementary roles in a full account of metaphysical modality, each helping to account for one aspect of it. Given the attractiveness of a Lewisian approach to laws, the thought then suggests itself that a roughly analogous account might be given for essences.

To have a name for the condition (or conditions) that this approach will lead us to identify, let us label it *systematic optimality*. If we then further say that an ontology is optimal just in case it is *systematically* optimal, then the account outlined in §5.4 may be rephrased as follows:

- (i) A property is an (abstract or concretized) essence just in case it is an (abstract or concretized) *O*-essence, for some systematically optimal ontology *O*.
- (ii) For any entity *x*, a property is essential to *x* just in case it is *O*-essential to *x*, for some systematically optimal ontology *O*.
- (iii) For any two entities *x* and *y*, *x* is individuated by *y* just in case *x* is *O*-individuated by *y*, for some systematically optimal ontology *O*.

The task for the rest of this chapter will be to arrive at an adequate account of systematic optimality. By way of a first approximation, let us begin with a relatively close analogue of Lewis's conception of a best system:

- (SO₀) An ontology *O* is *systematically optimal* if and only if it is as elegant as possible under the following two constraints: first, it has to be true, and second, all its atomic predicates should denote *basic attributes*.

This calls for two clarificatory remarks. First, I speak of *basic* rather than of 'perfectly natural' attributes because the notion of a basic attribute – which will be clarified in the next section – seems to me in the present context a preferable alternative to Lewis's notion of

perfect naturalness.³ Second, the requirement that is here formulated concerning the respective ontology’s atomic predicates should not be extended to the material that surrounds the schematic λ -expression in an individuating specification (since that material is “essentially boilerplate”⁴).

An important feature of the present conception of systematic optimality lies in the fact that it involves a trade-off between informativeness and simplicity: in order to be systematically optimal, an ontology need not entail a given class of facts if the corresponding gain in informativeness would require a disproportionate increase in complexity. This trade-off will be in place regardless of whether the ontologies in question are required to be individualational or allowed to be hybrid. In particular, even if the maximization of elegance that (SO₀) refers to is restricted in such a way that *only individualational* ontologies are taken into account, the following effect will still manifest itself: If, in order to entail a certain class of facts, an individualational ontology would have to contain a certain individuating specification that in turn requires the inclusion of numerous (and/or extremely complex) ontological restrictions, then a systematically optimal ontology would be one that does *not* include that specification.

³In an interesting discussion of Lewis’s account of lawhood, Barry Loewer (2007) has pointed out that the problem posed by ‘ $\forall x Fx$ ’ need not necessarily be solved by appeal to some ‘elite’ class of attributes. For one could instead say that the best system is not one that offers the best balance between simplicity and strength (in Lewis’s sense), but rather one that offers the best balance between simplicity and *informativeness*, where a system’s informativeness is given, roughly, by the amount of information that can actually be read off that system without going into the semantics of its atomic predicates. On this understanding of ‘informative’, the single statement ‘ $\forall x Fx$ ’ scores indeed quite poorly, precisely because ‘ F ’ is an atomic predicate; and so we have the desired result that that statement will no longer count as a best system.

Loewer’s proposal deserves more extensive discussion than I can here provide. It seems to me, however, that Lewis’s reaction to the mentioned problem – viz., to appeal to an ‘elite’ class of properties – is ultimately to be preferred. For in contrast to Lewis, Loewer’s approach does not penalize systems that are formulated in terms of extremely complex and (as Lewis might call them) gerrymandered attributes. This conceivably allows for the possibility that, by careful selection of such attributes, one could construct a system that offers an exceptionally good balance between simplicity and informativeness (where ‘informativeness’ is understood in Loewer’s sense). But because this system would by hypothesis be formulated in terms of extremely gerrymandered attributes, it would fail to ‘carve nature at its joints’: the relative syntactic simplicity of the axioms and theorems of the system would hide the metaphysical complexity of the attributes that its atomic predicates refer to. On the face of it, this makes them poor candidates for the role of laws. For, arguably, we strive for simple laws for more or less the same reason that we strive for simple explanations; and what we are interested in, when it comes to explanations, seems to be not merely syntactic but *metaphysical* simplicity. In other words, I take it that (holding the vocabulary fixed) we favor simpler explanations over more complex ones because *the world itself* can be less complicated and still render them true.

⁴See above, p. 68n.

Recall that we tried to achieve an analogous effect in §6.2 by allowing the ontologies in question to be hybrid. For if the ‘cost’, in terms of complexity, of including a certain individuating specification is too high, then an explanatorily optimal ontology will lack that specification, because it can still achieve comprehensiveness by non-individual means. (At least, that was the idea; but as we have seen, it is far from clear how the notion of comprehensiveness can be defined.) *Systematically* optimal ontologies, by contrast, will lack the specification in question because they will trade informativeness for simplicity.

Due to this fact, we are here not under the same pressure as we were in §6.2 to take into account both individual and hybrid ontologies. So it is, at least *prima facie*, open to us to choose whether hybrid ontologies should be taken into account, or whether the ontologies in question should all be required to be individual. My own inclination is in favor of this latter solution, though I don’t see any conclusive grounds on which to rule out the former. So, at least provisionally, I would propose that we understand the notion of systematic optimality in such a way that the maximization of elegance ranges only over individual ontologies. In effect, this amounts to adopting an additional constraint under which elegance is to be maximized: the ontologies in question now have to be not only true and such that all their atomic predicates denote basic attributes, but they also have to be individual (rather than hybrid). As we will see in §7.3 and later sections, this account still has to be revised quite substantially. But before we come to that, it will first have to be clarified what it means for an attribute to be basic.

7.2 Basic Attributes

7.2.1 Preliminaries

By way of a preliminary clarification, one could say that an attribute is *basic* just in case it is not constructed from other attributes. In the context of the present framework, this might be understood as saying that a basic attribute is one that is not derived, by way of λ -abstraction,

from a complex state of affairs, where a state of affairs counts as ‘complex’ if and only if it is built up from other states of affairs by way of negation, conjunction, existential quantification, or some combination thereof. Thus, for any properties P and Q , the properties $\lambda x \neg Q\langle x \rangle$, $\lambda x (P\langle x \rangle \wedge Q\langle x \rangle)$, and $\lambda x \exists y (P\langle x \rangle \wedge Q\langle y \rangle)$, for example, all fail to be basic. The hope behind this approach is that there will be some attributes that are not constructed in any such way, and those will be the basic ones.⁵

In the present framework, however, that hope is short-lived. For, according to the principles of chapters 2 and 3, any property P is identical with $\lambda x \neg Q\langle x \rangle$, where Q is $\lambda x \neg P\langle x \rangle$. Also, any property P is identical with $\lambda x (P\langle x \rangle \wedge Q\langle x \rangle)$, where Q is $\lambda x (x = x)$. And finally, any property P is identical with $\lambda x \exists y (P\langle x \rangle \wedge Q\langle y \rangle)$, where Q is, e.g., $\lambda x (x = P)$. So, on the proposal hinted at in the previous paragraph, no property (and similarly, no relation) will count as basic.

There are various possible responses to this difficulty. One option might be to give up on providing any reductive account of basicity, and to accept as ‘brute’ any fact as to whether a given attribute is basic. But this approach has a number of disadvantages. For instance, it would be mysterious why certain attributes that we would intuitively regard as non-basic, such as $\lambda x \exists y, z (y \in x \wedge z \in x \wedge y \neq z)$ (i.e., the property of having at least two members), do indeed fail to be basic. If their failure to be basic does not follow from some account of basicity, how are we supposed to decide that question?⁶ Also, one would have to wonder just why this special status of basicity should render basic attributes suitable for philosophically

⁵Cf. Lewis’s characterization of ‘fundamental’ properties in his (2009):

Fundamental properties are those properties that I have elsewhere called ‘perfectly natural’. They are not at all disjunctive, or determinable, or negative. They render their instances perfectly similar in some respect. They are intrinsic; and all other intrinsic properties supervene on them. They are not conjunctive or structural. (p. 204)

If we regard determinable properties as a subclass of those that involve existential quantification (taking, e.g., the property of being colored to be identical with the property of having *some* color), then what is missing from the present preliminary characterization seems to be only the reference to similarity and intrinsicity.

⁶Much the same epistemological worries could also be raised against Ted Sider’s (2011) recent proposal to regard a notion of joint-carving, or ‘structure’, as primitive. To be sure, Sider has a response to such worries. He says that he can offer “a broadly Quinean approach to the epistemology of metaphysics, and to the epistemology of joint-carving in particular” (p. 97). In his description of this Quinean approach earlier in the book, the key passage reads as follows:

interesting roles – in particular, why they should be given any prominent role in an account of essentiality.

A much more promising possibility would be to erect the present framework of attributes and states of affairs on the basis of a sparse ontology of tropes or universals. For instance, if we had at our disposal a sparse ontology of universals, then we could combine this with the resources of set theory to reconstruct all the attributes and states of affairs of the present framework as set-theoretic constructions out of universals and first-order entities, with the exception of those attributes that are simply *identified* with universals.⁷ An attribute would then be *basic* just in case it is either a universal or identified with a universal by the principles of our framework. Thus, if P is a universal, then P and $\lambda x \neg\neg P\langle x \rangle$ (the latter of which would be a set-theoretic construction with P as a constituent) would both count as basic, since $\lambda x \neg\neg P\langle x \rangle$ is under the principles of chapters 2 and 3 to be treated as identical with P .⁸

I confess that I am broadly sympathetic with this latter proposal, although it is not very clear that it can avoid all the disadvantages of the non-reductionist approach. In particular, it is not clear how the proposal would bear out some of the things that seem intuitively obvious, as for instance that the aforementioned property $\lambda x \exists y, z (y \in x \wedge z \in x \wedge y \neq z)$ is not basic. For it is not clear what sort of principle would allow us to say that this property is not

Quine's advice for forming *ontological* beliefs is familiar: believe the ontology of your best theory. [...] [B]ut a believer in structure can say more. A good theory isn't merely likely to be *true*. Its ideology is also likely to carve at the joints. For the conceptual decisions made in adopting that theory—and not just the theory's ontology—were vindicated; those conceptual decisions also took part in a theoretical success, and also inherit a borrowed luster. So we can add to the Quinean advice: regard the ideology of your best theory as carving at the joints. We have defeasible reason to believe that the decisions of successful theories correspond to something real: reality's structure. (p. 12)

Sider suggests here that successful theories should not only be taken to be true, but that we also have defeasible warrant for taking them to carve at the joints. But given that the notion of joint-carving is supposed to be primitive, it is simply not clear what reason we could have to believe that this particular epistemological claim (viz., that successful theories can reasonably be taken to carve at the joints) is true.

⁷In order for the reconstruction to work, there would also have to be universals corresponding to the relations of identity and set-membership. The assumption that there are such relations, or that there are universals that could play their role, is admittedly a highly controversial one, but it is also not easily dispensed with.

⁸Of course, since P would not strictly speaking *be* identical with $\lambda x \neg\neg P\langle x \rangle$, the talk of identity in (e.g.) the principle (S1) (p. 50) would have to be understood as correspondingly loose.

a universal.

Another reason why I think that this approach would be inadequate stems from the fact that, with the notion of a universal, it relies on metaphysical resources that are simply not needed for the present purposes. As we will see, it is possible to formulate a perfectly useful account of basicity within the comparatively leaner framework of attributes and states of affairs that we have been working with so far. So there will be no need to decide whether the present framework should rest on a concept of universal, or of trope, or on whatever other concept might be employed for this sort of foundational work.

The account of basicity that I would here like to propose makes use of λ -expressions as the principal means of referring to attributes, and also as a means of *describing* attributes, while staying neutral with regard to the particular relations by which attributes are related to their respective constituents.⁹ However, since λ -expressions and the formulas that occur in them occupy such a central role in the to-be-proposed account of basicity, and since I take the concept of basicity to be a purely metaphysical notion, it is clear that the account should not depend on any idiosyncrasies of the language in which the respective formulas and λ -expressions are constructed. For instance, the account should not depend on the richness of the language's basic vocabulary.

One way to ensure that the account will be independent of such linguistic factors is to make use of universal quantification over both languages and interpretations, as well as over variable-assignments (which will be needed since we allow formulas and λ -expressions to contain free variables). Another way would be to specify in advance that the λ -expressions and formulas that are quantified over in the account are meant to be constructed in a single powerful language \mathcal{L}^* , interpreted according to an interpretation I^* . In the present case, as will become clearer below, \mathcal{L}^* and I^* would have to be such that, for every single attribute, \mathcal{L}^* contains

⁹To clarify what is here meant by 'staying neutral', let us suppose, e.g., that a particular property is referred to as ' $\lambda x \exists y R(x, y)$ '. This expression evidently conveys some information about the denoted property: among other things, one may infer that the relation R is, in a certain intuitive sense, one of the property's constituents. But even so, the expression in question does not carry any commitment to any particular view as to the *exact nature* of the relation between that property and R . (Cf. §3.4 above.)

an atomic term (i.e., a constant or variable) to which I^* assigns that attribute as a referent. Given our abundant ontology of attributes and states of affairs, this means that I^* would have to assign proper-class many referents to equally many atomic terms. This might be achieved by construing \mathcal{L}^* as a Lagadonian language in which every attribute is treated as an atomic name of itself. Rather than to go to these lengths, however, I will here adopt an approach that is intermediate between the two just mentioned. More specifically, I will stipulate that, where the account quantifies over λ -expressions or formulas, these should be thought of as constructed in a certain language that merely contains proper-class many *variables*. The account will then not need to employ universal quantification over languages, interpretations, and variable-assignments. Instead, universal quantification over variable-assignments will be sufficient.¹⁰

7.2.2 An Account of Basicity

Let then \mathcal{L} be some formal language (in the sense of §3.1) that contains a proper class of variables. In particular, for every ordinal α , let \mathcal{L} contain a variable of the form ' x_a ', where ' a ' is replaced by a name of α .¹¹ If α is finite, take the relevant name to be the corresponding arabic numeral, so that \mathcal{L} contains the variables ' x_1 ', ' x_2 ', etc. If α is *not* finite, any name will do, since we will not make use of any subscripts that refer to infinite ordinals. Whenever we will in the following quantify over λ -expressions or formulas, these should be thought of as λ -expressions or formulas in \mathcal{L} . Further, let I be some interpretation of \mathcal{L} . For the present purposes, the details of I will not matter, but whenever a λ -expression will be said to denote something, this denoting should be thought of as relative to I and some variable-assignment. To avoid clutter, the relativization to I will always, and the relativization to a variable-assignment usually, be left implicit.

¹⁰A variable-assignment is a function, construed as a set of ordered pairs, from some of the language's variables to entities. (See above, §2.1f.) Since the language in question will contain proper-class many variables, this means that no variable-assignment will manage to assign a referent to every variable of the language. For the present purposes, this limitation will be harmless.

¹¹Note that all variables, even those with subscripts, count as atomic.

The next step is to introduce the notion of a *development*. Roughly, if L is some λ -expression and g some variable-assignment, then a development of L relative to g is the result of replacing in L zero or more atomic terms by other terms that denote, relative to g , the same entities as the former. If this replacement is done in the right way, then the resulting λ -expression will, by the principles of chapters 2 and 3, denote just the same attribute as the original. However, in order to ensure this result, two rules have to be adhered to.

First, bound variable-occurrences should never be replaced. For example, in constructing a development of the expression ' $\lambda x \exists y (x = y)$ ', no occurrence of either ' x ' or ' y ' should be replaced, since each of those occurrences is bound within that expression: the ' x ' by the ' λ ' and the ' y ' by the ' \exists '. And for similar reasons, when an occurrence of a term t is replaced by an occurrence of another term t' , then the replacing occurrence of t' should not contain any occurrences of variables – nor should it be itself an occurrence of a variable – that would become bound within the resulting λ -expression. For example, in ' $\lambda x (x = a)$ ', the ' a ' should not be replaced by an ' x ' nor by a ' $\lambda y (y = x)$ ', because the respective occurrences of ' x ' would become bound by the initial ' λ '.

In accordance with these remarks, the concept of a development can be defined as follows:

(D) For any λ -expression L and variable-assignment g , an expression L' is a *development of L relative to g* if and only if L' is a λ -expression that results from L by replacing zero or more occurrences of atomic terms by occurrences of other terms in such a way that the following three conditions are satisfied:

- (i) For any terms t and t' : if an occurrence of t is replaced by an occurrence of t' , then t and t' denote, relative to g , the same entity.
- (ii) No replaced occurrence is a bound-variable occurrence.
- (iii) No replacing occurrence o contains, or is itself, a variable-occurrence that is bound by an occurrence of an operator outside of o but within L' .

Note the trivial consequence that every λ -expression is a development of itself, relative to any variable-assignment whatsoever. Further, it can be seen that for any λ -expressions L and L' ,

if L' is a development of L relative to some variable-assignment g , then any development of L' will likewise be a development of L relative to g .

With this concept in place, our account of basicity can be formulated as follows:

- (B) An attribute A is *basic* if and only if, for every variable-assignment g and every formula φ : If, relative to g , the λ -expression $\ulcorner \lambda x_1, x_2, \dots \varphi \urcorner$ denotes A , then this λ -expression has (relative to g) a development whose matrix is equivalent to $\ulcorner F\langle x_1, x_2, \dots \rangle \urcorner$, where F is atomic.¹²

An obvious consequence of (B) is that, for any basic binary relation R , the converse of R is also basic. This follows from straightforward considerations of symmetry. More generally, we can say that, for any basic attribute A , every ‘permutation’ of A is likewise basic. A ‘permutation’ of an attribute is here, informally speaking, the result of permuting the attribute’s argument-places.¹³

Somewhat less obviously, (B) has the consequence that, if A is a basic attribute, then the negation of A – i.e., the attribute $\lambda x_1, x_2, \dots \neg A\langle x_1, x_2, \dots \rangle$ – will also be basic.¹⁴ This con-

¹²As usual, the *matrix* of a λ -expression is simply the formula that follows the list of variables after the ‘ λ ’.

¹³I take the term ‘permutation’, as applied to attributes, from Sider (1993, p. 31). In the present framework, we can define it as follows: If A is a κ -ary attribute and u_1, u_2, \dots are κ -many, pairwise distinct variables, then a *permutation* of A is any attribute that is denoted by a λ -expression of the form

$$\lambda u_1, u_2, \dots P\langle v_1, v_2, \dots \rangle,$$

where P is a term denoting A , and where the v_i are pairwise distinct variables such that $\{u_1, u_2, \dots\} = \{v_1, v_2, \dots\}$.

¹⁴*Proof sketch.* Suppose that A is basic. Further, let F be an atomic term that denotes A , and let G be an atomic term that denotes the negation of A . To reduce clutter, A will be taken to be a unary attribute (i.e., a property). In order to prove that the negation of A is a basic attribute, it suffices to show that, for every variable-assignment g and every formula φ , if the λ -expression $\ulcorner \lambda x \varphi \urcorner$ denotes relative to g the negation of A , then this λ -expression has (relative to g) a development whose matrix is equivalent to $\ulcorner G\langle x \rangle \urcorner$. So let g be some variable-assignment, and let φ be some formula such that the λ -expression $\ulcorner \lambda x \varphi \urcorner$ denotes, relative to g , the negation of A . In addition, let L be this λ -expression; i.e., $L = \ulcorner \lambda x \varphi \urcorner$. The goal will be to show that L has, relative to g , a development whose matrix is equivalent to $\ulcorner G\langle x \rangle \urcorner$. (In the following, relativizations to g will be suppressed for the sake of brevity.)

In a first step, we will establish the following thesis:

- (1) The matrix of L is equivalent to some development of $\ulcorner \neg F\langle x \rangle \urcorner$.

To see this, note that, by the principle (S2) (p. 53), there will have to be some sequence of formulas $\varphi_1, \dots, \varphi_n$ such that $\varphi_1 = \ulcorner \neg F\langle x \rangle \urcorner$, $\varphi_n = \varphi$, and for each i with $0 < i < n$: φ_{i+1} is either equivalent to φ_i or results from φ_i by a uniform substitution of co-referential terms. For each i with $0 < i < n$, if φ_{i+1} results from φ_i by way

sequence marks a strong contrast between the present account of basicity and the Lewisian conception of perfect naturalness. It can be argued, however, that the present concept of basicity is in fact better suited than Lewis's concept of naturalness to fill the role that this latter notion plays in his best-system account of lawhood;¹⁵ and it is to be expected that the same will hold for an account of essence that is constructed in rough analogy to such an account of lawhood. In order to alleviate any worries that the present conception of basicity may be overly liberal, I will now briefly discuss four classes of attributes whose members, at least in typical cases, fail to be basic.¹⁶

of a uniform substitution of terms, then let s_i be that substitution (conceived of as a function from formulas to formulas); otherwise let s_i be the identity function. Without loss of generality, we may assume that the various s_i are substitutions of atomic terms only. Let s^* be the result of composing these substitutions, i.e., $s^* = s_n \circ s_{n-1} \circ \dots \circ s_1$, and let φ^* be the result of applying s^* to $\ulcorner \neg F \langle x \rangle \urcorner$. It can then be shown by induction over n that φ is equivalent to φ^* . But φ^* is a development of $\ulcorner \neg F \langle x \rangle \urcorner$. Hence, the matrix of L , i.e., φ , is equivalent to the matrix of some development of $\ulcorner \lambda x \neg F \langle x \rangle \urcorner$. This completes the argument for (1).

Next, we have to establish the following:

- (2) Every development of $\ulcorner \lambda x \neg F \langle x \rangle \urcorner$ has a development whose matrix is equivalent to $\ulcorner G \langle x \rangle \urcorner$.

This may be seen as follows. First, let L' be an arbitrary development of $\ulcorner \lambda x \neg F \langle x \rangle \urcorner$. By the definition of 'development', L' will be $\ulcorner \lambda x \neg \chi \urcorner$, for some formula χ . Since L' denotes the negation of A , the expression $\ulcorner \lambda x \chi \urcorner$ denotes A itself, and will therefore, by the basicity of A , have a development $\ulcorner \lambda x \psi \urcorner$ whose matrix is equivalent to $\ulcorner F \langle x \rangle \urcorner$. Let now ψ' be the result of replacing in ψ every occurrence of F by $\ulcorner \lambda x \neg G \langle x \rangle \urcorner$: as a result, $\ulcorner \psi' \urcorner$ will be equivalent to $\ulcorner \neg G \langle x \rangle \urcorner$, and $\ulcorner \neg \psi' \urcorner$ will therefore be equivalent to $\ulcorner G \langle x \rangle \urcorner$. Furthermore, by the manner of construction, $\ulcorner \lambda x \neg \psi' \urcorner$ is a development of L' . So L' has a development whose matrix is equivalent to $\ulcorner G \langle x \rangle \urcorner$. This completes the argument for (2).

Recall now from (1) that the matrix of L is equivalent to the matrix of some development L^* of $\ulcorner \lambda x \neg F \langle x \rangle \urcorner$. By (2), L^* has a development D^* whose matrix is equivalent to $\ulcorner G \langle x \rangle \urcorner$. Let s be a substitution of atomic terms that, when applied to L^* , produces D^* . By applying s to L , one obtains a λ -expression whose matrix is equivalent to that of D^* and hence to $\ulcorner G \langle x \rangle \urcorner$. So L has a development whose matrix is equivalent to $\ulcorner G \langle x \rangle \urcorner$, which is what needed to be shown.

¹⁵Very roughly, the argument hinges on the idea that basic attributes are in a metaphysically relevant sense *simple* attributes. I hope to elaborate on this on another occasion.

¹⁶It might at first seem that a simpler account of basicity than the one presented here could be formulated as follows. First, suppose that a κ -ary attribute A is said to be 'pure' just in case the state of affairs $\exists x_1, x_2, \dots A \langle x_1, x_2, \dots \rangle$ is purely qualitative in the sense given in §6.3 above. Second, suppose that a κ -ary attribute A is said to be 'constructed' from the members of a given set S just in case there exists a formula φ and a variable-assignment g such that A is denoted by $\ulcorner \lambda x_1, x_2, \dots \varphi \urcorner$ relative to g , and φ does not contain any atomic terms that (relative to g) denote any entity not in S . It might then be thought that the *basic* attributes can be taken to be exactly those that are (i) pure and (ii) members of a set of attributes that is minimal under the condition that every pure attribute can be constructed from its members. (As usual, a set is 'minimal' under a condition C just in case the set itself satisfies the condition but no proper subset of it does.)

Note, first, that this conception of basicity has, just like the one given in the text, the consequence that for any basic attribute A , the negation of A will also count as basic. But it has still other consequences, some of which seem to me much less palatable. For example, suppose that P is a basic property under the account in question. It stands to reason that the identity relation I is also basic in this sense. Further, suppose that

7.2.3 Examples of Non-Basic Attributes

Compound attributes

Consider the relation that holds between two entities if the first is fond of the second and the second is a dog. This relation may be denoted by the following λ -expression:

$$\lambda x, y (\text{fond-of}\langle x, y \rangle \wedge \text{dog}\langle y \rangle). \quad (1)$$

If this relation, call it ‘ R ’, were basic, then the displayed λ -expression would have a development whose matrix is equivalent to $\lceil F\langle x, y \rangle \rceil$, for some atomic term F . Whether there exists such a development depends in part on the underlying metaphysics of attributes. Thus, suppose that R is denoted by the (somewhat artificial) atomic term ‘fond-of-dog’. If, now, the fond-of relation were *identical* with R , then the term ‘fond-of’ in the above λ -expression could be replaced by ‘fond-of-dog’. And if, in addition, the property of being a dog were nothing else than the property of being self-identical, then (1) would have the following development:

$$\lambda x, y (\text{fond-of-dog}\langle x, y \rangle \wedge \lambda x (x = x)\langle y \rangle),$$

and the matrix of this is straightforwardly equivalent to ‘fond-of-dog $\langle x, y \rangle$ ’. But of course, neither of the two metaphysical suppositions just mentioned is at all plausible. In particular, the property of being a dog is quite clearly not the same as self-identity. But if this and all similar

the set $\{P, I\}$ is minimal under the condition that every pure attribute can be constructed from its members. (If that set is not minimal, add further attributes and modify the following argument accordingly.) Let now Q be the property $\lambda x (P\langle x \rangle \wedge \forall y (y = x))$ (i.e., the property of having P and being such that there exists exactly one entity), and let Q' be the property $\lambda x (P\langle x \rangle \wedge \neg \forall y (y = x))$ (i.e., the property of having P and being such that there exist at least two entities). By the principle (S1) (p. 50), it follows that P is identical to $\lambda x (Q\langle x \rangle \vee Q'\langle x \rangle)$. Therefore, any pure attribute that can be constructed from the members of $\{P, I\}$ can also be constructed from the members of $\{Q, Q', I\}$. Under the proposed conception, then, the properties Q and Q' would both have to count as basic, which seems intuitively unattractive.

One might hope to avoid this consequence by exploiting the fact that $\{Q, Q', I\}$ is larger than $\{P, I\}$; in particular, one might think of modifying the present proposal in such a way that a set is understood to be ‘minimal’ under a given condition C just in case the set itself satisfies C , no proper subset of it does, *and* no smaller set satisfies C , either. However, this solution will not be of much help if – as seems possible – the smallest sets from whose members all pure attributes can be constructed are all infinitely large.

routes are blocked, i.e., if neither ‘fond-of’ nor ‘dog’ can *salva referentia* be replaced with a term that contains, besides logical vocabulary, only the term ‘fond-of-dog’, then there will evidently be no way to develop the original λ -expression into one whose matrix is equivalent to $\ulcorner F\langle x, y \rangle \urcorner$, where F is atomic.

Quantification-involving attributes

Next, let us consider the relation that holds between two entities if the first is R -related to something that in turn has the second entity as a member:

$$\lambda x, y \exists z (R\langle x, z \rangle \wedge y \in z). \quad (2)$$

Again, the question of whether this relation is basic depends entirely on the metaphysics of the relevant attributes. For instance, if R is the identity relation, then (2) will have a development whose matrix is equivalent to the simple formula ‘ $y \in x$ ’. In that case, the whole relation will turn out to be simply the converse of set-membership. And, as has been noted above, the converse of set-membership is basic just in case set-membership itself is basic. On the other hand, if R is not the identity relation, then it will be difficult to see how (2) could plausibly be developed into a λ -expression whose matrix is equivalent to $\ulcorner F\langle x, y \rangle \urcorner$, for any atomic term F .

Reflexive attributes

The property of being R -related to oneself (for some binary relation R) is an example of what might be called ‘reflexive’ attributes. Relative to a suitable variable-assignment, this property will be denoted by

$$\lambda x R\langle x, x \rangle. \quad (3)$$

The property will be basic if, e.g., R is the relation $\lambda x, y (P\langle x \rangle \wedge P\langle y \rangle)$, for some basic property P . For suppose we replace the ‘ R ’ in (3) with $\ulcorner \lambda x, y (F\langle x \rangle \wedge F\langle y \rangle) \urcorner$, where F is an atomic

term denoting P . The resulting development will then be

$$\lambda x (\lambda x, y (F\langle x \rangle \wedge F\langle y \rangle)\langle x, x \rangle),$$

whose matrix is equivalent to $\lceil F\langle x \rangle \rceil$. Hence, the property of being R -related to oneself is in this particular case a basic attribute. Needless to say, the property will also be basic if R is identical to, for instance, $\lambda x, y (P\langle x \rangle \vee P\langle y \rangle)$ or $\lambda x, y (P\langle x \rangle \rightarrow P\langle y \rangle)$, where P is again a basic property. Most relations, however, are not of this sort.

Impure attributes

For our last example, consider the property of being taller than Socrates:

$$\lambda x \text{ taller-than}\langle x, \text{Socrates} \rangle. \tag{4}$$

If the property denoted by this λ -expression is to be basic, the latter will have to have a development whose matrix is equivalent to $\lceil F\langle x \rangle \rceil$, for some atomic term F . As before, there are several ways in which this might turn out to be the case, but none of them (as far as I can see) has any metaphysical plausibility.

Let us here consider one possible way in which (4) may turn out to have a development of the sort in question. To begin with, suppose that the taller-than relation is identical with the relation

$$\lambda x, y (\text{taller-than-Socrates}\langle x \rangle \wedge \neg \text{taller-than-Socrates}\langle y \rangle),$$

where the atomic constant ‘taller-than-Socrates’ denotes the property of being taller than Socrates. Of course, this identification is quite implausible. But if it were correct, then the following would be a development of (4):

$$\lambda x \lambda x, y (\text{taller-than-Socrates}\langle x \rangle \wedge \neg \text{taller-than-Socrates}\langle y \rangle)\langle x, \text{Socrates} \rangle. \tag{5}$$

Suppose now further that both ‘Socrates’ and ‘taller-than-Socrates’ are co-referential with certain terms that are assigned the same referent under every interpretation. In particular, suppose that ‘Socrates’ is co-referential with ‘I’ (which means – implausibly – that Socrates is the identity relation), and that ‘taller-than-Socrates’ is co-referential with ‘ $\lambda x \neg I\langle x, I \rangle$ ’, which denotes the property of being distinct from the identity relation. Under these suppositions, (5) will have a development as follows:

$$\lambda x \lambda x, y (\text{taller-than-Socrates}\langle x \rangle \wedge \neg \lambda x \neg I\langle x, I \rangle\langle y \rangle)\langle x, I \rangle.$$

The matrix of this is equivalent to

$$\text{taller-than-Socrates}\langle x \rangle \wedge \neg \neg I\langle I, I \rangle,$$

which in turn is (since ‘I’ denotes the identity relation under every interpretation) equivalent to

$$\text{taller-than-Socrates}\langle x \rangle.$$

So, under the above suppositions, it does turn out that (4) has a development whose matrix is equivalent to $\ulcorner F\langle x \rangle \urcorner$, where F is atomic. But the metaphysical assumptions that have here been made in order to obtain this outcome are all quite implausible.

