January 2010

Essays on Money and Credit: A New Monetarist Approach

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ESSAYS ON MONEY AND CREDIT: A NEW MONETARIST APPROACH

by

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A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

May 2010
Saint Louis, Missouri

Abstract

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Chapter 1: Money and Credit with Limited Commitment and Theft

Credit contracts and fiat money seem to be robust means of payment in the sense that we observe both monetary exchange and credit transactions under a wide array of technologies and monetary policy rules. However, a common result in a large class of models of money and credit is that the optimal monetary policy -- usually the Friedman rule -- eliminates any transactions role for credit: money drives credit out of the economy. In this sense, money and credit are not robust in the model. We study the interplay among imperfect recordkeeping, limited commitment, and theft, in an environment that can support both monetary exchange and credit arrangements. Imperfect recordkeeping makes outside money socially useful, but it also permits theft of currency to go undetected, and therefore provides lucrative opportunities for thieves in decentralized exchange. First, we show that imperfect recordkeeping and limited commitment are not sufficient to account for the robust coexistence of money and credit. Then, we show that theft, together with imperfect recordkeeping and limited commitment, is sufficient to account for the robust coexistence, given that theft imposes a cost on monetary exchange. The Friedman rule is in general not optimal with theft, and the optimal money growth rate tends to rise as the cost of theft falls.

Chapter 2: Unsecured Loans and the Initial Cost of Lending

We study the terms of credit in a competitive market where sellers are willing to repeatedly finance the purchases of buyers by extending direct credit. Lenders (sellers) can commit to deliver any long-term credit contract that does not result in a payoff that is lower than that associated with autarky while borrowers (buyers) cannot commit to any contract. A borrower's ability to repay a loan is privately observable. As a result, the terms of credit within an enduring relationship change over time according to the history of trades. Although there is free entry of lenders in the credit market, each lender has to pay a cost to contact a borrower. We show that a lower cost makes each borrower better off from the perspective of the contracting date, results in less variability in a borrower's expected discounted utility, and makes each lender uniformly worse off *ex post*. As this cost approaches zero, the credit contract offered by a lender converges to a full-insurance contract.

Chapter 3: Costly Recordkeeping, Settlement System, and Monetary Policy

We study an arrangement in which the government provides a public settlement system to the private sector and evaluate its implications for the implementation of monetary policy. A key ingredient of the analysis is that it is costly for the government to operate a record-keeping technology which is necessary for the construction of a settlement system through which private loans and tax liabilities are settled. For this reason, the choice of the optimal size of a settlement system by the government is non-trivial. Another benefit

of such a system is that it allows the government to effectively control the money supply. We show that the Friedman rule is suboptimal. Money and credit coexist as means of payment at the optimum. The government relies on a credit system to implement an optimal policy because of the role of credit in relaxing cash constraints. As a result, money and credit are complementary in transactions: the existence of a credit system makes the operation of a monetary system more effective.

Acknowledgements

I would like to thank my advisor Stephen Williamson for teaching me the modern theory of money and banking. His guidance and dedication were essential for the completion of this dissertation.

I also would like to thank Gaetano Antinolfi, James Bullard, Rodolfo Manuelli, Costas Azariadis, David Andolfatto, Christopher Waller, Pedro Gomis-Porqueras, and Ping Wang for their helpful comments.

I dedicate this dissertation to my wife Ammanda Sanches who supported me throughout my doctorate. Without her this project would not be possible.

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1 Money and Credit with Limited Commitment and Theft[1]

1.1 Introduction

It is hard to find examples of economies in which we do not observe the use of both money and credit in transactions. Thus, we should think of money and credit as robust, in the sense that we will observe transactions involving both money and credit under a wide array of technologies and monetary policy rules. One goal of this paper is to help us understand what is required to obtain robustness of money and credit in an economic model. Then, given robustness, we want to explore the implications for monetary policy. As well, this paper will serve to tie together some key ideas in monetary economics.

As is now well known, barriers to the flow of information across locations and over time appear to be critical to the role that money plays in exchange. If there were no such barriers, in particular if there were perfect "memory", i.e. recordkeeping, then it would be possible to support efficient allocations in the absence of valued money – see Kocherlakota (1998). One can think of the models of Green (1987), or Atkeson and Lucas (1992), as determining efficient allocations with credit arrangements under private information, where the memory of past transactions by economic agents supports incentive compatible intertemporal exchange. As Kocherlakota (1998) points out, spatial separation of the type encountered in turnpike models – such as Townsend (1980) – or random matching models – such as Trejos and Wright (1995) – also yields efficient credit arrangements under perfect memory. Aiyagari and Williamson (1999) study an environment with private information *and* random matching where credit arrangements are efficient. Thus, neither private information nor spatial separation is a sufficient friction to provide a socially useful role for monetary exchange. Both

[1] Joint project with Stephen Williamson.

frictions mitigate credit arrangements, but not to the point where monetary exchange necessarily improves matters.

The work of Kocherlakota (1996, 1998) seems to suggest that limited commitment works much like private-information and spatial frictions, in that it in general implies less intertemporal exchange than would occur in its absence, but does not imply a welfare-improving role for money. However, in the case of limited commitment, this is not obvious. For example, suppose that two economic agents, A and B meet. Agent A can supply B with something that B wants, but all that B can offer in exchange is a promise to supply A with some object in the future. Agent B is unable to commit, and what A is willing to give to B will depend on $A's$ ability to punish B, or to have other economic agents punish B, if he or she fails to fulfill his or her promises in the future. The amount of credit that A is willing to extend to B will in general be limited. However, suppose that B has money to offer A in exchange. Possibly A and B can trade more efficiently using money, or by using money and credit, because monetary exchange is not subject to limited commitment.

First, we wish to construct a framework which can potentially permit monetary exchange, trade using credit under limited commitment, and the coexistence of valued money and credit. We build on the model of Rocheteau and Wright (2005) and Lagos and Wright (2005), which has quasilinear utility and alternating decentralized and centralized trading among economic agents. This lends tractability to our analysis, but we think that the basic ideas are quite general.

The first result is that, consistent with Kocherlakota (1998), limited commitment is in fact not sufficient to provide a social role for money in our model. The result hinges on the fact that lack of commitment applies to tax liabilities as well as private liabilities. If there is perfect memory and limited commitment matters, then limited commitment also may make the Friedman rule infeasible for the government. This is because agents may want to default on the tax liabilities that are required to support deflation at the Friedman rule rate. While it is possible in special cases for money to be valued in equilibrium with limited commitment and perfect memory, there is no welfare-improving role for money. This is similar to the flavor of some results in the monetary model without credit considered by Andolfatto (2008).

As mentioned above, one of our aims in this paper is to determine a set of frictions under which money and credit are both robust as means of payment. Clearly, perfect memory does not provide conditions under which monetary exchange is robust, so we need to add imperfect memory to provide a role for money. However, we do not want to shut down memory entirely, as is typical in some monetary models – e.g. Lagos and Wright (2005) – as this will also shut down credit. We use a hybrid approach, whereby decentralized meetings between buyers and sellers are either monitored, or are not. A monitored trade is subject to perfect memory, while there is no access to memory in non-monitored transactions – see also Deviatov and Wallace (2009).

In this context, our results depend critically on the punishments that are triggered by default in the credit market. For tractability, we consider global punishments, whereby default by a borrower will imply that all would-be lenders refuse to extend credit. At the extreme, this can result in global autarky. With global autarky as an off-equilibrium path supporting valued money and credit in equilibrium, higher money growth lowers the rate of return on money, and there is substitution of credit for money. Efficient monetary policy is either a Friedman rule, if incentive constraints do not bind at the optimum, or else optimal money growth is greater than at the Friedman rule and incentive constraints bind at the optimum. In either case, efficient monetary policy drives out credit. Money works so well that if the government gives money a sufficiently high rate of return there will be no lending in equilibrium.

We also consider off-equilibrium punishments that are less severe than autarky.

Much as in Aiyagari and Williamson (2000) or in Antinolfi, Azariadis, and Bullard (2007), we allow punishment equilibria to include monetary exchange. That is, if a borrower defaults, this triggers a global punishment where there is no credit, but agents can trade money for goods. Here, the only equilibrium that can be supported is one with no credit, and with a fixed stock of money (also implying a constant price level in our model). This is quite different from results obtained in Aiyagari and Williamson (2000) or Antinolfi, Azariadis, and Bullard (2007). A key difference in our setup is that we take account of the fact that the government cannot commit to punishing private agents through monetary policy. When a default occurs, the government adjusts monetary policy so that it is a best response to the decision rules that private agents adopt as punishment behavior.

Imperfect memory provides a role for money, but in the context of imperfect memory alone, money in some sense works too well in our model, relative to what we see in reality. That is, optimal monetary policy always drives credit out of the system. This is a typical result, which is obtained for example in Ireland's cash-in-advance model of money and credit – see Ireland (1994). Ireland's model has the property that a Friedman rule is optimal and, at the Friedman rule, all transactions are conducted with cash, which eliminates the costs of using credit. The intuition for this is quite clear. All alternatives to using currency in transactions come at a cost, for example there are costs of operating a debit-card or credit-card system, there are costs to clearing checks, etc. If it is costless to produce currency and to carry it around, then if the government generates a deflation that induces a rate of return on money equivalent to that on the best safe asset, then this should be efficient, and it should also eliminate the use of all cash substitutes in transactions.

Of course, in practice it is costly for the government to operate a currency system. For example, maintaining the currency stock by printing new currency to replace worn-out notes and coins is costly, as is counterfeiting and the prevention of counterfeiting. As well, a key cost of holding currency is the risk of theft, which has been studied, for example, in He, Huang, and Wright (2008). We model theft differently here, and do this in the context of monitored credit transactions. That is, we assume that it is possible for sellers, at a cost, to steal currency in non-monitored transactions, but theft is not possible if the transaction is monitored. This changes our results dramatically. Now, monetary policy will affect the amount of theft in existence, and theft will matter for how borrowers are punished in the event of default. For example, if the off-equilibrium punishment path involves no credit market activity and only monetary exchange, then the risk of theft is higher on the off-equilibrium path, and this reduces welfare in the punishment equilibrium.

At the optimum it will always be optimal for the government to eliminate theft. Theft will matter for policy, but in the model theft will not be observed in equilibrium. Because theft is potentially more prevalent with off-equilibrium punishment, however, money and credit will in general coexist at the optimum. When theft matters, the Friedman rule is not optimal, and the optimal money growth rate tends to increase as the cost of theft falls.

1.2 The Environment

Time is discrete and each period is divided into two subperiods: day and night. There are two types of agents in the economy, buyers and sellers, and there is a continuum of each type with unit measure. There is a unique perishable consumption good which is produced and consumed within each subperiod. During the day, a seller can produce one unit of the consumption good with one unit of labor. At night, a buyer is able to produce one unit of the consumption good with one unit of labor.

A buyer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - n_t] \tag{1.1}$$

where q_t is consumption during the day, and n_t is labor supply at night, with $\beta \in (0,1)$ the discount factor between night and day. Assume $u(\cdot)$ is strictly concave, strictly increasing, and twice continuously differentiable with $u(0) = 0$, $u'(0) = \infty$, and define q^* to be the solution to $u'(q^*) = 1$. A seller has preferences given by

$$\sum_{t=0}^{\infty} \beta^t (-l_t + x_t) \tag{1.2}$$

where l_t is labor supply during the day and x_t is consumption at night. Sellers and buyers discount at the same rate. Agents are bilaterally and randomly matched during the day and at night trade is centralized.

1.3 Planner's Problem

1.3.1 Full Commitment

First, consider what a social planner could achieve in this economy in the absence of money. Ultimately, money will consist of perfectly divisible and durable objects that are portable at zero cost, and can be produced only by the government. In this section assume there is complete memory and that each agent can commit to the plan proposed by the social planner at $t = 0$. If the planner treats all sellers identically and all buyers identically, then an allocation $\{(q_t, x_t)\}_{t=0}^{\infty}$ satisfies the participation constraints

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - x_t] \geq 0, \tag{1.3}$$

and

$$\sum_{t=0}^{\infty} \beta^t (-q_t + x_t) \geq 0, \tag{1.4}$$

which state that a buyer and a seller, respectively, each prefer to participate in the plan at $t = 0$. Then, if we confine attention to stationary allocations with $q_t = q$ and

$x_t = x$ for all t, we must have

$$u(q) - x \geq 0 \tag{1.5}$$

and

$$-q + x \geq 0 \tag{1.6}$$

The set of feasible stationary allocations is given by (2.5) and (3.4), and this set is non-empty given our assumptions. Further, the set of efficient allocations is also non-empty, satisfying (2.5), (3.4) and $q = q^*$.

1.3.2 Limited Commitment

Now, continue to assume complete memory, but now suppose that any agent can at any time opt out of the plan. The worst punishment that the planner can impose is zero consumption forever for an agent who deviates. Let v_t denote the utility of a buyer at the beginning of t, with w_t similarly denoting the utility of a seller. Then an allocation must satisfy the participation constraints (2.3) and (3.3) as before, as well as the incentive constraints

$$-x_t + \beta v_{t+1} \geq 0, \tag{1.7}$$

and

$$-q_t + x_t + \beta w_{t+1} \geq 0 \tag{1.8}$$

for $t = 0, 1, 2, ..., \infty$. Constraints (3.5) and (3.6) state that the buyer and seller, respectively, prefer to produce at each date rather than defecting from the plan.

Now, confining attention to stationary allocations, the planner's problem is then:

$$\max_{(q,x)\in\mathbb{R}_+^2} u(q) - x$$

subject to

$$x \leq \beta u(q),$$

and

$$-q + x \geq (1-\beta)\, w.$$

where w is the seller's lifetime utility. At the optimum, the seller's incentive constraint binds, so that $x = q + (1-\beta)\, w$. Now, let q^{**} denote the solution to $\beta u(q^{**}) = q^{**}$. We can then rewrite the buyer's incentive constraint as

$$\beta u(q) - q \geq (1-\beta)\, w. \tag{1.9}$$

Given that $w \geq 0$, we must have $q \in [0, q^{**}]$. Then, we can rewrite the planner's problem in the following way:

$$\max_{q \in [0,q^{**}]} u\,(q) - q - (1-\beta)\, w$$

subject to (3.7). The first-order conditions are

$$u'\,(q) - 1 + \lambda\,[\beta u'\,(q) - 1] \geq 0, \text{ with equality if } q < q^{**}, \tag{1.10}$$

and

$$\lambda\,[\beta u(q) - q - (1-\beta)\, w] = 0,$$

where $\lambda \geq 0$ is the Lagrange multiplier associated with the constraint (3.7).

Suppose that $q^{**} \geq q^{*}$. Then, (3.8) holds with equality. If $\lambda > 0$, then the buyer's incentive constraint (3.7) binds, and any $q \in [\hat{q}, q^{*}]$, together with $x = \beta u\,(q)$, is an efficient allocation. If $\lambda = 0$, then from (3.8) any $q = q^{*}$, together with $x \in [q^{*}, \beta u\,(q^{*})]$, is an efficient allocation. In Figure 1, the set of efficient allocations with limited commitment for the case $q^{**} \geq q^{*}$ is ABC, and the set of efficient allocations under full commitment is AD. Notice that the incentive constraint binds for the efficient allocations BC.

The perhaps more interesting case is when $q^{**} < q^{*}$, in which case the set of efficient allocations is $\{(q, x) : x = \beta u(q), q \in [\hat{q}, q^{**}], \beta u'(\hat{q}) = 1\}$. This case is depicted in

Figure 2, where the set of efficient allocations is AB. That is, in this case the incentive constraint for the buyer always binds for the efficient stationary allocation, and an efficient allocation with limited commitment is not efficient under full commitment.

1.4 Equilibrium Allocations with Perfect Memory

To establish a benchmark, we first assume that there is perfect memory. As we would expect from the work of Kocherlakota (1998), this will severely limit the role of money in this economy. An important element of the model will be the bargaining protocol carried out when a buyer and seller meet during the daytime. We assume that the seller first announces whether or not he or she is willing to trade with the buyer. If the seller is not willing to trade, then no exchange takes place. Otherwise, the buyer then makes a take-it-or-leave-it offer to the seller. This protocol in part allows us to focus on the limited commitment friction, as take-it-or-leave-it offers imply that there will be no bargaining inefficiencies.

When a buyer and seller are matched during the day, they continue to be matched at the beginning of the following night, after which all agents enter the nighttime Walrasian market.

1.4.1 Credit Equilibrium

Ultimately we will want to determine the role for valued money in this perfect-memory economy, but our first step will be to look at equilibria where money is not valued. Here, the daytime take-it-or-leave-it offers of buyers consist of credit contracts, with a loan made during the day and repayment at night. We will confine attention to stationary equilibria where sellers always choose to trade when they meet a buyer during the day. Let l denote the loan quantity offered by the buyer to the seller in the day. Then, letting v denote the lifetime continuation utility of the buyer after repaying the loan during the night, we have

$$v = \beta \max_{l} \left[u(l) - l + v\right] \tag{1.11}$$

subject to the incentive constraint

$$l \leq v - \hat{v},$$

and

$$l \geq 0.$$

Here, $\hat{v}$ is the buyer's continuation utility if he or she defaults on the loan, which triggers a punishment. In the equilibrium we consider here, $\hat{v} = 0$, so that the punishment for default is autarky for the defaulting buyer. On the off-equilibrium path, it is an equilibrium for no one to trade with an agent who has defaulted as, if agent A trades with agent B who has defaulted in the past, this triggers autarky for agent A. Here, note that the individual punishment for a buyer who defaults is identical to a global punishment whereby default by any buyer triggers global autarky. Letting $\psi_c(v)$ denote the right-hand side of (3.9), we get

$$\psi_c(v) = \beta u(v), \text{ for } 0 \leq v \leq q^*,$$

and

$$\psi_c(v) = \beta \left[u(q^*) - q^* + v\right], \text{ for } v \geq q^*.$$

An equilibrium is then a solution to $v = \psi_c(v)$. If $q^{**} < q^*$ then there are two equilibria. In the first, $v = 0$, and in the second $v > 0$, which are both solutions to $v = \beta u(v)$. Note that v is also the consumption of the buyer during the day, and of the seller during the night, with $v < q^*$. In this case, the incentive constraint for the buyer binds in either equilibrium. If $q^{**} \geq q^*$ then the $v = 0$ equilibrium still exists, and the equilibrium with $v > 0$ has

$$v = \frac{\beta}{1-\beta}\left[u(q^*) - q^*\right],$$

in which case consumption is q^* for any agent consuming at any date and the incentive constraint does not bind.

Note that, in equilibrium, a seller meeting a buyer during the daytime is always indifferent to trading or not. If he or she announces a willingness to trade, then the buyer makes an offer that leaves the seller with zero surplus, and utility is identical to what the seller would have achieved without trade. In the equilibrium we study, the seller always chooses to trade.

In Figure 3, panel (b) shows the case where the incentive constraint binds in equilibrium, and panel (a) the case where the incentive constraint does not bind. The nonmonetary credit equilibrium, by virtue of the bargaining solution we use, just picks out the efficient stationary allocation that gives all of the surplus to the buyer.

