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Strategic Communication Games: Theory and Applications

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by

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Abstract

The dissertation consists of three papers on the theory and application of strategic communication games. Strategic communication games are costless sender-receiver games, and address the question of how much information can be credibly transmitted in equilibrium, and what kind of communication environments facilitate information transmission.

The second chapter, "Multidimensional Cheap Talk with Sequential Messages," studies a two-dimensional cheap talk game with two senders and one receiver. The senders possess the same information and sequentially send messages about that information. In one-dimensional sequential message cheap talk games where the state space is unbounded, the information is fully transmitted under a self-serving belief, as suggested by Krishna and Morgan (2001b). However, this result depends crucially on the structure of the one-dimensional model. We show that a self-serving belief does not support full information transmission in two-dimensional models. We consider an extended self-serving belief, which implies full information transmission even if a self-serving belief cannot work. Then, We show that a necessary and sufficient condition for the existence of the fully revealing equilibrium is that the senders have opposing-biased preferences.

The third chapter, "A Characterization of Equilibrium Set of Persuasion Games without Single Crossing Conditions," considers a persuasion game between one sender and one receiver. The sender's private information is completely verifiable and the receiver has binary alternatives. However, the single crossing condition by Giovannoni and Seidmann (2007) is violated. That is, full information disclosure is impossible. We characterize the set of pure strategy equilibria by specifying the set of equilibrium ex ante expected utility of the receiver. The set is characterized by the maximum and the minimum utility of the receiver, and any value between them can be supported

as equilibrium ex ante expected utility of the receiver. In any equilibrium, the sender can conceal a part of unfavorable information, but cannot suppress all of the information if conflicts between the players happen frequently enough.

When mass media strategically suppress election-relevant information in order to influence public opinion, how do candidates and voters react to this media manipulation? To answer this question, the fourth chapter, "Manipulated News: Electoral Competition and Mass Media," develops a simple Downsian voting model including mass media. The media outlets observe the proposed policies by the two candidates, but the voter cannot. The voter then learns this information through reports from the outlets before voting occurs. The media outlets strategically choose either to disclose or withhold the information. In the model with single media outlet, we show that the voter's decision making could be incorrect ex post in any equilibrium when the media outlet's preference is sufficiently different from that of the voter. Appealing to the voter then becomes less attractive to the candidates. Furthermore, the candidates have an incentive to influence the media outlet's behavior through policy settings. Through the distortions in the behaviors of the voter and the candidates, the equilibrium outcomes are distorted in favor of the media outlet; that is, the median voter theorem could fail. This distortion mechanism can be observed even if there are multiple media outlets when their preferences are like-biased. However, if the media outlets' preferences are opposing-biased, then the median voter theorem holds.

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Chapter 1

Introduction

This chapter briefly overviews strategic communication games. Strategic communication games are costless sender-receiver games.[1] The sender has private information, and the receiver has to make some decision that affects both the sender's and receiver's payoffs. In order to make a correct decision, the receiver need to know the private information of the sender. So the receiver asks the sender to send a message about his private information. However, because the sender's and receiver's preferences are different, the sender tries to mislead the receiver by sending a message strategically. Because the receiver anticipates his misleading report, she does not naively believe the message. The receiver tries to infer the meaning behind the message. With this setup, strategic communication games address the question of how much information can be credibly transmitted in equilibrium, and what kind of communication environments facilitate information transmission.

There are two basic types of strategic communication games, depending on the properties of the sender's private information. In *cheap talk games*, initiated by Crawford and Sobel (1982), the

[1] As a matter of convention, we treat the sender as male and the receiver as female throughout the dissertation.

sender's private information is unverifiable. Because the private information is unverifiable, the sender can misreport the information without any cost. In *persuasion games*, first formalized by Milgrom (1981), the sender's private information is verifiable. The sender cannot misreport the information, but he can withhold his unfavorable information. Although the setups of cheap talk games and persuasion games have many similarities, the difference on private information generates nontrivial differences in model predictions. In cheap talk games, meaning of each message is defined by an equilibrium. That is, a message itself has no literal meaning because the sender's message space is independent from his private information. In other words, if the equilibrium that we focus on is different, then the same message could have different meaning. On the other hand, a message itself in persuasion games has literal meaning because the message space of the sender varies depending on his private information. That is, the receiver's inference from an observed message in persuasion games has less degree of freedom than that in cheap talk games. This is reason why we should distinguish these models clearly.

The dissertation contains both cheap talk games and persuasion games. The second chapter, "Multidimensional Cheap Talk with Sequential Messages," is based on cheap talk games. The third chapter, "A Characterization of Equilibrium Set of Persuasion Games without Single Crossing Conditions," and the fourth chapter, "Manipulated News: Electoral Competition and Mass Media," are based on persuasion games. Particularly, the fourth chapter is an application of the theory developed in the third chapter to contexts of electoral competitions.

1.1 References

1. Crawford, V. P., and J. Sobel. (1982), “Strategic Information Transmission,” *Econometrica*, **50**, 1431–1451.

2. Milgrom, P. (1981), “Good News and Bad News: Representation Theorems and Applications,” *Bell Journal of Economics*, **12**, 380–391.

Chapter 2

Multidimensional Cheap Talk with Sequential Messages

2.1 Introduction

This chapter studies a cheap talk game between two senders and one receiver with a two-dimensional unbounded state space. The senders share the same two-dimensional private information and sequentially send messages to the receiver. That is, the second sender can observe what the first sender sent before he or she chooses the message.[1] Sequential communication with several experts is often observable in our life; for example, seeking second opinions, a peer-review process and a debate are categorized into this communication structure. In this chapter, we consider whether full information transmission is possible in this setup and, if it is, how and when we achieve it.

[1] Note that the environment of this chapter is similar to that of Battaglini (2002), but this chapter studies sequential communication instead of simultaneous communication by Battaglini (2002).

The results are as follows. First, we show that Krishna and Morgan's (2001b) successful belief system, *self-serving belief*, which supports a *fully revealing equilibrium*, an equilibrium where the senders' private information is completely transmitted, in one-dimensional unbounded state space models does not work in the two-dimensional model. Their positive result crucially depends on the structure of the one-dimensional models. Second, we suggest a new belief system, *extended self-serving belief*, which supports a fully revealing equilibrium in the two-dimensional model. Finally, we show that the necessary and sufficient condition for the existence of fully revealing equilibria is that the senders' preferences are opposing biased, that is, they are biased in dissimilar directions. Therefore, we can conclude that the directions of preference biases remain important in the two-dimensional environment.

It is well known that if the private information is one-dimensional and the state space is unbounded, then full information transmission could be an equilibrium outcome. Krishna and Morgan (2001b) show that if the senders have opposing-biased preferences, then the receiver's self-serving belief supports a fully revealing equilibrium. By taking advantage of the conflicts between the senders, the receiver can make each sender check whether the other sends true messages. Because neither sender has an incentive to lie under the belief, full information transmission is realized as an equilibrium outcome.

However, their useful belief generally does not work in multidimensional environments; that is, one-dimensionality of the state space is necessary for the self-serving belief to work well. In two-dimensional models, the two senders can compromise more easily than in one-dimensional models even if the sender's preferences are opposing biased. The self-serving belief system is fragile in the case of such compromised deviations, and the deviations are omitted in one-dimensional models. Consider, for example, discussion of the tax on alcohol. If the situation is represented by a one-dimensional model, i.e., experts discuss only the total amount of taxes on alcohol, and their preferences are opposing biased, then any compromise is impossible. However, if we consider the same problem in a two-dimensional model, i.e., the experts discuss taxes on both whiskey and wine, then the experts who have opposing-biased preferences on the total amount of taxes could compromise. If the experts agreed with the low tax on wine, they would reach a compromise in terms of a lower wine tax and a higher whiskey tax than the socially optimal levels. One-dimensional models exclude such compromised deviations.

It is also well known that multidimensional cheap talk games have positive results on information transmission. Battaglini (2002) constructs a useful belief system that supports a fully revealing equilibrium. However, his belief system is fragile in the case of sequential communication; if messages are sequential, then his positive result can hold only in the special case. Basically, sequential cases are more difficult than simultaneous cases because of the sequential rationality of the second sender. Moreover, multidimensional models require us to check a number of possible strategies. The literature does not tell us whether full information transmission is possible in multidimensional sequential message models. To pursue the question, this chapter extends Krishna and Morgan's (2001b) results to a two-dimensional model.

This capter is structured as follows. In the next subsection, we discuss the related literature. Section 2.2 defines a two-dimensional sequential message cheap talk game model. In Section 2.3, we retest the result of Krishna and Morgan (2001b) in the two-dimensional model. We develop a new belief system that works well in the two-dimensional case, and characterize fully revealing equilibria in Section 2.4. We discuss extensions in Section 2.5, and conclude the chapter in Section 2.6.

2.1.1 Related literature

Crawford and Sobel (1982) study a one-dimensional cheap talk game with one sender and one receiver. Their result is that the degree of information transmission depends on the difference between the sender's and the receiver's preferences and, in particular, they show that full information transmission is impossible unless both players' preferences coincide. Following this study, several research streams that consider full information transmission in cheap talk games have developed. The research regarding multiple-sender models is one such stream.

Gilligan and Krehbiel (1989) define the one-dimensional bounded state space, $[0, 1]$, and analyze the situation where the two senders send messages simultaneously in the context of legislation. Krishna and Morgan (2001a) reexamine this problem and show that full information transmission is possible unless the conflict between the players is large. On the other hand, Krishna and Morgan (2001b) analyze the situation where the experts send messages sequentially. Because they also define the one-dimensional bounded state space, $[0, 1]$, they conclude that full information transmission is impossible. However, if the state space is defined as the real line, then the self-serving belief supports a fully revealing equilibrium.[2]

Battaglini (2002) defines a two-dimensional unbounded state space, $\mathbb{R}^2$, and analyzes simultaneous communication processes. He suggests a belief system that supports a fully revealing equilibrium. Under his belief system, the receiver makes each sender report only one element of the two-dimensional private information; for example, one sender sends a message about the x-coordinate of the private information, and the other sender sends a message about the y-coordinate. By aggregating both messages, the receiver acquires the true information. Battaglini (2002) shows

[2] All of the above papers as well as this chapter assume that experts are perfectly informed players. Austen-Smith (1993b) analyzes the situation where the experts are imperfectly informed.

that fully revealing equilibria exist unless the senders' preferences are biased in exactly the same direction. Therefore, he concludes that the important factor for full information transmission is not the degree of conflicts and the bias directions, but the multidimensionality itself.[3] Ambrus and Takahashi (2008) consider the same situation in bounded state space and point out that Battaglini's full revelation result depends crucially on the unboundedness of the type space. Furthermore, they show the necessary and sufficient condition for full information revelation for any state space.[4] [5]

Chapter 2 is different from the above literature in terms of the dimensionality of the state space and the communication process. Essentially, this chapter is an extension of Krishna and Morgan's (2001b) one-dimensional unbounded state space model into the two-dimensional environment, and it suggests a new belief system. The extended self-serving belief is more restricted than the original in order to prevent the compromised deviations mentioned above. It is also different from Battaglini's (2002) belief system in the sense that the receiver makes the senders send direct messages, and compares them to obtain true information.

2.2 The Model

We consider the following two-dimensional cheap talk game with sequential communication. There are three players: two senders and a receiver. We call the senders *expert 1* and *expert 2*, and the receiver the *decision-maker*.[6] The experts share private information about the state, which is denoted by a two-dimensional vector. Let $\Theta \equiv \mathbb{R}^2$ be the state space, and let $\theta = (\theta_1, \theta_2) \in \Theta$ be the realized

[3]Chakraborty and Harbaugh (2010) also claim the importance of the multidimensionality by showing the existence of informative equilibria with a state-independent preference sender.

[4]Austen-Smith (1993a), Battaglini (2004), and Levy and Razin (2007) study imperfectly informed experts models in multidimensional environments.

[5]Kawai (2011) and Zapechelnyuk (2011) are recent works in this research stream.

[6]As a matter of convention, we treat the experts as male and the decision-maker as female throughout this chapter.

value of the state, which is known to both experts but unknown to the decision-maker. This is the experts' private information.[7] Note that the state space is unbounded. Let $F(\cdot)$ be a differentiable prior probability distribution function on Θ with density $f(\cdot)$ such that $f(\theta) > 0, \forall\theta \in \Theta$. Let $S_i \equiv \Theta$ be expert i's message space, for $i = 1, 2$. Note that each expert uses direct messages and the message sent by expert i is denoted by $s_i = (s_1^i, s_2^i) \in S_i$.

Let $Y \equiv \mathbb{R}^2$ be the decision-maker's action set and let $y = (y_1, y_2)$ be the action chosen by the decision-maker. In this model, all players' preferences are different. We describe these differences by parameters $x_0, x_1,$ and x_2; let $x_i = (x_1^i, x_2^i) \in \mathbb{R}^2$ be the expert i's *preference bias*, and the decision-maker's preference bias, x_0, is normalized to be $(0, 0)$. Thus, x_i is a measure of how expert i's preference is biased compared with that of the decision-maker. We assume that $x_1 \neq x_2$ and $x_1, x_2 \neq (0, 0)$.

The decision-maker and the experts have von Neumann Morgenstern utility functions, $U^D : Y \times \Theta \to \mathbb{R}$, $U^{E_i} : Y \times \Theta \times \mathbb{R}^2 \to \mathbb{R}$, respectively, defined as follows:[8]

$$U^D(y, \theta) = -\sum_{j=1}^{2}(y_j - \theta_j)^2, \tag{2.1}$$

$$U^{E_i}(y, \theta, x_i) = -\sum_{j=1}^{2}(y_j - (\theta_j + x_j^i))^2. \tag{2.2}$$

From (2.1) and (2.2), when the value of θ is given, the decision-maker's and the experts' *ideal points* that represent the most preferable actions for each player are $\theta, \theta + x_1$ and $\theta + x_2$, respectively. For simplicity, we denote $\theta + x_i$ by O_i. We assume that all information except θ is common knowledge.

[7] We assume that both experts are perfectly informed players.

[8] We restrict our attention to the loss-quadratic utility case. This is the usual assumption in the literature on cheap talk games; see Crawford and Sobel (1982), Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b) and Battaglini (2002).

It is worth pointing out that the experts' messages do not directly affect all players' payoffs. Thus, this is a cheap talk game. In addition, because we focus on the direct message game with loss-quadratic utility functions, the experts' messages are interpreted as recommendations of the action that the decision-maker should choose.

The timing of the game is as follows. First, nature chooses state θ according to the distribution function $F(\cdot)$, and then both experts observe this value correctly. Second, expert 1 sends a message s_1, which is dependent on θ. Third, expert 2 sends a message s_2, which is dependent on both θ and s_1. Finally, the decision-maker chooses an action y after observing both messages s_1 and s_2.

Expert 1's pure strategy, $\mu_1 : \Theta \to S_1$, specifies a message s_1 that is sent in the state θ. Expert 2's pure strategy, $\mu_2 : \Theta \times S_1 \to S_2$, specifies a message s_2 that he sends in the state θ after observing expert 1's message s_1. The decision-maker's pure strategy, $y : S_1 \times S_2 \to Y$, specifies an action y that is chosen after observing both messages s_1 and s_2. Then, the decision-maker's posterior belief is denoted by $\mathcal{P} : S_1 \times S_2 \to \Delta(\Theta)$. This is a function from a pair of messages to a probability distribution on Θ.

The solution concept is the *perfect Bayesian equilibrium*(hereafter PBE) and we focus on pure strategy equilibria.

Definition 2.1 *A PBE is a strategy profile* $(\mu_1^*(\theta), \mu_2^*(\theta, s_1), y^*(s_1, s_2))$ *and posterior belief* $\mathcal{P}^*(\cdot|s_1, s_2)$*, such that:*

(1) $\forall\theta,\ \mu_1^*(\theta) = \arg\max_{s_1 \in S_1} U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1),$

(2) $\forall\theta, \forall s_1,\ \mu_2^*(\theta, s_1) = \arg\max_{s_2 \in S_2} U^{E_2}(y^*(s_1, s_2), \theta, x_2),$

(3) $\forall s_1,\ \forall s_2,\ y^*(s_1, s_2) = \arg\max_{y' \in Y} \mathbb{E}_{\mathcal{P}^*(\theta|s_1,s_2)}[U^D(y', \theta)],$

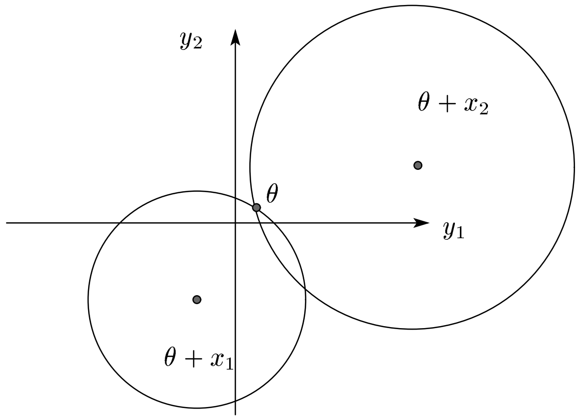


Figure 2.1: Both experts' indifference curves through $y = \theta$

(4) *$\mathcal{P}^*(\theta|s_1, s_2)$ is derived using $\mu_1^*(\theta)$ and $\mu_2^*(\theta, \mu_1^*(\theta))$ by Bayes's rule whenever it is possible. Otherwise, $\mathcal{P}^*(\cdot|s_1, s_2)$ is any probability distribution on Θ.*

Because we consider the loss-quadratic utility functions as defined in (2.1) and (2.2), expert i's indifference curve is a circle, the center of which is expert i's ideal point, O_i, and the radius of this circle is the norm of the preference bias, $||x_i||$, where $||\cdot||$ is the Euclidian norm. We use $I_i(\theta)$ to denote expert i's indifference curve through action $y = \theta$, $R_i(\theta)$ to denote the upper contour set of $I_i(\theta)$, and $P_i(\theta)$ to denote the strict upper contour set of $I_i(\theta)$. In other words:

$$I_i(\theta) \equiv \{y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) = U^{E_i}(\theta, \theta, x_i)\}, \tag{2.3}$$

$$R_i(\theta) \equiv \{y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) \geq U^{E_i}(\theta, \theta, x_i)\}, \tag{2.4}$$

$$P_i(\theta) \equiv \{y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) > U^{E_i}(\theta, \theta, x_i)\}. \tag{2.5}$$

By the definition of $I_i(\theta)$, $I_1(\theta)$ and $I_2(\theta)$ intersect at least at $y = \theta$, as in Figure 2.1.

The two-expert situations are divided into the following two cases: *like biases* and *opposing biases.*

Definition 2.2 *The experts have like biases if* $x_1 \cdot x_2 > 0$*. Otherwise, they have opposing biases; that is,* $x_1 \cdot x_2 \leq 0$*.*

In like-biases cases, the correlation coefficient of the vectors x_1 and x_2 are positive, so we can interpret this to mean that the experts' preferences are biased in similar directions. Geometrically, it is equivalent to $0° \leq \gamma < 90°$, where γ is the interior angle of x_1 and x_2. On the other hand, the correlation coefficient is nonpositive in opposing-biases cases, so we can say that the experts' preferences are biased in dissimilar directions. Geometrically, this is equivalent to $90° \leq \gamma \leq 180°$.

In the following sections of Chapter 2, we are particularly interested in an equilibrium in which the private information is perfectly transmitted. Then, we define this term.

Definition 2.3 *A fully revealing equilibrium is a PBE where, in each state, the experts' private information is perfectly transmitted. In other words, a fully revealing equilibrium satisfies the condition that for each state* θ*,* $\mathcal{P}^*(\theta|\mu_1^*(\theta), \mu_2(\theta, \mu_1^*(\theta))) = 1$*.*

2.3 Limitations of Self-serving Belief

In this section, we review the *self-serving belief* defined by Krishna and Morgan (2001b), and show that a straightforward application of the belief into the two-dimensional model does not support fully revealing equilibria.

2.3.1 One-dimensional unbounded state space model

We briefly review the one-dimensional unbounded state space model of Krishna and Morgan (2001b) in this subsection. Hence, we suppose that $\Theta, Y \equiv \mathbb{R}$ and $x_i \in \mathbb{R}$ for $i = 1, 2$ throughout this subsection. In the one-dimensional model, the opposing-biases cases are defined, without loss of generality, by the cases such that $x_1 < 0$ and $x_2 > 0$. In addition, we define the decision-maker and the experts' one-dimensional loss-quadratic utility functions, $u^D : Y \times \Theta \to \mathbb{R}$, $u^{E_i} : Y \times \Theta \times \mathbb{R} \to \mathbb{R}$, respectively, by $u^D(y, \theta) = -(y - \theta)^2$ and $u^{E_i}(y, \theta, x_i) = -(y - (\theta + x_i))^2$. The self-serving belief is defined as follows.

Definition 2.4 ***Self-serving belief (Krishna and Morgan (2001b))***

(1) *A message s_2 from expert 2 is self-serving if the adoption of the recommendation by expert 2 is strictly better for expert 2 than the adoption of the recommendation by expert 1, given that expert 1 sends the true messages. In other words,*

$$s_2 \text{ is self-serving if } u^{E_2}(s_2, s_1, x_2) > u^{E_2}(s_1, s_1, x_2). \tag{2.6}$$

(2) *The decision-maker has the self-serving belief if the posterior belief $\mathcal{P}(\cdot|s_1, s_2)$ satisfies the following conditions; for any $s_1 \in S_1$ and $s_2 \in S_2$:*

$$s_2 \text{ is self-serving} \quad \Rightarrow \quad \mathcal{P}(s_1|s_1, s_2) = 1, \tag{2.7}$$

$$s_2 \text{ is not self-serving} \quad \Rightarrow \quad \mathcal{P}(s_2|s_1, s_2) = 1. \tag{2.8}$$

That is, under the self-serving belief, the decision-maker believes expert 1's message for certain if expert 2's message is self-serving. Otherwise, she believes expert 2's message for certain. This belief system works when the experts have opposing biases.

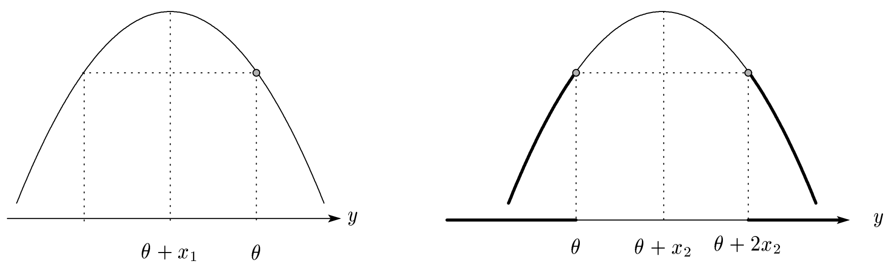


Figure 2.2: Expert 1 sends the true message.

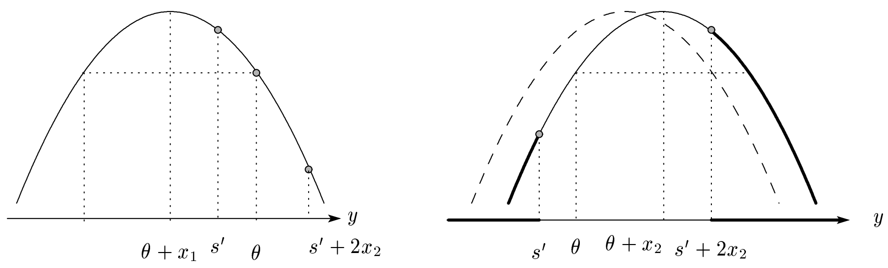


Figure 2.3: Expert 1 sends a false message $s'(< \theta)$.

Proposition 2.1 ***(Krishna and Morgan (2001b) footnote 9.)***

Consider the one-dimensional unbounded state space model. Suppose that the experts have opposing biases. Then, there exists a fully revealing equilibrium supported by the self-serving equilibrium.

Proof. See Krishna and Morgan (2001b). ∎

The intuition behind the result is described in Figure 2.2 and 2.3. Consider an opposing-biases case and suppose that expert 1 sends the true message. Given the message, expert 2 cannot improve his utility by lying because such messages are always self-serving; that is, the decision-maker never believes them. The bold region of Figure 2.2 is the set of actions that expert 2 can induce. Thus, by sending the true message, expert 1 can induce the first-best action $y = \theta$.

Next, suppose that expert 1 sends a false message, s', which is smaller than, but not far from, θ as described in Figure 2.3. Given the message s', expert 2 can always send a credible message, $s_2 = s' + 2x_2$. This induces the action $y = s' + 2x_2$, and it is better for expert 2 than $y = \theta$. Because both experts have opposing-biased preferences, $y = s' + 2x_2$ is worse for expert 1 than the

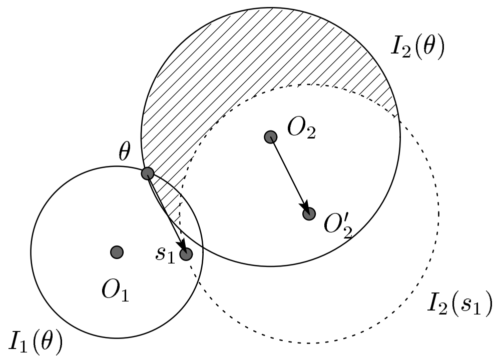


Figure 2.4: The self-serving belief.

first-best action, $y = \theta$. Similarly, if expert 1 sends a false message s'', which is larger than, but not far from, θ, then expert 2 agrees with the message and it is induced. However, it is worse for expert 1 than the first-best action. Thus, expert 1 has no incentive to lie. That is, the decision-maker can make each expert check whether the other expert's message is true, given the self-serving belief. On the equilibrium path, the self-serving belief is consistent with Bayes' rule. Therefore, in the one-dimensional opposing-biases case, the self-serving belief supports a fully revealing equilibrium.

2.3.2 Two-dimensional unbounded state space model

Now, we move back to the two-dimensional model defined in Section 2.2, and apply the self-serving belief into the two-dimensional opposing-biases cases, as shown in Figure 2.4. $I_i(s_1)$ represents expert i's indifference curve that the decision-maker faces when she believes that expert 1's message s_1 is true, and $P_i(s_1)$ and $R_i(s_1)$ are the strict and the weak upper contour sets of $I_i(s_1)$, respectively. We denote $s_1 + x_i$ by O_i'. By using the notation, the self-serving belief is described as follows; for

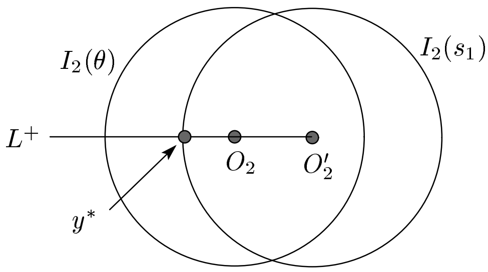


Figure 2.5: Lemma 2.1

any $s_1 \in S_1$ and $s_2 \in S_2$:

$$s_2 \in P_2(s_1) \Rightarrow \mathcal{P}(s_1|s_1, s_2) = 1, \tag{2.9}$$

$$s_2 \notin P_2(s_1) \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1. \tag{2.10}$$

Under the self-serving belief, if expert 1 sends $s_1 \neq \theta$, then, by inducing the action $y \in P_2(\theta)\backslash P_2(s_1)$, expert 2 will be better off than if $y = \theta$ is realized. The shaded region in Figure 2.4 is the set of such actions. If $O_2 \notin P_2(s_1)$, then expert 2's best response is trivial; he sends $s_2 = O_2$. If $O_2 \in P_2(s_1)$, then expert 2 cannot induce his ideal point O_2 under the self-serving belief. The next lemma characterizes the most preferred action that expert 2 can induce when $O_2 \in P_2(s_1)$. By this lemma, we can insist that expert 2's best response exists on the half-line through O_2' and O_2, the initial point of which is O_2'.

Lemma 2.1 *Fix s_1 such that $O_2 \in P_2(s_1)$, and let y^* be the closest point on $I_2(s_1)$ to expert 2's ideal point O_2. Then, y^* is the intersection of $I_2(s_1)$ and the half-line through O_2' and O_2, the initial point of which is O_2'.*

(Note that all proofs are in the Appendix 2-A.) Consider opposing-biases cases with $90° \leq \gamma < 180°$. In these cases, $P_1(\theta) \cap P_2(\theta)$, a set of actions that both experts strictly prefer to action $y = \theta$,

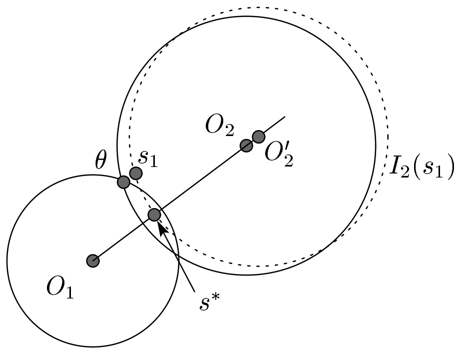


Figure 2.6: Expert 1's profitable deviation

is nonempty. Hence, the experts can induce an action in $P_1(\theta) \cap P_2(\theta)$ by deviating from truth-telling because the self-serving belief is fragile in the face of such compromised deviations, which is explained as follows. Suppose that expert 1 sends message s_1 such that $\overrightarrow{\theta s_1}$ is parallel to the line through O_1 and O_2, and $||\theta - s_1|| = \epsilon > 0$ where ϵ is small enough, as shown in Figure 2.6. Then, O_2' exists on the line through O_1 and O_2. By Lemma 2.1, expert 2's best response is sending $s_2 = s^*$, the intersection of $I_2(s_1)$ and the half-line $O_2'O_2$, starting at O_2'. Under the self-serving belief, it is not self-serving, so action $y = s^*$ is induced. However, because the deviation is so small, action $y = s^*$ exists in $P_1(\theta) \cap P_2(\theta)$ as shown in Figure 2.6. Because the experts strictly prefer the induced action to $y = \theta$, expert 1 has an incentive to deviate from truth-telling and expert 2 endorses expert 1's deviation. Therefore, we can say that, in the two-dimensional model, the self-serving belief cannot support fully revealing equilibria. The next proposition shows that the self-serving belief can support fully revealing equilibria only when the experts have perfectly opposing biases.

Proposition 2.2 *Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium supported by the self-serving belief if and only if $P_1(\theta) \cap P_2(\theta) = \emptyset$.*

Geometrically, the necessary and sufficient condition is that the experts' indifference curves $I_1(\theta)$

and $I_2(\theta)$ circumscribe at $y = \theta$; that is, $\gamma = 180°$, the perfectly opposing-biases case. In the two-dimensional model, there are other "intermediate" opposing-biases cases such that $90° \leq \gamma < 180°$. The self-serving belief is weak for these intermediate cases. However, because of the structure of the models, we only focus on the perfectly like-biases and the perfectly opposing-biases cases in one-dimensional models. In other words, the intermediate cases are ignored, so the self-serving belief system is enough to support fully revealing equilibria in one-dimensional models. Therefore, we can conclude that the positive result in the one-dimensional model crucially depends on the one-dimensional structure.

This raises a new question as to whether fully revealing equilibria exist when $P_1(\theta) \cap P_2(\theta) \neq \emptyset$. We find the following positive result in the literature.

Proposition 2.3 ***(Battaglini (2002) p.1395)***

Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium supported by Battaglini's belief system if and only if $x_1 \cdot x_2 = 0$.

Proof. See Battaglini (2002). ■

The case of $x_1 \cdot x_2 = 0$ is equivalent to $\gamma = 90°$ and it is an opposing-biases case. However, this belief system is also fragile in the face of the other intermediate cases.

In summary so far, we have already known that there exists a fully revealing equilibrium if the experts have either (i) perfectly opposing biases, i.e., $\gamma = 180°$, or (ii) orthogonal biases, i.e., $\gamma = 90°$. On the other hand, the literature does not answer the question of whether there exists a fully revealing equilibrium when the experts have (i) intermediate opposing biases, i.e., $90° < \gamma < 180°$ or (ii) like biases, i.e., $0° \leq \gamma < 90°$.

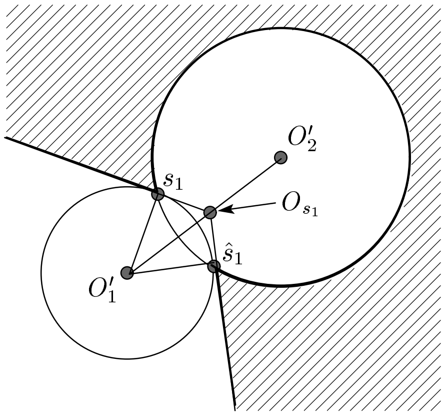


Figure 2.7: Extended self-serving belief

2.4 Extended Self-serving Belief and Fully Revealing Equilibria

This section studies the open questions discussed in the last paragraph. First, we suggest a new belief system, *extended self-serving belief*, and show that there exists a fully revealing equilibrium supported by the new belief system if the experts have opposing biases. Second, we show that there exist no fully revealing equilibria in the like-biases cases. That is, the existence of opposing-biased preferences is the necessary and sufficient condition for full information transmission.

Let us introduce some notation. Given expert 1's message s_1, let $\hat{s}_1$ be the other intersection of $I_1(s_1)$ and $I_2(s_1)$. Consider two tangents of $I_1(s_1)$ at s_1 and $\hat{s}_1$, and let O_{s_1} be the intersection of the tangents. Let $\mathcal{T}(s_1)$ be the interior of the cone, the vertex of which is O_{s_1} and the sides are the tangents of $I_1(s_1)$ at s_1 and $\hat{s}_1$. The extended self-serving belief is defined as follows.