7.3 Maximal Elegance?

According to the present account of essentiality, as stated in §7.1, a property is essential to a given entity just in case, for *some* systematically optimal ontology O , the property is O -essential to that entity. This proposal bears an obvious disanalogy to Lewis’s account of lawhood: according to him, a contingent generalization counts as a law only if it is a theorem or axiom in *every* deductive system that is maximally elegant (in the sense suggested on p. 112) among those that are true and formulated in a suitably austere vocabulary. To mark

this difference between the two approaches, one might say that Lewis's account of lawhood *universalizes*, whereas the present account of essentiality *particularizes*. I submit that, at least initially, a particularizing approach seems more natural, both in an account of essentiality and in an account of lawhood. For why should a true generalization not count as a law if it is a theorem or axiom in only *some* maximally elegant system? And why should a property not count as an essence if it is an *O*-essence only for *some* systematically optimal ontology *O*?

As far as the Lewisian account of lawhood is concerned, a possible answer might be that, under a particularizing version of that account, there might be two maximally elegant systems S_1 and S_2 such that (i) there exists a generalization L_1 that is a theorem in S_1 but not in S_2 , (ii) there exists another generalization L_2 that is a theorem in S_2 but not in S_1 , and (iii) there is no maximally elegant system S_3 that contains the *conjunction* of L_1 and L_2 as an axiom or theorem. Then, though both L_1 and L_2 would count as laws, their conjunction wouldn't; and this might be an unwelcome consequence. To be sure, this defect (if it is a defect) could be easily repaired by saying that a law is any sentence that appears in the deductive closure of any *union* of maximally elegant systems, but the added complication would render this option somewhat unattractive.

Another possible answer might be that Lewis's criterion can be naturally adapted to deal with cases in which *there is no* maximally elegant system (even if we only count those that are true and formulated in an austere vocabulary). Such a case would obtain if, for every true system that is formulated in an austere vocabulary, there is another one that is also true and formulated in an austere vocabulary, but more elegant than the former. This poses a problem, because it would certainly be unfortunate if, in such a case, one's account of lawhood had the consequence that there are no laws. Analogously for the present account of essentiality: if, for every maximally elegant ontology (among those that are true and whose atomic predicates all denote basic attributes), there is another one that is more elegant, then our account should certainly not *automatically* have the consequence that nothing has any essential properties.

Lewis can deal with his version of this problem by a slight modification of his criterion of lawhood. In particular, he can say that "a law must appear as a theorem in all sufficiently

good systems”.¹⁷ This looks *prima facie* preferable to the (particularizing) alternative account on which a law must appear as a theorem in *some* “sufficiently good system”. For this latter account would imply that a generalization G would count as a law if it appears as a theorem in some sufficiently elegant system S , even if it does not appear as a theorem in any system that is more elegant than S . In such a case, it would seem natural to say that G should not count as a law after all, and Lewis’s formulation accommodates this intuition.

Arguably, however, his formulation can still be improved. For suppose that there is a system S that does not contain a certain generalization G , and which barely manages to be sufficiently elegant, whereas every system that is more elegant than S does contain G . Then, on Lewis’s proposal, G will not count as a law, but this is counter-intuitive, for G would have counted as a law if the required level of elegance had been set only slightly higher. We might try to remedy this defect by saying that a generalization counts as a law if there exists *some* level of elegance λ such that G appears as a theorem in every system whose elegance exceeds λ . But now suppose that, no matter how high we set λ , there is always some system whose elegance exceeds λ but which does not contain G .¹⁸ Should this prevent G from counting as a law? Presumably not if, for every level λ , there is also a system that does contain G and whose elegance likewise exceeds λ . However, if there is a level λ such that (i) there exists at least one system whose elegance exceeds λ and (ii) *no* such system contains G , then this may plausibly count as a good reason to say that G is not a law. Here we seem to have hit on a passable necessary condition on lawhood. Can this condition also be regarded as sufficient? If so, we can say:

¹⁷*Op. cit.*, p. 73n. Lewis prefaces this proposal by expressing doubt that “our standards of simplicity would permit an infinite ascent of better and better systems”. However, the threat of such an infinite ascent need not exclusively arise from the possibility that we might have some perverse standards of simplicity; it may also come from the universe itself. For it is at least conceivable that (i) no deductive system is large enough to provide a full description of the entire universe, (ii) for every given system that describes a certain part or aspect of the universe, there is another part or aspect that can be described even more elegantly, and (iii) by describing this other part or aspect as well, a system can achieve greater elegance than the former. This admittedly looks like a remote possibility, but arguably it ought still to count as a deficiency if an account of lawhood (or of essence) should be unable to cope with it.

¹⁸For a suggestion of how this might be possible, see the previous footnote.

(L?) A generalization G is a law just in case, for every level of elegance λ : if there exists at least one system whose elegance exceeds λ , then there will also be at least one such system that contains G .

This may at first blush look reasonable enough, but we should still be skeptical. For it seems at least conceivable that, whenever we have a level λ and a system S whose elegance exceeds λ , we could stick any true generalization G into S and go on to improve the resulting system – e.g., by making it more informative – so that the end-product will again be a system whose elegance exceeds λ , but which also contains G . If so, then any generalization whatsoever will under (L?) count as a law.

To prevent this kind of mischief, it is no use theorizing at the level of whole systems; instead, one will have to look at the individual statements. In the light of these considerations, our initial definition (SO₀) (p. 114) appears relatively hopeless, given that it, like Lewis's account of lawhood, relies on the maximization of elegance. I shall therefore now pursue the alternative route of developing a number of conditions that have to be satisfied by the *individual statements* of an ontology in order for the latter to count as systematically optimal.

From the way in which the concept of systematic optimality will be defined in the following, it will become clear that the account of essentiality that is based on it has to be of the particularizing sort. For the definition will have the consequence that there exists a systematically optimal ontology O_0 that contains no individuating specifications whatsoever. Since this ontology contains no individuating specifications, every entity has the same O_0 -individuation graph, which consists of only a single node and no edges. Correspondingly, for any entity x , the fully concretized O_0 -essence of x will be simply the property of being identical with x . So the O_0 -essential entities of x will be limited to the properties whose instantiation by x is necessitated by x 's being self-identical. If, by contrast, the present account of essentiality were of the universalizing sort, so that a property P would be regarded as essential to x just in case it is O -essential to x for *every* systematically optimal ontology O , then x would on this account have no other essential properties than those that are O_0 -essential to it; and this would be a very unwelcome consequence. The upshot of this is that we should stick with the

particularizing account stated in §7.1.

7.4 Paying One's Dues

To return to the task at hand, the present goal is to develop a workable definition of 'systematically optimal', and to do so in such a way that the *definiens* consists, at least in part, of one or more conditions to be imposed on the respective ontology's statements. In order to get our bearings, it may help to begin by conceiving a similar definition of the notion of 'best system' that plays the familiar role in Lewis's account of lawhood.

Such a definition would have to consist, at least in part, of a set of constraints to be satisfied by the respective system's axioms. If we intend this account to be as faithful as possible to the spirit of Lewis's original conception, we will have to impose at least the following three: First, that the axioms should be true, second, that any atomic predicate they contain should denote a perfectly natural attribute, and third, that their respective complexity should be compensated by their respective contribution to the informativeness of the system of which they form a part.

Can we take an analogous route in constructing an account of systematic optimality? The analogy will certainly not be perfect. First of all, given that the statements in question will be statements of an individuation ontology,¹⁹ we cannot simply require that all of them should be true, for individuation specifications can only be evaluated collectively, as part of a whole ontology.²⁰ We will therefore instead have to require that the ontology as a whole should be true. Another disanalogy has to do with Lewis's reliance on his concept of a perfectly natural attribute: For the reasons mentioned in §7.2, I propose that we instead rely on the concept of a *basic* attribute. And finally, the requirement that the statements of an ontology should be formulated in terms of basic attributes can be obviated by a suitable conception of a statement's complexity, as we shall see in a moment.

¹⁹Cf. §7.1 (p. 116) above.

²⁰Cf. §4.7 above.

Of the three constraints listed two paragraphs back, our definition of systematic optimality will, in fact, only preserve the third. To a first approximation, this condition will be turned into the requirement that, for each statement of a given ontology, the statement's complexity should in some sense be compensated by its contribution to the ontology's informativeness. To have a shorter way of expressing this, one might say that each statement will be required to *pay its dues*. In order to clarify what this means, it will have to be specified, first, how to measure a statement's complexity, and second, how to assess the contribution that a statement makes to a given ontology's informativeness.

7.4.1 Informativeness and Complexity

By a statement's complexity, I will here mean the length of the statement's most economical translation into a standard vocabulary that fulfills the following two conditions: (i) each atomic predicate denotes a basic attribute; and (ii) the only operators are the usual quantifiers and truth-functional connectives.²¹ If we assess a statement's complexity in this way, it is clear that there will no longer be any need to require that the statement should itself be formulated in a standard vocabulary.

Next, it has to be specified how to measure the contribution that a statement makes to the informativeness of some ontology of which it forms a part. At first blush, this may seem to be only a technical problem, as if one merely had to find some way of measuring the 'amount of information' that is expressed by a given statement. However, in connection with this there are also some deeper issues that are worth paying attention to.

In assessing how much a particular statement contributes to the informativeness of a given ontology, it may be tempting to measure that contribution as the difference between two values V_1 and V_2 , where V_1 is the informativeness of the whole ontology and V_2 the informativeness that the ontology would have without the statement in question. Moreover, in assessing the

²¹Arguably, however, we should also include reasonably basic 'meta-ontological' vocabulary, which would include such terms as 'graph', 'node', 'edge', 'decoration', etc. (As a rule, if an ontology O talks of 'graphs' and 'decorations', these terms should be understood in the sense of ' O -graph' and ' O -decoration', respectively.)

informativeness of an ontology, it may be tempting to count each bit of information equally. But both of these ideas are problematic. First, if the values V_1 and V_2 are infinite, their difference will not in general be well-defined. Second, it can be argued that, in many cases, not every bit of information is equally relevant to the informativeness of an ontology ‘as an ontology’. Thus, an ontology may contain very detailed statements as to what there *isn’t* – e.g., if it contains statements to the effect that there are no talking swans and no polka-dotted zebras. But given that such statements do not tell us *what there is*, the information they provide should arguably not count toward the ontology’s informativeness *qua* ontology.

In order to address this second problem, I propose that the question of whether a given statement pays its dues should be made to depend on its contribution, not to the respective ontology’s informativeness, but rather to the ontology’s *discriminatory power*, where the latter is measured, very roughly, by the number of properties that the ontology correctly claims to be instantiated. A corollary of this proposal is that we have to focus only on the respective ontology’s individuating specifications, since it is only these that increase the number of properties that the ontology claims to be instantiated. More precisely (and here is where the first problem is addressed), my proposal is that, if σ is an individuating specification of an ontology O , then σ ’s contribution to O ’s discriminatory power can be computed according to the following algorithm:

1. Let O' be the ontology that results from O by deleting σ , and let O^σ be the ontology that contains σ as its only statement.
2. Let S be the set of instantiated abstract O' -essences.
3. For each essence E in S , let M_E be the set of all instantiated properties

$$\lambda x (E\langle x \rangle \wedge F\langle x \rangle),$$

where F is an abstract O^σ -essence.

4. If the cardinality of M_E is finite, let $\delta(E)$ be the cardinality of M_E minus 1;²² otherwise, let $\delta(E)$ be the cardinality of M_E .
5. The contribution of σ to the discriminatory power of O is given by the sum $\sum_{E \in S} \delta(E)$.

In a nutshell, one might say that σ 's contribution to O 's discriminatory power is the summed difference, computed over all instantiated abstract O' -essences, between the number of instantiated properties that are a conjunction of an O' -essence and an abstract O^σ -essence,²³ and the number of instantiated abstract O' -essences.

Although ontological restrictions do not contribute anything to an ontology's discriminatory power, they do add to the respective ontology's complexity; and their presence is needed because they have to filter out false existence claims that would otherwise result from the

²²By subtracting 1, I am taking into account that E itself is already an instantiated O' -essence. For example, if the cardinality of M_E is only 1, then this means that the addition of σ does not produce any additional instantiated O -essences. In such a case, $\delta(E)$ should be zero.

²³The fact that only these conjunctions are counted may seem puzzling, for it might have been thought that we should instead count, to a first approximation, all the instantiated abstract O -essences E' that 'entail' E , in the sense that for every entity x , $E'\langle x \rangle$ necessitates $E\langle x \rangle$. To prevent double counting, this proposal might have been refined in such a way that only all those instantiated abstract O -essences E' are counted that are such that E is the 'strongest' O' -essence entailed by E' , in the sense that (i) for every entity x , $E'\langle x \rangle$ necessitates $E\langle x \rangle$, and (ii) there is no O' -essence E'' distinct from E such that, for every entity x , $E'\langle x \rangle$ necessitates $E''\langle x \rangle$, and $E''\langle x \rangle$ necessitates $E\langle x \rangle$. But even by this latter proposal, we would still have counted far more O -essences than we would on the proposal stated in the text.

Although the proposal just stated may seem very natural, given that it does not leave any O -essences uncounted, it arguably has the disadvantage of leading to an overly liberal account of essentiality. For let O' be an ontology that has σ^\supset (i.e., the statement, 'Properties of the form ' $\lambda x (\alpha \in x)$ ' are individuating') as its only individuating specification, and let σ be some individuating specification that is such that the predicate derived from it, call it ' F ', denotes a property P that is instantiated only by urelements. Further, let O be a true ontology that contains both σ^\supset and σ . For any well-founded set x , consider now the O' -graph of x , and suppose that this graph is modified by adding F -edges to its terminal nodes, subject only to the constraint that (i) the number of nodes to which an F -edge is added should not exceed the number of entities that instantiate P , and (ii) the number of nodes to which *no* F -edge is added should not exceed the number of entities that do not instantiate P . The result of this modification will inevitably be an O -graph that expresses an instantiated abstract O -essence. In particular, if k is the number of terminal nodes of the graph in question, there will be up to 2^k instantiated abstract O -essences (or exactly 2^k , if k is both smaller than the number of entities that instantiate P and smaller than the number of entities that don't instantiate P , and provided that the graph does not exhibit any symmetries with respect to its terminal nodes).

In this way, almost any choice of σ will produce a vast overall increase in the number of instantiated abstract O -essences, as computed for the various instantiated abstract O' -essences. However, this increase should arguably not be attributed to any special virtues on the part of σ , but rather only to the 'promiscuousness' of set-theoretic construction, which allows sets to be constructed from any urelements whatsoever, regardless of the latter's properties. So, in order to prevent the effect just described, I propose to count only those instantiated properties that result from conjoining an abstract instantiated O' -essence with an abstract O^σ -essence.

ontology's individuating specifications. Because the latter often require the addition of ontological restrictions in order to keep the respective ontology from lapsing into falsehood, I would propose that a given specification's *contribution to the complexity of an ontology* should be computed as the sum of two values: first, the specification's 'inherent' complexity (as defined above), and second, the combined complexity of the ontological restrictions whose addition it makes necessary. Further, I would propose that, in assessing whether a given specification pays its dues, we have to consider not its inherent complexity, but rather its contribution to the complexity of a particular ontology. This has to be made more precise in the following.

7.4.2 Paying One's Dues, Relativized and *Simpliciter*

Suppose that O is an individual ontology and σ some individuating specification (not necessarily contained in O), and let O^+ be the simplest true ontology that results from O by adding to it σ as well as whatever ontological restrictions are needed in order to render the resulting ontology true. I will then say that σ *pays its dues relative to O* just in case the contribution that σ makes to the complexity of O^+ is 'outweighed' by its contribution to the discriminatory power of O^+ .

This definition naturally raises the question of how the notion of outweighing should be spelled out. Suppose we write ' $D_{\sigma,O}$ ' to denote σ 's contribution to O 's discriminatory power, and ' $C_{\sigma,O}$ ' to denote σ 's contribution to O 's complexity. On the most straightforward (i.e., linear) approach, there will then be two constants c_0 and c_1 , and $D_{\sigma,O}$ will be said to outweigh $C_{\sigma,O}$ just in case $D_{\sigma,O}$ exceeds the ratio $\frac{C_{\sigma,O}-c_0}{c_1}$. This proposal may not be adequate in cases where $D_{\sigma,O}$ is infinite, since we might not want to allow that an infinite $D_{\sigma,O}$ will outweigh every finite increase of the respective ontology's complexity. But even if we suppose that the linear approach is adequate, we have to face the question of how the parameters c_0 and c_1 are to be determined. There does unfortunately not seem to be any clear, non-arbitrary way of choosing a value for them. Consequently, the parameters could be set to whatever values are deemed appropriate, which introduces a certain subjective element into our definition of

‘systematically optimal’, and thus also into the present account of essentiality. Although the presence of this subjective element is not exactly welcome, I think that it can be tolerated, and also that it *should* be tolerated, at least by adherents of Lewis’s account of lawhood. For if one can accept a subjective element in one’s account of lawhood, it seems that one should also be able to live with such an element in one’s account of essence and essentiality.²⁴

Finally, we have to specify what it means for a given specification σ to pay its dues *simpliciter*. This additional step is necessary because we cannot simply say that, in order for an ontology to be systematically optimal, all its individuating specifications have to pay their dues relative to that particular ontology. For in that case, there would be a danger of violating the constraint (C) from §5.4, according to which

[f]or every set S of optimal ontologies, there exists an optimal ontology O that contains every individuating specification that is contained in any given member of S . (p. 96)

If we now said that an ontology O is systematically optimal only if all its individuating specifications pay their dues relative to O , this would give rise to the following difficulty: There might be pairs of individuating specifications σ_1 and σ_2 such that there exists a systematically optimal ontology that contains σ_1 , and also a systematically optimal ontology that contains

²⁴In his (1994), Lewis worries about the subjective element in his account of lawhood:

The worst problem about the best-system analysis is that when we ask where the standards for simplicity and strength and balance come from, the answer may seem that they come from us. (p. 479)

His way of alleviating this worry is to take resort to the “reasonable hope” that “nature is kind” and “the best system [is] *robustly* best – so far ahead of its rivals that it [comes] out first under any standards of simplicity and strength and balance” (*ibid.*). But not everyone is equally worried. For instance, Sider (2011), in commenting on Lewis’s account, seems quite comfortable with the idea that the question of “how much complexity can be tolerated to gain a given amount of strength” admits of different answers, which “correspond to different notions of law” (p. 22):

So let a “middling” assignment of cost to complexity be one that counts the generalizations of physics as laws, but only barely. This corresponds to a sense of ‘law’ in which there are laws of physics, but in which certain special-science generalizations do not count as laws. [Footnote:] I have in mind special-science generalizations that are physically contingent—perhaps because they depend on certain physically contingent “initial conditions”. [End of footnote.] If complexity is instead made cheaper, then those special-science generalizations no longer count as laws; all that remain are laws of metaphysics and logic. (*ibid.*)

It seems to me that one could reasonably come to accept a similar sort of scheme in a theory of essences. Another option would be to allow that essentiality ‘comes in degrees’. In the following, however, I shall not pursue these ideas any further.

σ_2 , but no systematically optimal ontology that contains both σ_1 and σ_2 . For it might turn out that, if σ_1 is added to an ontology that already contains σ_2 , then σ_1 's contribution to the resulting ontology's discriminatory power will be simply zero. If, then, on the account currently under consideration, there exists no systematically optimal ontology that contains both σ_1 and σ_2 , (C) would clearly be violated.

To avoid this outcome, I propose the following definition for the concept of a specification's paying its dues:

(PD) An individuating specification σ *pays its dues* if and only if there exists some systematically optimal ontology such that σ pays its dues relative to that ontology.

Given that this definition makes use of the concept of systematic optimality, and given that the latter will in turn be defined on the basis of the concept just now introduced (viz., that of a specification's paying its dues), one might think that (PD) is circular. In fact, however, it should be understood as being merely recursive.

This is possible because the concept of a systematically optimal ontology will here be defined in such a way that an individual ontology counts as systematically optimal if and only if (i) it is true and (ii) all of its individuating specifications fulfill certain conditions. Hence, an individual ontology that does not contain any individuating specifications at all will be systematically optimal as long as it is true. And in order to render a specificationless ontology true, all that is needed is, at most, an ontological restriction to the effect that there are no more than a certain number of entities. For if an individual ontology contains no individuating specifications, then any existence claim that it generates will only be to the effect that, for some set-sized cardinality κ , there are at least κ -many entities. Thus, if it should happen that there is no limit to the number of entities, no ontological restriction will be needed.

By what has just been said, there exists a systematically optimal ontology, call it ' O_0 ', that contains no individuating specifications. Using this ontology as a starting-point, one can construct a further systematically optimal ontology as follows. Suppose that there is

some individuating specification σ that pays its dues relative to O_0 . By (PD), σ will then count as paying its dues *simpliciter*, and thereby satisfy one of the four conditions that have to be met by the individuating specifications of any systematically optimal ontology. If σ also satisfies the three other conditions (to be formulated in the next three sections), then a true individual ontology that contains σ as its only individuating specification will be systematically optimal. Similarly, one could now ask what individuating specifications pay their dues relative to this new ontology. In this way, one can recursively construct ever larger systematically optimal ontologies. (Of course, the process will come to an end once no further specifications can be found that satisfy the mentioned conditions.)

It might at this point be asked why (PD) has to make reference to systematically optimal ontologies in the first place. Would it not have been enough simply to speak of *individual* ontologies, or perhaps of ones that are both true and individual? The definition would then have been much more straightforward. However, it would not have been enough to put ‘individual’ or ‘true individual’ where (PD) has ‘systematically optimal’, because it would then have been all-too easy for an individuating specification to count as paying its dues. For example, consider:

- (1) Properties of the form ‘ λx white $\langle x \rangle$ ’ are individuating.

As long as there are sufficiently many white things as well as non-white things, one can easily construct an ontology relative to which (1) seems to pay its dues by practically any reasonable standard. Thus, let N be some large number that is smaller than the total number of white things and also smaller than the total number of non-white things. Further, let x_1, \dots, x_N be pairwise distinct white things, and let y_1, \dots, y_N be pairwise distinct non-white things. For each $i = 1, \dots, N$, let now σ_i be the individuating specification

Properties of the form ‘ $\lambda x (x = X \vee x = Y)$ ’ are individuating,

where ‘ X ’ is replaced by a name of x_i and ‘ Y ’ by a name of y_i . Finally, let O be the ontology that consists of (1) together with $\sigma_1, \dots, \sigma_N$, and let O' be the ontology that consists only of $\sigma_1, \dots, \sigma_N$. Then it can be seen that the number of abstract O' -essences that O'

correctly claims to be instantiated is $N + 1$, and for every such O' -essence E , there are *two* relevant properties that O correctly claims to be instantiated, viz., $\lambda x (E\langle x \rangle \wedge \text{white}\langle x \rangle)$ and $\lambda x (E\langle x \rangle \wedge \neg \text{white}\langle x \rangle)$. So the contribution that (1) makes to the discriminatory power of O will be exactly $N + 1$.

What about the contribution that (1) makes to the *complexity* of O ? Let us say that P_1, \dots, P_N are the properties that are denoted by the predicates derived from the various σ_i . Then, in order not to be false, already O' has to contain an ontological restriction to the effect that none of the P_i are co-instantiated. But the addition of (1) does not require any further ontological restrictions. So the contribution that (1) makes to the complexity of O will be quite modest, and, at least if N is large enough, it should seem reasonable to suppose that the increase in complexity will be outweighed by the contribution that (1) makes to O 's discriminatory capacity. If so, then (1) will have to count as paying its dues relative to O , but it is very questionable that it should also count as paying its dues *simpliciter*.

Admittedly, (PD) is not an easy definition to work with. In particular, it makes it difficult to show that a given specification fails to pay its dues; for to show this, one would (at least on a brute-force approach) have to consider all systematically optimal ontologies. By contrast, the definition is much more user-friendly – at least in some cases – when it comes to showing that a specification σ does pay its dues. For we have already seen that there is a systematically optimal ontology that contains no individuating specifications whatsoever, and so we can show that σ pays its dues if we can show that it pays its dues relative to such an ontology. To do the latter, we have to establish that σ 's contribution to the complexity of O_σ is outweighed by its contribution to O_σ 's discriminatory power, where O_σ is the simplest true ontology that has σ as its only individuating specification. Given that this ontology contains no other specifications, σ 's contribution to O_σ 's discriminatory power is simply the number of instantiated abstract O_σ -essences (or the number of instantiated abstract O_σ -essences *minus 1*, if that number is finite).

The requirement that the individuating specifications of a systematically optimal ontology should pay their dues is needed if the present account of essentiality is not to become overly

liberal. However, it is not the only, or even the most important, condition that we have to impose. In order for the account to accommodate Fine's asymmetry, we still have to add three further requirements.

7.5 Parsimony

Suppose that, relative to some systematically optimal ontology, the following individuating specification pays its dues:

- (1) Properties of the form ' $\lambda x P\langle x \rangle$ ' are individuating,

where ' P ' is replaced by a constant that denotes a certain property P . In such a case, it may easily happen that the following specification also pays its dues:

- (2) Properties of the form ' $\lambda x (P\langle x \rangle \wedge Q\langle \alpha \rangle)$ ' are individuating,

where ' Q ' is replaced by the name of some property Q that is instantiated by only a handful of entities. If (2) pays its dues, and if our definition of 'systematically optimal' were by now complete, there would be no obstacle to the existence of a systematically optimal ontology that contains (2) among its individuating specifications. Let us suppose, then, that O is such an ontology, and further, that there exist two entities, Abel and Cain, such that Abel instantiates P and Cain instantiates Q . Evidently, the property $\lambda x (P\langle x \rangle \wedge Q\langle \text{Cain} \rangle)$ will then be O -essential to Abel, from which it further follows that it is O -essential to Abel to be such that Cain has Q . By hypothesis, O is systematically optimal, and so it will on the present account be *essential* to Abel to be such that Cain has Q . But in most cases, this seems highly counter-intuitive: it should not normally be essential to one entity to be such that *another* entity has a certain property.²⁵

²⁵A similar intuition has also been used by Fine in arguing against the modal account of essentiality (see above, §1.1). The qualification 'in most cases' is needed because, under certain circumstances, it does seem acceptable to say that the property of being such that Cain has Q is essential to Abel. In particular, suppose that it is essential to Abel to bear a certain relation R to Cain, and the fact that Abel bears R to Cain necessitates that Cain should have the property Q . It should then also seem acceptable to say that it is

For this reason, the requirement that the individuating specifications of a systematically optimal ontology should pay their dues is apparently insufficient: some further requirement is needed by which ontologies that contain specifications like (2) will be prevented from counting as systematically optimal. But what feature of (2) should be held responsible for the counter-intuitive consequences noted in the previous paragraph? On the face of it, the trouble with (2) is, to put it loosely, that the conjunct ‘ $Q\langle y \rangle$ ’ does not contain an ‘ x ’. Only it is not immediately clear how one should generalize from this.

I will here adopt the working hypothesis that (2) is a member of a much broader class of problematic specifications, which also contains the following:

- (3) Properties of the form ‘ $\lambda x (x \in \alpha \wedge (\beta = x \vee \beta \in x))$ ’ are individuating.
- (4) Properties of the form ‘ $\lambda x (x \in \alpha \wedge \beta \in x \wedge \beta \in \alpha)$ ’ are individuating.

As we will see in §8.2, under certain relevant circumstances (3) does succeed in paying its dues; and the same holds for (4). But neither of these two specifications should form a part of any

essential to Abel to be such that Cain has Q . In order to see this, it may help to focus for a moment on another property, viz., that of being such that Cain is borne R to by Abel, or in symbols:

$$\lambda x (x = x \wedge R\langle \text{Abel}, \text{Cain} \rangle).$$

(The inclusion of the ‘ $x = x$ ’ in the matrix of this λ -expression is needed for reasons stated in §2.7, p. 37.) I submit that it is acceptable to say that, if it is essential to Abel to bear R to Cain, then it is also essential to Abel to be such that Cain is borne R to by Abel. But how should this conditional be justified?

On the present account, the justification will run as follows: For any essential property P of any given entity x , there is some essence E of x such that $E\langle x \rangle$ necessitates $P\langle x \rangle$. If there is now some other property P' such that $P\langle x \rangle$ necessitates $P'\langle x \rangle$, then – given that necessitation is transitive – $P'\langle x \rangle$ will also be necessitated by $E\langle x \rangle$. This means that P' , too, is an essential property of x . So, if the property P is essential to x , and $P\langle x \rangle$ necessitates $P'\langle x \rangle$, then P' will be essential to x as well. From this, we can infer the following as a special case:

- (*) If the property $\lambda x R\langle x, \text{Cain} \rangle$ is essential to Abel, and Abel’s instantiation of this property necessitates his instantiation of the property $\lambda x (x = x \wedge R\langle \text{Abel}, \text{Cain} \rangle)$, then this latter property is also essential to Abel.

Further, it is certainly the case that Abel’s instantiation of the property $\lambda x R\langle x, \text{Cain} \rangle$ necessitates his instantiation of the property $\lambda x (x = x \wedge R\langle \text{Abel}, \text{Cain} \rangle)$: in fact, the two instantiations are one and the same state of affairs. So we can infer from (*) the aforementioned conditional. But if this justification is sound (and I am not aware of any better way of justifying that conditional), then it will be admissible to apply it generally. In particular, it will allow us to justify the following: If the property of bearing R to Cain is essential to Abel, and Abel’s instantiation of this property (i.e., the state of affairs $R\langle \text{Abel}, \text{Cain} \rangle$) necessitates his instantiation of the property of being such that Cain has Q , then this latter property, too, will be essential to Abel. In other words, it will in the described situation be essential to Abel to be such that Cain has Q . In view of that situation, however, I don’t think this will be unexpected or counter-intuitive.

systematically optimal ontology. For, if (3) were a part of such an ontology, then the property of being a member of {Socrates} would on the present account be essential to Socrates. And similarly, if (4) were a part of a systematically optimal ontology, then it would be essential to Socrates' singleton to be a member of the set {Socrates, {Socrates}}. Both consequences are intuitively unacceptable.

What appears to be wrong with (2), (3), and (4) is that they fail to be *parsimonious*, in the sense that the matrices of the embedded λ -expressions contain, as it were, redundant material. For example, the matrix in (2) contains the conjunct ' $Q\langle\alpha\rangle$ ', and if this conjunct were deleted, the result would be (1), which pays its dues just as well as (2) does. In this sense, the conjunct ' $Q\langle\alpha\rangle$ ' may be said to be redundant. Similarly, (3) contains the redundant conjunct ' $x \in \alpha$ ', and (4) contains at least two redundant conjuncts, viz., ' $x \in \alpha$ ' and ' $\beta \in \alpha$ '.