1.4.2 Monetary Equilibrium

Assume that money is uniformly distributed across buyers at the beginning of the first day. Subsequently the government makes equal lump-sum transfers at the beginning of the night to buyers, so that the money stock grows at the gross rate μ. Confine attention to stationary monetary equilibria, and consider only cases where $\mu \geq \beta$, as otherwise a monetary equilibrium does not exist. Let m denote the real money balances acquired by a buyer in the night, and γ the real value of a lump-sum transfer received by a buyer from the government during the night. Suppose that the buyer receives the lump-sum transfer before acquiring money balances during the night, and continue to let v denote the continuation utility for the buyer after receiving the lump-sum transfer. As in Andolfatto (2008), we treat the government symmetrically with the private sector, in that there is limited commitment with respect to tax liabilities as well as private liabilities.

Continue to assume complete memory and, as in the previous subsection, default by a buyer triggers autarky for that buyer. Since a seller will always be indifferent to trading with a buyer, sellers not only refuse to engage in credit contracts with a buyer who has defaulted; they also refuse to take his or her money. Note that the trigger to individual autarky is identical to a global punishment where, if an agent defaults, no seller will trade with any buyer. With global punishment, the value of money is zero on the off-equilibrium path.

In this case, we determine the continuation value v for the buyer by

$$v = \max_{l,m} \left\{ -m + \beta \left[u\left(\frac{1}{\mu} m + l \right) - l + \gamma + v \right] \right\} \tag{1.12}$$

subject to

$$l \leq \gamma + v - \hat{v}$$

$$l \geq 0.$$

Again, we have $\hat{v} = 0$. Here, note that we need to be careful about the lump-sum transfer the buyer receives. Should the buyer default on his or her debt, he or she will also not receive the transfer, or will default on current and future tax liabilities if $\gamma < 0$. In equilibrium, we have

$$\gamma = m\left(1 - \frac{1}{\mu}\right).$$

For $\mu > \beta$, the right-hand side of equation (3.10) is given by

$$\psi_m(v) = -v_1 + \beta u(v_1) + v, \text{ for } \max\left[0, m^*\left(\frac{1}{\mu} - 1\right)\right] \leq v \leq v_1,$$

$$\psi_m(v) = \beta u(v), \text{ for } v_1 \leq v \leq q^*,$$

$$\psi_m(v) = \beta\left[u(q^*) - q^* + v\right], \text{ for } v \geq q^*.$$

Here, m^* solves

$$u'\left(\frac{m^*}{\mu}\right) = \frac{\mu}{\beta},$$

and v_1 satisfies

$$u'(v_1) = \frac{\mu}{\beta},$$

For $\mu = \beta$, we have

$$\psi_m(v) = \beta[u(q^*) - q^*] - m(1-\beta) + \beta v, \text{ for } v \geq m^* \left(\frac{1}{\beta} - 1\right),$$

where

$$m \in [q^* - \min(q^*, v), \beta q^*].$$

Proposition 1 *If $q^{**} \geq q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta$.*

Proof. If $\mu < \beta$, a monetary equilibrium does not exist, for standard reasons. Suppose $\mu > \beta$. Define the function $\Gamma(v) = \Psi_m(v) - v$. Notice that $\Gamma(\cdot)$ is continuous and $\lim_{v\to\infty} \Gamma(v) = -\infty$. Moreover, $\Gamma(v) > 0$ for all $v \in [\max\{0, (1-\mu)v_1\}, q^*)$ and $\Gamma(\cdot)$ is strictly decreasing on (q^*, ∞). Hence, there exists a unique value $v \geq q^*$ such that $\Gamma(v) = 0$. However, money is not valued in this equilibrium. **Q.E.D.** ■

Proposition 2 *If $q^{**} \geq q^*$ and $\mu = \beta$, then a continuum of monetary equilibria exists with $v \in [\frac{\beta[u(q^*)-q^*]}{1-\beta} - \min\left\{\beta q^*, \frac{\beta u(q^*)-q^*}{1-\beta}\right\}, \frac{\beta[u(q^*)-q^*]}{1-\beta})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^*)-q^*}{1-\beta}$.*

Proof. Suppose $\mu = \beta$. It follows that $\Gamma(\cdot)$ is continuous everywhere except possibly at $v = q^*$ and $\lim_{v\to\infty} \Gamma(v) = -\infty$. We have $\Gamma(v) = \beta u(q^*) - q^* > 0$ for all $v \in [(1-\beta)q^*, q^*)$. At $v = q^*$ we have $\Gamma(q^*) = \beta u(q^*) - q^* - (1-\beta)m$. A necessary condition for the existence of a monetary equilibrium is $\Gamma(q^*) \geq 0$, which requires

$$m \leq \frac{\beta u(q^*) - q^*}{1-\beta}.$$

Hence, a necessary and sufficient condition for the existence of a monetary equilibrium is

$$m \leq \min\left\{\beta q^*, \frac{\beta u(q^*) - q^*}{1-\beta}\right\}.$$

Given a positive value of m satisfying the inequality above, there exists a unique value $v \geq q^*$ such that $\Gamma(v) = 0$, in which case money is valued in equilibrium. Therefore, there exists a continuum of monetary equilibria with

$$v \in [\frac{\beta[u(q^*) - q^*]}{1-\beta} - \min\left\{\beta q^*, \frac{\beta u(q^*) - q^*}{1-\beta}\right\}, \frac{\beta[u(q^*) - q^*]}{1-\beta}).$$

All of these equilibria support the allocation $(q, x) = (q^*, q^*)$. **Q.E.D.** ■

Proposition 3 *If $q^{**} < q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta u'(q^{**})$.*

Proof. Suppose $\mu > \beta u'(q^{**})$. Notice that $\Gamma(v) = \beta u(v_1) - v_1 > 0$ for all $v \in [\max\{0, (1-\mu)v_1\}, v_1)$, $\Gamma(v) > 0$ for all $v \in [v_1, q^{**})$, and $\Gamma(q^*) < 0$. Since $\Gamma(\cdot)$ is continuous and strictly decreasing on (q^{**}, ∞), it follows that $v = q^{**}$ is the unique value satisfying $\Gamma(v) = 0$. Since $v_1 < q^{**}$, it follows that money is not valued in equilibrium. Suppose $\mu \in (\beta, \beta u'(q^{**}))$. In this case, $\Gamma(v) < 0$ for all $v \geq (1-\mu)v_1$, so that a monetary equilibrium does not exist. Finally, assume $\mu = \beta$. Again, we find that $\Gamma(v) < 0$ for all $v \geq (1-\beta)q^*$, so that a monetary equilibrium does not exist. **Q.E.D.** ■

Proposition 4 *If $q^{**} < q^*$ and $\mu = \beta u'(q^{**})$, then a continuum of monetary equilibria exists with $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^{**}) - q^{**}}{1-\beta}$.*

Proof. Take $\mu = \beta u'(q^{**})$. Then, $\Gamma(v) = 0$ for all $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**}]$ and $\Gamma(v) < 0$ for all $v > q^{**}$. Hence, a continuum of monetary equilibria exists with $v \in [q^{**}[1 - \beta u'(q^{**})], q^{**})$. All of these equilibria yield the allocation $(q, x) = (q^{**}, q^{**})$. **Q.E.D.** ■

If the money growth rate is sufficiently high, that is if $\mu > \beta \max[1, u'(q^{**})]$, then the rate of return on money is sufficiently low that money is not held in equilibrium.

If $q^* \leq q^{**}$, it certainly seems clear why a monetary equilibrium will not exist when the money growth rate is sufficiently high. In this case, when money is not valued a credit equilibrium exists which is efficient and incentive constraints do not bind. Thus, there is clearly no role for money in equilibrium in relaxing incentive constraints in decentralized trade. Why money is not valued even when $q^* > q^{**}$ and $\mu > \beta u'(q^{**})$ is perhaps less clear. In this case, the only stationary equilibria that exist are the two credit equilibria: one where $v = 0$ and one with $v > 0$ and binding incentive constraints, as in Figure 3. Money cannot relax the binding incentive constraints, as in order to support a money growth rate sufficiently low as to induce agents to hold money, the government would have to impose sufficiently high taxes that buyers would choose to default on their tax liabilities. Thus, there is no role for money in improving efficiency.

If $\mu = \beta \max[1, u'(q^{**})]$, then in equilibrium buyers are essentially indifferent between using money and credit in decentralized transactions with sellers, and there exist a continuum of equilibria with valued money. Each of these equilibria supports the same allocation as does the credit equilibrium with $v > 0$. The continuum of equilibria is indexed by the quantity of real money balances held by buyers. Across these equilibria, as the quantity of real balances rises, the quantity of lending falls.

Our results are consistent with the ideas in Kocherlakota (1998), as they should be. With perfect memory, money is not socially useful. At best, money can be held in equilibrium. This equilibrium is either one where incentive constraints do not bind and the monetary authority follows a Friedman rule, or incentive constraints bind and the money growth rule is similar to what Andolfatto (2008) finds. In either case, money provides no efficiency improvement.

1.5 Imperfect Memory and Autarkic Punishment

As we have seen, with perfect memory there is essentially no social role for money, and it will only be held under special circumstances. As is well known, particularly given the work of Kocherlakota, we need some imperfections in record-keeping in order for money to be useful and to help it survive as a valued object. We will start by assuming that, during the day, there is no memory in some bilateral meetings, and perfect memory in other meetings. In particular, a fraction ρ of sellers has no monitoring potential, while a fraction $1-\rho$ does. In any day, a given buyer has probability ρ of meeting a seller with no monitoring potential, in which case there is no memory in the interaction between the buyer and seller. That is, each agent in such a meeting has no knowledge of his or her trading partner's history, and nothing about the meeting will be recorded. With probability $1-\rho$ a buyer meets a seller with monitoring potential. In this case, the buyer has the opportunity to choose to have his or her interaction with the seller monitored. Here, $0<\rho\leq 1$. If the buyer chooses a monitored interaction in the day, then his or her history is observable to the seller, and the interaction between that buyer and seller will be publicly observed during the day and through the beginning of the following night. Otherwise, the buyer's and seller's actions are unobserved during the day and the following night.

Trade is carried out anonymously in the Walrasian market that opens in the latter part of each night, in the sense that all that can be observed in the Walrasian market is the market price. Individual actions are unobservable. Here, the case where $\rho=1$ is the standard one in monetary models with random matching such as Lagos and Wright (2005). However, even in the case with $\rho=1$, we deviate from the usual assumptions, in that there is lack of commitment with respect to tax liabilities. We assume that each agent can observe the interaction between the government and all other agents. That is, default on tax liabilities is publicly observable.

We change the bargaining protocol between a buyer and seller during the day as follows. The buyer first declares whether interactions with the seller during the period will be monitored or not. If monitoring is chosen, then the seller learns the buyer's history of publicly-recorded transactions. Then, the seller decides whether or not to transact with the buyer. If the seller is willing to transact, the buyer then makes a take-it-or-leave-it offer.

Given our setup, if a buyer defaults on a loan made in a monitored trade, this will be public information. As well, default by a buyer on tax liabilities is public information. However, suppose that a seller were to make a loan to a buyer during the day in a non-monitored trade, and then the buyer defaulted on the loan during the night. In this case, it is impossible for that seller to signal to anyone else that default has occurred. The interaction between the buyer and seller is private information, and the individual seller cannot affect prices in any subsequent nighttime Walrasian market. Further, in the equilibria we study, a seller in a non-monitored trade during the day will never have the opportunity to engage in a monitored trade during any subsequent day and so will be unable to signal that a default has occurred.

1.5.1 Credit Equilibrium

First consider stationary equilibria where money is not valued, so that all exchanges in the day market involve credit. Here, in the case where a buyer does not have the opportunity to engage in a monitored transaction, there will be no exchange between the buyer and the seller, as the buyer will be able to default and this will be private information. Thus if money is not valued, then trade takes place during the day only in monitored transactions, and the buyer will always weakly prefer to have the interaction with a seller monitored. Here, v is determined by

$$v = \beta \left\{ (1-\rho) \max_{l} \left[u(l) - l \right] + v \right\} \tag{1.13}$$

subject to the incentive constraint

$$l \leq v - \hat{v},$$

and

$$l \geq 0.$$

As in the previous section, $\hat{v}$ is the continuation value when punishment occurs, and the punishment is autarky so $\hat{v} = 0$. Now, letting $\phi_c(v)$ denote the right-hand side of (2.13), we can rewrite (2.13) as

$$v = \phi_c(v),$$

with

$$\phi_c(v) = \beta \left[(1-\rho)u(v) + \rho v\right], \text{ for } 0 \leq v \leq q^*$$

and

$$\phi_c(v) = \beta \left\{(1-\rho)[u(q^*) - q^*] + v\right\}, \text{ for } v \geq q^*.$$

Let q^{***} denote the solution to

$$\frac{\beta(1-\rho)}{1-\rho\beta} u(q^{***}) = q^{***}$$

Proposition 5 *If $q^{***} < q^*$ then there are two credit equilibria, one where $v = 0$, and one where the incentive constraint binds, $l < q^*$ and $v = q^{***}$.*

Proof. Define $\Gamma_c(v) = \phi_c(v) - v$. Note that $\Gamma_c(\cdot)$ is continuous and $\lim_{v\to\infty} \Gamma_c(v) = -\infty$. Since $q^{***} < q^*$, it follows that $\Gamma_c(v) > 0$ for all $v \in (0, q^{***})$, $\Gamma_c(q^{***}) = 0$, and $\Gamma_c(v) < 0$ for all $v \in (q^{***}, \infty)$. This implies that $v = q^{***}$ is the unique positive value satisfying $\Gamma_c(v) = 0$. Since the incentive constraint binds, it follows that $l = q^{***}$ in such equilibrium. **Q.E.D.** ■

Proposition 6 *If $q^{***} \geq q^*$ then there are two credit equilibria, one where $v = 0$ and one where the incentive constraint does not bind, $l = q^*$, and*

$$v = \frac{\beta(1-\rho)[u(q^*) - q^*]}{1-\beta}.$$

Proof. In this case, $\Gamma_c(v) > 0$ for all $v \in (0, q^*)$ and $\Gamma_c(q^*) \geq 0$. Note that $\Gamma_c(\cdot)$ is strictly decreasing on (q^*, ∞), with $\lim_{v \to \infty} \Gamma_c(v) = -\infty$. Hence, there exists a unique positive value $v \geq q^*$ satisfying $\Gamma_c(v) = 0$. This means that

$$\beta \{(1-\rho)[u(q^*) - q^*] + v\} - v = 0,$$

so that

$$v = \frac{\beta(1-\rho)}{1-\beta}[u(q^*) - q^*],$$

and $l = q^*$ is the amount consumed by the buyer in a monitored meeting during the day. **Q.E.D.** ■

Now, since $q^{***} < q^{**}$ for $\rho > 0$, imperfect memory limits credit market activity, just as one might expect. Relative to the credit equilibrium with perfect memory there is in general less trade in a credit equilibrium with imperfect memory, and the quantity traded decreases as ρ increases. Of course, there is no credit market activity when $\rho = 1$.

1.5.2 Monetary Equilibrium

As in the previous section, publicly observable default triggers autarky for the agent who defaults. However, in this case autarkic punishment is carried out through a global punishment whereby, if a single buyer defaults, this triggers an equilibrium where no seller will trade during the day and therefore money is not valued.

Here, we solve for the equilibrium continuation value in a similar fashion to the previous section. That is,

$$v = \max_{m,l} \left(-m + \beta \left\{ \rho u\left(\frac{m}{\mu}\right) + (1-\rho)\left[u\left(\frac{m}{\mu} + l\right) - l\right] + \gamma + v \right\} \right) \tag{1.14}$$

subject to

$$l \leq \gamma + v - \hat{v},$$

$$l \geq 0.$$

Given autarkic punishment, $\hat{v} = 0$. In equilibrium, the real value of the government transfer is

$$\gamma = m\left(1 - \frac{1}{\mu}\right). \tag{1.15}$$

Here, and in the rest of the paper, it will prove to be more straightforward to define an equilibrium and solve for it in terms of the consumption quantities for the buyer in non-monitored and monitored trades, rather than solving for the continuation value v. Therefore, let x be the daytime consumption of a buyer in the non-monitored state, and y the buyer's daytime consumption in the monitored state. Then in the problem (2.14) above, we have $m = \mu x$, $\gamma = x(\mu - 1)$, and $l = y - x$. Thus from (2.14), we can solve for v in terms of x and y to get

$$v = -\mu x + \frac{\beta\left\{\rho\left[u(x) - x\right] + (1-\rho)\left[u(y) - y\right]\right\}}{1-\beta}.$$

We can then define an equilibrium in terms of x and y as follows.

Definition 1 *A stationary monetary equilibrium is a pair (x, y), where x and y are chosen optimally by the buyer,*

$$\rho u'(x) + (1-\rho)\, u'(y) = \frac{\mu}{\beta}, \tag{1.16}$$

x and y have the property that consumptions and the loan quantity are nonnegative, and consumptions do not exceed the surplus-maximizing quantity,

$$0 \leq x \leq y \leq q^*, \tag{1.17}$$

and (x, y) is incentive compatible,

$$\beta\left[\rho u(x) + (1-\rho)\, u(y)\right] - \rho\beta x - (1-\rho\beta)\, y \geq (1-\beta)\hat{v}, \tag{1.18}$$

where $y = q^*$ if (1.18) does not bind.

Proposition 7 *If $q^{**} \geq q^{*}$ then a unique stationary monetary equilibrium exists for $\mu \geq \beta$.*

Proof. Suppose $q^{*} \leq q^{***} \leq q^{**}$. In this case, we cannot have an equilibrium with a binding incentive constraint. Now, if the incentive constraint does not bind, then $y = q^{*}$ and

$$\rho u'(x) + 1 - \rho = \frac{\mu}{\beta}. \tag{1.19}$$

Note that $(1-\rho)\beta u(q^{*}) - (1-\rho\beta) q^{*} \geq 0$, so that the incentive constraint is always slack when $y = q^{*}$. Therefore, a unique stationary monetary equilibrium with a non-binding incentive constraint exists for any $\mu \geq \beta$, with x defined by (1.19) and $y = q^{*}$.

Suppose $q^{***} < q^{*} \leq q^{**}$. First, assume the incentive constraint does not bind. Then, there exists $\tilde{\mu} > \beta$ such that

$$\rho\beta[u(x) - x] \geq -(1-\rho)\beta u(q^{*}) + (1-\rho\beta) q^{*}$$

if and only if $\mu \in [\beta, \tilde{\mu}]$. Again, a unique stationary monetary equilibrium with a non-binding incentive constraint exists for $\mu \in [\beta, \tilde{\mu}]$. Let $\tilde{x}$ be the value of x satisfying (1.19) when $\mu = \tilde{\mu}$. For $\mu > \tilde{\mu}$, the incentive constraint binds, and a unique stationary monetary equilibrium exists with (x, y) satisfying

$$\rho\beta[u(x) - x] = -(1-\rho)\beta u(y) + (1-\rho\beta) y \tag{1.20}$$

and

$$\rho u'(x) + (1-\rho) u'(y) = \frac{\mu}{\beta}, \tag{1.21}$$

where $x < \tilde{x}$ and $q^{***} < y < q^{*}$. **Q.E.D.** ■

Proposition 8 *If $q^{**} < q^{*}$ then a unique stationary monetary equilibrium exists for $\mu \geq \beta u'(q^{**})$.*

Proof. Note that we cannot have an equilibrium with a non-binding incentive constraint because

$$\rho\beta\left[u\left(x\right)-x\right]<-\left(1-\rho\right)\beta u\left(q^{*}\right)+\left(1-\rho\beta\right)q^{*}$$

when $q^{**}<q^{*}$. Then, (x,y) satisfy (1.20) and (1.21), with $0\leq x\leq y<q^{*}$. Note that (1.20) requires that $y\leq q^{**}$. Then, a unique stationary monetary equilibrium exists for any $\mu\geq\beta u'\left(q^{**}\right)$. **Q.E.D.** ■

If $q^{***}\geq q^{*}$, which guarantees that $q^{**}>q^{*}$, as $q^{***}<q^{**}$, then the incentive constraint does not bind for all $\mu\geq\beta$. In this case, the buyer consumes q^{*} in all monitored trades where credit is used during the day, and consumes x in non-monitored trades, where $x\leq q^{*}$ and x is decreasing in μ. Therefore, the welfare of the buyer is decreasing in μ, while the seller receives zero utility in each period for all μ. Further, when $\mu=\beta$, then $x=y=q^{*}$, in which case the loan quantity is $l=y-x=0$, and no credit is used. As μ increases, then, the quantity of credit rises, that is credit is substituted for money in transactions.

