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Essays on the Impact of Commitment on Bargaining and Efficiency

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Essays on the Impact of Commitment on Bargaining and Efficiency

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Chapter 1: Bargaining with Revoking Costs

1 Introduction

A trade union leader who announces a demand in a negotiation with the management may risk losing his job if he accepts a lower share than the demand. The President of a country may face a tougher re-election prospect if she fails to achieve her publicly announced demand in a domestic or international bargaining situation. More generally, backing down from an initial demand made in some bargaining scenarios may entail a cost that depends on the amount conceded. While seemingly a weakness, these costs may actually confer greater bargaining power to the party facing these costs. If this cost makes the party prefer an impasse to concession, following incompatible offers, the said party can force a concession from her opponent who does not face such costs. The cost of revoking an earlier demand therefore gives a bargainer an ability to partially commit herself to a stated demand. The object of this study is to identify and characterize the relationship between such revoking costs and bargaining power.

Following the insights found in Schelling(1956) regarding the role of commitment tactics in bargaining, Crawford(1982) presents a formal model where a bargainer can revoke her stated demand at some cost. The game involves two stages with players simultaneously making demands in the first stage. Incompatible demands (add up to more than one) lead to a second stage where players decide simultaneously whether to revoke their demand or concede to the other offer. The present paper uses this basic framework but drops the assumption in Crawford(1982) that revoking costs are unknown at the demand stage and independent of the extent of concession, with each player getting to know their own revoking cost before the second stage. Instead, following Muthoo(1996)(henceforth M), the revoking costs are assumed to be known and increasing in the extent of the concession. Crawford(1982) studies the role such commitment tactics play in generating inefficiency in bargaining. This issue is also studied by Ellingsen and Miettinen(2008) where attempting commitment is assumed to be costly.[1] While the complete information structure in both M and the present paper results in efficiency readily, the goal here is to study the relationship between revoking costs and bargaining power.

M addresses this issue using a one shot simultaneous move demand game between two players over a unit sized pie. Following incompatible demands, the outcome is selected by the Nash Bargaining Solution (NBS) applied to a modified utility possibility set(UPS).[2] For a given pair of incompatible demands, a division of the pie is mapped to this UPS with players paying a cost, for a share lower than their initial demand. The analysis is carried out for convex cost functions and concave utility functions that are strictly increasing and twice continuously differentiable. In its unique efficient Nash Equilibrium outcome a player's equilibrium share is shown to increase in her marginal revoking cost. Leventoglu and Tarar(2005) (henceforth LT) conduct their analysis on the linear version of this game, by explicitly modeling the post incompatible offers stage as a Rubinstein bargaining game (Rubinstein(1982)) played over the modified UPS. Importantly they find that payoff efficiency is attained only

[1]Li(2010), in a related paper, shows that the inefficiency result potentially depends on whether the decisions regarding demand and attempting commitment were made simultaneously or sequentially.

[2]The result that the unique SPE in the Rubinstein bargaining game converges to the NBS when the discount factors are the same, and the time between offers converges to 0 is used to support the use of the NBS in Muthoo(1999).

at the limit when the equal discount factors converge to 1.

This paper extends M's analysis and results to a model where, instead of using the NBS, the bargainers, following incompatible offers, play a one-shot game like in Crawford(1982) to determine their payoffs. More precisely, the two players bargain over a unit sized pie in a two stage game. Each player announces a demand in the first stage. If the demands are compatible they split midway between their demands. Otherwise, in the second stage, each party chooses simultaneously whether to stick to their own demand or accept the other's offer. Both parties sticking to their incompatible demands results in an impasse. Accepting the other player's offer, however, is costly, with the cost increasing in the amount by which the accepted share is less than the demanded share. A set of efficient pure strategy subgame perfect equilibria emerges, which is characterized in terms of the cost and payoff functions. Unlike in M, no further convexity assumptions are required on these functions for equilibrium existence. Analogous to M's unique equilibrium behavior, however, the highest and lowest equilibrium payoffs for a given player are shown to increase with an increase in their revoking cost functions. Importantly, the set of equilibria is shown to shrink with higher cost functions. Indeed, as the cost functions are made arbitrarily high the limit of the equilibrium set is shown to make a unique equilibrium selection in the limiting Nash Demand Game(NDG). The model captures the insight that a bargainer wishes to make it difficult for herself to concede to a lower offer. Interestingly, it shows how making a greater demand for oneself results in making concession more difficult for the other party, giving the latter higher commitment ability. The equilibria, as a result, are characterized by a tradeoff between the twin needs of higher shares and greater commitment ability.

While M(implicitly) and LT capture scenarios where bargainers have the ability to renegotiate endlessly after their initial demand, the present model studies the opposite benchmark, where bargainers cannot make offers beyond the initial demand they partially commit to. The goal here is to provide a transparent and simple analysis of the tradeoff between higher demands and greater commitment in the presence of revoking costs while assuming away the influence of time preferences. The simple two stage framework, however, can be used as the stage game of a repeated game, to capture scenarios like international negotiations where each party gets to change their publicly announced demands after the failure of an earlier round of negotiation, but backing down from the most recent stated demand in a given round of negotiation incurs a cost. Given the general class of cost functions which the paper studies one could model scenarios where these cost functions change over time.

M suggests that allowing players to back down from incompatible demands at a cost can be seen as a perturbation of the commitment structure implicit in the NDG. Making these costs arbitrarily high, therefore, gives the NDG at the limit. The limit equilibrium prediction then makes an equilibrium selection in the NDG. I conduct a similar limit analysis of the two stage model. Surprisingly, the unique equilibrium selected in the NDG corresponds to the Proportional Bargaining Solution(PBS) of Kalai(1977), in contrast with the results of Nash(1953) and Carlsson(1991). The proportion is determined by the limiting ratio of the cost functions. Section 5.2 discusses this in detail.

Interestingly, the rationale for extreme divisions being ruled out in this model and the equilibrium strategies are similar in spirit to findings in Kambe(1999), Abreu and Gul(2000) and Compte and Jehiel(2002), where by making a lower demand a player can force her opponent to make the initial mass acceptance in the second stage war of attrition. The commitment possibilities in these papers, however, are generated by the presence of behavioral types.[3]

The rest of the paper is as follows. Section 2 presents the formal model. Section 3 analyzes the special case of the model where the payoff and cost functions are linear. The intuition behind the equilibrium strategies in the general model can be found here. Further, it is easier to foresee the comparative statics and limit arguments for the general model, by analyzing the linear case. Section 4 characterizes the equilibrium set for the general model. Section 5 deals with comparative statics and the limit predictions of the model as the cost functions are made arbitrarily high. Section 6 concludes. All proofs are collected in the appendix.

2 The Bargaining Game

Two players, 1 and 2, play a two stage game. In the first stage, player i chooses a level of demand $z_i \in [0, 1)$. Let $d = z_1 + z_2 - 1$ measure the excess of the aggregate demand over the size of the pie. If $d \leq 0$ the game ends with player i getting $x_i = z_i - d/2$, the amount demanded plus half the excess of the size of the pie over the aggregate demand. [4] The corresponding payoffs are $\pi_1(x_1), \pi_2(x_2)$, where π_i is the payoff function for player i. If $d > 0$ then the

[3]Li(2007) considers an infinite horizon alternating offers bargaining model where the ability to commit arises due to history dependent preferences. In particular, a player prefers an impasse to an agreement with a lower discounted utility than would have been achieved by accepting an earlier offer.

[4]All the results remain the same if the bargainers get their exact demands when the demand profile adds up to less than the pie size, as in Nash(1953)

following second stage simultaneous move game is played.

	Accept	*Stick*
Accept	$\pi_1(x_1) - c_1(z_1 - x_1), \pi_2(x_2) - c_2(z_2 - x_2)$	$\pi_1(1 - z_2) - c_1(d), \pi_2(z_2)$
Stick	$\pi_1(z_1), \pi_2(1 - z_1) - c_2(d)$	$\pi_1(0), \pi_2(0)$

The interpretation of this game is as follows. If and when the two players make incompatible demands ($d > 0$), player i must choose whether to stick to her own demand or accept j's offer, which must be less . However, there is a cost attached to accepting a division of the pie that is less than the share demanded in the first stage. This can happen if either player i *Accepts* while j *Sticks* or if both players choose *Accept*. This feature is captured by the cost function c_i for player i. So if player i had initially demanded z_i which was incompatible with player j's demand, z_j, then accepting j's offer in the second stage while j sticks to his offer would give player i a payoff of $\pi_i(1 - z_j) - c_i(z_i - (1 - z_j))$. If both players choose to *Accept* in the second stage following incompatible offers (z_1, z_2), then player i gets a compromise share $x_i = z_i - d/2$ with a payoff of $\pi_i(x_i)$ and also pays the cost for accepting a lower share, $c_i(z_i - x_i)$. Note that since the second stage game is played only if $d > 0$, it must be true that $x_i < z_i$. Finally if both players decide to stick to their incompatible demands they get their disagreement payoff, $(\pi_1(0), \pi_2(0))$. The following assumptions are met by the payoff and cost functions in the rest of the note.

A1. *For* $i \in \{1, 2\}$, π_i *is a strictly increasing and continuously differentiable function. Further* $\pi_i(0) = 0$ *and* $\pi_i(1)$ *is some finite value.*

A2. *For $i \in \{1,2\}$, $c_i : \Re_+ \to \Re_+$ is a strictly increasing and continuously differentiable function with $c_i(0) = 0$.*

This completes the description of the two stage bargaining game.

3 The Linear Model

In this section the payoff and cost functions are assumed to be linear. In particular, $\pi_i(x) = x$ and $c_i(d) = k_i d$ where $k_i > 0$. The second stage game, therefore, is as follows

	Accept	*Stick*
Accept	$x_1 - k_1(z_1 - x_1), x_2 - k_2(z_2 - x_2)$	$1 - z_2 - k_1(d), z_2$
Stick	$z_1, 1 - z_1 - k_2(d)$	$0, 0$

Proposition 1. $\frac{k_2}{1+k_1} \leq \frac{z_2^*}{z_1^*} \leq \frac{1+k_2}{k_1}$ *and* $z_1^* + z_2^* = 1$ *are necessary and sufficient conditions for* (z_1^*, z_2^*) *to be a pure strategy subgame perfect equilibrium outcome of the bargaining game with linear payoffs and costs.*

Figure 1 illustrates the intuition behind Proposition 1. BA represents demand profiles that add up to one. OE and EF represent $z_2/z_1 = (1+k_2)/k_1$ and $z_2/z_1 = k_2/(1+k_1)$, respectively. BD is the graph of $1-z_2-k_1(z_1+z_2-1) = 0$ while CA graphs $1 - z_1 - k_2(z_1 + z_2 - 1) = 0$. For points lying above (below) BD it must be that $1 - z_2 - k_1(z_1 + z_2 - 1) < (>)0$. Similarly points lying above (below) CA satisfy $1 - z_1 - k_2(z_1 + z_2 - 1) < (>)0$. Demand profiles below BA cannot be subgame perfect as both players will have an incentive

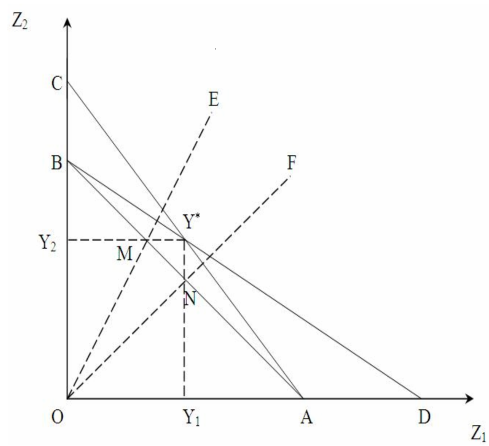

Figure 1: The Linear Model

to increase their demands. Demand profiles above BA, say (z_1, z_2), eventually result in some player j getting a payoff less than $1 - z_i$, where i is the other player, since $(Accept, Accept)$ is not a NE of the second stage game. Player j could then profitably deviate to demanding $1 - z_i$. Subgame perfect demands in the first stage must therefore lie on BA, as shown by Lemma A1 and A2.

I will now show that extreme divisions along this line, BA, can be eliminated by profitable deviations by the less favored player. Such deviations would lead to incompatible demands (points above BA), resulting in payoffs determined by equilibrium behavior in the second stage game. Incompatible demands can be separated into 4 regions, in terms of second stage equilibrium behavior. For points above CY^*D, both players prefer *Stick* to *Accept*. In the AY^*D region the unique NE in the second stage involves 1 playing *Accept* while 2 *Sticks*. Incompatible demands from the CBY^* region results in the unique NE $(Stick, Accept)$ in the second stage. Finally for first stage offers in BY^*A both $(Accept, Stick)$ and $(Stick, Accept)$ are NE of the second stage. Lemma A3 essentially shows that equilibrium demands cannot be in the AN

region since player 2 would then have the incentive to deviate to a point in AY^*D, forcing a concession from player 1 and getting a higher payoff. A symmetric argument rules out the BM region. Notice that player 1 making a high demand (greater than Y_1) gives player 2 greater commitment power. Indeed by making a demand that selects a point in AY^*D player 2 ends up making *Stick* her dominant strategy in the second stage game, while leaving player 1 enough room to prefer conceding to an impasse. Demand profiles that are not ruled out as above, therefore, lie on MN. The proof for Proposition 1 also specifies subgame perfect strategies to support these demands.

From Figure 1 it is easy to see how the equilibrium set changes with changes in k_i. An increase in k_1, for instance, moves the interval MN towards A, thereby increasing player 1's highest and lowest equilibrium payoffs. Notice also that increasing the k_i's result in both CA and BD shift towards BA, which makes Y^* move closer to BA. This, in turn, makes OE and OF get closer to each other. Indeed, the limit equilibrium set, as the costs are made arbitrarily high, consists of a single efficient demand profile. Consider, for example, $k_1 = c$ and $k_2 = \alpha c$. The limit equilibrium set as $c \to \infty$ consists of the unique demand profile (z_1, z_2) with $z_1 + z_2 = 1$ and $z_2/z_1 = \alpha$. This issue is discussed further in Section 5.

4 The General Model

In this section the only assumptions imposed on the payoff and cost functions are **A1** and **A2**.

Figure 2 captures the workings of Proposition 3. Note that the coordinates of a given point in the figure correspond to the shares of the pie demanded

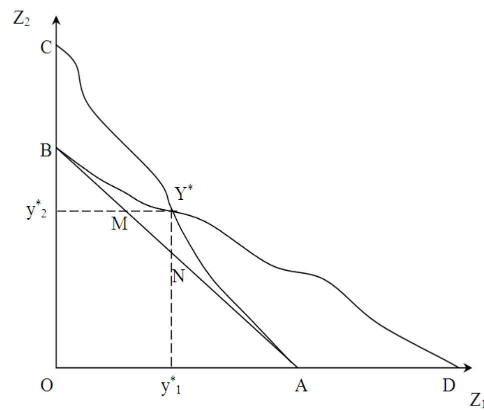

Figure 2: The General Model

by each party. BA is the same as in Fig. 1. BD is the graph for $\pi_1(1 - z_2) - c_1(z_1 + z_2 - 1) = 0$, while CA graphs $\pi_2(1 - z_1) - c_2(z_1 + z_2 - 1) = 0$. The intuition for why first stage demands must lie on MN is exactly the same as in the linear case, as can be seen by comparing this with Fig. 1. The only substantial addition for the general model is to show that the curves, BD and CA, which are generated by the particular payoff and cost functions, have a unique intersection point. This is indeed true given **A1** and **A2** and is established by Proposition 2.

Proposition 2. *There exists a unique* (y_1, y_2) *with* $y_i \in (0, 1)$ *that solves*

$$\pi_1(1 - y_2) = c_1(y_1 + y_2 - 1) \tag{1}$$

and

$$\pi_2(1 - y_1) = c_2(y_1 + y_2 - 1). \tag{2}$$

The uniqueness of the intersection point is driven by the fact that the curve BD must have a slope less than -1 while the slope of AC must be strictly greater than -1.

Let (y_1^*, y_2^*) be the unique solution to (1) and (2), guaranteed by Proposition 2.

Proposition 3. *Given* ***A1*** *and* ***A2*** *the demand profile in any pure strategy subgame perfect equilibrium of the bargaining game must be an element of* $\{(z_1^*, z_2^*)$ *s.t.* $z_1^* + z_2^* = 1, z_1^* \leq y_1^*$ *and* $z_2^* \leq y_2^*\}$.

It can be easily verified that $(\frac{1+k_1}{1+k_1+k_2}, \frac{1+k_2}{1+k_1+k_2})$ solves (1) and (2) in the linear model of Section 2. Proposition 3 then readily gives us the relevant inequalities of Proposition 1.

5 Implications

5.1 Comparative Statics

Corollary 3.1 makes precise how higher revoking cost functions lead to greater bargaining power. Higher revoking cost functions essentially give the player greater ability to commit to their stated demands. Consequently, fearing eventual concession to a low offer, the opponent must make a lower demand. It is also shown how the set of equilibria shrinks with higher cost functions.

Corollary 3.1. *Given* ***A1*** *and* ***A2*** *an increase in player i's revoking cost function, increases her highest and lowest equilibrium payoffs. Further the difference between the highest and lowest equilibrium cake share for either player decreases.*

5.2 Equilibrium Selection in the Nash Demand Game

M suggests how the perfect commitment implicit in the NDG can be perturbed by allowing players to back down from their stated demands, at some cost. Indeed, he shows that at the limit as the revoking cost is made arbitrarily high, a unique equilibrium in the NDG survives. The present model derives a similar result replacing the Nash Bargaining Solution by equilibrium behavior in the second stage game above to determine payoffs after incompatible demands. Surprisingly, the unique equilibrium selected in the NDG corresponds to the division selected by the Proportional Bargaining Solution of Kalai(1977) with the proportion equal to the ratio of the marginal revoking costs evaluated at 0+.

Let Π_{A1} and $\mathcal{C}$ be the sets of all functions that satisfy **A1** and **A2** respectively. Let $\Gamma^{c_1,c_2}(\pi_1,\pi_2)$ denote a two stage bargaining game as outlined in Section 2, with $\pi_i \in \Pi_{A1}$ and $c_i \in \mathcal{C}$. The corresponding set of subgame perfect payoff profiles is denoted by $\xi(\Gamma^{c_1,c_2}(\pi_1,\pi_2))$. Γ^{c_1,c_2}, therefore, maps any pair of payoff functions in Π_{A1} into a corresponding two stage bargaining game. Consider a sequence of such mappings $\{\Gamma^{c_1^n,c_2^n}\}_{n=1}^{\infty}$ such that as $n \to \infty$, $c_i^{n'}(0+) \to \infty$ (the right derivative of the cost functions at 0 becomes arbitrarily large) with $c_i^n \in \mathcal{C}$ for all n. Further, it is assumed that $\exists \epsilon > 0$ and an integer M such that $\forall d \in [0,\epsilon)$ and $\forall n > M$, $c_1^n(d)/c_2^n(d)$ (ratio of the revoking cost functions) is a constant. Along such a sequence, payoff function pairs are mapped into games where the amount a player can afford to concede becomes progressively smaller. Consequently, at the limit any pair of payoff functions is mapped to its corresponding NDG, where one does not have the ability to back down from incompatible demands. The assumption of the ratio of the revoking costs being constant for an arbitrarily small interval containing

0 guarantees that the limit of the equilibrium set exists.

Define,

$\xi^*_\gamma(\pi_1, \pi_2) = \lim_{n\to\infty} \xi(\Gamma^{c_1^n, c_2^n}(\pi_1, \pi_2))$ where $\gamma = \lim_{n\to\infty} c_1^n(0+)/c_2^n(0+)$. Given a pair of payoff functions, ξ^*_γ gives the limit equilibrium prediction of the two stage model when the revoking costs are made arbitrarily high with the parameter γ capturing the ratio of the revoking costs evaluated at 0+. ξ^*_γ therefore makes the equilibrium selection in the corresponding NDG.

Let $\Pi(\pi_1, \pi_2) = \{(u_1, u_2)|u_i = \pi_i(x_i), 0 \leq x_i + x_j \leq 1, x_i \geq 0, \forall i \in \{1, 2\}, j \neq i\}$ denote the set of feasible payoffs of the bargaining game and $d = (\pi_1(0), \pi_2(0)) = (0, 0)$, the disagreement point. $(\Pi(\pi_1, \pi_2), d)$ gives the familiar object of a bargaining problem from axiomatic bargaining theory. $\mathcal{B} = \{(\Pi(\pi_1, \pi_2), d)|\pi_1, \pi_2 \in \Pi_{A1}\}$, therefore, is the set of bargaining problems that can be generated by payoff functions that satisfy **A1**.

The PBS with proportions $(\gamma, 1)$, denoted by $\mathcal{K}_\gamma$, is defined as $\mathcal{K}_\gamma(\Pi, d) = \lambda(\Pi, d)(\gamma, 1)$, $\forall \Pi \in \mathcal{B}$ where $\lambda(\Pi, d) = \max\{t : t(\gamma, 1) \in \Pi\}$.[5]

Corollary 3.2. $\forall \pi_1, \pi_2 \in \Pi_{A1}$, $\xi^*_\gamma(\pi_1, \pi_2) = \mathcal{K}_\gamma(\Pi(\pi_1, \pi_2), d)$.

Kalai(1977) proves the remarkable result that any bargaining solution that satisfies the axioms of *Independence of Irrelevant Alternatives*, *Individual Monotonicity* and *Continuity* must be a Proportional Bargaining Solution. The analysis gives a family of bargaining solutions parameterized by the proportions, suggesting that finding the appropriate proportion needs looking beyond the information contained in the bargaining problem (Π, d). The present analysis does just that, with the proportion given by the ratio of the revoking cost functions at 0+. Such information is typically not considered in the axiomatic

[5]The dependence of Π on π_1 and π_2 is suppressed for notational convenience.

theory of bargaining. The simplicity of the NDG makes it all the more attractive that the PBS is supported by an equilibrium selection argument in the NDG.

To see the intuition behind the result, consider the following example. Let player 1's cost function always be α times player 2's cost function, for every pair in the sequence of cost functions. In particular, $c_1^n(d) = \alpha c_2^n(d)$ with $c_2^n \in \mathcal{C}$, $\forall n$, $\forall d > 0$ and $\alpha > 0$. Let the payoff functions be $\pi_1, \pi_2 \in \Pi_{A1}$. Notice first that the extreme points for the set of equilibrium demands, are given by $(1-y_2^n, y_2^n)$ and $(y_1^n, 1-y_1^n)$ where y_1^n and y_2^n solve (1) and (2) given the corresponding cost functions, c_1^n and c_2^n. The following crucial characterization of these extreme points is then evident,

$$\frac{\pi_1(1-y_2^n)}{\pi_2(1-y_1^n)} = \alpha, \qquad \forall n. \tag{3}$$

So the ratio of payoffs to the two players with each getting the least share of the pie that they get in any equilibrium is a fixed number, α. Further, $\alpha = \lim_{n\to\infty} c_1^n(0+)/c_2^n(0+)$. Along the sequence as the cost functions increase the solution to (1) and (2) requires progressively smaller amounts of $y_1^n + y_2^n - 1$. The assumption that $c_i^{n'}(0+) \to \infty$ as $n \to \infty$ therefore results in $\lim_{n\to\infty} y_1^n = \lim_{n\to\infty} 1 - y_2^n$. This delivers the result that at the limit there exists a unique demand profile that can be supported in equilibrium. Consequently at this limit the unique equilibrium demand share for player 1 is also her least equilibrium demand share. (3) then makes it necessary that the ratio of payoffs at this limit equilibrium must indeed be α. This unique efficient payoff profile therefore coincides with the PBS prediction given by $\mathcal{K}_\alpha(\pi_1, \pi_2)$. In fact, for *any* pair of payoff functions (π_1, π_2) the PBS, $\mathcal{K}_\alpha$, predicts the efficient payoff profile that has a ratio of α. To see that this is also the case with the equilibrium selection procedure, note that (3) must be satisfied irrespective of the particular pair of payoff functions the players are equipped with.

This result, however, is in sharp contrast with previous equilibrium selection arguments in the NDG which deliver the Nash Bargaining Solution as the unique outcome. Nash(1953), himself suggested the "smoothing argument" where the NDG is approached by a sequence of games in which the payoff following incompatible offers smoothly tapers off to zero. The limit equilibrium outcomes of the smoothed games as the amount of smoothing goes to zero would then converge to the NBS. Particular examples of such smoothing procedures can be found in Binmore(1987) and van Damme(1991). Carlsson(1991) gives a model closely related to Binmore(1987) in which the smoothing is generated by the fact that players tremble when making their demands. In all these papers the perturbations are of an informational nature, while in this paper and M it is the perfect commitment implicit in the NDG which is perturbed. Further in this paper, the payoffs following incompatible offers are generated by a complete information non-cooperative game between the players. Therefore the result is determined entirely by strategic incentives as opposed to informational features. Finally it should be noted that in this paper the payoffs following incompatible offers in the perturbed games do not smoothly taper off to zero. The curves BD and AC in figure 2 are instances where the payoffs are discontinuous. Therefore, the present analysis, does not satisfy the "smoothing argument" of Nash(1953).

6 Conclusion

The tradeoff between higher demands and higher commitment ability has been studied using a simple and transparent two stage non-cooperative model of bargaining. The ability to commit is generated by making backing down from a stated demand costly. When these costs are common knowledge and increasing in the extent of concession, higher cost functions yield greater bargaining power. The objective of this study has been to provide a simple tractable model to capture this relationship, which can then be applied to model scenarios where players have the ability to modify their commitments. While backing down in a negotiation may be costly, the breakdown of a negotiation (i.e. $(Stick, Stick)$) could lead to a new round of negotiation where the two parties get to choose new levels of demand to commit to. Given the general class of payoff and cost functions that the present analysis considers it would indeed be possible to consider the effect of changing cost structures over time in such scenarios. The limit prediction of the model as the revoking costs are made arbitrarily high has been used as an equilibrium selection argument in the Nash Demand Game, delivering the Proportional Bargaining Solution with proportion equal to the ratio of revoking costs as the outcome.