In order to make this notion of (non-)parsimony more precise, it will be helpful to introduce the concept of a *regimentation*, as applied, first, to λ -expressions, and then to individuating specifications. Thus, if L is a λ -expression denoting an attribute A , let us say that a regimentation of L is any λ -expression L' that

- (i) denotes A ,
- (ii) contains no atomic predicate that denotes a non-basic attribute, and
- (iii) is such that there exists no shorter λ -expression that fulfills both (i) and (ii).²⁶

²⁶This third condition is needed because it is theoretically possible that, for two basic properties P and Q , the conjunction of P and Q – i.e., the property $\lambda x (P\langle x \rangle \wedge Q\langle x \rangle)$ – is basic as well. (I am here using properties for the sake of simplicity, but the present point can be generalized to attributes of any arity.) To see the problem, let F , G , and H be atomic predicates (constants, not variables) that respectively denote P , Q , and the conjunction of P and Q , and suppose that all three of these properties are basic. Then, if it were not for condition (iii), the λ -expression

$$\lambda x ((F\langle x \rangle \wedge \neg G\langle x \rangle) \vee H\langle x \rangle) \tag{*}$$

would have to be regarded as a regimentation of $\ulcorner \lambda x F\langle x \rangle \urcorner$. This would not only be intuitively undesirable, but would also lead to counter-intuitive consequences in connection with the definition of 'parsimonious' to be proposed below. (In the rest of this footnote, I will presuppose familiarity with that definition, and also with the notion of *pruning* that the definition makes use of.) Thus, consider the following two specifications:

- (a1) Properties of the form ' $\lambda x F\langle x \rangle$ ' are individuating,
- (a2) Properties of the form ' $\lambda x (F\langle x \rangle \wedge G\langle x \rangle)$ ' are individuating,

where ' F ' is to be replaced by F and ' G ' by G . If (*) were to count as a regimentation of $\ulcorner \lambda x F\langle x \rangle \urcorner$, then (a2)

By extension, if σ is an individuating specification whose embedded λ -expression is L , then a *regimentation* of σ will be any specification whose embedded λ -expression is a regimentation of L . For example, suppose that P is a basic property, and consider the following specification:

(5) Properties of the form ‘ $\lambda x F\langle x, \alpha \rangle$ ’ are individuating,

where ‘ F ’ is replaced by an atomic predicate (a constant, not a variable) that denotes the relation $\lambda x, y (P\langle x \rangle \wedge P\langle y \rangle)$. A regimentation of (5) may then look as follows:

(5′) Properties of the form ‘ $\lambda x (P\langle x \rangle \wedge P\langle \alpha \rangle)$ ’ are individuating,

where ‘ P ’ is replaced by an atomic predicate denoting P .

Another useful concept for the present purposes is that of *pruning*, as applied to individuating specifications. Thus, the pruning of a specification σ will here be taken to be the act of replacing one or more of the formulas in the matrix of σ ’s embedded λ -expression with either ‘ \top ’ or ‘ \perp ’, where ‘ \top ’ denotes the logically necessary state of affairs $I = I$, and ‘ \perp ’ denotes the negation of the latter, i.e., $I \neq I$. For example, by pruning the specification (5′), we can obtain each of the following:

(6) Properties of the form ‘ $\lambda x (P\langle x \rangle \wedge \top)$ ’ are individuating,

(7) Properties of the form ‘ $\lambda x (P\langle x \rangle \wedge \perp)$ ’ are individuating,

(8) Properties of the form ‘ $\lambda x (\top \wedge P\langle \alpha \rangle)$ ’ are individuating,

(9) Properties of the form ‘ $\lambda x (\perp \wedge P\langle \alpha \rangle)$ ’ are individuating,

where ‘ P ’ is again replaced by an atomic predicate denoting P . Note here that (8) and (9) do not in fact count as individuating specifications, because the ‘ x ’ does not occur free in the

would in turn count as the result of a pruning of (a1), just as, conversely, (a1) is the result of a pruning of (a2). As a result, (a1) would not count as parsimonious if (a2) pays its dues, just as (a2) will not be parsimonious if (a1) pays its dues. So, if both (a1) and (a2) should happen to pay their dues, then neither of them would count as parsimonious, and would consequently be excluded from any systematically optimal ontology. And all this merely because, by hypothesis, the conjunction of P and Q is basic. Rather than to exclude both (a1) and (a2), it seems to me that the intuitively correct way of treating them is to regard both specifications as parsimonious, on account of the fact that the predicates derived from each of them denote basic properties.

matrices of the embedded λ -expressions.²⁷

We are now in a position to explicate the concept of parsimony alluded to above:

- (P) An individuating specification σ is *parsimonious* just in case no regimentation of σ can be pruned in such a way that the result of that pruning is a specification that pays its dues.

With this concept in place, I propose to include the following requirement in the present definition of ‘systematically optimal’: If an ontology is to be systematically optimal, then each one of its individuating specifications has to be parsimonious.

The definition (P) has the important corollary that any individuating specification σ is parsimonious if the predicate derived from it denotes a basic attribute. This can be seen from the fact that, in that case, every regimentation of σ will be of the form

Properties of the form ‘ $\lambda x P\langle \dots \rangle$ ’ are individuating,

where ‘ P ’ is replaced by an atomic predicate, and where the dots are replaced by a list of variables composed from ‘ x ’ and possibly some meta-variables. Given that the matrix of the embedded λ -expression contains only a single formula, it is evidently not possible to prune a specification of this form in such a way that the result still pays its dues.

For a simple example of how the proposed requirement affects the extension of ‘systematically optimal’, consider the following three specifications:

- (10) Properties of the form ‘ $\lambda x (R\langle x, \alpha \rangle \wedge P\langle \alpha \rangle)$ ’ are individuating,
(11) Properties of the form ‘ $\lambda x R\langle x, \alpha \rangle$ ’ are individuating,
(12) Properties of the form ‘ $\lambda x P\langle x \rangle$ ’ are individuating,

where ‘ R ’ is replaced by a constant denoting the basic binary relation R , and ‘ P ’ is replaced by a constant denoting the basic property P . The specification (11) is semantically equivalent

²⁷See above, §4.3. For the same reason, the two ‘ λ -expressions’ in (8) and (9) do not even count as λ -expressions in the sense of the definition given in §2.7 (p. 36), and can consequently not be regarded as ‘forms of properties’.

to the result of a pruning of (10), and therefore (10) will not be parsimonious if (11) pays its dues.²⁸ On the other hand, no way of pruning (10) will result in a specification that is semantically equivalent to (12), and hence, the question of whether (12) pays its dues is irrelevant to the parsimony of (10). So, while there will exist no systematically optimal ontology that contains both (10) and (11), it might very well be, for all we have said, that some systematically optimal ontology contains both (10) and (12). If that is the case, then entities that have P will have P essentially; and whenever an entity x is R -related to some other entity y , and y has P , then x will be essentially R -related to y , *even though* x might not be essentially R -related to a certain further entity z , to which x is also R -related but that does not have P .

7.6 No Trumping

So far, I have introduced only two requirements that have to be satisfied by the individutive specifications of a systematically optimal ontology: viz., that each specification should pay its dues, and that each of them should be parsimonious. However, this is not yet enough to accommodate Fine's asymmetry. In order to see what else is needed, it will be useful to begin by comparing two individuational ontologies.

7.6.1 A Problem Posed by Non-Well-Founded Sets

On the one hand, let O_{\exists} be the simplest true individuational ontology that contains no individutive specifications except for σ^{\exists} , according to which properties of the form ' $\lambda x (\alpha \in x)$ ' are individuating.²⁹ In order to be true, O_{\exists} will have to contain at least one ontological restriction. In particular, it will have to contain some version of an Axiom of Extensionality

²⁸The expression 'semantically equivalent' should here be understood in the sense that two individutive specifications σ and σ' are semantically equivalent if and only if the two individuational ontologies that (respectively) have σ and σ' as their only statements will entail exactly the same states of affairs.

²⁹Note that O_{\exists} is not identical with O^{\exists} (as described on p. 75): the latter contains σ^{\exists} as its only *statement*, not just as its only individutive specification.

that is compatible with the existence of multiple memberless entities (viz., the empty set and the urelements). If there are no non-well-founded sets, it will also have to contain some version of a Foundation Axiom. And if the number of urelements does not exceed every set-sized cardinality, it will have to contain some restriction to the effect that there are no more than so-and-so many memberless entities.³⁰

On the other hand, let O_ϵ be the simplest true ontology that contains the following as its only individuating specification:

Properties of the form ‘ $\lambda x (x \in \alpha)$ ’ are individuating.

Following a convention introduced in §4.4, we can refer to this specification as ‘ σ^ϵ ’. While the aforementioned ontology O_\exists has to include some ontological restrictions to keep it from falsehood, the need for such restrictions is even greater in the case of O_ϵ . For, to describe the effect that σ^ϵ has on the existence claims of this latter ontology, one could say that it stands the set-theoretic universe on its head. Where O_\exists claims that there are memberless entities, O_ϵ would claim, if it were not for its ontological restrictions, that there are entities that fail to be members of anything. Further, where O_\exists claims that there are singletons, O_ϵ would claim that some things are members of precisely one entity each; and so on. However, at least on standard assumptions about the abundance of sets, there are no such things as O_ϵ would claim there to be. For it is standardly assumed that, for any entity x and set a , there is a set $a \cup \{x\}$, which has x as a member; and if so, everything must be a member of as many sets as there are sets. So O_ϵ has to contain ontological restrictions that prevent it from making the sorts of existence claim just mentioned.

In this connection, it is instructive to consider what an O_ϵ -individuation graph of (say) Socrates would look like. Socrates, like every other entity, is a starting-point of infinitely long ascending \in -chains: for instance, he is a member of his singleton, which is a member of *its* singleton, and so on *ad infinitum*. In fact, he is a member of as many such \in -chains as

³⁰To be sure, this ontology does not provide the resources to distinguish, among the many memberless entities, the one entity that is the empty set. However, I do not think that such a set is really needed, for the role that it plays in classical set theory can equally well be played by any memberless entity.

there are sets. For this reason, if there existed an O_∞ -individuation graph of Socrates, that graph would have to contain more nodes and edges than any set-sized cardinality. But by definition, every graph consists of a set of nodes and a set of edges. Consequently, there can be no O_∞ -individuation graph of Socrates. Since these considerations can be applied to any entity whatsoever, it follows that no entity has an O_∞ -individuation graph. But if no entity has an O_∞ -individuation graph, then no O_∞ -graph has an O_∞ -decoration. If O_∞ is to be true, it will thus have to contain an ontological restriction that prevents it from making any existence claims at all.

We might suppose, however, that the cardinality-related difficulties described in the previous paragraph can be circumvented somehow, e.g., by relaxing our conception of graph (and relatedly, our conceptions of formula, state of affairs, and attribute). In that case, Socrates will have an O_∞ -individuation graph, after all: Figure 7.1 gives a rough impression of what that graph would look like. Since not only Socrates, but any entity whatsoever is a starting-point of as many ascending \in -chains as there are sets, this graph will be the O_∞ -individuation graph not only of Socrates, but of every other entity as well, except for those non-well-founded set that, directly or indirectly, have themselves as members. (The reason for this is that the O_∞ -individuation graph of such a set would have to contain at least one edge that leads back to the graph's point.)

To see what all this has to do with Fine's asymmetry, suppose that O_∞ is systematically optimal, and that Socrates has an O_∞ -individuation graph like the one partially depicted in Figure 7.1. Let us also assume that there exists an O_∞ -decoration of this graph that assigns Socrates to the graph's point (depicted at the bottom of Figure 7.1), and that Socrates has a fully concretized O_∞ -essence. The mentioned decoration will then assign Socrates' singleton to the node that, in Figure 7.1, is depicted directly above the point. Accordingly, Socrates' instantiation of his fully concretized O_∞ -essence will necessitate the state of affairs that he is a member of his singleton. Hence, if O_∞ were systematically optimal, then it would on the present account be essential to Socrates to be a member of $\{\text{Socrates}\}$, and this contradicts the second half of Fine's asymmetry. If this outcome is to be avoided, O_∞ must therefore fail

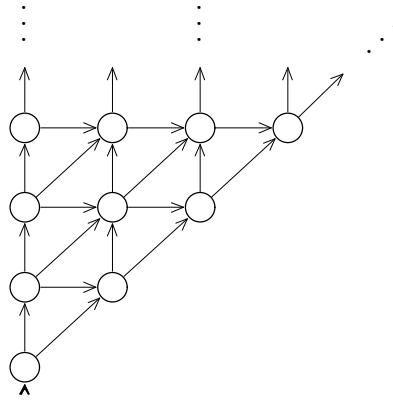


Figure 7.1: Extremely simplified sketch of a hypothetical O_ϵ -individuation graph of an urelement or well-founded set. The chevron indicates the graph’s point. For simplicity, nodes to which an O_ϵ -decoration would have to assign non-well-founded sets, or sets with more than three members (among many others), are not depicted.

to be systematically optimal. And on the supposition that our definition of ‘systematically optimal’ is by now complete, this means that at least one of the following must be the case: O_ϵ is not a true individuational ontology, or its single individuational specification σ^ϵ fails to pay its dues, or σ^ϵ is not parsimonious.

Now, by hypothesis, O_ϵ is a true individuational ontology. Further, there does not seem to be any compelling reason to think that σ^ϵ is not parsimonious. For the predicate derived from σ^ϵ , i.e., ‘ $\lambda x, \alpha (x \in \alpha)$ ’, simply denotes the relation of set-membership, and it certainly seems possible that this relation might be basic.³¹ And if it is basic, then, by what was said in the previous section (in particular, on p. 144), σ^ϵ will not fail to be parsimonious. Yet even if set-membership were *not* basic, we could not very well rely on this fact in order to save the present account’s ability to accommodate the second half of Fine’s asymmetry. For the intuitive appeal of Fine’s asymmetry does certainly not seem to stand or fall with the question of whether set-membership is basic. So now the only remaining hope for the account in its present form is that σ^ϵ might turn out not to pay its dues.

The question of whether σ^ϵ pays its dues depends on how many non-well-founded sets have, either directly or indirectly, themselves as members. If there are none, then there will also be

³¹However, see Lewis (1991) for an analysis of set-membership in terms of parthood and singleton-membership. (Sections 8.1 and 8.2 below contain discussion related to this analysis.)

no systematically optimal ontology with respect to which σ^ϵ pays its dues. For as we have seen above, if there are no non-well-founded sets that (directly or indirectly) have themselves as members, then everything will have the same abstract O_ϵ -essence, which we may call ' E_ϵ '. So, for every ontology O , and for every O -essence E , there will be exactly one instantiated property that is a conjunction of E and some O_ϵ -essence, namely, $\lambda x (E\langle x \rangle \wedge E_\epsilon\langle x \rangle)$. For any ontology that contains σ^ϵ , the contribution that σ^ϵ makes to that ontology's discriminatory power will therefore be exactly zero. And so, by any reasonable standard of what it takes for a specification to pay its dues relative to a given ontology, there will be no ontology (let alone a systematically optimal one) relative to which σ^ϵ pays its dues.

Suppose, however, that there *are* some non-well-founded sets that (directly or indirectly) have themselves as members. If we accept that there are some, we will presumably also have to accept that there are infinitely many, barring arbitrary restrictions. So we would have to accept that there are infinitely many instantiated abstract O_ϵ -essences. This in turn means that σ^ϵ will make a rather large contribution to O_ϵ 's discriminatory power; and so the specification will very plausibly count as paying its dues.³² But if σ^ϵ is parsimonious and pays its dues, then O_ϵ , being a true individuation ontology, will according to our definition, as so far developed, be systematically optimal; and, on the present account of essentiality, this will then mean that Socrates is essentially a member of {Socrates}.

One might want to avoid this outcome by insisting that there are no non-well-founded sets, and *a fortiori* none that (directly or indirectly) have themselves as members. But this would clearly not do. For the intuition that Socrates is not essentially a member of his singleton does not in the least seem to depend on the question of whether there are non-well-founded sets of any particular sort. Similarly, we should not try to avert the mentioned outcome by appealing to the fact that, due to the mentioned limitations of cardinality, Socrates does not have any O_ϵ -individuation graphs or O_ϵ -essences in the first place, because those limitations

³²Recall that any true individuation ontology that contains no individuating specifications at all is thereby systematically optimal. So, if σ^ϵ pays its dues relative to such an ontology (which it plausibly does if it makes an infinitely large contribution to O_ϵ 's discriminatory power), then it pays its dues relative to a systematically optimal ontology, and hence *simpliciter*.

are intuitively no less irrelevant to the issue of essentiality. This becomes clear if one switches the example to one that involves *singleton*-membership instead of set-membership: In this case, the analogous individuation graph of Socrates would need to contain only countably many nodes and edges, but nothing would change as far as the relevant intuitions of essentiality are concerned. The upshot is that the definition of ‘systematically optimal’, as it has so far been developed, is incomplete, and that at least one further requirement is needed.

7.6.2 A First Solution

Before this additional requirement can be formulated, we have to introduce a few auxiliary concepts. The first of these is the concept of permutation, as applied to individutive specifications. In §7.2.2, I have said that a permutation of an *attribute* is, roughly, the result of permuting the attribute’s argument-places.³³ Now, if σ and σ' are individutive specifications, let us say that σ' is a permutation of σ just in case there are attributes A and A' such that the predicate derived from σ denotes A , the predicate derived from σ' denotes A' , and A' is a permutation of A . For example, σ^\in and σ^\exists are in this sense permutations of each other.

Second, we will need the concept of *differentiation*. If x and y are two entities and O some individuation ontology, then O will be said to *differentiate* the pair (x, y) just in case x and y have distinct abstract O -essences. For example, O_\exists differentiates the pair (Socrates, {Socrates}), but O_\in doesn’t: whereas Socrates and his singleton have distinct abstract O_\exists -essences, they both have – at least if cardinality-related difficulties are set aside – one and the same abstract O_\in -essence, because they both have one and the same O_\in -individuation graph, as we have seen above. (If the mentioned cardinality-related difficulties are *not* set aside, one will have to say that Socrates and his singleton do not have any O_\in -essences at all, because they will then not have any O_\in -individuation graphs. So in this case, too, O_\in will fail to differentiate the pair (Socrates, {Socrates}).)

Let now σ and σ' again be two individutive specifications, and let O and O' be the

³³A more formal definition is given in footnote 13, p. 122.

individuational ontologies whose only statements are, respectively, σ and σ' . I will then say that σ *trumps* σ' just in case

- (i) σ is a permutation of σ' ,
- (ii) O differentiates every pair of entities that is differentiated by O' , and
- (iii) O' does *not* differentiate every pair of entities that is differentiated by O .

By extension, I will say that a given individuational specification is *trumped* just in case there exists some specification that trumps it. With this concept in hand, the requirement that I here wish to add to the definition of ‘systematically optimal’ can be formulated as follows: If an ontology O is to be systematically optimal, then none of its individuational specifications should be trumped.

As we will see shortly, the definition of ‘trumps’ that has just been proposed still requires some revision; but before we come to that, more has to be said about the motivation for condition (i). To see the need for this condition, consider once more Fine’s asymmetry, and in particular its first half, i.e., the thesis that it is essential to $\{\text{Socrates}\}$ to have Socrates as a member. If the present account of essentiality is to accommodate this thesis, it has to turn out that, for some systematically optimal ontology O , it is O -essential to $\{\text{Socrates}\}$ to have Socrates as a member. And a very straightforward way in which this could happen would be for O to contain the individuational specification σ^\exists . (We will see this in detail in §8.1 below.) But now, if the above definition of ‘trumps’ did not include that first condition, then it would be possible to conceive of scenarios in which σ^\exists would have to be regarded as trumped, while the intuitive appeal of the thesis that it is essential to $\{\text{Socrates}\}$ to have Socrates as a member would seem to persist even if we took the respective scenario to be actual. Given that σ^\exists would have to be regarded as trumped, it would on the present account of essentiality no longer be clear that, in such a scenario, the first half of Fine’s asymmetry still comes out true.

To make this point more vivid, it might help to imagine a concrete (albeit unrealistic) scenario. Suppose that two demons, Magos and Moros, set out to contemplate the richness

of both the physical and the set-theoretic universe. For every pair of entities x and y , Magos contemplates x to be a member of y if and only if x is in fact a member of y . Moros, on the other hand, is more interested in his fellow demon, and contemplates Magos to have the empty set as a member. By contrast, nothing – not even Moros himself – contemplates Moros to have anything as a member. And finally, let us suppose that Magos is the only entity by which any set is contemplated to have any members. Consider now the ontology O^* that contains only the following individuating specification:

(*) Properties of the form ‘ $\lambda x \text{Cmpl}\langle x, \alpha, \beta \rangle$ ’ are individuating,

where ‘Cmpl’ denotes the ternary relation whose instantiation by entities x, y, z (in this order) is the state of affairs that x is contemplated by y to have z as a member. By what has just been said, Moros and Magos have distinct abstract O^* -essences: e.g., it is O^* -essential to Moros, but not Magos, that nothing contemplates him to have anything as a member.

In addition, any two things (and in particular, any two sets) that have distinct abstract O^\exists -essences also have distinct abstract O^* -essences.³⁴ But of course, Moros and Magos, both being memberless entities, do not have distinct abstract O^\exists -essences. So here is a pair of entities that is differentiated by O^* but not by O^\exists . Consequently, if the above definition of ‘trumps’ did not include condition (i), then σ^\exists would have to be regarded as trumped by (*). Yet we would still want to say that it is in this scenario essential to {Socrates} to have Socrates as a member. The present account of essentiality can accommodate this intuition only because the definition of ‘trumps’ contains that first condition.³⁵

Let us briefly consider how the requirement that I have just now proposed to include in the definition of ‘systematically optimal’ – viz., the requirement that systematically optimal

³⁴Recall from §4.5 that O^\exists is the individualational ontology that has σ^\exists as its only statement.

³⁵The scenario just described might be thought to raise problems for the present account even with condition (i) included. For intuitively, it seems obvious that nothing is *essentially* such that some entity y contemplates it to have some entity z as a member. In order for our account to accommodate this intuition, it has to turn out that no ontology containing the specification (*) is systematically optimal. But it may certainly be that (*) pays its dues and is untrumped, and for all we can tell it might also be parsimonious. In order to avoid the unwelcome consequence that it is essential to a set to be contemplated by Magos to have such-and-such members, we thus have to add yet another requirement to our definition of ‘systematically optimal’. I will return to this problem in §7.7.

ontologies should not contain any trumped specifications – helps with accommodating the second half of Fine’s asymmetry. I will defer the details till §8.2, but the basic idea is quite simple. Suppose that O^ϵ is the individuation ontology that has σ^ϵ as its only statement. It then turns out that O^\exists differentiates every pair of entities that is differentiated by O^ϵ , whereas O^ϵ does not differentiate every pair of entities that is differentiated by O^\exists . By the requirement in question, it follows that σ^ϵ is trumped by σ^\exists , and can hence not be part of a systematically optimal ontology. Similar considerations hold, *mutatis mutandis*, for relevant variants of σ^ϵ , so that no systematically optimal ontology O will be such that it is O -essential to Socrates to be a member of {Socrates}. All this will be considered in greater detail in §8.2.

7.6.3 Mereological Troubles

Unfortunately, even with the ‘no trumping’ requirement that has just been introduced, our definition of ‘systematically optimal’ is not yet fully satisfactory, because in some relevant cases, the present account of essentiality does not manage to accommodate a plausible mereological analogue of Fine’s asymmetry. Making again use of Socrates for the sake of concreteness, we can state this analogous asymmetry as follows:

- (M1) It is essential to the fusion of Socrates’ body parts (henceforth Fusion) that it should have his nose as a part.³⁶
- (M2) But it is *not* essential to Socrates’ nose (henceforth Nose) that it should be a part of Fusion.

The *fusion* (or mereological sum) of some class of entities is here taken to be the entity that (i) has all the members of that class as parts and (ii) is such that each one of its parts overlaps, i.e., has some part in common with, at least one member of that class.³⁷

³⁶That is to say, if y is Socrates’ nose, then the property $\lambda x (y \preceq x)$ is essential to Fusion. The symbol ‘ \preceq ’ represents here the parthood relation.

³⁷This definition represents one of two slightly different (and logically inequivalent) ways of defining the concept of fusion that can be found in the literature. For discussion, see Hovda (2008).

I think that the conjunction of (M1) and (M2) will seem quite plausible under a conception of parthood that regards non-atomic fusions as in a certain sense ‘constructed’ from their respective proper parts. Under such a *combinatorial* conception, it also appears natural to adopt a principle of Universal Composition, according to which any class of entities whatsoever has a fusion, and to hold that there can even be fusions of abstract entities such as sets. This conception seems to be particularly congenial to classical mereology.³⁸ But there is also an alternative conception of parthood (and perhaps there are more), under which the proper parts of a given mereological whole are somehow ‘abstracted’ from the latter. Under this *abstractional* conception, it is much less natural to say that any class of entities whatsoever has a fusion, or to hold that there are mereological wholes that are composed of abstract objects. This latter conception is suggested by monistic world-views of the sort recently defended by Jonathan Schaffer (2007, 2010).

I am admittedly not sure whether our everyday notion(s) of parthood tend to be closer to the combinatorial or to the abstractional conception, but for present purposes, there is no need to decide this question. It will be enough to note that on the combinatorial conception, the two theses (M1) and (M2) are intuitively plausible. So, when those two theses are interpreted in accordance with that conception, an adequate account of essentiality should allow them to come out true.

It is possible to show that the present account accommodates (M1) if it can be shown that there exists some systematically optimal ontology whose domain contains both Socrates’ nose and the fusion of his body parts, and which contains the following individuating specification:

Properties of the form ‘ $\lambda x (\alpha \preceq x)$ ’ are individuating, (σ^{\succ})

³⁸For an overview, see Simons (1987). The thesis that mereological wholes have their respective parts essentially has first been articulated by Roderick Chisholm (1973). As applied to ordinary objects, such as artefacts and organisms, it is a highly controversial thesis. Nevertheless, as long as one operates with a combinatorial conception of parthood, it still strikes me as intuitively plausible. From the fact that mereological essentialism does not plausibly apply to ordinary objects, I would therefore draw the conclusion that ordinary objects should not be conceived of as fusions under a combinatorial conception of parthood.

where ‘ \preceq ’ symbolizes the parthood relation.³⁹ For if there exists some such ontology O , then it will be O -essential to Fusion to have Nose as a part. To have a name for the specification, let us call it ‘ σ^{\preceq} ’, and let O^{\preceq} be the ontology that contains this specification as its only statement.

The specification σ^{\preceq} has (apart from itself) only a single permutation, which I shall refer to as ‘ σ^{\preceq} ’:

Properties of the form ‘ $\lambda x (x \preceq \alpha)$ ’ are individuating. (σ^{\preceq})

It is clear that, if our account is to accommodate (M2), there must not be any systematically optimal ontology whose domain contains both Fusion and Nose and whose list of individuating specifications includes σ^{\preceq} . So, on the present account, it would have to be the case that σ^{\preceq} either fails to pay its dues or fails to be parsimonious, or is trumped.

To begin with, let O^{\preceq} be the ontology that contains σ^{\preceq} as its only statement, and let us consider whether this specification pays its dues. If there is no ‘atomless gunk’, and if there are only finitely many mereological atoms, then many things may differ from each other with respect to the number of entities of which they are a part, and hence with respect to their abstract O^{\preceq} -essences. This would in particular be the case if we assume the principle of Universal Composition.⁴⁰ Thus, if N is the number of atoms, then an atom will be a part of 2^{N-1} different entities, while a fusion of two atoms will be a part of only 2^{N-2} entities, and so on. Accordingly, the abstract O^{\preceq} -essence of an atom will be different from that of a fusion of two entities, and both will differ with respect to their abstract O^{\preceq} -essences from a fusion of three entities; and similarly for any two fusions that differ in the numbers of atoms they contain. And even if there are infinitely many atoms, there might be a large variety of instantiated abstract O^{\preceq} -essences: namely, if Universal Composition is in fact false. For in that case, it is entirely conceivable that one entity may be a part of exactly m entities, while

³⁹This new symbol can be incorporated into the present framework in the same manner by which we have in §2.5 introduced ‘ \in ’. In particular, let us first adopt ‘ P ’ as a predicate denoting the parthood relation. In the next step, we may then agree to treat any expression ‘ $t_1 \preceq t_2$ ’, for terms t_1 and t_2 , as an abbreviation of the formula ‘ $P(t_1, t_2)$ ’.

⁴⁰In addition, I here assume throughout (as usual) that parthood is reflexive, anti-symmetric, and transitive.

another is a part of exactly n entities, where $n \neq m$. And if so, entities may differ widely with respect to their abstract O^{\preceq} -essences.

In cases such as these, then, it seems entirely possible that σ^{\preceq} pays its dues relative to the systematically optimal ontology O_0 that contains no individuating specifications at all, and hence pays its dues *simpliciter*.⁴¹ It would not be of any help to the present account if we tried to show that such cases do not obtain, for instance by arguing that there are infinitely many atoms (or pieces of gunk) and that Universal Composition is true. For it seems clear that (M2) will retain its intuitive appeal even if we assume that there are only finitely many atoms, and that there is no gunk.⁴² For similar reasons, it would not help the account, either, to argue that σ^{\preceq} fails to be parsimonious. For it does fairly clearly not matter with respect to the intuitive appeal of (M2) whether parthood is a basic relation, and so the account should be able to accommodate (M2) even on the supposition that parthood is basic. But if parthood is basic, then σ^{\preceq} will be parsimonious. Thus, the only hope for the present account, as far as its ability to accommodate (M2) is concerned, lies in arguments to the effect that σ^{\preceq} is trumped.

However, this hope is disappointed in just the same cases in which we have above seen that σ^{\preceq} may succeed in paying its dues (and regardless of whether parthood is basic). First, if Universal Composition holds true, if the number of mereological atoms is finite, and if there is no atomless gunk, then O^{\preceq} will differentiate all the same pairs of entities that are differentiated by O^{\succ} . For the only way in which two entities may differ with respect to their abstract O^{\succ} -essences will then be by being fusions of different numbers of atoms. But whenever two entities differ in this way, the one that is a fusion of the greater number of atoms will be a part of a fewer number of other entities (given that we are assuming there to be only finitely many atoms, and no gunk), and will hence differ from the other entity with respect to its abstract

⁴¹In order to keep the discussion simple (relatively speaking), I have here omitted any considerations as to what contribution σ^{\preceq} will make, in various cases, to the complexity of the simplest true ontology that contains σ^{\preceq} as its only individuating specification.

⁴²Would (M2) retain its intuitive appeal also if we assumed that Universal Composition is false? Of this I am less sure. See the discussion of gunk in §8.2.4 below.

O^{\preceq} -essence as well. In this case, then, σ^{\preceq} will not be trumped.