Definition 2.5 ***Extended self-serving belief.***[9]

The decision-maker has the extended self-serving belief if the posterior belief $\mathcal{P}(\cdot|s_1, s_2)$ satisfies the

[9] I am very grateful to Nozomu Muto for suggesting this criterion. Originally, I used a stronger criterion in the sense that more conditions are needed to construct a fully revealing equilibrium.

following conditions; for any $s_1 \in S_1$ *and* $s_2 \in S_2$*:*

$$s_2 \in P_2(s_1) \cup \mathcal{T}(s_1) \Rightarrow \mathcal{P}(s_1|s_1, s_2) = 1, \tag{2.11}$$

$$s_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1. \tag{2.12}$$

Expert 2's messages in the shaded region or on the bold line in Figure 2.7 are credible under the extended self-serving belief. It restricts the set of credible messages for expert 2 more than the original. Under the extended self-serving belief, the decision-maker believes expert 1's message for certain if expert 2's message is $s_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$. Otherwise, she believes expert 2's message for certain.

The extended self-serving belief is interpreted as follows.[10] We define $s_1^*(s_1, s_2) \equiv \arg\min_{s \in \{s_1, \hat{s}_1\}} ||s_2 - s||$, given s_1 and s_2. We assume that there is a decision-maker who believes one or the other of experts' messages for certain. First, the decision-maker believes s_2 if $s_1 = s_2$, that is, expert 2 endorses expert 1. In addition, the decision-maker believes $s_2 = \hat{s}_1$ for certain; given the first point, expert 2 has no incentive to send $s_2 = \hat{s_1}$ if $s_1 = \theta$, i.e., expert 1 tells the truth. Because expert 2 is indifferent between $y = s_1$ and $y = \hat{s}_1$ as long as $s_1 = \theta$, the decision-maker knows that expert 2 has no incentive to send $s_2 = \hat{s}$ if expert 1 tell the truth. Hence, the decision-maker can infer that expert 1 misreports when she observes $s_2 = \hat{s}_1$, and never believes such messages by expert 1. Then, the decision-maker believes $s_2 \neq s_1, \hat{s}_1$ if and only if (i) it is not self-serving and (ii) the direction that expert 2 would like the decision-maker to move from s_1 or $\hat{s}_1$ never benefits expert 1. In other words, expert 2 must show that (i) his message is not self-serving as in the original, and (ii) the direction of a deviation from s_1^*, $\overrightarrow{s_1^* s_2}$, is "not like" the direction in which expert 1 would

[10]I really appreciate the advice of an anonymous referee who suggested this interpretation.

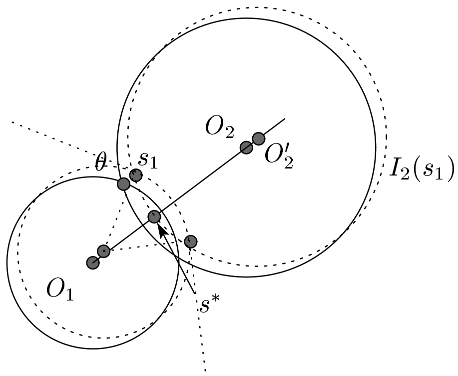


Figure 2.8: Preventing the deviation

like the decision-maker to move from s_1^*, $\overrightarrow{s_1^* O_1'}$, i.e., the interior angle of $\overrightarrow{s_1^* s_2}$ between $\overrightarrow{s_1^* O_1'}$ must be obtuse. The latter means that the vector $\overrightarrow{s_1^* s_2}$ never passes through $P_1(s_1)$; the complement of $\mathcal{T}(s_1)$ is the set of such messages by expert 2. Therefore, the decision-maker believes s_2 if and only if $s_2 \notin P(s_1) \cup \mathcal{T}(s_1)$.

We move back to the problematic deviation for the self-serving belief mentioned in the last section, and demonstrate how the extended self-serving belief prevents it. Under the self-serving belief, expert 1 can induce action $y = s^*$ in Figure 2.8 because, given s_1, expert 2's best response, $s_2 = s^*$, is credible for the decision-maker. However, under the extended self-serving belief, s_2 is not credible. Given s_1, expert 2's best response is either $s_2 = s_1$ or $s_2 = \hat{s}_1$. The decision-maker adopts expert 2's message, but the induced action $y = s_1$ or $\hat{s}_1$ is worse than $y = \theta$ for expert 1. Therefore, the extended self-serving belief can prevent the deviation.

We can show that, as long as the experts have opposing biases, any deviation from the truth never improves expert 1's payoff under the extended self-serving belief, so he has no incentive to deviate from $s_1 = \theta$.[11] Expert 2's best response given $s_1 = \theta$ is the endorsement of it, $s_2 = \theta$. Hence, at any state, the experts' private information is completely transmitted to the decision-maker. That

[11] You can find the formal proof in the Appendix 2-A.

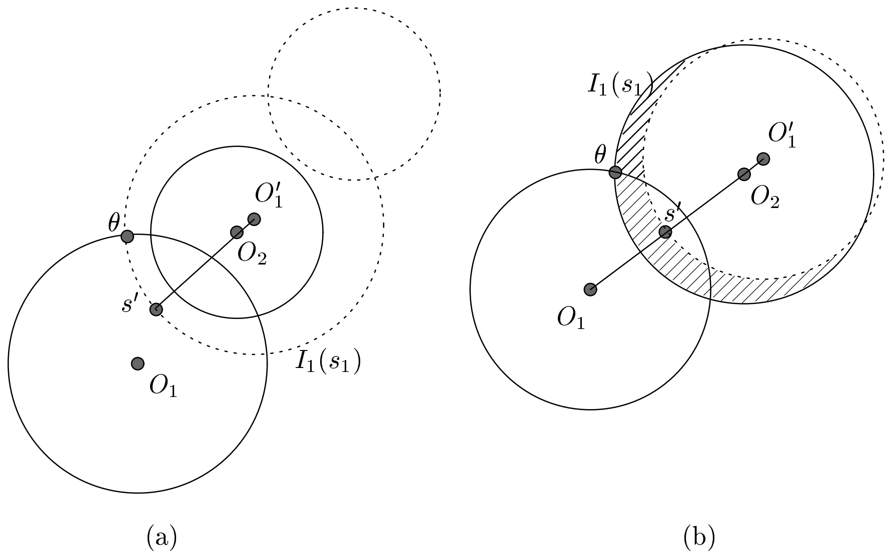


Figure 2.9: Examples of successful deviations

is, we have a positive answer to the first question proposed in the last section; there also exists a fully revealing equilibrium in the intermediate opposing-biases cases, i.e., $90° < \gamma < 180°$.

It is worthwhile to mention why it is insufficient to exclude only $P_1(s_1) \cup P_2(s_1)$. It may seem that when the experts have opposing biases, excluding $P_1(s_1)$ is sufficient to prevent deviations that induce better actions for expert 1. However, this is not correct; unboundedness of the excluded region is necessary. Consider the following belief system; the decision-maker believes a message s_2 if and only if $s_2 \notin P_1(s_1) \cup P_2(s_1)$. This belief system does not always support a fully revealing equilibrium even if the experts have opposing biases. If $||x_1|| > ||x_2||$, then expert 1 can deviate such that $P_1(s_1)$ includes $P_2(\theta)$, as shown in Figure 2.9-(a). We face the same problem that the original self-serving belief faces; given such s_1, expert 2's best response is $s_2 = s'$, which is on $I_1(s_1)$, and the decision-maker believes it. Therefore, it is a profitable deviation for expert 1. Moreover, if $||x_1|| \leq ||x_2||$ but $||x_2|| - ||x_1||$ is not large enough, then the same problem also occurs; expert 1 can induce action $y = s' \in P_1(\theta)$, as shown in Figure 2.9-(b). In addition to opposing biases, we need $||x_2 - x_1|| \geq 2||x_1||$ to support a fully revealing equilibrium under this belief system.[12] As long as the excluded region is bounded in the direction of O_1', we face the same problem, so we need additional conditions regarding the norms of the preference biases for supporting fully revealing equilibria. Thus, we have to exclude the unbounded region, like $\mathcal{T}(s_1)$, to avoid such constraints.

Next, we consider the second question; do fully revealing equilibria exist in the like-biases cases, i.e., $0° \leq \gamma < 90°$? Our answer is negative; there exists no fully revealing equilibrium in like-biases cases. The last part of this section demonstrates why full information transmission is impossible when the experts have like biases. Consider the line segment connecting θ with O_2, and call it *the line of endorsement at* θ.[13] As Lemma 2.1 shows, at any state θ, if expert 1 deviates to some $s_1 \neq \theta$ that lies on the line segment, then expert 2 always endorses the deviation; that is, $s_2 = s_1$ is the unique best responses of expert 2. In other words, by deviating in the direction of expert 2's preference bias, x_2, expert 1 can force expert 2 to endorse the deviation. The next lemma gives an equivalent condition for bias relations.

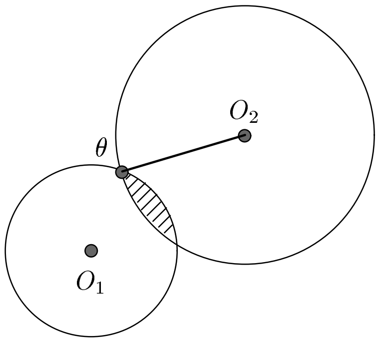
Lemma 2.2 *Consider the two-dimensional unbounded state space model. Then, the experts have opposing biases if and only if for any state θ, the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty.*[14]

The impossibility of full information transmission in like-biases cases comes from the nonemptiness of the intersection of $P_1(\theta)$ and the line of endorsement. That is, expert 1 is strictly better off by deviating to some point on the intersection of $P_1(\theta)$ and the line of endorsement, and no belief system ever prevents such deviations. Intuitively, consider a like-biases case in Figure 2.11. We

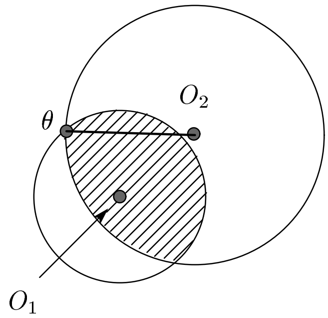
[12] The formal proof is available upon request.

[13] I thank the anonymous referee who suggested this notion.

[14] As long as we consider the loss-quadratic utility case, the same property holds for the line segment connecting O_2 and $\hat{\theta}$, which is the other intersection of $I_1(\theta)$ and $I_2(\theta)$.



(a) Opposing biases



(b) Like biases

Figure 2.10: Lemma 2.2

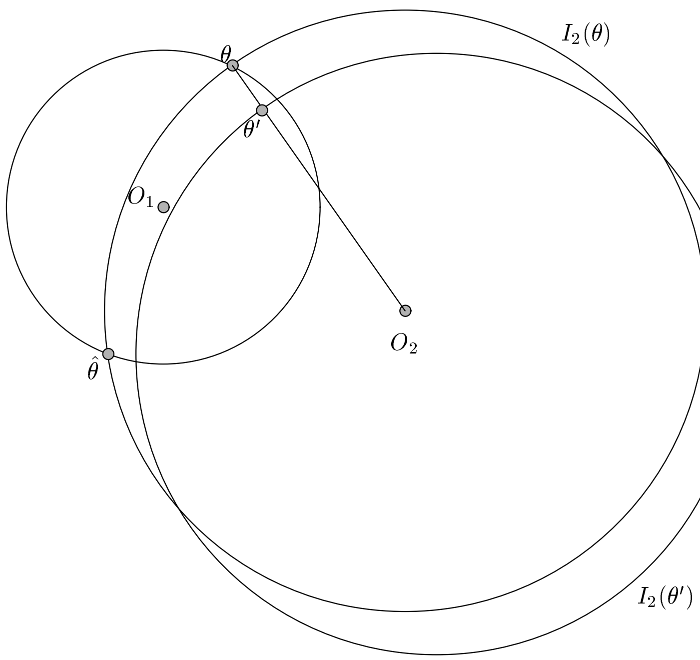


Figure 2.11: Impossibility in like-biases cases

suppose, in contrast, that there exists a fully revealing equilibrium in this case. If expert 1 pretends to be state θ' when the true state is θ, then endorsing this deviation is the unique best response of expert 2 because θ' lies in the line of endorsement at θ. Hence, action $y = \theta'$ is induced, and expert 1 strictly prefers it to $y = \theta$, which is a contradiction. Under any belief system, expert 2 never contests such deviations by expert 1, so we can conclude that there exists no fully revealing equilibrium in like-biases cases. Therefore, fully revealing equilibria in the two-dimensional unbounded state space model are characterized as follows.

Proposition 2.4 *Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium if and only if the experts have opposing biases.*

As shown in Lemma 2.2, the condition that the experts have opposing biases is equivalent to the condition that the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty, and this emptiness is crucial for the existence of fully revealing equilibria, as shown in Proposition 2.4. That is, properties of the intersection of $P_1(\theta)$ and the line of endorsement at θ provides an exact analogy with the definitions of Krishna and Morgan's (2001b) one-dimensional opposing/like biases. Therefore, this point makes clear the connection between one-dimensional and two-dimensional models, even though these are often discussed as somehow fundamentally different models in the literature.

2.5 Discussion and Extensions

2.5.1 Collusion

Sequential communication can be regarded as a situation where experts in a committee can collude before advising a decision-maker. That is, expert 2 can choose whether to collude with expert 1 before sending his recommendation. Hence, the fully revealing equilibrium supported by the extended self-serving belief can be said to robust with respect to this kind of collusion.

Zapechelnyuk (2011) studies collusion among experts in a committee under the framework of cheap talk games; there are n experts who share the same multidimensional private information, and the experts engage in bargaining before giving recommendations to a decision-maker. Instead of specifying the bargaining procedure, Zapechelnyuk (2011) imposes axioms that any bargaining solution must satisfy. In his environment, the decision-maker can elicit full information if and only if the outcome induced by a fully revealing equilibrium is not Pareto dominated by the experts in the committee.

Our result is a complement of Zapechelnyuk (2011). As Proposition 2.4 has shown, in our environment, the decision-maker can obtain full information even if the outcome induced by a fully revealing equilibrium is Pareto dominated by the experts as long as they have opposing biases. The discrepancy of the results in the two papers comes from the differences in how the experts deviate. Zapechelnyuk (2011) mainly focuses on *opting-out* deviations. That is, if expert $i \in \{1, 2, ..., n\}$ deviates, then the other $n-1$ experts have chances to react this deviation. In our setup, expert 1 is forced to follow the opting-out deviation, but expert 2 is not; expert 1 has no chance to react after expert 2's deviation. By the terminology of Zapechelnyuk (2011), expert 2 is forced to follow *defection* in this chapter. In other words, Zapechelnyuk (2011) focuses on the scenario of full opting-out or defection deviations. On the other hand, our bargaining procedure is a mixture of them.[15]

2.5.2 $N(>2)$-dimensional models.

The results can be extended to $N(>2)$-dimensional models as long as loss-quadratic utility functions are assumed. We can easily define the N-dimensional extended self-serving belief in a similar way to the two-dimensional model. The construction of the fully revealing equilibrium depends on the distance between the experts' true indifference curves and the illusionary ones. The assumption of two-dimensionality is not essential for this construction. Thus, we can obtain the same results in N-dimensional unbounded state space models. Moreover, because the decision-maker can elicit true information by taking advantage of the conflict between two experts, this means that two experts are sufficient for the existence of a fully revealing equilibrium even in N-dimensional models.

2.5.3 $n(>2)$-expert models

We consider a situation where there are $n(>2)$ experts who share the same two-dimensional private information, and send messages sequentially. The sufficiency of Proposition 2.4 can be easily extended to the n-expert model. Like and opposing biases in the n-expert model are defined as follows. The experts are said to have *like biases* if $x_i \cdot x_j > 0$ for any experts i and j. Otherwise, the experts are said to have *opposing biases.* If the experts have opposing biases, then we can find a fully revealing equilibrium that is essentially equivalent to that in the two-expert model. That is, because there exists a pair of experts i and j such that $x_i \cdot x_j \leq 0$, the decision-maker can elicit full information by caring only about the messages from experts i and j under the extended self-serving

[15] I thank Andriy Zapechelnyuk for conversations about it.

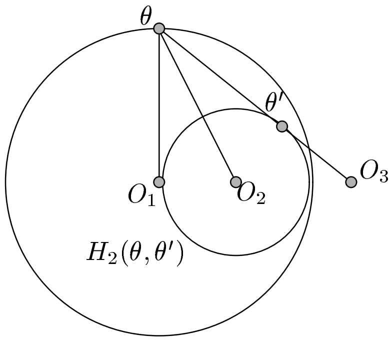


Figure 2.12: An example of Proposition 2.5

belief, and ignoring the other messages.

However, an extension of the necessary part of Proposition 2.4 is not straightforward. The difficulty is specifying how each expert reacts to the predecessors' behaviors without specifying the decision-maker's belief. This problem can be avoided in the two-expert model by considering a deviation along the line of endorsement; expert 2 endorses such a deviation whatever the decision-maker's belief is. However, in the n-expert model, because of the sequential rationality of the subsequent experts, finding a deviation that every expert endorses without specifying the decision-maker's belief is nontrivial, even if the experts have like biases. We have the following partial extension. Let $H_i(\theta, \theta') \equiv \{y \in Y | U^{E_i}(y, \theta, x_i) \geq U^{E_i}(\theta', \theta, x_i)\}$.

Proposition 2.5 *Consider the three-expert model, and suppose that for any $\theta \in \Theta$, there exists $\theta'(\neq \theta) \in \Theta$ such that (i) θ' lies on the line of endorsement of expert 3 at θ and (ii) $H_2(\theta, \theta') \subset P_1(\theta)$. Then, there exists no fully revealing equilibrium.*

Intuitively, the conditions in Proposition 2.5 mean that (i) each bias has the similar magnitude, and (ii) the interior angles of the biases are sufficiently small as shown in Figure 2.12. Hence, if the conditions are satisfied, then the experts have like biases.[16] Two open questions remain. First, we

[16]The formal proof is in the Appendix 2-B.

conjecture that a fully revealing equilibrium never exists in the general n-expert model under the similar conditions in Proposition 2.5. However, we need nontrivial modifications on the conditions and the proof. Second, even in the three-expert model, we do not have the answer when the experts are like biases but the conditions in Proposition 2.5 are not satisfied.

2.5.4 Mixed strategies

The necessary condition for the existence of fully revealing equilibrium does not change even if we adopt mixed strategies. Ambrus and Takahashi (2008) show that in simultaneous communication, allowing mixed strategies by the experts could generate a fully revealing equilibrium when there is no fully revealing equilibrium in pure strategies. That is, stochastic outcomes generated by mixed strategies prevent the experts from deviations. However, this logic does not hold in sequential communication because the decision problem that expert 2 faces is equivalent to that in pure strategies even if mixed strategies are allowed. In other words, expert 2's best response could depend on each realized message in sequential communication even if expert 1 undertakes mixed strategies. Because expert 2 is not forced to face stochastic outcomes in his decision making, the same logic used in pure strategies can be applied for showing the necessary condition. There exists no fully revealing equilibrium as long as the experts have like biases, even if the players are allowed to undertake mixed strategies.[17]

2.5.5 Noisy information

The fully revealing equilibrium supported by the extended self-serving belief is robust to the following type of noise. Suppose that expert 1 always observes the true state, but expert 2, instead of observing

[17] The formal statement is in the Appendix 2-B.

the true state for certain, privately observes a noisy signal $\sigma \in \Sigma \equiv \{\theta, \phi\}$; with probability $1-\epsilon$, expert 2 observes the correct state (i.e., $\sigma = \theta$), and with probability $\epsilon > 0$, he observes nothing (i.e., $\sigma = \phi$).

Proposition 2.6 *Consider the two-dimensional unbounded state space model with noisy observations. Then, there exists a fully revealing equilibrium if $x_1 \cdot x_2 > 0$ and ϵ is sufficiently small.*

Proposition 2.6 means that the fully revealing equilibrium supported by the extended self-serving belief is robust to noise such that the second sender could not be an "expert" with small probability. However, this fully revealing equilibrium is sensitive to other types of noise. For example, suppose that the observation of expert i is given by $\sigma_i \equiv \theta + \epsilon_i$, where ϵ_i follows a normal distribution with zero mean, and ϵ_1 and ϵ_2 are independent.[18] In this environment, disagreements among reports could happen with positive probability even if the experts truthfully report their own observations. Therefore, because the extended self-serving belief is discontinuous in messages, this belief system does not work well under such an information structure.

2.5.6 General preferences

The necessary part of Proposition 2.4 can be somewhat extended to more general preferences under the following additional assumption. To hold the logic used in the proof of Proposition 2.4, the property of Lemma 2.1 is needed; that is, a closest point on $I_2(s_1)$ from O_2 is on the intersection of line O_2O_2' and $I_2(s_1)$ given $O_2 \in P_2(s_1)$. This property may not be true without the assumption of the loss-quadratic utility functions.

[18]This noisy information structure is studied by Battaglini (2004) and Ambrus and Lu (2010).

Proposition 2.7 *Consider a two-dimensional unbounded state space model with players' single-peaked preferences, and suppose that a closest point on $I_2(s_1)$ from O_2 is on the intersection of half-line O_2O_2' starting from O_2' and $I_2(s_1)$ as long as $O_2 \in P_2(s_1)$. Then, there exists a fully revealing equilibrium only if the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty for any state θ.*

Proof. It is straightforward from the proof of Proposition 2.4. ■

A recent work by Kawai (2011) shows that there exists a fully revealing equilibrium if the experts have opposing biases in other than the loss-quadratic environment. Kawai (2011) assumes that the utility functions satisfy (i) quasi-concavity, (ii) single-peakedness, (iii) $U^{E_i}(y, \theta, x_i) = U^{E_i}(y + a, \theta + a, x_i)$ for any $y \in Y$, any $\theta \in \Theta$ and any $a \in \mathbb{R}^n$, and (iv) the hyperplanes of $I_2(\theta)$ at θ and $\bar{\theta}$ are parallel where $\bar{\theta}$ is the other intersection of $I_2(\theta)$ and the line through θ and O_2. Under this environment, Kawai (2011) shows that there exists a fully revealing equilibrium supported by other belief system than the extended self-serving belief. This is a generalization of the necessary part of Proposition 2.4.

2.5.7 Bounded state space

The unboundedness of the state space is also a crucial assumption for our results. If we consider a bounded type space, the extended self-serving belief may not imply a fully revealing equilibrium. This is consistent with Krishna and Morgan (2001b) and Ambrus and Takahashi (2008).

2.6 Conclusion

In this chapter, we have studied a sequential cheap talk game with two-dimensional unbounded state space. We have two main findings; first, the self-serving belief suggested by Krishna and Morgan (2001b) generally does not support fully revealing equilibria in the two-dimensional environment; it works if and only if the experts have perfectly opposing biases. In the two-dimensional environment, there exist outcomes where both experts are strictly better off than the first-best outcome even if the experts' preferences are biased in "not like" directions. The self-serving belief is fragile in the face of such "intermediate" opposing-biases cases.

Second, we characterize the necessary and sufficient condition for the existence of fully revealing equilibria in the two-dimensional environment, which is that the experts have opposing biases. As the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty in opposing-biases cases, an appropriate belief system can support fully revealing equilibria. We suggest the extended self-serving belief, under which the decision-maker believes expert 2's messages if and only if (i) it is not self-serving, and (ii) it "contests" expert 1's message, that is, the direction in which expert 2 recommends the decision-maker to move never benefits expert 1. On the other hand, if the experts have like biases, then the intersection of $P_1(\theta)$ and the line of endorsement at θ is not empty. Expert 1's deviation to some point on the intersection makes expert 1 strictly better off, and expert 2 always endorses such deviations under any belief system. Therefore, full information transmission is impossible.

2.7 Appendix 2-A: Proofs

First, we define the following notation. For any $a, b \in Y$, let $L(a,b) \equiv \{y \in Y | \exists \alpha \in \mathbb{R} \text{ s.t } \overrightarrow{ay} = \alpha\overrightarrow{ab}\}$, $L^+(a,b) \equiv \{y \in Y | \exists \alpha > 0 \text{ s.t } \overrightarrow{ay} = \alpha\overrightarrow{ab}\}$, and $\bar{L}(a,b) \equiv \{y \in Y | \exists \alpha \in [0,1] \text{ s.t } \overrightarrow{ay} = \alpha\overrightarrow{ab}\}$. Geometrically, $L(a,b)$ represents the line ab, $L^+(a,b)$ represents the half-line ab, the initial point of which is a, and $\bar{L}(a,b)$ represents the segment ab. Let $\hat{\theta}$ be the other intersection of $I_1(\theta)$ and $I_2(\theta)$.

Proof of Lemma 2.1

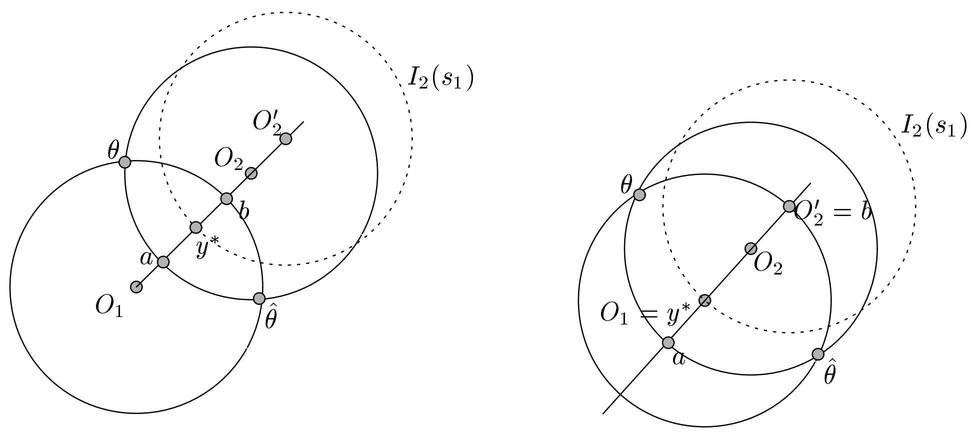
Without loss of generality, assume that $O_2 = (0,0)$ and $O_2' = (a,0)$, where $0 < a < ||x_2||$. Take an arbitrary point $(b,c) \in I_2(s_1)$. Then:

$$(b-a)^2 + c^2 = ||x_2||^2 \iff b^2 + c^2 = ||x_2||^2 - a^2 + 2ab. \tag{2.13}$$

If we let $f \equiv b^2 + c^2$, then f is the square of the distance from O_2 to the point (b,c). Thus, $f = ||x_2||^2 - a^2 + 2ab$, and f is minimized when b is minimized. By construction, $b = a - ||x_2||$. Because $\overrightarrow{O_2'O_2} = (-a, 0)$ and $\overrightarrow{O_2'y^*} = (-||x_2||, 0)$, $\overrightarrow{O_2'y^*} = \frac{||x_2||}{a}\overrightarrow{O_2'O_2}$. Then, $y^* \in L^+(O_2', O_2)$. Therefore, $y^* \in L^+(O_2', O_2) \cap I_2(s_1)$. That is, the closest point y^* is the intersection of $I_2(s_1)$ and the half-line through O_2' and O_2, the initial point of which is O_2'. ■

Proof of Proposition 2.2

(Sufficiency) Suppose that $P_1(\theta) \cap P_2(\theta) = \emptyset$. In other words, $R_1(\theta) \cap R_2(\theta) = \{\theta\}$ and neither $R_1(\theta) \subset R_2(\theta)$ nor $R_2(\theta) \subset R_1(\theta)$. If $s_1 = \theta$, then, from the self-serving belief system, expert 2's best response is $s_2 = \theta$. If $s_1 \neq \theta$, then $P_2(\theta) \backslash P_2(s_1) \neq \emptyset$. Because $P_1(\theta) \cap P_2(\theta) = \emptyset$ from



(i) $O_1 \notin P_2(\theta)$ (ii) $O_1 \in P_2(\theta)$

Figure 2.13: Proposition 2.2

the hypothesis, $y \in P_2(\theta)$ and $y \notin P_1(\theta)$ for all $y \in P_2(\theta)\backslash P_2(s_1)$. Hence, for such action y, $U^{E_1}(\theta, \theta, x_1) \geq U^{E1}(y, \theta, x_1)$. From the self-serving belief system, if $s_1 \neq \theta$, then expert 2 induces the action $y \in P_2(\theta)\backslash P_2(s_1)$. Because expert 1 cannot strictly improve his utility by sending false messages, he has no incentive to lie. Therefore, on the equilibrium path, both experts send messages involving the truth, and the self-serving belief is consistent with Bayes' rule. This is a fully revealing equilibrium.

(Necessity) By definition, $I_1(\theta) \cap I_2(\theta) \neq \emptyset$. Then, there are the following three cases: (i) $I_1(\theta) \cap I_2(\theta) = \{\theta, \hat{\theta}\}$, where $\hat{\theta} \neq \theta$, (ii) $I_1(\theta) \cap I_2(\theta) = \{\theta\}$ and either $R_1(\theta) \subset R_2(\theta)$ or $R_2(\theta) \subset R_1(\theta)$, and (iii) $I_1(\theta) \cap I_2(\theta) = \{\theta\}$ and neither $R_1(\theta) \subset R_2(\theta)$ nor $R_2(\theta) \subset R_1(\theta)$.

(i) case: Let $A \equiv \{y \in Y | L(O_1, O_2) \cap R_1(\theta) \cap R_2(\theta)\}$ and $D \equiv$ diam A. Because the set A is compact, $\exists a, b \in A$ such that $||a - b|| = D$.

Case 1: $O_1 \notin P_2(\theta)$**.** Suppose that expert 1 sends the message $s_1 \neq \theta$ such that $\overrightarrow{\theta s_1} = \epsilon \overrightarrow{O_1 O_2}$, where ϵ is such that $0 < ||\epsilon \overrightarrow{O_1 O_2}|| < D$ and $O_2 \in P_2(s_1)$. Then, $\overrightarrow{O_2 O_2'} = \epsilon \overrightarrow{O_1 O_2}$. By Lemma 2.1, $y^* \in L(O_1, O_2)$. Without loss of generality, assume that $a = y^* - \epsilon \overrightarrow{O_1 O_2}$. Suppose, by way of contradiction, that $y^* \notin P_1(\theta)$. That is $||O_1 - y^*|| \geq ||x_1||$. Because $O_1 \notin P_2(\theta)$, $||x_1|| = ||O_1 - a|| + D$. Hence;

$$\begin{aligned} ||x_1|| &\leq ||O_1 - y^*|| \leq ||O_1 - a|| + ||a - y^*|| = ||O_1 - a|| + ||\epsilon\overrightarrow{O_1O_2}|| \\ &< ||O_1 - a|| + D = ||x_1||, \text{a contradiction.} \end{aligned} \tag{2.14}$$

Then, $y^* \in P_1(\theta)$ must hold. That is, expert 1 has an incentive to lie. Therefore, the self-serving belief does not support fully revealing equilibria in this case.

Case 2: $O_1 \in P_2(\theta)$. Without loss of generality, assume that $||O_1 - a|| \leq ||O_1 - b||$. Consider the following message s_1 such that $\overrightarrow{\theta s_1} = \beta\overrightarrow{O_1O_2}$, where $\beta = ||O_1 - a||$. From Lemma 2.1, $y^* \in L(O_1, O_2)$. Note that this is the most preferred action for expert 2 that he can induce. By construction, $y^* = a - \overrightarrow{O_1a} = O_1$. That is, expert 1 can induce the most preferred action by sending this message. Then, under the self-serving belief, he has an incentive to lie.

(ii) case: By constructing the same deviation in case (i), expert 1 can become strictly better off. That is, the self-serving belief cannot support fully revealing equilibria.

Therefore, if there exists a fully revealing equilibrium supported by the self-serving belief, then this must be case (iii). In other words, it should be $P_1(\theta) \cap P_2(\theta) = \emptyset$. ■

Proof of Lemma 2.2

Note that $\bar{L}(\theta, O_2)$ represents the line of endorsement at θ.

(Necessity) Suppose, in contrast, that there exists $\theta \in \Theta$ such that $P_1(\theta) \cap \bar{L}(\theta, O_2) \neq \emptyset$ when the experts have opposing biases. That is, there exists $y \in P_1(\theta) \cap \bar{L}(\theta, O_2)$. As $y \in P_1(\theta)$, $x_1 \cdot \overrightarrow{\theta y} > 0$.

In addition, because $y \in \bar{L}(\theta, O_2)$ and the experts have opposing biases, there exists $\alpha \in (0,1]$ such that $\overrightarrow{\theta y} = \alpha x_2$. However, as the experts have opposing biases, $x_1 \cdot x_2 \leq 0$. That is, $x_1 \cdot (\frac{1}{\alpha}\overrightarrow{\theta y}) \leq 0$, or still $x_1 \cdot \overrightarrow{\theta y} \leq 0$, a contradiction. Therefore, for any state θ, $P_1(\theta) \cap \bar{L}(\theta, O_2) = \emptyset$.