A Appendix

Let (z_1, z_2) be the demands made in the first stage of a pure strategy subgame perfect equilibrium of the linear model.

Lemma A1. $z_1 + z_2 \not< 1$

Proof. This is immediate, since if $z_1+z_2 < 1$, player i can deviate by demanding

$1 - z_j$. Since $(1 - z_j, z_j)$ is still compatible, player i gets a payoff of $1 - z_j$ which is strictly higher than the original payoff z_i, as $z_1 + z_2 < 1$. □

Lemma A2. $z_1 + z_2 \not> 1$

Proof. Suppose $z_1 + z_2 > 1$. Let the payoffs in the second stage game, which must now be played, be (y_1, y_2). Due to the nature of the bargaining game the outcome must be determined by a pure strategy Nash Equilibrium of the second stage game. Note that $\{Accept, Accept\}$ could never be a Nash Equilibrium of the second stage game.

Suppose the Nash Equilibrium in the second stage game for this SPE involves the strategies $\{Stick, Accept\}$. Then $y_1 = z_1$ and $y_2 = 1 - z_1 - k_2(z_1 + z_2 - 1)$. Consider what happens if player 2 deviates to making the compatible demand $\tilde{z}_2 = 1 - z_1$, in the first stage. The payoffs from this deviation are $(z_1, 1 - z_1)$. Given that $1 - z_1 > y_2$, this is a profitable deviation. So if $z_1 + z_2 > 1$ and (z_1, z_2) are demands made in a subgame perfect equilibrium, the second stage Nash Equilibrium cannot involve $\{Stick, Accept\}$. A symmetric argument rules out $\{Accept, Stick\}$. If the second stage Nash Equilibrium is $\{Stick, Stick\}$ then $y_1 = y_2 = 0$. Player i could then profitably deviate by demanding $\tilde{z}_i = \epsilon$ where $0 < \epsilon = 1 - z_j$, thereby making a compatible offer and receiving a payoff of ϵ. So irrespective of the pure strategy Nash Equilibrium in the second stage game, there is always a profitable deviation for some player if $z_1 + z_2 > 1$.

□

Lemmas A1 and A2 imply that if (z_1, z_2) are demands made in a pure strategy SPE of the bargaining game, it must be that $z_1 + z_2 = 1$.

Lemma A3. *If (z_1, z_2) is the demand profile in a pure strategy SPE of the* *bargaining game with* $z_1 + z_2 = 1$ *then* $\nexists \epsilon > 0$ *and* $i \in \{1, 2\}$*such that*

$$1 - z_i - k_j \epsilon < 0 \tag{4}$$

and

$$1 - z_j - \epsilon - k_i \epsilon > 0. \tag{5}$$

Proof. Suppose not. Let $\epsilon > 0$ and let $1 - z_i - k_j \epsilon < 0$ and $1 - z_j - \epsilon - k_i \epsilon > 0$ for some $i \in \{1, 2\}$ with (z_i, z_j) being the demands made in an SPE of the bargaining game. I will show that player j has a profitable deviation. With the present demand profile, (z_i, z_j) the payoffs are also (z_i, z_j) due to compatibility. Now suppose player j deviates to making the incompatible offer $z_j + \epsilon$. Due to incompatible offers the second stage game would have to be played. If player i chooses *Accept* then player j is clearly better off choosing *Stick*. If player i chooses *Stick* then j's payoff from choosing *Accept* is $1 - z_i - k_j \epsilon$ which is strictly less than the 0 he gets if he *Sticks*, given the assumption above. So *Stick* strictly dominates *Accept* for player j. Given that player j will choose *Stick* player i would get $1 - z_j - \epsilon - k_i \epsilon$ if she chose *Accept* which is strictly greater than the 0 she would get if she chooses *Stick*. Consequently the unique Strict Nash Equilibrium of the second stage game following the deviation would involve i playing *Accept* and j playing *Stick*, with a payoff of $z_j + \epsilon$ for player j. Hence player j has a profitable deviation. □

Proposition 1

Necessity

Proof. Let (z_1^*, z_2^*) be the demands made in a pure strategy SPE of the bargaining game. From lemmas A1 and A2 it must be that $z_1^* + z_2^* = 1$. If an ϵ

satisfies the conditions of Lemma A3 for (z_1^*, z_2^*) it must be that $\epsilon > \frac{z_1^*}{k_1}$ (from (4), setting $i = 2$) and $\epsilon < \frac{z_2^*}{1+k_2}$ (from (5), setting $i = 2$). So it must be that $\frac{z_1^*}{k_1} < \epsilon < \frac{z_2^*}{1+k_2}$. Now, given that $\frac{z_2^*}{1+k_2}$ is bounded above by 1, such an ϵ will not exist iff

$$\frac{z_2^*}{1+k_2} \leq \frac{z_1^*}{k_1}. \tag{6}$$

A similar argument using (4) and (5) and setting $i = 1$ shows that for profitable deviations of the kind considered in Lemma 3 not to exist, it must also be true that

$$\frac{z_1^*}{1+k_1} \leq \frac{z_2^*}{k_2}. \tag{7}$$

Combining (6) and (7) gives us the necessary condition for (z_1^*, z_2^*) to be the equilibrium demands; namely

$$\frac{k_2}{1+k_1} \leq \frac{z_2^*}{z_1^*} \leq \frac{1+k_2}{k_1}. \tag{8}$$

□

Sufficiency

Proof. Let (z_1^*, z_2^*) satisfy (8) and $z_1^* + z_2^* = 1$. I will construct strategies that constitute an SPE of the bargaining game using these demands. In the first stage player 1 demands z_1^* while player 2 demands z_2^*. If the second stage game is reached and if player 2 demanded $z_2 > z_2^*$ in the first stage, then player 1 chooses $\{Stick\}$ while player 2 chooses $\{Accept\}$ if $1 - z_1^* - k_2(z_2 - z_2^*) > 0$ and $\{Stick\}$ otherwise. Similarly, if player 1 demanded $z_1 > z_1^*$ in the first stage, then player 2 chooses $\{Stick\}$ while player 1 chooses $\{Accept\}$ if $1 - z_2^* - k_1(z_1 - z_1^*) > 0$ and $\{Stick\}$ otherwise.

To see why these strategies constitute an SPE of the bargaining game, note first that neither player has any incentive to demand a lesser amount. Now consider player i's incentives to deviate by demanding $z_i > z_1^*$. If in the second stage player i is required by the strategies to play $\{Accept\}$ then it must be that $1 - z_j^* - k_i(z_i - z_i^*) > 0$. Given that player j's strategy requires j to $\{Stick\}$, i would do strictly worse by deviating to $\{Stick\}$. Further, given that i chooses $\{Accept\}$ player j can do no better than play $\{Stick\}$ as is required by his strategies. In other words the off equilibrium strategies induce a strict Nash Equilibrium of the second stage game when i demands $z_i > z_i^*$ and $1-z_j^*-k_i(z_i-z_i^*) > 0$. So by deviating to z_i, i gets a payoff of $1-z_j^*-k_i(z_i-z_i^*)$ which is strictly less than the payoff of $1 - z_j^*$ she was guaranteed under the original strategies. Now, if the deviation z_i is such that $1-z_j^*-k_i(z_i-z_i^*) < 0$ the strategies require i to $\{Stick\}$ which is indeed her dominant strategy in this case. By the fact that (z_1^*, z_2^*) satisfies (8) it must be the case that $\nexists \epsilon > 0$ such that $1 - z_j^* - k_i\epsilon < 0$ and $1 - z_i^* - \epsilon - k_j\epsilon > 0$. However the deviation z_i is such that setting $\epsilon = z_i - z_i^*$ we get $1 - z_j^* - k_i\epsilon < 0$. So (8) implies that $1 - z_i^* - \epsilon - k_j\epsilon \leq 0$. Substituting for ϵ we get $1 - z_i - k_j(z_j^* - (1 - z_i)) \leq 0$. The left hand term in this inequality is the payoff j gets from choosing *Accept* while choosing *Stick* gives him 0. Therefore, j's optimal action continues to be $\{Stick\}$ as suggested by the strategies. The pure Nash Equilibrium in the second stage after such deviations, thus, involve a payoff of $(0, 0)$, which makes i strictly worse off. As a result i has no incentive to deviate from the specified strategies. Hence, the strategies specified above constitute an SPE of the bargaining game. □

Lemma A4. *There exists a unique* $\bar{y}_j \in (0, 1)$ *such that* $\pi_i(1 - y_j) = c_i(y_j)$.

Proof. Let $g_i(y_j) = \pi_i(1 - y_j) - c_i(y_j)$

Note that $g_i(0) = \pi_i(1) = m_i > 0$ and $g_i(1) = -c_i(1) < 0$. Further, g_i is a strictly decreasing and continuous function. Consequently by the intermediate value theorem there exists $\bar{y}_j$ such that $g_i(\bar{y}_j) = 0$. Further, given that g_i is strictly decreasing, $\bar{y}_j \in (0, 1)$. $\square$

Proposition 2

Proof. Define the function $\hat{y}_1(y_2) = c_1^{-1}(\pi_1(1 - y_2)) + 1 - y_2$ for all $y_2 \in [0, 1]$ such that $\exists\ d > 0$ with $c_1(d) = \pi_1(1 - y_2)$.

Note that $\hat{y}_1(1) = 0$. By Lemma *A4* there exists $\bar{y}_2 \in (0, 1)$ such that $\hat{y}_1(\bar{y}_2) = 1$. Further, given **A1** and **A2**, $\hat{y}_1$ is a well defined, continuously differentiable and strictly decreasing function on $[\bar{y}_2, 1]$ with

$$\frac{\partial \hat{y}_1}{\partial y_2} = \frac{-\pi_1'(1 - y_2)}{c_1'(c_1^{-1}(\pi_1(1 - y_2)))} - 1 < -1. \tag{9}$$

Similarly define the function $\hat{y}_2(y_1) = c_2^{-1}(\pi_2(1 - y_1)) + 1 - y_1$ for all $y_1 \in [0, 1]$ such that $\exists\ d > 0$ with $c_2(d) = \pi_2(1 - y_1)$. By the same arguments as before, $\hat{y}_2$ is a continuously differentiable strictly decreasing function on the corresponding $[\bar{y}_1, 1]$ with

$$\frac{\partial \hat{y}_2}{\partial y_1} = \frac{-\pi_2'(1 - y_1)}{c_2'(c_2^{-1}(\pi_2(1 - y_1)))} - 1 < -1. \tag{10}$$

Let $\tilde{y}_2 : [0, 1] \to \Re$ be defined by $\tilde{y}_2(y_1) = \hat{y}_1^{-1}(y_1)$.

Note that $\tilde{y}_2(0) = 1$ while $\tilde{y}_2(1) = \bar{y}_2$. Also $\tilde{y}_2$ is a continuous and strictly decreasing function with

$$-1 < \frac{\partial \tilde{y}_2}{\partial y_1} = \frac{1}{\frac{-\pi_1'(1-y_2)}{c_1'(c_1^{-1}(\pi_1(1-y_2)))} - 1} < 0. \tag{11}$$

Therefore $\tilde{y}_2(\bar{y}_1) < 1$, since $\bar{y}_1 \in (0, 1)$.

Consequently $(\hat{y}_2 - \tilde{y}_2)(\bar{y}_1) = 1 - \tilde{y}_2(\bar{y}_1) > 0$.

Also, $(\hat{y}_2 - \tilde{y}_2)(1) = 0 - \bar{y}_2 < 0$.

Finally, the function $(\hat{y}_2 - \tilde{y}_2)$ is a strictly decreasing function of y_1 on $[\bar{y}_1, 1]$ as can be seen by subtracting the fraction in (11) from that in (10), the former being strictly greater than -1, the latter strictly less than -1 and both being negative.

Therefore by the intermediate value theorem and the fact that $(\hat{y}_2 - \tilde{y}_2)$ is a strictly decreasing function of y_1 on $[\bar{y}_1, 1]$, there exists a unique $y_1^* \in (\bar{y}_1, 1)$ such that $(\hat{y}_2 - \tilde{y}_2)(y_1^*) = 0$. Let $y_2^* = \hat{y}_2(y_1^*)$. $y_2^* \in (0, 1)$ since $y_1^* \in (\bar{y}_1, 1)$. Further, $y_2^* = \tilde{y}_2(y_1^*) \Rightarrow y_1^* = \hat{y}_1(y_2^*)$. Therefore, (y_1^*, y_2^*) solves (6) and (7) and does so uniquely amongst any (y_1, y_2) with $y_1 \in [\bar{y}_1, 1]$. The proof concludes by showing that (7) cannot hold for any $y_1 < \bar{y}_1$.

Let $y_1 < \bar{y}_1$. By the definition of $\bar{y}_1$, it must be that $\pi_2(1 - y_1) > c_2(y_1)$.

$\Rightarrow \pi_2(1 - y_1) > c_2(y_1 + y_2 - 1)$ for all $1 - y_1 \leq y_2 \leq 1$ as $c_2(\cdot)$ is a strictly increasing function. $\square$

Proposition 3

Proof. The argument for $z_1^* + z_2^* = 1$ is very similar to the linear case and is therefore skipped. I will first show that (z_i, z_j) with $z_i > y_i^*$ and $z_1 + z_2 = 1$ cannot be the demand profile of a pure strategy subgame perfect equilibrium. The payoffs generated by these demands are $(\pi_i(z_i), \pi_j(z_j))$. Further, $\tilde{y}_j(z_i)$ is well defined as $z_i > y_i^* > \bar{y}_i$ and satisfies $\tilde{y}_j(z_i) > z_j$. Now, given that $z_i > y_i^*$ it must be that $(\hat{y}_j - \tilde{y}_j)(z_i) < 0$.

$\Rightarrow \hat{y}_j(z_i) < \tilde{y}_j(z_i)$.

Since $\pi_j(1-z_i) = c_j(z_i + \hat{y}_j(z_i) - 1)$ by definition, it follows that

$$\pi_j(1-z_i) - c_j(z_i + \tilde{y}_j(z_i) - \epsilon - 1) < 0 \tag{12}$$

for a small enough $\epsilon > 0$.

On the other hand, since $\pi_i(1-\tilde{y}_j(z_i)) - c_j(z_i + \tilde{y}_j(z_i) - 1) = 0$ it must also be true that

$$\pi_i(1-(\tilde{y}_j(z_i) - \epsilon)) - c_j(z_i + \tilde{y}_j(z_i) - \epsilon - 1) > 0 \tag{13}$$

for a small enough $\epsilon > 0$.

Consider the deviation by player j involving a demand of $\tilde{y}_j(z_i) - \epsilon$ in the first stage. This leads to incompatible demands thereby leading to the second stage. Now, given (12) it is a dominant strategy for j to play {Stick}. Further (13) implies that player i would strictly prefer {Accept} to {Stick} conditional on j playing {Stick}. Consequently the unique Nash Equilibrium in the second stage would involve i accepting and j sticking to her offer. The payoff to j from this deviation is $\pi_j(\tilde{y}_j(z_i) - \epsilon)$ which is strictly greater than her original payoff. This profitable deviation rules out the possibility of the equilibrium demand profile being (z_i, z_j) with $z_i > y_i^*$ and $z_1 + z_2 = 1$.

Finally I construct a pure strategy SPE to support an element of the set $\{(z_1^*, z_2^*)\ s.t.\ z_1^* + z_2^* = 1, z_1^* \leq y_1^*\ and\ z_2^* \leq y_2^*\}$ as the first stage demand profile. Let $\{(z_1^*, z_2^*)$ be such an element. The strategies are as follows, Player i demands z_i^* in the first stage. If the second stage game is reached and if player j demanded $z_j > z_j^*$ in the first stage, then i chooses {Stick}, while j chooses {Accept} if $\pi_j(1-z_i^*) - c_j(z_i^* + z_j - 1) > 0$ and chooses {Stick} otherwise.

The above strategies can be verified to be subgame perfect, using arguments similar to the linear case. This concludes the proof. □

Corollary 3.1

Proof. Recall from Proposition 2 that (y_1^*, y_2^*) is the unique solution to (1) and (2). Proposition 3, then makes it clear that the highest share for player i in equilibrium is y_i^* and the lowest, $1 - y_j^*$. To see what happens to equilibrium shares if player i's cost function increases, consider the following setup. I fix player j's payoff and cost functions at π_j and c_j. Player i's payoff function is given by π_i, while two cost functions c_i and $\hat{c}_i$ are considered with $c_i(d) < \hat{c}_i(d)$, for all $d > 0$. Payoff and cost functions are assumed to satisfy **A1** and **A2** respectively. Let y_i^* and $1 - y_j^*$ be the highest and lowest equilibrium payoffs for i with cost function c_i. Let the corresponding payoffs for the cost function $\hat{c}_i$ be y_i^{**} and $1 - y_j^{**}$. Define $\hat{y}_i$ and $\bar{y}_j$ for the cost function c_i as in the proof for Proposition 2. Let $\hat{\hat{y}}_i$ and $\bar{\bar{y}}_j$ be the corresponding objects for $\hat{c}_i$. By definition,

$$\pi_i(1 - \bar{y}_j) = c_i(\bar{y}_j) \tag{14}$$

and

$$\pi_i(1 - \bar{\bar{y}}_j) = \hat{c}_i(\bar{\bar{y}}_j). \tag{15}$$

Given **A1**, **A2** and $c_i(d) < \hat{c}_i(d)$, for all $d > 0$, it must be true that $\bar{\bar{y}}_j < \bar{y}_j$. It is also easy to verify that $\hat{\hat{y}}_i(y_j) < \hat{y}_i(y_j)$ for all $y_j \in [\bar{y}_j, 1]$. By the definition of y_j^* it must be true that $(\hat{y}_i - \tilde{y}_i)(y_j^*) = 0$. Therefore $(\hat{\hat{y}}_i - \tilde{y}_i)(y_j^*) < 0$. On the other hand $(\hat{\hat{y}}_i - \tilde{y}_i)(\bar{\bar{y}}_j) = 1 - \tilde{y}_i(\bar{\bar{y}}_j) > 0$. Consequently there exists $x \in (\bar{\bar{y}}_j, y_j^*)$ such that $(\hat{\hat{y}}_i - \tilde{y}_i)(x) = 0$. In other words $y_j^{**} = x$. Importantly, note that $y_j^{**} < y_j^*$. Further, since $y_i^{**} = \tilde{y}_i(y_j^{**})$ with $\tilde{y}_i$ being a strictly decreasing function, it is true that $y_i^{**} > y_i^*$. Therefore increasing the cost function for player i from c_i^* to c_i^{**} increases both her lowest payoff from $1 - y_j^*$ to $1 - y_j^{**}$ and her highest payoff from y_i^* to y_i^{**}. In this sense, the more costly it is to back down from the first stage demand, the greater is the player's bargaining power.

Finally note that the difference between the highest and lowest equilibrium share for either player given the initial(modified) cost functions is equal to $y_1^* + y_2^* - 1$ $(y_1^{**} + y_2^{**} - 1)$. By definition, $\pi_j(1 - y_i^*) = c_j(y_1^* + y_2^* - 1)$ and $\pi_j(1 - y_i^{**}) = c_j(y_1^{**} + y_2^{**} - 1)$. Since $y_i^* < y_i^{**}$ it follows that $y_1^{**} + y_2^{**} - 1 < y_1^* + y_2^* - 1$. Therefore an increase in the cost functions shrinks the set of equilibria.

□

Corollary 3.2

Proof. $(\pi_1, \pi_2) \in \Pi_{A1}$ gives a corresponding $(\Pi(\pi_1, \pi_2), d) \in \mathcal{B}$. Now given the definition of $\mathcal{K}_\gamma$ it is clear that $\mathcal{K}_\gamma(\Pi(\pi_1, \pi_2), d) = (u_1, u_2)$ where $u_1 = \pi_1(x)$, $u_2 = \pi_2(1 - x)$ with $0 \leq x \leq 1$ and $\pi_1(x)/\pi_2(1 - x) = \gamma$.

Given (π_1, π_2) let (y_1^{*n}, y_2^{*n}) solve (1) and (2) for revoking cost functions c_1^n and c_2^n. Therefore,

$$\pi_1(1 - y_2^{*n}) = c_1^n(y_1^{*n} + y_2^{*n} - 1),$$

$$\pi_2(1 - y_1^{*n}) = c_2^n(y_1^{*n} + y_2^{*n} - 1).$$

As $n \to \infty$ by assumption $c_i^{n'} \to \infty$. Since π_i is bounded above it must be that $y_1^{*n} + y_2^{*n} - 1 \to 0$. Recalling the assumption that $\exists \epsilon > 0$ and an integer M such that $\forall d \in [0, \epsilon)$ and $\forall n > M$, $c_1^n(d)/c_2^n(d) = \gamma$, it must be that for high enough values of n, $\frac{\pi_1(1-y_2^{*n})}{\pi_2(1-y_1^{*n})} = \gamma$. So the limit of the solution to (1) and (2) as $n \to \infty$ is given by (y_1^{**}, y_2^{**}), such that $\frac{\pi_1(1-y_2^{**})}{\pi_2(1-y_1^{**})} = \gamma$ and $y_1^{**} + y_2^{**} = 1$. Also for high enough values of n,

$$\frac{\pi_1(1-y_2^{*n})}{\pi_2(y_2^{*n})} \leq \gamma \leq \frac{\pi_1(y_1^{*n})}{\pi_2(1-y_1^{*n})} \tag{16}$$

since $y_1^{*n} + y_2^{*n} - 1 > 0$ for every n. Now, by Proposition 3, for (z_1^n, z_2^n) to be a subgame perfect demand profile for $\Gamma^{c_1^n, c_2^n}(\pi_1, \pi_2)$ it must be that $z_1^n + z_2^n = 1$ with $z_1^n \leq y_1^{*n}$ and $z_2^n \leq y_2^{*n}$. In other words,

$$\frac{\pi_1(1-y_2^{*n})}{\pi_2(y_2^{*n})} \leq \frac{\pi_1(z_1^n)}{\pi_2(z_2^n)} \leq \frac{\pi_1(y_1^{*n})}{\pi_2(1-y_1^{*n})} \tag{17}$$

It is easy to see from (16) and (17) that (z_1^*, z_2^*) such that $z_1^* + z_2^* = 1$ and $\frac{\pi_1(z_1^*)}{\pi_2(z_2^*)} = \gamma$ is an element of the limit set of subgame perfect demand profiles as $n \to \infty$. To show that it is also the unique element in the limit set consider (z_1, z_2) such that $z_1 + z_2 = 1$ and $\frac{\pi_1(z_1)}{\pi_2(z_2)} = \gamma + \epsilon$ for some $\epsilon > 0$. Given that $lim_{n\to\infty} y_1^{*n} = y_1^{**}$ and the continuity of the π_i functions, it is true that $\frac{\pi_1(y_1^{*n})}{\pi_2(1-y_1^{*n})} \leq \frac{\pi_1(y_1^{**})}{\pi_2(1-y_1^{**})} + \epsilon/2 = \gamma + \epsilon/2$ for all $n > N$ where N is high enough. This implies that $\forall n > N$, $\frac{\pi_1(z_1)}{\pi_2(z_2)} = \gamma + \epsilon > \frac{\pi_1(y_1^{*n})}{\pi_2(1-y_1^{*n})}$, violating (17). Consequently, such a demand profile cannot be an element of the limit set. A similar argument eliminates demand profiles (z_1, z_2) such that $z_1 + z_2 = 1$ and $\frac{\pi_1(z_1)}{\pi_2(z_2)} = \gamma - \epsilon$ for some $\epsilon > 0$. Therefore the unique limit subgame perfect demand profile (z_1^*, z_2^*) is characterized by $z_1^* + z_2^* = 1$ and $\frac{\pi_1(z_1^*)}{\pi_2(z_2^*)} = \gamma$.

As a result, $\xi_\gamma^*(\pi_1, \pi_2) = (u_1, u_2)$ such that $u_1 = \pi_1(x)$, $u_2 = \pi_2(1-x)$ with $0 \leq x \leq 1$ and $\pi_1(x)/\pi_2(1-x) = \gamma$. □

References

[1] Abreu, D., Gul, F., 2000. Bargaining and reputation. Econometrica 68(1), 85-117.

[2] Binmore, K.G., 1987. Nash bargaining theory II, in: Binmore, K.G., Dasgupta, P.(Eds.), The economics of bargaining. Oxford:Basil Blackwell.

[3] Carlsson, H., 1991. A bargaining model where parties make errors. Econometrica 59(5), 1487-1496.

[4] Compte, O., Jehiel, P., 2002. On the role of outside options in bargaining with obstinate parties. Econometrica 70(4), 1477-1517.

[5] Crawford, V.P., 1982. A theory of disagreement in bargaining. Econometrica 50(3), 607-37.

[6] Ellingsen, T., Miettinen, T., 2008. Commitment and conflict in bilateral bargaining. Amer. Econ. Rev. 98(4), 1629-35.

[7] Kambe, S., 1999. Bargaining with imperfect commitment. Games Econ. Behav. 28(2), 217-37.

[8] Kalai, E., 1977. Proportional solutions to bargaining situations: Interpersonal utility comparisons. Econometrica 45(7), 1623-1630.

[9] Leventoglu, B., Tarar, A., 2005. Prenegotiation public commitment in domestic and international bargaining. Amer. Polit. Sci. Rev. 99(3), 419-33.

[10] Li, D., 2007. Bargaining with history-dependent preferences. J. Econ. Theory 136(1), 695-708.