If $q^{**}\geq q^{*}>q^{***}$, then the incentive constraint binds for $\mu>\tilde{\mu}$, where

$$\rho\beta\left[u\left(\tilde{x}\right)-\tilde{x}\right]=-\beta(1-\rho)u(q^{*})+(1-\rho\beta)q^{*}$$

with $\tilde{x}$ the solution to

$$\rho u'\left(\tilde{x}\right)+1-\rho=\frac{\tilde{\mu}}{\beta}.$$

The incentive constraint does not bind for $\beta\leq\mu\leq\tilde{\mu}$. Here, $\mu=\beta$ implies that $x=y=q^{*}$ and there is no credit, just as in the previous case. However, if the money growth rate is sufficiently high, then the incentive constraint binds. If the incentive constraint does not bind, then just as in the previous case $y=q^{*}$ and x falls as μ rises, so that the welfare of buyers falls with an increase in μ and credit is substituted for money in transactions. If the incentive constraint binds, then it is straightforward to show that an increase in μ causes both x and y to fall, with the loan quantity $l = y - x$ increasing. Thus, as in the other cases, the welfare of buyers must fall as μ rises, and the use of credit rises with an increase in the money growth rate.

Finally, if $q^{**} < q^*$, then the incentive constraint will always bind in a stationary monetary equilibrium. Here, when $\mu = \beta u'(q^{**})$, then $x = y = q^{**}$ and there is no credit. Again, it is straightforward to show that x, y, and the welfare of buyers decrease with an increase in μ, and the quantity of lending rises.

Which case we get (the incentive constraint never binds; the incentive constraint binds only for large money growth rates; the incentive constraint always binds) depends on $q^* - q^{***}$. While q^* is independent of β and ρ, q^{***} is increasing in β and decreasing in ρ. Thus, the incentive constraint will tend to bind the lower is β and the higher is ρ. Higher β tends to relax incentive constraints for typical reasons. That is, as buyers care more about the future, potential punishment is more effective in enforcing good behavior. Higher ρ implies that the imperfect memory friction becomes more severe, and credit can be used with lower frequency. In general, monetary exchange will be less efficient than credit, and so a reduction in the frequency with which credit can be used will tend to reduce the utility of a buyer in equilibrium. This will therefore reduce the relative punishment to a buyer if he or she defaults and thus tighten incentive constraints.

Proposition 9 *If $q^{**} \geq q^*$, then $\mu = \beta$ is optimal, and this implies that $l = 0$, the incentive constraint does not bind, and the buyer consumes q^* in all trades during the day.*

Proof. Suppose the government treats buyers and sellers equally. Then, the government chooses a money growth rate $\mu \geq \beta$ to maximize $\rho [u(x) - x] + (1 - \rho)[u(y) - y]$ subject to (2.16), (2.17), and (1.18). It follows that $\mu = \beta$ implies $x = y = q^*$, and the efficient allocation under full commitment is implemented. **Q.E.D.** ■

Proposition 10 *If $q^{**} < q^*$, then $\mu = \beta u'(q^{**})$ is optimal, and this implies that $l = 0$, the incentive constraint binds, and the buyer consumes q^{**} in all trades during the day.*

Proof. If $q^{**} < q^*$, the incentive constraint requires that $y \leq q^{**}$. It follows from (1.20) and (1.21) that setting $\mu = \beta u'(q^{**})$ implements the efficient allocation (q^{**}, q^{**}). **Q.E.D.** ■

Here, we have essentially generalized the results of Andolfatto (2008) to the case where credit is permitted in some types of bilateral trades. If the discount factor is sufficiently small, then the Friedman rule is not feasible and the incentive constraint binds at the optimum. In terms of our goal of constructing a model with robust money and credit, an undesirable feature of this setup is that optimal monetary policy drives credit out of the economy. Here, the only inefficiency in monetary exchange is due to the fact that buyers in general hold too little real money balances in equilibrium, and this inefficiency can be corrected in the usual way, with the caveat that too much deflation can cause agents to default on their tax liabilities. Ultimately, at the optimum money is equivalent to memory, in that an appropriate monetary policy achieves the same allocation that could be achieved by a social planner with perfect recordkeeping.

1.6 Imperfect Memory and Non-Autarkic Punishment

In the previous section, given the limited commitment friction, optimal monetary policy will yield an equilibrium allocation where credit is not used. Credit seems to be more robust than this in practice, so we would like to study frictions that potentially imply that money and credit coexist, even when monetary policy is efficient.

Here, we will assume the same information technology and bargaining protocol as in the previous section. However, we will consider a different equilibrium, where default does not trigger autarky, but instead triggers an equilibrium where money is valued. That is, a default results in reversion to an equilibrium where sellers will not trade if a buyer announces that he or she wishes the interaction to be monitored, but will exchange goods for money if the buyer announces that the interaction will not be monitored. The government is not able to commit to a monetary policy, so the money growth rate that is chosen by the government when punishment occurs is chosen optimally at that date given the behavior of private sector agents.

We restrict attention to punishment equilibria that are stationary. Further, a punishment equilibrium must be sustainable, in that no agent would choose to default on his or her tax liabilities in such an equilibrium. Letting $\hat{v}$ denote the continuation value in the punishment equilibrium, after agents receive their lump-sum transfers from the government, we have

$$\hat{v} = -m(\mu) + \beta \left[u\left(\frac{m(\mu)}{\mu}\right) + m(\mu)\left(1 - \frac{1}{\mu}\right) + \hat{v} \right]$$

where $m(\mu)$ is the quantity of real balances acquired by the buyer during the night, which solves the first-order condition

$$u'\left(\frac{m(\mu)}{\mu}\right) = \frac{\mu}{\beta}. \tag{1.22}$$

Now, for the punishment equilibrium to be sustainable, we require that

$$m(\mu)\left(1 - \frac{1}{\mu}\right) + \hat{v} \geq \hat{v}, \tag{1.23}$$

i.e. the equilibrium is sustained in the sense that, if an agent chooses not to accept the transfer from the government, then the punishment is reversion to the punishment equilibrium. Clearly, condition (1.23) implies that punishment equilibria are sustainable if and only if $\mu \geq 1$. That is, private agents need to be bribed to enforce the punishment with positive transfers, otherwise they would default on the tax liabilities.

The government will choose μ optimally in the punishment equilibrium, and it must choose a sustainable money growth factor, i.e. $\mu \geq 1$. Assume that the government weights the utility of buyers and sellers equally, though since sellers receive zero utility in any punishment equilibrium, it is only the buyers that matter. Therefore, the government solves

$$\max_{\mu}\left[u\left(\frac{m(\mu)}{\mu}\right)-\frac{m(\mu)}{\mu}\right]$$

subject to (1.22) and $\mu \geq 1$. Clearly, the solution is $\mu = 1$, so we have

$$\hat{v} = \frac{-\hat{m} + \beta u(\hat{m})}{1-\beta}, \tag{1.24}$$

where $\hat{m}$ solves

$$u'(\hat{m}) = \frac{1}{\beta} \tag{1.25}$$

When punishment occurs, the government would like to have been able to commit to an infinite growth rate of the money supply so as to make the punishment as severe as possible. However, given the government's inability to commit, once punishment is triggered the government chooses the sustainable money growth rate that maximizes welfare, consistent with the optimal punishment behavior of sellers in the credit market. Thus, the money growth rate is set as low as possible without inducing default on tax liabilities.

Now that we have determined the continuation value in a punishment equilibrium, we can work backward to determine what the equilibrium can be. For this purpose, we again define the stationary equilibrium in terms of (x, y), where x denotes the daytime consumption of a buyer in the non-monitored state, and y the buyer's daytime consumption in the monitored state. The definition of a stationary monetary equilibrium is the same as in the previous section, except now $\hat{v}$ is defined by (1.24) and (1.25).

Proposition 11 *The only monetary equilibrium is the punishment equilibrium.*

Proof. First, suppose that $y > x$ in equilibrium. Then, using Jensen's inequality,

$$\beta[\rho u(x)+(1-\rho)u(y)]-\rho\beta x-(1-\rho\beta)y < \beta u\left[\rho x + (1-\rho)y\right]-\rho x-(1-\rho)y \leq -\hat{m}+\beta u(\hat{m}),$$

by virtue of (1.25). Thus, given that an equilibrium must satisfy (1.18), we have a contradiction. Therefore, if an equilibrium exists, it must have $y = x$, in which case inequality (1.18) can be written, using (1.24),

$$-x + \beta u(x) \geq -\hat{m} + \beta u(\hat{m}),$$

but then by virtue of (1.25), (1.24) can only be satisfied, with equality, when $x = y = \hat{m}$, and this can be supported, from (2.16), only if the money growth factor is $\mu = 1$. **Q.E.D.** ■

Therefore, the only monetary equilibrium with non-autarkic punishment is one where no credit is supported. The incentive constraint is satisfied with equality and no seller is willing to lend to a borrower, even if the interaction is monitored. The optimal money growth factor, indeed the only feasible money growth factor, is $\mu = 1$.

Intuition might tell us that, in line with some of the ideas in Aiyagari and Williamson (2000) and Antinolfi, Azariadis, and Bullard (2007), the possibility of being banned from credit markets, but with punishment mitigated by the ability to trade money for goods, would tend to promote credit. That is, because the degree of punishment depends on money growth, the government might tend to produce inflation so as to increase the punishment for bad behavior in the credit market, thus reducing the payoff to holding money and causing buyers to substitute credit for money. In the context of this model, this intuition is wrong, in part because we take account here of the government's role as a strategic player, and its inability to commit to inflicting punishment.

Thus far, we have not arrived at a set of assumptions concerning the information structure under which credit is robust. Either efficient monetary policy will drive credit out of the system, or the only equilibrium that exists is one without credit. Thus, it appears that there must be another friction or frictions that are necessary to the coexistence of robust money and credit that we observe in reality.

1.7 Theft

One aspect of monetary exchange is that, due to anonymity, theft is easier in most respects than it is with exchange using credit. It seems useful to consider a framework where limited commitment makes credit arrangements difficult, and theft makes monetary exchange difficult. However, the fact that theft makes monetary exchange difficult may lessen the limited commitment friction in the credit market, as this will make default less enticing.

We will assume the same imperfect memory structure as in the previous section, but allow for a technology that permits the theft of cash. Suppose the following bargaining protocol. On meeting a seller in the daytime, the buyer first announces whether his or her interaction with the seller will be monitored or not. Recall that it is necessary that the seller have the potential for monitoring (occurring with probability $1-\rho$ from the buyer's point of view) in order for the interaction to be monitored. Then, the seller announces whether or not he or she is willing to trade. Following this, if the interaction is not monitored, the seller can pay a fixed cost τ to acquire a technology (a "gun"), which permits him or her to confiscate the buyer's money, if the buyer has any. Clearly, if the buyer's money is stolen in a non-monitored trade, the interaction with the seller ends there. Otherwise, the buyer makes a take-it-or-leave-it offer to the seller if the seller has agreed to trade.

With theft, an equilibrium can be characterized by (x, y, α) where, as before, x is consumption by the buyer in a non-monitored trade when theft does not occur, y is consumption when monitored, and α is the fraction of non-monitored daytime meetings where theft occurs, so that $\alpha \in [0, 1]$. In general, given the continuation value $\hat{v}$ in the punishment equilibrium, we can define a monetary equilibrium as follows.

Definition 2 *A monetary equilibrium is a triple* (x, y, α), *where* x *and* y *are chosen optimally by the buyer,*

$$\rho(1-\alpha)u'(x)+(1-\rho)u'(y)=\frac{\mu}{\beta}, \tag{1.26}$$

x and y have the property that consumptions and the loan quantity are nonnegative, and consumptions do not exceed the surplus-maximizing quantity,

$$0 \leq x \leq y \leq q^*, \tag{1.27}$$

(x, y, α) is incentive compatible,

$$\beta[\rho(1-\alpha)u(x)+(1-\rho)u(y)]-\rho\beta x-(1-\rho\beta)y \geq \hat{v}(1-\beta), \tag{1.28}$$

where $y = q^*$ if (1.28) does not bind. Further, x and α must be consistent with optimal theft by sellers in non-monitored trades, that is

$$\textit{if } \alpha = 0, \ \textit{then } x \leq \tau, \tag{1.29}$$

$$\textit{if } 0 < \alpha < 1, \ \textit{then } x = \tau, \tag{1.30}$$

$$\textit{if } \alpha = 1, \ \textit{then } x \geq \tau. \tag{1.31}$$

Conditions (1.29)-(1.31) state that in equilibrium there is either no theft, so sellers must weakly prefer not to steal in non-monitored trades, or sellers sometimes steal, so they must be indifferent to being honest, or sellers always steal, so they must weakly prefer theft.

Now, the government will choose μ so as to maximize welfare in equilibrium, where the utilities of sellers and buyers are weighted equally. Thus, in the stationary equilibria we study, the government wishes to maximize

$$W = \rho(1-\alpha)\left[u(x)-x\right]+(1-\rho)\left[u(y)-y\right]-\alpha\rho\tau$$

Lemma 12 *When the government chooses μ optimally, $\alpha = 0$.*

Proof. First, suppose that there exists an equilibrium with $\alpha = 1$, $y = \bar{y} < q^*$ and $x > \tau$, supported by $\mu = \bar{\mu}$. Then from the definition of equilibrium, we can construct another equilibrium with $\alpha < 1$, $y > \bar{y}$ and $x = \tau$, supported by some $\mu > \bar{\mu}$. In this other equilibrium, W must be larger. If there exists an equilibrium with $\alpha = 1$, $y < q^*$ and $x = \tau$ in equilibrium, we can accomplish the same thing except by holding x constant at τ. Similarly if $\alpha = 1$ and $y = q^*$ the same argument applies except that we do not increase y. Next, if $0 < \alpha < 1$ in equilibrium, we can construct another equilibrium with lower α, larger μ, and larger y if $y < q^*$ which achieves higher welfare. **Q.E.D.** ■

A smaller amount of theft necessarily increases the continuation value for the buyer and relaxes the incentive constraint, while increasing welfare. A smaller amount of theft can be achieved in this fashion as an equilibrium outcome with a higher money growth rate. The higher money growth rate discourages the holding of currency, and therefore reduces the payoff from theft. Note that this is true no matter what $\hat{v}$ is. Irrespective of the punishment that is imposed when a buyer defaults, efficient monetary policy must always drive out theft.

1.7.1 Autarkic Punishment

First, consider the case where default triggers autarky. In determining what is optimal for the government in this context, we know from the above arguments that we can restrict attention to equilibria where $\alpha = 0$, and search among these equilibria for the one that yields the highest welfare. The government then solves the following problem:

$$\max_{x,y,\mu} \{\rho [u(x) - x] + (1 - \rho) [u(y) - y]\} \tag{1.32}$$

subject to

$$\rho u'(x) + (1 - \rho) u'(y) = \frac{\mu}{\beta}, \tag{1.33}$$

$$x \leq \tau \tag{1.34}$$

$$0 \leq x \leq y \leq q^* \tag{1.35}$$

$$\beta[\rho u(x) + (1-\rho)u(y)] - \rho\beta x - (1-\rho\beta)y \geq 0, \tag{1.36}$$

where $y = q^*$ if the last constraint does not bind. We first have the following results.

Proposition 13 *If $q^* \leq q^{**}$ and $\tau \geq q^*$, then a Friedman rule is optimal, and this supports an efficient allocation in equilibrium.*

Proof. Suppose that we ignore the constraint (1.34) in the government's optimization problem. If $q^* \leq q^{**}$, then the solution to the problem is $x = y = q^*$ and $\mu = \beta$, i.e. the solution is what we obtained when we studied non-autarkic punishment with the same setup and no theft technology. However, for the constraint (1.34) not to bind at the optimum then requires $\tau \geq q^*$. **Q.E.D.** ■

Proposition 14 *If $q^* > q^{**}$ and $\tau \geq q^{**}$, then $\mu = \beta u'(q^{**})$ at the optimum, and this supports an efficient allocation in equilibrium.*

Proof. Again, suppose that we ignore the constraint (1.34) and solve the government's optimization problem in the case where $q^* > q^{**}$. Then the solution to the problem is $x = y = q^{**}$ and $\mu = \beta u'(q^{**})$, i.e. the solution is what we obtained when we studied non-autarkic punishment with the same setup and no theft technology. Now, for the constraint (1.34) not to bind at the optimum requires $\tau \geq q^{**}$. **Q.E.D.** ■

Thus, as should be obvious, if the cost of theft is sufficiently large that theft does not take place in equilibrium given the efficient monetary policy rules we derived in the absence of theft, then theft is irrelevant for policy. Of course, our interest is in what happens when theft is sufficiently lucrative, i.e. when τ is sufficiently small that (1.34) binds at the optimum.

Now, since $x = \tau$ at the optimum when theft matters, this makes solving the government's optimization problem easy. First, suppose that $q^* \leq q^{***} \leq q^{**}$ in which case theft matters if and only if $\tau \leq q^*$. Then $(x, y) = (\tau, q^*)$ must be optimal, as this satisfies (1.36) as a strict inequality, (1.35) is satisfied, and we can recover the money growth factor that supports this as an equilibrium from (1.33), i.e.

$$\mu = \beta \left[\rho u'(\tau) + 1 - \rho \right]. \tag{1.37}$$

Note that the optimal money growth rate rises as the cost of theft falls, as a lower cost of theft requires a higher money growth rate to drive out theft. An interesting feature of the efficient equilibrium is that money and credit now coexist. Indeed, the loan quantity is $l = q^* - \tau$, which increases as the cost of theft decreases. Essentially, money and credit act as substitutes. As the theft friction gets more severe, money becomes more costly to hold at the optimum (the optimal money growth rate rises), and buyers use credit more intensively.

Now, suppose that $q^{***} < q^* \leq q^{**}$ in which case theft matters if and only if $\tau \leq q^*$. Let $\bar{\tau} < q^*$ be the unique value of τ satisfying

$$\rho\beta \left[u\left(\bar{\tau}\right) - \bar{\tau} \right] + \beta(1 - \rho)u(q^*) - (1 - \rho\beta)q^* = 0.$$

Then, for $\tau \in (0, \bar{\tau}]$ the incentive constraint binds, and the optimal equilibrium allocation is $(x, y) = (\tau, \bar{y})$, where $\bar{y}$ is the solution to

$$\rho\beta \left[u\left(\tau\right) - \tau \right] + \beta(1 - \rho)u(\bar{y}) - (1 - \rho\beta)\bar{y} = 0. \tag{1.38}$$

The optimal money growth factor in this case is

$$\mu = \beta \left[\rho u'(\tau) + (1 - \rho)u'(\bar{y}) \right]. \tag{1.39}$$

For $\tau \in [\bar{\tau}, q^*]$, the incentive constraint does not bind, and the optimal equilibrium allocation is $(x, y) = (\tau, q^*)$ with the optimal money growth factor given by

$$\mu = \beta \left[\rho u'(\tau) + 1 - \rho \right].$$

Clearly, given $q^{***} < q^* \leq q^{**}$, x and y both decrease as τ decreases, at the optimum, so that the welfare of buyers falls. Further, it is straightforward to show that the quantity of lending, $y - x$ increases as τ falls, at the optimum, so that less costly theft promotes credit. As well, the optimal money growth factor decreases as the cost of theft rises.

