Summer 8-15-2015

Spectrum Management using Markov Decision Processes

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by
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A dissertation presented to the
Graduate School of Arts and Sciences
of Washington University in
partial fulfillment of the
requirements for the degree
of Doctor of Philosophy

August 2015
Saint Louis, Missouri

Table of Contents

List of Tables

List of Figures

Acknowledgments

The completion of my Ph.D culminates a life long achievement that has been the most humbling accomplishment of my life. The pursuit has enabled me to work with many wonderful and incredibly talented people. I want to thank God for guidance and help. My wife Cindy and children (Elizabeth, Jason, and Jared) made many sacrifices and offered unwavering encouragement enabling me to finish this work. I would like to dedicate this dissertation to my grandchildren (Jaden, Justin, James, and Braden).

My advisors, Roger Chamberlain and Chris Gill, are the most patient, supportive, and giving scientists that I could have hoped to work with. They have been understanding as life events continued to slow my progress over the past several years. I would also like to thank my other committee members, Paul Min, Dave Peters, and Angelina Lee, three professors that have encouraged me as I proposed and defended my dissertation. Thank you.

John Leo Meier

Washington University in Saint Louis

August 2015

ABSTRACT OF THE DISSERTATION

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by
John Leo Meier
Doctor of Philosophy in Computer Science
Washington University in St. Louis, 2015
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The advent of cognitive radio technology has enabled dramatically more options in the use of RF spectrum, allowing multiple transmitters to effectively share spectrum in ways that were previously unavailable (either due to technical limitations or regulatory restrictions). In this dissertation, we investigate approaches to managing RF spectrum use, with a focus on combining multiple control decisions in a mutually beneficial manner.

Our approach to making spectrum management decisions is grounded in Markov decision theory, which has a rich formal foundation and is frequently used to guide decision making in other disciplines. Here, we develop a set of Markov Decision Processes (MDPs) that model the RF spectrum management problem (in various forms). These MDPs are then queried to provide guidance for management decisions, including the combination of both admission and modulation decisions. This results in control decisions that are optimal in expectation.

To address the computational complexity inherent in computing these control decisions, we develop heuristic approaches that mimic the MDP's decisions based upon patterns observed in the MDP decision space. These heuristics are shown to closely approximate the optimal results from the the MDP.

Finally, we empirically assess the appropriateness of using Markov decision theory for RF spectrum management by comparing our MDPs to a discrete-event simulation model that relaxes several of the modeling assumptions made in the development of the MDPs.

Chapter 1

Introduction

Radio spectrum is a critical scarce resource, commanding billions of dollars [17] for wireless communication companies to enable improved user information transfer. In 1990, there were only 10 million cell phone subscribers worldwide [40], mostly using inefficient analog FM 1G (first generation) cellular RF technology, which provides very low data transfer rates (typically 2.4 kilobits/second) and assigns a channel of spectrum exclusively to each user for the duration of the call. Cell phone usage and user bandwidth demand has grown exponentially, causing new cell phones to rapidly transition to the 4G (fourth generation) standard [34], which uses digital modulation to provide approximately 1 gigabit per second (nearly 1 million times improvement over 1G performance). According to the International Telecommunications Union, the number of active cell phone accounts will soon exceed the world's population [36].

An important new wireless technology question, then, is how to provide more users with faster service, especially considering the need to improve efficiency in the face of wireless radio interference and other obstacles. A variety of emerging solutions, ranging from cognitive radios for coordinated multi-agency disaster response [49] to industrial process control [46], attempt to answer this question for individual applications. However, an effective spectrum allocation method that is able to be customized rigorously across differing features relevant to multiple application domains has not been developed. Furthermore, historically, rigid boundaries tend to divide the available radio frequency (RF) range coarsely into static non-overlapping blocks, each of which accommodates only a very limited number of users.