(Sufficiency) Fix $\theta \in \Theta$ and $y \in \bar{L}(\theta, O_2)$ arbitrarily. As $P_1(\theta) \cap \bar{L}(\theta, O_2) = \emptyset$, $y \notin P_1(\theta)$. So, $x_1 \cdot \overrightarrow{\theta y} \leq 0$. In addition, because $y \in \bar{L}(\theta, O_2)$, there exists $\alpha \in [0,1]$ such that $\overrightarrow{\theta y} = \alpha x_2$. Hence, $x_1 \cdot (\alpha x_2) \leq 0$ implies $x_1 \cdot x_2 \leq 0$. That is, it is an opposing-biases case. ■

Proof of Proposition 2.4

(Sufficiency) We show the if the experts have opposing biases, then there exists a fully revealing equilibrium supported by the extended self-serving belief. The proof is constructive; first, we specify expert 2's best response for several s_1 under the extended self-serving belief. Then, we show that expert 1 has no incentive to lie given expert 2's best response. We define the following notation: let $l(s_1)$ and $l(\hat{s}_1)$ be the tangents of $I_1(s_1)$ at $y = s_1$ and $\hat{s}_1$, respectively.[19]

$$s_A \equiv \{y \in \mathbb{R}^2 \mid \min_{y \in I_2(s_1)} ||O_2 - y||\} \tag{2.15}$$

$$s_B \equiv \{y \in \mathbb{R}^2 \mid \min_{y \in l(s_1) \cup l(\hat{s}_1)} ||O_2 - y||\} \tag{2.16}$$

$$s_C \equiv \{y \in \mathbb{R}^2 \mid \min_{y \in I_2(s_1) \setminus \mathcal{T}(s_1)} ||O_2 - y||\} \tag{2.17}$$

Similarly to the original case, if $s_1 = \theta$, then expert 2 cannot improve his own payoff by lying. Then suppose that $s_1 \neq \theta$, and divide all cases into the following five cases: (i) $O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1)$, (ii) $O_2 \in P_2(s_1)$ and $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$, (iii) $O_2 \in P_2(s_1)$ and $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$, (iv) $O_2 \notin P_2(s_1)$ and $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, and (v) $O_2 \notin P_2(s_1)$ and $O_2 \in \mathcal{T}(s_1)$ and

[19] Geometrically, s_B is the foot of the perpendicular from O_2 to either $l(s_1)$ or $l(\hat{s}_1)$.

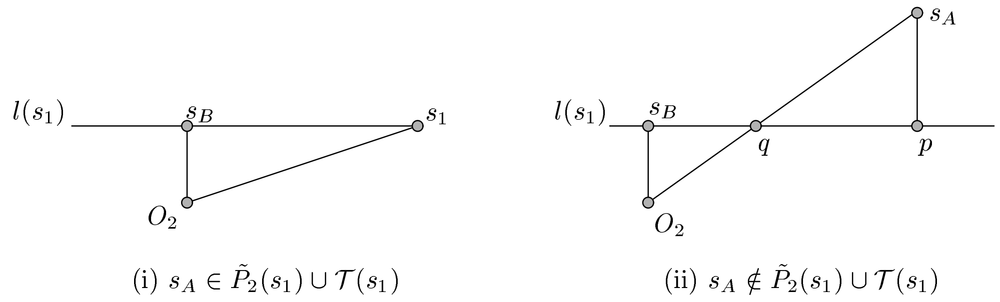


Figure 2.14: Lemma 2.3

$s_B \in P_2(s_1) \cup \mathcal{T}(s_1)$.[20]

In Case (i), expert 2's best response is $s_2 = O_2$ and action $y = O_2$ is induced because $O_2 \notin P_1(s_1) \cup \mathcal{T}(s_1)$. In Case (ii), because $O_2 \in P_2(s_1)$, expert 2 cannot induce action $y = O_2$, and the most preferable action that expert 2 can induce lies on $I_2(s_1)$. Given s_1, $s_2 = s_A$ is the best response, and action $y = s_A$ is induced because $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Similarly, as $O_2 \in P_2(s_1)$, the most preferred action for expert 2 that he can induce exists on $I_2(s_1)$ in Case (iii). If he sends $s_2 = s_A$, then action $y = s_1$ is induced because $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$. By construction, if expert 2 sends $s_2 = s_C$, then it is always believed and either action $y = s_1$ or $y = \hat{s}_1$ is induced. Hence, $s_2 = s_C$ is weakly better than $s_2 = s_A$; it is expert 2's best response.

In Cases (iv) and (v), expert 2's best responses are characterized by the following lemmas.

Lemma 2.3 *Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, expert 2's best response is $s_2 = s_B$.*

Proof of Lemma 2.3. Because $O_2 \in \mathcal{T}(s_1)$, expert 2 cannot induce action $y = O_2$, given s_1. Then, expert 2's best response is either $s_2 = s_A$, s_B or s_C.

Case (a): $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$**.** It is obvious that if $s_A = s_C$, then $s_A \notin \mathcal{T}(s_1)$. Then, $s_A \neq s_C$ in Case (a). If $s_2 = s_A$, then action $y = s_1$ is induced. As $s_1 \in I_2(s_1) \backslash \mathcal{T}(s_1)$, $s_2 = s_C$ is weakly

[20] An example of each case is represented in the figures of Appendix 2-C.

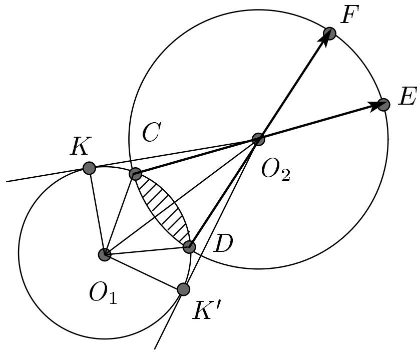


Figure 2.15: Lemma 2.4

better for expert 2 than is $s_2 = s_A$. Hence, we compare $||O_2 - s_B||$ with $||O_2 - s_C||$. Without loss of generality, assume that $s_B \in l(s_1)$ and $s_C = s_1$. Because $\overrightarrow{s_BO_2} \cdot \overrightarrow{s_Bs_1} = 0$, $||O_2 - s_B||^2 + ||s_B - s_1||^2 = ||O_2 - s_1||^2$. That is, $||O_2 - s_B|| \leq ||O_2 - s_1||$. Therefore, $s_2 = s_B$ is a best response.

Case (b): $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$**.** As $s_A = s_C$, it is sufficient to compare $||O_2 - s_A||$ and $||O_2 - s_B||$. Without loss of generality, assume that $s_B \in l(s_1)$. Because $s_A \notin \mathcal{T}(s_1)$ and $O_2 \in \mathcal{T}(s_1)$, O_2 and s_A are separated by $l(s_1)$. Then, there exists a point $q \in Y$ such that $\{q\} = l(s_1) \cap \bar{L}(O_2, s_A)$. Let $p \in Y$ be the point such that $p \in l(s_1)$ and $\overrightarrow{ps_A} \cdot \overrightarrow{py} = 0$ for any $y \in l(s_1)$. Note that $\overrightarrow{s_Bq} \cdot \overrightarrow{s_BO_2} = 0$ and $\overrightarrow{pq} \cdot \overrightarrow{ps_A} = 0$. Then, $||O_2 - q||^2 = ||O_2 - s_B||^2 + ||s_B - q||^2$, $||q - s_A||^2 = ||p - s_A||^2 + ||p - q||^2$, and $||O_2 - s_A|| = ||O_2 - q|| + ||q - s_A||$. Then:

$$\begin{aligned} ||O_2 - s_A||^2 &> ||O_2 - q||^2 + ||q - s_A||^2 \\ &= ||O_2 - s_B||^2 + ||s_B - q||^2 + ||p - s_A||^2 + ||p - q||^2 \qquad (2.18) \\ &> ||O_2 - s_B||^2. \end{aligned}$$

Therefore, $s_2 = s_B$ is one of the best response of expert 2. ■

Lemma 2.4 *Suppose that $O_2 \notin P_2(s_1), O_2 \in \mathcal{T}(s_1)$ and $s_B \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, expert 2's best* *response is* $s_2 = s_C$.

Proof of Lemma 2.4. Let $C(a, L^+(a,b), L^+(a,c))$ be the cone that have a vertex of a and sides of $L^+(a,b)$ and $L^+(a,c)$. Suppose, in contrast, that $O_2 \notin C(O_2', L^+(O_2', s_1), L^+(O_2', \hat{s}_1))$. That is, $O_2 \in \mathcal{T}(s_1) \backslash C(O_2', L^+(O_2', s_1), L^+(O_2', \hat{s}_1))$. There are two possibilities regarding the position of O_2: either (a) $O_2 \in C(s_1, l(s_1), L^+(O_2', s_1))$ or (b) $O_2 \in C(\hat{s_1}, l(\hat{s}_1), L^+(O_2', \hat{s}_1))$. For Case (a), $s_B \in l(s_1) \backslash (\bar{L}(O_{s_1}, s_1) \backslash \{s_1\})$ must be satisfied. This means that $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, a contradiction. For Case (b), we can imply a contradiction in a similar way. Then, $O_2 \in C(O_2', L^+(O_2', s_1), L^+(O_2', \hat{s}_1))$. From Lemma 2.1, $s_A \in L^+(O_2', O_2)$. Because $O_2 \in C(O_2', L^+(O_2', s_1), L^+(O_2', \hat{s}_1))$, $s_A \in I_2(s_1) \cap R_1(s_1)$. If $s_A \in I_2(s_1) \cap P_1(s_1)$, then $s_A \neq s_C$ because $P_1(s_1) \subset \mathcal{T}(s_1)$. Then, both $s_2 = s_A$ and $s_2 = s_B$ induce action $y = s_1$. As $s_1 \in I_2(s_1) \backslash \mathcal{T}(s_1)$, $s_2 = s_C$ is best for expert 2. If $s_A \in I_2(s_1) \cap I_1(s_1)$, then $s_A = s_C$. Similarly, as $s_2 = s_B$ induces action $y = s_1$, $s_2 = s_C$ is expert 2's best response. ■

Next, given expert 2's best response specified above, we show that truth telling is a best response for expert 1. If expert 1 sends $s_1 = \theta$, then $s_2 = \theta$ and action $y = \theta$ is induced. Hence, it is sufficient to show that for any deviation, the induced action is not included in $P_1(\theta)$. Consider the same five cases specified above. In Case (i), action $y = O_2$ is induced, but obviously $O_2 \notin P_1(\theta)$ because the experts have opposing biases. The following lemmas show that induced actions are not included in $P_1(\theta)$; Cases (ii), (iii), (iv), and (v) correspond to Lemmas 2.5, 2.6, 2.7, and 2.8, respectively.

Lemma 2.5 *Suppose that* $O_2 \in P_2(s_1)$ *and* $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$. *Then,* $s_A \notin P_1(\theta)$.

Proof of Lemma 2.5. By the construction of s_A, $s_A \in I_2(s_1)$. As $s_A \notin \mathcal{T}(s_1)$, $s_A \in I_2(s_1) \backslash \mathcal{T}(s_1)$. That is, $s_A \notin P_1(s_1)$. Suppose, by way of contradiction, that $s_A \in P_1(\theta)$. As $O_2 \in P_2(s_1)$ and by

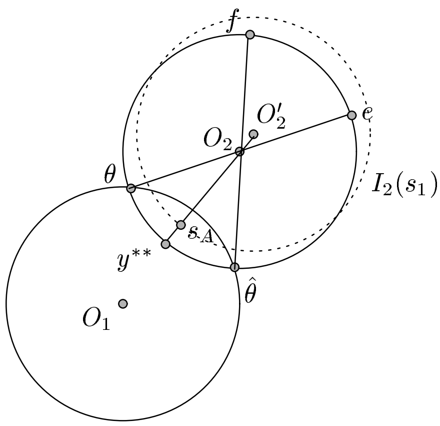


Figure 2.16: Lemma 2.5

Lemma 2.1, $s_A \in R_2(\theta)$. That is, $s_A \in R_1(\theta) \cap R_2(\theta)$. Let $e, f \in Y$ be the point such that $\{\theta, e\} = L(\theta, O_2) \cap I_2(\theta)$ where $e \neq \theta$ and $\{\hat{\theta}, f\} = L(\hat{\theta}, O_2) \cap I_2(\theta)$ where $f \neq \hat{\theta}$. From Lemma 2.1, $s_A \in L^+(O_2', O_2) \cap I_2(s_1)$. Because of opposing biases, $P_1(\theta) \cap \bar{L}(\theta, O_2) = \emptyset$ and $P_1(\theta) \cap \bar{L}(\hat{\theta}, O_2) = \emptyset$ from Lemma 2.2. Then, to hold $s_A \in R_1(\theta) \cap R_2(\theta)$, there exist $\alpha, \beta > 0$ such that $\overrightarrow{O_2O_2'} = \alpha\overrightarrow{O_2e} + \beta\overrightarrow{O_2f}$. This implies that there exists point $y^{**} \in Y$ such that $y^{**} \in R_1(\theta) \cap I_2(\theta)$ and $y^{**} + \overrightarrow{O_2'O_2} = s_A$. That is, $s_A \in R_1(s_1) \cap I_2(s_1)$. Because $R_1(s_1) \cap I_2(s_1) \subset R_1(s_1)$, $(R_1(s_1) \cap I_2(s_1)) \backslash \{s_1, \hat{s}_1\} \subset P_1(s_1)$. Because $\alpha, \beta > 0$, $s_A \neq s_1, \hat{s}_1$. Then, $s_A \in (R_1(s_1) \cap I_2(s_1)) \backslash \{s_1, \hat{s}_1\} \subset P_1(s_1)$. That is, we have $s_A \in P_1(s_1)$, which is a contradiction. Therefore, $s_A \notin P_1(\theta)$ must hold. ■

Lemma 2.6 *Suppose that $O_2 \in P_2(s_1)$ and $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_C \notin P_1(\theta)$.*

Proof of Lemma 2.6. As $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$, $s_A \in P_1(s_1) \cap I_2(s_1)$. Then, there must exist $\alpha, \beta > 0$ such that $\overrightarrow{O_2'O_2} = \alpha\overrightarrow{O_2's_1} + \beta\overrightarrow{O_2'\hat{s}_1}$. As $\overrightarrow{O_\theta O_{s_1}} = \overrightarrow{O_2O_2'}$, $\overrightarrow{O_\theta O_{s_1}} = -\alpha\overrightarrow{O_2's_1} - \beta\overrightarrow{O_2'\hat{s}_1}$. Hence,

$$\begin{aligned} \overrightarrow{O_\theta O_{s_1}} &= -\alpha\overrightarrow{O_2's_1} - \beta\overrightarrow{O_2'\hat{s}_1} = -\alpha(\overrightarrow{O_2'O_{s_1}} + \overrightarrow{O_{s_1}s_1}) - \beta(\overrightarrow{O_2'O_{s_1}} + \overrightarrow{O_{s_1}\hat{s}_1}) \qquad (2.19) \\ &= -\alpha\overrightarrow{O_{s_1}s_1} - \beta\overrightarrow{O_{s_1}\hat{s}_1} - (\alpha+\beta)\overrightarrow{O_2'O_{s_1}}. \end{aligned}$$

As the experts have opposing biases, there exist $\gamma, \delta \geq 0$ such that $\overrightarrow{O_{s_1}O_2'} = -\gamma\overrightarrow{O_{s_1}s_1} - \delta\overrightarrow{O_{s_1}\hat{s}_1}$. Then,

$$\begin{aligned} \overrightarrow{O_\theta O_{s_1}} &= (-\alpha - \gamma(\alpha+\beta))\overrightarrow{O_{s_1}s_1} + (-\beta - \delta(\alpha+\beta))\overrightarrow{O_{s_1}\hat{s}_1} \\ &= (-\alpha - \gamma(\alpha+\beta))\overrightarrow{O_\theta\theta} + (-\beta - \delta(\alpha+\beta))\overrightarrow{O_\theta\hat{\theta}_1}. \end{aligned} \tag{2.20}$$

Claim 2.1 *Suppose that there exists $\alpha, \beta < 0$ such that $\overrightarrow{O_\theta O_{s_1}} = \alpha\overrightarrow{O_\theta\theta} + \beta\overrightarrow{O_\theta\hat{\theta}}$. Then, $C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta})) \subseteq \mathcal{T}(s_1)$.*

Proof of Claim 2.1. Suppose that $\alpha, \beta < 0$. Take any $y \in C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta}))$. Then, there exist $\gamma_1, \delta_1 \geq 0$ such that $\overrightarrow{O_\theta y} = \gamma_1\overrightarrow{O_\theta\theta} + \delta_1\overrightarrow{O_\theta\hat{\theta}}$. Hence:

$$\begin{aligned} \overrightarrow{O_{s_1}y} &= \overrightarrow{O_{s_1}O_\theta} + \overrightarrow{O_\theta y} = -\alpha\overrightarrow{O_\theta\theta} - \beta\overrightarrow{O_\theta\hat{\theta}} + \gamma_1\overrightarrow{O_\theta\theta} + \delta_1\overrightarrow{O_\theta\hat{\theta}} \\ &= (-\alpha + \gamma_1)\overrightarrow{O_{s_1}s_1} + (-\beta + \delta_1)\overrightarrow{O_{s_1}\hat{s}_1}. \end{aligned} \tag{2.21}$$

As $-\alpha + \gamma_1, -\beta + \delta_1 > 0$, $y \in \mathcal{T}(s_1)$. Therefore, $C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta})) \subseteq \mathcal{T}(s_1)$. □

As $-\alpha - \gamma(\alpha+\beta), -\beta - \delta(\alpha+\beta) < 0$, $C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta})) \subseteq \mathcal{T}(s_1)$ from Claim 2.1. Since $s_C \notin \mathcal{T}(s_1)$, $s_C \notin C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta}))$. Because $P_1(\theta) \subset C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta}))$, $s_C \notin P_1(\theta)$. ■

Lemma 2.7 *Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_B \notin P_1(\theta)$.*

Proof of Lemma 2.7. Let $\mathcal{T}(\theta)$ be the interior of the cone $C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta}))$. As the experts have opposing biases, $O_2 \notin \mathcal{T}(\theta)$. Because $O_2 \notin \mathcal{T}(\theta)$ and $O_2 \in \mathcal{T}(s_1)$, there exist $\alpha, \beta < 0$ such that $\overrightarrow{O_\theta O_{s_1}} = \alpha\overrightarrow{O_\theta\theta} + \beta\overrightarrow{O_\theta\hat{\theta}}$. From Claim 2.1, $C(O_\theta, L^+(O_\theta,\theta), L^+(O_\theta,\hat{\theta})) \subseteq \mathcal{T}(s_1)$. As $s_B \notin \mathcal{T}(s_1)$,

$s_B \notin C(O_\theta, L^+(O_\theta, \theta), L^+(O_\theta, \hat{\theta}))$. Because $P_1(\theta) \subset C(O_\theta, L^+(O_\theta, \theta), L^+(O_\theta, \hat{\theta}))$, $s_B \notin P_1(\theta)$. ■

Lemma 2.8 *Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_C \notin P_1(\theta)$.*

Proof of Lemma 2.8. As we showed in the proof of Lemma 2.4, $O_2 \in C(O_2', L^+(O_2', s_1), L^+(O_2', \hat{s}_1))$. Also, we can say that $s_A \in \bar{L}(O_2, O_2')$. Hence, $s_A \in R_1(s_1) \cap I_2(s_1)$. Suppose, in contrast, that $s_A \in I_1(s_1) \cap I_2(s_1)$; that is, $s_A = s_C \in \{s_1, \hat{s}_1\}$. This means that either $O_2 \in L(O_2', s_1)$ or $O_2 \in L(O_2', \hat{s}_1)$. However, as we showed in the proof of Lemma 2.4, this implies that $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, which is a contradiction. Hence, $s_A \in P_1(s_1) \cap I_2(s_1)$ must hold. We can apply the same argument in the proof of Lemma 2.6, and then we can say that $s_C \notin P_1(\theta)$. ■

By Lemmas 2.5, 2.6, 2.7, and 2.8, we can say that for any state θ, any deviation from $s_1 = \theta$ never improves expert 1's payoff. Thus, given expert 2's and the decision-maker's strategies, truth telling is one of the best response of expert 1. It is obvious that, on the equilibrium path, the belief specified by the extended self-serving belief is consistent with Bayes' rule. Therefore, it is a PBE, a fully revealing equilibrium. In summary, the players' strategies are described as follows:

$$\mu_1^*(\theta) = \theta \tag{2.22}$$

$$\mu_2^*(\theta, s_1) = \begin{cases} \theta & \text{if } s_1 = \theta \\ O_2 & \text{if } O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \\ s_A & \text{if } O_2 \in P_2(s_1) \text{ and } s_A \notin P_2(s_1) \cup \mathcal{T}(s_1) \\ s_C & \text{if } O_2 \in P_2(s_1) \text{ and } s_A \in P_2(s_1) \cup \mathcal{T}(s_1) \\ s_B & \text{if } O_2 \notin P_2(s_1) \text{ and } O_2 \in \mathcal{T}(s_1) \text{ and } s_B \notin P_2(s_1) \cup \mathcal{T}(s_1) \\ s_C & \text{if } O_2 \notin P_2(s_1) \text{ and } O_2 \in \mathcal{T}(s_1) \text{ and } s_B \in P_2(s_1) \cup \mathcal{T}(s_1) \end{cases} \tag{2.23}$$

$$\mathcal{P}^*(\theta|s_1, s_2) = \begin{cases} 1 & \text{if } s_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \text{ and } \theta = s_2 \\ 0 & \text{if } s_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \text{ and } \theta \neq s_2 \\ 1 & \text{if } s_2 \in P_2(s_1) \cup \mathcal{T}(s_1) \text{ and } \theta = s_1 \\ 0 & \text{if } s_2 \in P_2(s_1) \cup \mathcal{T}(s_1) \text{ and } \theta \neq s_1 \end{cases} \tag{2.24}$$

$$y^*(s_1, s_2) = \begin{cases} s_2 & \text{if } s_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \\ s_1 & \text{if } s_2 \in P_2(s_1) \cup \mathcal{T}(s_1) \end{cases} \tag{2.25}$$

(Necessity) Suppose, by contrast, that there exists a fully revealing equilibrium $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$

in like-biases cases. By Lemma 2.2, $P_1(\theta) \cap \bar{L}(\theta, O_2) \neq \emptyset$. Pick a point $\theta' \in P_1(\theta) \cap \bar{L}(\theta, O_2)$, as in Figure 2.11. Because there exists a fully revealing equilibrium, on the equilibrium path, there exist messages s_1, s_2, s_1' and s_2' such that:

$$\mu_1^*(\theta) = s_1,\ \mu_2^*(\theta, s_1) = s_2,\ y^*(s_1, s_2) = \theta, \tag{2.26}$$

$$\mu_1^*(\theta') = s_1',\ \mu_2^*(\theta', s_1') = s_2',\ y^*(s_1', s_2') = \theta'. \tag{2.27}$$

First, show that $s_1 \neq s_1'$. Suppose, in contrast, that $s_1 = s_1'$. Because $y^*(s_1, s_2) \neq y^*(s_1', s_2')$, $s_2 \neq s_2'$. However, because $\theta' \in \bar{L}(\theta, O_2)$, expert 2 has an incentive to send s_2' at state θ after observing s_1, which is a contradiction. Therefore, $s_1 \neq s_1'$ must hold. Next, we show that, given expert 1's message s_1', there is no message s_2'' by expert 2 such that $y(s_1', s_2'') \in P_2(\theta')$. Suppose, by contrast, that there exists such a message s_2''. Then, expert 2's best response to the message s_1' at θ' is not sending s_2' because, by sending s_2'', expert 2's utility is strictly improved. This contradicts the message s_2' being on the equilibrium path. Thus, such a message s_2'' does not exist. Finally, we show that expert 1 has an incentive to deviate. Suppose that expert 1 sends message s_1' at the state θ. Given s_1', there is no message s_2'' such that $y(s_1', s_2'') \in P_2(\theta')$, as shown above. That is, expert 2 cannot induce the actions in $P_2(\theta')$ if expert 1 sends message s_1'. From the construction of θ' and Lemma 2.1, the most preferred action in $\mathbb{R}^2 \backslash P_2(\theta')$ for expert 2 is $y = \theta'$, and it can be induced by sending s_2', i.e., an endorsement of expert 1's deviation. However, action $y = \theta' \in P_1(\theta)$. This means that expert 1 has no incentive to send the message s_1, which thus contradicts the message s_1 being on the equilibrium path. Therefore, there is no fully revealing equilibrium.[21] ■

[21] Notice that the proof of the necessary part does not depend on the direct message game setting. That is, the impossibility result holds in both direct and indirect message games.

Proof of Proposition 2.5

Suppose, in contrast, that there exists a fully revealing equilibrium even if the conditions (i) and (ii) hold. Fix $\theta \in \Theta$ arbitrarily, and choose $\theta' \in \Theta$ satisfying the conditions. Because there exists a fully revealing equilibrium, there exist the following messages:

$$\mu_1^*(\theta) = s_1,\ \mu_2^*(\theta, s_1) = s_2,\ ,\mu_3^*(\theta, s_1, s_2) = s_3,\ y^*(s_1, s_2, s_3) = \theta; \tag{2.28}$$

$$\mu_1^*(\theta') = s_1',\ \mu_2^*(\theta', s_1') = s_2',\ ,\mu_3^*(\theta', s_1', s_2') = s_3',\ y^*(s_1', s_2', s_3') = \theta'. \tag{2.29}$$

Note that given s_1' and s_2', for any $\tilde{s}_3 \in S_3$, $y(s_1', s_2', \tilde{s}_3) \notin P_3(\theta')$; otherwise, expert 3 deviates from sending s_3' at state θ'. By Lemma 2.1 and conditions (i), sending s_3' is expert 3's unique best response after observing s_1' and s_2' at state θ.

Claim 2.2 $s_1 \neq s_1'$.

Proof of Claim 2.2. Suppose, in contrast, that $s_1 = s_1' = s^*$. Because $y^*(s_1, s_2, s_3) \neq y^*(s_1', s_2', s_3')$, $(s_2, s_3) \neq (s_2', s_3')$. If $s_2 = s_2'$, then $s_3 \neq s_3'$ must hold. However, in this scenario, expert 3 has an incentive to send s_3' instead of s_3 at state θ because θ' lies in his line of endorsement at θ. That is, $s_2 \neq s_2'$ must hold. By Condition (ii), because $\theta \notin P_1(\theta)$, $\theta \notin H_2(\theta, \theta')$. Now, given that expert 1 sends s^* at state θ, consider a situation where expert 2 sends s_2'. Because expert 3 sends s_3' after observing s^* and s_2' at state θ, action $y = \theta'$ is induced. Because $\theta' \in H_2(\theta, \theta')$ and $\theta \notin H_2(\theta, \theta')$, this is a profitable deviation for expert 2, which is a contradiction. Therefore, $s_1 \neq s_1'$. $\square$

By Claim 2.2, sending s_1' at state θ is a deviation by expert 1. Consider expert 2's response given observing s_1' at state θ. If $y^*(s_1', \tilde{s}_2, \mu_3(\theta, s_1', \tilde{s}_2)) \notin H_2(\theta, \theta')$ for any $\tilde{s}_2 \in S_2 \backslash \{s_2'\}$, then sending s_2'

is expert 2's unique best response. Because expert 3 sends s'_3 after observing s'_1 and s'_2 at state θ, action $y = \theta'$ is induced. Because $\theta' \in H_2(\theta, \theta')$, by Condition (ii), sending s'_1 at state θ is expert 1's profitable deviation. Therefore, to hold the fully revealing equilibrium, there must exists message $\hat{s}_2 \in S_2 \backslash \{s'_2\}$ such that $\mu_2^*(\theta, s'_1) = \hat{s}_2$ and $y^*(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \in H_2(\theta, \theta')$. Also, to hold the fully revealing equilibrium, $y^*(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \notin P_1(\theta)$; otherwise, expert 1 has an incentive to deviate. However, by Condition (ii), $y(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \in H_2(\theta, \theta') \backslash P_1(\theta)$ is impossible, which is a contradiction. Therefore, there exists no fully revealing equilibrium. ■

Proof of Proposition 2.6

A pure strategy and the belief of expert 2 is represented by $\mu_2 : \Sigma \times S_1 \to S_2$ and $\mathcal{P}_2 : \Sigma \times S_1 \to \Delta(\Theta)$, respectively. Define set $J \equiv \{y \in Y |$ there exist $\alpha_1 \in \mathbb{R}$ and $\alpha_2 \leq 0$ such that $y = O_2 + \alpha_1 \overrightarrow{O_\theta \theta} + \alpha_2 \overrightarrow{O_\theta \hat{\theta}}\}$. That is, set J is the half-space separated by line $\bar{l}$, which is parallel to $l(\theta)$ and go through O_2. Define $s_D \equiv L(O_1, \theta) \cap \bar{l}$. Let $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ be the fully revealing equilibrium specified in Proposition 2.4, and show that $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{y}; \tilde{\mathcal{P}}_2, \tilde{\mathcal{P}})$ defined as follows is a PBE when $x_1 \cdot x_2 > 0$ and $\epsilon < 1 - \frac{|U^{E_1}(\theta,\theta,x_1)|}{|U^{E_1}(s_D,\theta,x_1)|}$.[22]

$$\begin{aligned}
\tilde{\mu}_1(\theta) &= \theta && (2.30)\\
\tilde{\mu}_2(\sigma, s_1) &= \begin{cases} \mu_2^*(\theta, s_1) & \text{if } \sigma = \theta \\ s_1 & \text{if } \sigma = \phi \end{cases} && (2.31)\\
\tilde{y}(s_1, s_2) &= y^*(s_1, s_2) && (2.32)\\
\tilde{\mathcal{P}}_2(\tilde{\theta} | \sigma, s_1) &= \begin{cases} 1 & \text{if } [\sigma = \theta \text{ and } \tilde{\theta} = \theta] \text{ or } [\sigma = \phi \text{ and } \tilde{\theta} = s_1] \\ 0 & \text{otherwise} \end{cases} && (2.33)\\
\tilde{\mathcal{P}}(\tilde{\theta} | s_1, s_2) &= \mathcal{P}^*(\tilde{\theta} | s_1, s_2) && (2.34)
\end{aligned}$$

It is straightforward that $\tilde{y}$ is the decision-maker's best response given her belief $\tilde{P}$. We consider

[22]Note that because the experts have opposing biases, $0 < \frac{|U^{E_1}(\theta,\theta,x_1)|}{|U^{E_1}(s_D,\theta,x_1)|} < 1$.

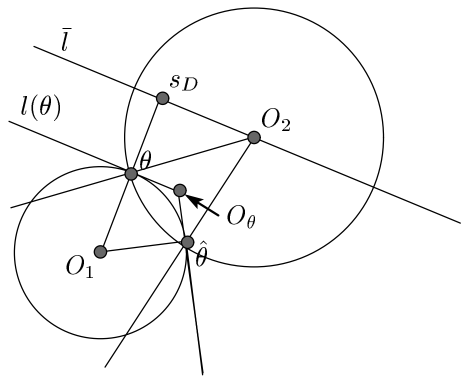


Figure 2.17: Proposition 2.6

expert 2's decision given his belief $\tilde{\mathcal{P}}_2$ and $\tilde{y}$. If $\sigma = \theta$, then this scenario is identical to the noiseless case. Hence, by the same arguments in Proposition 2.4, μ_2^* represents the optimal behavior of expert 2. If $\sigma = \phi$, then expert 2 believes that expert 1 reports the true state for certain. Given this belief and $\tilde{y}$ consistent with the extended self-serving belief, sending $s_2 = s_1$ is optimal because expert 2 cannot strictly improve his utility by sending $s_2 \neq s_1$. Hence, $\tilde{\mu}_2$ represents expert 2's best response.

Next, we consider expert 1's decision given $\tilde{\mu}_2$ and $\tilde{y}$. If expert 1 sends $s_1 = \theta$, then action $y = \theta$ is induced. That is, his utility is $U^{E_1}(\theta, \theta, x_1)$. If expert 1 sends $s_1 \neq \theta$, then his expected utility is $\tilde{U}(s_1) \equiv (1-\epsilon)U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1) + \epsilon U^{E_1}(s_1, \theta, x_1)$. Now, we suppose that $s_1 \notin P_1(\theta)$. Then, $U^{E_1}(\theta, \theta, x_1) \geq \tilde{U}(s_1)$ because $y^*(s_1, \mu_2^*(\theta, s_1)) \notin P_1(\theta)$ as shown in Proposition 2.4. Therefore, expert 1 has no strict incentive to send $s_1 \notin P_1(\theta)$. Then, we suppose that $s_1 \in P_1(\theta)$. The next lemma gives us an upper bound of $\tilde{U}(s_1)$.

Lemma 2.9 *Suppose that $x_1 \cdot x_2 > 0$. For any $\theta \in \Theta$ and $s_1 \in P_1(\theta)$,*

$U^{E_1}(s_D, \theta, x_1) \geq U^{E_1}(y^*(s_1, \mu_2(\theta, s_1)), \theta, x_1)$.

Proof of Lemma 2.9. Fix $\theta \in \Theta$ and $s_1 \in P_1(\theta)$ arbitrarily. First, we show that $y^*(s_1, \mu_2^*(\theta, s_1)) = s_A$ or O_2. It is obvious that $y^*(s_1, \mu_2^*(\theta, s_1)) = O_2$ if $O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, we assume that

$O_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$. Because $x_1 \cdot x_2 > 0$, there exist $\beta_1, \beta_2 < 0$ such that $\overrightarrow{O_\theta \theta} = \beta_1 \overrightarrow{O_\theta \theta} + \beta_2 \overrightarrow{O_\theta \hat{\theta}}$. Also, because $s_1 \in P_1(\theta)$, there exist $\gamma_1 \in \mathbb{R}$ and $\gamma_2 > 0$ such that $\overrightarrow{\theta s_1} = \gamma_1 \overrightarrow{O_\theta \theta} + \gamma_2 \overrightarrow{O_\theta \hat{\theta}}$. Therefore, if $O_2 \in \mathcal{T}(s_1)$, then $\gamma_2 < 0$ must hold, but it is impossible as long as $s_1 \in P_1(\theta)$. Thus, $O_2 \in P_2(s_1) \backslash \mathcal{T}(s_1)$. Now, we suppose, in contrast, that $s_A \in \mathcal{T}(s_1)$. By Lemma 2.1, if $s_A \in \mathcal{T}(s_1)$, then there exist $\delta_1, \delta_2 < 0$ such that $\overrightarrow{\theta s_1} = \delta_1 \overrightarrow{O_2 \theta} + \delta_2 \overrightarrow{O_2 \hat{\theta}}$. Hence:

$$
\begin{aligned}
\overrightarrow{\theta s_1} &= \delta_1(\overrightarrow{O_2 O_\theta} + \overrightarrow{O_\theta \theta}) + \delta_2(\overrightarrow{O_2 O_\theta} + \overrightarrow{O_\theta \hat{\theta}}) \\
&= (-\beta_1\delta_1 - \beta_1\delta_2 + \delta_1)\overrightarrow{O_\theta \theta} + (-\beta_2\delta_1 - \beta_2\delta_2 + \delta_2)\overrightarrow{O_\theta \hat{\theta}}. \qquad (2.35)
\end{aligned}
$$

However, because $-\beta_2\delta_1 - \beta_2\delta_2 + \delta_2 < 0$, which is a contradiction to $\gamma_2 > 0$. Then, $s_A \notin \mathcal{T}(s_1)$. Therefore, if $O_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$, then $y^*(s_1, \mu_2(\theta, s_1)) = s_A$.