Finally, consider the case where $q^* > q^{**}$, in which case theft matters if and only if $\tau \leq q^{**}$. Here, the incentive constraint always binds, and $(x, y) = (\tau, \bar{y})$, where $\bar{y}$ is the solution to (1.38), and the optimal money growth factor is given by (1.39). Just as in the other cases, x and y fall as τ falls, at the optimum, and welfare decreases. As well, the quantity of lending rises as τ falls at the optimum.

1.7.2 Non-Autarkic Punishment

Recall that, with non-autarkic punishment we are looking for a sustainable punishment equilibrium in which, if a buyer meets a seller and announces that the interaction will be monitored, the seller will not trade. Money will be valued in the punishment equilibrium, but all transactions between buyers and sellers will be non-monitored ones. The government cannot commit to a monetary policy rule, so when default occurs the government will choose the money growth factor that maximizes welfare in the punishment equilibrium.

Through arguments identical to what we used previously when theft was not an issue, any sustainable punishment equilibrium must have $\mu \geq 1$, as buyers need to be bribed with a transfer to sustain the punishment. Note that we cannot have $\alpha = 1$ in the punishment equilibrium since, if all sellers steal, no buyer would accumulate money balances, but if no buyer accumulates money balances there will be no theft. Let x denote the buyer's daytime consumption in the punishment equilibrium. Then, the punishment equilibrium is the solution to the following problem.

$$\max_{x,\alpha,\mu} (1 - \alpha) [u(x) - x] - \alpha\tau$$

subject to

$$(1-\alpha)u'(x) = \frac{\mu}{\beta}$$

$$0 \leq x \leq q^*$$

$$\alpha \in [0,1)$$

$$\mu \geq 1$$

$$\text{if } \alpha = 0, \text{ then } x \leq \tau$$

$$\text{if } \alpha > 0, \text{ then } x = \tau$$

Now, just as in the efficient equilibrium, it is straightforward to show that part of the solution to this problem is $\alpha = 0$. That is, if there is a sustainable equilibrium where $\alpha > 0$, then there is another equilibrium with a higher money growth factor, lower α, and higher welfare that is also sustainable. Given that $\alpha = 0$ is optimal (no theft in the punishment equilibrium), the government will choose the lowest money growth rate consistent with sustainability and no theft. Therefore, the solution to the above problem is

$$\text{If } \beta u'(\tau) \leq 1, \text{ then } x = \hat{m},\ \mu = 1, \text{ and } \hat{v} = \frac{\beta u(\hat{m}) - \hat{m}}{1-\beta}$$

$$\text{If } \beta u'(\tau) > 1, \text{ then } x = \tau,\ \mu = \beta u'(\tau), \text{ and } \hat{v} = \frac{\beta \left\{-\tau\left[(1-\beta)u'(\tau)+1\right] + u(\tau)\right\}}{1-\beta}$$

Here, recall that $u'(\hat{m}) = \frac{1}{\beta}$.

Now, suppose that $\beta u'(\tau) \leq 1$, that is $\tau \geq \hat{m}$. Then, given the same arguments as we used in the absence of the theft technology, the only incentive compatible equilibrium allocation is $x = y = \hat{m}$ and $\mu = 1$. Since $\tau \geq \hat{m}$, this is an equilibrium where there is only monetary exchange and no theft. It is identical to what we obtained when there was no theft technology.

The interesting case is the one where $\beta u'(\tau) > 1$, or $\tau < \hat{m}$. Here, in a manner similar to what we did in the last subsection, we are looking for an efficient equilibrium that is the solution to the government's problem:

$$\max_{x,y,\mu} \left\{ \rho \left[u(x) - x \right] + (1-\rho) \left[u(y) - y \right] \right\} \tag{1.40}$$

subject to

$$\rho u'(x) + (1-\rho) u'(y) = \frac{\mu}{\beta}, \tag{1.41}$$

$$x \leq \tau \tag{1.42}$$

$$0 \leq x \leq y \leq q^* \tag{1.43}$$

$$\rho\beta \left[u(x) - x \right] + \beta(1-\rho)u(y) - (1-\rho\beta)y \geq \beta \left\{ -\tau \left[(1-\beta)u'(\tau) + 1 \right] + u(\tau) \right\}. \tag{1.44}$$

Lemma 15 *If $\beta u'(\tau) > 1$, then with non-autarkic punishment, $x = \tau$ in an efficient equilibrium.*

Proof. Suppose not. Then, an increase in x will relax constraint (1.44), since $\tau < \hat{m}$. Therefore if there exists an equilibrium with $x < \tau$ and $y = q^*$, there exists another equilibrium with larger x and smaller μ such that the constraints in the above problem are all satisfied and the value of the objective function increases. Similarly, if there exists an equilibrium with $x < \tau$ and $y < q^*$, so that (1.44) holds with equality, then we can construct another equilibrium satisfying all of the constraints in the problem and increase the value of the objective function, simply because increasing x relaxes the incentive constraint and increases the value of the objective function, and we can find a value for μ that satisfies (1.41) and therefore supports this allocation as an equilibrium. **Q.E.D.** ■

Given the above lemma, we can write the incentive constraint (1.44) as

$$\beta(1-\rho)u(y) - (1-\rho\beta)y \geq \beta(1-\rho)[u(\tau) - \tau] - \beta(1-\beta)\tau u'(\tau) \tag{1.45}$$

Now, let $y(\tau)$ denote the value of y satisfying (1.45), given τ, where $y(\tau) = q^*$ if (1.45) does not bind. The function $y(\tau)$ is defined for $\tau \in [0, \hat{m}]$. We know that $y(0) = q^{***}$

and $y(\hat{m}) = \hat{m}$. Therefore, for example, if $q^{***} > \hat{m}$, then by continuity there are some values of τ for which a reduction in τ causes an increase in y. That is, a decrease in the cost of theft can increase the quantity of consumption in the monitored state, which makes this case much different from the one where the punishment equilibrium is autarky. It is straightforward to show that, if $\beta u'(\tau) > 1$, and $y = \tau$, then (1.45) is satisfied as a strict inequality, so that $y(\tau) > \tau$ for $\tau \in [0, \hat{m})$. Thus, as long as theft matters, an efficient equilibrium supports some credit, just as in the autarkic punishment case. Finally, the optimal money growth rate will be given by

$$\mu = \beta \left\{ \rho u'(\tau) + (1-\rho) u' [y(\tau)] \right\} < \beta u'(\tau).$$

With non-autarkic punishment, theft acts as a disciplining device. The opportunities are greater for thieves in the punishment equilibrium, since all exchange is carried out using money. Thus, the government needs to inflate at a higher rate in order to drive out thieves, which makes the punishment more severe. The efficient money growth rate is always smaller than it is in the punishment equilibrium. Therefore, buyers who default not only give up access to credit markets, but they will have to face a higher inflation tax.

1.8 Conclusion

In determining the roles for money and monetary policy, it is important to analyze models with credit. Credit and outside money are typically substitutes in making transactions, and an important aspect of the effects of monetary policy may have to do with how central bank intervention works through credit market relationships. In the model studied in this paper, limited memory provides a role for money, as in much of the recent monetary theory literature, and does this by reducing the role for credit. This role for credit is further mitigated by limited commitment.

In this context, monetary policy works too well, in the sense that efficient monetary policy drives out credit. In reality, money and credit appear to be robust, in that it is hard to imagine an economy where there are not some transactions carried out with both money and credit. To obtain this robustness in our environment, it is necessary that there be some cost to operating the monetary system. The cost we choose to model is theft, as we think that theft, or the threat of theft, is likely an empirically significant cost associated with monetary exchange. If the cost of theft is small enough to matter, then money and credit always coexist under an optimal monetary policy, and a reduction in the cost of theft acts to increase lending in the economy, though this depends to some extent on how bad behavior in the credit market is punished. In general, the Friedman rule is not optimal given theft, and the optimal money growth rate tends to increase as the cost of theft falls.

For convenience, we have modeled monetary intervention by the central bank as occurring through lump-sum transfers. Though we have not shown this in the paper, we think that the results are robust to how money injections occur. For example, it should not matter if money is injected through central bank lending or open market purchases. In the latter case, of course, we would have to take a stand on why government bonds are not used in transactions.

This model should be useful for evaluating the performance of monetary policy in the context of aggregate shocks. As well we could easily consider other types of costs of operating a monetary system, including counterfeiting, the costs of deterring counterfeiting, or the costs of replacing worn currency.

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2 Unsecured Loans and The Initial Cost of Lending

2.1 Introduction

The cost of starting a credit relationship has fallen significantly over the last few decades. For instance, Mester (1997) points out that the use of credit scoring has reduced significantly the time and cost in the loan approval process. Barron and Staten (2003) and Berger (2003) provide evidence suggesting that advances in information technology have significantly reduced the cost of processing information for lenders. An important question that needs to be addressed is the following. What is the impact of changes in the cost of starting a credit relationship on the supply of credit? Drozd and Nosal (2008) argue that such a drop in the initial cost of lending can account for several facts in the market for unsecured loans such as the significant increase in revolving lines of credit over the last two decades. To derive these results, they introduce a search friction into an incomplete markets model in which the terms of the contract offered by a lender are fixed. In a recent paper, Livshits, MacGee, and Tertilt (2009) also use an incomplete markets model to analyze the effect of technological progress on consumer credit.

Although both models are successful in reproducing some stylized facts of the market for unsecured loans, it is crucial to adopt a more fundamental approach by not restricting the space of contracts that can be offered by a lender in a competitive credit market. In this way, we can clearly analyze how changes in the initial cost of lending affect the endogenous credit contract offered by lenders. This is an essential aspect of the analysis because the dynamics of long-term credit arrangements is an important property of any model of credit. We emphasize precisely how changes in the initial cost of lending affect the dynamics of a credit relationship.

In this paper, we study the impact of changes in the cost of starting a credit relationship on the terms of the contract in a decentralized credit market where sellers are willing to repeatedly finance the purchases of buyers by extending direct credit. Our approach is consistent with the endogenously incomplete markets literature – see Sleet (2008) – where trading arrangements are derived from primitive frictions instead of assumed. The frictions we choose to model are the following. First, the environment is such that lenders are asymmetrically informed about a borrower's ability to repay a loan. Second, lenders can commit to some credit contracts while borrowers cannot commit to any contract. Third, transactions within each credit relationship are not publicly observable, which captures the idea that information is dispersed in the market for unsecured loans. Fourth, it is costly for a lender to contact a borrower in the credit market as in Drozd and Nosal (2008) and Livshits, MacGee, and Tertilt (2009). Given these frictions, we derive the terms of the contract that lenders offer to borrowers in a competitive credit market.

We build on the model of perfect competition by Phelan (1995). In his model, there is a particular mechanism for price formation in the credit market: lenders post the terms of the contract. One important difference is that we assume that lenders need to pay a one-shot cost to make a contact with a borrower in the credit market. This captures the idea that it is costly to start a credit relationship. Another crucial difference in our model is that we make the flow of payments associated with a credit contract explicit within each period – as opposed to net transfers. One important characteristic of a credit transaction is that settlement takes place at a future date: each transaction between a buyer and a seller necessarily creates a liability to the buyer that needs to be settled some time in the future. In an environment where buyers (borrowers) cannot commit to repay their loans, this results in *ex post* individual rationality constraints. Making the flow of payments explicit allows us to clearly characterize how the loan amounts within an enduring credit relationship evolve over time.

As in Phelan (1995), we assume that lenders can commit to deliver some credit contracts while borrowers cannot commit to any contract. The main difference from Phelan's work is that we assume that, although lenders can commit to deliver a long-term credit contract, they cannot commit to contracts that at any date result in a payoff that is lower than that associated with autarky. This assumption, together with the assumption that it is costly for lenders to contact borrowers, changes significantly the equilibrium outcome. If the initial cost of lending is positive, a borrower's expected discounted utility fluctuates over time as a result of variable terms of credit within an enduring relationship. As the cost approaches zero, the equilibrium credit contract converges to a full-insurance contract.

A lower cost of starting a relationship has the following impact on the equilibrium outcome: (i) each borrower is better off from the perspective of the contracting date; (ii) a borrower's expected discounted utility fluctuates within a smaller set; and (iii) each lender is uniformly worse off *ex post*. A lower cost of entry in the credit market leads to more competition among lenders, which in turn results in better terms of credit for each borrower. Another implication is that the terms of the contract are such that a borrower's expected discounted utility has less variability over time. Although the terms of credit change over time according to the history of trades, the loan and repayment amounts specified in a lender's contract are such that the space of expected discounted utilities for a borrower shrinks when the initial cost of lending falls. Finally, a lender's cost function under an entry cost of $k' < k$ is uniformly above a lender's cost function under the cost k, which necessarily means that lenders are uniformly worse off *ex post*.

The model in this paper relates to decentralized models of credit, such as Diamond (1990), Temzelides and Williamson (2001), Nosal and Rocheteau (2006), Koeppl, Monnet, and Temzelides (2008), and Andolfatto (2008), as opposed to centralized models of credit, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000). The model also builds on search-theoretic models of money, such as Shi (1997) and Lagos and Wright (2005). However, we depart from these models by weakening the assumption that agents cannot engage in enduring relationships. Finally, the analysis builds on dynamic contracting. Important papers in this literature include Green (1987), Thomas and Worrall (1990), Atkeson and Lucas (1992, 1995), Kocherlakota (1996), Aiyagari and Williamson (1999), and Krueger and Uhlig (2006).

2.2 The Model

Time is discrete and continues forever, and each period has two subperiods. There are two types of agents, referred to as borrowers and lenders. In the first subperiod, a lender is able to produce the unique perishable consumption good but does not want to consume, and a borrower wants to consume but cannot produce. In the second subperiod, we have the opposite situation: a borrower is able to produce but does not want to consume, and a lender wants to consume but cannot produce. Production and consumption takes place within each subperiod. This generates a double coincidence of wants and, for this reason, we refer to the first subperiod as the transaction stage and to the second subperiod as the settlement stage. The types (borrower and lender) refer to the agent's role in the transaction stage. The production technology allows each agent to produce one unit of the good with one unit of labor. Each agent receives an endowment of $h > 0$ units of time in each subperiod.

A lender's utility in period t is given by $-q_t^l + x_t^l$, where q_t^l is production of the good in the transaction stage and x_t^l is consumption of the good in the settlement stage. A borrower's momentary utility from consuming q_t^b units of the consumption good in the transaction stage is given by $u\left(q_t^b\right)$. Assume that $u : \mathbb{R}_+ \to D \subset \mathbb{R}$ is increasing, strictly concave, and continuously differentiable. Let H denote the inverse of u, and let $w^a \equiv u(0)$ denote the value associated with autarky. Producing y_t^b units of the good in the settlement stage generates utility $-y_t^b$ for a borrower. However, there is a friction that affects a borrower's ability to produce goods in the settlement stage. With probability π a borrower is unable to produce the consumption good and with probability $1-\pi$ a borrower can produce the good using the linear production technology. This productivity shock is independently and identically distributed over time. Each borrower learns his productivity shock at the beginning of the settlement stage, which is privately observed. Finally, let $\beta \in (0,1)$ be the common discount factor over periods.

Suppose that there is a large number of borrowers and lenders, with the set of lenders sufficiently large. There is a one-shot cost $k > 0$ in terms of the consumption good for a lender to post a credit contract in the credit market. A contract specifies consumption and production by each party as a function of the available information. Each lender can have at most one borrower – it is infinitely costly for a lender to contact two borrowers at the same time. Only the agents in a bilateral meeting observe the history of trades. Other agents in the economy observe a break in a particular match but do not observe the history of trades in that match. Notice that there are gains from trade since a lender can produce the consumption good for a borrower in the first subperiod (transaction stage) and a borrower can produce the good for a lender in the second subperiod (settlement stage). An important feature of the model is that, with probability π, a borrower is unable to produce the good in the second subperiod and settle his debt. This is equivalent to assuming that the settlement process involves a friction.

2.3 Equilibrium

In this section, we study an equilibrium allocation under a particular pricing mechanism: price posting by lenders. To enter the credit market, a lender needs to post a contract to attract a borrower and start a credit relationship. Although it is costly for a lender to make a contact with a borrower, there is free entry of lenders in the credit market. We characterize the terms of the contract in each credit relationship in the economy and restrict attention to a symmetric, stationary equilibrium in which each borrower receives a market-determined credit contract offered by a lender that promises him expected discounted utility w^*, from the perspective of the contracting date. Each lender needs to provide incentives to induce the desired behavior by a borrower given that a borrower's ability to repay a loan is not publicly observable.

We assume that lenders can commit to some credit contracts while borrowers cannot commit to any contract. Specifically, each lender can commit to deliver any contract that does not result at any moment in an expected discounted utility that is lower than that associated with autarky – recall that a lender has always the option of remaining inactive. On the other hand, borrowers cannot commit to any contract and can walk away from a credit relationship at any moment without any pecuniary punishment. As we will see, a lender's optimal contract results in a long-term relationship from which neither party wants to deviate.

The expected discounted utility w^* associated with the market contract must be such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive, from the perspective of the contracting date. As a result, some lenders post a contract and successfully match with a borrower while others do not post a contract and remain inactive. When offering her own contract, each lender takes as given the contracts offered by the other lenders. The only relevant characteristic about these contracts is the expected discounted utility w^* that each borrower associates with them. This is the utility that a borrower obtains by accepting a lender's contract, from the perspective of the signing date. The equilibrium is symmetric because every active lender offers the same credit contract.

The market contract must *always* result in an expected discounted utility for a borrower that is greater than or equal to w^*. If the market contract promises, in a given period, an expected discounted utility w' for a borrower which is less than w^*, the latter can do better by reneging on his current contract and starting a new credit relationship with another lender. Recall that inactive lenders observe the dissolution of a credit relationship and may be willing to enter the credit market. Given that there is free entry of lenders and limited commitment, we can have an equilibrium only if the lowest promised expected discounted utility at any moment is exactly w^*.

2.3.1 Recursive Formulation of the Contracting Problem

A contract specifies in every period a transfer of the good from the lender to the borrower in the transaction stage and a repayment – a transfer of the good from the borrower to the lender – in the settlement stage as a function of the available history of reports by the borrower. These are reports about a borrower's ability to produce goods in the settlement stage. Let $\gamma^{t-1} = \left(\gamma_0, \gamma_1, ..., \gamma_{t-1}\right) \in \{0, 1\}^t$ denote a partial history of reports, where $\gamma_\tau = 0$ means that a borrower is unable to produce the good in the settlement stage of period τ and $\gamma_\tau = 1$ means that he is able to produce it in the settlement stage of period τ.

In equilibrium, each active lender chooses to offer a long-term contract, which means that she matches with a borrower at the first date and keeps him in the credit relationship forever. The long-term contract specifies quantities produced and transferred within each subperiod. We say that in each period t there is a *transaction* between a borrower and a lender which consists of a loan amount from the lender to the borrower in the first subperiod (transaction stage) and a repayment amount in the second subperiod (settlement stage) contingent on the report of the productive state of nature ($\gamma_t = 1$).