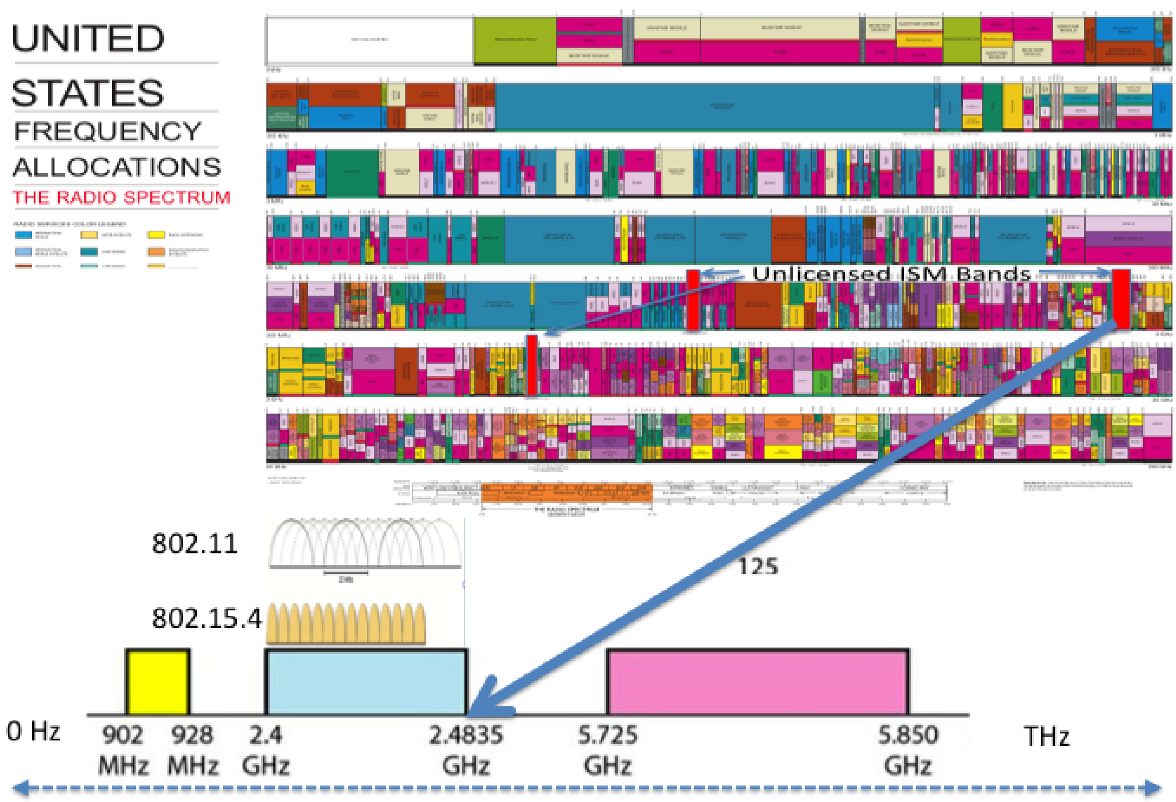


Figure 1.1: The (ISM) bands are a small fraction of US spectrum (from [35]).

In the United States, the radio spectrum is divided and controlled by the Federal Communications Commission (FCC) and the National Telecommunication and Information Administration (NTIA), which regulate available spectrum. Small sections of unlicensed shared spectrum have promoted the development of more innovative allocation methods.

To accommodate the rapid increases needed in scalability and data transfer rates, however, there is a need for allocation algorithms that can improve spectrum reuse even further. Studies suggest that RF spectrum can be used more efficiently by ignoring the arbitrary channel boundaries [49]. Our research explores efficient allocation methods that can be applied to a diverse set of applications using traditional and non-traditional spectrum regions, as depicted in Figure 1.1. Traditional spectrum regions typically support and enforce channel boundaries, which may limit efficiency, while non-traditional spectrum regions, e.g. shared spectrum, eliminate arbitrary spectrum boundaries to improve spectral reuse efficiency. Interoperability requirements often do not exist between standards regulating the spectrum, which introduces a further challenge to sharing spectrum efficiently. This lack of coordination can cause poor performance of the user devices due to interference in the small available shared unlicensed operational spectrum, as is also illustrated in Figure 1.1.

Numerous methods exist to manage traditional static licensed spectrum (e.g., cellular communications and military restricted bands) and unlicensed dynamic spectrum (e.g., ISM band). Regulatory agencies, such as the United States FCC, typically regulate the spectrum using fixed size channels (e.g., AM radio, FM radio, television channels). In traditional spectrum management, fixed channels use a single modulation technique, but they don't reuse the channel to transfer information. Enforcing the approved standard ensures consistent operation of wireless devices.

Much of the recent spectrum management innovation so far has centered upon small sections of unlicensed bands, such as those shown in Figure 1.1. Innovative dynamic allocation algorithms have the potential to significantly increase spectrum efficiency. Removing artificial channel boundaries can improve efficiency but creates interoperability challenges.