Next, we show that $y^*(s_1, \mu_2(\theta, s_1)) \in J$. By the above arguments, if $s_1 \in P_1(\theta)$, then $y^*(s_1, \mu_2(\theta, s_1)) = s_A$ or O_2. Because it is obvious that $O_2 \in J$, it is sufficient to show that $s_A \in J$. By Lemma 2.1, $s_A \in I_2(s_1) \cap L^+(O_2', O_2)$; that is, there exists $\eta > 0$ such that $s_A = O_2 - \eta\overrightarrow{\theta s_1}$. Hence, $s_A = O_2 - \eta\gamma_1\overrightarrow{O_\theta \theta} - \eta\gamma_2\overrightarrow{O_\theta \hat{\theta}}$. Because $-\eta\gamma_2 < 0$, $s_A \in J$. By construction, $||O_1 - J|| = ||O_1 - s_D||$. Because $s_D \in \bar{l}$ and $\bar{l}$ goes through O_2 and $x_1 \cdot x_2 > 0$, we can say that $U^{E_1}(s_D, \theta, x_1) \geq U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1)$. ■

By Lemma 2.9:

$$
\begin{aligned}
& U^{E_1}(\theta,\theta,x_1)-(1-\epsilon)U^{E_1}(y^*(s_1,\mu_2(\theta,s_1)),\theta,x_1)-\epsilon U^{E_1}(s_1,\theta,x_1) \\
= \;& (1-\epsilon)\big\{U^{E_1}(\theta,\theta,x_1)-U^{E_1}(y^*(s_1,\mu_2(\theta,s_1)),\theta,x_1)\big\}+\epsilon\big\{U^{E_1}(\theta,\theta,x_1)-U^{E_1}(s_1,\theta,x_1)\big\} \\
\geq \;& (1-\epsilon)\big\{U^{E_1}(\theta,\theta,x_1)-U^{E_1}(s_D,\theta,x_1)\big\}+\epsilon\big\{U^{E_1}(\theta,\theta,x_1)-U^{E_1}(O_1,\theta,x_1)\big\} \\
= \;& (1-\epsilon)\big\{U^{E_1}(\theta,\theta,x_1)-U^{E_1}(s_D,\theta,x_1)\big\}+\epsilon U^{E_1}(\theta,\theta,x_1). \qquad (2.36)
\end{aligned}
$$

Because $\epsilon < 1-\frac{|U^{E_1}(\theta,\theta,x_1)|}{|U^{E_2}(s_D,\theta,x_1)|}$, equation (2.36) is positive. Therefore, expert 1 has no strict incentive to send $s_1 \in P_1(\theta)$. Thus, $\tilde{\mu}_1$ is expert 1's best response. It is straightforward that $\tilde{\mathcal{P}}_2$ and $\tilde{\mathcal{P}}$ are consistent with Bayes' rule. Therefore, this is a PBE, and the decision-maker always knows the true state on the equilibrium path. That is, it is a fully revealing equilibrium. ■

2.8 Appendix 2-B: Supplemental Materials

B.1 n-expert models

Claim 2.3 *Consider the three-expert model. If the conditions of Proposition 2.5 are satisfied, then they have like biases.*

Proof. Suppose, in contrast, that they have opposing biases. By Condition (ii), because $H_2(\theta,\theta') \subset P_1(\theta)$, $O_2 \in P_1(\theta)$. That is, $x_1 \cdot x_2 > 0$. Thus, one of the following cases must occur: (i) $x_1 \cdot x_3 \leq 0$, or (ii) $x_1 \cdot x_3 > 0$ and $x_2 \cdot x_3 \leq 0$.

In the first scenario, because $x_1 \cdot x_3 \leq 0$, $\tilde{\theta} \notin P_1(\theta)$ for any $\tilde{\theta} \in \bar{L}(\theta, O_3)$. By Condition (i), $\theta' \in \bar{L}(\theta, O_3)$, and then $\theta' \notin P_1(\theta)$. However, by definition, $\theta' \in H_2(\theta,\theta')$ must hold, which is a contradiction to Condition (ii). In the second scenario, because $x_2 \cdot x_3 \leq 0$, $P_2(\theta) \subset H_2(\theta, \tilde{\theta})$ for any $\tilde{\theta} \in \bar{L}(\theta, O_3)$. By Conditions (i) and (ii), we can say that $P_2(\theta) \subset H_2(\theta, \theta') \subset P_1(\theta)$. To hold this relation, x_1 and x_2 must be linearly dependent. However, because $x_1 \cdot x_3 > 0$, $x_2 \cdot x_3 > 0$, which is a contradiction. Therefore, this case must be like biases. ■

B.2 Mixed strategies

With abuse of notation, let $\mu_1 : \Theta \to \Delta(S_1)$ and $\mu_2 : \Theta \times S_1 \to \Delta(S_2)$ be experts 1's and 2's strategies, respectively. Note that the decision-maker always undertakes a pure strategy because of the loss-quadratic utility function. We say that a PBE is a *fully revealing equilibrium* in the mixed-strategy environment if for any $\theta \in \Theta$, any $s_1 \in supp(\mu_1^*(\theta))$ and any $s_2 \in supp(\mu_2^*(\theta, s_1))$, $\mathcal{P}^*(\theta | s_1, s_2) = 1$.

Proposition 2.8 *Consider the two-dimensional unbounded state space model with mixed strategies. Then, there exists a fully revealing equilibrium if and only if the experts have opposing biases.*

Proof. (Sufficiency) Show that the strategies and belief specified in Proposition 2.4 is also an equilibrium in the mixed-strategy environment. It is straightforward that y^* is decision-maker's best response given $\mathcal{P}^*$. Because expert 2 observes the exact message that expert 1 sends before sending a message, μ_2^* specifed is expert 2's best response. Because sending $s_1 \neq \theta$ does not improve expert 1's utility given μ_2^* and y^* as shown in Lemmas 2.5 to 2.8, expert 1 has no incentive to randomize messages other than $s_1 = \theta$. Therefore, μ_1^* is expert 1's best response. It is obvious that, on the equilibrium path, the belief specifed by $\mathcal{P}^*$ is consistent with Bayes' rule. Therefore, it is a fully revealing equilibrium.

(Necessity) Suppose, in contrast, that there exists a fully revealing equilibrium when the experts have like biases. Fix $\theta \in \Theta$ arbitrarily. By Lemma 2.2, $P_1(\theta) \cap \bar{L}(\theta, O_2) \neq \emptyset$. Choose $\theta' \in P_1(\theta) \cap \bar{L}(\theta, O_2)$ arbitrarily. Because there exists a fully revealing equilibrium $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$, there exist a pair of messages (s_1, s_2) and (s_1', s_2') such that:

$$s_1 \in supp(\mu_1^*(\theta)),\ s_2 \in supp(\mu_2^*(\theta, s_1)),\ y^*(s_1, s_2) = \theta, \tag{2.37}$$

$$s_1' \in supp(\mu_1^*(\theta')),\ s_2' \in supp(\mu_2^*(\theta', s_1')),\ y^*(s_1', s_2') = \theta', \tag{2.38}$$

Claim 2.4 $s_1' \notin supp(\mu_1^*(\theta))$.

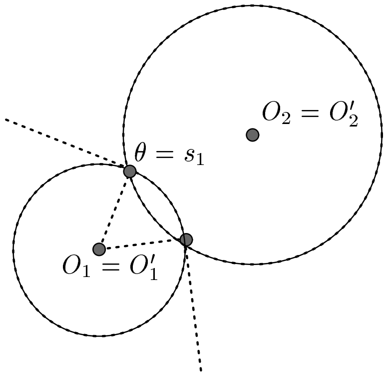
Proof of Claim 2.4. Suppose, in contrast, that $s_1' \in supp(\mu_1^*(\theta))$. Because $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ is a fully revealing equilibrium, there exists a message $\hat{s}_2 \in S_2$ such that $\hat{s}_2 \in supp(\mu_2^*(\theta, s_1'))$ and $y^*(s_1', \hat{s}_2) = \theta$. Hence, $U^{E_2}(\theta, \theta, x_2) \geq U^{E_2}(y^*(s_1', \tilde{s}_2), \theta, x_2)$ for all $\tilde{s}_2 \in S_2$ must hold; otherwise, it contradicts that $\hat{s}_2 \in supp(\mu_2^*(\theta, s_1'))$. However, because $\theta' \in \bar{L}(\theta, O_2)$ and $y^*(s_1', s_2') = \theta'$, $U^{E_2}(y^*(s_1', s_2'), \theta, x_2) > U^{E_2}(\theta, \theta, x_2)$, which is a contradiction. □

Claim 2.5 $\mu_2^*(\theta, s_1') = s_2'$.

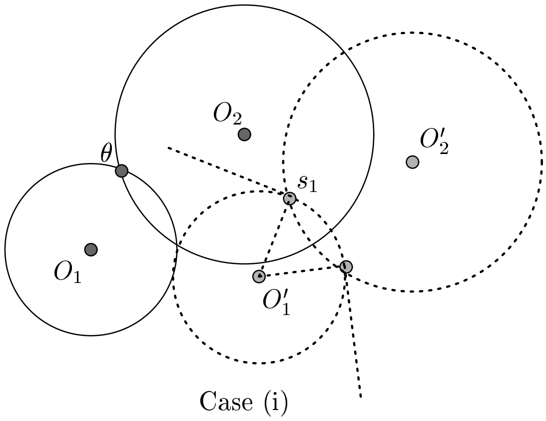
Proof of Claim 2.5. Note that for any $\tilde{s}_2 \in S_2$, $y^*(s_1', \tilde{s}_2) \notin P_2(\theta')$; otherwise, expert 2 never sends s_2' at state θ'. Because $\theta' \in \bar{L}(\theta, O_2)$ and by Lemma 2.1, sending s_2' is expert 2's unique best response when observing s_1' at state θ. That is, $\mu_2^*(\theta, s_1') = s_2'$. □

By Claim 2.4, sending s_1' at state θ' is a deviation from μ_1^*. By Claim 2.5, expert 2 sends s_2' after observing s_1' at state θ. That is, action $y = \theta'$ is induced. However, because $\theta' \in P_1(\theta)$, this is a profitable deviation for expert 1, which is a contradiction. Therefore, there exists no fully revealing equilibrium when the experts have like biases. ■

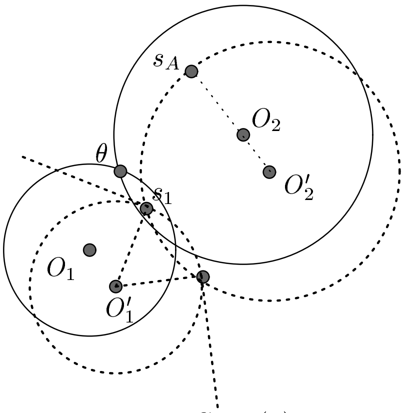
2.9 Appendix 2-C: Figures of Proposition 2.4. Cases 1 to 6.



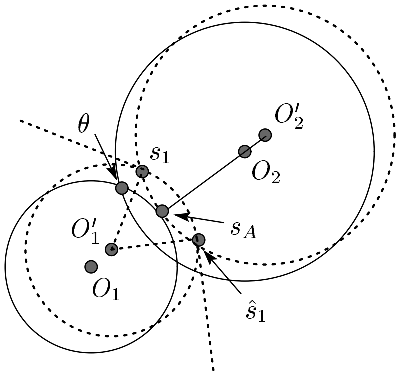
$s_1 = \theta$



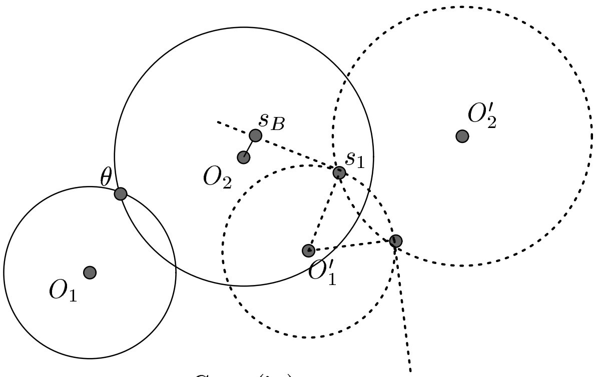
Case (i)



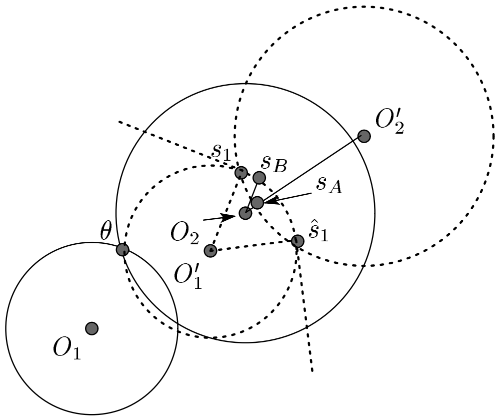
Case (ii)



Case (iii)



Case (iv)



Case (v)

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Chapter 3

A Characterization of Equilibrium Set of Persuasion Games without Single Crossing Conditions

3.1 Introduction

Persuasion games are costless sender-receiver games with verifiable private information. The sender sends a message about his private information to the receiver who chooses an action that affects the players' payoffs.[1] The sender can send any message costlessly, but he cannot misreport the information because the information is verifiable and the sender is required to submit evidence with his message.[2] Hence, the sender manipulates the information by concealing unfavorable information

[1] As a matter of convention, we treat the sender as male and the receiver as female throughout this chapter.
[2] In the literature, such information is called *hard information*.

instead of lying.

In this chapter, we consider a persuasion game between one sender and one receiver where the perfectly informed sender can completely verify any private information, and the receiver has two alternatives. However, the single crossing condition by Giovannoni and Seidmann (2007) is violated. It is well known that the *Giovannoni-Seidmann single crossing condition* (hereafter, GS single crossing condition) is the necessary and sufficient condition for full information disclosure in the class of completely verifiable persuasion games with perfectly informed senders. That is, full information disclosure is impossible in our environment. Intuitively, if the GS single crossing condition is violated, then the sender has a "cross-mimicking" incentive. In other words, type θ sender has an incentive to mimic type θ', but type θ' also has an incentive to mimic type θ. Because this cross-mimicking incentive prevents the receiver from inferring the motivation of the sender who conceals the information, full information disclosure breaks down.

Although most of the literature discuss the possibility of full information disclosure, we know little about how much information is transmitted in equilibrium when full information disclosure is impossible. That is, in our best knowledge, we do not know what is the most informative equilibrium (second-best), or what is the least informative equilibrium. However, characterizing these non-full disclosure equilibria is also an important issue, especially in applications. For example, imagine a situation where an investor asks a professional advice from a consultant who knows the state of the economy before she decides whether to invest or not. It could be reasonable that the consultant has an upward bias when the state of the economy is good, but he has a downward bias when the state of the economy is bad. If the sender has such a type-dependent bias, then the GS single crossing condition is violated. That is, full information transmission is impossible. What is the second-best (or the worst) scenario for the investor? The aim of this paper is making clear how much information is transmitted in equilibrium when the first best equilibrium is impossible.

We characterize the set of pure strategy equilibria from the viewpoint of the receiver's equilibrium ex ante expected utility. In other words, we characterize each equilibrium by its informativeness measured by the receiver's ex ante expected utility. First, we specify the maximum and the minimum of the possible equilibrium utility, and then show that any value in that range can be supported as equilibrium ex ante expected utility of the receiver. An implication is that in any equilibrium, the sender can conceal a part of unfavorable information, but cannot suppress all of them if conflicts between the players happen frequently enough.

This chapter is organized as follows. In the next subsection, we discuss the related literature. In Section 3.2, we outline the model. In Section 3.3, we characterize the set of equilibria. Section 3.4 concludes this chapter.

3.1.1 Related Literature

The seminal studies of persuasion games are those of Milgrom (1981) and Milgrom and Roberts(1986).[3] These papers assume that (i) the sender's preference is type-independent, and (ii) the receiver can distinguish whether the sender discloses all information. In this environment, full information disclosure can be supported in equilibrium by undertaking the sender's most unfavorable action as a punishment for withholding information. This is the well known *unraveling argument* in the literature. The subsequent researches revisit the above assumptions, and check the validity of the unraveling argument.

[3] The idea of verifiable information disclosure had been used in industrial organization theory before they formalized the concepts. See Grossman (1981) and Grossman and Hart (1980).

Seidmann and Winter (1997) and Giovannoni and Seidmann (2007) relax assumption (i). They assume that the sender's preference is also type-dependent. The players have single-peaked preferences in the receiver's action, and the bliss points vary depending on the sender's private information. These papers show that satisfying the single-crossing condition is the necessary and sufficient condition for the unraveling argument. On the other hand, Shin (1994a, 1994b), Lipman and Seppi (1995), Wolinsky (2003) and Mathis (2008) are categorized in the branch relaxing assumption (ii).[4] Because the receiver cannot correctly recognize whether the sender discloses everything, the unraveling argument becomes hard to hold. Mathis (2008) characterizes the necessary and sufficient condition for the unraveling argument, which is more demanding compared with that in the completely verifiable environments.

As a departure from the unraveling argument, Forges and Koessler (2008) geometrically characterize the set of all Nash and perfect Bayesian equilibrium in one-round and multi-round persuasion games with assumption (ii). Their characterization is quite general, and the results can apply to any finite persuasion games holding assumption (ii). However, as the cost of the generality, the characterization is abstract, and then it is hard to observe what happens in each equilibrium. Especially, it is hard to understand that how much information is transmitted in equilibrium, and that how this amount varies dependent on the parameters.

Glazer and Rubinstein (2004, 2006), Lanzi and Mathis (2008), Dziuda (2011) and Kamenica and Gentzkow (2011) characterize the non-full-disclosure behaviors more concretely. Glazer and Rubinstein (2004, 2006) characterize the optimal "persuasion rule" that minimizes the probability that the receiver acts incorrectly. Lanzi and Mathis (2008) characterize equilibria in a partially

[4]Shin (1994a, 1994b) study the situations where the sender is imperfectly informed. On the other hand, Lipman and Seppi (1995), Wolinsky (2003) and Mathis (2008) analyze the environments of *partial verifiability*, in which some private information is verifiable, but the others is unverifiable.

verifiable persuasion game where the receiver has binary alternatives. Dziuda (2011) considers the "strategic argumentation" model, in which the sender's private information represents the "number of arguments" that endorses each alternative, and discuss the properties of equilibria. Kamenica and Gentzkow (2011) characterize the sender-optimal equilibrium in the environment where the sender can choose his own informativeness about the state of nature before sending a message.

This chapter can be regarded as a complement of the above papers. First, this chapter is located in the branch of relaxing assumption (i) like Seidmann and Winter (1997) and Giovannoni and Seidmann (2007). However, the main objective of this chapter is characterizing the set of equilibria when the unraveling argument fails instead of focusing on full information disclosure. Second, while the model of Forges and Koessler (2008) is more general than ours, it is difficult to understand from their results that how much information is transmitted in equilibrium. Then, in order to make clear the informativeness of each equilibrium, we adopt the more specialized model. Finally, the motivation of this chapter is similar to those of Glazer and Rubinstein (2004, 2006), Lanzi and Mathis (2008), Dziuda (2011) and Kamenica and Gentzkow (2011), but the emphasized points are different. Although these papers relax assumptions (ii) to focus on the aspects of the sender's informativeness or verification ability, this chapter relaxes assumption (i) in order to focus on the preference aspect.[5]

[5]Our setup is similar to that of Lanzi and Mathis (2008). They focus on the partial verifiability of the private information in the model where the GS single crossing condition holds. While we assume the complete verifiability of the private information, we study a model without the GS single crossing condition to emphasize the preference aspects.

3.2 The Model

There is one sender and one receiver. The receiver chooses an action $y \in Y \equiv \{y_1, y_2\}$, but the outcome produced by action y_i for $i = 1, 2$ is the sender's private information. Let $\theta \in \Theta \equiv [0, 1]$ be the sender's private information. We interchangeably call set Θ the type space or state space. Let $F(\cdot)$ be the prior distribution function on the type space Θ with full support density function $f(\cdot)$; that is, $f(\theta) > 0, \forall \theta \in \Theta$.

Let $M(\theta) \equiv \{X \in \mathbb{P}(\Theta) | \theta \in X\}$ be the sender's message space when the sender's type is θ, where $\mathbb{P}(\Theta)$ is the power set of the type space Θ. Any available message must contain the true information θ. Define $M \equiv \cup_{\theta \in \Theta} M(\theta)$, where $m \in M$ represents a message sent by the sender. Note that for any subset $P \subseteq \Theta$, there exists a message m such that $m \in M(\theta)$ if and only if $\theta \in P$. That is, this is a *completely verifiable* environment.

We denote the receiver's and the sender's von Neumann-Morgenstern utility functions by $u : \Theta \times Y \to \mathbb{R}$ and $v : \Theta \times Y \to \mathbb{R}$, respectively. We assume that both $u(\theta, y)$ and $v(\theta, y)$ are continuous in θ for any $y \in Y$. Depending on conflicts between the players, the state space is partitioned into the following five regions:

$$
\begin{aligned}
A &\equiv \{\theta \in \Theta | u(\theta, y_1) \geq u(\theta, y_2) \text{ and } v(\theta, y_1) \geq v(\theta, y_2)\} \\
B &\equiv \{\theta \in \Theta | u(\theta, y_2) > u(\theta, y_1) \text{ and } v(\theta, y_2) > v(\theta, y_1)\} \\
C &\equiv \{\theta \in \Theta | u(\theta, y_1) > u(\theta, y_2) \text{ and } v(\theta, y_2) > v(\theta, y_1)\} \qquad (3.1) \\
D &\equiv \{\theta \in \Theta | u(\theta, y_2) > u(\theta, y_1) \text{ and } v(\theta, y_1) > v(\theta, y_2)\} \\
E &\equiv \Theta \backslash (A \cup B \cup C \cup D)
\end{aligned}
$$

It is worth mentioning that if θ lie in region $A \cup B \cup E$, then the sender and the receiver have no conflict. We call regions A, B and E *agreement regions.* On the other hand, if θ lies in region $C \cup D$, then there is conflict between the players. That is, if $\theta \in C$, then the receiver prefers y_1 but the sender prefers y_2. Similarly, if $\theta \in D$, then the receiver prefers y_2 but the sender prefers y_1. Hence, we call regions C and D *disagreement regions.* To avoid unnecessary complexity, we assume that each region is measurable, and regions A, B, C and D have positive measure but region E has zero measure. In other words, the sender and the receiver have nontrivial conflicts.

The timing of the game is as follows. First, nature chooses the state of nature $\theta \in \Theta$ according to the prior distribution $F(\cdot)$. Only the sender observes the state θ. Second, the sender sends a message $m \in M(\theta)$ given the state θ. Then, after observing the message, the receiver undertakes an action $y \in Y$.

The sender's pure strategy $\sigma : \Theta \to M$ specifies a message sent by the sender. The receiver's pure strategy $\mu : M \to Y$ describes an action that she chooses when she observes message m. Let $\mathcal{P} : M \to \Delta(\Theta)$ be the posterior belief of the receiver. This is a function from the entire message space M to the set of probability distributions on the type space Θ.[6]

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept and focus on pure strategy equilibria. Because of the verifiability of the information about the outcomes of alternatives, any message must contain the true information. In other words, the receiver can infer that the types of sender not included in the observed message never occur for certain. Thus, we must place a restriction on the receiver's equilibrium belief. Letting $S(\mathcal{P}(\cdot|m))$ be the support of the receiver's belief $\mathcal{P}(\cdot|m)$, this requirement is described below.

[6]In relaxed notation, let $\mathcal{P}(\cdot|m)$ represent a conditional probability function if the support of the posterior is countable, and a conditional density function if the support is uncountable.

Requirement 3.1 *Given a message m, $S(\mathcal{P}(\cdot|m)) \subseteq m$.*

Definition 3.1 ***PBE***

A triple $(\sigma^*, \mu^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions:

(i) $\sigma^*(\theta) \in \arg\max_{m \in M(\theta)} v(\theta, \mu^*(m)), \forall \theta \in \Theta$*;*

(ii) $\mu^*(m) \in \arg\max_{y \in Y} \mathbb{E}[u(\theta, y)|m], \forall m \in M$*;*

(iii) *$\mathcal{P}^*$ is derived by σ^* consistently from Bayes' rule whenever possible.*

Otherwise, $\mathcal{P}^*$ is any probability distribution satisfying Requirement 3.1.

3.3 Characterization of Equilibrium Set

3.3.1 Impossibility of full information disclosure

First, we show that full information disclosure is impossible in this setup as a preliminary result. Define $y^R(\theta) \equiv \arg\max_{y \in Y} u(\theta, y)$. We say that a PBE $(\sigma^*, \mu^*; \mathcal{P}^*)$ is a *full-disclosure equilibrium* if $\mu^*(\sigma^*(\theta)) = y^R(\theta)$ for any $\theta \in \Theta$. That is, in full-disclosure equilibrium, the receiver can always undertake her most preferred action.

Proposition 3.1 *There is no full-disclosure equilibrium.*

All proofs are in Appendix 3-A. This impossibility result is a corollary of Proposition 3.1 of Giovannoni and Seidmann (2007), which says that the necessary and sufficient condition for the existence of full-disclosure equilibrium is that the preferences satisfy the GS single crossing condition.[7] We say that the players' preferences satisfy the GS single crossing condition if for any pair of types

[7] Because our information structure is different from Giovannoni and Seidmann (2007), we cannot apply their result directly. We need straightforward modification.

$\theta, \theta' \in \Theta$, either $v(\theta, y^R(\theta)) \geq v(\theta, y^R(\theta'))$ or $v(\theta', y^R(\theta')) \geq v(\theta', y^R(\theta))$ holds. Because disagreement regions C and D are both nonempty, it is obvious that the GS single crossing condition fails in our environment. Thus, there exists no full-disclosure equilibrium.

3.3.2 Characterization of equilibrium set

In this subsection, we characterize the set of pure strategy equilibria by specifying the set of the receiver's equilibrium ex ante expected utility. First, we characterize the maximum and the minimum of the set, and then show that any value between the bounds can be supported as equilibrium utility of the receiver.

The maximum of the receiver's equilibrium ex ante expected utility is given by the following proposition.

Proposition 3.2 *There exists an equilibrium in which types in region $A \cup B \cup E$ disclose their own types, and the types in region $C \cup D$ are pooling. Moreover, this equilibrium is best for the receiver.*

This is an equilibrium in which the types who disagree with the receiver are completely pooling, and other types disclose their own types. Because our model has continuum types, one might be able to derive an equilibrium that dominates the equilibrium in Proposition 3.2 by cleverly partitioning the disagreement regions. However, as Proposition 3.2 shows, one need not consider more complicated partitions of the disagreement regions in order to find the best equilibrium for the receiver.

Unlike the maximum, the minimum of the receiver's equilibrium ex ante expected utility depends crucially on the receiver's utility function and the distribution of θ. Hereafter, to simplify representations, we write $\mathbb{E}[\cdot|Z] = \mathbb{E}[\cdot|\theta \in Z]$, and assume that $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$

and $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$, and specify the minimum in this case.[8] Intuitively, this is a situation where disagreement region C is so likely that agreement region B cannot entirely absorb disagreement region C with continuing to induce the preferred action by the types in region B when they are pooling. The minimum ex ante expected utility is characterized by the following proposition.

Proposition 3.3 *Suppose that* $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$ *and* $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$. *Then, there exists an equilibrium in which: (i) types in* $B \cup \bar{C}$ *are pooling and induce action* y_2*; (ii) types in regions* A, D *and* E *induce their own preferred actions; and (iii) types in* $C \backslash \bar{C}$ *induce action* y_1*, where* $\bar{C}$ *is a subset of disagreement region* C *such that* $\mathbb{E}[u(\theta, y_1)|B \cup \bar{C}] = \mathbb{E}[u(\theta, y_2)|B \cup \bar{C}]$. *Moreover, this equilibrium is worst for the receiver.*

In the worst equilibrium for the receiver, disagreement region D is completely pooling with agreement region A, and subset $\bar{C}$ of disagreement region C is pooling with agreement region B. Because disagreement region C is sufficiently likely, the receiver cannot ignore the effect of disagreement region C when all of disagreement region C is pooling with agreement region B. Hence, pooling region $B \cup C$ cannot be supported in equilibrium, but some of disagreement region C can be pooling with agreement region B. In other words, the worst equilibrium for the receiver is characterized by subset $\bar{C}$ of disagreement region C, which is pooling with agreement region B. That is, the receiver undertakes the unfavorable action in region $\bar{C}$, but undertakes the preferred action in $C \backslash \bar{C}$.

There are two implications from these results. First, in any equilibrium, the sender can successfully conceal part of the unfavorable information. In the best equilibrium for the receiver char-

[8] The minimum in the other cases are characterized by the similar way used in this case. The formal description is in Appendix 3-B.

acterized in Proposition 3.2, the types in the disagreement regions are completely pooling. Hence, the types belonging to one of the disagreement regions are successfully withheld. Because more information is withheld, less the receiver's utility becomes, we can conclude that the sender can conceal the information at least one of the disagreement regions in any equilibrium. Second, if the disagreement regions happen frequently enough, then the sender cannot completely suppress the unfavorable information in any equilibrium. As Proposition 3.3 shows, because the receiver can undertake the preferred action in the subset of the disagreement region in the worst equilibrium, the information that the types lies in the subset is disclosed to the receiver. Therefore, we can conclude that in any equilibrium, the sender cannot completely suppress the unfavorable information.

Given the results so far, we can characterize the set of pure strategy equilibria. Let $(\sigma^+, \mu^+; \mathcal{P}^+)$ be the best equilibrium for the receiver specified in Proposition 3.2, and let $(\sigma^-, \mu^-; \mathcal{P}^-)$ be the worst equilibrium for the receiver specified in Propositions 3.3. Define $U^+ \equiv \mathbb{E}[u(\theta, \mu^+(\sigma^+(\theta)))]$ and $U^- \equiv \mathbb{E}[u(\theta, \mu^-(\sigma^-(\theta)))]$. As shown in the following theorem, there are continuum equilibria in this setup, and any value between U^- and U^+ can be equilibrium ex ante expected utility of the receiver.

Theorem 3.1 *There exists an equilibrium $(\sigma, \mu; \mathcal{P})$ such that $\mathbb{E}[u(\theta, \mu(\sigma(\theta)))] = U$ if and only if $U \in [U^-, U^+]$.*

Let $X \subset C \cup R$ be the subset in which the receiver undertakes the preferred action given an equilibrium $(\sigma, \mu; \mathcal{P})$. Define X^+ and X^- by the subset in which the receiver undertakes the preferred action in the best and worst equilibria, respectively. To simplify the explanation, we assume that $\mathbb{E}[u(\theta, y_1)|C \cup D] \geq \mathbb{E}[u(\theta, y_2)|C \cup D]$. Then, $X^+ = C$ and $X^- = C \backslash \bar{C}$. As shown in Proposition 3.3, any partition of disagreement region C satisfying $\mathbb{E}[u(\theta, y_1)|B \cup \bar{C}] = \mathbb{E}[u(\theta, y_2)|B \cup \bar{C}]$ induces the minimum ex ante expected utility to the receiver. Hence, we begin with constructing a partition of disagreement region C with nicely defined subset $\bar{C}$, and continuously shrink the subset. Intuitively, for any $U' \in [U^-, U^+]$, we can find an appropriate subset $C' \subseteq \bar{C}$ through the process of the shrink such that if the receiver undertakes the preferred action on $X = C \backslash C'$, then her ex ante expected utility is U'. We can easily show that this partition can be supported in an equilibrium. That is, $[U^-, U^+]$ is the set of equilibrium ex ante expected utility of the receiver.

3.4 Conclusion

In this chapter, we have analyzed a persuasion game in which the GS crossing condition is violated. Because the GS single crossing condition fails, it is well known that there exists no full-disclosure equilibrium. Then, this chapter has characterized the set of pure strategy equilibria by specifying the set of equilibrium ex ante expected utility of the receiver. The set is characterized by the maximum and the minimum utility of the receiver, and any value between them can be supported as equilibrium ex ante expected utility of the receiver. That is, there are continuum equilibria.