The optimal contracting problem has a recursive formulation in which we can use a borrower's expected discounted utility $w \in D$ as the state variable. The optimal contract minimizes the expected discounted cost for a lender of providing expected discounted utility w to a borrower subject to incentive compatibility. Let $C_{(w^*,\bar{w})} : [w^*, \bar{w}] \to \mathbb{R}$ denote the expected discounted cost for a lender that satisfies the following functional equation:

$$C_{(w^*,\bar{w})}(w) = \min_{\varphi \in \Upsilon_{(w^*,\bar{w})}(w)} \left\{ \begin{array}{c} (1-\beta)\left[H(u) - (1-\pi) y_1\right] + \\ \beta\left[\pi C_{(w^*,\bar{w})}(w_0) + (1-\pi) C_{(w^*,\bar{w})}(w_1)\right] \end{array} \right\}. \tag{2.1}$$

Here, the choices are given by $\varphi = (u, y_1, w_0, w_1)$, where u denotes a borrower's momentary utility of consumption in the transaction stage, y_1 denotes his production in the settlement stage given that he is able to produce the good, and w_γ denotes his promised expected discounted utility at the beginning of the following period given that his report in the current period is $\gamma \in \{0, 1\}$. Recall that $\gamma = 0$ means that a borrower is unable to produce the good in the settlement stage and $\gamma = 1$ means that he is able to produce it. The constraint set $\Upsilon_{(w^*,\bar{w})}(w)$ consists of all φ in $D \times [0, h] \times [w^*, \bar{w}]^2$ satisfying a borrower's individual rationality constraints,

$$w_0 \geq w^*, \tag{2.2}$$

$$-(1-\beta) y_1 + \beta w_1 \geq \beta w^*, \tag{2.3}$$

a borrower's truth-telling constraint,

$$-(1-\beta) y_1 + \beta w_1 \geq \beta w_0, \tag{2.4}$$

and the promise-keeping constraint,

$$(1-\beta)\left[u - (1-\pi) y_1\right] + \beta\left[\pi w_0 + (1-\pi) w_1\right] = w. \tag{2.5}$$

It can be shown that, for any fixed lower bound w^* and upper bound $\bar{w}$, there exists a unique continuously differentiable, strictly increasing, and strictly convex function $C_{(w^*,\bar{w})} : [w^*, \bar{w}] \to \mathbb{R}$ satisfying the functional equation (3.1). Let $\hat{u} : [w^*, \bar{w}] \to D$,

$y : [w^*, \bar{w}] \to [0, h]$, and $g : [w^*, \bar{w}] \times \{0, 1\} \to [w^*, \bar{w}]$ denote the associated policy functions, which can be shown to be continuous and bounded.

Given our transformation of the state space, a borrower's expected discounted utility w now summarizes the partial history of reports. As mentioned before, the terms of credit for the current transaction are given by $\{H[\hat{u}(w)], y(w)\}$. The quantity $H[\hat{u}(w)]$ gives the loan amount from the lender to the borrower in the transaction stage, and the quantity $y(w)$ gives the repayment amount in the settlement stage contingent on the report of the productive state of nature. Both quantities depend on w.

Notice that a lender cannot commit to a contract that gives her at any moment an expected discounted utility that is lower than that associated with autarky. As a result, individual rationality for a lender requires that $C_{(w^*, \bar{w})}(w) \leq 0$ holds for all $w \in [w^*, \bar{w}]$. I show next that, for any given w^*, there exists an upper bound $\bar{w} = \bar{w}(w^*)$ on the set of expected discounted utilities that gives the highest promised expected utility to which a lender can commit to deliver given that the lowest expected utility that can be promised is w^*. As we will see later, the market utility w^* is determined endogenously and is such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive.

Lemma 16 *For any $w^* \geq w^a$ such that $C_{(w^*, w^*)}(w^*) \leq 0$, there exists an upper bound $\bar{w}(w^*)$ on the set of expected discounted utilities such that $C_{(w^*, \bar{w}(w^*))}[\bar{w}(w^*)] = 0$.*

Proof. Let $\bar{w}_F$ denote the expected discounted utility such that the expected discounted cost of providing $\bar{w}_F$ given full information equals zero. Define the function $\tau : [w^*, \bar{w}_F] \to [w^*, \bar{w}_F]$ as follows. For any given $w \in [w^*, \bar{w}_F]$, if there is no $w' \in [w^*, w]$ such that $C_{(w^*, w)}(w') = 0$, then $\tau(w) = w^*$. Otherwise, $\tau(w)$ equals the highest point w' in $[w^*, w]$ for which $C_{(w^*, w)}(w') = 0$. Notice that $C_{(w^*, w^*)}(w^*) \leq 0$ by assumption, which implies that $\tau(w^*) = w^*$. For any other $\bar{w}$ such that $\tau(\bar{w}) = \bar{w}$, it must be that $C_{(w^*,\bar{w})}(\bar{w}) = 0$.

Now, construct a sequence $\{w_t\}_{t=0}^{\infty}$ of candidates for the upper bound $\bar{w}$ in the following way. Let $w_0 = \bar{w}_F$. We have that $C_{(w^*,w_0)}(w_0) \geq 0$, with strict inequality if the truth-telling constraint (3.3) binds. Also, notice that $\Upsilon_{(w^*,w^*)}(w^*) \subseteq \Upsilon_{(w^*,w_0)}(w^*)$, which implies that $C_{(w^*,w_0)}(w^*) \leq C_{(w^*,w^*)}(w^*) \leq 0$. The first inequality is strict if the truth-telling constraint binds. Continuity implies that there exists $w_1 \in [w^*, w_0]$ such that $C_{(w^*,w_0)}(w_1) = 0$. This means that $w_1 = \tau(w_0) \leq w_0$. We proceed in the same fashion to define w_2. From the fact that $C_{(w^*,w_0)} \leq C_{(w^*,w_1)}$, it follows that $C_{(w^*,w_1)}(w_1) \geq C_{(w^*,w_0)}(w_1) = 0$. Given that $\Upsilon_{(w^*,w^*)}(w^*) \subseteq \Upsilon_{(w^*,w_1)}(w^*)$, we have that $C_{(w^*,w_1)}(w^*) \leq C_{(w^*,w^*)}(w^*) \leq 0$. Again, continuity implies that there exists $w_2 \in [w^*, w_1]$ such that $C_{(w^*,w_1)}(w_2) = 0$. This means that $w_2 = \tau(w_1) \leq w_1$. Notice then that $\{w_t\}_{t=0}^{\infty}$ is a non-increasing sequence on a closed interval. As a result, it converges to a point w_∞ in the interval $[w^*, \bar{w}_F]$. The Theorem of the Maximum guarantees that $\phi(w) \equiv C_{(w^*,w)}(w)$ moves continuously, which implies that w_∞ is the highest fixed point of τ. **Q.E.D.** ■

To ease notation, define $C_{w^*}(w) \equiv C_{(w^*,\bar{w}(w^*))}(w)$ and $D_{w^*} \equiv [w^*, \bar{w}(w^*)]$. Given that $C_{w^*}(w)$ is strictly increasing in w, it follows that $C_{w^*}(w) \leq 0$ for all w in the set D_{w^*}. This means that, for any given lower bound w^*, D_{w^*} gives the set of promised expected discounted utilities that are actually incentive-feasible. If the truth-telling constraint binds, then it follows that $\bar{w}(w^*) > w^*$ for any lower bound w^* satisfying $C_{(w^*,w^*)}(w^*) \leq 0$. I show next that the truth-telling constraint indeed binds for any w in D_{w^*}. But first notice that the truth-telling constraint (3.3), together with the constraint $0 \leq y(w) \leq h$, implies that $g(w,1) \geq g(w,0)$ for all $w \in D_{w^*}$, which means that the optimal contract needs to assign a higher promised expected discounted utility to a borrower contingent on the realization of the productive state of nature to effectively induce truthful reporting.

Lemma 17 *The truth-telling constraint (3.3) binds for any* $w \in D_{w^*}$.

Proof. Suppose that

$$-(1-\beta)\,y_1 + \beta w_1 > \beta w_0 \tag{2.6}$$

holds at the optimum. This implies that

$$-(1-\beta)\,y_1 + \beta w_1 > \beta w^* \tag{2.7}$$

must also hold at the optimum. Now, reduce the left-hand side of (3.4) and (3.5) by a small amount $\Delta > 0$ so that both inequalities continue to hold. Define $w_1' = w_1 - \pi\Delta$ and $w_0' = w_0 + (1-\pi)\,\Delta$. Notice that $\pi w_0' + (1-\pi)\,w_1' = \pi w_0 + (1-\pi)\,w_1$ and $w_1' - w_0' < w_1 - w_0$. The strict convexity of C_{w^*} implies that

$$\pi C_{w^*}(w_0') + (1-\pi)\,C_{w^*}(w_1') < \pi C_{w^*}(w_0) + (1-\pi)\,C_{w^*}(w_1)\,,$$

so that the value of the objective function on the right-hand side of (3.1) falls. Since all constraints continue to be satisfied, this implies a contradiction. **Q.E.D.** ■

An immediate consequence of the previous result is that $w^* < \bar{w}(w^*)$ for any given w^* such that $C_{(w^*,w^*)}(w^*) \leq 0$.

2.3.2 Existence and Uniqueness of Stationary Equilibrium

Now, we need to ensure that there exists a market-determined expected discounted utility w^* associated with a market contract that makes each lender indifferent between posting a contract and remaining inactive. This is equivalent to showing the existence of an equilibrium.

Formally, a stationary and symmetric equilibrium consists of a cost function $C_w : D_w \to \mathbb{R}$, policy functions $\hat{u} : D_w \to D$, $y : D_w \to [0, h]$, $g : D_w \times \{0, 1\} \to D_w$, and a market utility w^* such that: *(i)* C_{w^*} satisfies (3.1); *(ii)* $(\hat{u}, y, g)$ are the optimal policy functions for (3.1); and *(iii)* w^* satisfies the free-entry condition:

$$C_{w^*}(w^*) + (1-\beta)k = 0. \tag{2.8}$$

The market utility w^* gives the expected discounted utility for a borrower at the signing date. Due to limited commitment and free entry of lenders in the credit market, it is also the lower bound on the set of expected discounted utilities.

Lemma 18 *There exists a unique expected discounted utility w^* satisfying (3.6) provided that $k > 0$ is sufficiently small.*

Proof. First, notice that $C_{w^a}(w^a) < 0 < C_{\bar{w}_F}(\bar{w}_F)$. Suppose that $k > 0$ is sufficiently small such that $C_{w^a}(w^a)+(1-\beta)k < 0$. Given that $\hat{\phi}(w) \equiv C_w(w)$ is continuous in w, there exists $w^* \in [w^a, \bar{w}_F]$ such that $\hat{\phi}(w^*)+(1-\beta)k = 0$. To show uniqueness, define the mapping $\sigma : [w^a, \bar{w}_F] \to [w^a, \bar{w}_F]$ as follows. If $C_w+(1-\beta)k$ is always greater than zero on $[w, \bar{w}_F]$, then $\sigma(w) = w^a$. Otherwise, $\sigma(w)$ equals the point $w' \in [w, \bar{w}_F]$ for which $C_w(w')+(1-\beta)k = 0$. We claim that σ is a non-increasing function. To verify this claim, we need to show first that $\bar{w}(w)$ is non-increasing in w. Fix a lower bound w' in the set $[w^a, \bar{w}_F]$, and consider the associated upper bound $\bar{w}(w')$. Take another point $w'' > w'$ in the set $[w^a, \bar{w}(w')]$. Notice that $C_{(w', \bar{w}(w'))} \leq C_{(w'', \bar{w}(w'))}$. Thus, we have that $C_{(w'', \bar{w}(w'))}[\bar{w}(w')] \geq 0$ given that $C_{(w', \bar{w}(w'))}[\bar{w}(w')] = 0$ by the definition of $\bar{w}(w')$. This implies that $\bar{w}(w'') \leq \bar{w}(w')$, and we conclude that $\bar{w}(w)$ is indeed non-increasing in w. The fact that $\bar{w}(w)$ is non-increasing then implies that raising the lower bound w only tightens the constraint set $\Upsilon_{(w, \bar{w}(w))}(\cdot)$. As a result, the point at which $C_w + (1-\beta)k$ equals zero is a non-increasing function of the lower bound w, which means that σ can have at most one fixed point. **Q.E.D.** ■

Notice that *ex ante* each lender gets zero expected discounted utility by posting a contract. *Ex post* a lender gets a higher utility, given that $C_{w^*}(w^*) < 0$. Moreover, as the contract is executed, there is no history of reports by a borrower that gives a lender an expected discounted utility that is lower than that associated with autarky. For this reason, neither a lender nor a borrower finds it optimal to renege on the credit contract.

2.3.3 Properties of the Optimal Contract

We can rewrite the optimization problem on the right-hand side of (3.1) in the following way. The relevant constraints for the optimization problem are (3.2),

$$-(1-\beta)\, y_1 + \beta w_1 = \beta w_0, \tag{2.9}$$

and

$$(1-\beta)\,(u - y_1) + \beta w_1 = w. \tag{2.10}$$

Substituting (3.7) and (3.8) into (3.1), the optimization problem now consists of choosing y_1 and w_1 to minimize:

$$(1-\beta)\left[H\left(\frac{w-\beta w_1}{1-\beta} + y_1\right) - (1-\pi)\, y_1\right] + \beta \left\{ \begin{array}{c} \pi C_{w^*}\left[w_1 - \frac{(1-\beta)}{\beta} y_1\right] + \\ (1-\pi)\, C_{w^*}(w_1) \end{array} \right\},$$

subject to $w^* \leq w_1 \leq \bar{w}(w^*)$, $0 \leq y_1 \leq h$, and

$$w_1 - \frac{(1-\beta)}{\beta} y_1 \geq w^*. \tag{2.11}$$

The first-order conditions for the optimal choice of y_1 are

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta} + y(w)\right] - \pi C'_{w^*}\left[g(w,0)\right] + \frac{\lambda(w)}{\beta} \geq 1-\pi, \tag{2.12}$$

if $y(w) < h$, and

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta} + y(w)\right] - \pi C'_{w^*}\left[g(w,0)\right] + \frac{\lambda(w)}{\beta} \leq 1-\pi, \tag{2.13}$$

if $y(w) > 0$. The first-order condition for the optimal choice of w_1 is

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta} + y(w)\right] \geq \left\{ \begin{array}{c} \pi C'_{w^*}\left[g(w,0)\right] + \\ (1-\pi)\, C'_{w^*}\left[g(w,1)\right] - \frac{\lambda(w)}{\beta} \end{array} \right\}, \tag{2.14}$$

with equality if $g(w,1) < \bar{w}(w^*)$. Also, we have that

$$\lambda(w)\left[g(w,1) - \frac{(1-\beta)}{\beta}y(w) - w^*\right] = 0, \tag{2.15}$$

where $\lambda(w) \geq 0$ is the Lagrange multiplier on constraint (3.9). Finally, the envelope condition is given by

$$C'_{w^*}(w) = H'\left[\frac{w - \beta g(w,1)}{1-\beta} + y(w)\right], \tag{2.16}$$

for any value of w in the interior of the set D_{w^*}.

Now, we establish some properties of the optimal continuation value $g(w,\gamma)$ for each $\gamma \in \{0,1\}$. These give a borrower's expected discounted utility at the beginning of the following period associated with the market contract as a function of his initially promised expected discounted utility w and his report in the settlement stage of the current period. If a borrower's expected discounted utility falls in the subsequent period relative to the current period, this means that the terms of the contract become less favorable for him - and as a result more favorable for the lender.

Lemma 19 $g(w,1) \geq w$ *for all* $w \in D_{w^*}$.

Proof. Suppose that $g(w,1) < w$ for some w in the interior of D_{w^*}. Given that $g(w,1) < w \leq \bar{w}(w^*)$, it must be that

$$C'_{w^*}(w) = \pi C'_{w^*}[g(w,0)] + (1-\pi)C'_{w^*}[g(w,1)] - \frac{\lambda(w)}{\beta}.$$

Recall that $g(w,1) \geq g(w,0)$ and that $C_{w^*}(w)$ is strictly convex in w. As a result, we have that

$$C'_{w^*}(w) < C'_{w^*}(w) - \frac{\lambda(w)}{\beta} \leq C'_{w^*}(w),$$

where the last inequality follows because $\lambda(w) \geq 0$. But this results in a contradiction. Hence, we conclude that $g(w,1) \geq w$ for all w in the interior of D_{w^*}. The fact that

$g(w,1)$ is continuous implies that $g(w,1) \geq w$ holds for all $w \in D_{w^*}$ as claimed. **Q.E.D.** ■

A repayment by a borrower in the settlement stage results in at least the same terms of credit for future transactions within the credit relationship. If a borrower reports the productive state of nature in the settlement stage and as a result makes a repayment $y(w)$ to his lender, his expected discounted utility at the beginning of the following period $g(w,1)$ either rises or remains the same. This means that the terms of credit for all future transactions within the relationship either become more favorable or remain the same for him. This property of the optimal contract arises because a lender cannot observe a borrower's ability to repay a loan in the settlement stage. As a result, a lender needs to induce a repayment from a borrower who is currently productive in the settlement stage by promising him at least the same terms of credit for future transactions as those promised in the current period.

Lemma 20 *The function $g(w,0)$ has the following properties: (i) $g(w,0) < w$ for all $w > w^*$; (ii) $g(w^*,0) = w^*$; and (iii) there exists $\delta > 0$ such that $g(w,0) = w^*$ for all $w \in [w^*, w^* + \delta)$.*

Proof. First, notice that we must have $y(w) > 0$ for all $w \in D_{w^*}$. To verify this claim, suppose that $y(w) = 0$ for some $w \in (w^*, \bar{w}(w^*))$. Then, we must have $g(w,1) = g(w,0)$ given that (3.3) holds with equality. Moreover, either $g(w,1) = g(w,0) = \bar{w}(w^*)$ or $g(w,1) = g(w,0) < \bar{w}(w^*)$. If $g(w,1) = g(w,0) = \bar{w}(w^*)$, then (2.14) and (2.16) imply that $C'_{w^*}(w) \geq C'_{w^*}[\bar{w}(w^*)]$, which results in a contradiction. Suppose now that $g(w,1) = g(w,0) < \bar{w}(w^*)$. From (2.14) and (2.16), we conclude that $g(w,1) = g(w,0) = w$. Thus, we have that $C_{w^*}(w) = H(w) > H(w^a) = 0$, which implies a contradiction. Therefore, we must have $y(w) > 0$ for all $w \in (w^*, \bar{w}(w^*))$. Continuity then implies that $y(w^*) > 0$ and $y[\bar{w}(w^*)] > 0$, so that $y(w) > 0$ for all

$w \in D_{w^*}$ as claimed. As a result, $g(w, 1) > g(w, 0)$ for all $w \in D_{w^*}$.

Suppose that $g(w, 0) \geq w$ for some $w > w^*$. From (2.14) and (2.16), we have that

$$C'_{w^*}(w) \geq \pi C'_{w^*}[g(w, 0)] + (1 - \pi) C'_{w^*}[g(w, 1)] > C'_{w^*}(w),$$

where the last inequality follows from the fact that $C_{w^*}(w)$ is strictly convex in w and the fact that $g(w, 1) > g(w, 0)$. But we obtain a contradiction. Hence, we must have $g(w, 0) < w$ for all $w > w^*$. Since $g(w, 0)$ is continuous in w, it follows that $g(w^*, 0) = w^*$.