[11] Li, D., forthcoming. Commitment and compromise in bargaining. J. Econ. Behav. Organ.

[12] Muthoo, A., 1996. A bargaining model based on the commitment tactic. J. Econ. Theory 69(1), 134-52.

[13] Muthoo, A., 1999. Bargaining theory with applications. Cambridge: Cambridge University Press.

[14] Nash, J.F., 1953. Two-person cooperative games. Econometrica 21(1), 128-40.

[15] Rubinstein, A., 1982. Perfect equilibrium in a bargaining model. Econometrica 50(1), 97-109.

[16] Schelling, T.C., 1956. An Essay on Bargaining. Amer. Econ. Rev. 46(3), 281-306.

[17] van Damme, E., 1991. Stability and perfection of Nash equilibria. 2nd ed., Berlin/New York: Springer-Verlag.

Chapter 2: Bargaining with Uncertain Commitment: On the Limits of Disagreement

1 Introduction

Bargaining impasses entail significant costs. Whether they manifest as strikes, lockouts or war, the bargaining parties end up at a highly inefficient outcome. One explanation for the existence of such disagreement relies on the ability of rational bargaining agents to commit themselves to aggressive demands. An agent who credibly commits herself to an aggressive demand can force an uncommitted opponent to concede. The ability to commit arises from a (revoking)cost which rational agents must pay to back down from their stated demand. Uncertainty regarding the revoking cost results in uncertain commitment ability. Both players may then attempt commitment to aggressive demands hoping that they themselves face a high revoking cost while their opponent faces a low (or no) cost. Simultaneous attempts to commit to aggressive demands yield disagreement. This leads to the question that this paper formally addresses: When does the ability to attempt commitment to aggressive demands lead to disagreement in bargaining between two rational agents, given that the success of the commitment attempt is ex ante uncertain?

The above question has been answered in the asymmetric information environment by Crawford(1982). This paper extends the basic model of Crawford(1982) to analyze the symmetric information case. In particular, I study a two stage game with two players bargaining over a pie of size 1. In stage 1 the two players announce their demands simultaneously. If these demands are compatible (add up to no more than 1) then each agent gets her own demand

and half the remaining surplus, if any. If the demands are incompatible a second stage simultaneous move game is played. Each player can either *stick* to her demand or *accept* the other's offer. If one player sticks to her demand while the other player concedes (Accept), the former gets her first stage demand while the latter only gets what was offered by the former. In addition, the conceding player must pay his revoking cost. If both players concede then both get their opponents offer, pay their respective revoking costs and split in half the excess of the surplus over the sum of their offers. Both players sticking to their incompatible demands results in disagreement with a resulting payoff of 0 to both. When making their demands the two players only know the distribution of the revoking costs. These costs become commonly known only after the demand stage but before the second stage game. This feature gives rise to the uncertain commitment ability of players.

I study this basic model under two sets of informational assumptions. In the first, as in Crawford(1982), I assume that the revoking costs can take values of either 0 or some number greater than 1 (henceforth referred to as binary distributions). If the players face revoking costs which have independent and identical binary distributions then I find that disagreement can always be supported in equilibrium, irrespective of the particular probability of facing the high cost, q. Further, if facing a high cost is less probable, $0 < q < 1/2$, any equilibrium must involve disagreement. Disagreement continues to be supported in equilibrium even if the revoking cost distribution functions are identical and perfectly positively correlated (the two players face an identical but uncertain cost). These results are collected in *Proposition 2*, showing the pervasiveness of disagreement in the presence of binary distributions.

In the second set of informational assumptions, players *do not* believe that intermediate revoking costs are impossible. In particular, the density functions for the revoking costs are assumed to be strictly positive and continuous over an interval between and including 0 and some value greater than 1.[1] In addition it is assumed that before the second stage game each player gets to know the realized values of the revoking costs but with a small amount of noise. The equilibrium predictions of this model are analyzed for the limit case when the amount of noise is made arbitrarily small. *Proposition 3* shows that if the revoking cost distribution functions are identical and perfectly positively correlated, disagreement cannot be supported in equilibrium, *irrespective of the particular distribution function considered*. If the distribution functions are independent and First Order Stochastically Dominate the uniform distribution then two results hold. First, the efficient profile of each party demanding half the surplus can be supported in equilibrium. Second, disagreement cannot be supported in equilibrium.

Symmetric Information: The study of symmetric information environments in this paper is motivated by the observation that in bargaining settings where such commitment tactics are available the revoking costs often end up becoming (almost) commonly known before concession decisions are made. For example, in international or domestic political disputes revoking costs take the form of "audience costs" as discussed in Fearon(1994). The two leaders make public announcements of their demand while the domestic audiences assess the performance of the leadership. Backing down may entail a revoking cost in the form of a significantly lower chance of reelection. The particular cost is determined by the relevance of a particular negotiation to the domestic audience's

[1] Such density functions made the problem intractable in the asymmetric information setting of Crawford(1982).

assessment. While uncertain when the demands are made, these costs can be easily ascertained by all parties soon after.

A recent movement in India, for example, involved Anna Hazare and the Indian government making incompatible demands regarding the contents of an anti corruption bill to be passed in parliament. Given the unconstitutional nature of the Hazare demand on the one hand and the ineffective past anti corruption role of the government on the other, it was by no means certain which way public opinion would swing. The Hazare movement ended up with an unprecedented level of public support. Hazare's high realized revoking cost consisted of losing credibility in front of such a large group of supporters. The Indian Government garnered less sympathy and therefore stood to lose less by backing down. News outlets, opinion polls and visible public rallies made the costs apparent to all soon after the demands had been made public. Eventually the Indian Government backed down.

Similar examples can be found in the the debt ceiling debates of the Obama and Clinton administrations. In such instances lobbying groups are an important source of revoking costs for elected leaders. Importantly, in all these cases, the uncertainty regarding revoking costs when demands are made gets resolved almost entirely before the concession decisions are made. Many more illuminating examples of such bargaining instances are discussed in detail in Schelling(1960), Martin(1993) and Fearon(1994).

Ellingsen and Miettinen(2008)(henceforth EM) also analyze symmetric information settings, but with findings that contrast sharply with this paper. EM show that the presence of uncertain commitment *always* results in disagreement, with both parties demanding the entire surplus in equilibrium. In EM bargaining agents have access to independent random commitment devices, using which, following incompatible demands, an agent is forced to either back down or stick (*achieve commitment*) to her demand with exogenously fixed probabilities. The key modeling difference in the present paper is that *achieving commitment* is required to be the result of equilibrium behavior in the second stage game, as in Crawford(1982). An agent must *choose* to play *Stick* in order to *achieve commitment*. This modeling difference leads to very distinct implications. In particular, in EM, the probability of a successful commitment attempt is independent of the demands made. By contrast, in this paper, with continuous densities and noisy signals, equilibrium play results in a systematic dependence of second stage concession behavior on first stage demands. The particular dependence, so established, often eliminates the possibility of disagreement. In such a setting, demanding the entire surplus can never be supported in equilibrium.

Demands and Concession Behavior: The analysis of binary distributions in this paper gives results that are similar to EM. In particular, disagreement is shown to always be supportable in equilibrium. The reason for this lies in the existence of equilibria in these models in which the probability with which a player backs down in the second stage does not depend upon the first stage demands. Notice that a player has no option but to stick to her demand when her revoking cost is greater than 1. So, if one player faces a cost of 0 and the other faces the high cost, the dominance solvable outcome involves the latter playing *Stick* while the former plays *Accept*. The existence of multiple equilibria in the second stage game, when both player face 0 costs, makes supporting disagreement essentially a question of selecting an appropriate equilibrium. Making the particular equilibrium selection independent of first stage demands makes supporting disagreement in equilibrium possible.

In other words, such equilibria behave *as if* the probability of a successful commitment attempt were exogenous.

The analysis of continuous distributions with noisy signals, however, limits the possibility of disagreement considerably. To understand the intuition behind these results it will help to spell out the counteracting forces involved in the model. Disagreement arises if both parties make high demands that are incompatible, since there is always a state of the world where neither player can back down following such a demand profile. Player 1's incentive to make a higher demand is driven by the possibility that following incompatible demands she will face a high revoking cost (and therefore *achieve commitment*), while player 2 faces a low cost and is therefore better off conceding. The opposite scenario works as a disincentive for making higher demands. A second disincentive arises from the possibility that both face high costs and are unable to back down resulting in the loss of the entire surplus.

These features are present in both the binary and continuous distribution models. The continuous distribution models along with the global games information structure, by making concession behavior dependent on first stage demands, gives rise to another disincentive to making higher demands. A higher demand systematically makes it more difficult for one's opponent to concede thereby conferring a greater probability of success to the latter's commitment attempt. This in turn reduces the payoff an agent can hope to get by making the higher demand. It is the addition of this disincentive that results in the lack of disagreement in the continuous density models. Importantly, it is not merely the use of continuous densities that yields the agreement results. The presence of noise is critical for generating the global games argument. Section 4.2 gives an example of disagreement with continuous, identical and perfectly correlated density functions in the absence of noise.

The global games structure results in the risk dominant outcome of the second stage being played as a result of iterated elimination of dominated strategies whenever there would otherwise be multiple equilibria. This argument is especially acute for the case where both agents face the same (but uncertain) revoking cost. Given an incompatible demand profile, in equilibrium, if one player makes a (sufficiently) higher demand than the other, then in the second stage either both players stick to their demands (when the cost is high enough) or the player with the higher demand backs down while the one with the lower demand gets her way. So, in equilibrium, conditional on making incompatible demands, each player would want to make the smaller demand. Consequently there is always some player who wishes to deviate from an incompatible demand profile. When the distributions are independent, the players weigh the benefits of making a higher demand against the subsequent shrinking of the risk dominant region (of the state space) where she actually gets her demand. This systematic relationship between the probability of a successful commitment attempt and first stage demands makes the results of this analysis different from those with binary distributions or exogenous commitment probabilities.

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the disagreement results in informational settings involving binary distributions. Section 4 considers the continuous density case where both parties face an identical but uncertain cost. Section 5 deals with the independent continuous density case. Section 6 concludes. Proofs are collected in the appendix.

2 Related Literature

Commitment and Reputation in Bargaining: The basic framework of the present analysis is almost identical to that of a symmetric information version of Crawford(1982). The only difference is the payoffs that result following incompatible demands if both player's choose to back down. In Crawford(1982) the payoff is given by an exogenously set compromise payoff, while in the present model each player gets what the other offered and half the remaining surplus. This assumption is also made in Kambe(1999), Abreu and Gul(2000) and Compte and Jehiel(2002). To show that this difference preserves the arguments leading to disagreement in the asymmetric information model in Crawford(1982), the latter's disagreement results are replicated using the present model in Section 3.1. Given that the analysis gets rid of an additional parameter (the compromise solution), the disagreement result of Crawford(1982) can in fact be seen in a simpler setting.

While specific arguments regarding the role of commitment tactics in bargaining can be traced back to Schelling(1960), Crawford(1982) was the first to analyze this issue in a formal game theoretic setting. A number of papers have extended the asymmetric information model of Crawford(1982) in a way closely related to the notion of reputation. Kambe(1999) replaces the second stage one-shot game with an infinite horizon counterpart where players may either stick to their demand or lower it, giving rise to a war of attrition game. While focussing on binary distributions, the analysis rules out the possibility of delay. Wolitzky(2011) considers the same model as Kambe(1999), but focusses on minmax profiles and payoffs as opposed to sequential equilibria. The goal here is to characterize the highest payoff a player can guarantee herself by announcing a bargaining posture, with the only assumptions being that her opponent is rational and believes that she will be committed to her posture (face the high cost) with some given probability. In Myerson(1991), Abreu and Gul(2000) and Compte and Jehiel(2002), the irrational or obstinate types are given exogenously, and rational players attempt to increase their shares by mimicking these types. This is in contrast with the earlier papers where following the choice of any demand, the player could become obstinate with a given probability(the probability of facing the high revoking cost). Abreu and Gul(2000) show the possibility of delay when with positive probability a player could be an obstinate type. Compte and Jehiel(2002) show that the existence of outside options in this setting may cancel out the effects of these obstinate types.

Relation to the Global Games Literature: A few comments regarding the global games information structure, critical for the results in this paper, are in order. Firstly, while the paper heavily uses the methods developed in Carlsson and Van Damme(1993) (henceforth CvD), it is not possible to directly apply the results of CvD in the present setting. In CvD it is shown that for a certain kind of perturbation to a *fixed* complete information strategic game with multiple strict equilibria, as the perturbation is made arbitrarily small, the unique rationalizable strategy profile corresponds to the risk dominant profile. In the present paper multiplicity of equilibria is a potential problem in the second stage game. However, the second stage game is itself generated endogenously by the choice of demands in the first stage. In such a case it is by no means self evident that for a sufficiently small amount of noise, in equilibrium only the risk dominant profiles will be played in *all* second stage games. Indeed, the latter statement is false for any positive amount of noise. The crucial part comprises in proving that the class of games where the multiplicity is unresolved for a small enough amount of noise, has a sufficiently negligible effect on the choices made in the first stage. The non trivial nature of such an extension of the equilibrium selection argument to endogenously determined games in the global games literature along with a general result in this regard can be found in Chassang(2008). Unfortunately the particular game studied in this paper does not satisfy the required conditions of Chassang(2008) and must therefore be studied separately.

Secondly the equilibrium selection result implicit in this paper is not one involving the perturbation of a perfect information game. The original game in this study is already one of incomplete information. The equilibrium selection argument in this case applies to subgame perfect strategy profiles of the incomplete information game. Consequently the criticism of Weinstein and Yildiz(2007) does not apply in this case. The limit results in this paper involve the amount of private noise becoming arbitrarily small. The common uncertainty (public noise) regarding revoking costs shared by both players in the first stage is held fixed since it is an intrinsic part of the strategic environment studied here and not itself a perturbation of some complete information game. Any concern regarding the generality of the class of perturbations considered here would then have to do with the class of densities considered for private noise. The generality of this class can be assessed by evaluating assumptions *A2* and *A3* in Section 4.1.

3 Binary distributions and pervasive disagreement

This section shows that if the revoking costs are drawn from binary distributions, either 0 or some value greater than the size of the entire pie, then there always exist equilibria which result in a positive probability of disagreement. This is true irrespective of whether the revoking costs become known privately(asymmetric case) or publicly(symmetric case), following incompatible demands.

For the rest of the section the following basic model applies. Each subsection will add a different set of assumptions to this framework. Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1, 2\}$, with j being the other player, $j \in \{1, 2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0, 1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with $d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k_1, 1 - z_1 + d - k_2$	$1 - z_2 - k_1, z_2$
Stick	$z_1, 1 - z_1 - k_2$	$0, 0$

Table 1: Payoffs following incompatible demands

3.1 Asymmetric information case

The informational assumptions of this subsection are identical to that of Crawford(1982). The only modeling difference lies in the payoff specification when both players simultaneously concede following incompatible demands. In Crawford(1982) these payoffs are given exogenously, while it is endogenously determined here. The results below show that the disagreement results of Crawford(1982) are *not* weakened by this change. Moreover, in the absence of additional parameters representing exogenous compromise payoffs, the disagreement results can be seen more transparently.

Add to the game defined above, the assumption that players in the first stage do not know the value of k_i. They only know that they are independent random variables with $Pr(k_i > 1) = q$ and $Pr(k_i = 0) = 1 - q$. Following incompatible demands and before playing the second stage game, players get to know their own but not their opponent's revoking cost, k_i. Given these assumptions the following results hold.

Proposition 1. *(a) For any value of* $q \in (0, 1)$ *there exists an equilibrium with a positive,* q^2*, probability of disagreement.*
(b) If $0 < q < \frac{1}{2}$ *then any equilibrium must entail a positive probability of disagreement.*

Proposition 1(a) may seem like a stronger result than the disagreement result in Crawford(1982). In the latter paper it was shown that disagreement can be supported in equilibrium if q is small. The possibility of disagreement with high values of q was indeterminate. Proposition 1, on the other hand, shows that even if, ex ante, the probability of commitment is arbitrarily high (close to 1), the players may still choose incompatible demands and therefore lose the surplus with near certainty. However, Gori and Villanacci(2011) have shown that disagreement can be supported in the Crawford(1982) model even when q is large.

To understand the rationale behind Proposition 1(a), notice first that following incompatible demands if a player faces the high revoking cost her strictly dominant action (irrespective of the demands made) is to play *Stick*. Suppose player i plays *Accept* when her cost is 0. Then the two second stage choices available to j yield exactly the same payoff if both players made a demand of $z = \frac{q+1}{2}$. Further if player i makes a demand higher than z while still playing *Accept* when her cost is 0, player j must then optimally choose *Stick* when her cost is 0. Following a demand profile (z, z) each player can therefore play *Stick* with a high cost and *Accept* with a low cost in equilibrium. A higher demand by player i can be dissuaded by player j playing *Stick* irrespective of the cost, forcing i to concede when the cost is 0, resulting in a payoff loss. The strategies for the second stage Bayesian game with demands (z, z)) continue to be in equilibrium if one of the players makes a lower but still incompatible demand, giving the latter a lower payoff.

Given such a second stage strategy profile and initial demands of $z = \frac{q+1}{2}$ each, no player has an incentive to deviate. Such a demand profile, being incompatible, leads to disagreement with probability q^2. It may seem surprising that players would not want to deviate to simply making a compatible demand, especially when q is very high. Notice, though, that when q is really high, the share being offered by the other player is also sufficiently low, $\frac{1-q}{2}$. This low offer makes it a strictly better alternative for a player to make the higher incompatible demand and rely on the small probability with which she gets her stated demand.

Proposition 1(b) is driven by the fact that when $q < \frac{1}{2}$, if some player deviates from compatible demands to making a higher demand, the probability with which the entire surplus is lost, q^2, is less than the probability with which the deviating player gets her demand $q(1-q)$. If the deviating player's increase in demand is small enough, she can ensure that there is still enough room for the other player to back down upon facing a 0 cost. Given a compatible demand profile, the deviating player would be the one with the smaller of the two compatible demands.

3.2 Symmetric information case

Asymmetric information has been shown to give rise to inefficiency in numerous bargaining models. In studying the role of commitment it is important to ascertain if the disagreement results are an artifact of asymmetric information. Ellingsen and Miettinen(2008) have shown that, even without asymmetric information, when the probability of a successful commitment attempt is exogenous(and commonly known), disagreement is an immediate outcome. This subsection studies the symmetric information scenario by making the revoking costs publicly known following incompatible demands. However, the probability of a successful commitment attempt is derived endogenously from equilibrium behavior in the second stage game. The results below show that when the revoking costs are drawn from binary distributions, there always exist equilibria that support disagreement. This is true even if the players know for sure that they will face the *same* revoking cost in the second stage but are unsure about its value when making their demands.

In this subsection, in addition to the basic model outlined earlier, it is assumed that while the costs of backing down are uncertain to both players at the demand stage, they become common knowledge following incompatible demand profiles. In particular, in the first stage it is common knowledge that player i faces cost k_i which takes a value greater than 1 with probability q

while $Pr(k_i = 0) = 1 - q$.

Two settings are analyzed. In the first, the distribution functions for k_1 and k_2 are assumed to be independent. In the second it is assumed that both players face identical revoking costs, $Pr(k_1 = k_2) = 1$. Following incompatible demands the true values of k_1 and k_2 are made common knowledge before the second stage game is played. The departure from Section 3.1 lies in the elimination of asymmetric information in the second stage game. In this symmetric information setup the following results hold.

Proposition 2. *If the distribution functions for k_1 and k_2 are independent,*
(a) For $0 < q < 1$, the incompatible demand profile $(1, 1)$ can be supported in equilibrium, resulting in disagreement with probability q^2.
(b) For $0 < q < 1/2$, no efficient equilibrium exists.
If the players face the same revoking cost, $Pr(k_1 = k_2) = 1$,
(c) For $0 < q < 1$ the incompatible demand profile $(1, 1)$ can be supported in equilibrium, resulting in disagreement with probability q^2.

The disagreement results in Proposition 2 depend heavily on the multiplicity of Nash Equilibria in the second stage games following incompatible demands. The multiplicity allows for the construction of equilibria in which the probability with which a player backs down in the second stage *does not* depend upon the particular demands made in the first stage. It is this independence of second stage behavior from first stage demands that makes disagreement supportable in equilibrium.

Consider the setting with independent revoking costs. Following incompatible demands, three of the possible four second stage games are dominance solvable. If both players face high costs the unique profile is *(Stick, Stick)*. If player i faces the high cost and j the low cost, the dominance solvable profile involves i playing *Stick* and j playing *Accept*. If both players face 0 costs, however, there exist two strict pure strategy Nash Equilibria. The disagreement result of Proposition 2(a) relies on the appropriate equilibrium selection in these second stage games, following different incompatible demand profiles. In the subgame perfect equilibrium constructed to support the profile $(1, 1)$, the choice of second stage Nash Equilibrium for the case of $k_1 = k_2 = 0$ is entirely *independent of the first stage incompatible demands*. In particular, Player 1 plays *Stick* while Player 2 plays *Accept* following any incompatible profile when they both face a cost of 0. Player 2 cannot, for instance, force Player 1 to concede by making a lower demand since the second stage behavior is independent of the particular incompatible demand profile.

Proposition 2(c) further highlights the acuteness of the second stage multiplicity problem. In this case both players know that they will face identical revoking costs in the second stage. So the incentive to making a higher demand that arises from the possibility that one will find it too costly to back down while one's opponent wont simply does not exist. Disagreement is again supported by making appropriate equilibrium selection in the second stage games, independent of the first stage demands. If player 1 never backs down, irrespective of the revoking cost, then player 2 can do no worse by playing *Accept* when the cost is 0. Further if both players demand the entire pie, making a compatible offer does not help either. The rationale behind the non existence of efficient equilibria when the probability of facing a high revoking cost is low, as established in Proposition 2(b), is very similar to that for Proposition 1(b). Deviating from a compatible profile yields a gain with probability $q(1-q)$ and a loss of the entire surplus with probability q^2. When q is small, deviating to a demand of 1 results in a gain that outweighs the loss.

Interestingly, both players demanding the entire pie cannot be supported in the asymmetric information environment of Section 3.1. The second stage multiplicity in the symmetric information setting, in fact, makes it easier to support disagreement. As argued earlier, disagreement is easy to support if the probability of a successful commitment attempt can be made independent of the first stage demands. In the strategic environments described in Sections 4 and 5, it is precisely this independence of second stage behavior from first stage demands that collapses. Further, the particular dependence that is established overturns the disagreement results of this section.

4 Identical revoking costs with continuous density functions

This section studies the bargaining game in settings where the revoking cost can take values from an interval containing the points 0 and 1. The idea captured in this assumption is that players do not believe that intermediate values of revoking costs are impossible. The probability attached to such values, however, can be arbitrarily small.

Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1, 2\}$, with j being the other player, $j \in \{1, 2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0, 1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with $d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k, 1 - z_1 + d - k$	$1 - z_2 - k, z_2$
Stick	$z_1, 1 - z_1 - k$	$0, 0$

Table 2: Payoffs following incompatible demands

4.1 Noisy signals and agreement

In the first stage, when choosing their demands, players' prior regarding the cost of backing down k is given by a random variable K which takes values in $[0, \bar{k}]$ where $\bar{k} > 1$. Having announced their demands, each player i gets a noisy signal, k_i^ϵ about k before playing the simultaneous move game. In particular, player i observes a realization of the random variable K_i^ϵ that is defined by

$$K_i^\epsilon = K + \epsilon E_i, \qquad i = 1, 2$$

where E_i is a random variable taking values in $\mathbb{R}$ and $\epsilon > 0$ serves as the scale parameter for the noise. A strategy for player i, comprises of a demand $z_i \in [0, 1]$ and a measurable function $s_i(z_1, z_2)$ for every incompatible demand profile, that gives the probability of playing *Accept* as a function of the the observed cost of backing down k_i^ϵ. So, $s_i(z_1, z_2) : [-\epsilon, \bar{k} + \epsilon] \to [0, 1]$. Γ^ϵ is used to denote this two stage game for a particular value of ϵ.

The following assumptions are made on the parameters of the model.

A1. *K admits a density h that is continuously differentiable on $(0, \bar{k})$, strictly positive, continuous and bounded on $[0, \bar{k}]$.*

A2. *The vector (E_1, E_2) is independent of K and admits a density φ.*

A3. *The support of each E_i is contained in the interval $[-1, 1]$ in $\mathbb{R}$ and φ is continuous on $[-1, 1] \times [-1, 1]$.*

As a result of these assumptions the model acquires the structure of a global game as studied in CvD. I am interested in the perfect equilibrium prediction of Γ^ϵ for small values of ϵ. To this effect the following proposition holds.

Proposition 3. *Given $A1, A2, A3$, and for sufficiently small $\epsilon > 0$, if players use pure strategies for their first stage demands, there is never any disagreement in any perfect equilibrium of the game Γ^ϵ.*

The impossibility of disagreement in this setting is in sharp contrast with Proposition 2(c) which showed that disagreement can be supported in equilibrium irrespective of the revoking cost probability function. Notice that the assumptions for Proposition 3 allow for density functions that can arbitrarily approximate the two point random variables considered in Section 3.

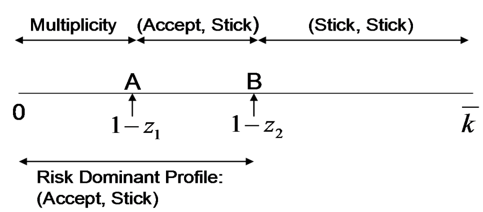

Figure 1: Second stage equilibrium behavior: Common Cost

To get some intuition for Proposition 3 consider Figure 1. Suppose player 1 makes the higher demand in an incompatible demand profile (z_1, z_2). The $0\bar{k}$ line represents the state space for the revoking cost. In the absence of noise ($\epsilon = 0$), the second stage game following the incompatible profile (z_1, z_2) would be one of complete information and would depend on the realized value (k) of the revoking cost, K. Now for all realizations of K in the $B\bar{k}$ region the dominant strategy for both players would be to play *Stick* since backing down would incur a cost strictly greater than the share received by playing *Accept*.