3.5 Appendix 3-A: Proofs

Proof of Proposition 3.1

Lemma 3.1 *In any equilibrium, for any types $\theta', \theta'' \in \Theta$, there exists an off-the-equilibrium-path message that is available to both types θ' and θ''.*

Proof of Lemma 3.1. Suppose, by contrast, that there exists an equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$, in which there exist two distinct types $\theta', \theta'' \in \Theta$ such that for all message $m \in M(\theta') \cap M(\theta'')$, there exists a type $\tilde{\theta} \in \Theta$ such that $\sigma^*(\tilde{\theta}) = m$. That is, the range of σ^* must contain $M(\theta') \cap M(\theta'')$. Let $\#(A)$ be the cardinality of set A. Because the range of σ^* contains $M(\theta') \cap M(\theta'')$, $c = \#(\Theta) \geq \#(M(\theta') \cap M(\theta''))$, where c represents the cardinality of $\mathbb{R}$. However, $\#(M(\theta') \cap M(\theta'')) = 2^{c-2} = 2^c$ and hence, $c = 2^{\aleph_0} < 2^c$, which is a contradiction. Therefore, such an equilibrium cannot exist. ■

Proof of Proposition 3.1. Suppose, by contrast, that there exists a full-disclosure equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$. Because $C \neq \emptyset$ and $D \neq \emptyset$, pick arbitrary types $\theta \in C$ and $\theta' \in D$. Note that the type-θ sender strictly prefers action y_2 to action y_1, and the type-θ' sender strictly prefers action y_1 to action y_2. Because $(\sigma^*, \mu^*; \mathcal{P}^*)$ is a full-disclosure equilibrium, $\mu^*(\sigma^*(\theta)) = y_1$ and $\mu^*(\sigma^*(\theta')) = y_2$. By Lemma 3.1, there exists an off-the-equilibrium-path message m that is available to both types θ and θ'. However, $\mu^*(m) = y_1$ implies a deviation of type θ', and $\mu^*(m) = y_2$ implies a deviation of type θ, which is a contradiction. Thus, there exists no full-disclosure equilibrium. ■

Proof of Proposition 3.2

Without loss of generality, we assume that $\mathbb{E}[u(\theta, y_1)|C \cup D] \geq \mathbb{E}[u(\theta, y_2)|C \cup D]$. We omit the the characterizations of PBEs and the related proofs. These are in Appendix 3-B. We denote the equilibrium by $(\sigma^*, \mu^*; \mathcal{P}^*)$. We show that the equilibrium attains the maximum ex ante expected utility to the receiver. Suppose, by contrast, that there exists an equilibrium $(\hat{\sigma}, \hat{\mu}; \hat{\mathcal{P}})$ such that:

$$\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))] > \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))]. \tag{3.2}$$

Claim 3.1 *In any equilibrium, $\mu(\sigma(\theta)) = y^R(\theta), \forall \theta \in A \cup B \cup E$.*

Proof of Claim 3.1. Suppose, by contrast, that there exists an equilibrium $(\tilde{\sigma}, \tilde{\mu}; \tilde{\mathcal{P}})$, in which there exists a type $\theta \in A \cup B \cup E$ such that $\tilde{\mu}(\tilde{\sigma}(\theta)) \neq y^R(\theta)$. However, this type has the same preference as the receiver, and he can prove his true type by sending the message $m = \{\theta\}$. So, this message induces the sender's preferred action; that is, it is a profitable deviation for him, which is a contradiction. Therefore, these types must induce preferred actions in any equilibrium. □

By (3.2):

$$\begin{aligned}
&\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|A]Pr(A) + \mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|B]Pr(B) \\
&\qquad + \mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|E]Pr(E) + \mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C \cup D]Pr(C \cup D) \\
&> \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))|A]Pr(A) + \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))|B]Pr(B) \\
&\qquad + \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))|E]Pr(E) + \mathbb{E}[u(\theta, y_1)|C \cup D]Pr(C \cup D).
\end{aligned} \tag{3.3}$$

By Claim 3.1:

$$\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C \cup D] > \mathbb{E}[u(\theta, y_1)|C \cup D]. \tag{3.4}$$

Define $W \equiv \{\theta' \in D | \hat{\mu}(\hat{\sigma}(\theta')) = y_2\}$.

Claim 3.2 *If equation (3.4) holds, then $W \neq \emptyset$.*

Proof of Claim 3.2. Suppose, by contrast, that $W = \emptyset$. That is, for any $\theta' \in D$, $\hat{\mu}(\hat{\sigma}(\theta')) = y_1$. By (3.4):

$$\begin{aligned} &\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|D]Pr(D|C \cup D) \\ &> \mathbb{E}[u(\theta, y_1)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|D]Pr(D|C \cup D). \end{aligned} \tag{3.5}$$

From the hypothesis:

$$\begin{aligned} &\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|D]Pr(D|C \cup D) \\ &> \mathbb{E}[u(\theta, y_1)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|D]Pr(D|C \cup D), \end{aligned} \tag{3.6}$$

or:

$$\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C] > \mathbb{E}[u(\theta, y_1)|C]. \tag{3.7}$$

However, as long as $\theta \in C, u(\theta, y_1) > u(\theta, y_2)$. Hence, $\mathbb{E}[u(\theta, y_1)|C] \geq \mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))|C]$, which is a contradiction to (3.7). $\square$

Define $Z \equiv \{\theta \in C | \hat{\mu}(\hat{\sigma}(\theta)) = y_1\}$.

Claim 3.3 *If equation (3.4) holds, then $Z \neq \emptyset$.*

Proof of Claim 3.3. Suppose, by contrast, that $Z = \emptyset$; that is, for all $\theta \in C$, $\hat{\mu}(\hat{\sigma}(\theta)) = y_2$. By (3.4), the hypothesis and the definition of W:

$$\begin{aligned} & \mathbb{E}[u(\theta, y_2)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_2)|W]Pr(W|C \cup D) + \mathbb{E}[u(\theta, y_1)|D\backslash W]Pr(D\backslash W|C \cup D) \\ > \;& \mathbb{E}[u(\theta, y_1)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|W]Pr(W|C \cup D) + \mathbb{E}[u(\theta, y_1)|D\backslash W]Pr(D\backslash W|C \cup D), \end{aligned}$$

or:

$$\begin{aligned} & \mathbb{E}[u(\theta, y_2)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_2)|W]Pr(W|C \cup D) \\ & \qquad > \mathbb{E}[u(\theta, y_1)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|W]Pr(W|C \cup D). \end{aligned} \tag{3.8}$$

Because $\mathbb{E}[u(\theta, y_1)|\theta \in C \cup D] \geq \mathbb{E}[u(\theta, y_2)|\theta \in C \cup D]$:

$$\begin{aligned} & \mathbb{E}[u(\theta, y_1)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_1)|W]Pr(W|C \cup D) + \mathbb{E}[u(\theta, y_1)|D\backslash W]Pr(D\backslash W|C \cup D) \\ \geq \;& \mathbb{E}[u(\theta, y_2)|C]Pr(C|C \cup D) + \mathbb{E}[u(\theta, y_2)|W]Pr(W|C \cup D) + \mathbb{E}[u(\theta, y_2)|D\backslash W]Pr(D\backslash W|C \cup D). \end{aligned} \tag{3.9}$$

By (3.8) and (3.9), $\mathbb{E}[u(\theta, y_1)|D\backslash W]Pr(W|C \cup D) > \mathbb{E}[u(\theta, y_2)|D\backslash W]Pr(W|C \cup D)$ must hold. If $Pr(W|C \cup D) = 0$, then, the above inequality does not hold, which is a contradiction. Hence, $Pr(W|C \cup D) \neq 0$. That is, $\mathbb{E}[u(\theta, y_1)|D\backslash W] > \mathbb{E}[u(\theta, y_2)|D\backslash W]$. However, as long as $\theta \in D, u(\theta, y_2) > u(\theta, y_1)$. Hence, $\mathbb{E}[u(\theta, y_2)|D\backslash W] > \mathbb{E}[u(\theta, y_1)|D\backslash W]$, which is a contradiction. □

By Claims 3.2 and 3.3, to hold (3.4), $W \neq \emptyset$ and $Z \neq \emptyset$; that is, there exists $\theta \in C$ and $\theta' \in D$ such that $\hat{\mu}(\hat{\sigma}(\theta)) = y_1$ and $\hat{\mu}(\hat{\sigma}(\theta')) = y_2$. By Lemma 3.1, there exists an off-the-equilibrium-path message $m \in M(\theta) \cap M(\theta')$. However, if $\hat{\mu}(m) = y_1$, then type θ' has an incentive to deviate, and if

$\hat{\mu}(m) = y_2$, then type θ has an incentive to deviate. This contradicts the notion that $(\hat{\sigma}, \hat{\mu}; \hat{\mathcal{P}})$ is an equilibrium. Therefore, an equilibrium satisfying (3.2) cannot exist; that is, equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$ is best for the receiver. ■

Proof of Proposition 3.3

Let $X \subset C \cup D$ be the subset in which the receiver undertakes action $y^R(\theta)$ for any $\theta \in X$ given an equilibrium $(\sigma, \mu; \mathcal{P})$.

Lemma 3.2 *Suppose that either* $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ *or* $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$. *Then, for any equilibrium* $(\sigma, \mu; \mathcal{P})$, $X \neq \emptyset$, *and either* $X \subseteq C$ *or* $X \subseteq D$.

Proof of Lemma 3.2. Suppose, by contrast, that there exists an equilibrium $(\tilde{\sigma}, \tilde{\mu}; \tilde{\mathcal{P}})$ such that $\tilde{X} = \emptyset$. That is, for all $\theta \in C \cup D$, $\tilde{\mu}(\tilde{\sigma}(\theta)) \neq y^R(\theta)$. In the equilibrium, types in region C are never pooling with types in D; otherwise, there exists a type $\theta \in C \cup D$ such that $\tilde{\mu}(\tilde{\sigma}(\theta)) = y^R(\theta)$. Hence, the following is necessary: (i) types in $B \cup C$ are pooling with the induction of action y_2; and (ii) types in $A \cup D$ are pooling with the induction of action y_1. However, given the assumption, $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ or $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$, either (i) the pooling of region $B \cup C$ induces action y_1 or (ii) the pooling of region $A \cup D$ induces action y_2 must hold, which is a contradiction. Therefore, for any equilibrium under the assumption, $X \neq \emptyset$.

Next, suppose, by contrast, that there exists an equilibrium $(\hat{\sigma}, \hat{\mu}; \hat{\mathcal{P}})$ such that $\hat{X} \cap C \neq \emptyset$ and $\hat{X} \cap D \neq \emptyset$. Choose $\theta \in \hat{X} \cap C$ and $\theta' \in \hat{X} \cap D$ arbitrarily. However, by Lemma 3.1, there exists an off-the-equilibrium-path message $m \in M(\theta) \cap M(\theta')$, and hence, there exists no incentive-compatible reaction to the message m; if $\tilde{\mu}(m) = y_1$, then type θ' has an incentive to deviate, and if $\tilde{\mu}(m) = y_2$, then type θ has an incentive to deviate, which is a contradiction. Therefore, either $X \subseteq C$ or $X \subseteq D$

must hold. ■

Proof of Proposition 3.3. We omit the the characterizations of PBE $(\sigma^*, \mu^*; \mathcal{P}^*)$ and the related proofs. These are in Appendix 3-B. We show that this equilibrium attains the minimum ex ante expected utility to the receiver. Define $c^- \equiv \inf C$ and $c^+ \equiv \sup C$. Because Θ is bounded, both c^- and c^+ are also bounded. Define $C_\delta \equiv \{\theta \in C | c^- \leq \theta \leq \delta\}$ for $\delta \in [c^-, c^+]$. Then, we define the following function:

$$G(\delta) \equiv \int_{B \cup C_\delta} (u(\theta, y_1) - u(\theta, y_2)) f(\theta) d\theta_1. \tag{3.10}$$

It is clear that $G(\cdot)$ is continuous in δ. Also note that $G(c^+) = \mathbb{E}[u(\theta, y_1) - u(\theta, y_2) | B \cup C] Pr(B \cup C)$. Because $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$, $G(c^+) > 0$. Because $G(c^-) < 0$, from the intermediate value theorem, there exists $\delta^* \in (c^-, c^+)$ such that $G(\delta^*) = 0$. That is, $\mathbb{E}[u(\theta, y_1) - u(\theta, y_2)|B \cup C_{\delta^*}] Pr(B \cup C_{\delta^*}) = 0$ is equivalent to $\mathbb{E}[u(\theta, y_1)|B \cup C_{\delta^*}] = \mathbb{E}[u(\theta, y_2)|B \cup C_{\delta^*}]$. Define $\bar{C} \equiv C_{\delta^*}$.

Because of Lemma 3.2, it is sufficient to show that: (i) there exists no equilibrium $(\hat{\sigma}, \hat{\mu}; \hat{\mathcal{P}})$ with $\hat{X} \subseteq C$ such that $\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))] < \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))]$; and (ii) there exists no equilibrium $(\hat{\hat{\sigma}}, \hat{\hat{\mu}}; \hat{\hat{\mathcal{P}}})$ with $\hat{\hat{X}} \subseteq D$ such that $\mathbb{E}[u(\theta, \hat{\hat{\mu}}(\hat{\hat{\sigma}}(\theta)))] < \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))]$.

Let $(\hat{\sigma}, \hat{\mu}; \hat{\mathcal{P}})$ be an arbitrary equilibrium with $\hat{X} \subseteq C$. Define $\hat{C} \equiv C \backslash \hat{X}$.

Claim 3.4 $\mathbb{E}[u(\theta, y_2)|B \cup \hat{C}] \geq \mathbb{E}[u(\theta, y_1)|B \cup \hat{C}]$.

Proof of Claim 3.4. Suppose, by contrast, that $\mathbb{E}[u(\theta, y_2)|B \cup \hat{C}] < \mathbb{E}[u(\theta, y_1)|B \cup \hat{C}]$. Note that $\hat{C} = \{\theta \in C | \hat{\mu}(\hat{\sigma}(\theta)) = y_2\}$. By the hypothesis, if types in $B \cup \hat{C}$ are pooling, then the receiver undertakes action y_1. Hence, to support the equilibrium, there must exist subsets $C' \subseteq \hat{C}$, $D' \subseteq D$

such that types in $C' \cup D'$ are pooling and the pooling induces action y_2. Because $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$ and $\hat{X} \subseteq C$, by Lemma 3.2, $\hat{X} \cap D = \emptyset$. However, for all $\theta \in D'$, $\hat{\mu}(\hat{\sigma}(\theta)) = y_2 = y^R(\theta)$, so $\hat{X} \cap D \neq \emptyset$, which is a contradiction. $\square$

By Claim 3.4, in order to minimize the receiver's ex ante expected utility, region C must be partitioned into $\tilde{C}$ and $C \backslash \tilde{C}$, where $\tilde{C}$ is a solution to the following problem.

$$\min_{\hat{C} \subseteq C} \mathbb{E}[u(\theta, y_2)|\hat{C}]Pr(\hat{C}) + \mathbb{E}[u(\theta, y_1)|C \backslash \hat{C}]Pr(C \backslash \hat{C}) \tag{3.11}$$
$$\text{subject to } \mathbb{E}[u(\theta, y_2)|B \cup \hat{C}] \geq \mathbb{E}[u(\theta, y_1)|B \cup \hat{C}].$$

Because the objective function of (3.11) is decreasing in the measure of $\hat{C}$ and because $\mathbb{E}[u(\theta, y_2)|B \cup \hat{C}] - \mathbb{E}[u(\theta, y_1)|B \cup \hat{C}]$ is decreasing in the measure of $\hat{C}$, the constraint of (3.11) must be binding at the optimal point. Hence, $\bar{C}$ is a candidate for the solution of (3.11). The following lemma guarantees that $\bar{C}$ is the solution to the maximization problem.

Lemma 3.3 *Let $\tilde{C}, \tilde{\tilde{C}} \subseteq C$ be subsets of region C such that $\mathbb{E}[u(\theta, y_1)|B \cup \tilde{C}] = \mathbb{E}[u(\theta, y_2)|B \cup \tilde{C}]$ and $\mathbb{E}[u(\theta, y_1)|B \cup \tilde{\tilde{C}}] = \mathbb{E}[u(\theta, y_2)|B \cup \tilde{\tilde{C}}]$. Then, $\mathbb{E}[u(\theta, y_2)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|C \backslash \tilde{C}]Pr(C \backslash \tilde{C}) = \mathbb{E}[u(\theta, y_2)|\tilde{\tilde{C}}]Pr(\tilde{\tilde{C}}) + \mathbb{E}[u(\theta, y_1)|C \backslash \tilde{\tilde{C}}]Pr(C \backslash \tilde{\tilde{C}})$.*

Proof of Lemma 3.3 Because $\mathbb{E}[u(\theta, y_1)|B \cup \tilde{C}] = \mathbb{E}[u(\theta, y_2)|B \cup \tilde{C}]$,

$$\mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|B \cup \tilde{C}]Pr(B \cup \tilde{C}) = 0$$
$$\Leftrightarrow \quad \mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|B]Pr(B) + \mathbb{E}[u(\theta, y_2) - u(\theta, y_2)|\tilde{C}]Pr(\tilde{C}) = 0. \tag{3.12}$$

Similarly, $\mathbb{E}[u(\theta, y_1)|B \cup \tilde{\tilde{C}}] = \mathbb{E}[u(\theta, y_2)|B \cup \tilde{\tilde{C}}]$ implies:

$$\mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|B]Pr(B) + \mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|\tilde{\tilde{C}}]Pr(\tilde{\tilde{C}}) = 0. \tag{3.13}$$

By using (3.12) and (3.13), we obtain $\mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|\tilde{C}]Pr(\tilde{C}) = \mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|\tilde{\tilde{C}}]Pr(\tilde{\tilde{C}})$. Note that:

$$\begin{aligned}
& \mathbb{E}[u(\theta, y_2) - u(\theta, y_1)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|C]Pr(C) \\
= \; & \mathbb{E}[u(\theta, y_2)|\tilde{C}]Pr(\tilde{C}) - \mathbb{E}[u(\theta, y_1)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|C\backslash\tilde{C}]Pr(C\backslash\tilde{C}) \\
= \; & \mathbb{E}[u(\theta, y_2)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|C\backslash\tilde{C}]Pr(C\backslash\tilde{C}).
\end{aligned}$$

Therefore, we conclude that $\mathbb{E}[u(\theta, y_2)|\tilde{C}]Pr(\tilde{C}) + \mathbb{E}[u(\theta, y_1)|C\backslash\tilde{C}]Pr(C\backslash\tilde{C}) = \mathbb{E}[u(\theta, y_2)|\tilde{\tilde{C}}]Pr(\tilde{\tilde{C}}) + \mathbb{E}[u(\theta, y_1)|C\backslash\tilde{\tilde{C}}]Pr(C\backslash\tilde{\tilde{C}})$. ■

By Lemma 3.3, any $\hat{C} \subseteq C$ satisfying $\mathbb{E}[u(\theta, y_1)|B \cup \hat{C}] = \mathbb{E}[u(\theta, y_2)|B \cup \hat{C}]$ is a solution of (3.11). Hence, $\bar{C}$ is also a solution. Therefore, any equilibrium with $X \subseteq C$ attains weakly better expected utility to the receiver than equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$.

Now, we show that there is no equilibrium with $X \subseteq D$ that attains less expected utility to the receiver than equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$. Suppose, by contrast, that there exists an equilibrium $(\hat{\sigma}, \hat{\mu}; \hat{\hat{\mathcal{P}}})$ with $\hat{\hat{X}} \subseteq D$ such that $\mathbb{E}[u(\theta, \hat{\mu}(\hat{\sigma}(\theta)))] < \mathbb{E}[u(\theta, \mu^*(\sigma^*(\theta)))]$. By Lemma 3.2, for all $\theta \in C$, $\hat{\mu}(\hat{\sigma}(\theta)) = y_2$. Because $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$, if types in $B \cup C$ are pooling, then the receiver responds by undertaking action y_1. Hence, part of region C must be pooling with types in region D. That is, there exists a partition $\hat{\hat{C}}_1, \hat{\hat{C}}_2$ of region C and a subset $X' \subseteq \hat{\hat{X}}$ with the following properties: (i) types in $B \cup \hat{\hat{C}}_1$ are pooling and induce action y_2; and (ii) types in $\hat{\hat{C}}_2 \cup X'$ are pooling and induce action y_2. Note that $Pr(X') \neq 0$; otherwise, the types in the pooling region $\hat{\hat{C}}_2 \cup X'$ induce action y_1. That is:

$$\mathbb{E}[u(\theta, y_1)|B \cup \hat{\hat{C}}_1] \leq \mathbb{E}[u(\theta, y_2)|B \cup \hat{\hat{C}}_1] \quad (3.14)$$

$$\mathbb{E}[u(\theta, y_1)|\hat{\hat{C}}_2 \cup X'] \leq \mathbb{E}[u(\theta, y_2)|\hat{\hat{C}}_2 \cup X'] \quad (3.15)$$

Multiplying both sides of (3.14) and (3.15) by $Pr(B \cup \hat{\hat{C}}_1)$ and $Pr(\hat{\hat{C}}_2 \cup X')$, respectively, and combining the results yields:

$$\mathbb{E}[u(\theta, y_1)|B \cup C \cup X'] \leq \mathbb{E}[u(\theta, y_2)|B \cup C \cup X']. \quad (3.16)$$

Claim 3.5 *Let $(\sigma, \mu; \mathcal{P})$ be an equilibrium with $X \subseteq C$ and $Pr(X) \neq 0$, and $(\tilde{\sigma}, \tilde{\mu}, \tilde{\mathcal{P}})$ be an equilibrium with $\tilde{X} \subseteq D$ and $Pr(\tilde{X}) \neq 0$. Then, $\mathbb{E}[u(\theta, \mu(\sigma(\theta)))] \geq \mathbb{E}[u(\theta, \tilde{\mu}(\tilde{\sigma}(\theta)))]$ is equivalent to $\mathbb{E}[u(\theta, y_1)|X \cup \tilde{X}] \geq \mathbb{E}[u(\theta, y_2)|X \cup \tilde{X}]$.*

Proof of Claim 3.5. Note that:

$$\begin{aligned}\mathbb{E}[u(\theta, \mu(\sigma(\theta)))] &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|X]Pr(X) \\ &\quad + \mathbb{E}[u(\theta, y_2)|C \backslash X]Pr(C \backslash X) + \mathbb{E}[u(\theta, y_1)|\tilde{X}]Pr(\tilde{X}) + \mathbb{E}[u(\theta, y_1)|D \backslash \tilde{X}]Pr(D \backslash \tilde{X}).\end{aligned}$$

Then, $\mathbb{E}[u(\theta, \mu(\sigma(\theta)))] \geq \mathbb{E}[u(\theta, \tilde{\mu}(\tilde{\sigma}(\theta)))]$ is equivalent to:

$$\begin{aligned} \mathbb{E}[u(\theta,y_1)|X]Pr(X) + \mathbb{E}[u(\theta,y_1)|\tilde{X}]Pr(\tilde{X}) &\geq \mathbb{E}[u(\theta,y_2)|X]Pr(X) + \mathbb{E}[u(\theta,y_2)|\tilde{X}]Pr(\tilde{X}) \\ \Leftrightarrow \mathbb{E}[u(\theta,y_1)|X \cup \tilde{X}]Pr(X \cup \tilde{X}) &\geq \mathbb{E}[u(\theta,y_2)|X \cup \tilde{X}]Pr(X \cup \tilde{X}) \\ \Leftrightarrow \mathbb{E}[u(\theta,y_1)|X \cup \tilde{X}] &\geq \mathbb{E}[u(\theta,y_2)|X \cup \tilde{X}]. \qquad \square \end{aligned}$$

By construction, it is clear that $Pr(C \backslash \bar{C}) \neq 0$. In addition, because $Pr(X') \neq 0$, $Pr(\hat{\hat{X}}) \neq 0$. Then, by Claim 3.5, the hypothesis is equivalent to $\mathbb{E}[u(\theta,y_1)|(C\backslash\bar{C}) \cup \hat{\hat{X}}] > \mathbb{E}[u(\theta,y_2)|(C\backslash\bar{C}) \cup \hat{\hat{X}}]$. Because $u(\theta,y_2) > u(\theta,y_1)$ for any $\theta \in \hat{\hat{X}}$, the above inequality implies that:

$$\mathbb{E}[u(\theta,y_1)|(C\backslash\bar{C}) \cup X'] > \mathbb{E}[u(\theta,y_2)|(C\backslash\bar{C}) \cup X']. \tag{3.17}$$

Moreover, by the definition of $\bar{C}$:

$$\mathbb{E}[u(\theta,y_1)|B \cup \bar{C}] = \mathbb{E}[u(\theta,y_2)|B \cup \bar{C}]. \tag{3.18}$$

Multiplying both sides of (3.17) and (3.18) by $Pr((C\backslash\bar{C}) \cup X')$ and $Pr(B \cup \bar{C})$, respectively, and combining the results yields:

$$\mathbb{E}[u(\theta,y_1)|B \cup C \cup X'] > \mathbb{E}[u(\theta,y_2)|B \cup C \cup X']. \tag{3.19}$$

However equations (3.16) and (3.19) are contradictory. Therefore, such an equilibrium $(\hat{\hat{\sigma}}, \hat{\hat{\mu}}; \hat{\hat{\mathcal{P}}})$ cannot exist. Thus, the equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$ attains the minimum ex ante expected utility to the receiver. ■

Proof of Theorem 3.1

Because U^- and U^+ are the bounds of the receiver's ex ante expected utility, for any equilibrium, the receiver's ex ante expected utility in equilibrium must be in the interval $[U^-, U^+]$. Thus, the necessary part is obvious; it remains to prove sufficiency.

Define the following function:

$$H(\delta) \equiv \int_{C_\delta} (u(\theta, y_2) - u(\theta, y_1)) f(\theta) d\theta + \int_C u(\theta, y_1) f(\theta) d\theta. \tag{3.20}$$

Clearly, this function is continuous in δ. Without loss of generality, we assume that $\mathbb{E}[u(\theta, y_1)|C \cup D] \geq \mathbb{E}[u(\theta, y_2)|C \cup D]$. Then, by Proposition 3.2:

$$\begin{aligned} U^+ &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|C]Pr(C) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) \\ &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) + H(c^-). \end{aligned} \tag{3.21}$$

Similarly, by Proposition 3.3:[9]

$$\begin{aligned} U^- &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) \\ &\quad + \mathbb{E}[u(\theta, y_2)|\bar{C}]Pr(\bar{C}) + \mathbb{E}[u(\theta, y_1)|C \backslash \bar{C}]Pr(C \backslash \bar{C}) \\ &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) + H(\delta^*). \end{aligned} \tag{3.22}$$

[9] In this proof, we assume that $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$ and $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$. The proofs for other cases are in Appendix 3-B.

We fix $U \in [U^-, U^+]$ arbitrarily. From the intermediate value theorem, there exists a $\delta_U \in [c^-, \delta^*]$ such that:

$$\begin{aligned} U &= \mathbb{E}[u(\theta,y_1)|A]Pr(A) + \mathbb{E}[u(\theta,y_2)|B]Pr(B) + \mathbb{E}[u(\theta,y_1)|D]Pr(D) + H(\delta_U) \\ &= \mathbb{E}[u(\theta,y_1)|A]Pr(A) + \mathbb{E}[u(\theta,y_2)|B]Pr(B) + \mathbb{E}[u(\theta,y_1)|D]Pr(D) \\ &\quad + \mathbb{E}[u(\theta,y_2)|C_{\delta_U}]Pr(C_{\delta_U}) + \mathbb{E}[u(\theta,y_1)|C \backslash C_{\delta_U}]Pr(C \backslash C_{\delta_U}). \end{aligned} \tag{3.23}$$

Similarly, we can show that there exists a PBE that supports the above partition. That is, U can be supported as the receiver's ex ante expected utility. ■

3.6 Appendix 3-B: Supplemental Materials

Characterization of the best equilibrium for the receiver

Without loss of generality, assume that $\mathbb{E}[u(\theta,y_1)|C \cup D] \geq \mathbb{E}[u(\theta,y_2)|C \cup D]$. We show that the following is a PBE:[10]

$$\begin{aligned} \sigma^*(\theta) &= \begin{cases} \{\theta\} & \text{if } \theta \in A \cup B \cup E \\ C \cup D & \text{if } \theta \in C \cup D \end{cases} \\ \mu^*(m) &= \begin{cases} y_1 & \text{if } \mathbb{E}[u(\theta,y_1)|S(\mathcal{P}^*(\cdot|m))] \geq \mathbb{E}[u(\theta,y_2)|S(\mathcal{P}^*(\cdot|m))] \\ y_2 & \text{otherwise} \end{cases} \\ \mathcal{P}^*(\theta|m) &= \begin{cases} 1 & \text{if } m \text{ is a singleton and } m \subset A \cup B \cup E \text{ and } \theta \in m \\ \frac{f(\theta)}{\int_{C \cup D} f(\theta)d\theta} & \text{if } m = C \cup D \text{ and } \theta \in C \cup D \\ 0 & \text{if } [m \text{ is a singleton and } m \subset A \cup B \cup E \text{ and } \theta \notin m], \text{ or} \\ & [m = C \cup D \text{ and } \theta \notin C \cup D] \end{cases} \\ S(\mathcal{P}^*(\cdot|m)) &= \begin{cases} m \cap C & \text{if } m \text{ is an off-path message and } m \cap C \neq \emptyset \\ m & \text{otherwise.} \end{cases} \end{aligned} \tag{3.24}$$

[10]Depending on whether an off-the-equilibrium-path message m is countable or uncountable, the description of $\mathcal{P}^*(\cdot|m)$ is different. To simplify the description, hereafter, we specify the support of the posterior for off-the-equilibrium-path messages. Any off-the-equilibrium-path belief having the specified support is consistent with the equilibrium strategies.

It is clear that μ^* is the best response given the belief $\mathcal{P}^*$. Next, given μ^*, we check the optimality of σ^*. Note that only types in region C potentially have an incentive to deviate because other types can induce their preferred actions by following the strategy σ^*. However, any type must contain the true type in messages and if the observed message contains something in region C, then the receiver undertakes action y_1. That is, for types in region C, any available message induces action y_1. Hence, these types have no incentive to deviate, and hence, σ^* is the best response to μ^*. In addition, it is clear that the belief is consistent with Bayes' rule on the equilibrium path. Therefore, it is a PBE.

■

Characterization of the worst equilibrium for the receiver

As we have mentioned in the body of the chapter, the characterization of the worst equilibrium for the receiver depends on the receiver's utility function and the prior distribution of θ. There are the following four cases:

Case (i) $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$ and $\mathbb{E}[u(\theta, y_1)|B \cup C] \leq \mathbb{E}[u(\theta, y_2)|B \cup C]$;

Case (ii) $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$ and $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$;

Case (iii) $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ and $\mathbb{E}[u(\theta, y_1)|B \cup C] \leq \mathbb{E}[u(\theta, y_2)|B \cup C]$;

Case (iv) $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ and $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$.

Case (i)

In this case, the disagreement regions are less likely than the agreement regions. Hence, the effects by the disagreement regions can be ignored when the disagreement regions and the agreement regions are pooling:

Proposition 3.4 *Suppose that* $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$ *and* $\mathbb{E}[u(\theta, y_1)|B \cup C] \leq \mathbb{E}[u(\theta, y_2)|B \cup C]$. *Then, there exists an equilibrium in which: (i) types in* $A \cup D$ *are pooling; (ii) types in* $B \cup C$ *are pooling; and (iii) types in* E *disclose. Moreover, this equilibrium is worst for the receiver in Case (i)*

Proof of Proposition 3.4. We show that the following is a PBE:

$$\begin{aligned}
\sigma^*(\theta) &= \begin{cases} A \cup D & \text{if } \theta \in A \cup D \\ B \cup C & \text{if } \theta \in B \cup C \\ \{\theta\} & \text{if } \theta \in E \end{cases} \\
\mu^*(m) &= \begin{cases} y_1 & \text{if } \mathbb{E}[u(\theta, y_1)|S(\mathcal{P}^*(\cdot|m))] \geq \mathbb{E}[u(\theta, y_2)|S(\mathcal{P}^*(\cdot|m))] \\ y_2 & \text{otherwise} \end{cases} \\
\mathcal{P}^*(\theta|m) &= \begin{cases} \frac{f(\theta)}{\int_{A\cup D} f(\theta)d\theta} & \text{if } m = A \cup D \text{ and } \theta \in A \cup D \\ \frac{f(\theta)}{\int_{B\cup C} f(\theta)d\theta} & \text{if } m = B \cup C \text{ and } \theta \in B \cup C \\ 1 & \text{if } m \text{ is a singleton and } m \subset E \text{ and } \theta \in m \\ 0 & \text{if } [m = A \cup D \text{ and } \theta \notin A \cup D] \text{ or } [m = B \cup C \text{ and } \theta \notin B \cup C] \\ & \text{or } [m \text{ is a singleton and } m \subset E \text{ and } \theta \notin m] \end{cases} \\
S(\mathcal{P}^*(\cdot|m)) &= m, \text{ for all off-the-equilibrium-path message } m
\end{aligned} \tag{3.25}$$

By a similar argument to that used in section B.1, we can show that it is a PBE.[11] In addition, it is straightforward that this equilibrium is worst for the receiver because the receiver undertakes unfavorable actions in disagreement region $C \cup D$. ■

Case (ii)

This case have been already discussed in the body of the chapter. Hence, we just specify the worst equilibrium here. We have to consider the following two cases.