Finally, to prove (iii), suppose that $g(w^* + \varepsilon, 0) > w^*$ for all $\varepsilon > 0$. Then, (2.14) and (2.16) require that

$$C'_{w^*}(w^* + \varepsilon) \geq \pi C'_{w^*}[g(w^* + \varepsilon, 0)] + (1 - \pi) C'_{w^*}[g(w^* + \varepsilon, 1)]$$

holds for all $\varepsilon > 0$, which in turn requires that $\lim_{\varepsilon \to 0} g(w^* + \varepsilon, 1) = w^*$. But this implies a contradiction. **Q.E.D.** ■

If a borrower fails to make a repayment to his lender in the settlement stage, then the terms of the contract become less favorable for him in all future transactions within the credit relationship. As a result of intertemporal allocation of resources by a risk-neutral lender, a delayed repayment is compensated by more favorable terms of credit for future transactions.

Notice that the envelope condition (2.16) implies that the loan amount to which a borrower is entitled in the transaction stage is strictly increasing in his promised expected discounted utility w. As we have seen, the optimal provision of incentives by a lender results in a lower promised expected discounted utility for a borrower who reports the unproductive state and as a result fails to make a repayment. Thus, the loan amount that a borrower receives from a lender in the subsequent transaction stage shrinks, given that $H[\hat{u}(w)]$ is a strictly increasing function. This shows how the loan amount that a borrower receives from a lender in the current transaction depends on the history of trades within the credit relationship.

It is useful to define a statistic that summarizes the terms of credit within an enduring relationship. Notice that the expected return to a lender on the current transaction is given by

$$R(w) \equiv \frac{(1-\pi)\, y(w)}{H[\hat{u}(w)]}, \tag{2.17}$$

which summarizes the terms of credit for the current transaction. Notice that the expected return to a lender depends on w and fluctuates over time as a result.

Lemma 21 *The statistic $R(w)$ defined by (2.17) is strictly decreasing in w.*

Proof. It remains to show that $y(w)$ is non-increasing on D_{w^*}. To verify this claim, suppose that there is an interval $\tilde{D} \subset D_{w^*}$ on which $y(w)$ is strictly increasing. Then, there is an interval $\hat{D} \subseteq \tilde{D}$ on which $0 < y(w) < h$. Notice that (3.10)-(2.14) imply that $g(w, 1)$ is constant on $\hat{D}$. Then, (3.7) implies that $g(w, 0)$ must be strictly decreasing on $\hat{D}$. This necessarily means that $\lambda(w) = 0$ for all $w \in \hat{D}$. As a result, we must have

$$C'_{w^*}(w) = \pi C'_{w^*}[g(w, 0)] + 1 - \pi$$

for all $w \in \hat{D}$. But this implies a contradiction. Therefore, it must be that $y(w)$ is non-increasing on D_{w^*} as claimed. **Q.E.D.** ■

The statistic $R(w)$, which is depicted in Figure 4, captures the evolution of the terms of credit according to the history of trades (summarized by w). This means that $R(w^*)$ gives the worst terms of credit for a borrower while $R[\bar{w}(w^*)]$ gives the best terms of credit. A lower value for w in D_{w^*} implies that $R(w)$ is relatively higher – closer to the upper bound $R(w^*)$. This means that the terms of credit for the current transaction are less favorable for the borrower – and more favorable for the lender

– because he has had a weak history of repayments within the relationship. Worse terms of credit for a borrower mean that he is entitled to a lower loan amount in the transaction stage and/or is required to make a bigger repayment in the settlement stage contingent on the realization of the productive state.

2.4 Changes in the Initial Cost of Lending

An important parameter in the model is the cost $k > 0$ that a lender has to pay in order to post a credit contract. We have seen that there exists a sufficiently small $\bar{k} > 0$ such that, for any $k < \bar{k}$, there exists a unique market utility $w^*(k)$ such that $\hat{\phi}[w^*(k)] + (1-\beta)k = 0$, where $\hat{\phi}(w) \equiv C_w(w)$. Again, $w^*(k)$ gives a borrower's expected discounted utility from the perspective of the signing date. Given that $\hat{\phi}(w)$ is a continuous function, for any k' in a neighborhood of k, there exists a unique $w^*(k')$ such that $\hat{\phi}[w^*(k')]+(1-\beta)k' = 0$. Moreover, if $k' > k$, we have that $w^*(k') < w^*(k)$; if $k' < k$, we have that $w^*(k') > w^*(k)$. In the proof of Lemma 3, we have established that the upper bound $\bar{w}(w^*)$ on the set of expected discounted utilities is a non-increasing function of the lower bound w^*. Thus, we have that $D_{w^*(k)} \subset D_{w^*(k')}$ if $k' > k$ and that $D_{w^*(k')} \subset D_{w^*(k)}$ if $k' < k$. This means that a lower value for k results in a smaller set of expected discounted utilities.

We have some important implications. First, a lower value for k makes each borrower better off from the perspective of the signing date because the expected discounted utility associated with the market contract rises – a lower cost of entry results in more competition in the credit market. Second, there is less variability in a borrower's expected discounted utility over time. The terms of the contract are such that a borrower's expected discounted utility fluctuates within a smaller set according to the history of trades. Third, a lender's cost function under $k' < k$ is uniformly above her cost function under k – see Figure 5. This means that a lower value for k makes each lender uniformly worse off *ex post*.

We can interpret $k > 0$ as the initial cost of lending per customer for a lender in the market for unsecured loans. If technological progress drives the cost to nearly zero, we should expect small fluctuations over time in a borrower's expected discounted utility. Another prediction of the model is that borrowers obtain more favorable terms of credit as the initial cost of lending approaches zero: they are promised a higher expected discounted utility at the signing date.

Notice that as $k \to 0$ the credit contract offered by a lender in equilibrium converges to a full-insurance contract for each borrower. The equilibrium allocation converges to a contract that delivers a constant loan amount in the transaction stage and requires a constant repayment amount in the settlement stage contingent on the realization of the productive state of nature. Finally, the terms of credit are converging to a constant value $R[w^*(0)]$.

2.5 Discussion

A property of the equilibrium allocation is that borrowers are differentiated by lenders exclusively according to their history of transactions – loan and repayment amounts – within each credit relationship. This means that two borrowers are treated differently by the lenders with whom they are paired only because they have had distinct histories of repayments (due to different histories of productivity shock). Recall that at the first date each lender offers the same contract to a borrower. In the model proposed in this paper, borrowers are *ex ante* identical and face variable terms of credit over time within their credit relationships as a result of different histories of productivity shock. This is different from other theories of unsecured credit that assume that borrowers are *ex ante* heterogeneous with respect to some characteristic. For instance, in Livshits, MacGee, and Tertilt (2009), borrowers differ *ex ante* with respect to a characteristic that affects their ability to repay a loan; in Chatterjee, Corbae, and Ríos-Rull (2008), households differ *ex ante* with respect to the likelihood of a loss in their wealth.

In Drozd and Nosal (2008), borrowers are *ex ante* identical and differ *ex post* with respect to their wealth and income. In their analysis, the terms of the contract are fixed over time within each relationship between a borrower and a lender. This is a sufficient condition for obtaining default in equilibrium so that it is possible to interpret some results as bankruptcy. The contribution of our paper is to perform the comparative statics exercise of changing the initial cost of lending and to establish some properties of the equilibrium allocation in an environment where no restriction on the space of contracts is imposed. Although some properties that we obtain are similar – a lower initial cost of lending makes each borrower better off – others arise precisely due to the fact that the form of the contract is completely endogenous.

An important prediction of the model is that the equilibrium contract offered by a lender in the credit market converges to a full-insurance contract. In the limit, the terms of credit, as measured by the statistic $R(w)$, converge to a constant value. This means that the expected return to a lender on each transaction will be constant over time. Each borrower in the economy should get nearly the same terms of credit within his credit relationship with a lender as a result of any technological progress that drives the initial cost of lending to nearly zero. The history of transactions within each credit relationship becomes irrelevant as the initial cost of lending approaches zero. This is an important property of the dynamics of the model that we obtain from the assumptions that it is costly to make a contact in the credit market and that lenders can only commit to long-term contracts that do not result in a payoff that is lower than that associated with autarky.

2.6 Long-Run Properties

In this section, we study the long-run properties of the equilibrium allocation. Specifically, we show that there exists a well-behaved long-run distribution of expected discounted utilities with mobility. Let $\Psi(D_{w^*}, \mathbf{D})$ be the space of all probability measures ψ on the measurable space $(D_{w^*}, \mathbf{D})$, where $\mathbf{D}$ is the collection of Borel subsets of D_{w^*}. Define the operator T^* on $\Psi(D_{w^*}, \mathbf{D})$ by

$$(T^*\psi)(D') = \pi \int_{\mathbf{Q}_0(D')} d\psi + (1-\pi) \int_{\mathbf{Q}_1(D')} d\psi,$$

for each $D' \in \mathbf{D}$, where, for each $\gamma \in \{0, 1\}$, the set $\mathbf{Q}_\gamma(D')$ is given by

$$\mathbf{Q}_\gamma(D') = \{w \in D_{w^*} : g(w, \gamma) \in D'\}.$$

Notice that a fixed point of the operator T^* corresponds to an invariant distribution over D_{w^*}.

Lemma 22 *The operator T^* has a unique fixed point ψ^*, and for any probability measure ψ in $\Psi(D_{w^*}, \mathbf{D})$, $T^{*n}\psi$ converges to ψ^* in the total variation norm.*

Proof. Let ψ_w denote the probability measure that concentrates mass on the point w. I will show that there exist $N \geq 1$ and $\varepsilon > 0$ such that $(T^{*N}\psi_w)(w^*) \geq \varepsilon$ for all $w \in D_{w^*}$. From Lemma 5, there exists $k > 0$ such that either $g(w, 0) \leq w - k$ or $g(w, 0) = w^*$ for all $w \in D_{w^*}$. Now, choose an integer $N \geq 1$ large enough so that $\bar{w}(w^*) - kN \leq w^*$. Then, the probability of moving from the point $\bar{w}(w^*)$ to the point w^* in N steps is at least π^N. Since $g(w, 0)$ is non-decreasing in w, such a transition to w^* is at least as probable from any other point in D_{w^*}. Thus, if $\varepsilon = \pi^N$, then the implied Markov process satisfies the hypotheses of Theorem 11.12 of Stokey, Lucas, and Prescott (1989), and the proof is complete. **Q.E.D.** ■

The existence of a non-degenerate long-run distribution derives from the fact that there is no absorbing point, which implies that the entire state space is an ergodic set. The role of limited commitment is to bound the set of promised utilities, which is necessary to obtain a non-degenerate long-run distribution. Specifically, the lower bound w^* on the set of expected discounted utility entitlements arises due to the fact that a borrower can defect from his current contract and sign with another lender at any moment. The upper bound $\bar{w}(w^*)$ is the highest expected discounted utility to which a lender can commit to deliver to a borrower given that the lowest expected discounted utility that can be promised is w^*.

2.7 Conclusion

We have characterized the terms of the contract that a lender offers to a borrower in a competitive credit market with the following characteristics: lenders are asymmetrically informed about a borrower's ability to repay a loan; lenders can commit to some credit contracts while borrowers cannot commit to any contract; the history of trades within each enduring credit relationship in the economy is not publicly observable; and it is costly for a lender to contact a borrower. These frictions result in a market contract whose terms vary over time according to the history of trades within each long-term credit relationship.

As the initial cost of lending goes to zero, the contract that a lender offers to a borrower in the credit market converges to a full-insurance contract. If technological progress drives the cost to nearly zero, we should expect small fluctuations over time in a borrower's expected discounted utility. Another prediction of the model is that a borrower obtains more favorable terms of credit as the initial cost of lending approaches zero: a market contract is such that each borrower is better off from the perspective of the contracting date. Although we do not exploit the model's quantitative implications in this paper, we provide important properties of a lender's optimal contracting problem in the market for unsecured loans.

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3 Costly Recordkeeping, Settlement System, and Monetary Policy[2]

3.1 Introduction

We study the implications of a government-provided settlement system for the implementation of monetary policy. A key ingredient of the analysis is that it is costly for the government to use a record-keeping technology which is necessary for the construction of a settlement system through which private loans and tax liabilities are settled. In modern economies, it is common to observe a financial arrangement in which the government provides a public settlement system from which the private sector benefits. For instance, the Fedwire system in the U.S. facilitates the clearing of private debt and provides an important service to a large number of financial institutions. Participants are required to pay a fee that is designed to finance the costs of providing such service.

A public settlement system is also useful for the collection of a tax liability. The government needs to keep track of agents' identities and trading histories in order to enforce the payment of any tax liability. As a result, a record-keeping technology is necessary not only to support credit arrangements in the private sector but also to the operation of a fiscal system. For instance, the Treasury Tax and Loan (TT&L) Service in the U.S. allows the government to effectively collect a tax liability through a centralized system whose operation also involves a cost. Hence, it is crucial to evaluate the benefits and costs involved in an institutional arrangement in which the government provides a settlement system by using a costly record-keeping technology and by enforcing the repayment of private loans and the collection of tax liabilities. Given that it is costly for the government to use a record-keeping technology, what is the optimal size of a public settlement system? What are the implications for optimal monetary policy?

[2] Joint project with Pedro Gomis-Porqueras.

The arrangement that we study in this paper is one in which the government provides a settlement system to the private sector in a centralized location. We build on the models of Lagos and Wright (2005) and Rocheteau and Wright (2005). In the Lagos-Wright model, the pattern of trade is such that private agents periodically visit a centralized location where the government can interact with them. Within this framework, Koeppl, Monnet, and Temzelides (2008) show that a settlement system in a centralized location is essential for the implementation of an efficient allocation. It is necessary to have an institutional arrangement in which a centralized agency keeps track of agents' trades and imposes a punishment on agents who default on their liabilities. One possible arrangement involves a government-provided settlement system. The novelty of our analysis is to assume that it is costly for the government to use a record-keeping technology which is necessary to the operation of a settlement system.

If individuals cannot commit to their future promises, it is difficult to support credit transactions within the private sector. However, some credit arrangements can be supported in equilibrium provided that the government can enforce the repayment of private loans in a centralized location. In this paper, we assume that the only punishment that the government can impose on an agent is the seizure of his or her assets. However, the government can seize an agent's assets only if it observes his or her identity, which means that it needs to use a record-keeping technology to monitor individuals in the private sector. Given that it is costly for the government to keep track of agents' transactions, it may not be socially optimal to monitor all transactions in the economy in order to support credit arrangements within the private sector. This means that the choice of the optimal size of a settlement system is non-trivial. Although a settlement system allows private agents to expand the set of feasible trades, its operation results in a social cost due to a costly record-keeping technology.

A settlement system is also essential for the implementation of monetary policy.

Suppose that the monetary authority intervenes in the economy through a lump-sum transfer or tax in a centralized location. Then, it can effectively control the money supply only if it uses a record-keeping technology. To expand the money supply, the monetary authority needs to keep track of whom has already received a nominal transfer in a given period. To contract the money supply, the monetary authority needs to collect a lump-sum nominal tax in a centralized location. To enforce the payment of a lump-sum tax, the government needs to use a record-keeping technology to identify an individual in the private sector and effectively impose a punishment on her if she refuses to pay her tax liability. Even if the government identifies an agent and requires her to pay a lump-sum tax, it must be an incentive-compatible scheme, as in Andolfatto (2008, 2009). As a result, a settlement system expands the set of feasible public policies available to the government.

Our main result is to show that the Friedman rule is suboptimal and that the government relies on a credit system to implement an optimal policy. At the Friedman rule there is no credit activity: all trade is carried out with fiat money. This happens because such a policy essentially eliminates the opportunity cost of holding money over periods and individuals do not economize on their money holdings. However, the implementation of the Friedman rule involves a social cost due to the fact that it requires the use of a record-keeping technology to enforce the payment of a tax liability. Then, we show that it is possible to construct a welfare-improving deviation from the Friedman rule. Moving away from the Friedman rule results in a lower rate of return on money holdings. However, a deviation from the Friedman rule allows the government to reduce the size of the settlement system and consequently minimize its social cost. We show that this deviation results in higher welfare because it induces credit transactions in the private sector, which permits the cash constraint in some transactions to be relaxed. In this sense, a credit system complements the operation of a monetary system.

The importance of a settlement system for the implementation of government policies in monetary economies has been emphasized by many authors, including Freeman (1996), Aiyagari and Williamson (2000), Temzelides and Williamson (2001), Williamson (2003), Nosal and Rocheteau (2006, 2009), Kanh and Roberds (2009), Williamson and Wright (2010a, 2010b), among others. Our paper contributes to this literature by exploiting the implications of a costly record-keeping technology for the effective operation of a monetary system.

3.2 Historical Background

Systematic recordkeeping has been common among large-scale societies, even those lacking widespread literacy. This includes simple record-keeping technologies, such as the Sumerian token dating back to 8,000 B.C. The Sumerians began using stone and baked clay tokens to symbolically represent agricultural commodities that had been physically transferred. By 4,000 B.C., complex incised tokens were used to signify manufactured goods. Shortly before 3,200 B.C., tokens began to be sealed inside hollow clay balls ("bullae") that protected against fraud by imprinting signatures of the transacting parties and witnesses (via seals) on the envelope's exterior. The bullae were then baked, making the records permanent and difficult to alter. Over time new forms of recordkeeping emerged such as the "tally stick", which was used for centuries in England and in rural France as recently as 1970, the "knotted string", and the double-entry bookkeeping – see Robert (1956). Common properties among all of these forms of recordkeeping are the different degrees of monitoring and enforcement needed to operate them as well as the costs involved.

In recent times, the innovations in information technology have significantly improved societies' ability to maintain up-to-date records of transactions. As a result, a large variety of payment systems has developed in industrialized economies. Many of these systems are government-sponsored, such as the Fedwire in the U.S. and TARGET in the Euro area, and involve a large number of transfers daily among financial institutions.

Recordkeeping is not only crucial for private transactions but also for tax collection purposes. This important use of a record-keeping technology has been observed throughout history. For instance, tomb paintings depict tax collectors in Egypt at least as early as 2,000 B.C. The Egyptians kept written records of title deeds and field sizes. To assess the farmers' wealth there were also cattle counts. But not everybody's means of livelihood could be taxed as easily as the farmers', and attempts were made to tax other parts of the population. In order to increase the tax base, Late Period Egyptians had to declare their income, and if any man did not make declaration of an honest way of living, he was punished with death. In modern societies, the fiscal authority faces essentially the same issues, and the punishment for default on a tax liability usually involves the confiscation of an individual's assets.

As we can see, regardless of the time period considered, a record-keeping technology is costly for society, is needed to inflict punishments on individuals, and is necessary for taxing economic activity. In this paper, we incorporate these important characteristics of a record-keeping technology into a search-theoretic model of money and study their effects on the design of optimal monetary policy.

3.3 The Model

3.3.1 Private Sector

There is a continuum of infinitely-lived buyers and sellers. Each buyer is indexed by $i \in [0, 1]$ and each seller is indexed by $j \in [0, 1]$. Time is discrete and each period is divided into two subperiods: day and night. Within each subperiod, there is a unique perishable consumption good that is produced and consumed. In the day subperiod, a seller does not want to consume but can produce one unit of the consumption good with one unit of labor. In the night subperiod, a seller wants to consume but is not able to produce. A buyer wishes to consume only in the day subperiod but can produce one unit of the consumption good with one unit of labor in the night subperiod. Neither a buyer nor a seller can commit to his or her promises. This structure generates an absence-of-double-coincidence problem so that a medium of exchange can expand the set of feasible trades.