The unique NE in the second stage for such values of K would thus be (*Stick*, *Stick*). If K takes a value in AB, then Player 2 has a strictly dominant action in *Stick* since it would be too costly for her to back down. Conditional on Player 2 backing down, the optimal choice for Player 1 is to play *Accept*, since the revoking cost is not higher than the share she would get by conceding. The unique NE for all such k in AB is thus (*Accept*, *Stick*). K taking a value in $0A$, however results in multiplicity. Both (*Accept*, *Stick*) and (*Stick*, *Accept*) are pure NE of the second stage game for such values of K. Since the revoking cost is low enough relative to the amount received by both players upon concession, the problem now becomes one of coordination. In the absence of noise, the choice of Nash Equilibrium in this region can be entirely arbitrary.

It turns out, however, that for all values of K in the region $0B$ the unique risk dominant profile is (*Accept*, *Stick*). In the presence of a small amount of noise the setting becomes a global game. Iterated elimination of strictly dominated strategies in the resulting Bayesian games results in the players coordinating on the risk dominant profile for every realization of K. This in turn implies that while (*Stick*, *Stick*) is played for all realizations in the region $B\bar{k}$, (*Accept*, *Stick*) would be played for all realizations in the $0B$ region.

Player 1 receiving a noisy signal sufficiently in the interior of $B\bar{k}$ would know for sure that the true state of the world is in fact in $B\bar{k}$ and would therefore play her strictly dominant action *Stick* for such observations. Similarly player 2 would play *Stick* following any observation sufficiently in the interior of $A\bar{k}$. Given that player 2 plays *Stick* for observations in $A\bar{k}$, player 1 upon observing a value sufficiently in the interior of AB would infer that player 2 must have observed a value greater than A. It would then be conditionally dominant for Player 1 to play *Accept* for such observations. So there emerges an interval where the profile (*Accept*, *Stick*) is played. The question now is what is the left limit of this interval. In other words, what is the highest observed value of K when one of these players choose to switch their actions from the (*Accept*, *Stick*) profile. For a small enough value of ϵ it turns out that this left limit cannot be greater than 0, resulting in the profile (*Accept*, *Stick*) being played for all values of K when earlier there was multiplicity.

A crucial part of this argument is the existence of a sufficiently large (with respect to ϵ) region AB. So if Player 1 makes a sufficiently larger demand than Player 2, given an incompatible profile, whenever some player does back down it must be Player 1. Since backing down always pays less than simply accepting the other parties offer, Player 1 would be better off making a compatible demand in the first stage. More importantly this shows that conditional on an incompatible demand being made each party would want to make the lower demand and force the other to concede. This applies to the case when the region AB is not that large. In this case one of the players would have a strict incentive to lower her demand marginally and force a concession from the other whenever the cost is low. Such a deviation may not be possible if lowering ones demand essentially leads to a compatible profile. However it is shown that for an incompatible demand profile that makes deviation to compatible positions unprofitable, it must be that both players are making sufficiently high demands. This in turn ensures the possibility of lowering ones demand and still make it incompatible.

The result, therefore, relies on these two features of equilibrium strategies in this game. Firstly for a given incompatible profile, if no player wants to simply deviate to a compatible demand then the original demands must be sufficiently high. Secondly, conditional on making incompatible demands that are sufficiently high, each player has a strict incentive to make a lower demand and force the other player to concede most of the time. These two features make the existence of an incompatible demand profile and consequently disagreement, in equilibrium, an impossibility.

It should be pointed out that the assumptions $A1, A2, A3$, are slightly weaker than the corresponding assumptions made for the one-dimensional case in CvD. In particular the noise density function is allowed to be discontinuous at the boundary points of its support in the present study, while this is ruled out by the assumptions in CvD.[2]

The outline of the proof is as follows. *Lemma 1* establishes a result that is crucial for the global game arguments used for the result. In particular the distribution of player 1's observation conditional on player 2's observation is symmetric to the distribution of player 2's observation conditional on player 1's observation, in the sense that they add up arbitrarily close to 1. *Lemma 2* establishes a continuity result. It shows that for a given profile of measurable strategies,$(s_i)_{i\in\{1,2\}}$, and for any incompatible demand profile, the probability with which player i chooses *Accept* and the expected value of the true revoking cost, k, conditional on player j making an observation,k_j, is continuous in player j's observation. *Lemma 3* shows how following an incompatible demand profile if player i observes a cost sufficiently larger than $1 - z_j$ her dominant action is to play *Stick*. It is then argued in *Lemma 4* that following incompatible demands (z_1, z_2) if z_i is sufficiently larger than z_j, then there will always be observation values for which the unique dominance solvable outcome would

[2]Indeed, the motivating example in CvD involves noise with a uniform density, and does not satisfy the assumptions of their paper. However the discontinuity at the boundary points merely requires a little more work as is done in *Lemma 2*, and does not endanger the equilibrium selection argument in CvD. I thank Hans Carlsson for helping me with my doubts regarding this issue.

involve i backing down while j plays *Stick*. *Lemmas 5 - 7* then show that following such an incompatible demand profile, either for all lower observations i will continue to back down with j playing *Stick*, or there will be two observation values particularly close to each other where the two players will switch their actions. *Lemma 8*, the critical part of the proof, then shows that if z_i is sufficiently larger than z_j, such switch points cannot exist and therefore player i will continue to back down with j playing *Stick*. This result is a consequence of the global games information structure that appears in the model for small enough $\epsilon > 0$. *Lemma 8* relies heavily on the properties of symmetry and continuity established in *Lemmas* 1 and 2. This result allows for a complete characterization of equilibrium second stage strategies and payoffs following incompatible demand profiles and is stated in *Lemma 9*.

I then consider the choice of first stage demands. It can be easily seen that demands that add up to less than 1 always allow for deviations. *Lemma 10*, in addition, also shows that incompatible profiles with one player making a sufficiently higher demand than the other cannot be supported. This is a natural implication of *Lemma 8* where the player with the higher demand was shown to always be the one to concede. Making a compatible demand would do strictly better than making such a high incompatible demand. Next, *Lemma 11* establishes a lower bound that the sum of the demands must satisfy to be an incompatible profile from which neither player wants to deviate to a compatible profile. Finally it is shown that if an incompatible profile of demands involves z_1 and z_2 that do not differ much in value (no demand is sufficiently greater than the other as in *Lemma 8*) but sum up to greater than the bound mentioned in *Lemma 11*, then there is always a player i who could strictly improve her payoff by making a lower but still incompatible demand.

This lower demand by i forces j, in equilibrium, to always be the one backing down in the second stage. These arguments together exhaust the possible set of incompatible demand profiles. Consequently it is shown that equilibria involving pure strategies in the first stage cannot involve incompatible demands, thereby eliminating the possibility of disagreement.

First I define a few terms for the game Γ^{ϵ} that allow the use of *Lemma 4.1* in Carlsson and van Damme(1993), henceforth (CvD). Let $F_i^{\epsilon}(k_j|k_i)$ and $f_i^{\epsilon}(k_j|k_i)$ be the distribution and density functions, respectively, of K_j^{ϵ} conditional on $K_i^{\epsilon} = k_i$. Let φ^{ϵ} be the joint density of $(\epsilon E_1, \epsilon E_2)$. Then,

$$f_i^{\epsilon}(k_j|k_i) = \frac{\int h(k)\varphi^{\epsilon}(k_1 - k, k_2 - k)dk}{\int\int h(k)\varphi^{\epsilon}(k_1 - k, k_2 - k)dk_j dk} \tag{1}$$

The following lemma is the one dimensional version of *Lemma 4.1* in *CvD* that applies to the present model. This symmetry result is critical for the proof of *Lemma 8*.

Lemma 1 (CvD). *Let $k_1, k_2 \in [-\epsilon, \bar{k} + \epsilon]$. Then there exists a constant $\kappa > 0$ such that for sufficiently small $\epsilon > 0$,*

$$|F_1^{\epsilon}(k_2|k_1) + F_2^{\epsilon}(k_1|k_2) - 1| \leq \kappa\epsilon \tag{2}$$

Next, it is shown that for a pair of measurable second stage strategies, player i's expectation regarding the true value of k and the probability with which j plays *Accept*, conditional on observing k_i^{ϵ} are continuous functions of k_i^{ϵ}. Given j's second stage strategy s_j, let the probability with which i, conditional on observing k_i^{ϵ}, expects that j will play *Accept* be denoted by

$Pr(A_j|k_i^\epsilon, s_j)$.[3] So,

$$Pr(A_j|k_i^\epsilon, s_j) = \int s_j(k_j) f_i^\epsilon(k_j|k_i^\epsilon) dk_j \tag{3}$$

Also, let i's expectation of k given her observation k_i^ϵ be denoted as $E^\epsilon(k|k_i^\epsilon)$.

Lemma 2. *For a given incompatible demand profile* (z_1, z_2) *and strategies* s_i, s_j, $Pr(A_j|k_i^\epsilon, s_j)$ *and* $E^\epsilon(k|k_i^\epsilon)$ *are continuous in player* i*'s observation* k_i^ϵ.

Equilibrium behavior in the second stage game following an incompatible demand profile is considered next. The payoffs specified in Table 2 make it evident that if the observed cost is high enough the player would strictly prefer to play *Stick*. The following lemma captures this immediate but useful implication of observing such high costs of backing down.

Lemma 3. *In equilibrium, following an incompatible demand profile* (z_1, z_2), *conditional on observing* $k_i^\epsilon > 1 - z_j + \epsilon$, Stick *is the strictly dominant action for player* i.

Lemma 3 shows that for high enough observation values (i.e. greater than $1 - \min\{z_1, z_2\} + \epsilon$) the unique dominance solvable outcome in the second stage game is (*Stick, Stick*).

The next lemma shows that if the higher of the two incompatible demands is sufficiently larger than the lower demand, there will be an interval of observations that would always lead to a unique dominance solvable outcome in the second stage game where the player with the higher demand plays *Accept* while the other plays *Stick*. This is the crucial *dominance solvable region* in CvD that has a remote influence on the rest of the state space.

[3] The dependence of s_j on the demand profile (z_1, z_2) is suppressed for notational convenience, but it should be noted that the arguments are for a given pair of incompatible demands.

Lemma 4. *For an incompatible demand profile* (z_1, z_2) *such that* $z_i - z_j > 4\epsilon$*, the unique dominance solvable outcome of the second stage game following both players making an observation in* $(1 - z_i + 3\epsilon, 1 - z_j - \epsilon)$*, involves* i *playing* Accept *and* j *playing* Stick.

Given an equilibrium of Γ^ϵ and a pair of incompatible demands (z_1, z_2) where $z_i - z_j > 4\epsilon$, let $k_i^{\epsilon*}$ denote the highest observation value k_i^ϵ below $1 - z_i + 3\epsilon$ for which i chooses to play *Stick*. Similarly let $k_j^{\epsilon*}$ denote the highest observation value k_j^ϵ below $1 - z_i + 3\epsilon$ for which j chooses to play *Accept*. It is assumed that if i following some observation strictly greater than $-\epsilon$ is indifferent between her actions she chooses to play *Stick* while when j is indifferent he plays *Accept*. The next lemma shows that $k_i^{\epsilon*}$ and $k_j^{\epsilon*}$ are well defined. In other words, it is shown that unless the players continue to play the strategies they used in the dominance solvable region of *Lemma 4* for even lower values of K, there must exist points (highest value of their respective observations) on the state space at which the players switch the strategies. The continuity result of *Lemma 2* is critical to establishing this result.

Let $B_i^\epsilon(z_1, z_2)$ denote the set of observations $k_i^\epsilon > -\epsilon$ such that $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ and i plays *Stick* for such observations (i.e. $s_i(k_i^\epsilon) = 0$). Similarly let $B_j^\epsilon(z_1, z_2)$ denote the set of observations $k_j^\epsilon > -\epsilon$ such that $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ and j plays *Accept* for such observations (i.e. $s_j(k_j^\epsilon) = 0$).

Lemma 5. *In any equilibrium of* Γ^ϵ *following a pair of incompatible demands* (z_1, z_2) *where* $z_i - z_j > 4\epsilon$*, either* $B_i^\epsilon(z_1, z_2)$ *is empty or* $k_i^{\epsilon*} = \max\{x | x \in B_i^\epsilon(z_1, z_2)\}$ *is well defined.*
Similarly, either $B_j^\epsilon(z_1, z_2)$ *is empty or* $k_j^{\epsilon*} = \max\{x | x \in B_j^\epsilon(z_1, z_2)\}$ *is well defined.*

The following lemma shows that if one player does not switch her second stage action at smaller values of observed cost from that used in the dominance solvable region of *Lemma 4*, then the other player would not make a switch either.

Lemma 6. *If $B_i^\epsilon(z_1, z_2)$ or $B_j^\epsilon(z_1, z_2)$ is empty then they are both empty.*

The next lemma establishes a relation between $k_i^{\epsilon *}$ and $k_j^{\epsilon *}$ when they are well defined. In particular it is shown that the switching points when they exist would be near each other.

Lemma 7. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j > 4\epsilon$ if the terms are well defined then, $k_i^{\epsilon *} < k_j^{\epsilon *} + 2\epsilon$.*

The next lemma contains the crucial argument that drives the result, since it shows that for incompatible demands with the higher demand sufficiently larger than the smaller one, the player with the higher demand always concedes whenever the observed cost is in the range that generated multiplicity in the complete information game. The symmetry of conditional beliefs guaranteed by *Lemma 1* plays a significant role here.

Lemma 8. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, the sets $B_i^\epsilon(z_1, z_2)$ and $B_j^\epsilon(z_1, z_2)$ are empty.*

Lemma 8 makes it immediate that following an incompatible demand profile (z_1, z_2), where $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, player j plays *Stick* irrespective of the observation k_j^ϵ. On the other hand player i plays *Stick* for $k_i^\epsilon > 1 - z_j + \epsilon$ while playing *Accept* for $k_i^\epsilon < 1 - z_j - \epsilon$. This allows for a characterization of the expected payoffs in the first stage, from making such incompatible demands.

Let $y_i(z_1, z_2)$ and $y_j(z_1, z_2)$ denote i and j's expected payoff in equilibrium from making demands z_i and z_j. The following lemma is delivered simply by calculating payoffs given the characterization of equilibrium behavior in the second stage discussed in *Lemmas 3, 4 and 8*.

Lemma 9. *In any equilibrium of* Γ^ϵ *following a pair of incompatible demands* (z_1, z_2) *where* $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$*, it must be that*

$$z_j F_i^\epsilon(1 - z_j - \epsilon) \leq y_j \leq z_j F_i^\epsilon(1 - z_j + \epsilon) \tag{4}$$

$$y_i \leq \int_0^{1-z_j} (1 - z_j - w)h(w)dw \tag{5}$$

The analysis can now turn to the choice of first stage demands. Let the set of demand profiles that can be supported by equilibrium strategies in Γ^ϵ be denoted by Eq^ϵ. Further let $\phi(d) = \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$. The following lemma shows how equilibrium demands could never add up to less than 1. Also, it states the immediate implication of *Lemma 8* that incompatible demands with one player making a significantly higher demand than the other cannot be supported in equilibrium.

Lemma 10. *If* (z_1, z_2) *satisfies either of the following conditions,*

1. $z_1 + z_2 < 1$
2. $z_1 + z_2 > 1$ *and* $|z_1 - z_2| \geq \phi(d)$

then, $(z_1, z_2) \notin Eq^\epsilon$*.*

Let $\hat{k} = \int \min\{k, 1\}h(k)dk$. The following lemma shows that for an incompatible demand profile to be supported in equilibrium, the excess demand must be above a positive lower bound. If this were not to be the case then at least one of the players would have a strict incentive to deviate to making a compatible demand.

Lemma 11. *If $z_1 + z_2 > 1$ and $d < \hat{k}/2$ then $(z_1, z_2) \notin Eq^\epsilon$.*

Recall that $\phi(d) = \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$. Let $\phi^* = \phi(\hat{k}/8)$. The next lemma shows that incompatible demands that are close to each other but result in an excess demand that exceeds the bound from *Lemma 11* cannot be supported in equilibrium. Since the demand profile satisfies the lower bound, the result relies on the existence of some player i who can lower her demand enough to force the j to always do the conceding, thereby generating a higher expected payoff for i.

Lemma 12. *If $z_1 + z_2 > 1$, $d \geq \hat{k}/2$ and $|z_1 - z_2| < \phi(d)$ then $(z_1, z_2) \notin Eq^\epsilon$ for small enough ϵ.*

Proof of Proposition 3

Proof. Proposition 3 follows immediately from the observation that *Lemmas 10, 11 and 12* exhaust the entire set of incompatible demand profiles. □

4.2 Example of disagreement in the absence of noise

With revoking costs perfectly correlated and identical across players, Proposition 3 shows that with continuous density functions there is no disagreement while Proposition 2 shows that there can always be disagreement with binary distributions. The critical difference that gives rise to the contrasting results, however, is the presence of noisy signals in the continuous density case.[4] This

[4]Global game arguments require the state space to be a continuum and therefore has no analog in the discrete case.

can be seen by observing that without the noise there may be disagreement even in the continuous density case. An example of such a scenario follows.

Consider the game outlined earlier in this section with the additional assumption that $\epsilon = 0$. In other words, both players, following incompatible demands get to know the precise value of the revoking cost. Let the distribution function for the revoking cost be given by F with the interval $[0, \bar{k}]$ as its support where $\bar{k} > 1$. It is assumed that $F(1/4) = 9/10$ and $F(q) = 2/5$ where $q \in (0, 1/4)$. As outlined earlier, the players choose their demands in the first stage, with common knowledge regarding the distribution of the revoking cost, k. Following the demand stage, both players get to know the realized value of k and then decide simultaneously whether to stick to their demand or back down.

Consider the following subgame perfect strategy profile that leads to a positive probability of disagreement. The players demand identical amounts, namely $z_1 = z_2 = 3/4$. In the second stage, if $k \geq 1/4$, both players play *Stick*. If $k \in (q, 1/4)$ then player 1 plays *Accept* while player 2 plays *Stick*. If $k \leq q$ then player 1 plays *Stick* while player 2 plays *Accept*. In the subgame following player i making a demand, $\tilde{z}_i > z_i$, player $-i$ plays *Stick* irrespective of the realized value of k, while player i plays *Accept* if $k \leq 1/4$ and *Stick* otherwise. In the subgame following player i making a demand, $\tilde{z}_i < z_i$ but $\tilde{z}_i > 1 - z_i$, the following profile is played. If $k \geq 1 - \tilde{z}_i$ both players play *Stick*. If $k \in 1/4, (1 - \tilde{z}_i)$ then player i plays *Stick* and player $-i$ plays *Accept*. Finally, if $k \leq 1/4$ then player i plays *Accept* while player $-i$ plays *Stick*.

The expected payoff to player 1, y_1, from the above strategy profile is given by,

$$y_1 = \frac{3}{4} \cdot \frac{2}{5} + \frac{1}{4} \cdot \frac{1}{2} - \int_{q}^{1/4} k f(k)\, dk$$

The expected payoff to player 2 is given by,

$$y_2 = \frac{3}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{2}{5} - \int_0^q k f(k)\, dk$$

Clearly, $y_1 > \frac{3}{10}$ and $y_2 > \frac{3}{8}$. It can be easily checked that the second stage strategies are all Nash Equilibria of the subgames induced by the different values of k. To see that no player can do better by changing the first stage demand, notice first that by making a compatible demand a player would get, at best, 1/4 which is lower than both y_1 and y_2. If a player deviates to making a higher demand than 3/4 then her expected payoff would fall to strictly less than 1/4, rendering it a loss making deviation. If either player makes a lower demand, then given the stated strategy profile her highest possible expected payoff must still be strictly less than $\frac{3}{4} \cdot \frac{1}{10}$, again less than both y_1 and y_2. The strategy profile outlined above is therefore subgame perfect and results in a positive probability of disagreement.

5 Independent revoking costs.

In this section I consider the opposite benchmark that involves the revoking costs being independently distributed. The first stage game is exactly as outlined in Section 4. Further the payoffs following incompatible demands are determined by the outcome of the game outlined in Table 1. In the first stage the players' common priors regarding the revoking costs k_1 and k_2 are given by the random vector K that takes values in $[0, \bar{k}_1] \times [0, \bar{k}_2]$ with $\bar{k}_i > 1$. Following incompatible demands both players observe the realized value of K before taking their second stage actions.

In the following analysis, whenever there is multiplicity in the second stage game, the risk dominant outcome will be selected. Instead of imposing it, this equilibrium selection criterion can indeed be derived by perturbing the model above to give it a global game information structure as was done in Section 4. The limit equilibrium prediction of such a perturbed model as the amount of noise is made arbitrarily small then delivers the equilibrium selection rule of the risk dominant outcome being played. The proof for this result, however, is largely of a technical nature and of marginal interest with respect to the results in CvD and Section 4.1 and is therefore omitted.[5] In particular, it requires the extension of the two dimensional version of the global games argument in CvD to the present game where the game itself is endogenously determined by the actions taken in the first stage.

One difference between the global games argument involved in Section 4.1 and the present section should be noted. In Section 4.1, when both players made equal demands that were incompatible, the global games argument could not resolve the subsequent second stage multiplicity. This is due to the lack of the required dominance solvable region. In this section, on the other hand, due to the independent distributions assumption, the required dominance solvable regions exist irrespective of the particular incompatible demand profile. This allows the expected payoff following any demand profile to be pinned down precisely.

Let $\Theta = [0, \bar{k}_1] \times [0, \bar{k}_2]$. While I do not explicitly solve the full global games model, the following assumption on the fundamentals of the model is required for the global game argument to work (with the addition of the noise parameters) and is therefore stated.

A1a. *K admits a density h that is strictly positive, continuously differentiable,*

[5]The proof for this result is available upon request.

and bounded and continuous on Θ.

Suppose (z_1, z_2) is an incompatible demand profile. Let $D(z_1, z_2)$, $D_1(z_1, z_2)$ and $D_2(z_1, z_2)$ denote the part of the state space where the dominance solvable outcome of the second stage game are $(Stick, Stick)$, $(Stick, Accept)$ and $(Accept, Stick)$, respectively. Formally,

$$D(z_1, z_2) = \{k \in \Theta | k_1 > 1 - z_2 \text{ and } k_2 > 1 - z_1\}. \tag{6}$$

$$D_i(z_1, z_2) = \{k \in \Theta | k_i > 1 - z_j \text{ and } k_j < 1 - z_i\} \tag{7}$$

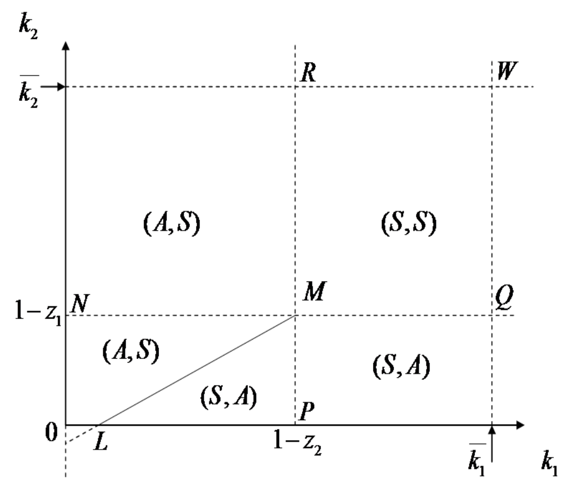

Figure 2: Second stage equilibrium behavior:Independent Costs

Figure 2 depicts the second stage equilibrium behavior over the entire state space, $0\bar{k}_1W\bar{k}_2$, following an incompatible demand profile (z_1, z_2) where $z_1 > z_2$. The dominance solvable regions $D(z_1, z_2)$, $D_1(z_1, z_2)$ and $D_2(z_1, z_2)$ correspond to $MQWR$, $MP\bar{k}_1Q$ and $MN\bar{k}_2R$. $0NMP$ marks the region where both $(Stick, Accept)$ and $(Accept, Stick)$ are strict Nash equilibria.

The equilibrium selection argument of risk dominance splits the state space (Θ) into three regions, following any incompatible demand profile, in terms of the action profile played in the second stage game. Let $R_i(z_1, z_2)$ denote the region of the state space where the risk dominant outcome in the second stage game following the incompatible demand profile (z_1, z_2) involves Player i playing *Stick* and Player j playing *Accept*. From Table 1 these regions can be completely characterized. In particular,

$$R_i(z_1, z_2) = \left\{ k \in \Theta | k_i < 1 - z_j \text{ } and \text{ } k_j < 1 - z_i \text{ } and \text{ } k_j < k_i \frac{d + 1 - z_i}{d + 1 - z_j} + \frac{d(z_j - z_i)}{d + 1 - z_j} \right\} \cup D_i(z_1, z_2) \tag{8}$$

In Figure 2, $R_1(z_1, z_2)$ and $R_2(z_1, z_2)$ correspond to $LMQ\bar{k}_1$ and $0LMR\bar{k}_2$. From (8) it can be seen that the line LM passes through the origin only if the two demands are the same.

Given this characterization it is possible to precisely pin down the payoffs following incompatible demands. In particular, following incompatible demands (z_1, z_2), if $k \in R_1(z_1, z_2)$, Player 1 gets z_1 while Player 2 gets $1 - z_1 - k_2$. Similarly if $k \in R_2(z_1, z_2)$, Player 1 gets $1 - z_2 - k_1$ while Player 2 gets z_2. Finally if $k \in D(z_1, z_2)$ then both players get 0. Notice that each player now faces a tradeoff between making a higher demand and increasing her risk dominant region where she actually receives her demand.