Case 1. $\mathbb{E}[u(\theta, y_1)|(C\backslash\bar{C}) \cup D] \geq \mathbb{E}[u(\theta, y_2)|(C\backslash\bar{C}) \cup D]$

The worst equilibrium is characterized as follows:

[11] Therefore, hereafter, we omit the proofs.

$$\sigma^*(\theta) = \begin{cases} \{\theta\} & \text{if } \theta \in A \cup E \\ B \cup \bar{C} & \text{if } \theta \in B \cup \bar{C} \\ (C \backslash \bar{C}) \cup D & \text{if } \theta \in (C \backslash \bar{C}) \cup D \end{cases}$$

$$\mu^*(m) = \begin{cases} y_1 & \text{if } \mathbb{E}[u(\theta, y_1)|S(\mathcal{P}^*(\cdot|m))] \geq \mathbb{E}[u(\theta, y_2)|S(\mathcal{P}^*(\cdot|m))] \\ y_2 & \text{otherwise} \end{cases} \tag{3.26}$$

$$\mathcal{P}^*(\theta|m) = \begin{cases} 1 & \text{if } m \text{ is a singleton and } m \subset A \cup E \text{ and } \theta \in m \\ \frac{f(\theta)}{\int_{B \cup \bar{C}} f(\theta) d\theta} & \text{if } m = B \cup \bar{C} \text{ and } \theta \in B \cup \bar{C} \\ \frac{f(\theta)}{\int_{(C \backslash \bar{C}) \cup D} f(\theta) d\theta} & \text{if } m = (C \backslash \bar{C}) \cup D \text{ and } \theta \in (C \backslash \bar{C}) \cup D \\ 0 & \text{if } [m \text{ is a singleton and } m \subset A \cup E \text{ and } \theta \notin m] \\ & \text{or } [m = B \cup \bar{C} \text{ and } \theta \notin B \cup \bar{C}] \\ & \text{or } [m = (C \backslash \bar{C}) \cup D \text{ and } \theta \notin (C \backslash \bar{C}) \cup D] \end{cases}$$

$$S(\mathcal{P}^*(\cdot|m)) = \begin{cases} m \cap (C \backslash \bar{C}) & \text{if } m \text{ is an off-path message and } m \cap (C \backslash \bar{C}) \neq \emptyset \\ m & \text{otherwise} \end{cases}$$

Case 2. $\mathbb{E}[u(\theta, y_1)|(C \backslash \bar{C}) \cup D] < \mathbb{E}[u(\theta, y_2)|(C \backslash \bar{C}) \cup D]$

Define $d^- \equiv \inf D$ and $d^+ \equiv \sup D$. Because Θ is bounded, both d^- and d^+ are finite. Define $D_\epsilon \equiv \{\theta \in D | d^- \leq \theta \leq \epsilon\}$ for $\epsilon \in [d^-, d^+]$. Then, define function $I(\epsilon)$ by: for $\epsilon \in [d^-, d^+]$,

$$I(\epsilon) \equiv \int_{(C \backslash \bar{C}) \cup D_\epsilon} (u(\theta, y_1) - u(\theta, y_2)) f(\theta) d\theta. \tag{3.27}$$

Clearly, this is continuous in ϵ, and $I(d^-) > 0$ and $I(d^+) < 0$. Hence, from the intermediate value theorem, there exists an $\tilde{\epsilon} \in (d^-, d^+)$ such that $I(\tilde{\epsilon}) = 0$; that is, $\mathbb{E}[u(\theta, y_1)|(C \backslash \bar{C}) \cup D_{\tilde{\epsilon}}] = \mathbb{E}[u(\theta, y_2)|(C \backslash \bar{C}) \cup D_{\tilde{\epsilon}}]$. Define $\tilde{D} \equiv D \backslash D_{\tilde{\epsilon}}$. Because $\mathbb{E}[u(\theta, y_1)|A \cup D] \geq \mathbb{E}[u(\theta, y_2)|A \cup D]$, $\mathbb{E}[u(\theta, y_1)|A \cup \tilde{D} \cup E] \geq \mathbb{E}[u(\theta, y_2)|A \cup \tilde{D} \cup E]$. The equilibrium is characterized as follows.

$$
\begin{aligned}
\sigma^*(\theta) &= \begin{cases} A \cup \tilde{D} \cup E & \text{if } \theta \in A \cup \tilde{D} \cup E \\ B \cup \bar{C} & \text{if } \theta \in B \cup \bar{C} \\ (C \backslash \bar{C}) \cup (D \backslash \tilde{D}) & \text{if } \theta \in (C \backslash \bar{C}) \cup (D \backslash \tilde{D}) \end{cases} \\
\mu^*(m) &= \begin{cases} y_1 & \text{if } \mathbb{E}[u(\theta, y_1)|S(\mathcal{P}^*(\cdot|m))] \geq \mathbb{E}[u(\theta, y_2)|S(\mathcal{P}^*(\cdot|m))] \\ y_2 & \text{otherwise} \end{cases} \\
\mathcal{P}^*(\theta|m) &= \begin{cases} \frac{f(\theta)}{\int_{A \cup \tilde{D} \cup E} f(\theta) d\theta} & \text{if } m = A \cup \tilde{D} \cup E \text{ and } \theta \in A \cup \tilde{D} \cup E \\ \frac{f(\theta)}{\int_{B \cup \bar{C}} f(\theta) d\theta} & \text{if } m = B \cup \bar{C} \text{ and } \theta \in B \cup \bar{C} \\ \frac{f(\theta)}{\int_{(C \backslash \bar{C}) \cup (D \backslash \tilde{D})} f(\theta) d\theta} & \text{if } m = (C \backslash \bar{C}) \cup (D \backslash \tilde{D}) \text{ and } \theta \in (C \backslash \bar{C}) \cup (D \backslash \tilde{D}) \\ 0 & \text{if } [m = A \cup \tilde{D} \cup E \text{ and } \theta \notin A \cup \tilde{D} \cup E] \\ & \text{or } [m = B \cup \bar{C} \text{ and } \theta \notin B \cup \bar{C}] \\ & \text{or } [m = (C \backslash \bar{C}) \cup (D \backslash \tilde{D}) \text{ and } \theta \notin (C \backslash \bar{C}) \cup (D \backslash \tilde{D})] \end{cases} \\
S(\mathcal{P}^*(\cdot|m)) &= \begin{cases} m \cap (C \backslash \bar{C}) & \text{if } m \text{ is an off-path message and } m \cap (C \backslash \bar{C}) \neq \emptyset \\ m & \text{otherwise} \end{cases}
\end{aligned}
\tag{3.28}
$$

Note that $\mu^*(B \cup \bar{C}) = y_2$ and $\mu^*(A \cup \tilde{D} \cup E) = \mu((C \backslash \bar{C}) \cup (D \backslash \tilde{D})) = y_1$.

Case (iii)

This case is essentially equivalent to Case (ii). Hence, we omit the proofs.

Proposition 3.5 *Suppose that* $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ *and* $\mathbb{E}[u(\theta, y_1)|B \cup C] \leq \mathbb{E}[u(\theta, y_2)|B \cup C]$. *Then, there exists an equilibrium in which: (i) types in* $A \cup \bar{D}$ *are pooling and induce action* y_1*; (ii) types in regions* $B, C,$ *and* E *induce their own preferred actions; and (iii) types in* $D \backslash \bar{D}$ *induce action* y_2*, where* $\bar{D}$ *is a subset of region* D *such that* $\mathbb{E}[u(\theta, y_1)|A \cup \bar{D}] = \mathbb{E}[u(\theta, y_2)|A \cup \bar{D}]$. *Moreover, this equilibrium is worst for the receiver in Case (iii).*

Case (iv)

Proposition 3.6 *Suppose that* $\mathbb{E}[u(\theta, y_1)|A \cup D] < \mathbb{E}[u(\theta, y_2)|A \cup D]$ *and* $\mathbb{E}[u(\theta, y_1)|B \cup C] > \mathbb{E}[u(\theta, y_2)|B \cup C]$. *Then, there exists either equilibrium* $(\sigma^*, \mu^*; \mathcal{P}^*)$ *or* $(\sigma^{**}, \mu^{**}; \mathcal{P}^{**})$ *specified in Propositions 3.3 and 3.5, respectively. Moreover, this equilibrium is worst for the receiver in Case (iv).*

Proof of Proposition 3.6. There are three cases: (a) $\mathbb{E}[u(\theta, y_1)|(C\backslash\bar{C})\cup(D\backslash\bar{D})] > \mathbb{E}[u(\theta, y_2)|(C\backslash\bar{C})\cup(D\backslash\bar{D})]$; (b) $\mathbb{E}[u(\theta, y_1)|(C\backslash\bar{C}) \cup (D\backslash\bar{D})] < \mathbb{E}[u(\theta, y_2)|(C\backslash\bar{C}) \cup (D\backslash\bar{D})]$; and (c) $\mathbb{E}[u(\theta, y_1)|(C\backslash\bar{C}) \cup (D\backslash\bar{D})] = \mathbb{E}[u(\theta, y_2)|(C\backslash\bar{C})\cup(D\backslash\bar{D})]$. Similarly, we can show that the equilibrium $(\sigma^*, \mu^*; \mathcal{P}^*)$ exists in (a), and that the equilibrium $(\sigma^{**}, \mu^{**}; \mathcal{P}^{**})$ exists in (b), and the both equilibria exist in (c). Moreover, by the same arguments as those used to prove Propositions 3.3 and 3.5, this is the worst equilibrium for the receiver in each case. ■

Proof of Theorem 3.1 other than Case (ii)

In the proof of Theorem 3.1 in Appendix 3-A, we only show the scenario of Case (ii). Here, we represent the formal proof for the other cases.

Case (i)

By Proposition 3.4:

$$\begin{aligned} U^- &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_2)|C]Pr(C) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) \\ &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) + H(c^+). \end{aligned} \tag{3.29}$$

We fix $U \in [U^-, U^+]$ arbitrarily. From the intermediate value theorem, there exists a $\delta_U \in [c^-, c^+]$ satisfying the desired property. ■

Case (iii)

Define:

$$J(\epsilon) \equiv \int_{A\cup D_\epsilon} (u(\theta, y_2) - u(\theta, y_1))f(\theta)d\theta. \tag{3.30}$$

It is clear that function $J(\epsilon)$ is continuous in ϵ. Also, note that $J(d^-) < 0$ and $J(d^+) > 0$. Then, by the intermediate value theorem, there exists $\epsilon^* \in (d^-, d^+)$ such that $\mathbb{E}[u(\theta, y_1)|A \cup D_{\epsilon^*}] = \mathbb{E}[u(\theta, y_2)|A \cup D_{\epsilon^*}]$. Define $\bar{D} \equiv D_{\epsilon^*}$.

Define:

$$K(\epsilon) \equiv \int_{D_\epsilon} (u(\theta, y_1) - u(\theta, y_2)) f(\theta) d\theta + \int_D u(\theta, y_2) f(\theta) d\theta. \tag{3.31}$$

Clearly, this function is continuous in ϵ. By Proposition 3.5:

$$\begin{aligned} U^- &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_2)|C]Pr(C) \\ &\quad + \mathbb{E}[u(\theta, y_1)|\bar{D}]Pr(\bar{D}) + \mathbb{E}[u(\theta, y_2)|D \backslash \bar{D}]Pr(D \backslash \bar{D}) \\ &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_2)|C]Pr(C) + K(\epsilon^*) \end{aligned} \tag{3.32}$$

Now, we show that there exists threshold $\hat{U} \in (U^-, U^+)$ such that for any $U \in [U^-, \hat{U}]$ can be supported as an equilibrium in which $X \subseteq D$, and for any $U \in [\hat{U}, U^+]$ can be supported as an equilibrium in which $X \subseteq C$. Define:

$$\begin{aligned} \hat{U} &\equiv \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) \\ &\quad + \mathbb{E}[u(\theta, y_2)|C]Pr(C) + \mathbb{E}[u(\theta, y_2)|D]Pr(D).^{10} \end{aligned} \tag{3.33}$$

Define function $L(\delta)$ by:

$$L(\delta) \equiv \int_{(C \backslash C_\delta) \cup D} (u(\theta, y_1) - u(\theta, y_2)) f(\theta) d\theta \tag{3.34}$$

[10] This is an equilibrium in which: (i) types in $A \cup E \cup D$ disclose; and (ii) types in $B \cup C$ are pooling. With appropriate off-the-equilibrium-path belief, types in D have no incentive to withhold.

Note that $L(\cdot)$ is continuous in δ and $L(c^+) < 0$. In addition, because $\mathbb{E}[u(\theta, y_1)|C \cup D] \geq \mathbb{E}[u(\theta, y_2)|C \cup D]$, $L(c^-) \geq 0$. Then, from the intermediate value theorem, there exists $\hat{\delta} \in [c^-, c^+)$ such that $L(\hat{\delta}) = 0$. Because $\mathbb{E}[u(\theta, y_2)|B \cup C] \geq \mathbb{E}[u(\theta, y_1)|B \cup C]$, we can say that $\mathbb{E}[u(\theta, y_1)|(C \backslash C_{\hat{\delta}}) \cup D] = \mathbb{E}[u(\theta, y_2)|(C \backslash C_{\hat{\delta}}) \cup D]$ and $\mathbb{E}[u(\theta, y_2)|B \cup C_{\hat{\delta}}] \geq \mathbb{E}[u(\theta, y_1)|B \cup C_{\hat{\delta}}]$. By construction:

$$\begin{aligned}
\hat{U} &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) \\
&\quad + \mathbb{E}[u(\theta, y_2)|C_{\hat{\delta}}]Pr(C_{\hat{\delta}}) + \mathbb{E}[u(\theta, y_1)|C \backslash C_{\hat{\delta}}]Pr(C \backslash C_{\hat{\delta}}) \\
&= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) + H(\hat{\delta}). \qquad (3.35)
\end{aligned}$$

Hence, for any $U \in [U^-, \hat{U}]$, there exists $\epsilon_U \in [d^-, \epsilon^*]$ such that:

$$\begin{aligned}
U &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_2)|C]Pr(C) + K(\epsilon_U) \\
&= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_2)|C]Pr(C) \\
&\quad + \mathbb{E}[u(\theta, y_1)|D_{\epsilon_U}]Pr(D_{\epsilon_U}) + \mathbb{E}[u(\theta, y_2)|D \backslash D_{\epsilon_U}]Pr(D \backslash D_{\epsilon_U}). \qquad (3.36)
\end{aligned}$$

We can construct an equilibrium in which: (i) types in $A \cup D_{\epsilon_U}$ are pooling with inducing action y_1; (ii) types in $B \cup C$ are pooling with inducing action y_2; and (iii) types in $(D \backslash D_{\epsilon_U}) \cup E$ disclose.

Similarly, for any $U \in [\hat{U}, U^{+}]$, there exists $\delta_U \in [c^{-}, \hat{\delta}]$ such that:

$$
\begin{aligned}
U &= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) + H(\delta_U) \\
&= \mathbb{E}[u(\theta, y_1)|A]Pr(A) + \mathbb{E}[u(\theta, y_2)|B]Pr(B) + \mathbb{E}[u(\theta, y_1)|D]Pr(D) \\
&\qquad + \mathbb{E}[u(\theta, y_2)|C_{\delta_U}]Pr(C_{\delta_U}) + \mathbb{E}[u(\theta, y_1)|C \backslash C_{\delta_U}]Pr(C \backslash C_{\delta_U}). \qquad (3.37)
\end{aligned}
$$

Note that because $(C \backslash C_{\hat{\delta}}) \subseteq (C \backslash C_{\delta_U})$ and $\mathbb{E}[u(\theta, y_1)|(C \backslash C_{\hat{\delta}}) \cup D] = \mathbb{E}[u(\theta, y_2)|(C \backslash C_{\hat{\delta}}) \cup D]$, $\mathbb{E}[u(\theta, y_1)|(C \backslash C_{\delta_U}) \cup D] > \mathbb{E}[u(\theta, y_2)|(C \backslash C_{\delta_U}) \cup D]$. Therefore, there exists an equilibrium in which: (i) types in $A \cup E$ disclose; (ii) types in $B \cup C_{\delta_U}$ are pooling with inducing action y_2; and (iii) types in $(C \backslash C_{\delta_U}) \cup D$ are pooling with inducing action y_1. ■

Case (iv)

We have to consider the following three cases:

Case 1: $\mathbb{E}[u(\theta, y_1)|(C \backslash \bar{C}) \cup D] \geq \mathbb{E}[u(\theta, y_2)|(C \backslash \bar{C}) \cup D]$

We can prove the statement by using the argument for Case (ii) mentioned in Appendix 3-A.

Case 2: $\mathbb{E}[u(\theta, y_1)|(C \backslash \bar{C}) \cup D] < \mathbb{E}[u(\theta, y_2)|(C \backslash \bar{C}) \cup D]$ **and there exists a subset** $\tilde{D} \subset D$ **such that** $\mathbb{E}[u(\theta, y_1)|A \cup \tilde{D}] \geq \mathbb{E}[u(\theta, y_2)|A \cup \tilde{D}]$ **and** $\mathbb{E}[u(\theta, y_1)|(C \backslash \bar{C}) \cup (D \backslash \tilde{D})] \geq \mathbb{E}[u(\theta, y_2)|(C \backslash \bar{C}) \cup (D \backslash \tilde{D})]$

We can prove the statement by using the argument for Case (ii) mentioned in Appendix 3-A.

Case 3: Otherwise

We can prove the statement by using the argument for Case (iii).

3.7 References

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Chapter 4

Manipulated News: Electoral Competition and Mass Media

4.1 Introduction

It is well accepted that mass media have substantial influence on political outcomes. In reality, media outlets report several types of information regarding elections, and their reports influence both candidates and voters. For example, media outlets report who is a candidate, what his/her proposed policy is, and to what extent the policy is endorsed by voters. Once we focus on information aspects, interactions between candidates and voters in real-world elections are indirect in the sense that mass media exist between candidates and voters, and mass media provide essential information for decision makings of candidates and voters. Most voters use the news as an information source for voting instead of directly acquiring relevant information. Candidates also care about polls conducted by mass media.

This chapter focuses on the situation where media outlets can omit part of election-relevant information from their reports. Suppression of information is one of the most frequent media manipulation in reality.[1] More or less relevant information is omitted through news generating process, and then the representation of the news could vary dependent on the emphasized facts chosen by media outlets, even if the same issues are reported.[2] Bagdikian (1997) supports this viewpoint.

> Different individuals writing about the same scene never produce precisely the same account. ... Every basic step in the journalistic process involves a value-laden decision: Which of the infinite number of events in the environment will be assigned for coverage and which ignored? Which of the infinite observations confronting the reporter will be noted? Which of the facts noted will be included in the story? ... None of these is a truly objective decisions.

For example, according to a study by the Project for Excellence in Journalism assessing the tone of Iraq war coverage in 2005, Fox segments had more positive tone than negative, but CNN segments were almost equally likely to be positive and negative in tone.[3] Different toned segments that describe the same issue are evidence for lack of relevant information.

By exploiting the manipulation through omitting information, media outlets potentially have an incentive to influence public opinion. For example, in the Iowa Presidential Straw Poll in August 2011, Michele Bachmann won Ron Paul with just 152 more votes. In despite of this tiny difference,

[1] Groseclose and Milyo (2005) argue that "Instead, for every sin of commission, such as those by Glass or Blair, we believe that there are hundreds, and maybe thousands, of sins of omission–cases where a journalist chose facts or stories that only one side of the political spectrum is likely to mention."

[2] Note that suppressing information is not fabrication of information.

[3] In Fox, the ratios of positive and negative tone were 38% and 14% of all segments, respectively. On the other hand, in CNN, the ratios of positive and negative tone were 20% and 23%, respectively.

152 votes out of 16892 cast, Bachmann's media exposure after the poll completely dominated that of Paul; according to Eddlem (2011), "Michele Bachmann managed to book herself on all five major Sunday national television political talk shows. But Ron Paul, who finished in a virtual statistical tie with Bachmann -just 152 votes and less than a one-percent difference- was booked on none of them. Zero." Carney (2011) argues that the reason why Ron Poul was ignored is "... the mainstream media and the Republican establishment wish he would just go away." Although this example seems extreme, this demonstrates that media outlets strategically withhold information to influence public opinion.

The main concern of this chapter is making clear of how rational candidates and voters react to such media manipulation by ideological media outlets. Especially, we focus on the situation where the media outlets suppress information about policies proposed by candidates. Because of this motivation, we develop a simple Downsian voting model including media outlets. In the model, there exist two candidates, single/multiple media outlets and one voter who are rational. Unlike standard models, we assume that the voter cannot directly observe the policies proposed by the candidates. Instead, the voter learns this information through reports from the media outlets. In other words, we consider the following two-stage game. In the first stage, the two candidates simultaneously propose policies that only the media outlets observe. In the second stage, the media outlets send news regarding those policies after which the voter chooses one of the candidates. The media outlets choose either disclose or withhold information about the proposed policies. Under this environment, we pursue the following question. When ideological media outlets, say Fox news, can strategically conceal the hardest policy of a conservative candidate to making the candidate seem more moderate, how do equilibrium outcomes are affected?

The results are as follows. In the model with single media outlet, we show that for any equilibrium, there exist a pair of policies on which the voter's decision is ex post incorrect when the preferences between the media outlet and the voter sufficiently diverge. When the voter can correctly infer why the media outlet withholds information, the voter's decision is ex post correct on the equilibrium path. However, to guarantee the correctness on the equilibrium path, the voter must give up ex post correct decision at some policy pair off the equilibrium path. Because of the voter's ex post incorrect decision making, appealing to the voter becomes less attractive to the candidates. The candidates then have an incentive to win the election by influence the media outlet's behavior through policy settings instead of appealing to the voter; one candidate tries to induce media manipulation, but the other tries to avoid. As a result, the equilibrium outcomes are distorted in favor of the media outlet when compared with no media model through the distortions in the voter's and the candidates's behaviors; that is, the median voter theorem could fail. This distortion mechanism could observe in the model with multiple media outlets. If there exist two media outlets and their preferences are like biased, then the same distortion mechanism can be observed. However, if the media outlets' preferences are opposing biased, then no information distortion occurs. That is, because of the monitoring by the media outlets, the median voter theorem holds again under this setup.

This chapter is organized as follows. In the following subsection, we briefly review the related literature. Through Sections 4.2 to 4.4, we focus on the model with single media outlet. Section 4.2 defines the formal model. In Section 4.3, we analyze a benchmark model without mass media, and consider a model with single outlet in Section 4.4. In Section 4.5, we consider the model with multiple media outlets, and conclude the chapter in Section 4.6.

4.1.1 Related literature

This chapter is based on several branches of economics. First, this chapter is positioned in the literature of political economy as a paper that demonstrates non-policy-convergence results by changing the basic setup. As the basic framework of the analysis, we adopt the Downsian voting model in which the candidates are fully office-motivated, as introduced by Downs (1957). In this environment, the equilibrium policies converge to the median voter's ideal policy. This is the well-known *median voter theorem*. As Roemer (2001) explains, because the policy convergence is inconsistent with observations in the real world, filling the gap between the model predictions and these observations is one of the main concerns in the literature. Included in this branch of work are several papers deriving policy divergence results by modifying the setup. One modification is introducing voter's uncertainty.[4] Kartik and McAfee (2007) and Kikuchi (2010) assume that the candidates have payoff-relevant private information.[5] Because policy announcing by the candidates is also a signal of the private information, the policy divergence results occur.

This chapter also introduces voters' uncertainty, and especially, focuses on the situation where the voters have imperfect information about proposed policies because of media manipulation. McKelvey and Ordeshook (1985) also study the situation where the voters face uncertainty about proposed policies. In their model, because the uninformed voters can partially learn the information about proposed policies by observing the poll that reflects behaviors of the informed voters, the policy convergence result holds. However, to what extent the information is released by the poll is exoge-

[4]For other examples, Palfrey (1984) derives the divergence results by introducing the entrant of the third candidate. Calvert (1985) assumes that the candidates are policy-motivated and face uncertainty about the location of the median voter, and then derives the divergence.

[5]In Kartik and McAfee (2007), the private information is the candidates' "character," so the candidates could have different private information. On the other hand, in Kikuchi (2010), the private information is state of nature, and then the candidates have identical private information.

nously fixed, and it seems a demanding assumption. Therefore, this chapter studies a model where the information released to the voters are endogenously determined by the strategic media outlets, and then the limited information setup derives the divergence result.

Second, this chapter is mostly related to mass media economics. Recent but growing literature discusses how mass media as information providers affect electoral outcomes.[6] That is, asymmetric information between candidates (or media outlets) and voters is essential. This literature can be divided into the following two cases depending on the roles of mass media. On the one hand, media outlets are modeled as "watchdogs" on elections. In other words, in the models, the voters update their beliefs about payoff-relevant uncertainty by observing both candidates' behaviors and reports from the media outlets. Chan and Suen (2008, 2009) consider a two-candidate election model where the candidates are policy-motivated, and the media outlets give cheap talk endorsements to the voters. Ashworth and Shotts (2010) consider a retrospective voting model where the incumbent politician has reputational concern, and the media outlets provide cheap talk endorsements. Because additional information is provided by the media outlets, the candidates (or the incumbent) could behave more friendly to the voters.

On the other hand, media outlets are modeled as a intermediary in information transmission process. That is, voters' observations could be distorted by media manipulation. Bernhardt et al. (2008) consider a political campaigning model where the media outlets can conceal negative news about the candidates, and show that the media manipulation could exaggerate political polarization than the actual distribution of ideological preferences. Duggan and Martinelli (2010) consider a

[6] According to the recent survey in Prat and Strömberg (2011), there exist other theoretical researches; the analyses of (i) media capture by the government (Besley and Prat 2006); (ii) how mass media affect government's public policy (Strömberg 2004); and (iii) how media bias is generated (Mullainathan and Shleifer 2005; Baron 2006; Gentzkow and Shapiro 2006).

retrospective voting model where the media outlets reduce two-dimensional information about the challenger's policy into a one-dimensional "story" as media manipulation. Then, they characterize the optimal distortion for the pro and anti-incumbent media outlets. Finally, Adachi and Hizen (2011) analyze a similar model to Ashworth and Shotts (2010), but the rolls of the media outlets are different. In Adachi and Hizen (2011), the media outlets add noise into the voters' observations instead of giving additional endorsements. They show that media bias, even anti-incumbent bias, never improves social welfare.

This chapter is also located in the second branch. While the papers in the second branch make clear how the media manipulation affect the voters' behaviors, these papers do not well explain effects on the candidates's behaviors. That is, the literature does not fully answer to the question of how rational candidates respond to media manipulation that distorts interactions between candidates and voters. Bernhardt et al. (2008) and Duggan and Martinelli (2010) exogenously fix the candidates' proposed policies. Adachi and Hizen (2011) also consider strategic aspects of candidates, only the incumbent behaves strategically. Then, interactions between candidates are still unclear. Therefore, this chapter presents a model that treats candidates, media outlets and voters are fully rational to answer the above question.

Third, to describe suppression of information by media outlets, this chapter adopt a persuasion game framework in the literature of strategic communication. Persuasion games are sender–receiver games with hard private information, as first formalized by Milgrom (1981), for which there is now a voluminous literature, for example, Milgrom and Roberts (1986), Seidmann and Winter (1997) and Giovannoni and Seidmann (2007). In contrast with cheap talk games, like Crawford and Sobel (1982), the sender's private information in this framework is verifiable, so the information cannot be misreported, but the sender can conceal unfavorable information. The analysis in this chapter is based on Miura (2012). Furthermore, from the viewpoint of this literature, this chapter analyzes a hierarchical persuasion game in which the sender's private information is affected by the others' strategies.[7]

4.2 The Model

4.2.1 Setup

There are four players in our model: candidates 1 and 2, one media outlet and one (median) voter.[8] They play the following two stage game. In the first stage, called the *policy setting stage*, each candidate simultaneously proposes a policy, and only the media outlet can observe the proposed policies. In the second stage, called the *information disclosure stage*, the media outlet sends a message about the proposed policies to the voter. After observing the message, the voter casts the ballot for one of the candidates. The winning candidate then implements the proposed policy.

Let $X \equiv \{l, 0, r\} \subset \mathbb{R}$ be the set of available policies for the candidates with $l < 0 < r$ and $|l| > |r|$. Let $x_i \in X$ be the policy proposed by candidate $i \in \{1, 2\}$, and $x \equiv (x_1, x_2) \in X^2 \subset \mathbb{R}^2$ describe a pair of the proposed policies by the candidates. We assume that the information regarding x is *hard information*; that is, verifiable information. In addition, we assume that the media outlet correctly observes x, but the voter cannot. Hence, the information about x is the media outlet's private information at the information disclosure stage.

[7] Other types of hierarchical communication are studied in the literature. See, for example, Ivanov (2010), Li (2010) and Ambrus et al. (2011).

[8] We define the model with single outlet here. The model with multiple outlets is defined in Section 4.5, which can be easily defined as an analogy of the single outlet model. Throughout the chapter, we treat the candidates and the voter as male and the media as female.

Let $M(x) \equiv \{x, \phi\}$ be the message space of the media outlet when she observes policy pair x in the policy setting stage. The element x represents the media outlet's disclosure behavior. That is, the media outlet tells the voter what she observes. On the other hand, the element ϕ represents the withholding of information by the media outlet. That is, the media outlet completely conceals what she observes and tells nothing relevant to the voter. Note that the media outlet cannot say that the observed policy pair is x' when she observes $x \neq x'$ because the information is verifiable. Let $M \equiv \cup_{x \in X^2} M(x)$ be the universal message space, and $m \in M$ be the generic notation of the media outlet's message. Let $Y \equiv \{y_1, y_2\}$ be the voter's action space, where y_i represents that the voter casts the ballot for candidate $i \in \{1, 2\}$, and $y \in Y$ describes the generic notation of the voter's action.

We assume that there are two types of candidates: a *non-policy type* and a *policy type*. The non-policy type is the standard office-motivated strategic type of candidate. On the other hand, the policy type is a behavioral type of candidate that always proposes his preferred policy. We assume that if candidate 1 (resp. 2) is the policy type, then he always proposes policy r (resp. l). That is, we assume an asymmetry between the candidate.[9] Let $\Theta \equiv \{\theta_N, \theta_P\}$ be the candidates' type space, and θ_N (resp. θ_P) represent the non-policy (resp. policy) type. We assume that candidate i's type $\theta_i \in \Theta$ is candidate i's private information, and θ_1 and θ_2 are independently determined. Let $p > 0$ be the probability that each candidate is the non-policy type, and assume this is common knowledge.

The players' preferences are defined as follows. Define the non-policy-type candidate i's von

[9] We discuss this assumption in Appendix 4-B.

Neumann–Morgenstern utility function $u_i : Y \to \mathbb{R}$ by:

$$u_i(y) \equiv \begin{cases} 1 & \text{if } y = y_i \\ 0 & \text{Otherwise} \end{cases} \tag{4.1}$$

We assume that the media outlet and the voter have single-peaked preferences. Define the voter's von Neumann–Morgenstern utility function $v : X^2 \times Y \to \mathbb{R}$ by:

$$v(x, y) \equiv \begin{cases} -|x_1| & \text{if } y = y_1 \\ -|x_2| & \text{if } y = y_2. \end{cases} \tag{4.2}$$

Similarly, define the media outlet's von Neumann–Morgenstern utility function $w : X^2 \times Y \times \mathbb{R} \to \mathbb{R}$ by:

$$w(x, y, b) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1 \\ -|x_2 - b| & \text{if } y = y_2. \end{cases} \tag{4.3}$$

The voter's ideal policy is 0, but that of the media outlet is $b > 0$. Hence, the parameter b represents the difference between the preferences of the voter and the media outlet. We refer to this parameter throughout the chapter as *preference bias*. We assume that the level of preference bias is common knowledge.

The timing of the game is formalized as follows. At the policy-setting stage, nature chooses candidate i's type $\theta_i \in \Theta$ according to the prior distribution p, and only candidate i correctly learns his own type θ_i. Then, given θ_i, each candidate simultaneously proposes a policy $x_i \in X$. Only the media outlet can correctly observe the pair of proposed policies $x \in X^2$. At the information disclosure stage, given the observed pair x, the media outlet sends a message $m \in M(x)$. After observing the message, the voter undertakes an action $y \in Y$. The policy announced by the winning candidate is then implemented.

The players' strategies are defined as follows. The non-policy-type candidate i's strategy $\alpha_i \in \Delta(X)$ is a probability distribution over the policy space for $i \in \{1, 2\}$. This is represented by $\alpha_i = (\alpha_i^0, \alpha_i^r, 1 - \alpha_i^0 - \alpha_i^r)$ where α_i^j represents the probability that candidate i of the non-policy type proposes policy j. With some abuse of notation, a pure strategy of the non-policy-type candidate i is simply described as $\alpha_i = x_i$. The media outlet's strategy $\beta : X^2 \to M$ is a function from an observed policy pair to a message. The voter's strategy $\gamma : M \to \Delta(Y)$ is a function from an observed message to a probability distribution over the voter's action set Y. The voter's strategy is represented by $\gamma(m) = (q(m), 1 - q(m))$, where $q(m)$ represents the probability that the voter casts the ballot for candidate 1 when he observes message m. With further abuse of notation, the voter's pure strategy is simply represented by $\gamma(m) = y$. Let $\mathcal{P} : M \to \Delta(X^2)$ represent the voter's posterior belief, which is a function from an observed message to a probability distribution over the set of possible policy pairs X^2.

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept. Because the voter knows that only the media outlet that observes policy pair x' can send message $m = x'$, we insert the following requirement as a restriction to off-equilibrium-path beliefs.

Requirement 4.1 *For any $x' \in X^2$, if the voter observes a message $m = x'$, then the voter's posterior belief satisfies $\mathcal{P}(x = x'|m = x') = 1$.*

Definition 4.1 ***Perfect Bayesian Equilibrium***

A quintuple $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *is a PBE if it satisfies the following conditions;*

(i) *For every* $x_i \in supp(\alpha_i^*)$ *and* $i, j \in \{1, 2\}$ *with* $j \neq i$*,* $x_i \in \arg\max_{x_i' \in X} \mathbb{E}[u_i(\gamma^*(\beta^*(x_i', \alpha_j^*)))]$*;*

(ii) *For every* $x \in X^2$*,* $\beta^*(x) \in \arg\max_{m \in M(x)} w(x, \gamma^*(m), b)$*;*

(iii) *For every* $m \in M$ *and* $y \in supp(\gamma^*(m))$*,* $y \in \arg\max_{y' \in Y} \mathbb{E}[v(x, y')|m]$*;*

(iv) $\mathcal{P}^*$ *is derived by* α_1^*, α_2^* *and* β^* *consistently with Bayes' rule whenever it is possible. Otherwise,* $\mathcal{P}^*$ *is any probability distribution satisfying Requirement 4.1.*

In addition, we assume the following tie-breaking rules; one for the voter, and the other for the media outlet. Then, we focus on PBEs satisfying the tie-breaking rules in the subsequent analysis.