Second, if Universal Composition is false, there might be pairs of entities that are perfectly similar to each other with respect to their internal mereological structure, but differ from each other with respect to their abstract O^{\preceq} -essences. For example, it is at least conceivable that two bricks may be perfectly similar with respect to their mereological make-up (so that they, e.g., have exactly the same number of parts, and if the one brick has n parts with a certain mereological make-up, then so does the other); but at the same time, one of them may be a part of a house, whereas the other may not be a part of anything at all. Here, then, would be a pair of entities that is differentiated by O^{\preceq} but not by O^{\succeq} , and in this case, σ^{\preceq} would again not be trumped.

From these considerations, we see that, given the definition of ‘systematically optimal’ as developed so far, the present account of essentiality does in some relevant cases not manage to accommodate (M2). This suggests that that definition is still in need of revision.

7.6.4 A Better Solution

Intuitively, the way in which a mereological fusion is related to its parts is not dissimilar from the way in which a set is related to its members: just as a set may be thought of as ‘constructed’ from its members, so a fusion can – at least on the combinatorial conception of parthood – be thought of as formed from its parts. The use of words like ‘constructed’ and ‘formed’ expresses in this context a sense of ontological priority, and it is easy to suspect that this priority of parts and members over (respectively) fusions and sets is at the heart of both Fine’s asymmetry and the mereological asymmetry mentioned above.

Of course, it might be said that the notion of ontological priority is, at least initially, just as obscure as that of essentiality, and so cannot be used in an explication of the latter. One might indeed think that it is equally possible to take the opposite route, and to explain the notion of ontological priority by saying that an entity x is *ontologically prior* to an entity y just in case x is essentially related to y , but y not essentially related to x . As far as I can

see, this is an entirely viable route to take. But even so, it seems that there exists a certain deeper similarity between, on the one hand, the way in which a set is essentially related to its members (but not vice versa) and, on the other hand, the way in which a fusion is essentially related to its proper parts. To express this a little more precisely: it would be somewhat surprising if the correct account of essentiality had multiple clauses, such that one clause is responsible for the result that Socrates is not essentially a member of his singleton, while *another* clause is responsible for the result that Socrates' nose is not essentially a part of the fusion of his body-parts.

The “clause” that is responsible for the result that, on the present account, Socrates is not essentially a member of his singleton is the requirement that a systematically optimal ontology should not contain any specifications that are trumped by any of their permutations. So, if the intuition expressed in the previous paragraph is by and large correct, it should be this same requirement that we have to modify in order to obtain the result that, on the modified account, Socrates' nose is not essentially a part of the fusion of his body-parts.

In order to see how best to modify this ‘no-trumping’ requirement, it might be useful first to redescribe it in relatively vague and informal terms, so as to make explicit the intuitive idea behind it. To this end, one might say that the requirement amounts to a prohibition against individuating specifications that *account less well for the diversity of things* than some of their permutations. For example, σ^ϵ may be said to account less well for the diversity of things than σ^\exists because there are pairs of entities that have the same abstract O^ϵ -essence yet different abstract O^\exists -essences, whereas every two things that have different abstract O^ϵ -essences also have different abstract O^\exists -essences.

Can we now also identify a sense in which σ^\lessdot “accounts less well for the diversity of things” than σ^\gtrdot ? Suppose that there are only finitely many mereological atoms (and no gunk), and that Universal Composition holds true. Further, for some natural number n , let Agnes and Bertha be two distinct fusions of n atoms. (They may overlap, but not all the atoms of Agnes should be atoms of Bertha, or vice versa.) Then the two fusions will have the same abstract O^\lessdot -essence, but also the same abstract O^\gtrdot -essence. So, when it comes to differentiating the

pair (Agnes, Bertha), in the sense of ‘to differentiate’ given above (p. 151), it turns out that there is no difference between O^{\preceq} and O^{\succeq} : both ontologies simply fail to differentiate the pair. However, it seems that this description slides over an important difference between O^{\preceq} and O^{\succeq} , which I will now try to make more salient.

Let us say that, for any individuational ontology O , an entity x is *ultimately O -individuated* by an entity y just in case x is O -individuated by y , and every entity that O -individuates y is itself O -individuated by y . For example, the singleton of Socrates’ singleton is ultimately O^{\exists} -individuated by Socrates, and the non-well-founded set that is its own singleton is, if it exists, ultimately O^{\exists} -individuated by itself. Applying this concept to our two fusions Agnes and Bertha, we find that they are ultimately O^{\succeq} -individuated by the atoms from which they are respectively composed, and by the hypothesis that the atoms that compose Agnes are not all the same as those that compose Bertha, it follows that the set of those that ultimately O^{\succeq} -individuate Agnes is distinct from the set of those that ultimately O^{\succeq} -individuate Bertha. By contrast, all entities, including Agnes and Bertha, are ultimately O^{\preceq} -individuated by one and the same entity, viz., the fusion of everything there is. As a result, we can draw the following contrast between the two ontologies: Agnes and Bertha differ from each other with respect to the sets of entities by which they are ultimately O^{\succeq} -individuated, but they do not differ from each other with respect to the sets of entities by which they are ultimately O^{\preceq} -individuated. In this sense, I think we can say that σ^{\preceq} “accounts less well for the diversity of things” than σ^{\succeq} . We now have to consider how the above definition of ‘trumps’ has to be modified in order to become sensitive to this difference.

Roughly put, what has to be changed in that definition is its reliance on the concept of an abstract O -essence; for, as we have seen, two things may be ultimately O -individuated by different sets of entities and yet have one and the same abstract O -essence. So we have to introduce a new conception of O -essence that is sensitive to the set of entities by which a given thing is ultimately O -individuated. This can be done in a way that is largely analogous to the way in which we have (in §5.2 above) defined the concept of a concretized O -essence.

Thus, let G be the O -individuation graph of an entity x , let $\chi_{O,G}$ be the O -individuation

formula that corresponds to G , and let f be an injective mapping of G 's nodes onto a set V of variables, chosen in such a way that f can be used in constructing $\chi_{O,G}$. (As already mentioned in earlier sections, the details of this construction are described in Appendix C.) Further, let d be an injective O -decoration of G that assigns x to G 's point, and suppose that h is an assignment of entities to the variables in V , such that an entity z is assigned to a variable u just in case

- (i) The decoration d assigns z to the node that f maps to u , and
- (ii) x is ultimately O -individuated by z .

From this second condition, it follows that, if an entity does not ultimately O -individuate x , then h will not assign that entity to any of the variables. Moreover, h will assign an entity to a given variable *only* if the node that f maps to that variable is such that d assigns to it an entity by which x is ultimately O -individuated; in which case that latter entity will be the one that h assigns to the variable in question. Given such an assignment h , we may say that a property P is a *semi-concretized O -essence* of x just in case P is, relative to h , denoted by

$$\lambda w \exists_{O} u_1, u_2, \dots (w = v \wedge \chi_{O,G}),$$

where w is a variable that does not occur free in $\chi_{O,G}$, v is the variable to which f maps G 's point, and where the u_1, u_2, \dots are those variables in V to which h does not assign any entity. (Note that v will be one of the u_i unless x is ultimately individuated by itself.)

The concept of a semi-concretized O -essence, thus defined, may at first seem very artificial, but it is really far more intuitive than the name (or the definition) may suggest. To put an intuitive gloss on it, one could say that a semi-concretized O -essence is nothing else than the property of *being constructed in such-and-such a way from such-and-such ultimate constituents*, where the terms 'constructed' and 'constituent' are both to be understood in a sense that is in part determined by the ontology O . What is somewhat counter-intuitive here (given the gloss just provided) is only that, relative to some ontologies O , some entities (such

as certain non-well-founded sets) may have no ultimate ‘constituents’ – i.e., no entities by which they are ultimately O -individuated – yet still have a semi-concretized O -essence, which will in this case be identical with their respective *abstract* O -essence.

We can now further define a new concept of differentiation that is based on that of a semi-concretized O -essence. For any entities x and y , an individuational ontology O will be said to *differentiate_{sc}* the pair (x, y) just in case x and y have distinct semi-concretized O -essences.⁴³ So, for example, if x and y are any two mereological atoms, then O^{\supset} differentiates the pair (x, y) , because x and y are ultimately O^{\supset} -individuated only by themselves, so that the only entity by which x is ultimately O^{\supset} -individuated (viz., x itself) is distinct from the only entity by which y is ultimately O^{\supset} -individuated. By contrast, if we assume that for every class of things, there is a fusion of that class, it follows that x and y are ultimately O^{\supset} -individuated by the same entity, viz., the fusion of everything there is. By further symmetry considerations, it can then be seen that x and y , if they have any O^{\supset} -essences at all, will have one and the same semi-concretized O^{\supset} -essence.⁴⁴ So O^{\supset} , but not O^{\supset} , differentiates the pair (x, y) .

By perfectly analogous considerations, one can see that some pairs of entities are differentiated_{sc} by O^{\supset} but not by O^{\supset} . Thus, any given urelement is ultimately O^{\supset} -individuated only by itself, and so any two urelements have distinct semi-concretized O^{\supset} -essences. By contrast, if any two urelements have any O^{\supset} -essences at all (which might not be the case, for the cardinality-related reasons mentioned above), they will have exactly the same abstract O^{\supset} -essence. For this reason, and because nothing is ultimately O^{\supset} -individuated by anything, the two urelements in question must have one and the same semi-concretized O^{\supset} -essence, or none at all. In either case, they will not be differentiated_{sc} by O^{\supset} .

We are now in a position to formulate an improved definition of ‘trumps’:

(Tr) If σ and σ' are individuational specifications, and O and O' individuational ontologies that have, respectively, σ and σ' as their only statements, then σ *trumps* σ' just in case

⁴³The subscript ‘sc’ stands for ‘semi-concretized’.

⁴⁴Just as in the case of O^{\supset} -individuation graphs and O^{\supset} -essences, there are analogous cardinality-related reasons to think that no entity has an O^{\supset} -individuation graph or an O^{\supset} -essence. (See above, p. 147.)

- (i) σ is a permutation of σ' ,
- (ii) O differentiates_{sc} every pair of entities that is differentiated_{sc} by O' , and
- (iii) O' does *not* differentiate_{sc} every pair of entities that is differentiated_{sc} by O .

As above, I will further say that an individuating specification is ‘trumped’ just in case it is trumped by some specification. To complete our definition of ‘systematically optimal’, we have to impose only one further requirement.

7.7 Principiality

Consider again the somewhat fantastic scenario described in §7.6.2 (p. 152), in which two demons contemplate the richness of the set-theoretic and physical universe. In this connection, we have discussed the following individuating specification:

- (1) Properties of the form ‘ $\lambda x \text{Cmpl}\langle x, \alpha, \beta \rangle$ ’ are individuating,

where the constant ‘Cmpl’ denotes the relation that is instantiated by entities x, y, z just in case x is contemplated by y to have z as a member. Let now O_1 be a true ontology that contains (1) as its only individuating specification. In the mentioned scenario, (1) certainly pays its dues. Moreover, for all that has just been said, the specification may also be parsimonious, and we may in addition suppose that it is untrumped. In other words, O_1 may well satisfy all the conditions that we have so far included in the *definiens* of ‘systematically optimal’. This further means that, if the definition of that phrase were by now complete, it would on the present account be essential to any given set to be contemplated by Magos to have such-and-such members. As unrealistic as the scenario may be, it still seems that this latter result should be avoided.

Fortunately, it is also fairly clear just why that consequence is so counter-intuitive: It cannot be essential to a set to be contemplated by Magos to have such-and-such members, because the set *already exists* when Magos sets out to contemplate it! Moreover, in order to explain how Magos is able to contemplate such-and-such sets to have such-and-such members,

one would presumably have to appeal to some way of differentiating between sets. This might be done, e.g., by making use of their respective O_{\exists} -essences. An adequate explanation as to why O_1 is true will therefore not merely consist of O_1 itself, but will have to go back to an ontology like O_{\exists} . To a first approximation, we might thus say that the specification (1) fails to be ‘principal’ (or perhaps: explanatorily fundamental) in the following sense:

(Pr₀) If σ is an individuating specification and O_{σ} a true ontology that contains σ as its only specification, then σ is *principal* just in case an adequate explanation of the fact that O_{σ} is true is provided by O_{σ} itself.

However, this is not yet satisfactory. Consider, e.g., the ontology O_{\exists} : We do want to say that O_{\exists} is in some sense explanatorily fundamental; certainly, in order to explain why this ontology is true, it does not seem as if we should have to appeal to another ontology of sets. However, we do have to appeal to an at least rudimentary ontology of *urelements*, for O_{\exists} does after all make claims about the number of memberless entities, just as it makes claims about the number of singletons, etc. So in what sense, exactly, does O_1 but not O_{\exists} fail to be explanatorily fundamental?

To answer this, I would suggest that we look once more at the discriminatory power of the ontologies in question. While O_{\exists} does not differentiate any pairs of urelements, it does differentiate pairs of sets, and so does O_1 . Hence we can say that the truth of O_1 has to be explained by appeal to some ontology O (which in this case is O_{\exists}) that is not only such that an adequate explanation of O ’s own truth need not make any appeal to O_1 ,⁴⁵ but also such that at least one pair of entities that is differentiated by O_1 is likewise differentiated by O . (By contrast, at least if set-membership is basic, the same will plausibly not be true of O_{\exists} , as we will see in §8.1.1 below.) In this sense, I would suggest that O_1 fails to be explanatorily fundamental. Accordingly, I propose to redefine the notion of principiality as follows:

⁴⁵It might be thought that explanatory dependence can never be reciprocal, but this is not so clear. Consider, e.g., a pair of true ontologies that have as their respective domains two species that have co-evolved (coyotes and jack rabbits, say). Then, in order adequately to explain the truth of either ontology, one may conceivably have to appeal to the respectively other member of the pair.

(Pr) If σ is an individuating specification and O_σ a true ontology that contains σ as its only specification, then σ is *principal* just in case no adequate explanation of the fact that O_σ is true relies on any ontology O that satisfies the following two conditions:

- (i) No adequate explanation of the fact that O is true relies on O_σ ; and
- (ii) Some pair of entities that is differentiated by O_σ is also differentiated by O .

This might be paraphrased by saying that a specification σ is principal just in case it allows us to differentiate a pair of entities *only if* that pair ‘had not been differentiated before’, which is to say, only if that pair is not differentiated by any ontology that is relied on by some adequate explanation as to why O_σ is true, but whose own truth cannot be adequately explained on the basis of O_σ . A noteworthy consequence of this definition is that, in cases where O_σ does not differentiate any pairs of entities at all, σ will invariably count as principal.⁴⁶

It may also be worth noting that the references to differentiation that this definition contains cannot be replaced by talk of differentiation_{sc}, on pain of losing any assurance that σ^\exists should be regarded as principal. The reason for this is that an ontology O of urelements might quite possibly, as in the case of O_0 , be such that every set is ultimately O -individuated only by itself. Such an ontology will thus manage to differentiate_{sc} every single pair of sets. Hence, if it should happen that an explanation of the truth of O_\exists has to rely on that ontology, then σ^\exists will no longer count as principal under the definition in question.

Admittedly, (Pr) is still in need of clarification. In particular, more has to be said as to what is meant by an ‘adequate explanation’ of a given ontological fact. This is a difficult notion to explicate, but an intuitive grasp will here hopefully be sufficient.⁴⁷ The final requirement

⁴⁶It is partly for this reason that, in the present context, I prefer the term ‘principal’ over ‘explanatorily fundamental’. In view of the consequence just mentioned, the latter term would have sounded like a gross misnomer, whereas the semantics of the rarely-used term ‘principal’ seems rather more malleable.

⁴⁷If pressed to explicate the notion, I would start with the ideas sketched in §6.1 above. Of course, the present *explicandum* is not quite the same, since we are here concerned not with explanations of ‘what there is’, but rather with explanations of more circumscribed ontological facts, viz., of facts that consist in the truth of a particular ontology. Thus, if F is such a to-be-explained fact, then a best explanation of F may, roughly, be taken to be a theory that (i) in some suitable sense entails F , and (ii) is a part of the simplest possible theory that is true, comprehensive, and formulated in an austere vocabulary in which all atomic predicates

to be included in our definition of ‘systematically optimal’ can then be stated as follows: If O is a systematically optimal ontology, then all of O ’s individuating specifications should be principal.

It might be thought that, in the presence of this last requirement, both the parsimony and the no-trumping requirement of the previous two sections are superfluous. Thus, to begin with the no-trumping requirement, one might think that (e.g.) the fact that σ^ϵ is trumped by σ^\supset should in the first place be taken to mean that O_ϵ ’s truth has to be explained on the basis of O_\supset , given that the latter allows for finer distinctions between sets. And further, if O_ϵ does not differentiate any pairs of entities at all (which will be the case if there are no non-well-founded sets), then σ^ϵ will not pay its dues, though it will be principal. But if O_ϵ *does* differentiate at least one pair of entities, then any such pair will also be differentiated by O_\supset (as we will see in §8.2), so that σ^ϵ will fail to be principal, given that – as it seems plausible to hold – an adequate explanation of O_ϵ ’s truth has to rely on O_\supset . So, at least when it comes to excluding σ^ϵ from the class of systematically optimal ontologies, there does not seem to be any need for the no-trumping requirement.

A similar concern could be raised for the parsimony requirement. For example, consider the non-parsimonious specification discussed at the beginning of §7.5:

(2) Properties of the form ‘ $\lambda x (P\langle x \rangle \wedge Q\langle \alpha \rangle)$ ’ are individuating,

and compare it with the simpler specification

(3) Properties of the form ‘ $\lambda x P\langle x \rangle$ ’ are individuating.

Suppose that O_2 and O_3 are true ontologies that contain (2) and (3), respectively, as their only specifications. Given that (3) results from a pruning of (2), O_3 is correspondingly simpler

refer to basic attributes.

A theory may here be understood to be *comprehensive* (relative to F) if and only if it entails all the facts, or at least all the purely qualitative ones, in addition to all those that involve any entity that is involved in F . (For the relevant meaning of ‘involves’, see the footnote on p. 105 above.) Since the theories in question are here not intended to reveal essences, this proposal does not lead to the sort of problem that we encountered at the end of §6.3. However, it does lead to a certain cardinality-related difficulty, for the class of facts just mentioned might conceivably be so large and complex that no single theory manages to entail all the facts in that class. I do not know how this difficulty might best be dealt with, but it is perhaps reasonable to hope that the possibility that gives rise to it does not obtain.

than O_2 , and it seems natural to say that some adequate explanations of the fact that O_2 is true will rely on O_3 . Moreover, as long as there exists at least one entity that has P , one entity that lacks P , and also at least one entity that has Q , there will be a pair of entities that is differentiated by both O_2 and O_3 . In such a case, then, the specification (2) will fail to be principal. So one might think that the parsimony requirement does not do any work that is not already done by the principiality requirement.

It would certainly be a welcome result if both the parsimony and the no-trumping requirement should in this way turn out to be superfluous: our definition of ‘systematically optimal’ could then be drastically simplified. However, as long as we have not yet settled on a precise explication of ‘adequate explanation’, our definition of ‘systematically optimal’ will benefit from the fact that it also includes the two other requirements, since these will help to delineate this latter concept more sharply.

7.8 Putting the Pieces together

The principiality requirement completes the list of conditions that I mean to impose on the individuating specifications of a systematically optimal ontology; as far as I can see, no further conditions have to be added. Accordingly, I propose to define the concept of systematic optimality as follows:

(SO) An ontology is *systematically optimal* just in case it is a true individual ontology, and each one of its individuating specifications pays its dues and is parsimonious, untrumped, and principal.

This concept can now be used to complete the present account of essentiality (as well as of essence and individuation) in the way already indicated above. In particular, I propose that an ontology should be taken to be ‘optimal’ if and only if it is *systematically optimal* in the sense just specified. The resulting account of essence, essentiality, and individuation has already

been stated in §7.1, but it may be worthwhile to repeat it here:⁴⁸

- (E1) A property is an (abstract or concretized) essence just in case it is an (abstract or concretized) *O*-essence, for some systematically optimal ontology *O*.
- (E2) For any entity *x*, a property is essential to *x* just in case it is *O*-essential to *x*, for some systematically optimal ontology *O*.
- (In) For any two entities *x* and *y*, *x* is individuated by *y* just in case *x* is *O*-individuated by *y*, for some systematically optimal ontology *O*.

To be sure, this formulation hides a great deal of complexity on the deeper levels of the account, and this complexity might raise doubts about the latter's plausibility. One might be reminded of the efforts that have been made, in the wake of Gettier (1963), to provide an acceptable alternative to the justified-true-belief analysis of the concept of knowledge, and of the common response: If *that* is what knowledge is, then why should we care about it? The secondary reaction, in the case of this response, would be either to dismiss the proposed account on the ground that it cannot do justice to the intuition that 'knowledge matters', or to regard the concept of knowledge itself as irrelevant.⁴⁹ Could one not justifiably have either of these two reactions to the present account of essentiality as well?

When considering this worry, it seems first of all clear that complexity itself is not the problem, for there is *prima facie* no compelling reason to think that only simple concepts can be philosophically significant. The problem seems to lie rather with a certain sort of theoretical disunity, for which complexity constitutes neither a sufficient nor a necessary condition. An account should arouse suspicion if it presents itself as a "more or less ad hoc sprawl" (to use Williamson's phrase), but not if it is merely complex.

If this consideration is applied to the above account of essentiality, it will at first seem that the account exhibits a certain amount of "ad hoc sprawl", namely insofar as it lists five

⁴⁸I use labels in boldface to distinguish the present statements from those of §5.4 above.

⁴⁹For an example of the first reaction, see Williamson (2000, p. 31); for an example of the second, see Beckermann (2001).

very different conditions that an individual ontological ontology has to meet in order to count as systematically optimal. In particular: (i) the ontology has to be true, (ii) all its specifications have to pay their dues, (iii) all its specifications have to be parsimonious, (iv) all of them have to be untrumped, and (v) all of them have to be principal. Despite their differences, however, these five requirements do have one important feature in common: each of them can be sensibly regarded as a condition that an ontology has to fulfill in order to form part of a *best explanation of the diversity of things*, and thereby also tells us something about what such an explanation would have to look like. If those requirements are thought of in this way, it becomes clear how they complement each other:

- (i) From the requirement that the ontology should be true, one can infer that the explanation should not rest on any falsehood.
- (ii) From the requirement that the ontology's specifications should all 'pay their dues', one can infer that the explanation should be in a certain sense worthwhile: the amount of diversity that it manages to explain should be sufficient to compensate for the explanation's complexity. (Otherwise, that diversity will be better regarded as accidental.)
- (iii) The parsimony requirement tells us that, *ceteris paribus*, an explanation is better if it is simpler.
- (iv) The no-trumping requirement effectively says that, *ceteris paribus*, an explanation is better if it is more powerful, where 'more powerful' means – given that the explanandum is "the diversity of things" – a greater number of explained differences.⁵⁰
- (v) And last but not least, the principality requirement says, roughly, that the explanation

⁵⁰The phrase 'explained difference' must here be taken with a grain of salt. In particular, if the explanation in question is some individual ontological ontology O , and if x and y are two entities, then it should be understood that 'the difference between x and y ' is 'explained' by O just in case x and y have distinct semi-concretized O -essences. This will be trivially the case if x and y are ultimately O -individuated only by themselves, even though there will then be no real sense in which O provides an *explanation* for their distinctness; their distinctness will be effectively treated as 'brute'. But none the less, there is certainly an important contrast between this case and one in which x and y are ultimately O -individuated in the same way *and by the same entities*. (Cf. p. 159 above.)

should ‘start at the beginning’, rather than with an ontology whose truth is adequately explained by relying on some further ontology.

It seems fair to say that each of these requirements contributes a crucial piece to the picture of what a best explanation of the diversity of things would have to look like. In this way, we find that the various intuitions about particular cases that have made necessary so many adjustments during the development of the present definition of ‘systematically optimal’ have not, after all, led us to an *ad hoc* sprawl, but rather to a quite cohesive account.

This is of course a welcome result, in part because it makes it easier to see why we should care about essentiality, if essentiality is what the present account says it is. A little more specifically, it is at least in part through its cohesiveness that this account promises to satisfy the second desideratum formulated in §1.3, according to which an account of essentiality should do justice to “the sense of philosophical significance that has traditionally been attached to the notion of essence” (p. 7). But moreover, the cohesiveness of this account is a welcome result also because the various intuitions that have led to it now appear as a body of *converging evidence*: although one may certainly be able to think of other accounts that manage to accommodate those intuitions, it would be surprising if any such account should turn out to be equally cohesive, or equally well able to meet the mentioned desideratum.

Let us now consider in detail how the present account meets the *first* desideratum formulated in §1.3: viz., to “provide an answer to the question of why Fine’s asymmetry holds”.

Chapter 8

Essence Explicated

The first desideratum listed in §1.3 requires that an account of essentiality should “provide an answer to the question of why Fine’s asymmetry holds” (p. 7). In slightly more technical terms, this means that the account – possibly in conjunction with independently plausible assumptions – should have Fine’s asymmetry as a consequence. For ease of reference, I will label the two ‘halves’ of the asymmetry as follows:

(F1) It is essential to {Socrates} to have Socrates as a member.

(F2) It is not essential to Socrates to be a member of {Socrates}.

In order to show that the present account of essentiality satisfies the mentioned desideratum, I will have to argue that each of these two theses follows from the account, at least when the latter is conjoined with certain independently plausible assumptions. This task will be taken up in the first two sections of this chapter. In §8.3, I will then address the question of whether the present account is ‘cognitively adequate’, and in the final section, I shall argue in somewhat greater detail that the account also satisfies the second desideratum formulated in §1.3, namely, that an account of essentiality should “do justice to the sense of philosophical significance that has traditionally been attached to the notion of essence”.

8.1 Why {Socrates} Essentially Has Socrates as a Member

According to the present account of essentiality, (F1) is true if and only if the property of having Socrates as a member is O -essential to {Socrates}, for some systematically optimal ontology O .¹ In order to show that there exists such an ontology, I will distinguish between two cases: in the first case, set-membership is basic, and in the second, it is analyzable in the way proposed by Bunt (1985) and Lewis (1991). As we will see, the structure of the argument is in both cases quite similar.

8.1.1 First Case: Set-Membership Is Basic

Let us for now assume that set-membership is basic. To show that, in this case, the present account of essentiality has (F1) as a consequence, we will proceed in two steps. First, it will be shown that,

- (1) For any ontology O that contains the specification σ^{\ni} and whose domain contains both Socrates and his singleton, it is O -essential to {Socrates} to have Socrates as a member.

And in the second step, we will show that

- (2) There exists at least one systematically optimal ontology that contains σ^{\ni} and whose domain contains both Socrates and his singleton.

From (1) and (2) taken together, it follows that there exists at least one systematically optimal ontology O such that it is O -essential to {Socrates} to have Socrates as a member, which is all that needs to be shown.

Let us begin, then, with the argument for (1). As may be recalled from earlier chapters, σ^{\ni} is the following individuating specification:

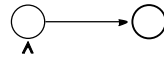
Properties of the form ' $\lambda x (\alpha \in x)$ ' are individuating.

¹Cf. p. 167 above.

To see that (1) is true, let O be any ontology that contains σ^\exists and whose domain contains both Socrates and his singleton. It then follows that the fully concretized O -essence of $\{\text{Socrates}\}$ must be of the form

$$\lambda z (z = \text{Singleton} \wedge \text{Socrates} \in \text{Singleton} \wedge \dots), \quad (*)$$

where ‘Singleton’ is a constant denoting $\{\text{Socrates}\}$. This can be seen as follows. Suppose that O contains σ^\exists as its *only* individuating specification. The O -individuation graph of $\{\text{Socrates}\}$ can then be depicted by the following diagram:



The graph’s point is here indicated by the chevron underneath the left-hand node. One can easily read off the abstract O -essence of $\{\text{Socrates}\}$, which is the property

$$\lambda z \exists_O x, y (z = x \wedge y \in x \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = y) \wedge \forall y_1 \neg(y_1 \in y) \wedge x \neq y)$$

Being an O -entity x that has an O -entity y as its only member, which itself has no members and is distinct from x .²

The fully concretized O -essence of $\{\text{Socrates}\}$ can be obtained from this by getting rid of existential quantifiers and substituting constants for variables:

$$\lambda z (z = \text{Singleton} \wedge \text{Socrates} \in \text{Singleton} \wedge \forall y_1 (y_1 \in \text{Singleton} \rightarrow y_1 = \text{Socrates}) \wedge \forall y_1 \neg(y_1 \in \text{Socrates}) \wedge \text{Singleton} \neq \text{Socrates})$$

Being identical with Singleton (i.e., with $\{\text{Socrates}\}$) and such that Socrates is the only member of Singleton, nothing is a member of Socrates, and Socrates is distinct from Singleton.

The λ -expression that has here been used to denote the fully concretized O^\exists -essence of Socrates’ singleton conforms to the above schema (*), and consequently it is O -essential to

²Cf. §5.2 above.

{Socrates} to have Socrates as a member. If O were to contain any additional individuating specifications, this would only add further conjuncts to the matrix of the λ -expression; and so it would still follow that it is O -essential to {Socrates} to have Socrates as a member. This completes the present argument for (1).

Next, we have to show that (2) holds as well; i.e., we have to show that there exists a systematically optimal ontology that contains σ^\supset and whose domain contains both Socrates and his singleton. To achieve this, it will in particular have to be shown that σ^\supset pays its dues and is parsimonious, untrumped, and principal. Once this has been done, it will be clear that there must exist a systematically optimal ontology of the sort just described. For the only remaining requirement that an ontology has to meet, if all its individuating specifications pay their dues and are parsimonious, untrumped, and principal, is the requirement that it must be true. But the latter can be easily met regardless of what individuating specifications the ontology may contain, simply by adding suitable ontological restrictions. Hence, if σ^\supset satisfies the four mentioned conditions, there cannot fail to exist a systematically optimal ontology that contains it, and whose domain contains both Socrates and his singleton. So let us now turn to those four conditions.

Paying one's dues

Due to the “subjective element” in the explication, in §7.4 above, of what it means for an individuating specification to pay its dues (see p. 136), an argument for the thesis that a particular specification pays its dues can never be perfectly conclusive. In the case of σ^\supset , however, the argument's conclusion is nevertheless difficult to avoid. For, if an ontology O contains σ^\supset and no other individuating specifications, then in one fell swoop, O will entail the existence of as many sets as the underlying ‘background ontology’ of graphs (or formulas) will allow.³ Given the vast number of abstract O -essences that the ontology will therefore correctly claim to be instantiated, the contribution that σ^\supset will make to O 's discriminatory power is

³For the relevant notion of entailment, see §6.1 above. On the background ontology of graphs, see §4.4.

correspondingly large. Opposed to this is only a very modest cost in terms of complexity, for all that has to be added to O are at most three ontological restrictions to keep it from lapsing into falsehood (as mentioned in §7.6, p. 146). This makes it difficult to avoid the conclusion that σ^{\supset} pays its dues relative to the systematically optimal ontology O_0 that contains no individuating specifications at all, and hence pays its dues *simpliciter*.

Parsimonious

The predicate derived from σ^{\supset} denotes the converse of set-membership. But we are here supposing that set-membership is a basic relation. Its converse is therefore also basic, and so, by the corollary stated at the end of §7.5 (p. 144), it follows that σ^{\supset} is parsimonious.