Figure 3 shows the changes in second stage behavior when Player 1 lowers her demand from z_1 to a still incompatible $\bar{z}_1$. Player 1, therefore, receives the lower share $\bar{z}_1$ whenever k takes a value in her risk dominant region. However, her risk dominant region itself has now increased because of her lower demand, from $LMQ\bar{k}_1$ to $UTV\bar{k}_1 0$. Her greatest gain comes from converting the $TVQM$

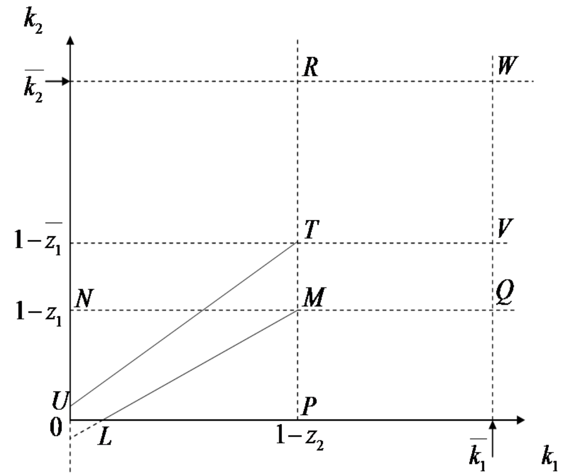

Figure 3: Lower demand and larger risk dominant region

region which earlier resulted in the full surplus being lost, to a region where she gets her exact demand. It is this tradeoff that prevents players from making arbitrarily high demands and results in the agreement results below.

Proposition 4. *If **A1a** is satisfied and K_1 and K_2 are independently distributed,with distribution functions F_1 and F_2, then the efficient demand profile* $(1/2, 1/2)$ *can be supported in equilibrium, for any pair of F_i that* First Order Stochastically Dominate *the uniform distribution.*

This efficiency argument is further strengthened by the non existence of equilibria supporting disagreement for the same range of distribution functions. In particular, the following result holds.

Proposition 5. *If **A1a** is satisfied and K_1 and K_2 are independently distributed, with distribution functions F_1 and F_2, then disagreement can not be supported in equilibrium for any pair of F_i that* First Order Stochastically Dominate *the uniform distribution.*

Example of Disagreement when FOSD Relation Fails: The following example was numerically computed using a program that calculated expected payoffs following incompatible demands exactly as outlined above, on Mathematica. Let K_1 and K_2 be identically and independently distributed according to a Beta distribution, $F(\alpha, \beta)$, with $\alpha = 2$ and $\beta = 15$. Observe that F does not FOSD the uniform distribution. Let the two players make equal demands of $z_1 = z_2 = 0.5985$. Following any incompatible demand profile (z_1, z_2) and observations of (k_1, k_2), the corresponding unique risk dominant profile is played in the second stage. It can be checked that such a strategy profile satisfies subgame perfection. Being incompatible, such a demand profile gives rise to disagreement with positive probability.

6 Conclusion

The ability to attempt commitment to aggressive demands *does not* necessarily lead to disagreement in bargaining between two rational agents, when the success of the commitment attempt is ex ante uncertain. Firstly, it is important to specify the cause of such commitment ability. If players have access to exogenous random commitment devices, then disagreement would necessarily follow, as shown in EM. If the ability to commit arises from the presence of uncertain revoking costs, then the possibility of disagreement depends on the finer details of the players beliefs about such uncertainty. If the players believe that revoking costs can only take values of 0 or some number greater than the surplus, then disagreement can always be supported in equilibrium, even if they know that their revoking costs are identical (though uncertain). However, if the players' believe that the revoking costs can take all possible intermediate values as well then the possibility of disagreement is significantly limited. If the revoking costs are identical (but uncertain) then disagreement cannot obtain, irrespective of the particular distribution chosen. Even when the revoking costs are independent across players there cannot be any disagreement if the distribution functions FOSD the uniform distribution. In a sense if the ex ante probability of facing a high revoking cost is high enough, disagreement cannot occur.

Secondly, the key factor influencing the different results is the dependence of concession behavior on first stage demands. Binary distributions for revoking costs or the use of exogenous commitment devices result in equilibria where the probability of a successful commitment attempt does not depend on the demands made in the first stage. Continuous densities with noisy signals force equilibrium behavior in the game to establish a systematic dependence of concession behavior on first stage demands. In particular a higher demand always increases the success probability of the opponents commitment attempt while reducing one's own. Equilibria are therefore determined by the tradeoff between making a larger demand and increasing the probability of actually getting one's own demand. Such incentives often rule out the possibility of disagreement.

The analysis in this paper also highlights a particular feature of modeling behavioral types. In particular, models of behavioral types tend to be discrete in the sense that players are either fully rational or a specified type, due to the use of binary distributions. Allowing for the density, instead, to be continuous in the cost that must be paid to deviate from the actions of some type, necessarily makes the model a continuous one. In the present analysis this distinction led to sharply contrasting results. Whether such contrast applies more widely remains to be ascertained.

A Appendix

Proposition 1(a)

Proof. Fix $q \in (0,1)$. Let $z = \frac{q+1}{2}$. Following an incompatible demand profile (z_1, z_2), in the second stage Bayesian game, player i must always play the strictly dominant action *Stick* when $k_i > 1$. Equilibrium behavior when $k_i = 0$ needs to be pinned down. In this regard notice that playing *Accept* when $k_i = 0$ for both i, would constitute a Bayesian Nash Equilibrium if the following two inequalities hold.

$$q(1 - z_2) + (1 - q)(1 - z_2 + d) \geq (1 - q)z_1 \tag{9}$$

$$q(1 - z_1) + (1 - q)(1 - z_1 + d) \geq (1 - q)z_2 \tag{10}$$

The left hand (right hand) side of the inequalities gives the expected payoff to the player with $k_i = 0$ from playing *Accept* (*Stick*) when her opponent's strategy involves playing *Accept* when the cost is zero and *Stick* when it is greater than 1. (9) and (10) hold with equality if $z_1 = z_2 = z = \frac{q+1}{2}$.[6] Clearly the demand profile (z, z) is incompatible.

Consider now the following strategies. Each player demands z. Following the demand profile (z, z) player i plays *Accept* when $k_i = 0$ and *Stick* when $k_i > 1$. Following a demand profile where $z_i = z$ but $z_j > z$, player i plays *Stick* irrespective of k_i while j plays *Accept* when $k_j = 0$ and *Stick* when $k_j > 1$. Following an *incompatible* demand profile where $z_i = z$ but $z_j < z$,

[6]Note that $d(z, z) = \frac{q}{2}$

both players play *Accept* when their cost is 0 and *Stick*, when it is high. The strategies also subscribe actions that constitute a BNE for any subgame not considered above. It will be shown that such a strategy profile constitutes a Perfect Bayesian Nash Equilibrium of the game.

Consider first, behavior in the second stage subgames. Only the behavior of the types facing $k_i = 0$ needs to be checked, since i must always play *Stick* when $k_i > 1$ as it is the strictly dominant action in that case. Following the profile (z, z) both players with 0 cost play *Accept*. It has been shown earlier that for this to be a BNE (9) and (10) must be satisfied. Given the derivation of z, this is in fact the case. For incompatible demand profiles where $z_i = z$ and $z_j > z$, the strategies suggest that the low type of player i should play *Stick* while player j with $k_j = 0$ should play *Accept*. Given j's strategy i's low type choice would be optimal if

$$q(1 - z_j) + (1 - q)(1 - z_j + d) < (1 - q)z \tag{11}$$

Given that this relation holds with equality when $z_j = z$ and that the left hand side is strictly decreasing in z_j, it must be that for $z_j > z$, (11) is indeed satisfied. Further given that player i plays *Stick* always, player j does strictly better by playing *Accept* when $k_j = 0$. Finally for incompatible demand profiles with $z_i = z$ and $z_j < z$, notice that the inequalities (9) and (10) continue to be satisfied. As a result the strategies involving low cost types playing *Accept* does induce a BNE in such subgames. As for the first stage decisions, consider player 1. The expected payoff to 1 from demanding z when 2 demands z is given by $q(1-q)z+(1-q)[q(1-z)+(1-q)(1-z+(2z-1)/2)]$. If 1 demands less than z, $(z_1 < z)$ her expected payoff is $q(1 - q)z_1 + (1 -$

$q)[q(1-z)+(1-q)(1-z+(z+z_1-1)/2)]$ which is clearly less than her payoff from not deviating. If 1 demands $z_1 > z$ then her expected payoff is merely $(1-q)(1-z)$, again strictly less than if she had not deviated. It remains to be shown that no player would want to deviate from the profile (z, z) to making the compatible demand $1-z$. Suppose this is a profitable deviation. Then it must be that,

$$
\begin{aligned}
&q(1-q)z + (1-q)[q(1-z) + (1-q)(1-z+d)] < 1-z \\
\Rightarrow &q(1-q)z + (1-q)(1-z) + (1-q)^2 d < 1-z \\
\Rightarrow &q(1-q)z - q(1-z) + (1-q)^2 \frac{q}{2} < 0 \\
\Rightarrow &z - zq - 1 + z + \frac{(1-q)^2}{2} < 0 \\
\Rightarrow &2z - 1 - zq + \frac{(1-q)^2}{2} < 0 \\
\Rightarrow &q - \frac{q+1}{2} q + \frac{(1-q)^2}{2} < 0 \\
\Rightarrow &2q - q^2 - q + 1 - 2q + q^2 < 0 \\
\Rightarrow &q > 1 \qquad (12)
\end{aligned}
$$

(12) contradicts the initial assumption of $q \in (0,1)$. As a result no player would want to deviate to making a compatible offer, from the incompatible profile (z, z). □

Proposition 1(b)

Proof. Suppose not. Let the compatible demand profile supported in equilibrium be (z_1, z_2) where $z_1 + z_2 = 1$. WLOG let $z_1 \leq z_2$. Notice that substituting z_1 and z_2 into the inequalities (9) and (10) makes the inequalities strict. Further $d(z_1, z_2) = 0$. In particular, $q(1-z_2) + (1-q)(1-z_2) > (1-q)z_1$.

Consequently if player 1 makes a higher demand, $z_1 + \delta$, the inequality will still be satisfied for small enough values of δ. Indeed, to satisfy the inequality (9), δ should satisfy, $q(1-z_2)+(1-q)(1-z_2+(\delta/2)) \geq (1-q)(z_1+\delta)$, which in turn implies that,

$$\delta \leq \frac{2qz_1}{1-q} \tag{13}$$

To ensure that such a deviation maintains the second inequality it must be that, $q(1-z_1-\delta)+(1-q)(1-z_1-\delta+(\delta/2)) \geq (1-q)z_2$. This in turn, simplifies to,

$$\delta \leq \frac{2qz_2}{1+q} \tag{14}$$

So if δ satisfies both (13) and (14), then following such a deviation, the subgame involving the incompatible demand profile, $(z_1+\delta, z_2)$, would involve both players playing *Stick* when the cost is high and *Accept* when it is 0. To see that no other BNE exists in the second stage game, note that both low types playing *Stick* cannot occur in equilibrium. Further given that the inequalities (13) and (14) are satisfied, if one of the low types plays *Accept* then the low type of the other player must also play *Accept*. The expected payoff to player 1 from such a profile would therefore be, $q^2(0)+q(1-q)(z_1+\delta)+(1-q)[q(1-z_2)+(1-q)(1-z_2+(\delta/2))]$. For this deviation to be profitable it must be that,

$$\begin{aligned}
&[q(1-q)+(1-q)]z_1 + q(1-q)\delta + (1-q)^2(\delta/2) > z_1 \\
\Rightarrow& q(1-q)\delta + (1-q)^2(\delta/2) > z_1 q^2 \\
\Rightarrow& (1-q^2)\delta > 2z_1 q^2 \\
\Rightarrow& \delta > \frac{2z_1 q^2}{1-q^2} \qquad (15)
\end{aligned}$$

Let $z_1 > 0$. Then for such a deviation to exist, it simply needs to be shown that there exists $\delta > 0$ that simultaneously satisfies (13), (14) and (15). Notice that $\frac{2z_1q^2}{1-q^2} < \frac{2qz_1}{1-q} \Leftrightarrow \frac{q}{1+q} < 1$, and is satisfied for all $q > 0$. Further $\frac{2z_1q^2}{1-q^2} < \frac{2qz_2}{1+q} \Leftrightarrow \frac{z_1q}{1-q} < z_2$. Given that $z_1 \leq z_2$, this is satisfied for all $q < 1/2$. Consequently, if $z_1 > 0$ and $0 < q < 1/2$, there always exists a profitable deviation for player 1.

For the case where $z_1 = 0$ and $z_2 = 1$. If 1 deviates by demanding $\delta > 0$ that satisfies $\delta < \frac{2q}{1+q}$, the inequality (9) would be reversed and hold strictly. In other words following the demand profile $(\delta, 1)$, if player 2 plays *Accept* when $k_2 = 0$ and *Stick* otherwise, then player 1 would play *Stick* always. Also, given that 1 plays *Stick* always, 2's optimal action when $k_2 = 0$ is indeed to play *Accept* since it gives a payoff of $1 - \delta$ as opposed to the payoff of 0 if *Stick* is played. So these strategies constitute a BNE of the subgame following $(\delta, 1)$. Both players playing *Stick* always is not a BNE of this subgame since the low type of player 2 would strictly prefer to play *Accept*, as just described. The low types of both players playing *Accept* cannot happen due to the strict reversal of the inequality (9). So the only other potential BNE of this subgame involves player 2 playing *Stick* always while the low type of player 1 plays *Accept*. This would require the low type of player 2 to choose *Stick*, requiring, $q(1-\delta) + (1-q)(1-\delta+(\delta/2)) \leq (1-q)(1)$. But, this inequality is violated if $\delta < \frac{2q}{1+q}$. The only BNE following a deviation to δ, therefore involves player 1 always playing *Stick* with the low type for player 2 playing *Accept*. Since this deviation gives a strictly positive payoff to player 1 it is a profitable deviation.

So it has been shown that given any compatible demand profile (z_1, z_2) with $z_1 \leq z_2$ as long as $0 < q < 1/2$, there always exists a profitable deviation for player 1. Clearly, a symmetric argument applies for $z_2 \leq z_1$. Consequently with $0 < q < 1/2$ there cannot be any equilibrium involving compatible demands. □

Proposition 2(a)

Proof. Consider the following strategies. Both players demand 1 in the first stage. Following any incompatible demand profile (z_1, z_2), player i plays *Stick* when $k_i > 1$. If $k_i = 0$ and $k_j > 1$, then player i plays *Accept*. If $k_1 = k_2 = 0$, then player 1 plays *Stick* while player 2 plays *Accept*.

Table 1 makes it clear that the strategies outlined above induce a Nash Equilibrium in every subgame following incompatible demand profiles. Notice that these subgames are dominance solvable except for the case where $k_1 = k_2 = 0$. In the latter case both $(Accept, Stick)$ and $(Stick, Accept)$ are Nash Equilibria. The particular selection made in this case is entirely arbitrary, but sufficient to support the incompatible profile as an equilibrium outcome.

The expected payoff to player 1 from the strategies above is $q(1-q)(1) + (1-q)(1-q)(1)$. Deviating to any lower incompatible demand z_1 gives an expected payoff, $q(1-q)(z_1) + (1-q)(1-q)(z_1)$, while making a compatible demand gives a payoff of 0. So player 1 has no incentive to deviate. Player 2's expected payoff from the stated strategies is $q(1-q)(1)$. Deviating to a lower but still incompatible demand, z_2, gives her $q(1-q)z_2$. Finally deviating to a compatible demand gives her 0. As a result player 2 also has no incentive to deviate. □

Proposition 2(b)

Proof. Suppose not. Let (z_1, z_2) be supported in equilibrium, where $z_1 + z_2 = 1$. Suppose player i deviates to demanding $\tilde{z}_i = 1$. Player i's expected payoff from such a deviation must be no less than $q^2(0) + q(1-q)(1) + (1-q)q(1-z_j) + (1-q)^2(1-z_j) = q(1-q) + (1-q)z_i$. For such a deviation to not be profitable it must be that $z_i \geq q(1-q) + (1-q)z_i$. This implies, $z_i \geq 1-q$. Given that $q < 1/2$ and $z_1 + z_2 = 1$, it must be that for some $i \in \{1,2\}$, $z_i < 1-q$ holds. Such a player i would then do strictly better by deviating to a demand of 1. □

Proposition 2(c)

Proof. Let $k_1 = k_2 = k$. When $k > 1$, the unique Nash Equilibrium in the second stage game involves both players playing *Stick*. $k = 0$, on the other hand, results in two pure strategy NE, namely $(\textit{Accept}, \textit{Stick})$ and $(\textit{Stick}, \textit{Accept})$. Consider the following strategies. Both players demand 1. Following any incompatible demand profile (z_1, z_2), if $k = 0$, player 1 plays *Stick* while 2 plays *Accept*. Facing $k > 1$, both players play *Stick*. As mentioned earlier, the subgame strategies constitute Nash Equilibria. Player 1 gets an expected payoff of $1-q$. By deviating to making any other demand z_1, the expected payoff would become strictly less, $(1-q)z_1$. Player 2, on the other hand, would always get 0 irrespective of her first stage demand and therefore has no incentive to deviate. Consequently the strategies support the demand $(1,1)$ in equilibrium. The subsequent probability of disagreement is therefore q^2. □

Lemma 1(CvD)

Proof. Let $l = \max_{k\in[0,\bar{k}]} |h'(k)|$, where $h'(k)$ is the derivative of the function h at k for $k \in (0, \bar{k})$ with $h'(0)$ and $h'(\bar{k})$ defined as $\lim_{k\to 0} h'(k)$ and $\lim_{k\to\bar{k}} h'(k)$, respectively. Given **A1**, l is well defined with $l \geq 0$. Let $\nu = \min_{k\in[0,\bar{k}]} h(k)$. Given that h is continuous and strictly positive on $[0, \bar{k}]$, ν is well defined with $\nu > 0$. Let ϵ be such that $l\epsilon < \nu/2$. Then (1) leads to the following inequality for all $k_i, k_j \in [0, \bar{k}]$,

$$f_i^\epsilon(k_j|k_i) \leq \frac{(h(k_i)+l\epsilon)\int \varphi^\epsilon(k_1-k, k_2-k)dk}{(h(k_i)-l\epsilon)\int\int \varphi^\epsilon(k_1-k, k_2-k)dk_j dk} = \frac{(h(k_i)+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(k_i)-l\epsilon}$$

ψ^ϵ is the density function for $\epsilon E_1 - \epsilon E_2$ and is equal to the integral in the numerator of the second term for given values of k_1 and k_2 . Note that the double integral in the denominator of the second term above is equal to 1. Similarly, $\frac{(h(k_i)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(k_i)+l\epsilon} \leq f_i^\epsilon(k_j|k_i)$. For $k_i \in [-\epsilon, 0]$ the relevant inequality is $\frac{(h(0)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(0)+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)-l\epsilon}$. If $k_i \in [\bar{k}, \bar{k}+\epsilon]$ then the inequality is $\frac{(h(\bar{k})-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(\bar{k})+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})-l\epsilon}$. Therefore,

$$(1-\frac{2l\epsilon}{h(k_i)+l\epsilon})\psi^\epsilon(k_1-k_2) \leq f_i^\epsilon(k_j|k_i) \leq (1+\frac{2l\epsilon}{h(k_i)-l\epsilon})\psi^\epsilon(k_1-k_2)$$ [7]

Further let $\kappa = \frac{8l}{\nu}$. Now,

$$\begin{aligned} 1+\frac{2l\epsilon}{h(k_i)-l\epsilon} &\leq 1+\frac{2l\epsilon}{\nu-l\epsilon} \\ &\leq 1+\frac{2l\epsilon}{\nu/2} \end{aligned}$$

Also,

$$\begin{aligned} 1-\frac{2l\epsilon}{h(k_i)+l\epsilon} &\geq 1-\frac{2l\epsilon}{h(k_i)-l\epsilon} \\ &\geq 1-\frac{2l\epsilon}{\nu-l\epsilon} \\ &\geq 1-\frac{2l\epsilon}{\nu/2} \end{aligned}$$

[7] For values of k_i in $[-\epsilon, 0]$ and $[\bar{k}, \bar{k}+\epsilon]$ replace $h(k_i)$ by $h(0)$ and $h(\bar{k})$, respectively.

Then,

$$\psi^\epsilon(k_1 - k_2)(1 - (\kappa\epsilon)/2) \leq f_i^\epsilon(k_j|k_i) \leq \psi^\epsilon(k_1 - k_2)(1 + (\kappa\epsilon)/2) \tag{16}$$

$$\Rightarrow \int_{y \leq k_2} \psi^\epsilon(k_1 - y)dy - (\kappa\epsilon)/2 \leq F_1^\epsilon(k_2|k_1) \leq \int_{y \leq k_2} \psi^\epsilon(k_1 - y)dy + (\kappa\epsilon)/2 \tag{17}$$

(16) also implies,

$$\int_{z \leq k_1} \psi^\epsilon(z - k_2)dz - (\kappa\epsilon)/2 \leq F_2^\epsilon(k_1|k_2) \leq \int_{z \leq k_1} \psi^\epsilon(z - k_2)dz + (\kappa\epsilon)/2$$
$$\Rightarrow \int_{z \geq k_1} \psi^\epsilon(z - k_2)dz + (\kappa\epsilon)/2 \geq 1 - F_2^\epsilon(k_1|k_2) \geq \int_{z \geq k_1} \psi^\epsilon(z - k_2)dz - (\kappa\epsilon)/2$$
$$\Rightarrow \int_{y \leq k_2} \psi^\epsilon(k_1 - y)dy + (\kappa\epsilon)/2 \geq 1 - F_2^\epsilon(k_1|k_2) \geq \int_{y \leq k_2} \psi^\epsilon(k_1 - y)dy - (\kappa\epsilon)/2 \tag{18}$$

Subtracting (18) from (17) gives the required inequality.

□

Lemma 2

Proof. The continuity of φ^ϵ is implied by the continuity of φ assumed in *A2*. Consider the numerator in the expression for $f_i^\epsilon(k_j|k_i)$ as expressed in (1). WLOG take a sequence k_1^n that converges to k_1, such that $k_1^n \in [-\epsilon, \bar{k}+\epsilon]$ for all n. Given the continuity of φ^ϵ it is immediate that holding k_2 fixed, $h(k)\varphi^\epsilon(k_1^n - k, k_2 - k) \to h(k)\varphi^\epsilon(k_1 - k, k_2 - k)$, *almost everywhere* in $[0, \bar{k}]$. Further $h(k)\varphi^\epsilon(k_1^n - k, k_2 - k) \leq h(k)\bar{\varphi}^\epsilon$ for all n and k, where $\bar{\varphi}^\epsilon$ is the maximum value taken by the function φ on $[-1, 1] \times [-1, 1]$. Consequently by the *Dominated Convergence Theorem*, $\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk = \lim_{n\to\infty} \int h(k)\varphi^\epsilon(k_1^n - k, k_2 - k)dk$. In other words, $\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk$ is continuous in k_i.

For the denominator in (1), consider first the marginal density. Fix k. Let $k_1 \notin \{k-\epsilon, k+\epsilon\}$. Then for any sequence k_1^n that converges to k_1 it must be the case that $\varphi^\epsilon(k_1^n-k, k_2-k) \to \varphi^\epsilon(k_1^n-k, k_2-k)$ for all values of k_2, by **A3**. Again by the *Bounded Convergence Theorem*, the marginal $\int \varphi^\epsilon(k_1^n-k, k_2-k)dk_2$ for a given value of k is found to be continuous at all k_1 other than potentially two points, $k-\epsilon$ and $k+\epsilon$. Consequently for any sequence k_1^n that converges to k_1, it is true that $h(k)\int \varphi^\epsilon(k_1^n-k, k_2-k)dk_2 \to h(k)\int \varphi^\epsilon(k_1-k, k_2-k)dk_2$ for all values of k other than possibly $k_1-\epsilon$ and $k_1+\epsilon$. Further, $h(k)\int \varphi^\epsilon(k_1^n-k, k_2-k)dk_2 \leq h(k)\bar{\varphi}^\epsilon$ for all k, n. By the *Dominated Convergence Theorem*, it must be that $\int h(k)\int \varphi^\epsilon(k_1^n-k, k_2-k)dk_2dk$, the denominator in (1), is continuous in k_1. Given **A1** and the additive structure of the noise, the denominator is also strictly positive for all $k_1 \in (-\epsilon, \bar{k}+\epsilon)$. Therefore for all $k_1, k_2 \in [-\epsilon, \bar{k}+\epsilon]$, $f_i^\epsilon(k_j|k_i)$ is continuous in k_i. $f_i^\epsilon(k_j|k_i)$ is also continuous in k_j, since k_j does not affect the denominator of (1), while its influence on the numerator is symmetric to that of k_i. So let $\bar{f}_\epsilon$ be the maximum value taken by $f_i^\epsilon(k_j|k_i)$ for $k_1, k_2 \in [-\epsilon, \bar{k}+\epsilon]$. Then for any measurable function s_j, it must be that $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \to s_j(k_j)f_i^\epsilon(k_j|k_i)$ if $k_i^n \to k_i$ and $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \leq s_j k_j \bar{f}_\epsilon$, for all values of k_j. Therefore by the *Dominated Convergence Theorem*, $Pr(A_j|k_i^\epsilon, s_j) = \int s_j(k_j)f_i^\epsilon(k_j|k_i^\epsilon)dk_j$ is continuous in k_i^ϵ.