Requirement 4.2 ***Tie-breaking Rules***

(i) *Given the voter's posterior belief* $\mathcal{P}$*, if* y_1 *and* y_2 *are indifferent for the voter, then he votes for each candidate with probability* $\frac{1}{2}$*.*

(ii) *Given a policy pair* x *such that* $x_1 = x_2$*. Then, the media outlet discloses the information.*

In the subsequent analysis, we consider whether the median voter theorem holds as a reference point. We define the median voter theorem in this context as follows.

Definition 4.2 ***Median Voter Theorem***

(i) *We say that the strict median voter theorem holds if there exists a unique PBE in which* $\alpha_1^* = \alpha_2^* = 0$*.*

(ii) *We say that the weak median voter theorem holds if there exists a PBE in which* $\alpha_1^* = \alpha_2^* = 0$*.*

That is, we require the existence of a PBE in which both the non-policy-type candidates propose the voter's ideal policy. To simplify the description, an equilibrium in which the non-policy-type candidates propose x_1 and x_2 for certain is called (x_1, x_2) *equilibrium*.

4.2.2 Discussion of the model

Verifiability of information about policies

We assume that information about policies is verifiable. In reality, this information is explicitly included in the manifesto of each candidate, which anyone can check if he wishes. That is, there exists the objective evidence that proves whether the media outlet's message is true.

Impossibility of fabrication

As Groseclose and Milyo (2005) argue, media manipulation by fabrication of information is thought to be less likely than manipulation by omission. Hence, this assumption seems reasonable to capture standard behaviors of mass media. Moreover, even if the media outlet are allowed to fabricate the information, this option would not be used because of the verifiability of this information. Because of the verifiability, fabrication is easy to be disclosed. Once fabrication is disclosed to the public, then the media outlets would bear severe bad reputation. That is, fabrication seems too costly in this setup.

No direct message from candidates

In this model, the candidates cannot send message directly to the voter. Behind the model, we assume that the voter in this model represents swing voters who have a big impact on the election. That is, we assume that the voter is interested in the election, but does not acquire the information by himself because the cost of information acquisition is too costly. Hence, even though the candidates try to send messages directly to the voter (e.g., stump speech), these messages hard to reach to the voter. This assumption is consistent with the viewpoints of Downs (1957).

Simplified message structure

The assumption that the media outlet choose either fully disclose or fully withhold might seem extreme and discrepant to our motivation. Actually, we can easily extend the model to allow the media outlet to partially disclose the information about the policies. However, we obtain similar results. Accordingly, to simplify the analysis, we restrict the media outlet's message space as above.

Tie-breaking rule for the media outlet

We assume that the media outlet discloses the information if the proposed policies are convergent. This is an assumption to avoid the serious multiplicity of equilibrium. As we show in Appendix 4-B, the equilibria supported by the tie-breaking rule are robust to perturbation in the media outlet's behavior.

4.3 Benchmark Model: No Media

In this section, we analyze the model without the media outlet as a benchmark model; that is, the voter can directly observe the proposed policies. In the benchmark model, the median voter theorem holds, and thus, we can say that the voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the policy type.

The voter's equilibrium strategy is straightforward. Because the voter can directly observe the

probability	(θ_1, θ_2)	proposed policy pair	winner	equilibrium policy
p^2	(θ_N, θ_N)	$(0, 0)$	1 or 2	0
$p(1-p)$	(θ_N, θ_P)	$(0, l)$	1	0
$(1-p)p$	(θ_P, θ_N)	$(r, 0)$	2	0
$(1-p)^2$	(θ_P, θ_P)	(r, l)	1	r

Table 4.1: Equilibrium outcomes in the benchmark model

proposed policies, the voter can cast the ballot for the candidate whose policy is closer to his ideal point for certain. Then, the argument for the non-policy-type candidates is the same as in the standard Downsian models. That is, because the voter can directly observe policy pair x, proposing the voter's ideal policy, i.e., $\alpha_i = 0$, is the dominant strategy for the strategic candidates. In other words, the $(0, 0)$ equilibrium is the unique equilibrium. The equilibrium outcomes are summarized in Table 4.1. We can see that the voter's ideal policy can be supported as the equilibrium policy unless both candidates are of the policy type. The following proposition summarizes the results in the benchmark model.

Proposition 4.1 *Consider the benchmark model. Then,*

(i) $(0, 0)$ *equilibrium is the unique equilibrium, i.e., the strict median voter theorem holds.*

(ii) *The voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the policy type.*

Proof. All proofs are in Appendix 4-A. ■

4.4 Manipulated News Model

Now, we move back to the model involving the single media outlet. We refer to this as the *manipulated news model.* We show that the equilibrium outcomes are distorted in favor of the media outlet through the following two channels. The first channel is the distortion in the voter's behavior; the voter's decision making could be incorrect ex post in any equilibrium when the preference bias is not small. The second channel is the distortion in the candidates' behaviors; the candidates have an incentive to win the election by influence the media outlet's behavior through policy settings. That is, the weak median voter theorem could fail. This is the distortion mechanism generated by the media outlet that can manipulate the information about the proposed policies.

4.4.1 Information disclosure stage

In this subsection, we analyze a persuasion game between the media outlet and the voter given the candidates' proposed policies. First, it is worthwhile to make clear the voter's uncertainty at the beginning of the information disclosure stage. The voter faces uncertainty about the proposed policy pair because of the uncertainty about the candidates' types. For example, suppose that $\alpha_1^* = \alpha_2^* = 0$ are the non-policy-type candidates' equilibrium strategies. The voter knows that either one of the pairs, $(0,0), (0,l), (r,0)$ or (r,l) is proposed in the equilibrium, but he cannot specify which policy pair is actually proposed. This is the voter's uncertainty at the beginning of the information disclosure stage.[10] Therefore, the news from the media outlet is crucial for the voter to choose the correct candidate in the manipulated news model.

Next, we define, and characterize a *full-disclosure equilibrium* as a reference point. Let $Z(\alpha_1^*, \alpha_2^*)$ be the support of the voter's equilibrium prior, i.e., the set of possible policy pairs from the viewpoint of the voter given the equilibrium strategies α_1^* and α_2^*. This is defined by $Z(\alpha_1^*, \alpha_2^*) \equiv \{x' \in$

[10]In other words, as long as we use the Nash concepts, players correctly expect the others' strategies in equilibrium. That is, no one faces strategic uncertainty in equilibrium. In the manipulated news model, the policies proposed are the strategies of the candidates and so the voter correctly expects the candidates' strategies in equilibrium. However, because the voter does not know the types of candidates, she faces uncertainty about the proposed policy pair.

$X^2 | Pr.(x = x' | \alpha_1^*, \alpha_2^*) > 0\}$. Let $y^v(x)$ be the voter's ex post correct decision making defined by:

$$y^v(x) = \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2| \end{cases} \tag{4.4}$$

Then, we define a full-disclosure equilibrium as follows:

Definition 4.3 ***Full-Disclosure Equilibrium***

A PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *is a full-disclosure equilibrium if* $\gamma^*(\beta^*(x)) = y^v(x), \forall x \in Z(\alpha_1^*, \alpha_2^*)$.

That is, the full-disclosure equilibrium is a PBE where the voter chooses the preferred candidate for certain on the equilibrium path. There are a few remarks about the full-disclosure equilibrium to be made. First, the full-disclosure equilibrium only requires that the voter's decision making is correct ex post on the equilibrium path. Hence, we do not care about the correctness of the voter's decision making off the equilibrium path. Second, we also do not care about the media outlet's behavior. If the media outlet "directly" discloses the information, i.e., $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$, then this is obviously the full-disclosure equilibrium. However, even if the media outlet withholds the information, then this behavior by the media outlet could support the full-disclosure equilibrium. For instance, if the media outlet withholds the information on the equilibrium path only when the media outlet observes policy pair $x' \in Z(\alpha_1^*, \alpha_2^*)$, then withholding the information itself is a signal about policy pair x'. That is, information about policy pair x' is "indirectly" disclosed.

Now, we begin to characterize the equilibrium strategies of the voter and the media outlet. The voter's equilibrium behavior is straightforward. If the media outlet sends a message $m = x$, then

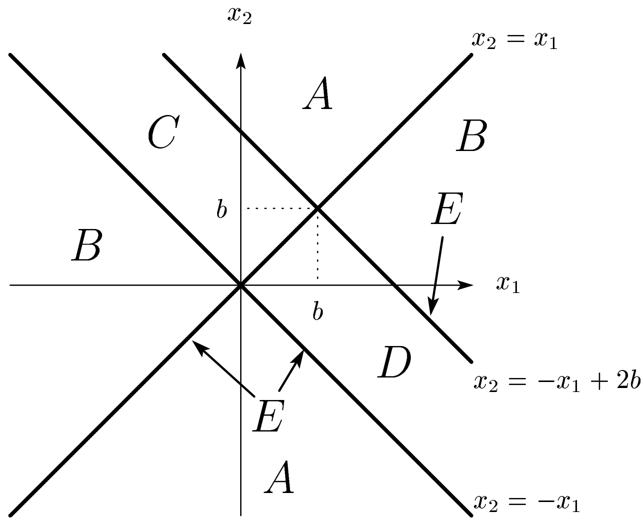

Figure 4.1: Distribution of preferences

the voter completely learns the proposed policies. Hence, the voter's decision making is correct. On the other hand, if the media outlet sends a message $m = \phi$, then the voter's uncertainty about the policy pair could remain. The voter's decision making is based on the posterior belief $\mathcal{P}(\cdot|\phi)$. Therefore, the voter's best response to message $m = \phi$ is characterized as follows:

$$\gamma^*(\phi) = \begin{cases} (1,0) & \text{if } \mathbb{E}[|x_1||\phi] < \mathbb{E}[|x_2||\phi] \\ (\frac{1}{2},\frac{1}{2}) & \text{if } \mathbb{E}[|x_1||\phi] = \mathbb{E}[|x_2||\phi] \\ (0,1) & \text{if } \mathbb{E}[|x_1||\phi] > \mathbb{E}[|x_2||\phi] \end{cases} \tag{4.5}$$

Given the voter's equilibrium strategy, consider the media outlet's strategy. Because we define the preferences of the voter and the media outlet as (4.2) and (4.3), given policy pair x, the voter prefers y_1 to y_2 if and only if $|x_1| \leq |x_2|$, and the media outlet prefers y_1 to y_2 if and only if $|x_1 - b| \leq |x_2 - b|$. Hence, the space $\mathbb{R}^2$ is divided into the following six regions, as shown in Figure 4.1:

$$A \equiv \{(x_1, x_2) \in \mathbb{R}^2 | [x_2 > x_1 \text{ and } x_2 > -x_1 + 2b] \text{ or } [x_2 < -x_1 \text{ and } x_2 < x_1\} \quad (4.6)$$

$$B \equiv \{(x_1, x_2) \in \mathbb{R}^2 | [-x_1 + 2b < x_2 < x_1] \text{ or } [x_2 < -x_1 \text{ and } x_2 > x_1]\} \quad (4.7)$$

$$C \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 > x_1 \text{ and } -x_1 < x_2 < -x_1 + 2b\} \quad (4.8)$$

$$D \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 < x_1 \text{ and } -x_1 < x_2 < -x_1 + 2b\} \quad (4.9)$$

$$E \equiv \{(x_1, x_2) \in \mathbb{R}^2 | x_2 = x_1 \text{ or } x_2 = -x_1 \text{ or } x_2 = -x_1 + 2b\} \quad (4.10)$$

We call regions A, B and E *agreement regions*, and regions C and D *disagreement regions*. If a proposed policy pair lies in the agreement regions, then the voter's and media outlet's preferences do not strictly conflict. In region A, both the voter and the media outlet strictly prefer y_1 to y_2, and in region B, they agree with strictly preferring y_2 to y_1. In region E, either the voter or the media outlet is indifferent between y_1 and y_2. On the other hand, if a proposed pair lies in the disagreement regions, then the voter's and the media outlet's preferences strictly conflict. In region C, the voter strictly prefers y_1 to y_2, but the media outlet strictly prefers y_2 to y_1. Similarly, in region D, the voter strictly prefers y_2 to y_1, but the media outlet strictly prefers y_1 to y_2.

If the media outlet observes a policy pair in the agreement regions, then disclosing the information is one of the best responses. Conversely, if the media outlet observes a policy pair in the disagreement regions, then withholding is weakly better than disclosing for herself. Hence, the media outlet's equilibrium strategy is characterized as follows:

$$\beta^*(x) = \begin{cases} x & \text{if } x \in (A \cup B \cup E) \cap X^2 \\ \phi & \text{if } x \in (C \cup D) \cap X^2 \end{cases} \quad (4.11)$$

Hereafter, we focus on equilibria satisfying (4.11) when we construct equilibria.[11]

The full-disclosure equilibrium is characterized as follows:

Proposition 4.2 *Consider the manipulated news model with single media outlet. There exists full-disclosure equilibrium* $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *if and only if either* $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ *or* $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$*.*

Whether the full-disclosure equilibrium exists depends on whether the voter can correctly infer the media outlet's motivation behind the withholding. For example, suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. That is, at the beginning of the information disclosure stage, the voter can infer that the media outlet wants to conceal the information only in the disagreement region C. Given the prior belief of the voter, the withholding itself is a signal showing that the proposed policy pair lies in the disagreement region C. Then, the voter chooses candidate 1 if he observes the withholding. In other words, because the voter can correctly infer the media outlet's motivation for withholding, the media outlet does not successfully conceal the unfavorable information on the equilibrium path, and full information disclosure is then possible.

However, if the voter is ambiguous about the media outlet's motivation behind the withholding, then the media outlet successfully conceals part of the unfavorable information on the equilibrium path. Suppose that $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. In the voter's prior belief, there are two explanations for the withholding. The voter cannot distinguish whether the media outlet tried to conceal the policy pair in disagreement region C or disagreement region D from observing the withholding. Because of this indeterminacy, the voter's decision making is incorrect with positive probability on the equilibrium path. That is, full information disclosure is impossible.[12]

[11] Of course, (4.11) is not the unique best response for the media outlet. If we show the impossibility results, we do

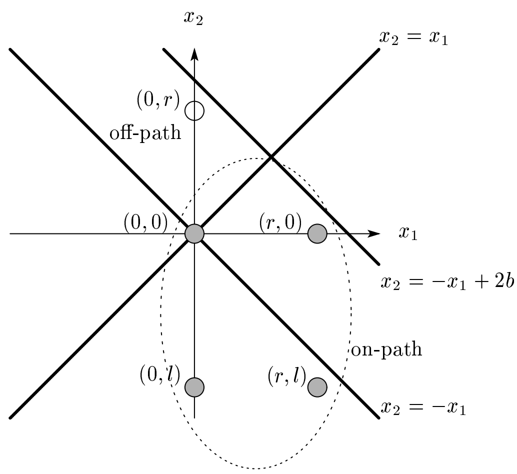

Figure 4.2: Incorrect decision making for off the equilibrium path policy.

As a corollary of Proposition 4.2, we obtain the following result.

Corollary 4.1 *Consider the manipulated news model with single media outlet. Suppose that* $b > \frac{1}{2}r$*. Then, in any equilibrium, there exists, at least, one policy pair* $x \in X^2$ *such that* $\gamma^*(\beta^*(x)) \neq y^v(x)$*.*

That is, the existence of the media outlet certainly distorts the voter's decision making for some policy pair. For a non-full-disclosure equilibrium, this claim is obvious. However, this claim is also true for the full-disclosure equilibrium. In any full-disclosure equilibrium with sufficiently large preference bias, the voter's decision making at some off the equilibrium path policy pair must be incorrect. In other words, full information disclosure on the equilibrium path is supported by the voter's incorrect decision making off the equilibrium path.

Suppose, for example, that $\alpha_1^* = 0$ and $\alpha_2^* = 0$. By Proposition 4.2, in this scenario the full-disclosure equilibrium exists because $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ as shown in Figure 4.2. To support this equilibrium, the voter's response to the withholding must be $\gamma^*(\phi) = y_2$. In this equilibrium, policy

not then restrict the media outlet's strategy to (4.11).

[12] In the terminology of persuasion games, if $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then there does not exist the *worst case inference* for message $m = \phi$.

pair $x = (0, r)$ is off the equilibrium path, and the voter and the media outlet disagree with the outcome given that policy pair; the voter prefers candidate 1, but the media outlet prefers candidate 2. Thus, given the voter's response $\gamma^*(\phi) = y_2$, the media outlet that observes policy pair $x = (0, r)$ withholds the information, and so candidate 2 wins for certain. That is, the voter's decision making at policy pair $x = (0, r)$ is incorrect ex post.

In summary, the media outlet successfully conceals part of the unfavorable information when the preference bias is not small. Because of this manipulation, the voter's decision making is certainly distorted at some policy pair. In the following subsection, we see how the voter's incorrect decision making affects the behavior of the non-policy-type candidates.

4.4.2 Policy setting stage

In this subsection, we analyze how the non-policy-type candidates behave in the policy setting stage. The media manipulation generates the following two effects. First, it is less beneficial for the candidates to propose the voter's ideal policy. Second, the candidates have an incentive to win the election by influencing the media outlet's behavior through policy settings. As a result, even the weak median voter theorem does not hold in the manipulated news model when the preference bias is not small.

Depending on the magnitude of the preference bias, we consider the following two cases. Suppose that $0 < b \leq \frac{1}{2}r$. In this case, the preference bias is so small that the media outlet's and the voter's preferences never conflict. Thus, the result is the same as that of the benchmark because all information is disclosed. That is, the strict median voter theorem holds. Then, hereafter, we suppose that $b > \frac{1}{2}r$. In this scenario, the voter and the media outlet conflict only over policies 0 and r. Hence, policy pairs $(0, r)$ and $(r, 0)$ are in the disagreement regions.

Proposition 4.3 *Consider the manipulated news model with single media outlet, and suppose $b > \frac{1}{2}r$. Then the strict median voter theorem does not hold. Moreover, the weak median voter theorem holds if and only if $p \leq \frac{1}{2}$.*[13]

There are the following two contrasts with the benchmark results. First, there exist multiple equilibria. In addition to the $(0, 0)$ equilibrium, there exist $(0, r)$, (r, r) and mixed strategy equilibria.[14] Second, the $(0, 0)$ equilibrium does not always exist; we require the condition that the policy-type candidates are more likely than the non-policy-type candidates.

The multiplicity of equilibrium arises because appealing to the voter becomes less attractive to the candidates because of ex post incorrect decision making by the voter. In the benchmark model, proposing the voter's ideal policy is the dominant strategy for both candidates. Because the voter correctly observes the proposed policy pair, appealing to the voter by proposing $x_i = 0$ is the unique way to maximize the winning probability. Hence, only the $(0, 0)$ equilibrium exists. However, in the manipulated news model, the voter could not correctly recognize the attractiveness of the candidate that proposes the voter's ideal policy because of the media manipulation. As a result, the candidate who proposes ex post less attractive policy for the voter could win with positive probability. Therefore, from the viewpoint of the candidates, proposing other than the voter's ideal policy is not dominated by proposing the voter's ideal policy.

In this case, there exists (r, r) equilibrium supported by $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. For candidate 2, the winning probability from $x_2 = r$ is $\frac{1}{2}$ in both the benchmark and manipulated news models. Suppose

[13]This is the necessary and sufficient condition under the tie-braking rules specified in Requirement 4.2.
[14]The characterization of each equilibrium is in Appendix 4-B.

that candidate 2 deviates to proposing $x_2 = 0$. In the benchmark, this deviation strictly improves his winning probability; candidate 2 wins for certain when $x = (r, 0)$. Consequently, the (r, r) equilibrium never exists. However, in the manipulated news model, this deviation does not strictly improve his winning probability. For candidate 2, he is indifferent between proposing $x_2 = r$ and $x_2 = 0$. The winning probability from proposing $x_2 = 0$ is also $\frac{1}{2}$ because the information at $x = (r, 0)$ is withheld by the media outlet, and then the voter chooses each candidate equally likely. That is, the voter's ex post incorrect decision making at $x = (r, 0)$ because of the manipulation makes this deviation less attractive to candidate 2. Therefore, the (r, r) equilibrium exists. For the same reason, there exists a mixed strategy equilibrium in which the candidates randomize policies $x_i = 0$ and $x_i = r$.

Unlike the benchmark, the $(0, 0)$ equilibrium does not exist when $p > \frac{1}{2}$ in the manipulated news model. This fragility of the $(0, 0)$ equilibrium arises from the incentive of the candidates to win the election by influencing the media outlet's behavior through policy settings instead of appealing to the voter. For example, suppose that an equilibrium is supported by $\gamma^*(\phi) = y_1$. Given the voter's response, candidate 1 prefers and candidate 2 dislikes media manipulation. Hence, through policy setting, candidate 1 has an incentive to lead the manipulation, which candidate 2 has an incentive to avoid. We refer to these incentives as *influence incentives*. The influence incentives are the main force in breaking down the $(0, 0)$ equilibrium; the $(0, 0)$ equilibrium is collapsed when the influence incentive dominates the incentive to appeal to the voter.

When $b > \frac{1}{2}r$, the $(0, 0)$ equilibrium is supported by $\gamma^*(\phi) = y_2$ when $p \leq \frac{1}{2}$. Given the voter's response, candidate 2 has the strong influence incentive to lead the manipulation. Because candidate 2 wins for certain when the proposed policy pair lies in the disagreement regions, candidate 2 wants to induce either policy pair $x = (r, 0)$ or $x = (0, r)$. If $p \leq \frac{1}{2}$, proposing $x_2 = 0$ is compatible with the influence incentive of candidate 2 because candidate 1 is more likely to propose $x_1 = r$. However, if $p > \frac{1}{2}$, then candidate 1 is more likely to propose $x_1 = 0$. Hence, proposing $x_2 = 0$ is no longer compatible with the influence incentive of candidate 2; proposing $x_2 = r$ is his best response. That is, the $(0, 0)$ equilibrium is collapsed by the candidate 2's influence incentive when $p > \frac{1}{2}$. Therefore, in this case, the weak median voter theorem does not hold. For the same reason, the candidate 2's influence incentive to avoid the manipulation supports the $(0, r)$ equilibrium when $p \leq \frac{1}{2}$.

In summary, the media manipulation distorts the behaviors of the non-policy-type candidates through the discount of the benefit from appealing to the voter and the influence incentives. With the growth of preference bias, the incentive of appealing to voters becomes weaker, but the influence incentive becomes stronger. Hence, if the preference bias is sufficiently large, the latter dominates the former. Therefore, policy convergence to the voter's ideal policy does not always hold, and several policy pairs can be supported in equilibrium.

4.4.3 Comparison of equilibrium outcomes

Given the analysis so far, we compare equilibrium outcomes of the manipulated news and the benchmark models. We have already shown that the equilibrium outcomes are distorted when the preference bias is sufficiently large. In this subsection, we consider the question of how equilibrium outcomes are distorted because of the media manipulation.

For ease of explanation, we focus on the mixed strategy equilibrium in which $\alpha_1^* = (\frac{1}{2p}, 1 - \frac{1}{2p}, 0)$ and $\alpha_2^* = (\frac{1}{2}, \frac{1}{2}, 0)$ when $b > \frac{1}{2}r$ and $p > \frac{1}{2}$. The equilibrium outcomes are summarized in Table 4.2. We can observe two kinds of distortions in this equilibrium. The first distortion is from the

probability	proposed policy pair	media	winner	equilibrium policy
$\frac{1}{4}p$	$(0, 0)$	discloses	1 or 2	0
$\frac{1}{4}p$	$(0, r)$	withholds	1 or 2	0 or r
$\frac{1}{2}(1-p)$	$(0, l)$	disclose	1	0
$\frac{1}{4}p$	$(r, 0)$	withholds	1 or 2	0 or r
$\frac{1}{4}p$	(r, r)	discloses	1 or 2	r
$\frac{1}{2}(1-p)$	(r, l)	discloses	1	r

Table 4.2: Equilibrium outcomes in the mixed strategy equilibrium

distortions in the candidates' behavior, and the second one is from the distortions in the voter's behavior. The first is *indirect distortion* and the second is *direct distortion*.

With indirect distortion, the equilibrium outcomes are distorted through distortions in the candidates' behavior. As already mentioned, the non-policy-type candidates have an incentive to propose other than the voter's ideal policy because of the media manipulation. As a result of the distortions in the candidates' behavior, the policy pairs proposed on the equilibrium path are changed, and so the winning policy is also changed. This is the indirect distortion. In the mixed strategy equilibrium, the indirect distortion appears in the fifth row of Table 4.2. As shown in Table 4.1, policy pair $x = (r, r)$ is never proposed on the equilibrium path in the benchmark model. However, because the non-policy-type candidates randomize policies, that policy pair can be proposed on the equilibrium path. Therefore, policy r becomes the winning policy, even if candidate 1 is the non-policy type.

In contrast, the direct distortion is distortion through the voter's behavior. As shown in Proposition 4.2, the voter's decision making could be incorrect on the equilibrium path. That is, because of the media manipulation, the voter chooses the unfavorable candidate with positive probability. As a result of this incorrect decision making, the winning policy is different from that of the benchmark model. This is the direct distortion. In the mixed strategy equilibrium, we can observe the direct distortion in the fourth row of Table 4.2. Policy pair $x = (r, 0)$ is proposed on the equilibrium path in both the benchmark and the manipulated news models. As shown in Table 4.1, policy 0 is the winning policy in the benchmark model. However, the winning policy is r with positive probability in the mixed strategy equilibrium because of the voter's incorrect choice.

We can observe either of the above distortions in all of the equilibria except for the $(0, 0)$ equilibrium. The voter's ex ante expected utility in the manipulated news model is then less than that of the benchmark model, and the winning policy is distorted to the media outlet's ideal policy with positive probability. Therefore, we can conclude that if we measure social welfare by the voter's ex ante expected utility, the presence of the strictly biased media outlet reduces social welfare.

4.5 Multiple Media Outlets

In this section, we consider a model with multiple media outlets. The model is modified as follows. There are two media outlets with preference biases $b_1, b_2 \neq 0$. The two media outlets correctly observe the proposed policy pair x, and each media outlet j simultaneously sends a message $m_j \in M(x)$ to the voter for $j \in \{1, 2\}$. We assume that the voter correctly observes both m_1 and m_2 before voting occurs. We say that the media outlets are *like biased* if $b_1 \cdot b_2 > 0$, and *opposing biased* if $b_1 \cdot b_2 < 0$. The results of the multiple outlet model crucially depend on the directions of the preference biases. In the like biased case, equilibrium outcomes are distorted because of the media manipulation similar to the single outlet model. However, if the media outlets have opposing-biased preferences, then the information about policy pairs is completely transmitted to the voter; that is the strict median voter theorem holds like the benchmark model.

4.5.1 Like-biased cases

Without loss of generality, we assume that $0 < b_1 < b_2$.[15] In the like-biased cases, we can obtain the identical results to those obtained in the model with single media outlet. Because a message from media outlet 2 never conveys extra information when a message from media outlet 1 does not disclose, the voter's and the candidates' decision do not change from the case where only a message from media outlet 1 is available. That is, if b_1 is large enough, then equilibrium outcomes are distorted to the direction of the preference biases through the direct and indirect distortions.

Proposition 4.4 *Consider the manipulated news model with like-biased multiple media outlets. Then the equilibrium outcomes are identical to those obtained in the manipulated news model with single media outlet.*

4.5.2 Opposing-biased cases

We assume that $b_2 < 0 < b_1$ without loss of generality. In the opposing-biased cases, the results are completely different from those in the model with single media outlet. If the media outlets have opposing-biased preferences, then the voter can learn all information by observing both messages. In other words, if one media outlet has an incentive to withhold the information, then the other outlet definitely has an incentive to disclose it. Suppose, for example, that the proposed policy pair is $x = (r, 0)$. Given the policy pair, media outlet 1 wants to withholds the information, but media outlet 2 discloses this information because the voter and the media outlet 2 share the same preference. Because the voter completely learns the relevant information by observing both m_1 and m_2, the media manipulation observed in the model with single media outlet never occurs. As a

[15] We can obtain the similar results with trivial modification if we assume that $b_1, b_2 < 0$. The details are available from the author upon the request.

result, the strict median voter theorem holds.

Proposition 4.5 *Consider the manipulated news model with opposing-biased multiple media outlets. Then, the strict median voter theorem holds.*

This proposition says that monitoring by mass media works well if there exist multiple media outlets with opposing-biased preferences. Any deviation that makes the voter worse off is completely reported at least one of the media outlet. Hence, the prediction of the model goes back to that in the benchmark model. This phenomenon is mentioned by Milgrom and Roberts (1986) in the literature of persuasion games.

4.6 Conclusion

This chapter has studied how mass media affect electoral competitions by analyzing the Downsian voting model including the media outlets that can suppress the information released to the voter, and specified the distortion mechanism of equilibrium outcomes.

In the manipulated news model with single media outlet, we have shown that equilibrium outcomes are distorted compared with the model without media outlets through the distortions in the voter's and the candidates' behaviors. When the preference bias is not small, the media outlet successfully conceals part of the unfavorable information in any equilibrium. Then, the voter's decision making at some concealed policy pair must be incorrect ex post. Because of the ex post incorrect decision making, appealing to the voter becomes more difficult than the model without media outlets. We can then observe a variety of policy distributions in equilibrium. Moreover, the non-policy-type candidates also have influence incentives. With the growth of preference bias, the influence incentives dominate the incentives of appealing to the voter. That is, when the preference bias is large enough, the candidates choose policies in order to influence the media outlet's behavior, not to appeal to the voter. The weak median voter theorem then fails. This is the distortion mechanism derived by the media outlet that strategically omits election-relevant information. Even if there exist multiple media outlets, we can observe the identical distortion mechanism when the media outlets are like biased. However, if the media outlets have opposing-biased preferences, then no information distortion occurs. As a result, the strict median voter theorem holds like standard Downsian voting models.

As part of the conclusion, we now briefly discuss some possible extensions. First, future research should revisit the multiple outlet model. In this chapter, we conclude that the strict median voter theorem holds in the model with opposing-biased multiple outlets. However, this conclusion crucially depends on the assumption that the voter correctly observes two messages without any cost. Therefore, we have to check how this conclusion is robust once this assumption is relaxed.

Consider the following modified model where the voter's observations are noisy. That is, even if the media outlet j send message $m_j = x$, this message might not reach to the voter with positive probability, and the voter cannot distinguish whether the media outlet withholds the information or the informative message does not reach by noise. Obviously, the voter's ex post incorrect decision making is guaranteed in this setup. However, it is not clear whether the influence incentive dominates the incentive to appealing to the voter. It is an interesting question to characterize the necessary and sufficient condition that the first incentive dominates the latter.

Second, future research should examine different voting model with including mass media. For example, instead of assuming fully office-motivated, we assume that the strategic candidates are also policy-motivated like Wittman (1973). Furthermore, we can relax the full commitment assumption of the winning candidate like Banks (1991) and Harrington (1992). Although these extensions are reasonable, it is nontrivial to check how the distortion mechanism specified in this chapter changes.

4.7 Appendix 4-A: Proofs

Proof of Proposition 4.1

(i) We show that the following is a PBE:

$$\begin{aligned} \alpha_1^* &= \alpha_2^* = 0 \\ \gamma^* &= \begin{cases} (1,0) & \text{if } |x_1| < |x_2| \\ (\frac{1}{2},\frac{1}{2}) & \text{if } |x_1| = |x_2| \\ (0,1) & \text{if } |x_1| > |x_2|. \end{cases} \end{aligned} \tag{4.12}$$

As already mentioned in the body of the chapter, it is easily shown that for the non-policy-type candidates, proposing $x_i = 0$ is the dominant strategy; for the non-policy-type candidate 1, it is the weakly dominant strategy, and for the non-policy-type candidate 2, it is the strictly dominant strategy. Therefore, policy pair $x = (0,0)$ is the unique equilibrium policy by the non-policy-type candidates. That is, the median voter theorem holds. (ii) It is obvious from Table 4.1. ■

Proof of Proposition 4.2

(Necessity) Suppose, in contrast, that there exists the full-disclosure equilibrium when $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Pick $x' \in C \cap Z(\alpha_1^*, \alpha_2^*)$ and $x'' \in D \cap Z(\alpha_1^*, \alpha_2^*)$, arbitrarily. Then,

$\gamma^*(\beta^*(x')) = (1,0)$ and $\gamma^*(\beta^*(x'')) = (0,1)$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$. That is, at least, one of the media outlet that observes policy pair either x' or x'' discloses the information. Without loss of generality, assume that $\beta^*(x') = x'$.

(1) $\beta^*(x'') = x''$**.** In this scenario, $m = \phi$ is an off-the-equilibrium-path message. Let $\gamma^*(\phi) = (q, 1-q)$ be the voter's response to the off-the-equilibrium-path message $m = \phi$, where $q \in [0,1]$. Because the media outlet that observes policy pair x'' choses $m = x''$ on the equilibrium path, $q = 0$; otherwise the media outlet has an incentive to deviate from $m = x''$ to $m = \phi$. However, given $\gamma^*(\phi) = (0,1)$, the media outlet that observes policy pair x' deviates from $m = x'$ to $m = \phi$, a contradiction.