Untrumped

Given that set-membership is a binary relation, σ^{\supset} has (apart from itself) only a single permutation, which will in the following be referred to as ' σ^{\in} ':

Properties of the form ' $\lambda x (x \in \alpha)$ ' are individuating. (σ^{\in})

As in §7.6, let O_{\supset} and O_{\in} be true ontologies whose only specifications are, respectively, σ^{\supset} and σ^{\in} . We have already seen an argument for the conclusion that O_{\in} , but not O_{\supset} , fails to differentiate_{sc} pairs of distinct urelements.⁴ That conclusion, however, supports the thesis that σ^{\supset} is untrumped only under the assumption that there are at least two urelements. To avoid having to make this assumption, we can instead argue that O_{\in} , but not O_{\supset} , fails to differentiate_{sc} the pair $(\emptyset, \{\emptyset\})$, where \emptyset is the empty set. This latter pair is not differentiated_{sc} by O_{\in} , because both the empty set and its singleton have one and the same abstract O_{\in} -essence

⁴See §7.6.4 above. For the present purposes, it does not matter whether we consider O_{\supset} and O_{\in} , which *ex hypothesi* contain whatever ontological restrictions are needed to make them true, or instead O^{\supset} and O^{\in} , which have σ^{\supset} and σ^{\in} (respectively) as their only statements. For the difference between O^{\supset} and O_{\supset} , and likewise the difference between O^{\in} and O_{\in} , only lies in the fact that the second member of each pair contains certain ontological restrictions, which do not have any influence on what pairs of entities the respective ontologies differentiate (nor on what pairs they differentiate_{sc}).

or none at all, and neither of them is ultimately O_{ϵ} -individuated by anything. By contrast, those two entities do have distinct abstract, and therefore also distinct semi-concretized, O_{\supset} -essences. As a result, the pair $(\emptyset, \{\emptyset\})$ is differentiated_{sc} by O_{\supset} but not by O_{ϵ} . It follows that σ^{\supset} is not trumped by σ^{ϵ} , and is consequently – since it has no permutations other than itself and σ^{ϵ} – untrumped.

Principial

Finally, we have to consider whether σ^{\supset} is principial. This will be the case if and only if the fact that O_{\supset} is true has no adequate explanation that relies on an ontology O such that (i) no adequate explanation of the fact that O is true relies on O_{\supset} , and (ii) some pair of entities that is differentiated by O_{\supset} is also differentiated by O .⁵ At first blush, this requirement does not appear to pose any problems. For, on the present assumption that set-membership is basic, it is hard to see what ontology should be appealed to in order to give an adequate explanation of the fact that O_{\supset} is true, except for an ontology of urelements.⁶ (By an ‘ontology of urelements’, I here mean an ontology whose individuating specifications do not make any reference, tacit or otherwise, to set-membership.⁷) But what plausible ontology of urelements will differentiate a pair of sets?

This last question points to a difficulty that is somewhat related to Frege’s Julius-Caesar problem.⁸ For suppose that a certain ontology of urelements differentiates between different

⁵See above, §7.7.

⁶The need for this exception has already been stated in §7.7: O_{\supset} will have to make claims to the effect that there are at least so-and-so many memberless entities, and such claims may call for an explanation.

⁷What does it mean to say that an individuating specification makes reference to set-membership? An obvious case would be a specification that, in its embedded λ -expression, makes use of a set-membership predicate. However, in order to cover cases of ‘tacit’ reference as well, we may say that an individuating specification σ *makes reference* to set-membership just in case the predicate derived from σ denotes an attribute whose existence necessitates the existence of the set-membership relation.

⁸This problem can be understood as an objection to ‘abstractionist’ definitions, examples of which are discussed in §62ff. of the *Grundlagen*. In a nutshell, the problem is that this type of definition leaves the *definiendum* without a determinate reference. Frege’s eventual definition of what is meant by ‘the number associated with the concept F ’ (*die Anzahl, welche dem Begriffe F zukommt*) does not suffer from this problem, as long as we can determinately refer to extensions of concepts, or more generally, to classes. But a metaphysician might now still be forgiven for asking why Julius Caesar (say) is not identical with some class

entities according to the number of mereological atoms from which they are respectively composed, and suppose that Julius Caesar and M. Junius Brutus are mereological fusions composed from different numbers of atoms. For the sake of the example, suppose further that Caesar and Brutus are not only mereological fusions but also *sets*. In particular, let us say that Caesar is identical with the von-Neumann ordinal 1 (i.e., the set $\{\emptyset\}$), whereas Brutus is identical with the ordinal 2 (i.e., the set $\{\emptyset, \{\emptyset\}\}$). Then the mentioned ontology of urelements will differentiate between Caesar and Brutus, and hence, given the identities just stipulated, between the two sets $\{\emptyset\}$ and $\{\emptyset, \{\emptyset\}\}$. This would be enough to show that O_{\exists} is not principial, if indeed that particular ontology of urelements has to be appealed to in order to explain the truth of O_{\exists} , and if the fact that that ontology is true does not in turn require an explanation on the basis of O_{\exists} .

At least on the assumption that set-membership is basic, however, it will not be plausible to say that any two sets are at the same time non-atomic fusions, differing from each other with respect to the number of atoms from which they are composed – unless the relevant parthood relation (if it can be called that) has an analysis in terms of set-membership. For example, if we were assuming not only that set-membership is basic but also that the parts of a set are precisely its subsets, we could not very well maintain that the subsets of a given set S bear a *basic* parthood relation to S . Rather, we would have to think of the ‘parthood’ relation that is at play here as nothing else than the subset relation. For otherwise we would face the difficult question of *why* a set should have exactly its subsets as parts, or indeed, why it should have any parts at all, except for itself. So the ‘parthood’ relation could in this case not be assumed to be basic, but would rather have to be taken to be a set-theoretic relation; and any ontology that, in its individuating specifications, makes use of a predicate denoting such a relation will *eo ipso* not be an ontology of urelements. But apart from such cases, where parthood can be analyzed in terms of set-membership, it appears quite implausible to say that any set should be anything else than a mereological atom (or equivalently: that any

of entities. It is this latter, metaphysical question that is relevant for the present passage.

non-atomic fusion should be anything else than an urelement).

This point generalizes to relations other than parthood. For just as it is implausible, if set-membership is basic, to suppose that sets differ among each other with respect to the numbers of the mereological atoms from which they are composed (unless the relevant parthood relation is really a set-theoretic relation), so it will also be implausible to suppose that sets differ from each other with respect to physical characteristics like location and momentum, or whatever other attributes might be referred to in an ontology of urelements. Sets may of course be said to differ from each other in terms of ‘location’ and ‘momentum’, if by ‘location’ and ‘momentum’ one understands the somehow aggregated location and momentum of their members. But in this case, the relevant attributes are clearly to be analyzed in terms of set-membership (among other things), and can therefore not be referred to in the individuating specifications of an ontology of urelements.

From these considerations, we can conclude that σ^{\exists} is principal, after all, and thus satisfies the last of the four conditions that an individuating specification has to satisfy in order to form part of a systematically optimal ontology. The argument for (2) is thus complete. On the assumption that set-membership is basic, there exists at least one systematically optimal ontology O such that it is O -essential to $\{\text{Socrates}\}$ to have Socrates as a member.

8.1.2 Second Case: Set-Membership Is Analyzable into Singleton-Membership and Parthood

Let us now suppose that set-membership is *not* basic, but is instead analyzable in the way proposed by Bunt (1985) and Lewis (1991). According to this analysis, to be a member of a given set just amounts to having a singleton that is a part of that set. Using our apparatus of attributes and states of affairs, this might be put by saying that set-membership is nothing else than the relation

$$\lambda x, y \exists z (\mathbf{S}\langle z, x \rangle \wedge \mathbf{P}\langle z, y \rangle), \quad (**)$$

where ‘ $\mathbf{S}\langle z, x \rangle$ ’ means that z is a singleton of x , and ‘ $\mathbf{P}\langle z, y \rangle$ ’, that z is a part of y .

The proposal to analyze the concept of set-membership in terms of singleton-membership and parthood can be understood in a sense that is entirely neutral with regard to the metaphysics of set-membership, viz., in a sense according to which ‘S’ and ‘P’ (or whichever other symbols may be chosen instead) are merely two undefined predicates in one’s formal system. For the sake of the present discussion, however, I shall adopt a more metaphysical reading of the proposal, according to which set-membership is not a basic relation, whereas singleton-membership and parthood are.

It is of course possible to hold that set-membership is analyzable in the way just described without assuming that either singleton-membership or parthood are basic relations. Perhaps these latter relations are only ‘more basic’ than set-membership, and can themselves be analyzed in terms of further attributes. (I take it that, with the help of the concept of basicity, the notion of ‘more basic than’ can be adequately explicated, as long as one doesn’t insist that it has to correspond to anything more than a partial ordering.) This theoretical possibility does, however, seem fairly remote, for it is not easy to see in what terms singleton-membership might be analyzed if not in terms of set-membership.⁹ So it appears plausible to assume that, if singleton-membership is not analyzable in terms of set-membership, then singleton-membership is indeed basic.

Similarly, I take it to be a plausible assumption that, if the Bunt–Lewis analysis of set-membership is correct (on a metaphysical reading, so that parthood is, like singleton-membership, ‘more basic’ than set-membership), then parthood is basic. The reason for this is slightly more complicated than in the case of singleton-membership, because we have to distinguish between two broad conceptions of parthood, viz., those that I have in §7.6.3 called the *combinatorial* and the *abstractional* conception, respectively.¹⁰ As we will see below, it can be argued that the Bunt–Lewis analysis of set-membership requires that the parthood relation be conceived of in accordance with the combinatorial conception. But if parthood is

⁹For some swift criticism of potential options, see Lewis (1991, §2.7).

¹⁰To recapitulate, under the combinatorial conception, non-atomic mereological fusions are in a certain sense constructed from their proper parts, whereas, on the abstractional conception, the proper parts of a given mereological whole are as it were abstracted from the latter.

conceived of in this way, it will be difficult to see (just as it was for singleton-membership) in terms of what other attributes this relation might be analyzed; and so appears plausible to assume that parthood is basic.

Let us suppose, then, that set-membership is not basic, that it can be analyzed in the way shown in (**), and that both parthood and singleton-membership are basic relations. Under these assumptions, σ^{\supset} can no longer be regarded as principial, for it would seem that an adequate explanation of O_{\supset} 's truth would have to rest on an ontology that contains the following two specifications:

(a1) Properties of the form ' $\lambda x S\langle x, \alpha \rangle$ ' are individuating.

(a2) Properties of the form ' $\lambda x P\langle \alpha, x \rangle$ ' are individuating.

If O is a true ontology whose only individuating specifications are (a1) and (a2), then any pair of entities that is differentiated by O_{\supset} will also be differentiated by O ; and while an adequate explanation of the truth of O_{\supset} would have to rely on O , the converse seems fairly clearly not to be the case. If so, σ^{\supset} will not be principial, which means that, on the above suppositions, the previous subsection's claim (2) (p. 171) is false. Consequently, to show that the first half of Fine's asymmetry is a consequence of the present account of essentiality even under the above suppositions, we will need an argument that does not rely on (2).

Fortunately, it is relatively easy to construct an alternative argument. In exact analogy to the previous one, we can – instead of (1) and (2) – establish the following two claims:

(1') For any ontology O that contains the specifications (a1) and (a2) and whose domain contains both Socrates and his singleton, it is O -essential to $\{\text{Socrates}\}$ to have Socrates as a member.

(2') There exists at least one systematically optimal ontology that contains (a1) and (a2) and whose domain contains both Socrates and his singleton.

These two claims together entail that there exists a systematically optimal ontology O such that it is O -essential to $\{\text{Socrates}\}$ to have Socrates as a member. So, if we can establish that

they are true, we will have shown that the present account of essentiality accommodates the first half of Fine’s asymmetry even under the assumption that set-membership is not basic but rather analyzable in terms of parthood and singleton-membership.

The argument for (1′) is perfectly analogous to the earlier argument for (1). Thus, if O is an ontology that contains (a1) and (a2) and whose domain contains both Socrates and his singleton, it can be seen that the fully concretized O -essence of $\{\text{Socrates}\}$ must be of the form

$$\lambda z (z = \text{Singleton} \wedge \mathbf{S}\langle \text{Singleton}, \text{Socrates} \rangle \wedge \mathbf{P}\langle \text{Singleton}, \text{Singleton} \rangle \wedge \dots),$$

where ‘Singleton’ is again a constant denoting $\{\text{Socrates}\}$. It will then follow that the state of affairs $E\langle \text{Singleton} \rangle$, where E is the fully concretized O -essence of $\{\text{Socrates}\}$, necessitates the state of affairs $P\langle \text{Singleton} \rangle$, where P is the property

$$\lambda x \exists z (\mathbf{S}\langle z, \text{Socrates} \rangle \wedge \mathbf{P}\langle z, x \rangle).$$

But under the Bunt–Lewis analysis of set-membership, this is just the same as the property of having Socrates as a member. So, if O is systematically optimal, it follows that that property is essential to $\{\text{Socrates}\}$.

It now only remains to establish (2′). To this end, we will have to show that (a1) and (a2) both pay their dues and are parsimonious, untrumped, and principal.

Paying one’s dues

To show that (a1) and (a2) both pay their dues, I will argue that they both pay their dues relative to the systematically optimal ontology O_0 , which contains no individuating specifications at all. (If they are relative to this ontology, they will by definition pay their dues *simpliciter*.) Thus, let O_1 and O_2 be the simplest true ontologies that have (a1) and (a2), respectively, as their only individuating specifications. O_1 will need to contain ontological restrictions to the

effect that (i) nothing is a singleton of more than one thing, and that (ii) nothing has more than one singleton, as well as – possibly – the following two: (iii) that there are no infinitely long descending chains of singleton-membership, and (iv) that there are no more than κ -many non-singletons (for some set-sized cardinality κ). In order for (a1) to pay its dues relative the specificationless ontology O_0 , the complexity of O_1 has to be outweighed by the number of abstract O_1 -essences that this ontology claims to be instantiated. That number is \aleph_0 , i.e., countable infinity. This should be enough to compensate for the complexity of an ontology that consists of only a single individuating specification and a few restrictions. Plausibly, then, (a1) does pay its dues.

Turning now to O_2 , the question of what ontological restrictions it has to contain depends heavily on what mereological principles are true, and the same goes for the number of abstract O_2 -essences that O_2 claims to be instantiated. That number will be smallest if the set of correct mereological principles contains all of the following:¹¹

1. Reflexivity: Everything is a part of itself.
2. Antisymmetry: If x is a part of y , and y is a part of x , then $x = y$.
3. Transitivity: If x is a part of y , and y is a part of z , then x is a part of z .
4. Universal Composition: For every class of entities, there exists a fusion of that class.¹²
5. Uniqueness of Composition: For every class of entities, there exists at most one fusion of that class.
6. There are no more than κ -many atoms,

¹¹Note that I do not mean to commit myself to all of the following principles, but only want to consider the ‘worst-case scenario’ with respect to the number of instantiated abstract O_2 -essences. Admittedly, the inclusion of Antisymmetry in this list is strictly speaking redundant, since this principle is entailed by Uniqueness of Composition together with Transitivity. However, the principle is sufficiently central to the characterization of parthood that its inclusion nevertheless seems warranted.

¹²For the sake of brevity, I will often use plurals rather than to make explicit reference to a class. Thus, if C is the class of all F s, I will sometimes speak of the ‘fusion of the F s’, meaning the fusion of C .

where κ is the number of (mereological) atoms, assuming that there is such a number.¹³ If these six principles are all true, then κ will at the same time be the number of instantiated abstract O_2 -essences, because any two such O_2 -essences will differ from each other only with respect to the number of atoms that it takes to instantiate them.¹⁴

Some might at this point think that κ is zero, on the ground that everything is made up of ‘atomless gunk’. If this hypothesis were correct, then everything would have exactly the same abstract O_2 -essence. But one can maintain this hypothesis only by ignoring (or denying the existence of) certain kinds of abstract entities, such as singletons, which would still have to be regarded as mereological atoms.¹⁵ To be sure, it is possible to deny the existence of singletons, but then there would also be no problem of accommodating Fine’s asymmetry. On the other

¹³Using the concept of parthood as primitive, the concepts of atom and fusion may be defined in the usual way: An *atom* is an entity that has only itself as a part, and something is a *fusion* of entities x_1, x_2, \dots just in case it has all of x_1, x_2, \dots as parts, and each part of it overlaps (i.e., has a part in common with) at least one of them. (Cf. above, p. 153.)

¹⁴To see how the first five principles limit the number of instantiated abstract O_2 -essences, note, first, that Reflexivity rules out the existence of entities that are not parts of themselves, and second, that Antisymmetry and Transitivity together limit the length of parthood-cycles to one: everything is a part of itself, but for any $n > 1$, there are no n things things such that the first is a part of the second, the second a part of the third, \dots , and the n th thing a part of the first. Because there are no such cycles, there are no corresponding instantiated O_2 -essences.

Further, to see the relevance of Universal Composition and Uniqueness of Composition, suppose that x is the fusion of bricks in a house and y the fusion of trees in a forest, where the house and the forest consist of exactly the same number of mereological atoms. (Let us assume that the forest is quite small and/or that the house is very large.) By Transitivity, x will have as its parts also all the atoms that make up the house, and y will have as parts all the atoms that make up the forest. Again by Transitivity, each brick in the house will have as parts all the atoms from which it is made up, and the same holds for each tree in the forest. So x will be the *fusion* of the atoms that make up the house, because it has all of them as parts, and each one of its parts overlaps at least one of them. Analogously, y is the fusion of all the atoms that make up the forest. Now, for each non-empty proper subset S of the set of atoms that make up the house, there exists (by the set-theoretical Axiom Schema of Separation) another set S' such that the union $S \cup S'$ is the set of all the atoms that make up the house, and by Universal Composition, there exists a fusion of S , a fusion of S' , and also a fusion of the latter two fusions. By Transitivity, this third fusion, which has the fusion of S as a part, must be a fusion of all the atoms that make up the house, and so – by Uniqueness of Composition – must be identical with x . Hence, for any subset of the set of those atoms, x has their fusion as a part. Analogously, y has as a part the fusion of the atoms contained in any given subset of the atoms that make up the forest. As a result, it emerges that x and y must have the same abstract O_2 -essence, given that both contain the same number of atoms.

¹⁵Although I take it to be very plausible that singletons are mereological atoms, it is admittedly not easy to say why this should hold. Closer reflection leads us once more into the vicinity of the Julius-Caesar problem. David Lewis argues for the atomicity of singletons in §4.2 of his (1991), but his argument there relies heavily on his axiom of “Distinctness”, which in effect says that no singleton has a part in common with any other singleton or with anything that does not have any singletons as parts. Anyone who doubts the atomicity of singletons will presumably also have doubts about this axiom.

hand, if we do accept the existence of singletons, we will (on pain of having to postulate an arbitrary limit) have to accept that there are infinitely many of them, and so κ will have to be at least \aleph_0 . The number of instantiated abstract O_2 -essences will consequently be at least as high, and so it seems that the contribution that (a2) makes to the discriminatory power of O_2 should be regarded as outweighing its complexity. In this way, as in the case of (a1), we can conclude that (a2) does pay its dues.

Parsimonious

Since I have been assuming that both singleton-membership and parthood are basic, it follows from the corollary stated at the end of §7.5 that both (a1) and (a2) are parsimonious.

Untrumped

In order to see that (a1) is not trumped, let O'_1 be the ontology that consists only of (a1)'s sole permutation, viz.,

(a1') Properties of the form ' $\lambda x \mathbf{S}\langle\alpha, x\rangle$ ' are individuating.

Just like O_ϵ and O_\exists in the analogous portion of §8.1.1 above, O'_1 but not O_1 fails to differentiate_{sc} the pair $(\emptyset, \{\emptyset\})$. Hence, (a1) is not trumped by (a1') and is accordingly untrumped.

We have already seen an analogous argument for the conclusion that (a2) is not trumped, either. In §7.6.3, a notational variant of (a2) was called ' σ^\succeq ', and the ontology that has (a2) as its only statement was accordingly referred to as ' O^\succeq '. Further, the non-identical permutation of σ^\succeq was labeled ' σ^\preceq ', and the name ' O^\preceq ' was used for the ontology that contains σ^\preceq as its only statement. The argument (in §7.6.4, p. 161) ran as follows: Under the assumption of Universal Composition, any two mereological atoms x and y have the same semi-concretized O^\preceq -essence or none at all, and so the pair (x, y) will not be differentiated_{sc} by O^\preceq . By contrast, the pair *is* differentiated_{sc} by O^\succeq , since each of those atoms is ultimately O^\succeq -individuated only by itself. So σ^\succeq is not trumped by σ^\preceq , and is hence untrumped. Of course, σ^\succeq 's notational

variant (a2) will then be untrumped as well.

A weakness of this argument lies in the fact that it relies on the principle of Universal Composition, which might coherently be doubted. Perhaps some mereological atoms are parts of only a finite number of things, whereas others – singletons in particular – are parts of as many things as there are sets. However, we can improve the argument very easily by letting x and y be, not just any two mereological atoms, but two *singletons*. For in that case, whenever there is a fusion of some entities and x , it would seem quite arbitrary to say that there is no fusion of those same entities and the union of x and y . So, whenever x is a part of some fusion F , there will be an entity of which both F and y form a part (at least under the Bunt–Lewis analysis of parthood, since y is a subset, and will hence count as a part, of the union of x and y), and vice versa: whenever y is a part of some fusion F' , there will be an entity of which both F' and x form a part. By this consideration, we see that x and y must be ultimately O^{\preceq} -individuated by exactly the same entities, and moreover, there is absolutely no difference between x and y with respect to the ways in which they are mereologically related to those entities. So they must either have one and the same semi-concretized O^{\preceq} -essence, or none at all. Consequently, O^{\preceq} fails to differentiate_{sc} the pair (x, y) , and σ^{\succeq} again turns out to be untrumped.

Principial

Finally, in order to show that both (a1) and (a2) are principial, we can proceed in roughly the same way as we did in the case of σ^{\exists} in §8.1.1. As above, let O_1 and O_2 be two true ontologies that have (a1) and (a2), respectively, as their only individuating specifications. In §8.1.1, we said that, on the assumption that set-membership is basic, “it is hard to see what ontology should be appealed to in order to give an adequate explanation of the fact that O_{\exists} is true, except for an ontology of urelements” (p. 175). Something exactly analogous can be said of O_1 and O_2 , given that – as we are now assuming – singleton-membership and parthood are basic. In particular, it is plausible that the only sort of ontology on which we would have to rely in order to explain the truth of O_1 will be an ontology of non-singletons, and that the only sort of

ontology on which we would have to rely in order to explain the truth of O_2 will be an ontology of mereological atoms. (By an ‘ontology of non-singletons’, I here mean an ontology whose specifications do not make any reference, tacit or otherwise, to singleton-membership, while an ‘ontology of mereological atoms’ is one whose specifications do not make any reference to parthood.¹⁶) And now, in further analogy to the argument in §8.1.1, we can ask: What plausible ontology of non-singletons will distinguish between singletons? And what plausible ontology of mereological atoms will distinguish between fusions?

As above, these questions lead us into the vicinity of the Julius-Caesar problem. For suppose it were suggested that some singletons, say $\{\emptyset\}$ and $\{\{\emptyset\}\}$, are respectively identical with two entities (perhaps an electron and a positron) that can be differentiated by other than set-theoretic means. Then those two singletons could be differentiated without making reference to singleton-membership; and something analogous might be said for non-atomic fusions.

In both of these cases, the suggested identifications are implausible, but the challenge is to explain why they should be false. However, I do not think that we have here the resources available to take up this challenge, and so I shall simply have to appeal to intuition in order to defend the claim that no two singletons are identical with such things as electrons and positrons, or to any other pairs of entities that can be differentiated without making reference to the relation of singleton-membership. In effect, the same considerations that we have above (p. 176) applied to sets can here be analogously applied to singletons (on the assumption that singleton-membership is basic), with the result that no ontology of non-singletons will differentiate between singletons. If this is correct, it follows that (a1) is principal.

We can proceed similarly with respect to (a2) and non-atomic fusions. At least on a conception of parthood that regards such fusions as combinatorial ‘constructions’ built up from their parts, it seems highly implausible to say that non-atomic fusions should be differentiable without reference to parthood. For if one does indeed regard them as constructions, one

¹⁶For a formal definition of what it means for a specification to ‘make reference’ to a given attribute, see footnote 7, p. 175 above.

would naturally conceive of them as abstract entities, in the same way in which sets are usually regarded as abstract. But if so, the same considerations that we have above applied to sets will again apply here. In particular, non-atomic fusions will appear to be ineligible for the instantiation of purely physical attributes such as location and momentum. One might admittedly want to insist that non-atomic fusions inherit their location and momentum from their proper parts. However, this move will be admissible only if it is granted that the relevant attributes of location and momentum have an analysis in terms of parthood (among other things), for it would otherwise be deeply mysterious on what sort of evidence one could say that non-atomic fusions inherit their location or momentum from their proper parts.

Nor do these considerations only apply to physical attributes. Just as it would be implausible to say that two non-atomic fusions differ with respect to purely physical characteristics, it would also be implausible to say that any pair of non-atomic fusions can be differentiated by reference to the relation of singleton-membership, e.g., in virtue of one fusion's being a singleton of some further entity while the other is a singleton of nothing. So it appears that we can quite generally say that no ontology of mereological atoms will differentiate between non-atomic fusions, whence it follows that (a2) is principal.

The considerations of the previous two paragraphs turn heavily on what I have called the 'combinatorial conception' of parthood, under which non-atomic fusions are best regarded as a species of abstract object. But what if one adopted instead an *abstractional* conception, under which the proper parts of any given mereological whole are thought of as (in a certain sense) 'abstracted' from the latter?

Under this latter conception, it might turn out that there are no mereological atoms other than abstract entities (such as singletons), because it would be possible that the whole physical world consists of atomless gunk. (For it might turn out that, from every part that can be 'abstracted' from a greater mereological whole, it is possible to abstract yet further parts.) Suppose that this is in fact the case. It seems that even then, there would have to be at least one entity that instantiates purely physical attributes, for it certainly wouldn't do to say that there is nothing more to the world than that it is a piece of atomless gunk. Monists will

presumably hold that there is only *one* entity that instantiates purely physical properties, viz., the whole physical world. But it is also conceivable that there are multiple such entities, and even that some of them are mereological atoms.¹⁷ By contrast, it is much less plausible to say that the bearers of the purely physical attributes should be any of the various mereological wholes (or atoms, if there are any) that are proper parts of the physical world, for presumably these are merely ‘abstractions’ from the whole physical world itself. If this is correct, then, while an adequate explanation of O_2 ’s truth will on the one hand have to rest on an ontology that is formulated entirely in terms of purely physical attributes, this latter ontology will, on the other hand, not differentiate any pair of entities that is also differentiated by O_2 . Consequently, the fact that O_2 ’s truth has to be explained on the basis of another ontology will not pose any danger to the principiality of (a2).

This is not to say, however, that we could continue to take (a2) to be principial if we adopted the abstractional conception of parthood. For it might be – and in fact it seems plausible – that the parthood relation that this conception corresponds to is not basic. If so, (a2) might fail to be principial in the same way in which we have above seen that σ^\exists fails to be principial under the (metaphysically interpreted) Bunt–Lewis analysis of set-membership.

But still, it is not without reason that we have been assuming that parthood is basic. After all, if the Bunt–Lewis analysis is to have any merit, it has to be applicable to sets, and sets have to be fusions of the singletons of their members. Yet under the abstractional conception of parthood, it is very doubtful that singletons have any fusions at all. For in what sense could we say that a singleton like $\{\emptyset\}$ is ‘abstracted’ from anything? There would be little to recommend the Bunt–Lewis analysis if we had no idea how the parthood relation behaves with respect to singletons. Given that, under the abstractional conception, it is not even clear that any singleton is a part of anything, the analysis would lose all plausibility. For this reason, it appears that the Bunt–Lewis analysis of set-membership can only be plausibly adopted in

¹⁷The reason for this is that the physical world – conceived of, roughly, as the thing of which all observed phenomena are a part – might be nothing else than the instantiation of a hugely complex attribute by a certain number of entities. Since this would not require that those entities have any parts themselves, it is possible, for all we can tell, that they might be atomic.

conjunction with the combinatorial conception of parthood. But under *this* conception, it is difficult to see in terms of what other attributes parthood might be analyzed.¹⁸ This in turn suggests that, under the Bunt–Lewis analysis of set-membership, the parthood relation must in fact be regarded as basic. So, again, there is no apparent reason to doubt the principality of (a2).

* * *

This completes the argument for (2′). We can accordingly conclude that, on the assumption that set-membership is analyzable in the way proposed by Bunt and Lewis, it still follows that it is essential to {Socrates} to have Socrates as a member.

The present argument and that of §8.1.1 together cover what appear to be the two most plausible theoretical options with regard to the question of whether set-membership is basic, or how it should be analyzed if it isn’t. If this is correct, it will be fair to say that the present account does indeed accommodate the first half of Fine’s asymmetry.

8.2 Why Socrates Is Not Essentially a Member of {Socrates}

Let us now turn to the second half. Here we have to show that the property of being a member of {Socrates} is on the present account *not* essential to Socrates, or in other words: there is no systematically optimal ontology O such that the property of being a member of {Socrates} is O -essential to Socrates. In order to see that this holds, we have to ask, first, what individuating specifications an ontology O has to contain in order for the property of being a member of {Socrates} to be O -essential to Socrates, and, second, whether any ontology that contains such individuating specifications is systematically optimal.

¹⁸A potential candidate might have been set-membership, but this is in the present context ruled out, given that parthood is here assumed to be more basic than set-membership.

8.2.1 The Specification σ^ϵ

The most straightforward way in which it might turn out that it is O -essential to Socrates to be a member of $\{\text{Socrates}\}$ is for O to contain the specification σ^ϵ , according to which properties of the form ‘ $\lambda x(x \in \alpha)$ ’ are individuating. As already noted in §7.6.1, the effect that σ^ϵ has on an ontology’s existence claims can be described by saying that it “stands the set-theoretic universe on its head” (p. 146): An ontology that contains σ^ϵ and no ontological restrictions will claim (among other things) that there are entities that are not members of anything, as well as that there are entities that are members of precisely one thing each. By contrast, such an ontology will *not* claim that there are memberless entities, singletons, or any well-founded sets.

To rule out the existence of systematically optimal ontologies that contain σ^ϵ , we (in §7.6) modified the definition of ‘systematically optimal’ by adding the requirement that no individuating specification of a systematically optimal ontology should be trumped. This was necessary because, under certain relevant circumstances, it turns out that σ^ϵ both pays its dues and is parsimonious, and thereby satisfies both of the requirements that had been introduced up to that point. Since the final definition of ‘systematically optimal’ also requires an ontology’s individuating specifications to be principial, there are now two potential obstacles to the existence of a systematically optimal ontology that contains σ^ϵ : Namely, that this specification might be trumped, or that it might fail to be principial. For the sake of brevity, I will here not try to show that σ^ϵ fails to be principial, but will instead focus only on arguing that it is trumped. We have already seen above (p. 175) that there are some pairs of entities – as for instance $(\emptyset, \{\emptyset\})$ – that are differentiated_{sc} by O^\supset but not by O^ϵ . So, to establish that σ^\supset trumps σ^ϵ , it only remains to show that O^\supset differentiates_{sc} *all* the pairs of entities that are differentiated_{sc} by O^ϵ .