To show that $E^\epsilon(k|k_i^\epsilon)$ is continuous in k_i^ϵ consider first the conditional density of the true k given an observation k_i.

$$f_i^\epsilon(k|k_i) = \frac{\int h(k)\varphi^\epsilon(k_1-k, k_2-k)dk_j}{\int\int h(k)\varphi^\epsilon(k_1-k, k_2-k)dk_j dk} \qquad (19)$$

Continuity of the denominator of (19) in k_i has already been established before. The numerator for a given k is the product of the strictly positive $h(k)$ and the marginal density of k_i. It has been shown earlier that for a given k the marginal density of k_i is continuous at all k_i other than possibly when $k_i \in \{k-\epsilon, k+\epsilon\}$, the boundary points. As a result, for a given k, $f_i^\epsilon(k|k_i)$ is continuous for all k_i other than the two boundary points. Therefore for a sequence k_i^n that converges to k_i, $kf_i^\epsilon(k|k_i^n) \to kf_i^\epsilon(k|k_i)$ for all k other than possibly when $k \in \{k_i-\epsilon, k_i+\epsilon\}$. Further since the denominator in (19) is bounded below and the numerator bounded above, the *Dominated Convergence Theorem* delivers the continuity of $E^\epsilon(k|k_i^\epsilon) = \int kf_i^\epsilon(k|k_i^\epsilon)dk$ in k_i^ϵ. □

Lemma 3

Proof. Given the payoffs in Table 2, it is clear that whenever j chooses *Accept*, i always does strictly better by choosing *Stick*. Upon observing $k_i^\epsilon > 1-z_j+\epsilon$ player i knows that for all the possible values that k can take she would get a strictly negative payoff by playing *Accept* if j plays *Stick*. As a result i would still strictly prefer to play *Stick* since it guarantees a payoff of 0 as opposed to the negative expected payoff from playing *Accept*, when j plays *Stick*. Consequently, upon observing $k_i^\epsilon > 1-z_j+\epsilon$, *Stick* is the strictly dominant action for player i. □

Lemma 4

Proof. From lemma 3 it is already known that j plays *Stick* for every observation $k_j^\epsilon > 1-z_i+\epsilon$. Player i making an observation $k_i^\epsilon \in (1-z_i+3\epsilon, 1-z_j-\epsilon)$ learns two things. Firstly, she knows that j must have observed $k_j^\epsilon > 1-z_i+\epsilon$ and must therefore be playing the strictly dominant *Stick*. Secondly, she knows that the true state k must lie in the interval $(1-z_i+2\epsilon, 1-z_j)$. Conditional on j playing *Stick* for any such value of k, playing *Accept* strictly dominates playing

Stick for i. The dominance solvable outcome following such an observation, therefore, involves i playing *Accept* while j plays *Stick*. □

Lemma 5

Proof. Suppose the statement is false for player i, who makes the higher demand. This means that $B_i^\epsilon(z_1, z_2)$ is non empty but $y = \sup\{x|x \in B_i^\epsilon(z_1, z_2)\} \notin B_i^\epsilon(z_1, z_2)$. So there exists a sequence of observations k_i^n that converge to y, with i playing *Stick* for all n but she plays *Accept* upon observing y. i's expected payoff from playing *Accept* following an observation k_i is given by $1 - z_j - E^\epsilon(k|k_i) + dPr(A_j|k_i)$ while it is $z_i Pr(A_j|k_i)$ from playing *Stick*. Given that i plays *Stick* for all observations in the sequence k_i^n it must be that $z_i Pr(A_j|k_i^n) \geq 1 - z_j - E(k|k_i^n) + dPr(A_j|k_i^n)$. By *Lemma 2*, $E^\epsilon(k|k_i)$ and $Pr(A_j|k_i)$ are continuous in k_i for all measurable strategies, s_j. So if $k_i^n \to y$ it must be that $z_i Pr(A_j|y) \geq 1 - z_j - E(k|y) + dPr(A_j|y)$. Given the tie break rule mentioned earlier this implies that i would play *Stick* upon observing y. This contradicts the earlier claim and proves the lemma for i. A symmetric argument proves the lemma for player j. □

Lemma 6

Proof. Let $B_i^\epsilon(z_1, z_2)$ be empty. Then for all observations $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ player i chooses to play *Accept*. In that case whenever player j receives a signal $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ it is conditionally dominant for him to play *Stick*. This would imply that $B_j^\epsilon(z_1, z_2)$ is empty.

Now if $B_j^\epsilon(z_1, z_2)$ is empty then for all observations $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ player j chooses to play *Stick*. Player i following an observation $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ knows that the true value of k is such that $1 - z_j - k > 0$. Consequently conditional on j playing *Stick*, she is strictly better off playing *Accept*. As a result $B_i^\epsilon(z_1, z_2)$ is empty. □

Lemma 7

Proof. Let $k_j^{\epsilon*} + 2\epsilon \leq k_i^\epsilon \leq 1 - z_i + 3\epsilon$. Conditional on such an observation player i knows that for all the possible values of k, $1 - z_j - k > 0$ and hence she would strictly prefer to play *Accept* if j plays *Stick*. Further such an observation implies that j has observed $k_j^\epsilon > k_j^{\epsilon*}$ implying that j would certainly play *Stick*. Consequently i's conditionally dominant action is to play *Accept*. □

Lemma 8

Proof. Suppose not. Then, by *Lemmas 5 and 6*, $k_i^{\epsilon*}, k_j^{\epsilon*} > -\epsilon$ are well defined. Let player i's payoff from playing *Accept* and *Stick* upon observing $k_i^{\epsilon*}$ be denoted as $u_i(A_i|k_i^{\epsilon*})$ and $u_i(S_i|k_i^{\epsilon*})$ respectively. Given the payoffs in Table 2, $u_i(A_i|k_i^{\epsilon*}) = 1 - z_j - E^\epsilon(k|k_i^{\epsilon*}) + dPr(A_j|k_i^{\epsilon*})$. Also $u_i(S_i|k_i^{\epsilon*}) = z_i Pr(A_j|k_i^{\epsilon*})$. Given that i chooses *Stick* after such an observation, it must be that $u_i(S_i|k_i^{\epsilon*}) \geq u_i(A_i|k_i^{\epsilon*})$. This in turn implies the following inequality,

$$Pr(A_j|k_i^{\epsilon*}) \geq \frac{1 - z_j - E^\epsilon(k|k_i^{\epsilon*})}{z_i - d} \tag{20}$$

Similarly, player j choosing *Accept* upon observing $k_j^{\epsilon*}$ implies that $u_j(A_j|k_j^{\epsilon*}) \geq u_j(S_j|k_j^{\epsilon*})$. Writing out the payoffs, $1 - z_i - E^\epsilon(k|k_j^{\epsilon*}) + dPr(A_i|k_j^{\epsilon*}) \geq z_j Pr(A_i|k_j^{\epsilon*})$. This gives rise to the following inequality,

$$Pr(A_i|k_j^{\epsilon*}) \leq \frac{1 - z_i - E^\epsilon(k|k_j^{\epsilon*})}{z_j - d} \tag{21}$$

Now, player j plays *Stick* following any observation $k_j^\epsilon > k_j^{\epsilon*}$. Therefore, it must be that,

$$Pr(A_j|k_i^{\epsilon*}) \leq F_i^{\epsilon}(k_j^{\epsilon*}|k_i^{\epsilon*}) \tag{22}$$

On the other hand, player i plays *Accept* for observations $k_i^{\epsilon} > k_i^{\epsilon*}$ as long as $k_i^{\epsilon} < 1 - z_j - \epsilon$. For values of k_i^{ϵ} that are within 2ϵ of $k_j^{\epsilon*}$ it must be that $k_i^{\epsilon} < 1 - z_j - \epsilon$ since $k_j^{\epsilon*} \leq 1 - z_i + \epsilon$ by *Lemma 3* and $1 - z_i + \epsilon < 1 - z_j - 2\epsilon$ by assumption. As a result the following inequality holds.

$$Pr(A_i|k_j^{\epsilon*}) \geq 1 - F_j^{\epsilon}(k_i^{\epsilon*}|k_j^{\epsilon*}) \tag{23}$$

Subtracting (23) from (22) and using (2) from *Lemma 1* gives the inequality,

$$Pr(A_j|k_i^{\epsilon*}) - Pr(A_i|k_j^{\epsilon*}) \leq \kappa\epsilon \tag{24}$$

Finally combining (20), (21) and (24) gives,

$$\kappa\epsilon \geq \frac{1 - z_j - E^{\epsilon}(k|k_i^{\epsilon*})}{z_i - d} - \frac{1 - z_i - E^{\epsilon}(k|k_j^{\epsilon*})}{z_j - d} \tag{25}$$

$$\geq \frac{1 - z_j - k_i^{\epsilon*} - \epsilon}{z_i - d} - \frac{1 - z_i - k_j^{\epsilon*} + \epsilon}{z_j - d} \tag{26}$$

$$> \frac{1 - z_j - k_j^{\epsilon*} - 3\epsilon}{z_i - d} - \frac{1 - z_i - k_j^{\epsilon*} + \epsilon}{z_j - d} \tag{27}$$

(25) $\Rightarrow$ (26) by the fact that $E^{\epsilon}(k|k_i^{\epsilon*}) \leq k_i^{\epsilon*} + \epsilon$ and $E^{\epsilon}(k|k_j^{\epsilon*}) \geq k_j^{\epsilon*} - \epsilon$. While the inequality from *Lemma 7*, namely $k_i^{\epsilon*} < k_j^{\epsilon*} + 2\epsilon$, makes (26) $\Rightarrow$ (27).

(27) $\Rightarrow$

$$
\begin{aligned}
&\kappa\epsilon(z_i - d)(z_j - d) > (z_j - z_i)(1 - k_j^{\epsilon *} + d) - (z_j^2 - z_i^2) - 3\epsilon z_j - \epsilon z_i + 4\epsilon d \\
\Rightarrow &\kappa\epsilon(1 - (z_i - z_j)^2) > (z_j - z_i)(1 - k_j^{\epsilon *} - (z_i + z_j) + d) + \epsilon(z_i - z_j) - 2\epsilon \\
\Rightarrow &\kappa\epsilon(1 - (z_i - z_j)^2) > (z_i - z_j)(k_j^{\epsilon *} + d + \epsilon) - 2\epsilon \\
\Rightarrow &k_j^{\epsilon *} + d + \epsilon < \frac{\kappa\epsilon(1 - (z_i - z_j)^2)}{z_i - z_j} + \frac{2\epsilon}{z_i - z_j} \\
\Rightarrow &k_j^{\epsilon *} < -\epsilon + \frac{\kappa\epsilon}{z_i - z_j} + \frac{2\epsilon}{z_i - z_j} - d - \kappa\epsilon(z_i + z_j) \\
\Rightarrow &k_j^{\epsilon *} < -\epsilon + \frac{(\kappa + 2)\epsilon}{z_i - z_j} - d \qquad (28)
\end{aligned}
$$

Given that $k_j^{\epsilon *}$ must be a value strictly greater than $-\epsilon$, (28) delivers a contradiction to the initial claim if,

$$
z_i - z_j \geq \frac{(\kappa + 2)\epsilon}{d} \qquad (29)
$$

The premise in the lemma satisfies (29) and therefore it must be that $B_j^{\epsilon}(z_1, z_2)$ is empty. *Lemma 6* then guarantees that $B_i^{\epsilon}(z_1, z_2)$ is empty too. □

Lemma 10

Proof. (a) is immediate, since player i has an incentive to demand $1 - z_j$ and strictly increase her payoff by $1 - z_j - z_i > 0$. *Lemma 9* shows that following an incompatible demand profile such as (b), the player with the higher demand, say i, has an expected payoff $y_i \leq \int_0^{1-z_j}(1 - z_j - w)h(w)dw < 1 - z_j$ and could do strictly better by simply making the compatible demand $1 - z_j$. □

Lemma 11

Proof. Following an incompatible demand profile, the payoffs are determined by outcomes in the second stage game described in Table 2. Notice that following any possible realization, k, the maximum total payoff would be $\max\{1-k,0\}$. As a result the expected payoffs from making incompatible demands must satisfy, $y_1 + y_2 \leq 1 - \hat{k}$. Now for the incompatible profile (z_1, z_2) to be supported as an equilibrium in Γ^{ϵ}, it must be that neither player gains by making a compatible demand instead. This means, $y_i \geq 1 - z_j$. Summing across the two players gives, $y_1 + y_2 \geq 2 - z_1 - z_2$, which in turn implies, $2 - z_1 - z_2 \leq 1 - \hat{k}$. Given that $d = (z_1 + z_2 - 1)/2$ it must be that $d \geq \hat{k}/2$. □

Lemma 12

Proof. Equilibrium behavior in the second stage game involves a total payoff of 0 if both parties play *Stick* or $1-k$ if (*Accept*, *Stick*) or (*Stick*, *Accept*) is the outcome. Players using mixed strategies results in the total payoff lying in the interval $[0, \max\{0, 1-k\}]$. *Lemma 2* makes it clear that if $k > 1 - \min\{z_1, z_2\} + 2\epsilon$ then the players would always play (*Stick*, *Stick*). So it can be said for certain that following an incompatible demand profile, the total expected payoff in equilibrium must be no more than $(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon)$. This in turn implies that following incompatible demands there exists i with an expected payoff,

$$y_i \leq \frac{1}{2}(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon) \quad (30)$$

$d \geq \hat{k}/2$ implies $z_i + z_j - 1 \geq \hat{k}$. Also by the definition of ϕ, it must be that

$\phi(d) \leq \phi^*$ since $d \geq \hat{k}/2$. So,

$$\begin{aligned}
&|z_i - z_j| < \phi(d) \leq \phi^* \\
\Rightarrow &2\min\{z_1, z_2\} + \phi^* - 1 \geq \hat{k} \\
\Rightarrow &\min\{z_1, z_2\} \geq \frac{1}{2} + \frac{\hat{k}}{2} - \frac{\phi^*}{2}
\end{aligned} \tag{31}$$

Let ϵ be small enough such that $\phi^* < \frac{\hat{k}}{8}$.

Then,

$$(31) \Rightarrow \min\{z_1, z_2\} > \frac{1}{2} + \frac{7}{16}\hat{k} \tag{32}$$

Now consider what happens if player i, who receives the payoff mentioned in (30), deviates to making a *still incompatible* demand of $\tilde{z}_i = 1/2$. Note that $z_j - \tilde{z}_i > \frac{7}{16}\hat{k} > \phi^*$. Further $d(\tilde{z}_i, z_j) > \frac{7}{32}\hat{k}$ which implies that $\phi(d) \leq \phi^*$. Therefore $z_j - \tilde{z}_i > \phi(d(\tilde{z}_i, z_j))$. As a result, the new demand profile satisfies the condition of *Lemma 9*, which implies that player i following such a deviation must expect a payoff $\tilde{y}_i$,

$$\tilde{y}_i \geq \frac{1}{2}F_i^{\epsilon}(\frac{1}{2} - \epsilon) \geq \frac{1}{2}H(\frac{1}{2} - 2\epsilon) \tag{33}$$

Player i's initial payoff inequality described in (30) along with (32) implies,

$$y_i < \frac{1}{2}H(\frac{1}{2} - \frac{7}{16}\hat{k} + 2\epsilon) \tag{34}$$

For small enough values of ϵ, it is clear that $y_i < \tilde{y}_i$. Given that such a profitable deviation exists, $(z_1, z_2) \notin Eq^{\epsilon}$. □

Proposition 4

Proof. Consider the following strategies. Both players demand 1/2 in the first stage. If player j makes a demand higher than 1/2 then in the second stage, in the event of multiplicity, both players play actions in accordance to the risk dominant outcome of the second stage game. Further if the state of the world (k_1, k_2) lies in the region $D(1/2, z_j)$ then both players play *Stick*. Given these strategies it is easy to see that second stage behavior satisfies equilibrium behavior since it either involves playing the unique dominance solvable action profile or playing one of the Nash Equilibria; in particular, the risk dominant action profile. However, it must be checked if any player has an incentive to deviate in the first stage. Deviating to a smaller demand is obviously less profitable to the deviator and hence ruled out.

Suppose Player 1 deviates to making a higher demand $z_1 > 1/2$. By doing so, Player 1 would gain a higher payoff for every k that lies in her new risk dominant region, $R_1(z_1, 1/2)$. Denote this gain by G. From (8), it must be that,

$$G \leq (z_1 - 1/2)(1 - F_1(1/2))F_2(1 - z_1) + 1/2(z_1 - 1/2)F_1(1/2)F_2(1/2). \quad (35)$$

On the other hand Player 1 ends up losing her share of 1/2 in the new disagreement region, $D(z_1, 1/2)$, while also paying her revoking cost in her opponents risk dominant region $R_2(z_1, 1/2)$. Denote this loss by L. From (8) and (6), it must be that,

$$L > \frac{1}{2}(1 - F_1(1/2))(1 - F_2(1 - z_1)) \quad (36)$$

The proposition will be first proven for F_1 and F_2 being uniform. If F_1 and F_2

are uniform then (35) can be rewritten as,

$$G \leq (z_1 - 1/2)\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{1 - z_1}{\bar{k}_2}\right) + \frac{1}{2}(z_1 - 1/2)\left(\frac{(1/2)^2}{\bar{k}_1\bar{k}_2}\right) \tag{37}$$

Similarly (36) implies,

$$L > \frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{\bar{k}_2 - 1/2}{\bar{k}_2}\right) + \frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{z_1 - 1/2}{\bar{k}_2}\right) \tag{38}$$

Note that $z_1 > 1/2$ and $\bar{k}_i > 1$. Consequently, for such a deviation to be profitable it must be that $G > L$. This in turn, from (37) and (38), implies that a profitable deviation must involve,

$$\begin{aligned}&\frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{\bar{k}_2 - 1/2}{\bar{k}_2}\right) < \frac{1}{2}(z_1 - 1/2)\left(\frac{(1/2)^2}{\bar{k}_1\bar{k}_2}\right)\\ \Rightarrow &1 < z_1 - 1/2\end{aligned} \tag{39}$$

The impossibility of (39) rules out any profitable deviation for Player 1. A symmetric argument rules out any profitable deviation for Player 2. Consequently the strategies outlined above constitute a Subgame Perfect Equilibrium when the F_i are uniform distributions. To see how the argument then extends to any pair of distributions that *FOSD* the uniform distribution, notice that to arrive at the contradiction above, it was shown that,

$$\left(z_1 - \frac{1}{2}\right)\left(1 - F_1\left(\frac{1}{2}\right)\right)F_2(1-z_1) + \left(z_1 - \frac{1}{2}\right)F_1\left(\frac{1}{2}\right)F_2\left(\frac{1}{2}\right) < \frac{1}{2}(1-F_1(1/2))(1-F_2(1-z_1)) \tag{40}$$

when the F_i are uniform. It is easy to see that if the F_i *FOSD* the uniform distribution, the right hand side of (40) would be even higher, while the left hand side even lower than in the uniform case. Consequently the relationship $L > G$ would hold for all such distributions. The result follows. □

Proposition 5

Proof. First, note that an incompatible demand profile with at least one player making a demand of $z_i = 1$ cannot be supported in equilibrium. Following such an incompatible profile, player i either backs down in the second stage or the entire surplus is lost, since player j will never back down. In other words, $R_i(z_i = 1, z_j) = \emptyset$. Therefore player i's expected payoff must be strictly less than $1 - z_j$, which in turn makes the first stage deviation to a compatible demand a profitable one if $z_j < 1$. If, however, $z_1 = z_2 = 1$, then each player is better off making a demand of $1/2$ instead. The demand profile $(1, 1)$ yields a payoff of 0 to both players. If player 1 makes a demand of $1/2$ instead her expected payoff becomes, $(1/2)F_2(1/2)$ which is clearly payoff improving.

Having eliminated the possibility of a demand of 1 in equilibrium, the result shall first be proven for the F_i being uniform distributions. It will then be shown that the arguments generalize easily to any pair of distributions that each First Order Stochastically Dominates the uniform distribution.

The statement is proved by contradiction. Suppose (z_1, z_2) is an incompatible demand profile that is supported in equilibrium with F_i being a uniform distribution. It must be true then that neither player can have her payoff strictly increases by making a compatible demand in the first stage. Consider the options for Player 2. If she deviates to making a compatible demand she gains $1 - z_1$ in the region $D(z_1, z_2)$. She also gains the revoking cost she would have had to pay following incompatible demands in the region $R_1(z_1, z_2)$. The total expected gain from such a deviation is denoted by G where $G \geq (1 - z_1)[1 - F_2(1 - z_1)][1 - F_1(1 - z_2)] + E(k_2|k_2 \leq 1 - z_1)[1 - F_1(1 - z_2)]F_2(1 - z_1)$. For the purpose of this proof the inequality, $G \geq (1 - z_1)[1 - F_2(1 - z_1)][1 - F_1(1 - z_2)]$ will suffice. Such a deviation, however, results in a loss of $z_2 - (1 - z_1)$ in the region $R_2(z_1, z_2)$. Denote the expected loss by L where $L \leq [z_2 - (1 - z_1)]F_1(1 - z_2)$.

Since the F_i are uniform distributions the relevant inequalities become,

$$G \geq (1 - z_1)\left(\frac{\bar{k}_2 - (1 - z_1)}{\bar{k}_2}\right)\left(\frac{\bar{k}_1 - (1 - z_2)}{\bar{k}_1}\right) \tag{41}$$

$$L \leq (z_2 - (1 - z_1))\frac{1 - z_2}{\bar{k}_1} \tag{42}$$

Given that such a deviation is not profitable by assumption it must be that $L \geq G$. Since $k_i > 1$, $L \geq G$ implies the following inequality,

$$(z_1 + z_2 - 1)(1 - z_2) \geq (1 - z_1)z_2 z_1 \tag{43}$$

A symmetric argument shows that for Player 1 to not be strictly better of from deviating to a compatible demand, the following inequality must hold.

$$(z_1 + z_2 - 1)(1 - z_1) \geq (1 - z_2)z_1 z_2 \tag{44}$$

Now suppose $z_1 \geq z_2$. Then to satisfy (44) it must be that $z_1 + z_2 - 1 \geq z_1 z_2$, which in turn implies $z_i \geq 1$. On the other hand if $z_2 \geq z_1$ then satisfying (43) would require $z_i \geq 1$. Since the possibility of $z_i = 1$ in equilibrium was ruled out earlier this delivers the contradiction.

The uniform distribution case was proved by essentially showing that at least one of the inequalities,

$$(1 - z_i)[1 - F_j(1 - z_i)][1 - F_i(1 - z_j)] > [z_j - (1 - z_i)]F_i(1 - z_j) \tag{45}$$

for $i \in \{1, 2\}$, holds if both players are not demanding 1 each. This made the deviation to a compatible demand a profitable one for Player j. Notice that for any other pair of distribution functions, F_1 and F_2 that FOSD the uniform distribution, this inequality would continue to hold since it would simply decrease the right hand side of (45) while increasing the left hand side. This would make the required inequality, $G > L$, hold for all such distributions. $\square$

References

[1] Abreu, Dilip and Faruk Gul. (2000): "Bargaining and Reputation," *Econometrica*, 68(1), 85-117.

[2] Carlsson, Hans and Eric van Damme. (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989-1018.

[3] Chassang, Sylvain. (2008): "Uniform Selection in Global Games," *Journal of Economic Theory*, 139(1), 222-41.

[4] Compte, Oliver and Philippe Jehiel. (2002): "On the Role of Outside Options in Bargaining with Obstinate Parties," *Econometrica*, 70(4), 1477-1517.

[5] Crawford, Vincent P. (1982): "A Theory of Disagreement in Bargaining," *Econometrica*, 50(3), 607-37.

[6] Ellingsen, Tore and Topi Miettinen. (2008): "Commitment and Conflict in Bilateral Bargaining," *American Economic Review*, 98(4), 1629-35.

[7] Fearon, James D. (1994): "Domestic Political Audiences and the Escalationof International Disputes," *American Political Science Review*, 88(3), 577-92.

[8] Gori, Michele and Antonio Villanacci. (2011): "A Bargaining Model in General Equilibrium," *Economic Theory*, 46, 327-75.

[9] Kambe, Shinsuke. (1999): "Bargaining with Imperfect Commitment," *Games and Economic Behavior*, 28(2), 217-37.

[10] Martin, Lisa L. (1993): "Credibility, Costs and Institutions: Cooperation on Economic Sanctions," *World Politics*, 45(3), 406-32.

[11] Myerson, Roger(1991): *Game Theory: Analysis of Conflict*, Cambridge, MA: Harvard University Press.

[12] Schelling, Thomas C.(1960): *The Strategy of Conflict*, Cambridge, MA: Harvard University Press.

[13] Weinstein, Jonathan and Muhamet Yildiz. (2007): "A Structure Theorem for Rationalizability with Application to Robust Predictions of Refinements," *Econometrica*, 75(2), 365-400.

[14] Wolitzky, Alexander. (2011): "Reputational Bargaining Under Knowledge of Rationality," *mimeo*.

Chapter 3: Coordinating by Not Committing: Efficiency as the Unique Outcome

1 Introduction

The role of commitment in determining the outcome of strategic interactions has been studied in a variety of settings since the classic work of Schelling(1960). From the simple observation that the Stackelberg outcome of a game need not be a Nash Equilibrium to the commitment folk theorem of Kalai et al(2010), the influence of commitment ability is seen to be both significant and diverse. Evidently the exact role of commitment crucially depends upon the precise description of the commitment ability itself.