(2) $\beta^*(x'') = \phi$**.** By the hypothesis, $\gamma^*(\phi) = (0,1)$. However, given the voter's best response, the media outlet that observes policy pair x' deviates to $m = \phi$, a contradiction.

Therefore, if there exists the full-disclosure equilibrium, then either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*)$.

(Sufficiency) Suppose that either $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ or $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$. Let $S(\mathcal{P}^*(\cdot|\phi))$ be the support of the voter's posterior after observing $m = \phi$. Then, we show the there exists the full-disclosure equilibrium supported by the media outlet's strategy specified by (4.11).

(1) $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$**.** Because any point in $Z(\alpha_1^*, \alpha_2^*)$ is in the agreement regions, $\beta^*(x) = x$ for all $x \in Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full-disclosure equilibrium because the voter chooses the preferred candidate for certain on the equilibrium path.

(2) $C \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$**.** Given the media outlet's equilibrium strategy specified by (4.11), $S(\mathcal{P}^*(\cdot|\phi)) \subset C$. Hence, $\gamma^*(\beta^*(x)) = (1,0)$ for any $x \in C \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, because the media outlet discloses the information about policy pairs in the agreement regions, this is the full-disclosure equilibrium.

(3) $C \cap Z(\alpha_1^*, \alpha_2^*) = \emptyset$ **and** $D \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$**.** Similar to case (2), $S(\mathcal{P}^*(\cdot|\phi)) \subset D$ given the media outlet's equilibrium strategy specified by (4.11), and then $\gamma^*(\beta^*(x)) = (0, 1)$ for all $x \in D \cap Z(\alpha_1^*, \alpha_2^*)$. Therefore, this is the full-disclosure equilibrium. ■

Proof of Corollary 4.1

Suppose that $b > \frac{1}{2}r$. Then, policy pair $(0, r)$ lies in disagreement region C, and policy pair $(r, 0)$ lies in disagreement region D. By Requirement 4.2, the voter's best response to $m = \phi$ must be either $\gamma^*(\phi) = (1, 0), (\frac{1}{2}, \frac{1}{2})$ or $(0, 1)$. If $\gamma^*(\phi) = (1, 0)$, then the media outlet that observes policy pair $(r, 0)$ withholds the information, and then $\gamma^*(\beta^*((r, 0))) \neq y^v((r, 0))$. Similarly, if $\gamma^*(\phi) = (0, 1)$, then $\gamma^*(\beta^*((0, r))) \neq y^v((0, r))$. If $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$, then $\gamma^*(\beta^*(x)) \neq y^v(x)$ for $x = (0, r)$ and $(r, 0)$ because the media outlet that observes policy pair $(0, r)$ or $(r, 0)$ strictly prefers the withholding. ■

Proof of Proposition 4.3

It is enough to show that there exist other equilibria other than the $(0, 0)$ equilibrium for proving the failure of the strict median voter theorem. When $p \leq \frac{1}{2}$, there exist $(0, 0), (0, r), (r, r)$ and mixed strategy equilibria. When $p > \frac{1}{2}$, there exist (r, r) and mixed strategy equilibria. The characterizations of such equilibria are in Appendix B. Hence, we show the necessary and sufficient condition for the existence of the $(0, 0)$ equilibrium.

(Sufficiency) Suppose that $p \leq \frac{1}{2}$. We show that the following is a PBE. Note that only policy pairs $x = (0, r)$ and $(r, 0)$ lie in the disagreement regions.

$$
\begin{aligned}
\alpha_1^* &= \alpha_2^* = 0 \\
\beta^*(x) &= \begin{cases} \phi & \text{if } x = (0,r) \text{ or } (r,0) \\ x & \text{otherwise} \end{cases} \\
\gamma^*(m) &= \begin{cases} (1,0) & \text{if } m = (0,r), (0,l) \text{ or } (r,l) \\ (\frac{1}{2},\frac{1}{2}) & \text{if } m = (0,0), (r,r) \text{ or } (l,l) \\ (0,1) & \text{if } m = (r,0), (l,0), (l,r) \text{ or } \phi \end{cases} \\
\mathcal{P}^*(x|m) &= \begin{cases} 1 & \text{if } [m = x' \text{ and } x = x' \text{ for any } x' \in X^2] \text{ or } [m = \phi \text{ and } x = (r,0)] \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\tag{4.13}
$$

It is obvious that $\gamma^*(\cdot)$ and $\beta^*(\cdot)$ is the best responses of the voter and the media outlet, respectively. For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1 - p$ and $\frac{1}{2}(1-p)$, respectively. Hence, candidate 1 does not deviate from $x_1 = 0$. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $1 - \frac{1}{2}p$, $\frac{1}{2}(1+p)$ and 0, respectively. Because $p \leq \frac{1}{2}$, candidate 2 does not deviate from $x_2 = 0$. Obviously, $\mathcal{P}^*(\cdot)$ is consistent with Bayes' rule on the equilibrium path. Hence, this is a PBE.

(Necessity) Suppose, in contrast, that there exists the $(0,0)$ equilibrium when $p > \frac{1}{2}$. Because $Z(\alpha_1^*, \alpha_2^*) = \{(0,0), (0,l), (r,0), (r,l)\}$, the following two scenarios are possible:

(1) Full disclosure scenario. By Proposition 4.2, the full-disclosure equilibrium is possible; that is, the voter's decision making is always correct on the equilibrium path. To support the full-disclosure equilibrium, $\gamma^*(\phi) = (0,1)$ is needed; otherwise, $\gamma^*(\beta^*((r,0))) \neq y^v((r,0))$. Hence, the media outlet sends $m = \phi$ when she observes policy pair $x = (0,r)$; this is off the equilibrium path. Given the voter and the media outlet's strategies, the winning probabilities of the candidates are same to the equilibrium characterized in the sufficiency part, so if $p > \frac{1}{2}$, then candidate 2 deviates to $\alpha_2 = r$, a contradiction.

(2) Withholding scenario. Suppose that the media outlet that observes policy pair $x = (r,0)$ is pooling with the media outlet that observes the policy pair either $x = (r, l)$ or $(0, l)$ by sending $m = \phi$. Because (r, l) and $(0, l)$ are in agreement region A, $\gamma^*(\phi) = (1, 0)$ is needed to hold this equilibrium; otherwise, the media outlet that observes either $x = (r, l)$ or $(0, l)$ deviates. Given the voter and the media outlet's strategies, candidate 1's winning probability from proposing $x_1 = 0$ is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = r$ is 1. Hence, candidate 1 deviates to $x_1 = r$, a contradiction.

Therefore, to hold the $(0, 0)$ equilibrium, $p \leq \frac{1}{2}$ is needed. ∎

Proof of Proposition 4.4

Because $0 < b_1 < b_2$, there are the following three cases: (i) $b_2 \leq \frac{1}{2}r$; (ii) $b_1 \leq \frac{1}{2}r < b_2$; and (iii) $b_1 > \frac{1}{2}r$. In cases (i) and (ii), media outlet 1 discloses all information, and then the voter completely learns the information regardless of the message from media outlet 2. In case (iii), both m_1 and m_2 withholds the policy pairs in the disagreement regions. Because, in each case, the voter's posterior after observing both m_1 and m_2 is equivalent to that after observing only m_1, the voter's decision making is identical in both scenarios. Because the voter's decision making does not change, the behaviors of the non-policy-type candidates also do not change. As a result, the equilibrium outcomes are identical to those derived in the manipulated news model with single media outlet. ∎

Proof of Proposition 4.5

First, let us introduce additional notations. Let $\beta_j : X^2 \to M$ be the strategy of media outlet j for $j \in \{1, 2\}$, and $\gamma : M^2 \to \Delta(Y)$ be the voter's strategy. A PBE is defined as an analogy of the model with single media outlet.

It is enough to show that for any policy pair $x \in X^2$, the voter's decision making is correct ex post in any equilibrium. If $b \leq \frac{1}{2}r$, then this statement is obvious. Hence, we assume $b > \frac{1}{2}r$ hereafter. Suppose, in contrast, that there exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta_1^*, \beta_2^*, \gamma^*; \mathcal{P}^*)$ such that there exists a policy pair $x \in X^2$ satisfying $\gamma^*(\beta_1^*(x), \beta_2^*(x)) \neq y^v(x)$. By Requirement 4.2-(ii), if $x_1 = x_2$, then $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = y^v(x)$. Hence, $x_1 \neq x_2$ is needed. Because $x_1 \neq x_2$, $y^v(x)$ is either $(1, 0)$ or $(0, 1)$. Without loss of generality, assume that $y^v(x) = (1, 0)$. That is:

$$|x_1| < |x_2|. \tag{4.14}$$

By the hypothesis, $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (0, 1)$ or $(\frac{1}{2}, \frac{1}{2})$. Suppose that $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (0, 1)$. To hold $\gamma^*(\beta_1^*(x), \beta_2^*(x)) \neq y^v(x)$, it is necessary that $\beta_1^*(x) = \beta_2^*(x) = \phi$; otherwise, the voter's decision making about x must be correct ex post. Because both media outlets 1 and 2 withhold the information, it implies that:

$$|x_2 - b_1| \leq |x_1 - b_1|; \tag{4.15}$$

$$|x_2 - b_2| \leq |x_1 - b_2|. \tag{4.16}$$

However, there exists no policy pair $x \in X^2$ satisfying (4.14), (4.15) and (4.16) simultaneously, which is a contradiction. In the case of $\gamma^*(\beta_1^*(x), \beta_2^*(x)) = (\frac{1}{2}, \frac{1}{2})$, we can derive a similar contradiction. Hence, in any equilibrium, the voter's decision making must be correct ex post for any $x \in X^2$. Therefore, $\alpha_i = 0$ is the unique way to maximize the winning probability for the candidates. As a result, only $(0, 0)$ equilibrium exists. ■

4.8 Appendix 4-B: Supplemental Materials

Characterizations of equilibria in the manipulated news model

We construct an equilibrium in which the media outlet's strategy is specified by (4.11). To avoid trivial repetitions, we, hereafter, omit descriptions of the media outlet's equilibrium strategy, the voter's posterior and the best response after observing the disclosure message.

(1) $(0, r)$ **equilibrium.** Suppose that $p \leq \frac{1}{2}$. Then:

$$\begin{aligned} \alpha_1^* &= 0 \\ \alpha_2^* &= r \\ \gamma^*(\phi) &= (1, 0) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} 1 & \text{if } x = (0, r) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4.17}$$

For candidate 1, the winning probability from proposing $x_1 = 0$ is 1, so candidate 1 has no incentive to deviate. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}p$, $\frac{1}{2}(1-p)$ and 0, respectively. Thus, this is a PBE as long as $p \leq \frac{1}{2}$. □

(2) (r, r) **equilibrium.** For any $p \in (0, 1)$:

$$\begin{aligned} \alpha_1^* &= \alpha_2^* = r \\ \gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\ \mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0, r) \text{ or } (r, 0) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4.18}$$

For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Thus, this is a PBE. □

(3) Mixed strategy equilibrium. Suppose that $p \geq \alpha_2^0$. Then:

$$\begin{aligned}
\alpha_1^* &= (\frac{\alpha_2^0}{p}, 1 - \frac{\alpha_2^0}{p}, 0) \\
\alpha_2^* &= (\alpha_2^0, 1 - \alpha_2^0, 0) \\
\gamma^*(\phi) &= (\frac{1}{2}, \frac{1}{2}) \\
\mathcal{P}^*(x|\phi) &= \begin{cases} \frac{1}{2} & \text{if } x = (0, r) \text{ or } (r, 0) \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{4.19}$$

For candidate 1, the winning probabilities from proposing $x_1 = 0, r$ and l are $1 - \frac{1}{2}p, 1 - \frac{1}{2}p$ and $\frac{1}{2}(1-p)$, respectively. For candidate 2, the winning probabilities from proposing $x_2 = 0, r$ and l are $\frac{1}{2}, \frac{1}{2}$ and 0, respectively. Obviously, $\mathcal{P}^*(\cdot|\phi)$ is consistent with Bayes' rule on the equilibrium path. Thus, this is a PBE. $\square$

Robustness

In this subsection, we discuss how the results in the model with single media outlet is robust by relaxing the assumptions.

Asymmetry between the candidates: symmetric setup

We have assumed that the candidates are asymmetric in the sense that the preferred policies of the policy-type candidates are different. This asymmetry is an essential assumption to the results; the asymmetry generates the influence incentives. In order to consider the importance of the asymmetry, we modify the model as follows. For $i \in \{1, 2\}$, if candidate i is the policy type, then he proposes $x_i = r$ for certain.[16] That is, the candidates are completely symmetric. Except for this modification, the model setup is identical. The result is as follows.

[16] As long as $b > 0$, if candidates 1 and 2 of the policy type choose policy l, then the media outlet trivially discloses all information.

Proposition 4.6 *Consider the manipulated news model with single media outlet and the symmetric candidates. Then, the weak median voter theorem always holds; that is, the* $(0,0)$ *equilibrium always exists.*

Proof. When $0 < b \leq \frac{1}{2}r$, the media outlet never withholds the information. Obviously, the weak median voter theorem then holds. Hence, hereafter, we assume that $b > \frac{1}{2}r$. Suppose that $\alpha_1^* = \alpha_2^* = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0,0),(0,r),(r,0),(r,r)\}$. We also focus on the media outlet's strategy specified by (4.11). Because $b > \frac{1}{2}r$, policy pair $x = (r,0)$ is in disagreement region C and policy pair $x = (0,r)$ is in disagreement region D. Then, $\beta^*((r,0)) = \beta^*((0,r)) = \phi$. Given the candidates and the media outlet's strategy, the voter's consistent belief after the withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{1}{2} & \text{if } x = (r,0) \text{ or } (0,r) \\ 0 & \text{otherwise} \end{cases} \tag{4.20}$$

Given the consistent posterior, the voter's best response to the withholding is $\gamma^*(\phi) = (\frac{1}{2}, \frac{1}{2})$. Given the media outlet and the voter's strategies, it is indifferent for the non-policy-type candidate 1 to propose $x_1 = 0$ and $x_1 = r$; both strategies provide the same winning probability $\frac{1}{2}$. The winning probability from proposing $x_1 = l$ is 0. Therefore, candidate 1 has no incentive to deviate from $x_1 = 0$. By the symmetry between the candidates, candidate 2 also never deviates from $x_2 = 0$. Therefore, $(0,0)$ equilibrium always exists without any restrictions. ■

In contrast with the asymmetric setup, the weak median voter theorem is persistent in the symmetric setup; that is, there exist multiple equilibria, but the $(0,0)$ equilibrium always exists without any restrictions. As in the asymmetric setup, because the benefit from proposing the voter's ideal policy is discounted, multiple equilibria exist. The persistence of the policy convergence result is the consequence of the symmetric candidates. Because the candidates are symmetric, the candidates do not have the enough influence incentive, which is the main force breaking down the $(0, 0)$ equilibrium. Therefore, we can conclude that the asymmetric setup is essential to have the fragility of the $(0, 0)$ equilibrium. In the next subsection. we discuss what kind of asymmetry is needed to generate the strong influence incentives.

Asymmetry between the candidates: distance vs direction

In the body of the chapter we have assumed the following two kinds of asymmetry between the candidates. The first is the *asymmetry in distance* in the sense that the voter has the strict preference on the policy pair by the policy-type candidates. The second is the *asymmetry in direction* in the sense that one candidate prefers a positive policy, but the other prefers a negative policy when they are the policy type. In this subsection, we show that the asymmetry in distance could generate the stronger influence incentive than that generated in the asymmetry in direction

First, we consider the asymmetry in distance by the following one-sided setup. The policy space is defined by $X \equiv \{0, r, 2r\}$ with $r > 0$, and assume that if candidate 2 is the policy type, then he always proposes policy $x_2 = 2r$. That is, in this one-sided setup, the candidates are only asymmetric in distance. As long as we focus on the fragility of the $(0, 0)$ equilibrium, the one-sided setup replicates the similar results obtained in the body of the chapter; the strict median voter theorem holds when $0 < b \leq \frac{1}{2}r$, and the weak median voter theorem could fail when $\frac{1}{2}r < b \leq r$. Moreover, we obtain the stronger result when $b > r$; the $(0, 0)$ equilibrium never exists regardless of the parameters.

Proposition 4.7 *Consider the manipulated news model with single media outlet and the one-sided* *setup. Suppose $b > r$, then the weak median voter theorem never holds.*

Proof. First, we prove the following lemma.

Lemma 4.1 *Fix an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $(C \cup D) \cap Z(\alpha_1^*, \alpha_2^*) \neq \emptyset$, arbitrarily. Then, for any policy pairs $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*(x')) = \gamma^*(\beta^*(x''))$.*

Proof of Lemma 4.1. Suppose, in contrast, that there exists an equilibrium such that for some policy pairs $x', x'' \in (C \cup D) \cap Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$.

(1) $x', x'' \in C$ or $x', x'' \in D$. Without loss of generality, assume that $x', x'' \in C$. Because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. If $\beta^*(x') = x'$ and $\beta^*(x'') = x''$, then $\gamma^*(x') = \gamma^*(x'') = (1, 0)$. Hence, exactly one of either x' or x'' must send $m = \phi$. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$. Then, $\gamma^*(x') = (1, 0)$. In addition, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the media outlet that observes policy pair x' has an incentive to deviate to $m = \phi$, a contradiction.

(2) $x' \in C$ and $x'' \in D$. Again, because $\gamma^*(\beta^*(x')) \neq \gamma^*(\beta^*(x''))$, $\beta^*(x') \neq \beta^*(x'')$ must hold. From Proposition 4.2, the media outlet's full-disclosure behavior cannot be supported in equilibrium. Then, the media outlet must withhold the information for exactly one of the policy pair x' or x''. Without loss of generality, assume that $\beta^*(x') = x'$ and $\beta^*(x'') = \phi$. Because $\gamma^*(x') = (1, 0)$, $\gamma^*(\phi)$ assigns positive probability to choosing y_2. However, given this voter's behavior, the media outlet that observes policy pair x' deviates to $m = \phi$, a contradiction. ■

Proof of Proposition 4.7. Suppose, by contrast that there exists the $(0, 0)$ equilibrium. That is, $\alpha_1^* = 0$ and $\alpha_2^* = 0$. Then, $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, 2r), (r, 0), (r, 2r)\}$. By Proposition 4.2, full disclosure is impossible. Then, consider the following two cases: (i) $r < b \leq \frac{3}{2}r$ and (ii) $b > \frac{3}{2}r$.

(i) Suppose that $r < b \leq \frac{3}{2}r$. That is, policy pairs $(0, r), (0, 2r), (r, 0)$ and $(2r, 0)$ are in the disagreement regions.

(1) $(r, 0)$ and $(r, 2r)$ are separating. Suppose, by contrast, that $\beta^*((0, 2r)) \neq \beta^*((r, 0))$. Because policy pair $x = (r, 0)$ is not pooling with any policy pairs in $Z(\alpha_1^*, \alpha_2^*)$, $\gamma^*(\beta^*((r, 0))) = (0, 1)$. By Lemma 4.1, $\gamma^*(\beta^*((r, 0))) = \gamma^*(\beta^*((0, 2r))) = (0, 1)$. Then, $\beta^*((0, 2r)) = \beta^*((r, 2r)) = \phi$; otherwise, $\gamma^*(\beta^*((0, 2r))) = (1, 0)$. However, if $\gamma^*(\phi) = (0, 1)$, then the media outlet that observes policy pair $x = (r, 2r)$ deviates to $m = (r, 2r)$, a contradiction. Therefore, in this sub-case, $\beta^*((0, 2r)) = \beta^*((r, 0)) = \phi$ and $\beta^*((r, 2r)) = (r, 2r)$ must hold. The voter's posterior after observing $m = \phi$ is $\mathcal{P}^*((0, 2r)|\phi) = \mathcal{P}^*((r, 0)|\phi) = \frac{1}{2}$. Because $2\mathcal{P}^*((0, 2r)|\phi) > \mathcal{P}^*((r, 0)|\phi)$, $\gamma^*(\phi) = (1, 0)$. Given the voter and the media outlet's strategies, for candidate 1, the winning probability from $x_1 = 0$ is $1 - \frac{1}{2}p$. However, the winning probability from $x_1 = r$ is 1, which is a contradiction.

(2) $(r, 0)$ and $(r, 2r)$ are pooling. That is, $\beta^*((r, 0)) = \beta^*((r, 2r)) = \phi$. To hold this equilibrium, $\gamma^*(\phi) = (1, 0)$ must hold; otherwise, the media outlet that observes policy pair $x = (r, 2r)$ deviates. However, if $\gamma^*(\phi) = (1, 0)$, then candidate 1 deviates to $x_1 = r$, as shown in sub-case (1), a contradiction.

(ii) Suppose that $b > \frac{3}{2}r$. That is, any divergent policy pairs are in the disagreement regions. By Lemma 4.1, $\gamma^*(\beta^*((0, 2r))) = \gamma^*(\beta^*((r, 2r))) = \gamma^*(\beta^*((r, 0)))$ must hold.

(1) $\beta^*(x) = \phi$ for $x = (0, 2r), (r, 0)$ and $(r, 2r)$. Given the candidates and the media outlet's strategies, the voter's consistent belief after withholding is:

$$\mathcal{P}^*(x|\phi) = \begin{cases} \frac{p}{1+p} & \text{if } x = (0, 2r) \text{ or } (r, 0) \\ \frac{1-p}{1+p} & \text{if } x = (r, 2r) \\ 0 & \text{otherwise} \end{cases} \tag{4.21}$$

Given the posterior, the voter's best response to withholding is $\gamma^*(\phi) = (1, 0)$. Then, for candidate 1, his winning probability from proposing $x_1 = 0$ is $1 - \frac{1}{2}p$. However, if candidate 1 proposes $x_1 = r$, then his winning probability is 1. Then, candidate 1 has an incentive to deviate, a contradiction.

(2) $\beta^*((r, 2r)) = \beta^*((r, 0)) = \phi$ **and** $\beta^*((0, 2r)) = (0, 2r)$**.** Because $\gamma^*(\beta^*((0, 2r))) = (1, 0)$, by Lemma 4.1, $\gamma^*(\phi) = (1, 0)$ is needed. However, given $\gamma^*(\phi) = (1, 0)$, candidate 1 has an incentive to deviate to $x_1 = r$ as shown in (1), a contradiction.

(3) $\beta^*((0, 2r)) = \beta^*((r, 0)) = \phi$ **and** $\beta^*((r, 2r)) = (r, 2r)$**.** We can derive a contradiction by the same argument in (2).

(4) $\beta^*((0, 2r)) = \beta^*((r, 2r)) = \phi$ **and** $\beta^*((r, 0)) = (r, 0)$**.** Because $\gamma^*(\beta^*((r, 0))) = (0, 1)$, by Lemma 4.1, $\gamma^*(\phi) = (0, 1)$. However, because $S(\mathcal{P}^*(\cdot|\phi)) \subset C$, the voter's reaction must be $\gamma^*(\phi) = (1, 0)$, which is a contradiction.

Therefore, the $(0, 0)$ equilibrium does not exist. ■

Next, we discuss the asymmetry in direction by considering the following setup. The policy space is defined by $X \equiv \{-r, 0, r\}$, and assume that candidate 2 of the policy type always proposes policy $x_2 = -r$. Except for this modification, the setup is same to that used in the body of the chapter.

Note that the candidates are only asymmetric in direction. In this symmetric two-sided setup, the fragility of the $(0, 0)$ equilibrium is equivalent to that obtained in the body of the chapter; if $b > \frac{1}{2}r$ and $p > \frac{1}{2}$, then there exists no $(0, 0)$ equilibrium.

In summary, there exists no essential difference between the asymmetry in direction and the asymmetry in distance in the sense that the $(0, 0)$ equilibrium becomes fragile, though the asymmetry between the candidates is necessary. However, when the candidates have only the asymmetry in distance, their influence incentive could become stronger, and then the stronger nonexistence result holds.

Tie-breaking rules

The tie-breaking rules specified in Requirement 4.2 seem to be crucial to the results. While the tie-breaking rule for the voter is well accepted in the literature, for the media outlet it would seem to be much more controversial. We have assumed that the media outlet discloses the information whenever the proposed policies are convergent, but there is no strong justification for this behavior. However, if the media outlet withholds the information even when the proposed policies are convergent, then we face the serious multiple-equilibrium problem; any randomization between policies 0 and r can be an equilibrium strategy.

Although this multiplicity is a serious problem, most of the equilibria are not robust with respect to small perturbation in the media outlet's behavior. Instead of assuming full disclosure or full withholding, we thus assume that the media outlet discloses the information about convergent policies with probability $\epsilon \in (0, 1)$. That is, the media outlet that observes the convergent policy pairs randomizes disclosure and withholding.[17] For easy reference, we call this tie-breaking rule the

[17] Because the result of the election is indifferent for the media outlet when the proposed policy is convergent, such

ϵ-*randomization rule*, and the original the *disclosure rule*. We can show that even if the probability of disclosure ϵ is sufficiently small, then the set of equilibrium policy pairs under the ϵ-randomization rule is equivalent to that under the disclosure rule. Therefore, we can justify focusing on the equilibria satisfying the disclosure rule from the viewpoint of the robustness.

Let us introduce additional notations. Let EP^{ϵ} be the set of equilibrium strategies of the non-policy-type candidates under the ϵ-randomization rule.[18] Note that the disclosure rule is the 1-randomization rule. The set of policy pairs X^2 is partitioned into the following three groups:

$$CP \equiv \{(0,0),(r,r),(l,l)\}; \tag{4.22}$$

$$AR \equiv \{(0,l),(l,0),(r,l),(l,r)\}; \tag{4.23}$$

$$DR \equiv \{(0,r),(r,0)\}. \tag{4.24}$$

That is, CP, AR and DR are the sets of convergent policy pairs, divergent policy pairs in the agreement regions and divergent policy pairs in the disagreement regions when $b > \frac{1}{2}r$, respectively. Let β^{ϵ} be the generic notation of the media outlet's strategy satisfying the ϵ-randomization rule. Especially, with abuse of notation, $\beta^{\epsilon}(x) = (t, 1-t)$ represents that the media outlet that observes policy pair x discloses the information with probability $t \in [0,1]$, and withholds with probability $1-t$. Let $\mathcal{P}^{\epsilon}(\cdot|m)$ and $\gamma^{\epsilon}(m)$ be the voter's posterior belief and the best response given a message m under the ϵ-randomization rule, respectively. Let $\mu_i(x|\alpha_1,\alpha_2)$ be the probability of policy pair x occurs from the viewpoint of candidate i of the non-policy-type under strategies α_1, α_2.

randomization can be supported as one of the best responses of the media outlet.

[18]Formally, $EP^{\epsilon} \equiv \{(\alpha_1^*, \alpha_2^*) \in (\Delta(X))^2|$ there exists$\beta^*, \gamma^*, \mathcal{P}^*$ s.t. $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE, where β^* satisfies the ϵ-randomization rule.$\}$.

Proposition 4.8 *Consider the manipulated news model with single media outlet. Suppose that $b > \frac{1}{2}r$. Then, for any $\epsilon \in (0,1)$, $EP^1 = EP^\epsilon$.*

Proof. Fix $\epsilon \in (0,1)$, arbitrarily. First, show that $EP^1 \subseteq EP^\epsilon$. Take $(\alpha_1, \alpha_2) \in EP^1$, arbitrarily. That is, there exist β^1, γ^1 and $\mathcal{P}^1$ such that $(\alpha_1, \alpha_2, \beta^1, \gamma^1; \mathcal{P}^1)$ is a PBE. There are the following cases: (i) $\gamma^1(\phi) = (1,0)$, (ii) $\gamma^1(\phi) = (0,1)$ and (iii) $\gamma^1(\phi) = (\frac{1}{2}, \frac{1}{2})$.

Consider the case where $\gamma^1(\phi) = (1,0)$. Given $\gamma^1(\cdot)$, $\beta^1(\cdot)$ is characterized as follows:[19]

$$\beta^1(x) = \begin{cases} (1,0) & \text{if } x \in AR \cup CP \\ (0,1) & \text{if } x \in DR. \end{cases} \tag{4.25}$$

Then, define $\beta^\epsilon(\cdot)$ as follows:

$$\beta^\epsilon(x) = \begin{cases} (1,0) & \text{if } x \in AR \\ (\epsilon, 1-\epsilon) & \text{if } x \in CP \\ (0,1) & \text{if } x \in DR. \end{cases} \tag{4.26}$$

Now, we show that given α_1, α_2 and β^ϵ, $\gamma^\epsilon(m) = \gamma^1(m)$ for any $m \in M$. If $m = x$, then it is obvious

[19] While disclosing and withholding are indifferent for the media outlet if the proposed policy pair is $(0, r)$, without loss of generality, we can focus on this strategy.

that $\gamma^\epsilon(x) = \gamma^1(x)$ for any $x \in X^2$. By Requirement 4.2-(i), because $\gamma^1(\phi) = (1,0)$:

$$\begin{aligned}
& \sum_{x \in Z(\alpha_1,\alpha_2)} |x_1| \mathcal{P}^1(x|\phi) < \sum_{x \in Z(\alpha_1,\alpha_2)} |x_2| \mathcal{P}^1(x|\phi) \\
\iff \quad & \sum_{x \in DR} |x_1| Pr.(x|\alpha_1,\alpha_2) < \sum_{x \in DR} |x_2| Pr.(x|\alpha_1,\alpha_2) \\
\iff \quad & \sum_{x \in DR} |x_1| Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon) \sum_{x \in CP} |x_1| Pr.(x|\alpha_1,\alpha_2) \\
& \quad < \sum_{x \in DR} |x_2| Pr.(x|\alpha_1,\alpha_2) + (1-\epsilon) \sum_{x \in CP} |x_2| Pr.(x|\alpha_1,\alpha_2) \\
\iff \quad & \sum_{x \in Z(\alpha_1,\alpha_2)} |x_1| \mathcal{P}^\epsilon(x|\phi) < \sum_{x \in Z(\alpha_1,\alpha_2)} |x_2| \mathcal{P}^\epsilon(x|\phi)
\end{aligned} \tag{4.27}$$

By Requirement 4.2-(i), $\gamma^\epsilon(\phi) = (1,0)$. Therefore, given α_1, α_2 and β^ϵ, $\gamma^\epsilon(m) = \gamma^1(m)$ for any $m \in M$.

Next, show that given α_j, β^ϵ and γ^ϵ, α_i is the best response of candidate $i \in \{1,2\}$. The winning probability of candidate 1 given α_2, β^1 and γ^1 is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + \frac{1}{2}\sum_{x \in CP} \mu_1(x|\alpha_1,\alpha_2)$, where $AR_1 \equiv \{(0,l),(r,l)\}$. Similarly, the winning probability of candidate 2 given α_1, β^1 and γ^1 is $\sum_{x \in AR_2} \mu_2(x|\alpha_1,\alpha_2) + \frac{1}{2}\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$, where $AR_2 \equiv \{(l,0),(l,r)\}$. Note that given α_j, β^1 and γ^1, proposing policy l with positive probability is never an equilibrium strategy of each candidate. Then, the objective functions of the candidates 1 and 2 are $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + \frac{1}{2}[1 - \sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2)]$, and $\frac{1}{2}\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2)$, respectively. Because $(\alpha_1,\alpha_2) \in EP^1$:

$$\sum_{x \in AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) \geq \sum_{x \in AR_1 \cup DR} \mu_1(x|\alpha_1',\alpha_2) \text{ for any } \alpha_1' \in \Delta(X) \tag{4.28}$$

$$\sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2) \geq \sum_{x \in CP} \mu_2(x|\alpha_1,\alpha_2') \text{ for any } \alpha_2' \in \Delta(X) \tag{4.29}$$

The winning probability of candidate 1 given α_2, β^ϵ and γ^ϵ is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1,\alpha_2) + (1 -$

$\frac{1}{2}\epsilon) \sum_{x \in CP} \mu_1(x|\alpha_1, \alpha_2)$. Similarly, the winning probability of candidate 2 given α_1, β^ϵ and γ^ϵ is $\sum_{x \in AR_2} \mu_2(x|\alpha_1, \alpha_2) + \frac{1}{2}\epsilon \sum_{x \in CP} \mu_2(x|\alpha_1, \alpha_2)$. Because α_1 does not put any positive probability on proposing policy l, candidate 2's winning probability becomes $\frac{1}{2}\epsilon \sum_{x \in CP} \mu_2(x|\alpha_1, \alpha_2)$. By (4.29), we can say that α_2 is the best response to α_1, β^ϵ and γ^ϵ. Furthermore, given α_2, β^ϵ and γ^ϵ, proposing policy l with positive probability is never an equilibrium strategy of candidate 1. Then, candidate 1's winning probability is $\sum_{AR_1 \cup DR} \mu_1(x|\alpha_1, \alpha_2) + (1 - \frac{1}{2}\epsilon)[1 - \sum_{AR_1 \cup DR} \mu_1(x|\alpha_1, \alpha_2)]$. By (4.28), α_1 is the best response of candidate 1 given α_2, β^ϵ and γ^ϵ. Therefore, because $(\alpha_1, \alpha_2, \beta^\epsilon, \gamma^\epsilon; \mathcal{P}^\epsilon)$ is a PBE, $(\alpha_1, \alpha_2) \in EP^\epsilon$. For the cases of $\gamma^1(\phi) = (0, 1)$ and $(\frac{1}{2}, \frac{1}{2})$, similarly we can show that $(\alpha_1, \alpha_2) \in EP^\epsilon$. Thus, $EP^1 \subseteq EP^\epsilon$. The converse is also proven similarly; that is, $EP^\epsilon \subseteq EP^1$. Therefore, $EP^1 = EP^\epsilon$. ■

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