Let us say that a *circular set* is any non-well-founded set that, whether directly or indirectly, has itself as a member. Suppose further that (x, y) is a pair of entities that is differentiated_{sc} by O^ϵ . If neither x nor y were a circular set, they would have the same semi-

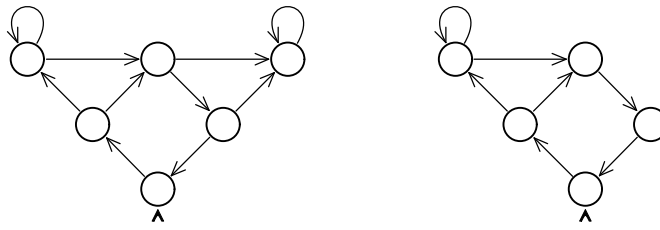


Figure 8.1: An accessible pointed graph and its circular part.

concretized O^ϵ -essence, and so this case is already ruled out. The only remaining possibilities are as follows: either (i) one of the two entities x and y is a circular set while the other isn't, or (ii) both of them are circular sets. So, for both of these cases, we have to show that the pair (x, y) is differentiated_{sc} by O^\exists . The first case is trivial, since the abstract (and hence the semi-concretized) O^\exists -essence of a circular set will obviously be different from that of a non-circular set or urelement.

The second case is only slightly more complicated. For any given pointed graph, let us say that the *circular part* of it is that subgraph that consists of all those cycles that lead back to the graph's point. This is illustrated in Figure 8.1: the right-hand side of the diagram depicts the circular part of the pointed graph that is shown on the left-hand side. Now, by hypothesis, x and y have different semi-concretized O^ϵ -essences. Since neither of them is ultimately O^ϵ -individuated by anything at all, this means that they also have different *abstract* O^ϵ -essences, and hence different O^ϵ -individuation graphs.¹⁹ It is further clear that, if the O^ϵ -individuation graphs of any two entities differ from each other, then these graphs must differ with respect to their circular parts; for any two entities are exactly alike with respect to their set-theoretic 'superstructures', except only for the cycles by which these entities (if they happen to be circular sets) are directly or indirectly members of themselves. But then it follows that, if any two entities differ with respect to their O^ϵ -individuation graphs, their O^\exists -individuation graphs must also differ. So x and y will have different abstract O^\exists -essences, and this in turn means that they will have different semi-concretized O^\exists -essences, which is what we needed to

¹⁹Of course, for the purpose of assessing whether two graphs differ from each other, the intrinsic structure of their nodes should be left out of account.

show.

This completes the argument for the thesis that no systematically optimal ontology contains σ^\in . However, this conclusion still leaves open the possibility that there may exist some other specification, or more generally, some *set* of specifications, that fulfills the following two conditions:

(C1) For any ontology O that contains the specification(s) in question, and whose domain contains both Socrates and his singleton, the property of being a member of $\{\text{Socrates}\}$ is O -essential to Socrates; and

(C2) There exists at least one such ontology that is systematically optimal.

If there exists any specification (or set thereof) that satisfies these conditions, then it will on the present account be essential to Socrates to be a member of $\{\text{Socrates}\}$. So the goal for the rest of this section will be to show that there are no such specifications.

There are two ways in which one might think that a specification (or set thereof) that fulfills (C1) and (C2) can be constructed. First, it might be thought that the formula ' $\alpha \in x$ ' from the matrix of the λ -expression in σ^\in could be incorporated in some more complex specification that, unlike σ^\in itself, is *not* trumped by any of its permutations. Second, one might think that the Bunt–Lewis analysis of set-membership offers hope for the construction of individuating specifications that satisfy the two conditions. Let us consider these two hypotheses in turn.

8.2.2 Variants of σ^\in

The following specification is a complex variant of σ^\in that is not trumped by any of its permutations:

(b1) Properties of the form ' $\lambda x (x \in \alpha \wedge (\beta = x \vee \beta \in x))$ ' are individuating.

The predicate derived from this is the ternary ' $\lambda x, \alpha, \beta (x \in \alpha \wedge (\beta = x \vee \beta \in x))$ ', whose instantiation by entities x, y, z is the state of affairs that x is a member of y , and z is either

identical with x or a member of x . Let O_1 be a true ontology that contains (b1) as its only specification, and whose domain contains both Socrates and his singleton.

For any entity x , an O_1 -individuation graph of x would have to contain a node both for each entity that has x as a member and for each entity that is a member of x (as well as for each entity that has a member of x as a member, etc.). It thus follows, as it does in the case of σ^ϵ , that no entity will have an O_1 -individuation graph, unless the relevant conception of graph is relaxed so as to allow graphs that have as many nodes and edges as there are sets. Suppose, then, that such a more liberal conception of graph, as well as correspondingly liberal conceptions of assignment and attribute, can be coherently adopted, so that entities may be said to have O_1 -individuation graphs as well as O_1 -essences. It can then be seen that (b1) satisfies condition (C1): for Socrates' instantiation of his fully concretized O_1 -essence will necessitate the state of affairs that Socrates is a member of {Socrates}, and so it is O_1 -essential to Socrates to be a member of {Socrates}. It is clear that this holds not only for O_1 but for any ontology that has (b1) as an individuating specification and whose domain contains both Socrates and his singleton.

Further, it can be seen that any two entities that have distinct abstract O_\exists -essences also have distinct abstract O_1 -essences. Accordingly, (b1) will plausibly not fail to pay its dues relative to the specificationless ontology O_0 , nor, therefore, *simpliciter*. Nor will it be trumped, as can be verified by considering its various permutations. And finally, it can be argued that it is principal.

That (b1) should be principal might at first be doubted, because it might be thought that an adequate explanation of O_1 's truth has to rely on O_\exists , due to the relative simplicity of this latter ontology. However, O_\exists does not entail any state of affairs that is not also entailed by O_1 , and this makes it somewhat implausible to say that an adequate explanation of O_1 's truth will rely on O_\exists .²⁰ But if an adequate explanation of O_1 's truth need not appeal to O_\exists or, for

²⁰Against this, some might raise the objection that, on the face of it, the truth of any given conjunction is to be explained by its conjuncts, or (more generally) by any of the smaller conjunctions that are made up from those conjuncts. From here, one could easily generalize to the thesis that, if an ontology O entails a proper subset of the states of affairs that are entailed by another ontology O' , then an adequate explanation

that matter, to any other set-theoretic ontology, then there seems to be no good reason why (b1) should not count as principal.

Nevertheless, there will still be no systematically optimal ontology containing (b1), because this specification is not parsimonious. To see this, it is enough to consider that the following specification results from a pruning of (b1):

(b1') Properties of the form ' $\lambda x (\top \wedge (\perp \vee \beta \in x))$ ' are individuating.

This specification is semantically equivalent to σ^\exists , which (as has been argued in §8.1.1 above) pays its dues. Consequently, it pays its dues as well, and so (b1) is not parsimonious.

In §8.2.1, we have seen that σ^\in is parsimonious but not untrumped; and now (b1), while untrumped, has turned out not to be parsimonious. This naturally raises the question of whether there are any variants of σ^\in that satisfy (C1) and are *both* parsimonious and untrumped.

To see how one might construct such a variant, let us begin by considering how one would have to modify (b1) in order to obtain a parsimonious specification. In part, the reason why (b1) fails to be parsimonious lies in the fact that the matrix of the embedded λ -expression contains the disjunct ' $\beta \in x$ '. If this disjunct were simply omitted (or rather, replaced by a ' \perp '), the resulting specification would be trumped in the same way in which σ^\in is trumped, viz., due to the fact that it still contains the formula ' $x \in \alpha$ '. So we might instead try replacing the first formula, i.e., ' $\beta \in x$ ', with something else. But here we seem to be caught in a dilemma between producing a specification that is trumped and producing a specification that fails to be parsimonious. For to ensure that the resulting specification is not trumped, the formula that is used to replace ' $\beta \in x$ ' must, roughly put, contain an ' x ' on the right-hand side of an ' \in ', and a meta-variable (like ' α ' or ' β ') on the left-hand side. There is of

of the truth of O' will have to appeal to O . But this reasoning cannot be quite right, for it would be absurd to claim that, whenever a given proposition is logically weaker than another, the truth of the latter has to be explained on the basis of the former. I would instead suggest that an adequate explanation of the fact that a given proposition p is true always has to provide some *additional* information, unless that fact is already adequately explained by p itself (which has to be possible, given that explanations have to come to an end somewhere). To apply this to ontologies: for any ontology O , an adequate explanation of the fact that O is true will have to consist in a set of statements (possibly another ontology) that entails some state of affairs that is not entailed by O , unless O 's truth is already adequately explained by O itself.

course some scope for variation here. One might, for instance, replace ‘ $\beta \in x$ ’ with the formula ‘ $\exists y (\beta \in y \wedge y \in x)$ ’, but the general pattern will be clear. However, if one uses such a formula to replace the original ‘ $\beta \in x$ ’, the result will always fail to be parsimonious. Suppose, e.g., that we replace that formula with the mentioned ‘ $\exists y (\beta \in y \wedge y \in x)$ ’. This yields

(b2) Properties of the form ‘ $\lambda x (x \in \alpha \wedge (\beta = x \vee \exists y (\beta \in y \wedge y \in x)))$ ’ are individuating.

This specification, in turn, can be pruned to produce

(b2′) Properties of the form ‘ $\lambda x (\top \wedge (\perp \vee \exists y (\beta \in y \wedge y \in x)))$ ’ are individuating,

which can be seen to pay its dues just like (b1′) does. Consequently, (b2) fails to be parsimonious: this illustrates the second horn of the dilemma.

To avoid the dilemma, let us now backtrack somewhat and ask why (b1) has to contain that second conjunct – i.e., ‘ $(\beta = x \vee \beta \in x)$ ’ – in the first place. The immediate reason for this is that (b1) would otherwise be trumped; and the reason for *this* is that (b1) contains the formula ‘ $x \in \alpha$ ’. So one might naturally wonder whether this last formula cannot be replaced with something else. But here our hands are tied by the constraint (C1): if the specification that results from such a replacement is to pose any threat to the present account’s ability to accommodate the second half of Fine’s asymmetry, then that specification has to be such that, if an ontology O contains it (and if O ’s domain contains both Socrates and his singleton), it will be O -essential to Socrates to be a member of {Socrates}. For this reason, any formula that is used to replace ‘ $x \in \alpha$ ’ will have to contain an ‘ \in ’ that is flanked on the left-hand side by ‘ x ’ and on the right-hand side by either (i) a meta-variable or (ii) a constant denoting {Socrates}. (Again, of course, there is some scope for variation.) The first option invariably leads to the dilemma that we just encountered, viz., between a specification that is trumped and one that is non-parsimonious. So we are left with only the second option.

Using ‘Singleton’ as a constant denoting {Socrates}, we can replace the ‘ $x \in \alpha$ ’ with, e.g., ‘ $x \in \text{Singleton}$ ’, and omit the second conjunct in (b1)’s λ -expression altogether (since there is now no danger of the specification’s being trumped). The result will be

(b3) Properties of the form ‘ $\lambda x (x \in \text{Singleton})$ ’ are individuating.

This specification obviously satisfies the constraint (C1), and since the predicate derived from it is unary, it is itself its only permutation and therefore trivially untrumped. Moreover, it is clearly parsimonious. On the negative side, however, it does not pay its dues (by virtually any standard), nor is it principal. The reason why (b3) does not pay its dues lies in the fact that the predicate derived from it is satisfied by only a single entity, viz., Socrates. As a result, the contribution that (b3) makes to the discriminatory power of any given ontology will at most be 1, which is hardly worth the added complexity.

Further, suppose that O_3 is a true ontology that contains (b3) as its only individuating specification. On the face of it, the fact that O_3 is true calls for an explanation. For O_3 claims, among other things, that there exists exactly one entity that is a member of “Singleton”, and so one might ask why there should exist such a thing as this “Singleton”. And it seems that an adequate explanation of this fact can only be provided by a more encompassing ontology of people and their sets. An admittedly quite rudimentary example of such an ontology would be one that contains the following two as its only individuating specifications:

(c1) Properties of the form ‘ $\lambda x (x = \text{Socrates})$ ’ are individuating.

(c2) Properties of the form ‘ $\lambda x (\alpha \in x)$ ’ are individuating.

If C is a true ontology that contains both of these specifications, then one of the claims that C entails will be to the effect that there exists an entity that has Socrates as its only member. So this ontology C can be used in explaining why there exists such a thing as “Singleton” (viz., if one adds the ‘bridge principle’ that what O_3 refers to as ‘Singleton’ is nothing else than what in C is known as the entity that has Socrates as its only member), and so can be used in an adequate explanation of the fact that O_3 is true. On the other hand, O_3 has apparently no place in an adequate explanation of the fact that C is true. And further, while O_3 differentiates between Socrates and anything that is not Socrates, the same is true for C . So, apart from not paying its dues, (b3) also fails to be principal.

From the above considerations, it may seem impossible to modify (b1) in such a way that the result is a specification that satisfies (C1) and at the same time pays its dues and is parsimonious, untrumped, and principal. However, all the modifications that we have discussed so far make some use of a set-membership predicate. It might thus still be thought an open question whether the Bunt–Lewis analysis of set-membership affords a way of constructing specifications that jointly satisfy (C1) and (C2).

8.2.3 Specifications Using Predicates of Singleton-Membership and Parthood

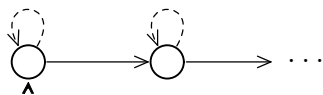
In the rest of this section, I shall again assume (as in §8.1.2 above) that set-membership is not a basic relation but can rather be analyzed in the way proposed by Bunt and Lewis, and that both singleton-membership and parthood are basic.²¹ The analysis of set-membership in terms of singleton-membership and parthood makes it possible to construct, without any direct reference to set-membership, an ontology O that is such that it is O -essential to Socrates to be a member of {Socrates}. In particular, suppose that O is a true ontology whose domain contains both Socrates and his singleton, and whose list of individuating specifications includes the following two:

- (d1) Properties of the form ‘ $\lambda x S\langle\alpha, x\rangle$ ’ are individuating.
- (d2) Properties of the form ‘ $\lambda x P\langle x, \alpha\rangle$ ’ are individuating.

We have already encountered these specifications in earlier sections. In §7.6.3 and §8.1.2, a notational variant of (d2) was referred to as ‘ σ^{\neq} ’, and in the latter section, (d1) was labeled ‘(a1’)’. (To recapitulate, the constants ‘S’ and ‘P’ can be read as ‘singleton-of’ and ‘part-of’, respectively.)

²¹As I have argued above, it is plausible to regard both singleton-membership and parthood as basic if, as per the metaphysical reading of the Bunt–Lewis analysis, both singleton-membership and parthood are taken to be ‘more basic’ than set-membership.

If it is to be O -essential to Socrates to be a member of the singleton {Socrates}, then Socrates' O -individuation graph will have to contain a node for each entity of which he is a part. In addition, the graph will have to contain a node for the singleton, a node for every entity of which the singleton is a part, a node for every singleton of such an entity, and so on. As above, let us here assume that the huge number of nodes and edges that such a graph would have to contain poses no obstacle to its existence. Then a very small fragment of Socrates' O -individuation graph may be depicted as follows:



Here the solid arrows represent singleton-membership, while the dashed arrows represent parthood. An O -decoration of this graph that assigns Socrates to the left-hand node would thus have to assign his singleton to the right-hand node. Given this information, we can infer from the diagram that the property

$$\lambda x \exists y (\mathbf{S}\langle y, x \rangle \wedge \mathbf{P}\langle y, \text{Singleton} \rangle)$$

is O -essential to Socrates. But under the Bunt–Lewis analysis, this property is nothing else than the property of being a member of {Socrates}. Hence, if O were systematically optimal, it would on the present account be essential to Socrates to be a member of his singleton. What thus needs to be show is that O is *not* systematically optimal, which means that we have to show that either (d1) or (d2) fails to pay its dues or is non-parsimonious, trumped, or non-principial.

Now, both (d1) and (d2) are parsimonious, for – given that both singleton-membership and parthood are basic, as we are here assuming – each of those two specifications is such that the predicate derived from it denotes a basic attribute. Further, there are relevant cases in which both (d1) and (d2) will plausibly pay their dues. For let O_1 and O_2 be true ontologies that have (d1) and (d2), respectively, as their only individuating specification. Then (d1) will plausibly pay its dues if singleton-membership admits of cycles of any finite length, because

there will then be infinitely many instantiated abstract O_1 -essences. And (d2) will plausibly pay its dues if Universal Composition is false and sufficiently many entities differ from each other with respect to the number of mereological wholes of which they form a part.²² Since these two scenarios are compatible with each other, we can coherently conceive of cases where both (d1) and (d2) appear to pay their dues. And these cases are relevant for the present purposes because, even if we assume that singleton-membership admits of cycles and Universal Composition fails, it will still be intuitively plausible that Socrates is not essentially a member of his singleton.

It is fortunate, then, that arguably neither (d1) nor (d2) is untrumped. To see this, consider their respective non-identical permutations:

(d1') Properties of the form ' $\lambda x \mathbf{S}\langle x, \alpha \rangle$ ' are individuating.

(d2') Properties of the form ' $\lambda x \mathbf{P}\langle \alpha, x \rangle$ ' are individuating.

Like (d1) and (d2), these two specifications are already familiar from earlier sections: in §8.1.2, (d1') was labelled '(a1)', and in §7.6.3, a notational variant of (d2') was referred to as ' σ^{\succ} '. Let now O'_1 and O'_2 be true ontologies that respectively contain (d1') and (d2') as their only individuating specifications.

An argument for the conclusion that (d1) is trumped can be easily derived from the argument by which we have in §8.2.1 shown that σ^{\in} is trumped. All that has to be done is to restrict the scope of those considerations to singletons rather than sets, and to replace the references to set-membership with references to singleton-membership. In this way, it can be argued that

- (1) O'_1 differentiates_{sc} some pairs of entities that are not differentiated_{sc} by O_1 ,

and moreover that

- (2) O'_1 differentiates_{sc} every pair of entities that is differentiated_{sc} by O_1 .

²²Cf. above, p. 155.

As for (1), it has already been mentioned that the pair $(\emptyset, \{\emptyset\})$ is differentiated_{sc} by O'_1 but not by O_1 .²³ To establish that (2) holds as well, let us first introduce the notion of a ‘circular singleton’ and say that a singleton is *circular* just in case it is, directly or indirectly, a singleton of itself. Further, suppose that (x, y) is a pair of entities that is differentiated_{sc} by O_1 . Since any two entities have the same semi-concretized O_1 -essence unless at least one of them is a circular singleton, we have to consider only the following two cases: Either (i) one of the entities x and y is a circular singleton while the other isn’t, or (ii) both of them are circular singletons. So what we have to show is that, in both of these cases, any pair (x, y) that is differentiated_{sc} by O_1 is also differentiated_{sc} by O'_1 . But this can be done in a way that is exactly analogous to the argument given in §8.2.1, and so I will here omit the rest of the argument.

Having established that (d1) is trumped, there is no need also to show that (d2) is trumped in order to argue that there exists no systematically optimal ontology that contains both (d1) and (d2). However, the question of whether (d2) is trumped is of some independent interest in connection with the mereological analogue of Fine’s asymmetry that we first discussed in §7.6.3, and so it should be worthwhile to address it here as well. According to that mereological asymmetry, it is on the one hand essential to the fusion of a given class of entities to have x as a part, where x is any member of that class; but on the other hand it is *not* essential to x to be a member of that fusion. We may call these two theses the ‘first’ and the ‘second’ half of the mereological asymmetry, respectively.

Both halves seem very plausible under a combinatorial conception of parthood. By contrast, under an *abstractional* conception, at least the second half is arguably much less plausible. For if the proper parts of a given entity are thought of as ‘abstractions’ from the latter, it will be natural to think of them as being in a certain sense dependent on that entity, which in turn suggests that it is essential to each of them to be a part of it, thereby contradicting the second half of the mereological asymmetry. In the following discussion, I will therefore

²³See above, p. 183. (But note that the reference of the names ‘ O_1 ’ and ‘ O'_1 ’ has here been switched.)

presuppose a combinatorial conception of parthood.

Roughly speaking, one could say that (d2) is for the mereological asymmetry what σ^{\in} is for Fine's asymmetry. That is, if there should exist any systematically optimal ontology that contains (d2), then, whenever an entity x is a part of another entity y , it will on the present account be essential to x to be a part of y , which again contradicts the second half of the mereological asymmetry. However, just as σ^{\in} is trumped by its permutation σ^{\ni} , it can be argued that (d2) is trumped by (d2'), which will then mean that no systematically optimal ontology contains (d2), after all.

To show that (d2) is trumped by (d2'), we have to establish the following two theses:

- (3) O'_2 differentiates_{sc} some pairs of entities that are not differentiated_{sc} by O_2 .
- (4) O'_2 differentiates_{sc} every pair of entities that is differentiated_{sc} by O_2 .

The first thesis (3) has already been established in §8.1.2, when we saw that any given pair of distinct singletons is differentiated_{sc} by O^{\supset} but not by O^{\preceq} .²⁴ The differences between O_2 and O^{\preceq} , and between O'_2 and O^{\supset} , do not matter for the present purposes: all four contain only one individuating specification each, and the specifications of O_2 and O'_2 , i.e., (d2) and (d2'), are merely notational variants of σ^{\preceq} and σ^{\supset} . The above argument for the conclusion that any pair of distinct singletons will fail to be differentiated_{sc} by O^{\preceq} admittedly rests on the thesis that a set has its subsets as parts. But, at least on a combinatorial conception of parthood, this seems entirely acceptable.²⁵

To turn now to (4), we have to show that any two entities that differ with respect to their semi-concretized O_2 -essences also have different semi-concretized O'_2 -essences. This thesis can be easily seen to hold under the following three assumptions:

- (i) Parthood is reflexive, anti-symmetric, and transitive.
- (ii) If an entity x contains every proper part of an entity y , then y is a part of x .

²⁴See p. 184 above.

²⁵Fine (2010, p. 579) suggests that the 'canonical' parts of a set are its members, but nevertheless allows that the subsets of a set may be regarded as parts of the latter, albeit only in a derivative sense.

(iii) There is no atomless gunk.

In classical mereology, the first two of these theses are generally accepted as true.²⁶ Under these assumptions, if x and y are any two entities, then the class of atoms that compose x will be distinct from the class of atoms that compose y . And since x and y are ultimately O'_2 -individuated by their atoms, this further means that they will have different semi-concretized O'_2 -essences. Hence, every pair of distinct entities, and *a fortiori* every pair of entities that is differentiated_{sc} by O_2 , is also differentiated_{sc} by O'_2 .

8.2.4 An Objection from Gunk

At this point, it may be objected that (iii), i.e., the assumption of atomism, may very well be false, and in fact various philosophers have thought that it *is* false.²⁷

There are two things to be said in reply to this objection. First, under the combinatorial conception of parthood that I am here presupposing, non-atomic fusions are best regarded as a species of abstract object, much like sets. They are ‘constructions’ whose existence, given the existence of their proper parts, is not due to some additional fact of nature. What sort of fact *is* responsible for their existence will depend on what exactly the correct general metaphysics of abstract objects happens to be. Perhaps they should be regarded as mere fictions (in which case they do not strictly speaking exist at all), or their existence should be taken to depend on facts about our minds, or on facts about a special Platonic realm. In any event, as already noted above (p. 186), it does not seem plausible to think of them as instantiating any purely physical attributes. Hence, if any non-atomic fusions should instantiate what we would regard as physical attributes, such as location or momentum, then these attributes must be regarded as analyzable in terms of parthood (among other things). And so, the fact that a given non-atomic fusion F should have such-and-such a location (say) is not a purely physical fact, but should rather be taken to be a matter of the locations of F ’s atomic parts. The ‘location’

²⁶Cf. Simons (1987, ch. 1), where (ii) is discussed under the name ‘Proper Parts Principle’.

²⁷A prominent example is Whitehead (1925), though he would change his view in later years. (For a concise account of the development of Whitehead’s metaphysics, see Simons (2009).)

attributes that we refer to when we talk of the (so-called) location of a non-atomic fusion must therefore be regarded as distinct from the attributes that we refer to when we talk of the location of mereological atoms. The former, but not the latter, are partly analyzable in terms of parthood. The same goes for other physical characteristics, such as momentum and mass.

If this much is correct, and if the non-atomic objects that we take ourselves to be surrounded with – such as chairs and tables – are non-atomic fusions, as conceived of in accordance with a combinatorial conception of parthood, then none of those objects can plausibly be regarded as atomless gunk; for the evident physical characteristics of any such object would have to be ultimately inherited from its atomic parts. More generally, if it is true that every part of the physical world has some physical characteristics, then it follows that (plausibly) no part of the physical world is a piece of atomless gunk. It might thus be easiest to accept the existence of gunk if one thinks of each piece of gunk as entirely devoid of physical characteristics, in the same way in which one might think of pure non-well-founded sets. It is not clear, however, how one would motivate the existence of such things.²⁸

²⁸Incidentally, it seems that there may be more to be said in favor of pure non-well-founded sets than there is (under a combinatorial conception of parthood) to be said for gunk. The reason for this has to do with the fact that parthood, but not set-membership, is generally accepted to be transitive. What motivates the view that parthood is transitive? The underlying idea appears to be one of a ‘collapse of levels’: e.g., that there should not be a difference between, on the one hand, the fusion of $\{a, b, c, d\}$ and, on the other hand, the fusion of the fusions of, respectively, $\{a, b\}$ and $\{c, d\}$. An obvious consequence of this collapse is that any two fusions must differ from each other with respect to what is found on the bottom-most level, i.e., with respect to the atoms from which they are ultimately composed, provided there is such a level. But of course, there is no such level in the case of gunk. As a result, what we find when we collapse all the levels of a piece of gunk is precisely nothing. It thus emerges that there is no real difference between a piece of gunk and the controversial ‘null individual’ that, in some mereological systems, is supposed to be a part of everything whatsoever. Further, since every piece of gunk has by definition other pieces of gunk as proper parts, it follows that, if there is any gunk at all, there will have to be infinitely many pieces of gunk, each of which is essentially a null individual. Thus, if already the postulation of a single null individual strikes us as counter-intuitive, the postulation of gunk should seem even more bizarre.

By contrast, if we accept an *abstractional* conception of parthood, it will seem less natural to motivate the transitivity of parthood by considerations as to how fusions differ among each other. Instead, the focus can be expected to lie on the parthood relation itself, which, under this conception, is understood as holding between two entities just in case the one is ‘abstracted’ from the latter. So, for instance, one might find it plausible to say that, if two abstraction operations are concatenated, the result will also be an abstraction operation. It would then follow that, if x is abstracted from a certain mereological whole y , and y is abstracted from a still greater whole z , then x will thereby also count as abstracted from z , and will hence count as a proper part of z .

Secondly, even if atomless gunk exists, this will not yet be sufficient to falsify (4). In the first place, it does not follow that any two pieces of gunk have the same semi-concretized O'_2 -essence. It is true that any two pieces of gunk would have infinitely many other pieces of gunk as parts; but there are different kinds of infinity, and so pieces of gunk may still differ with regard to their respective numbers of parts, and hence with regard to their semi-concretized O'_2 -essences. But let us suppose that there is at least one pair of entities (x, y) such that x and y are pieces of gunk that have exactly, say, \aleph_0 parts. Then this pair will not be differentiated by O'_2 . It still does not follow, however, that (4) is false, because nothing we have said so far requires that x and y should differ with respect to their semi-concretized O_2 -essences. Broadly speaking, there are two possible ways in which it might happen that they do.

First, it might happen that the set of things by which x is ultimately O_2 -individuated differs from the set of things by which y is ultimately O_2 -individuated. (By way of an example, suppose that x is a part of some entity X whereas y is a part of some other entity Y , and that neither X nor Y are parts of anything but themselves.) Second, it might be that x and y are ultimately O_2 -individuated by the same set of things, but that the way in which x is related to these things by its semi-concretized O_2 -essence differs from the way in which y is related to them by *its* semi-concretized O_2 -essence. To put this more precisely, let $\{z_1, z_2, \dots\}$ be the set of entities by which x and y are ultimately O_2 -individuated. Then the possibility in question amounts to there being a relation R such that (i) the state of affairs $E_x\langle x \rangle$ necessitates $R\langle x, z_1, z_2, \dots \rangle$ while (ii) the state of affairs $E_y\langle y \rangle$ does not necessitate $R\langle y, z_1, z_2, \dots \rangle$, where E_x and E_y are the semi-concretized O_2 -essences of, respectively, x and y .

Both of these two possibilities require the failure of Universal Composition. For if Universal Composition holds, then there will be a fusion of everything, and everything will be ultimately O_2 -individuated by this universal fusion; so the first possibility will then be ruled out. And further, for any two pieces of gunk that have the same number of parts (just as for any two atoms), there will be no difference in the ways in which they are mereologically related to the universal fusion, so that the second possibility is also ruled out. In order to claim that (4) is false, one would therefore have to be prepared to deny Universal Composition.

Under the combinatorial conception of parthood, however, this principle enjoys a great deal of plausibility.²⁹

We have now seen that anyone who denies (4) will have to be willing to accept two theses that, under a combinatorial conception of parthood, are both unattractive: first, the thesis that there is atomless gunk, and second, the denial of Universal Composition. It is worth noting that neither of these two theses will seem particularly problematic if one adopts a *abstractional* conception of parthood. Thus, the thesis that there is atomless gunk will not raise any obvious problems because, under such a conception, it will merely amount to the claim that, from every part that can be ‘abstracted’ from some mereological whole, it is possible to abstract yet further parts. Nor will it seem particularly problematic to deny Universal Composition: For why should there not exist two or more *cosmoi* from which everything else is abstracted, but which are not themselves abstracted from anything, and are in this sense not proper parts of anything? The only obvious consideration that tells against this possibility is a preference for ontological parsimony.

For these reasons, it is tempting to suspect that, if anyone should find attractive the existence of gunk and the denial of Universal Composition, then he or she will most likely be operating not with a combinatorial, but rather with an abstractional conception of parthood. But under this latter conception, the second half of the mereological asymmetry (i.e., the thesis that a proper part of a given entity is not essentially a part of that entity) will no longer seem intuitively compelling, as already mentioned above (p. 199). So the present response to the objection from gunk can now be summarized as follows: In order to deny (4), one has to be prepared to accept two theses that can be plausibly accepted only if one adopts a conception of parthood under which the mereological asymmetry is no longer intuitively compelling.

This response may at first not seem satisfactory. Could it not be that, even under a combinatorial conception of parthood, the two theses in question (viz., that there is atomless gunk, and that Universal Composition fails) simply *happen to be true*? And wouldn’t we even

²⁹For a brief but persuasive defense of Universal Composition, see Lewis (1991, pp. 79–81).

in such a case say that the proper parts of any given entity are not essentially parts of that entity? If the answer to both questions is ‘yes’, then the present account of essentiality will apparently miss the heart of the matter. But it is far from clear that the first question is to be answered in the affirmative. For it is debatable whether, under a combinatorial conception of parthood (as opposed to an abstractional one), both atomism and Universal Composition can even be coherently denied.³⁰ In other words, it is not obvious that we will still have a firm grasp on the combinatorial conception if we regard it as conceivable that both atomism and Universal Composition fail to hold. It thus seems entirely possible that, if we give up both of these principles, we will no longer be left with a conception of parthood under which it is intuitively plausible to say that a proper part of any given entity is not essentially a part of that entity. The objector would have to show that this is not the case, but I do not see how this could be done.