One form of commitment ability comprises in the capability of agents to limit their own set of choices available for a given strategic interaction. Cited examples of such ability range from armies burning their bridges thereby eliminating the option of a retreat to making a sunk contribution towards a joint project making it impossible to contribute less than the latter amount. Renou(2009) studies such commitment ability by adding a commitment stage to strategic games. Players, in the first stage, simultaneously choose a subset of the (finite set of) actions available to them in the strategic game. In the second stage they play the new strategic game with only the restricted action sets available to each player. Bade et al.(2009) analyze a game in which two players can repeatedly and irreversibly rule out their own actions over a fixed number of periods and simultaneously choose from the remaining actions in the final period. The initial action space in this setup is a closed interval of the real line, while players are allowed to commit to closed convex subsets of the original space. The payoff functions are assumed to be continuous and strictly quasi concave. Importantly they find that an outcome that can be supported in a commitment game lasting T period can be supported in a one period commitment game. One could therefore without loss of generality study single period commitment games. While both these papers involve players voluntarily restricting their own choice sets, Nava(2008) considers games where players can restrict the set of action profiles (not merely their own actions) if they unanimously choose to do so. The requirement of unanimity retains the voluntary nature of commitment ability of the earlier studies. The analysis is carried out for both single and multiple rounds of commitment. A common finding in all these papers is that such commitment ability typically increases the set of outcomes supported by equilibrium arguments. While efficient outcomes that were not Nash Equilibria(NE) of the original game may now be achievable through commitment, the original set of NE continues to be outcomes supported by SPE strategies in the commitment games analyzed in these papers. Nava(2008), going further, shows that if the players have multiple rounds to make their joint commitments a folk theorem holds. Such a folk theorem also holds when players have the ability to make conditional commitments as in Kalai et al.(2010).

The present paper contributes to this research program by analyzing two player finite games, as in Renou(2009), where the players can repeatedly rule out actions in their own choice set, as in Bade et al.(2009). However, there are three points of departure from earlier studies. Firstly, committing to a smaller set of actions is assumed to be costly. This assumption is informed by the casual observation that whether such commitment involves the physical elimination of an option (burning bridges) or rendering an option infeasible by making it too costly (a President announcing publicly that he would veto a particular bill, making backing down prohibitively costly for re election prospects), the very act of constraining one's choices may involve a cost. Importantly, this cost may be arbitrarily small in comparison to the payoffs involved in the actual strategic game. While the cost of making a public announcement may be negligible in comparison to the payoffs involved in a particular bill being passed or not, the cost is still strictly positive. Secondly, players have the ability to *commit to not commit*. This assumption is interesting for both normative and positive reasons. On the one hand it is common for people to take a *"no further comments"* stand when making a comment would potentially constrain their future choices. On the other hand, the normative analysis in this paper shows that having the ability to rule out future commitments may be hugely beneficial to both parties concerned, allowing them to avoid Pareto inefficient Nash Equilibrium outcomes. Finally, the number of periods for which the game continues is determined endogenously. The players can continue to commit to progressively smaller subsets of their choice sets. The final strategic game is played following a period when all players who retain the ability to commit choose not to constrain their choices further.

Given a finite strategic game between two players this paper embeds it in a larger multi stage game, referred to as a *dynamic commitment game*. In the first period the players decide whether they want to make a *strict commitment* by irreversibly eliminating some actions from their original choice sets. Alternatively a player could *commit to not commit* thereby giving up the possibility of making any future commitments. A player could also play *passive*, involving no strict commitments, while leaving the option of future commitments open. The two players make their commitment decisions *simultaneously*. In the next period players who can still make commitments face the same set of commitment options as the previous period and make their decisions simultaneously. Only now their choice set does not contain the actions eliminated by them earlier. The game continues in this way until a period is reached when all players who have the ability to commit either play passive or commit to not commit. In the subsequent period a strategic game is played with each player choosing from actions still available to them in their constrained choice sets. Given a strategy profile for the *dynamic commitment game*, a player gets the payoff from the outcome of the induced strategic game while paying the total cost for all the *strict commitments* she made on the equilibrium path. The cost of making a strict commitment is assumed to be some small constant. So if the player made *strict commitments* in four periods on the equilibrium path, she incurs a cost of four times the said constant. The analysis focuses on the outcomes of the final period strategic game induced by subgame perfect strategy profiles for the *dynamic commitment game* when the cost of making a *strict commitment* is made arbitrarily small. Such outcomes are called *supportable*.

An immediate but important feature of the *dynamic commitment game* outlined above is that Nash Equilibria of the original game need not be *supportable*. To see why this is the case consider the strategic game depicted in *Example 1*.

Example 1: Nash Equilibrium not supportable.

	a_2	b_2
a_1	$3,3$	$0,7$
b_1	$2,0$	$1,1$

Example 1 is in fact a dominance solvable game with (b_1, b_2) as its unique Nash Equilibrium. The unique supportable profile, however, turns out to be

(a_1, a_2). To see why (b_1, b_2) is not supportable note first that both players simultaneously committing to a single action each cannot be part of an SPE. For instance if players 1 and 2 commit to $\{b_1\}$ and $\{b_2\}$ respectively, the outcome would be (b_1, b_2) with a payoff of $1 - \epsilon$ where ϵ is the cost of making a *strict commitment*. If player 2, on the other hand, deviates to *committing to not commit*, the strategic game $\Gamma(\{b_1\}, \{a_2, b_2\})$ must be played in the next period, resulting in the same outcome (b_1, b_2) but giving player 2 a payoff of 1, thereby making it a profitable deviation. Since the game has just two actions per player, eliminating simultaneous commitments to a single action each, suggests that any SPE must involve no more than one player making a strict commitment in the first period. If player 2 is the one making the commitment she must get at least $3 - \epsilon$ since she can simply commit to $\{a_2\}$ guaranteeing herself her Stackelberg payoff, thereby rejecting possible support of the NE, (b_1, b_2). If player 1 is the only one to make a commitment in period 1 and chooses $\{b_1\}$ the resultant payoff would be $1 - \epsilon$. Again deviating to playing either *passive* or *committing to not commit* would result in the original game being played in the next period, resulting in the same outcome. Player 1, however, would save on the ϵ cost from this deviation, thereby making the stated strategy profile fail subgame perfection. Finally consider the strategy profile involving the players choosing either PS or NC in the first period, followed by the profile (b_1, b_2) in the subsequent strategic game. Player 2 would again have the strictly profitable deviation to committing to $\{a_2\}$, forcing the outcome (a_1, a_2) in the subsequent strategic game, and getting a payoff of $3 - \epsilon$ as opposed to the original 1. (b_1, b_2), as a result, fails to be supportable.

While not all Nash Equilibria of the original game are supportable, it turns out that if there exists a Nash Equilibrium that Pareto dominates all other outcomes then it will be supportable. In fact such a Nash Equilibria can be supported without any player making any *strict commitments* in equilibrium. This leads to the main results of the paper which show that for two classes of strategic games not only is the Pareto dominant Nash Equilibrium supported it is also the *unique* supportable outcome. The two classes are games of pure coordination and $n \times n$ games with n Nash Equilibria. These results mark a significant departure from the earlier commitment results. In Renou(2009), Bade et al.(2009) and Nava(2008) efficiency is often made achievable by the commitment ability but it is by no means guaranteed due to the multiplicity of equilibria, in particular the inefficient Nash Equilibria that are always supportable. Pure coordination games pose a particularly frustrating problem here, since there are usually a number of Pareto dominated Nash Equilibria despite the fact that the preferences of the two players are perfectly aligned across all outcomes.

The definition of pure coordination games used in this paper is a purely ordinal one, requiring that if one player prefers outcome x to y then so does the other player. The uniqueness result is surprising also in that the ruling out of inefficient equilibria does not require players making *strict commitments* in equilibrium; the possibility alone suffices. Pure coordination games have been studied extensively and a number of studies have analyzed ways in which the players could achieve the Pareto dominant outcome. Lagunoff and Matsui(1997) show that if a (cardinal) pure coordination game is repeated infinitely, but the players move asynchronously in the stage games, the Pareto dominant outcome is achieved uniquely for sufficiently patient players. Calcagno and Lovo(2010) and Kamada and Sugaya(2010) consider a setup more similar to the present analysis where the strategic game is played just once at the end. In these studies players have the ability to revise their actions stochastically till a deadline is reached when the most recent actions are played. In games that are sufficiently similar to pure coordination games, the unique outcome is found to be the Pareto dominant Nash Equilibrium. Caruana and Einav(2008) consider games where the players get to revise their actions sequentially in some predetermined order before a deadline when the final actions are played. Revising actions, however, is costly with the cost increasing as the deadline approaches. They find a unique equilibrium for generic 2×2 games that is independent of the order and timing of moves as long as each party gets to revise their actions frequently enough. This is a remarkable result since the commitment ability of players is determined endogenously but still the outcome is invariant to the particular revision protocol used. A crucial aspect in all these papers, however, is the asynchronicity of moves made by the players. While explicit in Lagunoff and Matsui(1997), it is implicit in the *idiosyncratic posting inefficiency* of Calcagno and Lovo(2010), the independent stochastic arrival of revision possibilities in Kamada and Sugaya(2010) and the requirement of the revision process to be sequential in Caruana and Einav(2008). All these studies acknowledge that this asynchronicity is crucial for their results. The present paper, in contrast, involves the players making their commitment decisions *simultaneously.* As a result the rationale for why the Pareto dominant NE is the unique outcome of *dynamic commitment games* is markedly different.

Indeed the logic behind the uniqueness result relies on a signalling effect that the choice of not making any commitments has(discussed in greater detail in Section 4), similar in spirit to the role played by the possibility of *burning money* in Ben-Porath and Dekel(1992). In the latter paper the ability of one player to burn money before the strategic game is played can lead to her most preferred outcome being played. In a pure coordination game this outcome would be the Pareto dominant Nash Equilibrium. Further, the option of burning money alone suffices, no money is burnt in equilibrium. However this result does not hold if both players can simultaneously burn money, and further the argument relies on the iterated elimination of weakly dominated strategies. In the present analysis the equilibrium concept used is that of subgame perfection, while allowing for simultaneous commitments to be made.

An efficiency result similar to the present paper can be found in Jackson and Wilkie(2005). The latter paper studies two stage games where the players simultaneously announce binding outcome contingent transfers in the first stage, and play the so modified strategic game in the second stage. While not explicitly mentioned in the paper, a natural consequence of their characterization results is that if a strict 2 player strategic game has a Nash Equilibrium that Pareto dominates all other outcomes, then it will be the unique supportable outcome of their two stage game.[1] Note that this result does not require the strategic game to be either one of pure coordination or an $n \times n$ game with n Nash Equilibria and therefore has a larger scope. The rationale behind this result, however, is quite distinct from that of the present paper. In particular the Jackson and Wilkie(2005) result relies on the ability of a given player, say i, to make the Pareto dominant Nash Equilibrium action for player $-i$, her strictly dominant action by promising a sufficient amount of transfer for any outcome that involves $-i$ playing the said action. Interestingly while all other outcomes are ruled out, the unique supportable outcome itself may be supported without the use of transfers. Again, the ability alone to make such

[1] I thank Matthew Jackson for his help in my understanding of this issue.

transfers suffices. One implicit assumption necessary for this result, however, should be pointed out. The transfers, in a sense, do not need to satisfy any budget constraints. In particular making the Pareto dominant NE action a dominant action may require transfers that are larger than any possible payoff in the original game. Therefore making such a commitment credible requires the availability of such amounts to the players from sources outside the game.

The rest of the paper is as follows. Section 2 formally introduces *dynamic commitment games*. Section 3 presents a couple of results relating to general strict games. In particular it is shown that while Nash Equilibria of the original game may or may not be supportable, some outcomes related to Nash Equilibria of the original game are never supportable. Section 4 presents the result for ordinal pure coordination games while section 5 does the same for $n \times n$ games with n Nash Equilibria. Section 6 discusses the role of the assumptions made, in particular pointing out the necessity of the ability to *commit to not commit*. The role of multiple rounds of commitment is discussed here too. Section 7 discusses how the commitment ability studied in this game does not necessarily lead to more efficient outcomes in general games. Section 8 concludes.

2 Model

$N = \{1, 2\}$ is used to denote the set of 2 players. The letter i is used to refer to a generic player in this set, while $-i$ is used for the *other* player. Each player i has a finite set of actions denoted by X_i. Let $X \equiv X_1 \times X_2$ and let $\Delta(X)$ denote the set of all probability functions over elements of X. The payoff function for player i is given by $u_i : X \to \mathbb{R}$. Let $\mathbb{C}$ be the set of all

2 component vectors c with $c_i \in \{0, 1\}$ for all $i \in N$. Fix a small $\epsilon > 0$. A dynamic commitment game $g^\epsilon(A, u, c)$ with $A = X$ and $c = q \in \mathbb{C}$ is defined as follows.

If $q_i = 1$, $\forall i \in N$, then the simultaneous move game, $\Gamma(X, u)$ is played, following which the game ends. If $\exists i \in N$ such that $q_i = 0$, then all players who can still commit *simultaneously* choose from their feasible actions (specified below), following which the game moves on to the next period and $g^\epsilon(A, u, c)$ is played with $A = X'$ and $c = q'$.

If $q_i = 0$ then player i has the ability to make commitments (*can still commit*), and can choose to do one of the following.

- Make a *strict commitment* (*SC*), by choosing some $X'_i \subset X_i$. This results in $q'_i = 0$ if $|X'| \geq 2$ and $q'_i = 1$ if $|X'| = 1$.[2]
- *Commit to not commit*(*NC*), resulting in $q'_i = 1$ while $X'_i = X_i$.
- Play *passive*(*PS*) in which case $X'_i = X_i$.
- Choose a mixed strategy involving a probability function over the above options.

If $q_i = 0$ and player i plays *PS*, then $q'_i = 0$ if $X'_{-i} \neq X_{-i}$ and $q'_i = 1$ otherwise. In other words, player i playing *passive* leaves her commitment options open next period as long as the other player makes a *strict commitment*. However, if every player with commitment ability either plays *PS* or *NC* then the induced simultaneous move game is played in the next period.

Finally *committing to not commit* as the name suggests, results in no further commitment opportunities. Formally, if $q_i = 1$ and $q_{-i} = 0$ then $q'_i = 1$

[2]The symbol $\subset$ is used to denote *is a non empty strict subset of* while $|A|$ denotes the cardinality of the set A.

and $X'_i = X_i$. Notice that if $q_i = 1$ for all $i \in N$ then the induced simultaneous move game is played, implicitly ending further commitment abilities.

The number of periods for which the game continues, given the specification above, is endogenously determined, though necessarily finite. A typical non terminal history of the game $g^\epsilon(A, u, c)$ comprises of a sequence of feasible action sets and commitment vectors, $((X^0, c^0), (X^1, c^1), \ldots, (X^T, c^T))$, with $X^0 = A$ and $c^0 = c$. Given the specification of the game above and for any *feasible* non terminal history $((X^0, c^0), (X^1, c^1), \ldots, (X^T, c^T))$, the following must be true for all $t \in \{0, 1, \ldots, T-1\}$.

$$c_i^t = 1 \Rightarrow c_i^{t+1} = 1, X_i^{t+1} = X_i^t.$$

$$X_i^{t+1} \subset X_i^t \Rightarrow c_i^t = 0$$

$$X_i^{t+1} \subset X_i^t \& |X_i^{t+1}| \geq 2 \Rightarrow c_i^{t+1} = 0$$

$$c^t \neq (1, 1)$$

The last equation captures the fact that if for a given non terminal history neither player can commit in period t, then it must be that $t = T$, since the simultaneous move game $\Gamma(X^t, u)$ should be played in the next period.

A non terminal history with $c^T = (1, 1)$ is called a *semi terminal history.* Notice that following a semi terminal history, the induced simultaneous move game, $\Gamma(X^T, u)$ is played. A strategy σ_i, for player i specifies a mapping from the set of all non terminal histories to her available choices described earlier.

A terminal history, $h = ((X^0, c^0), (X^1, c^1), \ldots, (X^T, c^T), x)$ comprises of a *semi terminal history* followed by action choices made in the induced simultaneous move game, $x \in X^T$. The payoff to player i at such a terminal history is given by

$$\pi_i(h) = u_i(x) - z_i(h)\epsilon \tag{1}$$

where $z_i(h)$ is the number of times player i made a *strict commitment* in the history, h.

A strategy profile $\sigma = (\sigma_i)_{i \in N}$ generates a probability function over the set of all terminal histories. Player i's expected payoff from such a strategy profile is given by,

$$\pi_i(\sigma) = E(\pi_i(h)|\sigma). \tag{2}$$

The probability function over the elements of X generated by a strategy profile, σ, in the dynamic commitment game, $g^\epsilon(X, u, c = (0, 0))$ is denoted by $\mu(\sigma)$.

The object of this study is to identify action profiles that are played in the simultaneous move games induced by subgame perfect equilibrium strategies of the dynamic commitment game, when ϵ is made arbitrarily small.

Definition 1. *Given X and u, a probability function $\varphi \in \Delta$ is said to be **supportable in** $g^\epsilon(X, u, c = (0, 0))$ if there exists a subgame perfect equilibrium strategy profile, σ for the game $g^\epsilon(X, u, c = 0_N)$ such that $\varphi = \mu(\sigma)$.*

The set of *supportable* probability functions in $g^\epsilon(X, u, c = (0, 0))$ is denoted by, $\mathcal{O}(g^\epsilon(X, u))$. An outcome $x \in X$ is called *supportable* in $g^\epsilon(X, u, c = (0, 0))$ if there exists $\varphi \in \mathcal{O}(g^\epsilon(X, u))$ such that $\varphi(x) = 1$. Through a minor abuse of notation such outcomes would be characterized by the inclusion, $x \in \mathcal{O}(g^\epsilon(X, u))$. The set of all such outcomes is denoted by the set $\mathcal{O}^P(g^\epsilon(X, u)) = \{x \in X | x \in \mathcal{O}(g^\epsilon(X, u))\}$.

Definition 2. *Given X and u, a probability function $\varphi \in \Delta$ is said to be **supportable** if there exists a sequence of $\epsilon^k > 0$, with $\lim_{k \to \infty} \epsilon^k = 0$ such that*

$\varphi \in \lim_{k\to\infty}(\mathcal{O}(g^{\epsilon}(X,u)))$.

The set of all *supportable* probability functions given X and u is denoted by $\mathcal{O}(g(X,u))$. Again, $x \in \mathcal{O}(g(X,u))$ implies that there exists $\varphi \in \mathcal{O}(g(X,u))$ such that $\varphi(x) = 1$. Such outcomes are collected in $\mathcal{O}^P(g(X,u)) = \{x \in X | x \in \mathcal{O}(g(X,u))$. The set of Nash Equilibria of a simultaneous move game $\Gamma(X,u)$ is denoted by $NE(\Gamma(X,u))$.

Finally, a *dynamic commitment game*, $g^{\epsilon}(X,u,c)$ with a finite X and $c \in \mathbb{C}$ is a finite extensive form game with perfect recall. As a result for all such games, the existence of an SPE is guaranteed.

3 Results for General Strict Games

A simultaneous move game $\Gamma(X,u)$ is called a strict game if $\forall x, y \in X$, $x \neq y \Rightarrow u_i(x) \neq u_i(y), \forall i \in \{1,2\}$. This paper focuses entirely on the class of strict games. The first observation highlights the role that the ϵ cost of commitment plays in deterring players from making non payoff improving commitments. If one player chooses to commit to a single action, then the other player, in the same period, can never do worse by playing *NC*. Indeed by ruling out further commitment ability not only does she retain greater flexibility in choosing her best response, she saves on the cost of making a strict commitment.

Lemma 1. *Given a strict game* $\Gamma(X,u)$ *and its corresponding dynamic commitment game* $g^{\epsilon}(X,u,c=(0,0))$, *an SPE strategy profile for the latter cannot have both players making a* strict commitment *to a single action each after any history.*

Proof. Consider a subgame where both players still have the ability to make commitments, $g(X^t, u, c = 0)$.[3] Suppose both players commit to a single action each x_i. The payoff to player i at this subgame would then be $u_i(x) - \epsilon$. If player i deviates to playing NC, the game must move to the next period where $g(X^{t+1}, u, c = (1, 1))$ is played with $X_i^{t+1} = X_i^t$ and $X_{-i}^{t+1} = \{x_{-i}\}$. Note that this subgame is in fact the simultaneous move game, $\Gamma(X^{t+1}, u)$. The outcome would then be $y = (z, x_{-i})$ where z is player i's unique best response to x_{-i} in the set $X_i^{t+1} = X_i^t$, resulting in a payoff of $u_i(y)$ to player i in the original subgame, $g(X^t, u, c = 0)$. Since z is player i's best response to x_{-i} in the set X_i^{t+1} and $z, x_{-i} \in X_i^{t+1}$ it must be that $u_i(y) \geq u_i(x)$. The above deviation must then be strictly profitable since $u_i(y) > u_i(x) - \epsilon$. Consequently, simultaneously committing to a single action each by both players cannot be part of SPE strategies following any history. □

The second observation establishes a relationship between pure strategy Nash Equilibria of a simultaneous move game with outcomes of the corresponding dynamic commitment game. As shown in Example 1, a Nash Equilibrium outcome of a simultaneous move game is not necessarily an outcome of the dynamic commitment game. Interestingly, however, such a Nash Equilibrium outcome would systematically eliminate the possibility of some other related outcomes to be supportable in the dynamic commitment game.

Proposition 1. *Given a strict game,* $\Gamma(X, u)$*, if* $(x_1, x_2) \in NE(\Gamma(X, u))$ *then* $\forall y \in X_{-i}$ *such that* $y \neq x_{-i}$*,* (x_i, y) *is not supportable.*

Proof. Let $(x_1, x_2) \in NE(\Gamma(X, u)$. Consider a strategy profile σ that induces a unique simultaneous move game with outcome (y, x_i), where $y \neq x_{-i}$. Further let the semi terminal history that precedes the simultaneous move game

[3] Remember that $c = 0 \Rightarrow |X_i^t| \geq 2$.

be $h^T = ((X^0, c^0), \ldots, (X^T, c^T))$. It must be that $x_i \in X_i^t$, $\forall 0 \leq t \leq T$. Consider a deviation by player $-i$ in the first period involving the strict commitment to the single action set $\{x_{-i}\}$. The resulting subgame would be $g^\epsilon(\{x_{-i}\}, X_i^1, u, c_i = 0, c_{-i} = 1)$. If player i does not make any strict commitment in this subgame, her payoff would be determined by the (necessarily unique) Nash Equilibrium outcome in the induced simultaneous move game, $\Gamma(\{x_{-i}\}, X_i^1, u)$, namely (x_1, x_2). Any other strategy of player i would give her no more than $u_i(x_{-i}, y) - \epsilon$, with $y \in X_i^1$. Since $u_i(x_{-i}, y) - \epsilon < u_i(x_1, x_2)$, such strategies would violate subgame perfection. Player i's optimal choice thus involves not making a strict commitment. Consequently, player $-i$'s initial deviation guarantees her a payoff of $u_{-i}(x_1, x_2)$ that is strictly higher than her payoff of $u_{-i}(x_i, y)$, since x_{-i} is her unique best response to x_i in X_{-i}^0. Therefore the strategy profile σ cannot be subgame perfect. □

If a Nash Equilibrium of the strategic game Pareto dominates all other outcomes then it is in fact supportable. Subgame perfect strategy profiles required to support such an outcome does not need any *strict commitments* to be made on the equilibrium path. Indeed the simplest strategy profile suffices and is outlined below. Note that an outcome that Pareto dominates all other outcomes must necessarily be a Nash Equilibrium outcome.

Proposition 2. *Given a strict game,* $\Gamma(X, u)$ *if* $\exists x \in X$ *such that* x *Pareto dominates all* $y \in X \backslash \{x\}$*, then* x *is supportable. Further there exists a subgame perfect strategy profile that supports* x *involving no strict commitments on the equilibrium path.*

Proof. Consider the following strategies. Both players play passive in period 1. In the original strategic game played in period 2, the Pareto dominant

Nash Equilibrium profile x is played. Subgame perfect strategies are used for every other subgame. It is clear that no deviation by any player, given these strategies, can give the said player a higher payoff. In fact deviation to any strict commitment would give the deviating player a strictly lower payoff. □

4 Ordinal Pure Coordination Games

A simultaneous move strict game $\Gamma(X, u)$ is said to be an *ordinal pure coordination game*(OPC game) if

$$\forall x, y \in X, \qquad u_1(x) > u_1(y) \Leftrightarrow u_2(x) > u_2(y).$$

A feature of OPC games is the existence of a Nash Equilibrium outcome that Pareto dominates all other outcomes. Given an OPC game $\Gamma(X, u)$, its Pareto dominant outcome is denoted by $\mathcal{P}(\Gamma(X, u))$. Notice that following any sequence of strict commitments in an OPC game, the resulting game is also an OPC game. The following lemma shows that for small enough values of ϵ, in a dynamic commitment game induced by an OPC game $\Gamma(X, u)$, if only one player has the ability to make commitments, the unique supportable outcome is in fact the Pareto dominant one. Further, there exists an SPE in which no strict commitment is made.