A buyer has preferences given by

$$u(q_i) - n_i, \tag{3.1}$$

where q_i is his consumption during the day and n_i is his labor supply at night. Assume that $u : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly concave, increasing, and continuously differentiable, with $u(0) = 0$ and $u'(0) = \infty$. A seller has preferences given by

$$-n_j + q_j, \tag{3.2}$$

where n_j is her labor supply during the day and q_j is her consumption at night. Buyers and sellers have a common discount factor between periods which we denote by $\beta \in (0, 1)$.

Agents are randomly and bilaterally matched in the day subperiod in such a way that each buyer meets a seller. In the night subperiod, agents interact in a centralized location. The terms of trade in the day market are determined by the following bargaining protocol. In each bilateral meeting, both agents simultaneously announce their willingness to trade. If both agree to trade, then the buyer makes a take-it-or-leave-it offer to the seller, who either accepts or rejects it. Otherwise, no trade takes place. In the night market, there is a Walrasian market in the centralized location. This sequential market structure, together with quasilinear utility with respect to labor supply, results in a degenerate end-of-period distribution of money balances across the population of buyers, as in Lagos and Wright (2005).

3.3.2 Government

There is a record-keeping technology that allows the government to observe the identities of a fixed fraction of sellers and record their individual trading histories. We say that a monitored seller is one that has her identity and trading history observed by the government. The identities and transactions of their trading partners are also recorded, as in Cavalcanti and Wallace (1999) and Sanches and Williamson (2009). As a result, a buyer who trades with a monitored seller in the decentralized market has his identity and transactions revealed in the day and night markets of the current period. If the government wishes to keep track of a fixed fraction ρ of sellers, the flow cost per seller in terms of the consumption good is given by $\kappa + \epsilon\rho$, where κ and ϵ are positive constants. This cost is paid in the centralized location in the night subperiod.

If the government decides to use the record-keeping technology, it can share the information with the private sector without any additional cost. The government can interact with agents only in the centralized location and can seize an agent's assets provided that it identifies such an agent in the current period. This means that the government can seize the assets of a particular agent only if it uses a record-keeping technology.

3.4 Discussion

Given that there is no additional cost for the government to share the information about identities and transactions that it obtains by using a record-keeping technology, it can construct a settlement system through which private debt and tax liabilities can be settled. First, notice that, by making the identities and trading histories of agents publicly observable, a record-keeping technology makes credit arrangements within the private sector feasible. Recall that agents cannot commit to their future promises, which makes it difficult to support credit transactions. However, some credit arrangements can be supported in equilibrium provided that the government can enforce the repayment of private loans in the centralized location. The only punishment that the government can impose on an agent is the seizure of his or her assets. However, the government can seize an agent's assets only if it observes his or her identity, which means that it needs to use a record-keeping technology to monitor agents in the private sector.

One way that the government can enforce the repayment of a private loan is to announce that any seller who trades with a buyer who has defaulted on his private loan will have her assets seized in the centralized location. This means that a buyer who has defaulted on his private loan will only be able to trade with anonymous sellers in the decentralized market. The threat of this punishment induces cooperation among buyers. In this way, the government provides a settlement system in the centralized location through which private debt is settled. Hence, a benefit of the record-keeping technology is that it provides a service to private agents that permits them to expand the set of feasible trades.

Second, a record-keeping technology allows the government to collect a tax liability in the centralized location. For instance, if the government wishes to shrink the money supply, it could levy a lump-sum nominal tax on buyers. Due to lack of commitment, the government needs to impose a punishment on private agents to enforce the payment of a tax liability in the centralized location. Again, it can announce that it will seize the assets of any seller who trades with a buyer who has defaulted on his tax liability. The threat of this punishment can induce buyers to pay a lump-sum tax in the centralized location. As a result, the government can effectively control the money supply only if it uses the record-keeping technology, which means that the settlement system is also useful for the implementation of public policies.

Finally, notice that the government is able to precisely infer a monitored seller's money holdings at the end of each subperiod, which means that it can effectively impose a punishment on each one of them. The government can infer a monitored seller's money holdings by simply keeping track of her transactions. These agents have all of their transactions publicly observable and as a result the government can keep track of their money holdings over time.

3.5 Monetary Equilibrium

Suppose that each buyer is endowed with one unit of fiat money at the beginning of the first day. We restrict attention to monetary equilibria in which the money supply grows at the gross rate $\mu \geq \beta$ and aggregate real money balances are constant over time. The government injects new money in the centralized location through lump-sum transfers to buyers. To receive a lump-sum transfer, a buyer needs to identify himself in the centralized location. Those buyers who traded with a monitored seller in the decentralized market of the current period already have their identities publicly observable. Those buyers who remained anonymous in the current period – those who traded with anonymous sellers in the decentralized market – have an incentive to reveal their identities and receive a nominal transfer from the government.

If the government wishes to shrink the money supply, it needs to levy a lump-sum tax on buyers. The government is able to collect a nominal tax only from buyers who are currently being monitored – those who traded with a monitored seller in the decentralized market. A buyer who remained anonymous in the current period has no incentive to voluntarily identify himself and pay a lump-sum tax, so that the government does not expect to receive a nominal payment from him.

If the government wishes to monitor sellers, it needs to pay a flow cost $\kappa + \epsilon\rho$ per seller in terms of the consumption good. There is a fee τ_b in terms of the consumption good that is designed to finance the use of a record-keeping technology. The settlement system provided by the government allows it to effectively collect such a fee in the centralized location. To enforce the payment of a tax liability, the government announces that it will seize the assets of any seller who decides to trade with a buyer who has defaulted on his tax liability. This punishment implies that a buyer who has defaulted on his tax liability is effectively banished from the public settlement system. Only anonymous sellers accept to trade with such a buyer, in which case they require fiat money in exchange for goods.

Let v denote a buyer's expected discounted utility at the end of the night subperiod. Let τ denote the real value of the lump-sum transfer to each buyer in the centralized location. Suppose first that $\tau < 0$, so that the government announces a lump-sum tax. Thus, a buyer's problem can be formulated in terms of the following Bellman equation:

$$v = \max_{(m.l)\in\mathbb{R}_+^2} \left\{ -m + \beta \left\{ \rho \left[u\left(\frac{m}{\mu} + l\right) - l - \tau_b + \tau \right] + (1-\rho)\, u\left(\frac{m}{\mu}\right) + v \right\} \right\},$$

subject to the incentive constraint,

$$-l - \tau_b + \tau + v \geq \hat{v}, \tag{3.3}$$

where m denotes a buyer's real money balances, l denotes a loan amount from a seller in a monitored meeting, and $\hat{v}$ denotes the value of defection. In the decentralized market, a buyer who trades with a monitored seller hands out all of his money balances to such a seller and also obtains a loan amount. His identity is revealed and his transactions are recorded: the settlement system ensures that the repayment of a loan as well as the payment of tax liabilities – a fee τ_b and a lump-sum transfer τ – become publicly observable. To enforce the repayment of a private loan, the government announces that any monitored seller who trades with a buyer who has defaulted on his private loan will have her assets seized in the centralized location.

Suppose now that $\tau \geq 0$. Then, the Bellman equation for a buyer's problem becomes:

$$v = \max_{(m.l)\in\mathbb{R}_+^2} \left\{ -m + \beta \left\{ \rho \left[u\left(\frac{m}{\mu} + l\right) - l - \tau_b \right] + (1-\rho)\, u\left(\frac{m}{\mu}\right) + \tau + v \right\} \right\},$$

subject to (3.3). If the government announces a lump-sum transfer in the centralized location, then the buyers who remained anonymous in the decentralized market are willing to identify themselves in the centralized location and receive the nominal transfer from the government, so that all buyers receive a lump-sum transfer from the government.

The government chooses the money growth factor $\mu \in [\beta, \infty)$ and the size of the settlement system $\rho \in [0,1]$. In a monetary equilibrium, the government's budget constraints are given by

$$\tau_b = \begin{cases} \kappa + \epsilon\rho \text{ if } \rho \in (0,1], \\ 0 \text{ if } \rho = 0, \end{cases}$$

and

$$\tau = \begin{cases} \frac{m}{\rho}\left(1 - \frac{1}{\mu}\right), \text{ if } \mu \in [\beta, 1) \text{ and } \rho \in (0,1], \\ m\left(1 - \frac{1}{\mu}\right), \text{ if } \mu \in [1, \infty) \text{ and } \rho \in [0,1], \end{cases}$$

where m denotes the real money balances that each buyer holds at the end of each period.

Consider now the value of defection $\hat{v}$. If a buyer defaults on either his tax liabilities or private loan, he will only be able to trade in anonymous meetings in the decentralized market using currency. Given that the government announces that it will seize a monitored seller's money holdings if she decides to trade with a defaulter, only anonymous sellers will find it profitable to trade with him. For a given buyer, an anonymous meeting happens with probability $1-\rho$. The value of defection $\hat{v}$ satisfies the Bellman equation:

$$\hat{v} = \max_{\hat{m}\in\mathbb{R}_+} \left\{ -\hat{m} + \beta \left[(1-\rho)\, u\left(\frac{\hat{m}}{\mu}\right) + \hat{v} \right] \right\}.$$

It follows that

$$(1-\beta)\,\hat{v} = -\mu z + \beta\,(1-\rho)\, u(z), \tag{3.4}$$

where the buyer's consumption z after defection is given by

$$u'(z) = \frac{\mu}{\beta(1-\rho)}. \tag{3.5}$$

Notice that z is strictly decreasing in both μ and ρ. A higher inflation rate reduces the value of defection for a buyer because he will only be able to use currency in transactions. A higher fraction of monitored sellers reduces the value of defection for a buyer because he will only be able to trade with anonymous sellers in the decentralized market.

Let x denote a buyer's consumption in an anonymous meeting in the decentralized market and let y denote his consumption in a monitored meeting. In a monetary equilibrium, a buyer's expected discounted utility v can be written as

$$(1-\beta)v = -(1-\beta)\mu x + \beta\rho[u(y) - y - \kappa - \epsilon\rho] + \beta(1-\rho)[u(x) - x].$$

Now, we can formally define a monetary equilibrium for the whole economy.

Definition 3 *Given $\mu \in [\beta, 1)$ and $\rho \in (0, 1]$, a stationary monetary equilibrium is a triple (x, y, z), with $0 \le x \le y \le q^*$, satisfying the first-order conditions (3.5) and*

$$\rho u'(y) + (1-\rho)u'(x) = \frac{\mu}{\beta}, \tag{3.6}$$

and satisfying the incentive constraint,

$$\begin{aligned} &\beta\rho u(y) - (1-\beta+\rho\beta)(y+\kappa+\epsilon\rho) + \beta(1-\rho)[u(x)-x] \\ \ge\ &(1-\mu)(1-\beta)\left(\rho^{-1}-1\right)x + (1-\beta)\hat{v}, \end{aligned} \tag{3.7}$$

where $y = q^*$ if (3.7) does not bind and $\hat{v}$ is given by (3.4). Given $\mu \ge 1$ and $\rho \in [0, 1]$, a stationary monetary equilibrium is a triple (x, y, z), with $0 \le x \le y \le q^*$, satisfying the first-order conditions (3.5) and (3.6) and satisfying the incentive constraint,

$$\beta\rho u(y) - (1-\beta+\rho\beta)(y+\kappa+\epsilon\rho) + \beta(1-\rho)[u(x)-x] \ge (1-\beta)\hat{v}, \tag{3.8}$$

where $y = q^*$ if (3.8) does not bind.

We need to show the existence and uniqueness of a stationary monetary equilibrium, especially an unconstrained equilibrium – one in which a buyer's incentive constraint does not bind and the efficient quantity q^* is traded in each monitored meeting in the decentralized market. We show next that a unique unconstrained monetary equilibrium exists for any money growth factor $\mu \geq \beta$ provided that the size of the settlement system is not too small.

Proposition 23 *Suppose that $u(q^*) - q^* - \kappa - \epsilon > 0$. Then, for any $\mu \in [\beta, \infty)$, there exists $\tilde{\rho}(\mu) \in (0, 1)$ such that an unconstrained monetary equilibrium exists provided $\rho \in [\tilde{\rho}(\mu), 1]$. In such an equilibrium, it follows that $y = q^*$, x is given by (3.6), and z is given by (3.5).*

Proof. Suppose that $\mu \in [\beta, 1)$. If the incentive constraint (3.7) is slack, then we have $y = q^*$. Notice that there exists a unique $\hat{\rho} \in (0, 1)$ such that

$$\beta\rho u(q^*) - (1 - \beta + \rho\beta)(q^* + \kappa + \epsilon\rho) \geq 0$$

if and only if $\rho \in [\hat{\rho}, 1]$ provided that $\beta \in (\bar{\beta}, 1)$, where $\bar{\beta}$ is such that $\bar{\beta}u(q^*) - q^* - \kappa - \epsilon = 0$. Second, notice that, for any $\mu \geq \beta$ and $\rho \in (0, 1)$, we have that $\beta(1 - \rho)[u(x) - x] > \beta(1 - \rho)u(z) - \mu z$, with x given by

$$u'(x) = \left(\frac{\mu}{\beta} - \rho\right)\frac{1}{1 - \rho} \tag{3.9}$$

and z given by (3.5). Notice that the term $(1 - \mu)(1 - \beta)(\rho^{-1} - 1)x$ on the right-hand side of the incentive constraint (3.7) goes to zero as $\rho \to 1$ from below. Hence, there exists $\tilde{\rho}(\mu) \in (0, 1)$ such that a unique unconstrained monetary equilibrium exists provided that $\rho \in [\tilde{\rho}(\mu), 1]$.

Suppose now that $\mu \in [1, \infty)$. Then, the incentive constraint (3.8) is satisfied for any $\rho \in [\hat{\rho}, 1]$. Then, there exists $\tilde{\rho}(\mu) \leq \hat{\rho}$ such that a unique unconstrained monetary equilibrium exists provided that $\rho \in [\tilde{\rho}(\mu), 1]$. **Q.E.D.** ■

For any given money growth factor $\mu \geq \beta$, a bigger size of the settlement system makes it harder to satisfy a buyer's incentive constraint because a bigger fee τ_b is needed to finance the use of the record-keeping technology. If the government wishes to implement a deflationary policy, there is an additional term on the right-hand side of a buyer's incentive constraint due to a lump-sum nominal tax in the centralized location. Notice that a bigger size of the settlement system in fact reduces the real value of the lump-sum tax and makes it easier to satisfy a buyer's incentive constraint. For this reason, there exists a minimum size of the settlement system for which an unconstrained monetary equilibrium exists when the money growth factor μ lies in $[\beta, 1)$.

3.6 Optimal Monetary Policy

A government's policy involves the choice of the money growth factor μ and the size ρ of the settlement system. Throughout the analysis, we assume that the government can induce the unique unconstrained monetary equilibrium with a choice of the policy instruments. The social welfare associated with a stationary monetary equilibrium (x, y, z) is given by

$$\rho\left[u\left(y\right)-y\right]+\left(1-\rho\right)\left[u\left(x\right)-x\right]-\rho\left(\kappa+\rho\epsilon\right). \tag{3.10}$$

Notice that a settlement system results in a social welfare loss, which is expressed by the last term in (3.10). Society needs to use real resources to keep track of agents' trading histories.

The benefits of using a record-keeping technology are that it allows credit arrangements within the private sector and permits the government to increase the rate of return on money holdings. In an unconstrained monetary equilibrium, it follows that $y = q^*$, so that the efficient quantity is traded in each monitored meeting in the decentralized market. If the government decides to implement the Friedman rule by setting

$\mu = \beta$, we have that $x = q^*$, so that the efficient quantity is also traded in each anonymous meeting in the decentralized market. This policy maximizes social welfare for any given choice of ρ. However, we need to verify whether such a policy is in fact feasible given that it requires a lump-sum tax from buyers.

Lemma 24 *There exists $\tilde{\rho}(\beta) \in (0,1)$ such that the Friedman rule is feasible if and only if $\rho \in [\tilde{\rho}(\beta), 1]$.*

Proof. Let $B = \{(\rho, \mu) : \rho \in [0,1] \text{ and } \mu \geq \beta\rho\} \subset \mathbb{R}^2_+$. Define the functions $h : B \to \mathbb{R}$ and $g : B \to \mathbb{R}$ by

$$h(\rho, \mu) = \beta\rho u(q^*) - (1 - \beta + \rho\beta)(q^* + \kappa + \epsilon\rho) + \beta(1 - \rho)\{u[x(\rho, \mu)] - x(\rho, \mu)\}$$

and

$$g(\rho, \mu) = \beta(1 - \rho) u[z(\rho, \mu)] - \mu z(\rho, \mu) + (1 - \mu)(1 - \beta)\left(\rho^{-1} - 1\right) x(\rho, \mu),$$

where $x : B \to \mathbb{R}_+$ and $z : B \to \mathbb{R}_+$ are given by $x(\rho, \mu) = (u')^{-1}\left[\left(\beta^{-1}\mu - \rho\right)(1 - \rho)^{-1}\right]$ and $z(\rho, \mu) = (u')^{-1}\left[\mu\beta^{-1}(1 - \rho)^{-1}\right]$. Also, define $\sigma : B \to \mathbb{R}$ by $\sigma(\rho, \mu) \equiv h(\rho, \mu) - g(\rho, \mu)$. Notice that

$$h(\rho, \beta) = \beta u(q^*) - q^* - (1 - \beta + \rho\beta)(\kappa + \epsilon\rho),$$

where $(\partial h/\partial\rho)(\rho, \beta) < 0$ for all ρ, $h(0, \beta) = \beta u(q^*) - q^* - (1 - \beta)\kappa$, and $h(1, \beta) = \beta u(q^*) - q^* - \kappa - \epsilon$. Also, we have that

$$g(\rho, \beta) = \beta\{(1 - \rho) u[z(\rho, \beta)] - z(\rho, \beta)\} + (1 - \beta)^2\left(\frac{1}{\rho} - 1\right) q^*,$$

where $(\partial g/\partial\rho)(\rho, \beta) < 0$ for all ρ. Finally, notice that $g(\cdot, \beta) \to \infty$ as $\rho \to 0$ from above and $g(\cdot, \beta) \to 0$ as $\rho \to 1$ from below. Hence, there exists a unique $\tilde{\rho}(\beta) \in (0,1)$ such that $\sigma(\rho, \beta) = 0$ if and only if $\rho = \tilde{\rho}(\beta)$ and $\sigma(\rho, \beta) > 0$ if and only if $\rho \in (\tilde{\rho}(\beta), 1]$.

Therefore, the Friedman rule is a feasible policy if and only if $\rho \in [\tilde{\rho}(\beta), 1]$. **Q.E.D.** ■

The policy combination $(\mu, \rho) = (\beta, \tilde{\rho}(\beta))$ implies that the efficient quantity q^* is always traded in the decentralized market. If the government wants to implement the Friedman rule, it is optimal for the government to choose the minimum size of the settlement system consistent with the feasibility of such a policy, which is given by $\rho = \tilde{\rho}(\beta)$. To verify this claim, notice that at the Friedman rule there is no credit activity because $y = x = q^*$ – there is only monetary exchange in the decentralized market. This means that the only reason for the government to use a record-keeping technology is that it allows the government to effectively collect a lump-sum nominal tax that is required to generate a deflation in the economy. As a result, it is optimal for the government to minimize the loss in social welfare associated with the implementation of the Friedman rule. However, such a policy is infeasible if the size of the settlement system is too small – a smaller size of the settlement system makes it more expensive for a buyer to pay for the lump-sum tax.