* * *

The above considerations complete the argument for the thesis that both (d1) and (d2) are trumped. Consequently, no systematically optimal ontology will contain either (d1) or (d2), so that at least in this way, the Bunt–Lewis analysis of set-membership cannot be made to yield the consequence that, on the present account of essentiality, Socrates is essentially a member of {Socrates}.

This only leaves the possibility that there might exist some *variants* of (d1) and (d2) that jointly satisfy the above conditions (C1) and (C2) (p. 191). However, here we can apply considerations analogous to those of §8.2.2 to close off these remaining loopholes as well. In that earlier section, we found it impossible to produce a variant of σ^ϵ that would satisfy (C1) and at the same time pay its dues and be parsimonious, untrumped, and principial. And by considerations that are almost exactly analogous, it can now be shown that there are no variants of (d1) and (d2) that jointly satisfy (C1) while also meeting those other constraints. So we can conclude that, on the present account of essentiality, Socrates is not essentially a

³⁰I think that this holds in particular for a denial of atomism; see above, footnote 28.

member of {Socrates}. The account thus turns out to accommodate not only the first, but also the second half of Fine's asymmetry.

8.3 Cognitively Adequate?

In the previous two sections, we have seen that the present account of essentiality, when conjoined with independently plausible assumptions, has Fine's asymmetry as a consequence. The account thus provides an answer to the question of why Fine's asymmetry holds, and thereby satisfies the first desideratum listed in §1.3 (p. 7).

To be sure, if the present account of essentiality is thought of as a conceptual analysis, in the sense of aiming to describe what most people believe when they believe that a given property is essential to a given entity, then this analysis can presumably not be regarded as adequate. For most philosophers' beliefs about essential properties are most likely not due to beliefs about individuation ontologies. Some might therefore prefer a simpler account, according to which questions of essentiality are entirely a matter of convention, of how we find it natural to talk about sets and persons, and/or of what we find conceivable.³¹ But if we made *this* the basis for our account of essentiality, we would inevitably lose much of the philosophical significance attached to that concept.³² So we have to dig deeper, and make conjectures as to why we might find it natural to talk about certain properties as 'essential' to certain entities, while other properties are considered accidental. And there is no compelling theoretical reason why those conjectures should have to be framed in terms that are already well-familiar to us, since the mechanisms that lead us to find a certain sort of judgment intuitively plausible need by no means be cognitively transparent. So I think it is at least theoretically possible that, when we judge a particular property to be essential to a given entity, we are led to this judgment by our grasp of certain facts that (however nebulous they may appear to us) are

³¹E.g., see Sidelle (1989) and Sveinsdóttir (2008). Also see Wiggins (1980; 2001), whose account of essentiality has already been discussed in §1.4.1 above. The first few sections of Paul (2006) provide further relevant discussion, introducing a distinction between 'shallow' and 'deep' essentialism.

³²I will expand on this in the next section.

best described within a framework of individuation ontologies. Perhaps it is not entirely improbable that the present account of essentiality may be considered adequate at least in this sense.

But suppose it isn't. It might be, for instance, that what has led us to find it natural to think of {Socrates} as essentially having Socrates as a member is merely that society has conditioned us to think in this way. Clearly, it would still not follow that a good account of essentiality should strive to reflect these sorts of causal factor. The causal factors that it should strive to reflect are rather those that would in some sense be 'good' or 'appropriate' causes for our finding it natural to think of an entity as having such-and-such properties essentially. Very broadly, I would suggest that such a causal factor is a 'good' one only if (i) it consists in some insight into the way things are, and (ii) this insight is suitably reflected in the respective thinker's disposition to find it natural to regard some properties as essential to their bearers and others as accidental. What the present account of essentiality offers, then, is not an account of what sort of thing we have meant by the term 'essential property' all along, but rather a description of what would be a good thing for this term to mean.

Some might object to the present account on the ground that it apparently makes it very difficult to know certain things that we seem to know with considerable ease.³³ For instance, we seem to know that a table that is painted green will have the property of being green only accidentally.³⁴ If the present account of essentiality is correct, how could we come to know this? We would first have to investigate what sorts of individuation specification are contained in a systematically optimal ontology of tables. But *prima facie*, we can know that green tables are only accidentally green without having conducted any such investigation.

In response to this objection, I would in the first place suggest that it is not all that obvious that green tables are only accidentally green. The main reason for this is that we do not even know *what tables are*, in the sense that we do not know the essences of particular tables. For, as can be seen by reflecting on tables and other 'ordinary objects', various metaphysical

³³Thanks to Thomas Hofweber for pressing this objection.

³⁴I am taking this example from Copi (1954, p. 709f).

puzzles arise in this connection that collectively can lead one to doubt that there even *are* such things as tables. Daniel Korman (2011) summarizes the situation as follows:

Reflection on Michelangelo's *David* and the piece of marble of which it is made threatens to lead to the surprising conclusion that these would have to be two different objects occupying the same location and sharing all of their parts. Reflection on the availability of microphysical explanations for events that we take to be caused by ordinary objects threatens to lead to the conclusion that ordinary objects—if they do exist—never themselves cause anything to happen. And reflection on the possibility of alternative conceptual schemes, which “carve up the world” in radically different ways, makes our own conception of which objects there are seem intolerably arbitrary. Taken together, the various puzzles that arise in connection with ordinary objects make a powerful case for their *elimination*. And, in many cases, what seem to be the best responses to these puzzles require the postulation of legions of objects that we fail to notice despite their being right before our eyes.

Korman here alludes to three kinds of problem that beset the metaphysics of ordinary objects: problems of coincidence, the problem of causal overdetermination, and the problem of strange kinds. In addition, there are problems that arise in connection with vagueness, problems that arise from cases of fission or fusion, and the ‘problem of the many’.³⁵

These various problems may or may not have convincing solutions. To mention only one possible approach, there has in recent years been an increased interest in neo-Aristotelian (hylomorphic) solutions to problems of coincidence.³⁶ It is safe to say, however, that so far no general view of the metaphysics of ordinary objects has been firmly established, and for this reason I think it is also safe to say that we do not know the essences of tables, and consequently, that we do not know what properties are essential or accidental to particular tables. Hence, insofar as we are tempted to say that a green table is only accidentally green, I think that this belief will best be treated as an *intuition* rather than as a piece of knowledge.

Even so, it is certainly desirable to accommodate intuitions of this sort. But this does still not constitute an objection to the present account of essentiality, because, partly as a result of the problems just mentioned, the metaphysics of ordinary objects is at the present stage far

³⁵All of these are also mentioned in Korman's article. Problems arising from fission or fusion take on particular interest when the objects in question are persons; see, e.g., Lewis (1976) and Parfit (1984, pt. 3).

³⁶See, e.g., Fine (2008) and Sattig (2010).

from settled. Consequently, rather than to invite a *reductio* of the present account of essentiality, intuitions about what is or isn't essential to particular ordinary objects may instead be fruitfully combined with the present account to formulate constraints on the metaphysics of ordinary objects, and thereby help to answer the question of what sort of thing a table (or a chair, or an organism, etc.) is. For instance, when it comes to the intuition that green tables are only accidentally green, the corresponding constraint would require that no entity be counted as a table if, according to some systematically optimal ontology O , the property of being green is O -essential to it.

8.4 Philosophically Significant?

We now have to consider in somewhat greater detail how the present account of essentiality is able to meet the second desideratum listed in §1.3, viz., that it should “do justice to the sense of philosophical significance that has traditionally been attached to the notion of essence” (p. 7). There are at least four features of the account that help it satisfy this condition.

First, the account avoids conventionalism and subjectivism. Admittedly, the definition of ‘systematically optimal’ that we have arrived at in the previous chapter contains a “subjective element”, lodged in the requirement that each one of a systematically optimal ontology’s specifications should pay its dues. Hence, to a certain very limited extent, the present account of essentiality may be regarded as subjectivist. But this ‘subjectivism’ is clearly a far cry from the extreme view that takes the essential properties of a given entity to be simply those that ‘we care about’, or those that are salient in the respective context of utterance. It is also a far cry from an extreme form of conventionalism, according to which a property is essential to an entity of a certain sort S if and only if the property is (i) instantiated by that entity and (ii) a member of a certain class of properties that, by general (possibly tacit) agreement, are apt to be referenced in ‘canonical’ descriptions of entities that fall under S .

In these extreme forms of subjectivism and conventionalism, convention and subjective factors are given free rein in determining what is essential to what. In the present account,

by contrast, they are only given (as it were) a single degree of freedom, in that they merely determine how much discriminatory power a particular individuating specification has to contribute to an ontology in order for this contribution to outweigh the combined complexity of the specification itself and the ontological restrictions that are needed to keep the ontology from falsehood. Consequently, the question of whether a given property is essential to a particular entity is under the present account chiefly determined by metaphysical facts, while convention plays only a minor role.

Second, the account is applicable not only to abstract entities, but also to whatever entities make up the physical world. If the account had been applicable only to abstract entities, in the sense of being by design incapable of ascribing non-trivial essences to concrete entities, this would arguably have detracted from its ability to do justice to the philosophical significance of the concept of essence. For in that case, this concept would on the present account have no import beyond the metaphysics of abstract objects. In fact, however, nothing prevents the account from being applicable to concrete entities as well as abstract ones.

Admittedly, the structure of the physical world does not seem to be as neatly ordered as the set-theoretic universe, and this might at first raise doubts about the account's applicability to concrete entities and their properties. But that lack of neatness does in fact not pose any obstacle to the existence of individuating ontologies that accurately describe the physical world. And if there are such ontologies, then some of them will be systematically optimal. In the worst case, of course, the only such ontology will be O_0 , i.e., the true ontology that contains no individuating specifications at all. But I see no reason why not at least some individuating specifications should pay their dues and be parsimonious, untrumped, and principial, and at the same time be such that the predicates derived from them are satisfiable by concrete entities. If there are such specifications, then there will also exist at least one systematically optimal ontology O that differentiates pairs of concrete entities. The only apparent difficulty is that of knowing what such an ontology would look like.

Third, the present account can be regarded as relatively unified. To recapitulate, a property P is on this account essential to an entity x just in case, for some systematically optimal

ontology O , P is O -essential to x . This is of course only the ‘top level’ of the account, but at least as far as this level is concerned, there does not seem to be the slightest disunity: no disjunctiveness, no stitching together of disparate sub-accounts. So we have to consider the two main concepts on which the account is based, viz., those of systematic optimality and O -essentiality (i.e., of essentiality relative to an ontology). The definition of the latter appears equally unified. It is in turn based on the concepts of necessitation and fully concretized O -essence (p. 90), whose definitions do not seem to exhibit any disunity, either.³⁷ Hence, if the account suffers from any lack of unity, the reason for this must lie solely in the definition of ‘systematically optimal’.

That definition is indeed fairly complex: According to it, an ontology is systematically optimal just in case it is a true individuational ontology, each one of whose individuating specifications pays its dues and is parsimonious, untrumped, and principial. However, even though these four conditions are motivated by very different considerations, we found in §7.8 that they can be regarded as jointly amounting to a single requirement, viz., to the effect that a systematically optimal ontology should offer a best-possible explanation for the diversity of things (p. 169). In this way, it turned out that the definition of ‘systematically optimal’, and thus also the present account of essentiality, exhibits a reasonably high degree of internal unity, which should help it do justice to the sense of philosophical significance associated with the concept of essence.

Fourth and probably most important, the account closely links the concept of essence to central concerns of metaphysics. In particular, as just mentioned, it links the concept of essence to the problem of explaining the diversity of things. This is not an altogether novel connection. In medieval metaphysics, it was a common doctrine – held by Avicenna and various scholastic philosophers – that “every nature or essence that comes to be exemplified in real, concrete singulars somehow preexists as an idea in the divine mind”.³⁸ Explaining the

³⁷The relevant definitions can be found on pp. 49 and 88, respectively.

³⁸Black (1999, p. 52). Interestingly, this view seems to have some of its roots in the Qur’an, in particular in the following two verses from sura 36: “But His command, when He intendeth a thing, is only that He saith unto it: Be! and it is” (transl. Pickthall); see Nasr (1989), Wisnovsky (2003, ch. 7). The view may also

diversity of things naturally goes hand in hand with explaining their existence, and at least in a medieval context, the obvious way to explain the existence of a thing is to give an account of its creation. If, then, an account of creation makes reference to pre-existing essences, then an explanation of the diversity of things that is in keeping with such an account will likewise have to appeal to essences. So it is not a novel idea to link the latter to the former.³⁹

What *is* new, by contrast, is the parallel that is in the present account drawn between essences and laws of nature. As may be recalled from §7.1, the idea that there is such a parallel – that essences are a sort of *de re* analogue to *de dicto* laws – is the starting point for the development of the present definition of ‘systematically optimal’. Traditionally, talk of laws is not often mixed with talk of essences. As long as Western philosophers were interested in essences, laws of nature and nomic explanations did not play a prominent role, and when this had finally changed, essences had largely fallen out of fashion (together, at least for a while, with the subject of metaphysics itself) as a relic of Aristotelianism.⁴⁰ But if what has here been said about essentiality is correct, the connection between those two concepts is in fact a fairly intimate one.

remind one of the account of creation given in Plato’s *Timaeus*, in which the Demiurge creates the world after an eternal model (29a). Moreover, it has obvious roots in Plato’s theory of Forms.

To be sure, not all scholastic philosophers accepted the view that that the essences of things are prior to their existence. For example, it was rejected, naturally enough, by the nominalists, and later notably also by Francisco Suárez. (For a concise historical overview, see Witt (2011). Also see Wippel (2010) and Gilson (1962).) In Islamic philosophy, the Persian master Mulla Sadra has likewise attacked a doctrine of the priority of essence (or more accurately, quiddity) over existence, which he found in the teachings of other post-Avicennian philosophers, most notably Suhrawardi and Mir Damad. (See, e.g., Izutsu (1971, pt. 4) and Talgharizadeh (2000).) A few centuries later, the motto ‘essence precedes existence’ would be attacked again by Sartre, who gave it, however, very much his own meaning. (Obvious differences notwithstanding, Izutsu and others have observed certain fundamental similarities between European existentialism and the considerably older philosophy of Mulla Sadra.) In still more recent times, the motto seems to have enjoyed a kinder fate, having been explicitly endorsed by Lowe (2008, p. 45).

³⁹At this point, another historical remark may be in order with respect to the connection that the present account draws between the concept of essence and the explanation of the diversity of things. In his *Topics*, Aristotle says that the essence of a thing – where a ‘thing’ may be both an individual object and a kind – is given by its ‘definition’ (101b38), and further, that a definition is given by specifying a genus and a differentia (e.g., 141b26). So already in Aristotle, we can observe a close connection between essences and the ways in which things differ from each other. It is clear, however, that one should not make too much of this parallel, since the connection that the present account draws between essence and diversity is of a very different sort.

⁴⁰Cf. Gilson (1962, esp. p. 166f.). The exact origins of the modern concept of a law of nature are a contentious issue. For a fascinating early account, see Zilsel (1942). For more recent discussion, see, e.g., Des Chene (2006), Lehoux (2006), and Ott (2009).

To summarize, the present account of essentiality is relatively free from subjectivism and conventionalism, is applicable to both concrete and abstract entities, and exhibits a high degree of theoretical unity. In addition, it links the concept of essence to the long-standing metaphysical concern of explaining the diversity of things. These four characteristics seem to me sufficient to render the account capable of doing justice to the sense of philosophical significance that has traditionally been attached to the concept of essence.

Chapter 9

Metaphysical Modality

9.1 What Needs to Be Explained?

Arguably the most important application of the present account of essentiality is the elucidation of *de re* modal discourse. Philosophers who have grown used to the Kripkean framework of possible worlds might wonder why such an elucidation should be needed: what it could possibly achieve that is not already achieved by Kripke's framework. But on reflection, it is not hard to see how the need for an elucidation might arise. It arises, in particular, from such questions as, 'Under what conditions is it possible for x to exist?', where x may be any particular entity. We may ask, for example: 'Under what conditions might Socrates have existed?', or, more specifically: 'Might Socrates have existed if Phaenarete had never borne a child?'. Questions like these will of course not be answered by a purely semantic framework. But it would arguably not be too much to ask of a framework of modal semantics that it provide us with a way of interpreting sentences of the form ' x might have existed if φ ' in such a way that the interpretation does not itself make use of *de re* modal notions.¹

Admittedly, it is possible to have quite different views on this issue. Some might hold that no piece of *de re* modal discourse can be interpreted without the use of *de re* modal notions. In

¹A similar point has been made by McMichael (1983b, p. 63). One can discern the same worry also in some of Quine's attacks on quantified modal logic; e.g., see Quine (1972).

such a case, it might not be possible to provide any further foundation for Kripke’s framework. But it is also possible to think that *de re* modal discourse *can* be elucidated without using *de re* modal notions, and in the following, I want to explore a way in which this might be done on the basis of a concept of essentiality. The larger metaphysical framework will be the same as the one that has been employed in the previous chapters. So, in particular, I shall make use of a rudimentary background ontology of attributes and states of affairs, as described in chapters 2 and 3.

The sort of *de re* modality that I intend to explicate in the following can more specifically be thought of as *metaphysical* rather than, say, epistemological. I will begin by sketching an account of metaphysical modality *de dicto*, before I turn to the *de re* aspect in §9.3. The iteration of modal operators, as in ‘Possibly, it is necessary that grass is green’, gives rise to some delicate issues that will be discussed in §9.4. In the final section, I will close the dissertation with a comment on the broadly Humean character of the overall project.

9.2 *De Dicto*

Let \mathfrak{M} be a non-empty class of facts that can by some standard or other be regarded as ‘metaphysical laws’, where ‘law’ is understood in a sense that is applicable to states of affairs rather than sentences.² On this basis, a first approximation to an account of metaphysical necessity *de dicto* might be formulated as follows:

(MN₀) A state of affairs is *metaphysically necessary* just in case it is necessitated by some state of affairs that is a conjunction of elements of \mathfrak{M} .

Note the consequence that every logically necessary state of affairs – i.e., every state of affairs that is denoted by a valid constant or formula – is also metaphysically necessary, since every logically necessary state of affairs is necessitated by any state of affairs whatsoever.

²I will here not take a stance as to how these laws are to be selected. One possibility would be to adopt a deflationistic approach, à la Sider (2011, ch. 12).

Corresponding to the concept of metaphysical necessity, one may also introduce a concept of metaphysical impossibility. In chapter 3, the concepts of logical necessity and (im)possibility have not been treated as interdefinable, due to difficulties that would otherwise arise in connection with contingently existing entities; the same will here have to go for their metaphysical counterparts. For the state of affairs (Socrates = Socrates) should arguably be treated as metaphysically contingent, given that Socrates' existence cannot plausibly be held to be necessary. But if we then simply treated metaphysical possibility and necessity as dual to each other, and accordingly said that a state of affairs is metaphysically impossible just in case its negation is metaphysically necessary, then the state of affairs (Socrates \neq Socrates), which is not even logically possible, would nevertheless have to be counted as metaphysically possible. Since this would be an unwelcome consequence, we will need a separate account of metaphysical impossibility. But such an account is not far to seek. We might say, for instance, that

(MI₀) A state of affairs is *metaphysically impossible* just in case it necessitates the negation of some metaphysically necessary state of affairs.

According to this explication, the state of affairs (Socrates \neq Socrates) is metaphysically impossible, because it necessitates $\exists x (x \neq x)$, which, by the principle (S1) (p. 50), is identical with the negation of the (logically necessary, and hence also metaphysically necessary) state of affairs that everything is self-identical.

The next task is to expand this account so as to accommodate *de re* modal truths.

9.3 *De Re*

We first have to modify the above definition of metaphysical necessity.³ For it may happen that some entities exist by metaphysical necessity, and if such an entity has a certain property *P* essentially, we should arguably also count it as metaphysically necessary that the entity in

³Thanks to Thomas Sattig for alerting me to the fact that (MN₀) is insufficient.

question has P . The reason for this lies in the traditional link between the notions of essence and modality, as exemplified in the modal account of essentiality. Thus, an entity has its essential properties in every ‘possible world’ in which it exists, and consequently, if the entity exists necessarily, it must be necessary that it should have those properties. It seems advisable to respect this connection between essence and modality regardless of whether one analyzes the concept of essentiality in modal or in ontological terms.

Suppose now that the existence of a certain entity x is entailed by some conjunction of elements of \mathfrak{M} . Then, by (MN_0) , x exists necessarily. If, moreover, some property P is essential to x (in the sense of our account of essentiality), then it may well turn out that the state of affairs $P\langle x \rangle$ is not itself necessitated by any conjunction of elements of \mathfrak{M} . Under (MN_0) , that state of affairs will then not count as metaphysically necessary. But by what has been said in the previous paragraph, it *should* count as metaphysically necessary, given that x exists necessarily, and P is essential to x . Consequently, we have to adopt a more liberal account of metaphysical necessity. It will be convenient to construct this account in a recursive fashion.

To see the need for recursion, suppose again that a certain entity (say, God) exists by metaphysical necessity, and that some property P is essential to God. The state of affairs $P\langle \text{God} \rangle$ should then also count as metaphysically necessary. (And more generally, any state of affairs should count as metaphysically necessary if it is necessitated by the conjunction of $P\langle \text{God} \rangle$ and some other metaphysically necessary state of affairs.) But if $P\langle \text{God} \rangle$ should happen to be a relational state of affairs, e.g., $R\langle \text{God}, \text{Gabriel} \rangle$ – as will be the case if, for some relation R , P is identical with $\lambda x R\langle x, \text{Gabriel} \rangle$ –, then Gabriel will now also count as existing by metaphysical necessity, since his existence is necessitated by $P\langle \text{God} \rangle$. Further, if Gabriel has some essential property Q , then the state of affairs $Q\langle \text{Gabriel} \rangle$ should count as metaphysically necessary by the same reasoning as above. And if, for some relation R' , $Q\langle \text{Gabriel} \rangle$ is identical with $R'\langle \text{Gabriel}, \text{Michael} \rangle$, then we will have to acknowledge Michael as yet another necessary entity; and so on.

By this line of reasoning, we are naturally led to adopt a recursive account of metaphysical

necessity, such as the following:

(MN) A state of affairs s is *metaphysically necessary* just in case at least one of the following two conditions is satisfied:

- (i) s is necessitated by some conjunction of elements of \mathfrak{M} .
- (ii) For some entity x that exists by metaphysical necessity, some property P that is essential to x , and some metaphysically necessary state of affairs s' , s is necessitated by the conjunction of $P\langle x \rangle$ and s' .

In addition, we also have to modify the above definition of metaphysical impossibility. For, to take as an example again Socrates' singleton, it can plausibly be regarded as impossible that this singleton should exist without having Socrates as a member, given that the property of having Socrates as a member is essential to it. The reason for this lies again in the traditional connection between the notions of essence and modality: if P is an essential property of x , it should be impossible for x to exist without having P . More generally, then, one may say that a state of affairs is metaphysically impossible whenever there exists an entity x , a property P that is essential to x , and a metaphysically necessary state of affairs s' (which might just be the logically necessary state of affairs $I = I$), such that the conjunction of s and s' necessitates $\neg P\langle x \rangle$. To take account of this, we can modify (MI₀) by adding a second clause, as follows:

(MI) A state of affairs s is *metaphysically impossible* just in case at least one of the following two conditions is satisfied:

- (i) s necessitates the negation of some metaphysically necessary state of affairs.
- (ii) For some entity x , some property P that is essential to x , and some metaphysically necessary state of affairs s' , the conjunction of s and s' necessitates $\neg P\langle x \rangle$.

This account provides us with a way of interpreting sentences of the form ' x might have existed if φ ' in such a way that the interpretation does not take recourse to *de re* modal notions. To apply it to an example, consider the sentence

(1) Socrates might have existed even if Phaenarete had never borne a child.

In a first step, this may be interpreted as asserting that the following state of affairs is not metaphysically impossible: Socrates exists ('exists' being read tenselessly), and Phaenarete never bore a child. Let us refer to this state of affairs as ' s '. In the next step, we have to apply (MI) to the assertion that s is not metaphysically impossible.

According to (MI), s fails to be metaphysically impossible just in case (i) s does not necessitate the negation of any metaphysically necessary state of affairs, and (ii) there exists no entity x such that, for some property P that is essential to x , and for some metaphysically necessary state of affairs s' , the conjunction of s and s' necessitates $\neg P\langle x \rangle$. So, in order to assess whether s is metaphysically impossible, one has to evaluate these two conditions.

It is fairly safe to say that condition (i) is satisfied, because s does not seem to violate anything that would plausibly be regarded as a metaphysical law. (E.g., one would hardly regard it as a metaphysical law that Phaenarete should have borne a child.) So let us turn to the second condition. For the sake of the example, we may suppose that one of Socrates' essential properties is that of *being a child of Phaenarete's*. Let us call this property ' P '. Further, let us suppose that the state of affairs that Phaenarete never bore a child (which is necessitated by s) necessitates, either by itself or in conjunction with some metaphysically necessary state of affairs, that nothing is a child of Phaenarete's. Then s will likewise necessitate, either by itself or in conjunction with some metaphysically necessary state of affairs, that Socrates is not a child of Phaenarete's, i.e., $\neg P\langle \text{Socrates} \rangle$. If so, the second condition of (MI) will be satisfied, given that P is essential to Socrates. Consequently, s will under the present assumptions be metaphysically impossible, and (1) will be false.

9.4 Iterated Modalities

One of the great virtues of the possible-worlds framework is that it provides an elegant formal semantics for sentences in which modal operators occur within the scope of other (occurrences of) modal operators, such as 'Necessarily, it's possible that grass is green' or 'It's possibly

contingent that Socrates is pale'. In the present framework, these sorts of statements could in principle be treated just as smoothly, provided that there is a suitable abundance of 'modal' states of affairs, i.e., of states of affairs that would typically be expressed by using modal notions, such as those of possibility, necessity, or necessitation.⁴ For example, suppose that there exists such a thing as the modal state of affairs *that it's possible that grass is green*; and let us refer to this state of affairs as 's'. It would then be obvious how to interpret the sentence

(2) Necessarily, it's possible that grass is green;

for one could understand it very straightforwardly as an assertion to the effect that *s* is necessary. And analogously for other cases of iterated modality. On the other hand, if there is no such thing as the state of affairs that it's possible that grass is green, then it would apparently be quite unclear how (2) should be understood, since there would be nothing of which (2) could be understood as asserting that it is necessary.

For this reason, it might be thought that, if we want to regard sentences like (2) as intelligible within the present framework, we have to accept the existence of modal states of affairs. However, as has been argued in §3.4, we cannot very well admit modal states of affairs unless we are ready to countenance a property of *obtainment* (i.e., a property of being an obtaining state of affairs), which would in turn threaten to give rise to a version of the Liar paradox. Supposing that we do not wish to accept a property of obtainment, and consequently cannot admit the existence of modal states of affairs, how should we deal with sentences containing iterated modal operators?⁵

It seems to me that the best we can do in such a case is to accept the consequence that sentences like (2) are strictly speaking unintelligible. In defense of this move, one might argue that there are plenty of grammatically well-formed sentences that, after more or less reflection, turn out to be nonsensical. It is only to be expected that in some cases the nonsensical

⁴For a slightly more precise definition, see §3.4, p. 54.

⁵To simplify the discussion, I will here assume that all the modal operators in question are to be understood as operators of either logical or metaphysical modality.

character will be less obvious than in others, and (2) may be one of the less-obvious ones. It could be objected, however, that we sometimes do seem to make perfectly good sense of sentences in which modal operators are iterated.⁶ Consider, for example:

(3) I might have had a sister who married a pianist, but who might also have married a physicist.⁷

(4) I might have had an older brother whom I could have asked for advice whenever I wanted.

In (3), the second ‘might’ occurs within the scope of the first; in (4), the ‘could’ occurs within the scope of the ‘might’. Both sentences, then, make use of iterated modal operators. But on the face of it, both are perfectly intelligible. So what should we say about them?

It would not be altogether absurd, I think, to insist that (3) and (4) are strictly speaking meaningless, even though they can play a useful role in communication. For example, we could say that, in uttering (3), the speaker conveys the information that a certain non-modal state of affairs is possible, viz., one in which he has a sister who married a pianist, but in which the sister’s circumstances are moreover such that anyone who imagines her in those circumstances would be led to conclude that she might also have married a pianist. This is admittedly very rough; in particular, the occurrences of ‘her’ and ‘she’ after ‘anyone who imagines’ should not be taken to refer to any actual entity, but should instead be read as ‘pretended’ references to a fictional entity, viz., the imagined sister. To get rid of these pretended references, we might try replacing them by a universally quantified variable. This would yield the following, slightly more formal version of the intended paraphrase:

(3′) There is a (non-modal) property P such that:

- (i) it is possible that I might have had a sister who married a pianist and who had P ,
- and

⁶Cf. McMichael (1983b).

⁷This example is inspired by Rayo (MSb).

- (ii) for any x : anyone who believes that x has P would (under ideal conditions) be led to conclude that x might have married a physicist.

It does not strike me as absurd to hold that someone who utters (3) will typically – i.e., in non-philosophical, everyday contexts – want to say nothing more than what is expressed in (3'). If so, then the latter should count as an acceptable paraphrase of (3). A perfectly analogous paraphrase is available for (4).

Suppose, however, that there are such things as modal states of affairs. Even though this would require the existence of an obtainment property, it need not lead to semantic paradox. For instance, one might say that instantiations of the obtainment property, and any other states of affairs that have this property as a constituent, belong to the class of (at least) *second-order* states of affairs, which lie outside of the range of the quantifiers employed in first-order states of affairs, and which do not occur as constituents of the latter. In something like this way, one might construct a hierarchy of classes of states of affairs, which would in effect be an analogue of a Tarskian hierarchy of languages.

If modal states of affairs are admitted, sentences containing iterated modal operators can be given the straightforward interpretation alluded to at the beginning of this section.⁸ However, the possibility of such an interpretation has its own unwelcome consequences. For

⁸As mentioned in the previous paragraph, in order for there to be modal states of affairs, there will have to be an obtainment property. It is worth noting, however, that modal states of affairs are not the only things that an obtainment property helps provide. For if we also accept the existence of an instantiation relation (such as, in particular, a relation that holds between a property P , an entity x , and the state of affairs that is the instantiation of P by x), we can help ourselves to the state of affairs that 'alien' properties not only exist, but are also instantiated. Thus, let \mathcal{O} be the obtainment property, and let \mathcal{I} be the instantiation relation, so that, for any entities x, y, z , $\mathcal{I}\langle x, y, z \rangle$ is the state of affairs that x is the instantiation of y by z . Further, let A be the property of being an 'alien' basic property, i.e., a basic property that is distinct from P_1, P_2, \dots , where these are all the basic properties that actually exist. (As long as there are not too many of these, there will also be a property A as just specified.) Then the state of affairs

$$\exists x, y, z (\mathcal{I}\langle x, y, z \rangle \wedge \mathcal{O}\langle x \rangle \wedge A\langle y \rangle)$$

will be the state of affairs that there exists an alien basic property y that is instantiated by some entity z .