Lemma 2. *Given X and u such that $\Gamma(X, u)$ is an ordinal pure coordination game, the unique supportable outcome in $g^{\epsilon}(X, u, c_i = 0, c_{-i} = 1)$ is $\mathcal{P}(\Gamma(X, u))$, for all small enough ϵ. There exists SPE strategies that support this outcome involving no strict commitments.*

Proof. In the first period of the game $g^{\epsilon}(X, u, c_i = 0, c_{-i} = 1)$, only player i

has the ability to make strict commitments. It is clear that $u_i(\mathcal{P}(\Gamma(X,u))) > u_i(y)$, for all $y \in X \setminus \{\mathcal{P}(\Gamma(X,u))\}$. To show that no other outcome can be supportable in this game, consider strategies that result in a different outcome, say $y \in X$ such that $y \neq \mathcal{P}(\Gamma(X,u)$. Consider the outcome if player i deviates by committing to the single action $\mathcal{P}_i(\Gamma(X,u))$. Following such a commitment the game $\Gamma(X_{-i}, \{\mathcal{P}_i(\Gamma(X,u))\}, u)$ is played in the next period. The outcome, $\mathcal{P}_i(\Gamma(X,u))$, from such a deviation brings player i a payoff of $u_i(\mathcal{P}(\Gamma(X,u))) - \epsilon$. Given that her original payoff was $u_i(y)$, the deviation must be profitable for small enough values of ϵ since $\mathcal{P}(\Gamma(X,u))$ strictly Pareto dominates all other outcomes.

To show that there exists SPE strategies supporting the Pareto dominant outcome involving no strict commitments on the equilibrium path consider the following. Player i plays passive in the first period. In the subsequent subgame $g^\epsilon(X,u,c=1_N)$, the profile $\mathcal{P}(\Gamma(X,u))$ is played. Subgame perfect strategies are played following any other subgame. It is easy to see that these strategies constitute an SPE of $g^\epsilon(X,u,c_i=0,c_{-i}=1)$, since any deviation must result either in a Pareto dominated outcome or in an outcome involving a commitment cost of ϵ. □

Notice that the lemma above ruled out the scenario where player i plays passive and then ends up coordinating with $-i$ on a Pareto inefficient Nash Equilibrium. As can be seen in the proof, such a scenario is ruled out as player i would then prefer to make a strict commitment, instead of playing passive, that would ensure the Pareto dominant profile to be the outcome. Since such a deviation is always available to player i the only outcome that can result following player i not making a strict commitment, in equilibrium, would have to be the Pareto dominant one.

The following proposition shows how this Pareto dominant outcome is in fact the unique outcome of the corresponding dynamic commitment game when both players can commit, for any small enough value of ϵ. Before delving in to the formal proof of the proposition, it may help to consider a particularly subtle role played by the commitment ability studied in this paper in avoiding Pareto inefficient Nash Equilibria. Consider what is required by subgame perfection following a first period choice of no commitment by both players. In a setup without commitment ability the two players could coordinate on an inefficient outcome. Player i could believe that player $-i$ wants to take the inefficient equilibrium action, and simply best responds to it. i believes that $-i$ will make such a choice because i believes that $-i$ believes that i will play the inefficient equilibrium action and so on. In a dynamic commitment game of an OPC game, if neither player makes a commitment in the first period then such beliefs that end up supporting inefficient equilibria cannot be sustained. In particular if i believes that $-i$ will take the inefficient equilibrium action in period 2, then it must be because i believes that $-i$ believes that i will take the inefficient equilibrium action in period 2. However, if this were to be true then $-i$ should have made a payoff improving commitment to the Pareto dominant profile action in period 1. Since $-i$ did not make such a commitment in period 1, it must mean that $-i$ believes that i intends to coordinate on the Pareto dominant profile. Consequently $-i$ not making a commitment in period 1 credibly signals that she herself intends to coordinate on the Pareto dominant profile. This implies that following no commitments in period 1 only the Pareto dominant profile can be played in period 2 with strategy profiles that are subgame perfect.

Proposition 3. *Given X and u such that $\Gamma(X, u)$ is an ordinal pure coordi-*

nation game, $\mathcal{P}(\Gamma(X,u))$ *is the unique supportable outcome.*

Proof. The set of all strategy profiles (σ_1, σ_2) for the game $g^\epsilon(X, u, c = 0)$ can be categorized in the following way. Given any strategy profile, in the first period either player i makes a strict commitment and $-i$ does not or both players make a strict commitment or neither player makes a strict commitment. Consider a strategy profile that results in an outcome $y \neq \mathcal{P}(\Gamma(X,u))$ that involves only player i making a strict commitment in the first period. Such a strategy profile results in a payoff of at most $u_i(y) - \epsilon$ for player i. If player i deviates by making a strict commitment to the single action $\mathcal{P}_i(\Gamma(X,u))$, then subsequently the subgame $g(\{\mathcal{P}_i(\Gamma(X,u))\}, X_{-i}, u, c_i = 1, c_{-i} = 0)$ must be played. *Lemma 2* shows that the outcome from such a subgame must be $\mathcal{P}(\Gamma(X,u))$ for small enough ϵ. Player i's payoff from such a deviation is therefore $u_i(\mathcal{P}(\Gamma(X,u))) - \epsilon$ which is strictly higher than her original payoff. This profitable deviation rules out the possibility of strategy profiles with only player i making a strict commitment in the first period resulting in a Pareto dominated outcome from being subgame perfect.

Now consider strategy profiles that involve both players making a strict commitment in the first period of $g^\epsilon(X, u, c = 0)$ that results in a Pareto dominated outcome y. The actions still available to the two players in the second period are captured by the sets X_1^1 and X_2^1. Note that X^1 and u also define a game of ordinal pure coordination. Subgame perfection then dictates that $y = \mathcal{P}(\Gamma(X^1, u))$. To see why this must be the case consider the opposite scenario where $y \neq \mathcal{P}(\Gamma(X^1, u))$. The payoff to player i in this case is at most $u_i(y) - \epsilon$. Player i, however, can deviate in the first period to making a strict commitment to $\mathcal{P}_i(\Gamma(X^1, u))$. This in turn, by *Lemma 2*, would lead to the outcome $\mathcal{P}(\Gamma(X^1, u))$ in the subsequent subgame $g^\epsilon(\{\mathcal{P}_i(\Gamma(X^1, u))\}, X^1_{-i}, u, c_i = 1, c_{-i} = 0)$ for small enough ϵ. Such a deviation yields a payoff of $u_i(\mathcal{P}(\Gamma(X^1, u)) - \epsilon$ to player i and is therefore a profitable deviation. It is clear therefore that if both players make strict commitments resulting in X^1, by subgame perfection, the outcome must be $\mathcal{P}(\Gamma(X^1, u))$, and player i gets a payoff of $u_i(\mathcal{P}(\Gamma(X^1, u)) - \epsilon$. Consider now player i's ability to deviate in the first period by committing to not commit. Such a deviation would give rise to the subsequent subgame $g^\epsilon(X^0_i, X^1_{-i}, u, c_i = 1, c_{-i} = 1)$. Again by *Lemma 2*, the unique outcome in such a subgame must be $\mathcal{P}(\Gamma(X^0_i, X^1_{-i}, u))$ resulting in a payoff of $u_i(\mathcal{P}(\Gamma(X^0_i, X^1_{-i}, u)))$ to player i. Given that X and u define an OPC game, it must be that $u_i(\mathcal{P}(\Gamma(X^0_i, X^1_{-i}, u))) \geq u_i(\mathcal{P}(\Gamma(X^1_i, X^1_{-i}, u)))$. As a result player i can profitably deviate by committing to not commit in the first period and save herself at least the ϵ cost of commitment, if not more. This implies that both players making strict commitments in the first period that finally result in a Pareto dominated outcome y cannot satisfy subgame perfection.

Strategy profiles involving both players not making any strict commitments in the first period must necessarily involve the outcome, $\mathcal{P}(\Gamma(X, u))$ in the subsequent simultaneous move game $\Gamma(X, u)$, to satisfy subgame perfection. This is true since for any other outcome, player i would have a strict incentive in the first period to make a strict commitment to $\mathcal{P}_i(\Gamma(X, u))$. By Lemma 2, the subsequent subgame yields $\mathcal{P}(\Gamma(X, u))$ as the unique SPE outcome.

This concludes the proof of why no Pareto dominant outcome is supportable in an OPC game.

Finally it is shown by construction that $\mathcal{P}(\Gamma(X, u))$ is in fact supportable. Consider a strategy profile for $g^\epsilon(X, u, c = 0)$ which involves both players playing *Passive* in the first period and playing $\mathcal{P}_i(\Gamma(X, u))$ in the subsequent game, $\Gamma(X,u)$. The profiles involve subgame perfect strategies following every other subgame. It is easy to see that such a profile does in fact satisfy subgame perfection. The proof concludes by noting that the SPE profile involves no strict commitments on the equilibrium path. □

5 n × n games with n Nash Equilibria

A simultaneous move strict game $\Gamma(X,u)$ is said to be an *$n \times n$ game with n Nash Equilibria*(n-Eq game) if $|X_i| = n$ and $|NE(\Gamma(X,u)| = n$. A particular feature of *n-Eq* games is that,

$$\forall x_i \in X_i, \exists \text{ a unique } x_{-i} \in X_{-i}, s.t.(x_i, x_{-i}) \in NE(\Gamma(X,u). \tag{3}$$

This feature, in turn, implies that if one of the Nash Equilibria Pareto dominates the others then it also Pareto dominates all other outcomes. It is clear from Proposition 1 and (3) that outcomes that are not a Nash Equilibrium of *n-Eq* games cannot be *supportable*. The following proposition shows that if an *n-Eq* game has a Pareto dominant outcome then the latter will also be the unique *supportable* outcome.

Proposition 4. *Given X and u such that $\Gamma(X,u)$ is an n-Eq game, if there exists a Pareto dominant Nash Equilibrium then it is the unique supportable outcome.*

Proof. Let $\mathcal{P}(X,u)$ be the Pareto dominant Nash Equilibrium of the *n-Eq* game $\Gamma(X,u)$. Consider first strategy profiles involving at least one player, say $-i$ not making a strict commitment in the first period. Subgame perfection would require the outcome from all such profiles to be $\mathcal{P}(X,u)$. Otherwise player i could deviate by choosing the single action commitment $\{\mathcal{P}_i(X,u)\}$ in the first period. Such a deviation gives rise to the subsequent subgame, $g^\epsilon(\{\mathcal{P}_i(X,u)\}, X_{-i}, c_i = 1, c_{-i})$. The argument used in Lemma 2 applies here as well, requiring that the only supportable outcome in such a subgame must in fact be $\mathcal{P}(X,u)$. The payoff to player i from such a deviation is $u_i(\mathcal{P}(X,u)) - \epsilon$. Any other outcome would result in a payoff strictly less than $u_i(\mathcal{P}(X,u))$ for player i. The deviation would therefore be profitable for all small enough ϵ.

Now consider strategy profiles involving both players making *strict commitments* in the first period. Suppose the strict commitments are X_1^1 and X_2^1 and let the outcome from such a strategy profile be some $y \neq \mathcal{P}(X,u)$. Again from Proposition 1 and (3) it must be that $y \in NE(\Gamma(X,u))$ which in turn implies that $y \in NE(\Gamma(X^1,u))$. It will be shown that such a profile must violate subgame perfection. Let $\overline{x}_i$ denote the Nash Equilibrium profile of $\Gamma(X_i^0, X_{-i}^1)$ that gives player i her highest payoff. Formally,

$$\hat{x}^i \equiv \arg \max_{y \in NE(\Gamma(X_i^0, X_{-i}^1))} u_i(y).$$

If $\hat{x}^1 = \hat{x}^2$, then both players have a strict incentive to deviate to playing *NC*. In particular, consider player i's deviation to playing *NC*. The resulting subgame is $g^\epsilon(X_i^0, X_{-i}^1, c_i = 1, c_{-i} = 0)$. First notice that $\hat{x}^1 = \hat{x}^2 \in X_i^0 \times X_{-i}^1$. Since $-i$ is the only player with commitment abilities, she can guarantee herself a payoff of $u_{-i}(\hat{x}^{-i})$ by committing to the single action set, $\{\hat{x}_{-i}^{-i}\}$. The possibility of any other outcome can be ruled out by noting that,

$$\forall X \subset X_{-i}^1, \qquad u_{-i}(\hat{x}^{-i}) \geq \max_{y \in NE(\Gamma(X_i^0, X))} u_{-i}(y).$$

Therefore $\hat{x}^1 = \hat{x}^2$ is the unique outcome for the subgame $g^\epsilon(X_i^0, X_{-i}^1, c_i =$

$1, c_{-i} = 0)$. Remember that the initial strategy profile resulted in $y \in NE(\Gamma(X_i^1, X_{-i}^1, u))$. So it must be the case that

$$u_i(\hat{x}_i) \geq u_i(y)$$

As a result by deviating to playing NC in the first period player does strictly better saving, at the very least, on the cost of commitment ϵ. This ends the proof of why simultaneous *strict commitments* in the first period that support a Pareto dominated outcome does not satisfy subgame perfection when $\hat{x}^1 = \hat{x}^2$.

Consider the remaining case of a strategy profile that supports a Pareto dominated Nash Equilibrium, y, involving simultaneous commitments to X_1^1 and X_2^1 in the first period with $\hat{x}^1 \neq \hat{x}^2$. It must be that $y \neq \hat{x}^i$ for some $i \in \{1, 2\}$. WLOG let $y \neq \hat{x}^1$. It is clear that $u_1(y) < u_1(\hat{x}^1)$. Player 1 can then profitably deviate to making the single action commitment of $\{\hat{x}_1^1\}$, thereby forcing the outcome, $\hat{x}^1$. This deviation rules out the possibility of the stated strategy profile satisfying subgame perfection. □

6 Discussion regarding the assumptions

The role played by the ϵ cost of making a *strict commitment* is obvious from Example 1. Without such a cost, all Nash Equilibria of the original game would be supportable. A few comments regarding the particular structure of these commitment costs, however, are in order. The analysis deals with arbitrarily small values of this cost to capture the idea that if a player prefers outcome x to y then she would continue to prefer x to y even if the latter requires a large but finite number of commitments to achieve. In a sense, with small commitment costs, a players preference over final outcomes does not get changed if she takes into account the cost she must incur to achieve those outcomes. At the same time the existence of these costs makes sure that players choose to use *strict commitments* only if it strictly increases their payoff. The fact that all strict commitments yield the same cost is not necessary for any of the results in this paper. For a given set A and any subset B, the commitment from A to B could potentially depend upon the identity of the two sets, say a function $f(A, B)$. The requirement would then be that to arrive at the cost such a function should be scaled down enough to represent a small enough cost as discussed earlier. This could be done by letting the cost be $\epsilon f(A, B)$ where the ϵ serves as a scale variable. All the results in this paper would hold for any arbitrary strictly positive function f, for small enough values of ϵ. Some of the strategy profiles outlined in examples, however, may then need to be suitably modified.

An implicit assumption that *does* play a crucial role is that *committing to not commit* is costless. The assumption required for the results to hold, however, is that the cost of *committing to not commit* is less than that of making a *strict commitment*. It is very possible that in certain strategic environments this assumption would in fact fail. The normative message of the paper however would still hold in pointing out the benefit of making a *committing to not commit* option available and cheap for certain games.

The critical role played by the option of *committing to not commit* can be seen in the following example.

Example 2: The importance of committing to not commit.

	a_2	b_2	c_2	d_2
a_1	$5, 7$	,	,	,
b_1	,	$4, 3$	,	,
c_1	,	,	$2, 2$	,
d_1	,	,	,	$10, 13$

The cells which have been left empty can be filled with any set of values as long as they are all less than 2. This would make all the (filled in) diagonal elements strict Nash Equilibria. Example 2 can then be used not only as an example of $n \times n$ games with n Nash Equilibria but also a pure coordination games (with suitably chosen values). Suppose the option of *committing to not commit* is not available to players. They may only choose between *strict commitments* and playing *passive*. In such a commitment game it can be shown that the Pareto inefficient Nash Equilibrium profile (b_1, b_2) can be supportable.

Consider the following strategy profile that supports (b_1, b_2). In period 1 players 1 and 2 commit to $\{b_1, c_1\}$ and $\{b_2, c_2\}$ respectively. In period 2 both players play *passive*. In the resulting third period strategic game, the profile (b_1, b_2) is played. If player i deviates by playing *passive* in period 1 then in period 2 player i commits to $\{b_i, c_i\}$, while player $-i$ plays passive. In period 3 along this history, both players play *passive* and play the profile (b_1, b_2) in the period 4 strategic game. In the period 2 subgame after player i's deviation in period 1 to playing *passive*, if player i deviates by playing *passive* then in period 3 the strategic game with action sets $\{a_i, b_i, c_i, d_i\}$ and $\{b_{-i}, c_{-i}\}$ is played. In this strategic game the profile (c_1, c_2) is played. Subgame perfect strategies are used for every other subgame. The resulting strategy profile is subgame perfect with a payoff of $4-\epsilon$ to player 1 and $3-\epsilon$ to player 2. Notice that the only way player i can do any better is by not making a commitment and saving on the ϵ cost. Such a deviation in period 1, however, only leads to a subgame where player i must now commit to what her original commitment should have been. The reason why the profile is subgame perfect in this particular subgame is that if player i were to not make a commitment, it would result in a strategic game being played where the worse Nash Equilibrium would be played. So player i makes the commitment in period 1 because she knows that playing *passive* would simply mean she would have to make the same commitment in the next period, and she must make the latter commitment in the next period since failing to do so would result in coordinating on an even worse Nash Equilibrium profile, namely (c_1, c_2). If player i had the ability to commit to not commit, then she would simply deviate to NC in the first period, and not be forced to make the commitment in the subsequent period. The argument in this example can be extended to show that every Nash Equilibrium but the worst one can be supported in an $n \times n$ game with n Pareto ranked Nash Equilibria, if players cannot *commit to not commit*.

The ability to make *strict commitments* in multiple periods has a critical bearing on the set of supportable outcomes. Thus unlike in Bade et al.(2009) there is a loss of generality in restricting the commitment ability of players to a single period. This feature of *dynamic commitment games* can be seen clearly in the following example.

Example 3: Efficiency achievable with multiple commitment periods but not with just one period.

	a_2	b_2	c_2
a_1	$3, 3$	$0, 7$	$-1, 2$
b_1	$2, 0$	$1, 1$	$0, -1$
c_1	$7, -1$	$-1, 0$	$-1, -2$

The unique Nash Equilibrium of the strategic game in Example 3 is the profile (b_1, b_2). Allowing for players to make a single round of commitment as in Renou(2009) does not allow the efficient profile (a_1, a_2) to be supportable. On the other hand the following strategy profile in the *dynamic commitment game* studied in this paper does support the efficient profile. In period 1, player 1 makes a strict commitment to $\{a_1, b_1\}$ while player 2 plays *passive*. In period 2, player 1 plays *passive* while player 2 makes a strict commitment to $\{a_2, c_2\}$. In period 3 both players play passive resulting in the strategic game $\Gamma(\{a_1, b_1\}, \{a_2, c_2\})$ being played in period 4. In the said strategic game the unique Nash Equilibrium profile, (a_1, a_2) is played. The payoff to both players from this profile is $3 - \epsilon$.

7 Commitment and Efficiency

Does the commitment ability studied in this paper always have an efficiency enhancing effect on a general strategic game? The answer to this question is no. The possibility of making commitments may lead players to reach an outcome that is Pareto inefficient even though the unique Nash Equilibrium in the original game involved a Pareto efficient outcome.

Example 4: Inefficiency due to commitment.

	a_2	b_2	c_2
a_1	$8, 8$	$10, 0$	$4, 2$
b_1	$0, 10$	$6, 6$	$3, 5$
c_1	$2, 4$	$5, 3$	$-1, -1$

Example 4 is again a dominance solvable game. The unique Nash Equilibrium, (a_1, a_2), is an efficient outcome. While (a_1, a_2) continues to be supportable, the *dynamic commitment games* also allow the inefficient profile (b_1, b_2) to be supportable. Consider the following subgame perfect strategy profile. Players 1 and 2 commit to the subsets (b_1, c_1) and (b_2, c_2), respectively, in the first period. In the second period both players choose to play *passive*. In the subsequent strategic game $\Gamma(\{(b_1, c_1), (b_2, c_2)$ the unique Nash Equilibrium (b_1, b_2) is played. Subgame perfect strategies are played following every other subgame. The payoff to each player is $6 - \epsilon$. To see why this strategy profile is subgame perfect consider the deviation possibilities available to players on the equilibrium path. For player i to do any better by making some other strict commitments she needs the outcome (a_1, b_2) to be played with some positive probability. Committing to the $\{a_i\}$ does not work since it yields the outcome (a_i, c_{-i}). The two remaining options are $\{a_i, b_i\}$ and $\{a_i, c_i\}$. Committing to $\{a_i, b_i\}$ gives rise to the subgame $g^\epsilon(\{a_i, b_i\}, \{b_{-i}, c_{-i}\}, c = (0, 0))$. No further commitments would result in outcome (a_i, c_{-i}) again. Lemma 1 shows how both players will not simultaneously commit to one action each in any subgame. So the only options remaining involve only one of the players making a strict commitment. The resulting subgame perfect outcome if player i commits further would be (b_1, b_2). If player $-i$ makes the commitment the outcome would be (a_i, c_{-i}). As a result the highest payoff i can guarantee herself by deviating to some other strict commitment is $6 - \epsilon$. The only other deviation possibilities for player i in period 1 involves playing *passive* or *committing to not commit*. The latter option would result in the outcome (a_i, c_{-i}) with a payoff of 2 to player i. Playing passive results in the subgame, $g^\epsilon(\{a_i, b_i, c_i\}, \{b_{-i}, c_{-i}\}, c = (0, 0))$. Again the only way i can get more than $6 - \epsilon$ is by achieving the outcome (a_i, b_{-i}). However as long as a_i is part of her choice set it would also continue to strictly dominate her other actions. So in the induced strategic game she must play a_i with probability 1. Player $-i$'s best response to a_i, though, is c_{-i}, with the consequent outcome, (a_i, c_{-i}), giving player i a payoff no more than 2. This eliminates any profitable deviation in period 1. The argument concludes by noticing that given the strategy profile if there were profitable deviations in period 2 there would be a profitable deviation in period 1.

It has been shown that a Nash Equilibrium that Pareto dominates all other outcomes is supportable. However, it turns out that supportability does not necessarily extend to all Pareto efficient Nash Equilibria. The following example provides a case in point.

Example 5: Pareto Efficient Nash Equilibrium not supportable.

	a_2	b_2
a_1	$1, 0$	$4, 3$
b_1	$2, 4$	$5, 1$

Example 5 is again a dominance solvable game with a unique Pareto efficient Nash Equilibrium, (b_1, a_2). The unique supportable profile, however, turns out to be (a_1, b_2). Both players playing *passive* or *committing to not commit* and then playing (b_1, a_2) is not subgame perfect as player 1 could deviate by committing to $\{a_1\}$. Again by Lemma 1, both players making a strict commitment can be ruled out. If player 1 were the only one making a strict commitment the preferred option would be $\{a_1\}$. If player 2 were the only one making a strict commitment, then the option to commit to $\{a_2\}$ would not be subgame perfect, since the outcome would be (b_1, a_2) with a payoff of $4-\epsilon$. Player 2 could save the ϵ cost by playing *passive* resulting in the original strategic game being played with the resultant unique Nash Equilibrium outcome, (b_1, a_2). The outcome (a_1, b_2) on the other hand is supported by player 1 making a strict commitment to $\{a_1\}$ while player 2 plays *passive* in period 1. Period 2 involves neither party making any strict commitments, while period 3 involves the profile (a_1, b_2) being played in the induced strategic game. Off the equilibrium path, if player 1 were to not make any strict commitments, the Nash Equilibrium profile (b_1, a_2) is played. Subgame perfect strategies are used for every other subgame.

8 Conclusion

The simultaneity of moves in strategic games often results in a multiplicity of equilibria. It has been shown that for two classes of two player games it is possible to avoid the Pareto inefficient equilibria without resorting to an asynchronous move structure. If the players can eliminate actions from their feasible choice sets at some arbitrarily small cost and also have the ability to commit to not making any further eliminations then they can coordinate on the Pareto efficient outcome. The two classes of games where it applies, pure coordination games and $n \times n$ games with n Nash Equilibria, are particularly plagued with multiple Nash Equilibria. The result relies on the use of committing to not commit as a signalling device but does not require asynchronous moves as in the earlier money burning papers. The cost of committing to eliminate some actions, the presence of endogenously determined number of rounds to make such commitments and the ability to commit to not commit are all necessary for the uniqueness result. For any strategic game, while its Nash Equilibria may or may not be *supportable*, each of these Nash Equilibria systematically eliminate a set of outcomes from being *supportable*.

Dynamic commitment games do not always have an efficiency enhancing effect on the underlying game. The precise class of games for which it delivers a unique efficient prediction is yet to be characterized. It also remains to be seen how the arguments presented here carry over to Bayesian games.

References

[1] Bade, S., Haeringer, G., Renou, L., 2009. Bilateral commitment. Journal of Economic Theory 144, 1817-1831.

[2] Ben-Porath, E., Dekel, E., 1992. Signaling future actions and the potential for sacrifice. Journal of Economic Theory 57, 36-51.

[3] Calcagno, R., Lovo, S., 2010. Preopening and equilibrium selection. *mimeo*

[4] Caruana, G., Einav, L., 2008. A theory of endogenous commitment. Review of Economic Studies 75, 99-116.

[5] Jackson, M., Wilkie, S., 2005. Endogenous games and mechanisms: Side payments among players. Review of Economic Studies 72, 543-566.

[6] Kalai, A.T., Kalai, E., Lehrer, E., Samet, D., 2010. A commitment folk theorem. Games and Economic Behavior 69, 127-137.

[7] Kamada, Y., Sugaya, T., 2010. Asynchronous revision games with deadline: Unique equilibrium in coordination games. *mimeo*

[8] Lagunoff, R., Matsui, A., 1997. Asynchronous choice in repeated coordination games. Econometrica 65(6), 1467-1477.

[9] Nava, F., 2008. Joint commitment: Implementation without public randomization. *mimeo*

[10] Renou, L., 2009. Commitment games. Games and Economic Behavior 66, 488-505.

[11] Schelling, T., 1960. The strategy of conflict. Harvard University Press, Cambridge, MA.