It is straightforward to show that the policy combination $(\beta, \tilde{\rho}(\beta))$ dominates any other policy combination (μ, ρ) such that $\mu \geq \beta$ and $\rho \geq \tilde{\rho}(\beta)$. All of these combinations imply a lower expected payoff from trade for a buyer and a higher loss in social welfare associated with the use of a record-keeping technology. This means that $\tilde{\rho}(\beta)$ is an upper bound for the optimal size of a settlement system. Moreover, the smaller the cost of this system – the smaller the values for κ and ϵ – the smaller is the upper bound $\tilde{\rho}(\beta)$. However, the policy combination $(\mu, \rho) = (\beta, \tilde{\rho}(\beta))$ does not achieve the highest level of social welfare. We show next that it is possible to construct a welfare-improving deviation from the Friedman rule.

Lemma 25 *The Friedman rule is suboptimal.*

Proof. By the Implicit Function Theorem, there exist open intervals U and V, with $\tilde{\rho}(\beta) \in U$ and $\beta \in V$, such that there exists a unique function $\tilde{\rho} : V \to U$ such that $\sigma[\tilde{\rho}(\mu), \mu] = 0$ for all $\mu \in V$. Moreover, $\tilde{\rho}$ is continuously differentiable with

$$\tilde{\rho}'(\beta) = -\frac{\partial \sigma[\tilde{\rho}(\beta), \beta]}{\partial \mu} \left\{ \frac{\partial \sigma[\tilde{\rho}(\beta), \beta]}{\partial \rho} \right\}^{-1}.$$

We have that $(\partial \sigma / \partial \mu)[\tilde{\rho}(\beta), \beta] > 0$ and $(\partial \sigma / \partial \rho)[\tilde{\rho}(\beta), \beta] > 0$, which implies $\tilde{\rho}'(\beta) < 0$. Since $\tilde{\rho}$ is continuously differentiable, there exists $\Delta > 0$ such that $\tilde{\rho}'(\mu) < 0$ for all $\mu \in [\beta, \beta + \Delta)$, so that $\tilde{\rho}$ is strictly decreasing on $[\beta, \beta + \Delta)$.

In any unconstrained monetary equilibrium, social welfare is given by

$$W(\rho, \mu) = \rho[u(q^*) - q^* - \kappa - \epsilon\rho] + (1 - \rho)\{u[x(\rho, \mu)] - x(\rho, \mu)\},$$

for all $\mu \geq \beta$ and $\rho \in [0, 1]$. Let $\delta_1 > 0$ and $\delta_2 > 0$. Given that $\tilde{\rho}$ is strictly decreasing in μ on $[\beta, \beta + \Delta)$, the direction $(d\rho, d\mu) = (-\delta_1, \delta_2)$ is feasible from $(\rho, \mu) = [\tilde{\rho}(\beta), \beta]$ provided that δ_1 and δ_2 are sufficiently small. The total variation in welfare is given by

$$[\kappa + 2\epsilon\tilde{\rho}(\beta)]\,\delta_1 > 0,$$

which means that there exists a welfare-improving deviation from the Friedman rule.

Q.E.D. ■

Moving away from the Friedman rule results in a lower rate of return on money holdings. However, such a deviation allows the government to reduce the size of the settlement system and consequently reduce its social cost. We have shown that the latter effect dominates and such a deviation results in higher welfare: the gain at the extensive margin more than compensates for the loss at the intensive margin. An immediate implication of this result is that money and credit coexist as means of payment under the optimal payment arrangement. There is a role for credit in transactions in the sense that credit helps in relaxing a cash constraint.

If there is no cost of using a record-keeping technology, then for any given value of ρ an efficient allocation can be implemented by setting $\mu = \beta$ (the Friedman rule) – see Sanches and Williamson (2009). In this case, there is no credit activity at the optimum: all trade is carried out with fiat money. The government essentially eliminates the opportunity cost of holding money balances by setting $\mu = \beta$, so that individuals do not economize on their money holdings and the efficient quantity is traded in the decentralized market. As a result, the government's optimal policy does not rely on the existence of a credit system.

If a record-keeping technology is costly, the implementation of the Friedman rule involves a social cost. One alternative for the government is to induce credit transactions in the economy through the settlement system by deviating from the Friedman rule. This choice permits the government to reduce the social cost associated with the use of a record-keeping technology. Although the efficient quantity will not be traded in each anonymous meeting in the decentralized market, such a quantity will be traded in each monitored meeting because of the role of credit in relaxing a cash constraint. This essentially means that the loss in the intensive margin associated with a deviation from the Friedman rule happens only in anonymous transactions, which are exclusively carried out with fiat money. For this reason, welfare increases as a result of the proposed deviation.

In an environment where a record-keeping technology is costly, the government relies on a credit system to implement an optimal policy. In this sense, money and credit are complementary in transactions: the existence of a credit system makes the operation of a monetary system more effective.

3.7 Conclusion

We study an arrangement in which the government provides a settlement system through which private loans and tax liabilities are settled. The existence of a credit system requires a record-keeping technology to enforce credit contracts within the private sector, which in turn results in a social cost due to the fact that it is costly to use a record-keeping technology. Fiat money is an alternative to credit as a means of payment, but there is an opportunity cost of holding money balances over periods. One way to reduce this cost is by generating a deflation to increase the rate of return on money holdings. In our environment, the implementation of a deflationary policy requires the use of a record-keeping technology to enforce the payment of a tax liability, which means that the effective operation of a monetary system also involves a social cost.

There exists a minimum size of a settlement system that is consistent with the feasibility of the Friedman rule. As a result, the efficient implementation of the Friedman rule involves the choice of such a minimum size in order to minimize the social cost associated with its implementation. However, it is possible to construct a welfare-improving deviation from the Friedman rule. The benefit of reducing the size of the settlement system more than compensates the higher opportunity cost of holding money over periods – the effect on the extensive margin dominates the effect on the intensive margin at the Friedman rule. This happens because of the role of credit in relaxing a cash constraint in some transactions: a deviation from the Friedman rule induces credit transactions through the settlement system. As a result, the government relies on a credit system to implement an optimal policy.

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Figure 1 - Efficiency when q**>q*

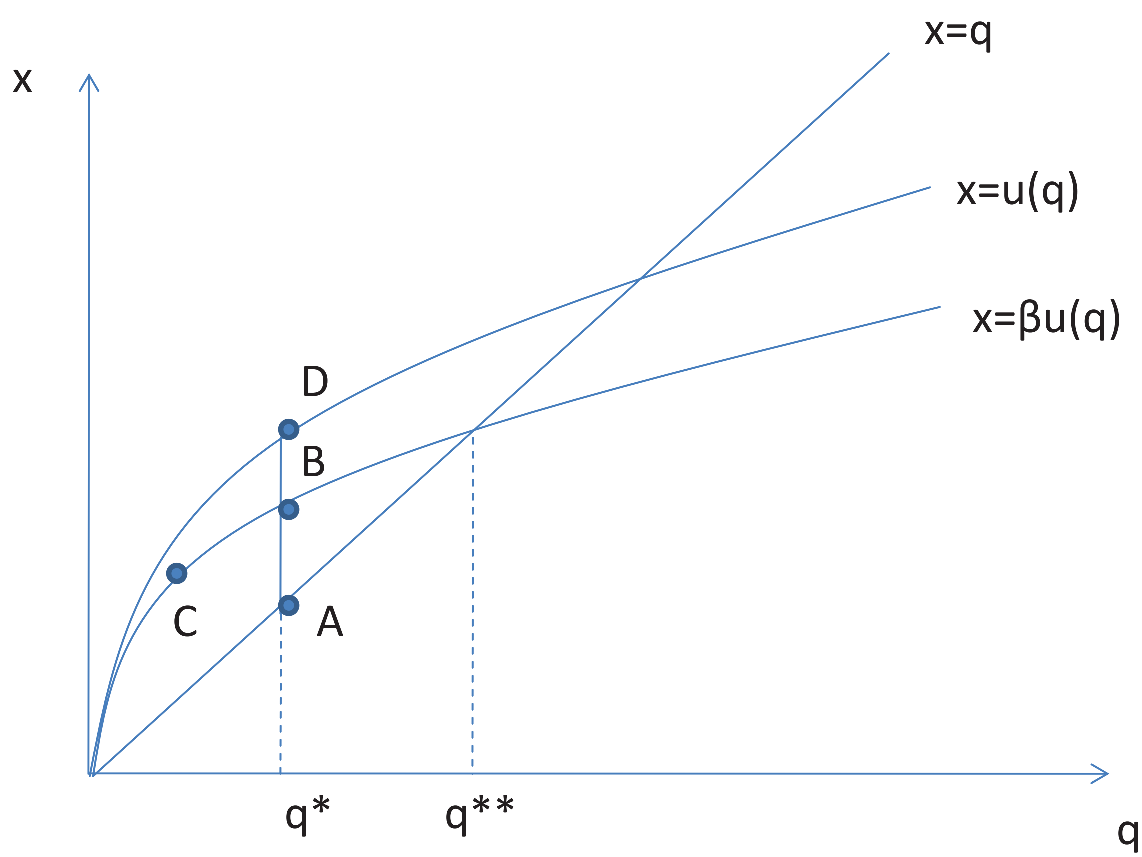

Figure 2 - Efficiency when q**<q*

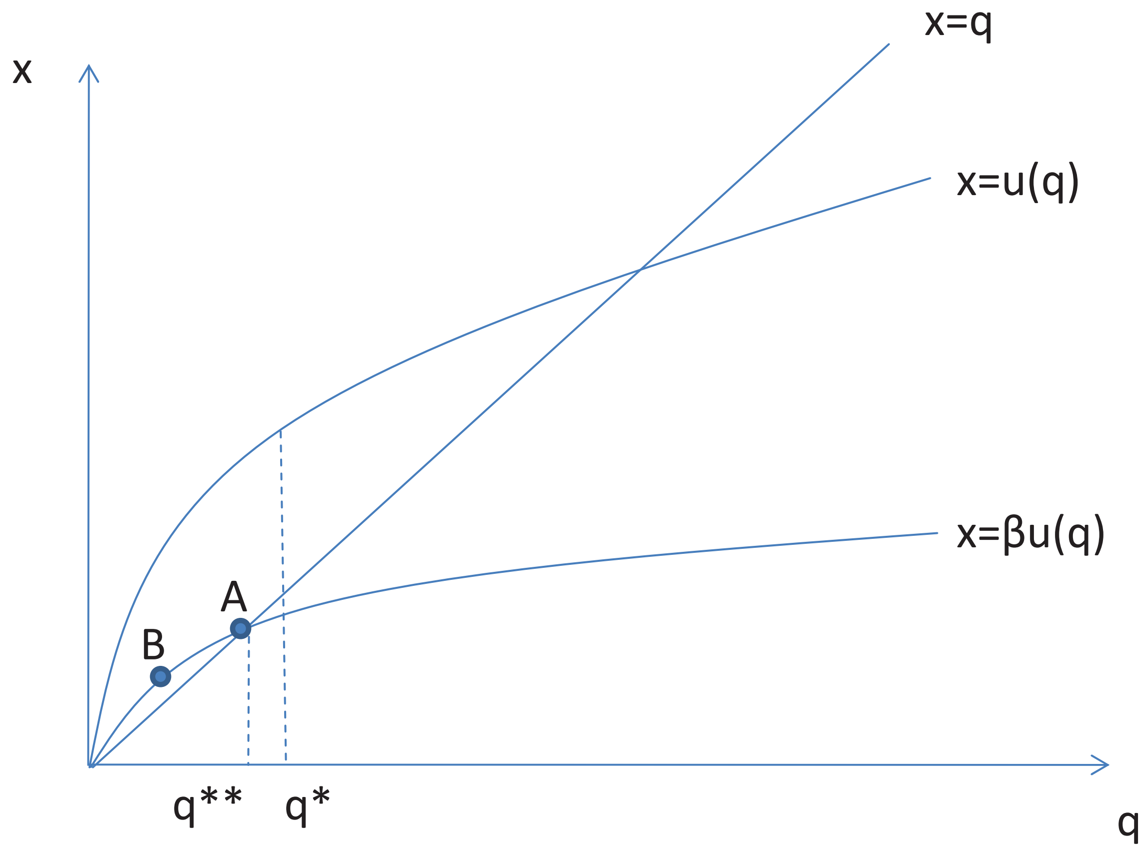

Figure 3 - Equilibrium with Credit

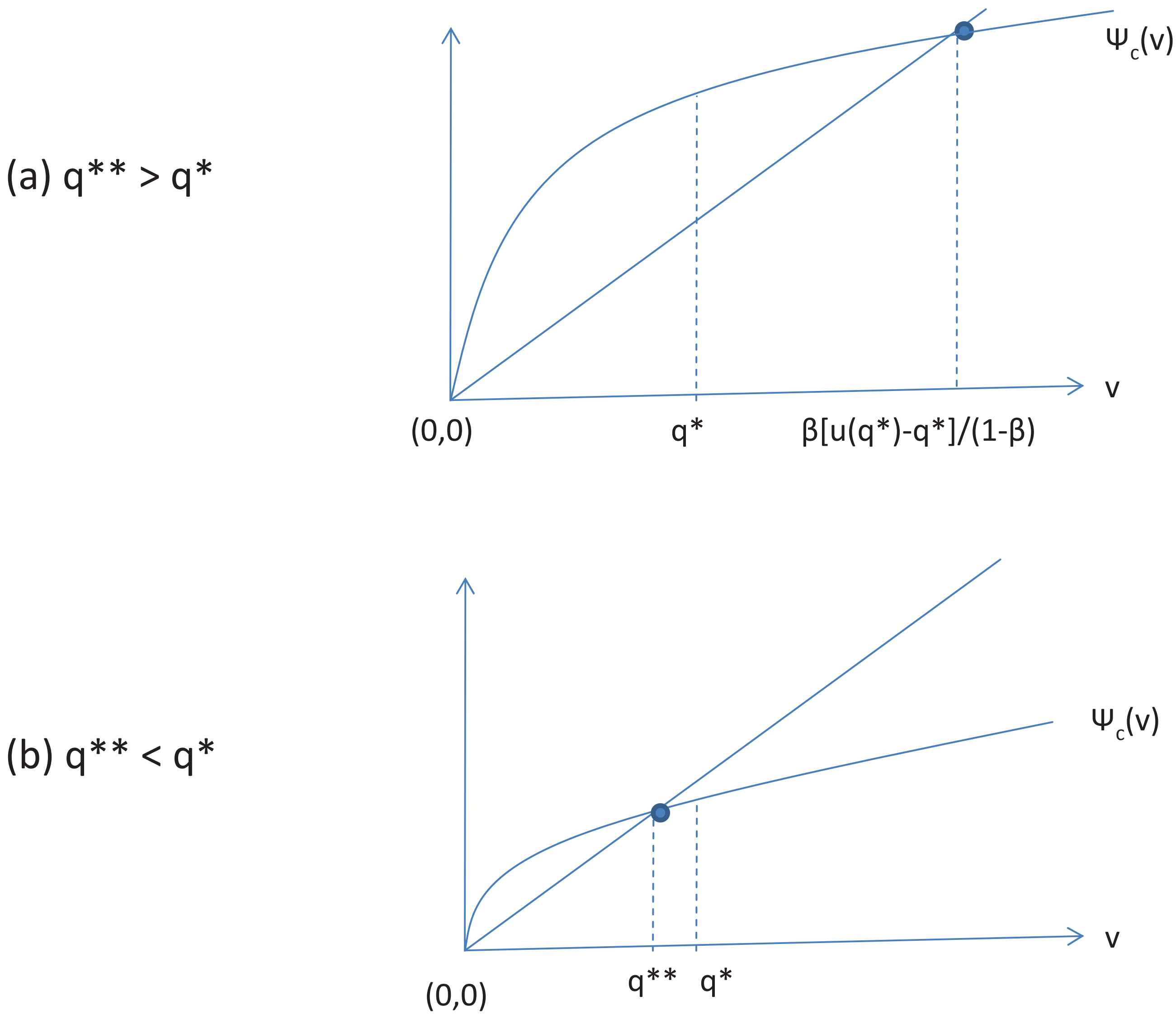

Figure 4 - Terms of Credit

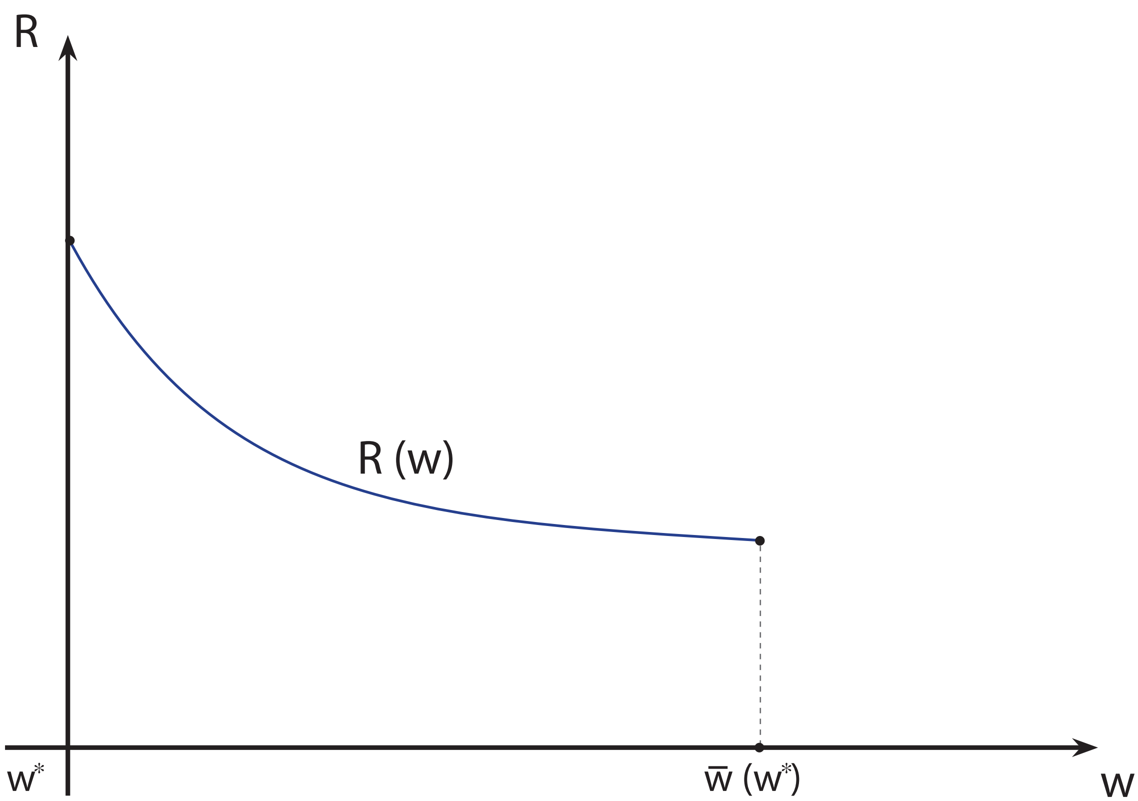

Figure 5 - Lender's Cost Function