If we accept the existence of such states of affairs, we will thus be able to say that it is possible that some entities instantiate alien basic properties. Intuitively, this is of course what one would ordinarily *want* to say (cf. Lewis 1986a, esp. p. 159); but conceivably, in view of the additional complications, a theorist might reasonably choose to make do with a somewhat sparser ontology of states of affairs, and accordingly adopt a more restrictive conception of what is possible.

the concept of essentiality, as explicated in the previous chapters, is itself a modal concept, given that the concept of *O*-essentiality on which it is based is in turn defined on the basis of the concept of (logical) necessitation.⁹ Hence, as long as there are no modal states of affairs, there will also be no states of affairs as to what is essential to what, and a sentence like the following will have to count as strictly speaking meaningless:

(5) Possibly, Socrates is not essentially human.

For if there are no modal states of affairs, there will also be no state of affairs to the effect that Socrates is not essentially human. Since one cannot meaningfully ascribe a modal status to a non-existing state of affairs, (5) must then count as meaningless.

By contrast, if there *are* modal states of affairs, there will apparently be nothing to prevent the existence of the state of affairs that Socrates is not essentially human: i.e., the state of affairs that no systematically optimal ontology *O* is such that it is *O*-essential to Socrates to be human. Further, nothing we have said rules out that such a state of affairs may be metaphysically possible, even if we count no state of affairs as metaphysically possible in which Socrates fails to be human. Indeed, if we admit modal states of affairs, it is only to be expected that there will be some cases in which an entity is in fact essentially *F* while it is nevertheless possible that this same entity might not be essentially *F*. For whether a property is on the present account essential to a given entity depends on what individuating specifications pay their dues and are parsimonious, untrumped, and principial; and nothing that has here been said rules out that it may be a metaphysically contingent matter which specifications meet these conditions.

To illustrate what this means, let us suppose that Socrates is essentially human and that it is nevertheless possible that he should not be essentially human. It may then turn out that, while it is not possible for Socrates to be non-human, it is on our account still *possibly* possible for him to be non-human. For let *s* be the state of affairs that Socrates is non-human, and let us write ' $\diamond s$ ' to denote the state of affairs that *s* is metaphysically possible. By (MI)

⁹See §5.3, p. 90.

(p. 218), $\diamond s$ is identical with the state of affairs that s neither necessitates the negation of some metaphysically necessary state of affairs, nor necessitates (whether by itself or in conjunction with some metaphysically necessary state of affairs) the negation of any entity's instantiation of any property essential to that entity. Under the assumption that Socrates is essentially human, $\diamond s$ does of course not obtain, because s necessitates (indeed, is identical with) the negation of human(Socrates). But if it is metaphysically possible that Socrates might not be essentially human, nothing will prevent $\diamond s$ from being metaphysically possible as well.¹⁰

On the face of it, this is an unwelcome consequence. If Socrates is essentially human, so that it is impossible for him to be non-human, one would normally not expect that it is *possibly possible* for him to be non-human. What seems to be at work here is an intuition to the effect that, if a property P is essential to a given entity x , then it will not be possible that P should fail to be essential to x . To express this intuition more succinctly, one might say that essentiality is 'modally stable'.

Suppose that we want to accommodate this intuition. There seem to be three general ways in which this might be done. First, we might go back to denying the existence of modal states of affairs, and consequently deny the iterability of modal operators. Second, we might modify our account of metaphysical impossibility. In particular, we might add a clause to the effect that a given state of affairs s is metaphysically impossible if, for some entity x and some property P that is essential to x , s necessitates that P should fail to be essential to x . And finally, we might modify our account of essentiality in such a way that, whenever a property P is essential to a given entity, the property of *having P essentially* will also count as essential to it. The first option will seem attractive to those who already have strong doubts about the existence of modal states of affairs. But to everyone else, it should seem much less appealing, given that it threatens to curtail our expressive powers. As for the other two

¹⁰To see the relevance of the antecedent, note that $\diamond s$ necessitates that Socrates is not essentially human. (For if Socrates were essentially human, it would not be possible for him to be non-human, so that $\diamond s$ would not obtain.) Hence, if it were impossible that Socrates should fail to be essentially human, then $\diamond s$ would necessitate a metaphysically impossible state of affairs. As can be seen from (MI), this would in turn mean that $\diamond s$ itself is metaphysically impossible.

options, I think that they are unattractively *ad hoc*. For a friend of modal states of affairs, perhaps the best option will be simply to abandon the intuition that essentiality is modally stable, and to accept instead that some of an entity's essential properties might under other circumstances not have been essential to it. But admittedly, others may regard this intuition as non-negotiable, and may therefore prefer one of the three options just listed. It is not clear to me that this issue can be decisively settled.

9.5 Concluding Remark

In this chapter, I have only given a brief outline of how *de re* modal discourse might be elucidated on the basis of the present account of essentiality. In a fuller treatment, the problem of iterated modalities would have to be discussed in considerably greater detail, and it would also be desirable to compare the present account to other reductive accounts of metaphysical modality, for example those by Peacocke (1997; 1999) and Sider (2011).¹¹ I here have to leave both of these tasks for another occasion.

I want to close by briefly remarking on the Humean character of the approach that I have in this dissertation taken to essentiality and thus also to *de re* modality. What mainly contributes to that character is the fact that the present approach avoids any use of concepts with which one might wish to answer the question of what is 'ultimately responsible' for the way things are. For Avicenna and Aquinas, the answer to this sort of question lay in the divine mind. Contemporary philosophers will typically formulate their answers in less fanciful terms, and instead choose to speak of causal powers or sources of necessity. To have a convenient label for all such concepts, we may call them 'concepts of ultimate responsibility'. Those who are opposed to the Humean approach may hold that an account of essence should preferably be formulated by using some concept (or concepts) of ultimate responsibility, or even take the concept of essence itself to be a primitive within this category. Arguably, however, such

¹¹Other noteworthy proposals include, e.g., those by McMichael (1983a), Lycan and Shapiro (1986), Bealer (2006), Jubien (2009), and Stalnaker (2012).

an approach would make it difficult to see why Fine's asymmetry holds, and would thus also make it difficult to know *that* it holds.

This situation is similar to the one we face when considering laws of nature. Intuitively, it may seem that an account of lawhood should incorporate some concept (or concepts) by which we might describe what is 'ultimately responsible' for the way things are; perhaps the concept of law itself should be taken to be a primitive within this category. But if we do so, it becomes difficult to know what generalizations in fact *are* laws.¹² To overcome this difficulty, it is natural – as Lewis has done – to conceive of lawhood in different terms, and to leave aside any concepts of ultimate responsibility. In effect, we have here done the same for essences.¹³

¹²See, e.g., Earman and Roberts (2005a;b) and Schaffer (2008, §4).

¹³I am grateful to Thomas Sattig and Kit Fine for alerting me to this analogy.

Appendix A

Rebirth of the Modal Account?

First Attempt

It might be thought that the concept of necessitation that has been introduced at the end of §3.2 provides the basis for a modal account of essentiality that manages quite straightforwardly to accommodate Fine's asymmetry. For consider the state of affairs – call it '*s*' – that there exists something that has Socrates as its only member. Is this not simply the existence of Socrates' singleton? Let us assume for the moment that this suggestion is correct. We can then make the following two observations:

(1) The state of affairs *s* necessitates the existence of Socrates.

(2) The existence of Socrates does *not* necessitate *s*.

The first observation stems simply from the fact that *s* is denoted by the formula

$$\exists x \forall y (y \in x \leftrightarrow y = \text{Socrates}), \quad (*)$$

which can be seen to entail ' $\exists x (x = \text{Socrates})$ '. For the second observation, we have to consider whether there are any formulas φ and ψ such that φ denotes the existence of Socrates, ψ denotes *s*, and φ entails ψ .

Given the principle (S2) (p. 53), it seems quite unlikely that there should be such a pair of formulas. After all, it would have to follow from the principles adopted in chapters 2 and 3 (together with information about the denotation of constants, variables, and existential quantifiers) that, first, φ denotes the same state of affairs as ‘ $\exists x (x = \text{Socrates})$ ’ and, second, that ψ denotes the same state of affairs as (*); and φ would have to entail ψ . Now the formula (*), which denotes s , is in unabbreviated form written as

$$\exists x \neg \exists y \neg (\neg(\mathbf{E}\langle y, x \rangle \wedge \neg \mathbf{I}\langle y, \text{Socrates} \rangle) \wedge \neg(\mathbf{I}\langle y, \text{Socrates} \rangle \wedge \neg \mathbf{E}\langle y, x \rangle)), \quad (**)$$

and it is clear from this that ‘ $\exists x (x = \text{Socrates})$ ’ does not entail (*). Consequently, we would have to choose as our φ and ψ some other formulas than ‘ $\exists x (x = \text{Socrates})$ ’ and (*). What mainly prevents ‘ $\exists x (x = \text{Socrates})$ ’ from entailing (*) is the occurrence of the constant ‘ \mathbf{E} ’ in the latter’s expansion (**). Since ‘ \mathbf{E} ’ is assigned different referents (or even no referent at all) under different interpretations, (*) will not be entailed by any formula that does not itself contain ‘ \mathbf{E} ’. But it is deeply unclear what other formulas could be chosen for φ and ψ such that the above requirements are satisfied. So, at least for the purposes of this discussion, let us grant that the second observation is correct, and that Socrates’ existence therefore fails to necessitate the state of affairs that there exists something that has Socrates as its only member.

The two mentioned observations might give rise to the hope that it may be possible to use the concept of necessitation to construct a modal account of essentiality that manages to accommodate Fine’s asymmetry. The relevant explication of what it means for an entity to have a certain property essentially can be quickly stated as follows:

(E_M) An entity x has a property P *essentially* if and only if the existence of x necessitates the instantiation of P by x .

In order to establish that this account of essentiality accommodates Fine’s asymmetry, we would now have to show that the following holds:

- (1) The existence of Socrates' singleton necessitates the state of affairs that the latter has Socrates as a member; and
- (2) Socrates' existence does not necessitate his being a member of his singleton.

One might think that (1) is virtually immediate. For if the existence of Socrates' singleton is, as we have been assuming, simply the state of affairs that there is something that has Socrates as its only member, then the existence of Socrates' singleton cannot obtain unless something has Socrates as its only member – and what could that “something” be if not Socrates' singleton? As for (2), it will seem obvious from what has been said above that it is satisfied as well. For if Socrates' existence does not even necessitate the existence of his singleton, then his existence will *a fortiori* not necessitate his being a member of that singleton.

Now, does not all this show that it *is* possible, after all, to construct a modal account of essence that accommodates Fine's asymmetry?

It doesn't. The main error in the above line of reasoning is the assumption that the existence of Socrates' singleton is identical with s , i.e., with the state of affairs that there exists something that has Socrates as its only member. To see the mistake, consider that the existence of Socrates' singleton is denoted by the formula $\ulcorner \exists x (x = t) \urcorner$, where t may be any term that denotes Socrates' singleton. But now, a *term* is either a variable or a constant, formula, or λ -expression.¹ Further, while formulas only denote states of affairs, λ -expressions only denote attributes; and it seems at least *prima facie* reasonable to assume that sets are neither states of affairs nor attributes. So the only terms that might denote Socrates' singleton are variables and constants. If the formula $\ulcorner \exists x (x = t) \urcorner$ is to denote the existence of Socrates' singleton, t must accordingly be either a variable or a constant.

Let us now turn to the question of whether the existence of Socrates' singleton is the same state of affairs as s . Here we have to consider whether it follows from the principles of chapters 2 and 3, together with information as to what constants, variables, or quantifiers

¹See §2.3, p. 28.

denote the same as what other terms or quantifiers, that the formulas $\lceil \exists x (x = t) \rceil$ and $(*)$ denote the same state of affairs. But it can easily be seen that they don't. For, under the assumption that Socrates and his singleton are neither states of affairs nor attributes, neither Socrates nor his singleton will be denoted by any terms other than constants and variables. Consequently, as far as t and the constant 'Socrates' are concerned, the mentioned information will consist only in sentences of the form ' a denotes the same as b ', where ' a ' is replaced by a name of either t or 'Socrates', and where ' b ' is replaced simply by the name of some variable or constant.

What this sort of sentence allows us to infer, together with the principles of chapters 2 and 3, is only that $\lceil \exists x (x = t) \rceil$ denotes the same state of affairs as $\lceil \exists x (x = u) \rceil$, where u is a constant or variable that denotes the same as t (which would be Socrates' singleton), or that ' $\exists x \forall y (y \in x \leftrightarrow y = \text{Socrates})$ ' – i.e., $(*)$ – denotes the same as $\lceil \exists x \forall y (y \in x \leftrightarrow y = u') \rceil$, where u' is a constant or variable denoting the same as 'Socrates'. But since already the two original formulas fail to entail each other, the same will also hold for any pair of these alternative formulas, in which t and/or 'Socrates' have been replaced by other constants or variables. In this way, we can see that it does not follow from (S1) (p. 50), nor from any combination of the other principles, that $\lceil \exists x (x = t) \rceil$ and $(*)$ denote the same state of affairs. And if it doesn't follow, then, by (S2), it is not the case. In other words, the existence of Socrates' singleton is not identical with the state of affairs that there exists something that has Socrates as its only member.

Once this is clear, it can further be seen by very similar considerations that the existence of Socrates' singleton does not necessitate the state of affairs that this entity has Socrates as a member. For the former is denoted by $\lceil \exists x (x = t) \rceil$ and the latter by $\lceil \text{Socrates} \in t \rceil$, and neither Socrates nor his singleton can be denoted by any term other than a constant or variable. So the account of essentiality given by (E_M) does not accommodate Fine's asymmetry, after all.

Second Attempt

As the above discussion shows, one of the reasons as to why the existence of $\{\text{Socrates}\}$ does not necessitate Socrates' being a member of $\{\text{Socrates}\}$ is the fact that the latter is not denoted by any terms other than constants or variables. But here it might be thought that this is only due to arbitrary restrictions. For, with only slight adjustments to the definitions in §2.3 and §3.2, the existence of Socrates' singleton may very well have counted as necessitating Socrates' being a member of the singleton.

To illustrate this idea, suppose that the definition of 'term' in §2.3 had been framed in such a way that terms include not only variables, constants, formulas, and λ -expressions, but also 'set terms', which might have been introduced by the following rule:

(ST) If a_1, a_2, \dots (κ -many variables) are terms, then the expression $\ulcorner \{a_1, a_2, \dots\} \urcorner$ will be a *set term*, and have a denotation if and only if there are entities x_1, x_2, \dots that are respectively denoted by a_1, a_2, \dots ; in which case the expression will denote the set that has exactly x_1, x_2, \dots as members.

With this modification in place, the existence of Socrates' singleton could be simply denoted by the formula ' $\exists x (x = \{\text{Socrates}\})$ '. However, the existence of Socrates' singleton would not yet necessitate the state of affairs that this entity has Socrates as a member. To ensure this, we would have to make a further slight change, viz., to the definition of 'interpretation'. In particular, we would have to stipulate that any interpretation I of a formal language \mathcal{L} should satisfy not only the conditions (i) and (ii) listed on p. 46, but also the following:

(iii) To the constant ' \mathbf{E} ', I assigns the relation of set-membership.

As a result, the formula ' $\exists x (x = \{\text{Socrates}\})$ ' would then count as entailing ' $\text{Socrates} \in \{\text{Socrates}\}$ ', which means that the existence of Socrates' singleton would count as necessitating Socrates' being a member of the singleton, as desired.

Unfortunately for this second attempt at saving the modal account, the formula ' $\text{Socrates} \in \{\text{Socrates}\}$ ' would under the proposed changes to the relevant definitions not only be entailed

by ' $\exists x (x = \{\text{Socrates}\})$ ' but also by ' $\exists x (x = \text{Socrates})$ ', so that even Socrates' own existence (and not just that of his singleton) would count as necessitating the state of affairs that he is a member of his singleton. Consequently, Socrates would under (E_M) count as being essentially a member of $\{\text{Socrates}\}$, which is precisely what we have to avoid in order to accommodate the second half of Fine's asymmetry. And even if this difficulty can be overcome somehow, the strategy of adjusting the definitions of 'term' and 'interpretation' in the way just illustrated seems quite clearly to be an *ad hoc* strategy. It is limited only to set-theoretic properties, and seeks to accommodate our intuitions about the essentiality and non-essentiality of those properties without any attempt at investigating what it is about them that makes them essential or non-essential to their respective bearers.

The lesson to be drawn from this discussion is that the concept of necessitation introduced in §3.2 does not help to save the modal account of essentiality.

Appendix B

Bibliographical Note on ‘Essence’ and ‘Individuation’ in Recent Metaphysics

Jonathan Lowe (2003) notes that the term ‘individuation’ is in contemporary discussion often used in an “epistemic or cognitive sense”, having to do with the ‘singling out’ of an object “as a distinct object of perception, thought, or linguistic reference” (p. 75). He proceeds to contrast this with a decidedly metaphysical sense, which has palpable connections with the traditional notion of essence. There is also, however, a second metaphysical sense that is much less ambitious than the former. In this second metaphysical sense, to individuate an entity also means to ‘single it out’, but not necessarily in a way that has anything to do with perception, thought, or linguistic reference. Thus, Strawson (1959) introduces the notion of an *individuating fact* as follows:

To know an individuating fact about a particular is to know that such-and-such a thing is true of that particular and of no other particular whatever. (p. 20)

Ambitious metaphysical uses of the term ‘individuation’, such as the one that Lowe is alluding to, can be found, e.g., in Castañeda (1975) and Gracia (1988), as well as in Lowe’s own work. According to Castañeda, the *individuator* of an entity x is the thing that “accounts for” or “constitutes” the “individuality” of x ; but unfortunately it remains unclear from his discussion what exactly individuality is. This latter question has been extensively discussed

by Jorge Gracia (*op. cit.*), who in the end equates individuality with *noninstantiability*. (This answer allows him to say that the principle of individuation – what accounts for the noninstantiability of entities – is existence itself.) Somewhat similarly to Castañeda, Lowe takes the individuator of an object to be the thing that “*explains* why that individual object is the very object that it is” (2003, p. 93). He stresses that ‘to explain’ should here not be read as ‘to render intelligible’, but rather as “to *account for*”; but he leaves unclear what it could mean for something to account for an object’s being the very object that it is. (We are apparently hearing echos of the Aristotelian phrase, *to ti ên einai*, that has come to be translated as ‘essence’, or of paraphrases thereof, such as Thomas Aquinas’ “that on account of which something is what it is”¹.) What Lowe presumably has in mind, as suggested by references earlier in the article, is his notion of *identity-dependence*, as explicated in chapter 6 of his (1998). He there first explains this notion informally by saying that

To say that the identity of x depends on the identity of y [...] is to say that *which* thing of its kind y is fixes (or at least helps to fix) *which* thing of its kind x is. (p. 147)

A little further below, he gives a formal definition:

The identity of x depends on the identity of y =_{df} Necessarily, there is a function F such that it is part of the essence of x that x is the F of y . (p. 149)

From this definition, it becomes quite clear how the concept of individuation is in Lowe’s thinking connected to that of essence. As he notes himself, however, his definition will not be “fully vindicated” until we have “a perspicuous account of the notion of ‘essence’” (*ibid.*), which he does not undertake to develop.

The idea that the notions of essence and individuation are closely connected to each other is perhaps the most suggestive if the term ‘individuation’ is understood in an ambitiously metaphysical sense, as exemplified in the work of Castañeda and Lowe. However, we can find such a connection also, and quite explicitly, in the writings of authors whose concept

¹In Gyula Klima’s translation of *De ente et essentia*, ch. 1. The original Latin phrase reads: “*hoc per quod aliquid habet esse quid*”.

of individuation is not at all of this sort. Thus, Barcan Marcus (1971) speaks of ‘individuating essentialism’ which she distinguishes from its more standard ‘Aristotelian’ counterpart; the term ‘individuation’ is here understood in the unambitious metaphysical sense identified above. For another example, Wiggins (1980; 2001) defends a position he calls ‘individuating essentialism’, taking the term ‘individuation’ in what Lowe would call the “epistemic or cognitive sense”.²

In the case of Marcus and Wiggins, the link between their respective notions of individuation and essentiality is fairly clear (though, in the case of Marcus, extremely weak). On the other hand, if one’s understanding of ‘essential’ follows the standard modal account (according to which a thing’s essential properties are simply those that it has in every world in which it exists), and if one’s concept of individuation is the unambitious metaphysical concept, then it will be comparatively obscure what that connection might consist in. But even in such cases, the idea that there must be some connection between individuation and essentiality has seemed curiously attractive. In particular, consider the following two sentences:

- (1) Sets are individuated by their members.
- (2) A set has its members essentially.

In the case of (1), talk of individuation is typically understood in the *unambitious* metaphysical sense mentioned above: for a set to be individuated by its members simply means that no two sets have all their members in common. On the other hand, the concept of essentiality that is relevant in (2) is typically taken to be the modal concept. It is obvious that, under these readings, (2) cannot be derived from (1). But as some authors have noted, this inference nevertheless seems to have intuitive appeal. Here, e.g., is van Cleve:

²On Wiggins, also cf. §1.4.1 above. Plantinga (1974) similarly draws a strong connection between essence and individuation:

We could say that an essence of Socrates *individuates* him; it must be a property nothing else has. But this is not sufficient [...]. If *E* is an essence of Socrates, then [...] it must be impossible that there *should have been* an object distinct from Socrates that had *E*. (p. 70)

As noted in §1.2 (p. 7), the view that essences are unique to their bearers can apparently be traced back at least to Boethius.

Same members, same set; different members, different sets. It is because a set is individuated in this way by its members, one might think, that it has its particular membership essentially. (1985, p. 586)

He is quick to note that the inference is fallacious, and then proceeds to look for ways of saving it. Eventually he suggests three principles from which, taken together, (2) may be derived; but one of these principles is in effect a strengthening of (1), and so constitutes a departure from the ordinary, unambitious understanding of ‘individuation’.³

³For other attempts to justify (2) on the basis of considerations revolving around (1), see Sharvy (1968), Fine (1981, p. 179f.), and Wiggins (2001, §4.4).

Appendix C

Improving the Semantic Principle

In chapter 4, I have spelled out the semantics of individuating specifications in terms of graphs and decorations. The advantage of this was that I could utilize already-established mathematical vocabulary, and that I could also make use of the fact that graphs easily lend themselves to visualization. But it could be argued that that approach was somewhat circuitous and even misleading, since the central ‘meaning-postulate’ (S) interprets individuating ontologies as making claims that are in the first place only about graphs and decorations. For this reason, I shall here propose an alternative principle that relies on graphs only indirectly and instead makes reference to *formulas*. In order to be able to state this principle, I will need to construct an algorithm that, given an individuating ontology O , turns O -graphs into formulas.

First of all, we have to introduce the concept of a *raw formula*. For any individuating ontology O , let G be some O -graph, and let f be an injective function from G ’s nodes to variables. Then the raw formula ρ that corresponds to G will be a conjunction that, for each edge (P, ν_1, \dots, ν_n) in G , contains a conjunct $\lceil P\langle \nu_1, \dots, \nu_n \rangle \rceil$ where $\nu_i = f(\nu_i)$ for each $i \leq n$. In addition, for each node ν in G that is not an end point of *any* edge in G , ρ will contain a conjunct $\lceil v = v \rceil$, where $v = f(\nu)$. These are all the conjuncts that ρ contains.

By way of example, consider the simple graph described on p. 75, whose two edges are (P, ν_1, ν_2) and (P, ν_2, ν_2) with $P = \lambda x, \alpha (\alpha \in x)$. If our function f maps ν_1 and ν_2 , respectively, to the variables ‘ x ’ and ‘ y ’, then that graph will, relative to f , yield the following raw

formula:

$$\lambda x, \alpha (\alpha \in x) \langle x, y \rangle \wedge \lambda x, \alpha (\alpha \in x) \langle y, y \rangle. \quad (\text{C.1})$$

Let us say that a *raw formula of O* is any formula that corresponds in this way to some *O*-graph.¹

For the present purposes, the raw formula ρ will be represented as a ‘contracted conjunction’

$$\bigwedge_{\varphi \in \Phi} \varphi, \quad (\text{C.2})$$

where Φ is the set of ρ ’s conjuncts.² In the following paragraphs, I shall describe a procedure by which ρ can be transformed into the corresponding *O-individuation formula*. This latter formula will be satisfied by *O*-entities, i.e., by members of the domain associated with *O*, just in case the graph *G* has an injective *O*-decoration – except that the formula has the advantage of not expressing any commitment to graphs and decorations (except *per accidens*).³

Most of the complexity of the procedure results from the fact that, in order for a given assignment of entities to nodes to count as an *O*-decoration of *G*, there has to be, for each individuating specification σ that is such that the predicate *P* derived from it has an arity greater than 1, a “tuple-wise matching” between each node’s *P*-children and the σ -individuators of the entity assigned to that node.⁴ This requirement can be divided into two parts: First, it demands (very roughly) that, for each edge of *G*, there should be entities that satisfy the appropriate predicate. This part is already taken care of by the raw formula ρ . Second, however, the requirement demands that, for each node of *G*, the entity assigned to that node should not be related to any *further* entities than is indicated by the edges that have that node as

¹Note that, although (C.1) is logically equivalent to ‘ $y \in x \wedge y \in y$ ’, the latter will here *not* be regarded as a raw formula.

²Some may wish to read ‘ Φ_ρ ’ instead of ‘ Φ ’, since the value of this variable will depend on the choice of ρ . The same will be true for many of the variables to be introduced below, but for simplicity, I will tend to omit such subscripts, since the relevant dependencies will usually be clear from the context.

³In chapters 4, 5, and 7, the *O*-individuation formula that corresponds to a graph *G* is in general referred to as ‘ $\chi_{O,G}$ ’.

⁴Cf. §4.5, p. 74.

their parent. (This is again very rough: it should be kept in mind that the only relations that are relevant here are those that are given by the individuating specifications of O .) To capture this second part of the requirement, we have to construct, for each one of ρ 's free variables, a clause that is then conjoined with ρ itself, i.e., with the conjunction (C.2).

Let now V be the set of the free variables in ρ , and let v be the variable in V for which we intend to construct the clause just mentioned. The general form of this clause is that of a conjunction

$$\bigwedge_{\psi \in \Psi_v} \psi,$$

where, for each individuating specification σ of O , the set Ψ_v contains a formula that is constructed according to the following algorithm.

We begin with the simple formula $\ulcorner P\langle v, u_1, \dots, u_{n-1} \rangle \urcorner$, where P is derived from σ , n is the arity of P , and none of the variables u_1, \dots, u_{n-1} are members of V . This formula, which I will in the rest of this paragraph refer to as ' χ ', can be regarded as something of a 'blueprint' against which the formulas in Φ are to be matched. In particular, a formula φ in Φ is said to *match* χ just in case φ agrees with χ with respect to both its predicate and the first variable, so that $\varphi = \ulcorner P\langle v, \dots \rangle \urcorner$. Let Δ be the (possibly empty) set of those formulas in Φ that match χ . Each formula δ in Δ can be written as $\ulcorner P\langle v, v_1^\delta, \dots, v_{n-1}^\delta \rangle \urcorner$, with $v_1^\delta, \dots, v_{n-1}^\delta$ being members of V . (The ' δ '-superscripts serve here to indicate that the assignment of indices depends on the respective member of Δ .)

Having thus constructed the formulas in Δ , we may write the formula to be included in Ψ_v as follows:

$$\forall u_1, \dots, u_{n-1} \left(P\langle v, u_1, \dots, u_{n-1} \rangle \rightarrow \bigvee_{\delta \in \Delta} \bigwedge_i u_i = v_i^\delta \right). \quad (\text{C.3})$$

To make sense of this, it is worth recalling that the formulas in Δ correspond to edges of G that have a common parent-node. Note also that empty conjunctions can be treated as tautologies, and that empty disjunctions can be treated as contradictions. Hence, if Δ is

empty, then (C.3) will simplify to

$$\forall u_1, \dots, u_{n-1} \neg P\langle v, u_1, \dots, u_{n-1} \rangle.$$

For each one of O 's individuating specifications, then, Ψ_v contains one such formula. If we proceed in this way for each variable v in V and eventually conjoin the resulting clauses with (C.2), we obtain

$$\bigwedge_{\varphi \in \Phi} \varphi \wedge \bigwedge_{v \in V} \bigwedge_{\psi \in \Psi_v} \psi. \quad (\text{C.4})$$

Further, let us say that the *O-existential closure* of a formula γ is the existential closure that is constructed using the quantifier ' \exists_O ', i.e., a quantifier whose domain is restricted to O -entities (see p. 71 above).⁵ From the above construction, it can be seen that the O -existential closure of (C.4) will be true in precisely those cases in which G has an O -decoration. To obtain a formula whose O -existential closure is true just in case G has an *injective* O -decoration, we will have to add only one more clause, namely:

$$\bigwedge_{v \neq u} v \neq u, \quad (\text{C.5})$$

i.e., a conjunction of all formulas ' $v \neq u$ ' for any two distinct variables v and u , where both v and u are taken from V . The addition of this clause results in

$$\bigwedge_{\varphi \in \Phi} \varphi \wedge \bigwedge_{v \in V} \bigwedge_{\psi \in \Psi_v} \psi \wedge \bigwedge_{v \neq u} v \neq u, \quad (\text{C.6})$$

which is the individuation formula that we have wanted to construct.

We are now able to reformulate (S) without reference to decorations. For any raw formula ρ of a given ontology O , the procedure just described yields the corresponding individuation formula. Let us abbreviate the O -existential closure of this formula as ' $\mathcal{I}_O \rho$ ' (i.e., ' \mathcal{I}_O ' is here

⁵Roughly speaking, the O -existential closure of γ will be a sentence of the form ' $\exists_O x_1, x_2, \dots \gamma$ ', where ' x_1, x_2, \dots ' is replaced by a list of γ 's free variables. But the notation would here have to be adjusted in order to accommodate an infinite number of variables.

used as an operator that implicitly binds all the free variables in ρ). The improved version of (S) can then be stated as follows:

(S') For any raw formula ρ of O , O makes the existence-claim expressed by $\ulcorner \mathfrak{I}_O \rho \urcorner$, unless the latter is incompatible with O 's ontological restrictions.

For example, suppose that O is an ontology containing an individuating specification to the effect that properties of the form ' $\lambda x (\alpha \in x)$ ' are individuating, and let ρ be the raw formula (C.1). Then $\ulcorner \mathfrak{I}_O \rho \urcorner$, i.e., the O -existential closure of the corresponding individuation formula, will be a formula equivalent to

$$\exists_O x, y (y \in x \wedge y \in y \wedge \forall y_1 (y_1 \in x \rightarrow y_1 = y) \wedge \forall y_1 (y_1 \in y \rightarrow y_1 = y) \wedge x \neq y),$$

which says that the domain associated with O contains at least two things x and y that both have y as their only member.

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