Implementation and use of more advanced collision detection and avoidance policies are needed to use improve spectrum utilization [50]. Newer modulation types, such as (direct sequence) spread spectrum (SS), simultaneously use wide-band, pseudo-noise (noise-like) signals that are hard to detect, intercept, demodulate or interfere with, when compared with narrow band modulated signals. Narrow band modulation, such as Orthogonal Frequency Division Multiplexing (OFDM), results in a much higher signal level and lower noise level than SS, exhibiting a high SNR. Spread spectrum wideband and OFDM narrowband signals can occupy the same channels, with a low probability of interference, enabling more signals to co-exist simultaneously.

Dynamic spectrum management protocols used in the ISM band, such as Zigbee (802.15.4) and WLAN (802.11), use narrowband and wideband modulated signals to minimize conflicts between users, as is also illustrated in Figure 1.1. Selecting the correct modulation combinations at the appropriate time is a key challenge for spectrum reuse.

The motivation of this research is to use the available spectrum efficiently, resulting in improved wireless information transfer rate and scalability. The challenge is that there are frequently many design and control decisions to be made that often have interacting impacts on one another, and the current approaches to making informed decisions are primarily ad hoc and empirically based. This dissertation attempts to provide a formal approach to decision making in the space of RF spectrum management, and the formalism that we investigate is the Markov Decision Process (MDP) [39]

Specifically, we develop a series of MDP models, initially based on a simple RF channel model which we call the Bernoulli model, followed up with a different RF channel model which we call the Shannon model (because it is based on Shannon capacity theory [48]). These MDPs are used to guide control decisions in RF spectrum management.

One benefit of MDP models is that they provide decision guidance that is long-term optimal (in expectation); however, this comes at high computational cost (exponential in the size of the model's state space). Following the model development, we design heuristics that have bounded execution time (i.e., $O(1)$) yet effectively mimic the design guidance provided by the formal model. These heuristics exploit patterns that regularly occur in the policies that come out of the original models.

In addition, the use of MDP models is validated by comparing with a discrete-event simulation model.

1.1 Research Questions

The central question of this dissertation is to assess the viability of using Markov decision theory in the management of radio frequency (RF) spectrum. We investigate that question by addressing the following set of hypotheses.

1. Markov decision process models can be developed for the purpose of RF spectrum management, with specific management decisions guided by the value-optimal policies determined from the MDPs. Specifically, we hypothesize that MDP models can be developed to guide both admission and modulation decisions effectively with respect to relevant throughput measures.

2. It is possible to formulate efficient and effective heuristics that mimic the value-optimal policies of the MDP models we consider. These heuristics are based on the discovery of efficiently computable boundaries between regions that are characterized by a common action.

3. There is a reasonable correspondence between the Markovian models used and the physical RF spectrum being managed. Here, we specifically hypothesize that:

   (a) throughput can be characterized simply by the mean of message durations and is relatively insensitive to their distribution;

   (b) even though imperfect channel allocations will occur in any real system, they are infrequent enough that ignoring them does not have a significant impact on an MDP model's ability to predict throughput accurately; and

   (c) value-optimal policy decisions made by the MDP are at least locally optimal as determined by a discrete-event simulation model.

1.2 Contributions and Dissertation Overview

This dissertation provides evidence in support of each of these hypotheses, as follows. The specific contributions (and their locations within the dissertation) are:

- The development of Markov decision process models to guide control decisions in RF spectrum management.

  – The design and evaluation of a basic Bernoulli MDP that guides admission decisions. This includes the specification of the state space, action set, transition system, reward function, and discount factor (Section 3.2) [32].

  – Revalidating, using the above MDP, the already known result that narrowband modulation techniques, such as OFDM, work best when each transmitter uses a unique channel. This revalidation demonstrates that the MDP can be used to confirm previously known results (Section 3.2).

  - The design and evaluation of an extended Bernoulli MDP that supports messages that occupy multiple channels (Section 3.3).
  - The development of empirical evidence that greedy allocation algorithms are sufficiently good that ideal allocations can be assumed within these MDP models (Section 5.4) [32].
  - The design and evaluation of a Shannon MDP that guides admission and modulation decisions (Section 3.4) [30].
  - The execution of a parametric empirical evaluation of the value-optimal policies that result from the Shannon MDP (Section 4.1) [30].

- The development of efficient heuristics that effectively mimic the value-optimal policies of the MDPs.

  - The discovery of regular patterns in the value-optimal policies of the MDPs. There are boundaries the separate regions of the state space that have common actions in the value-optimal policies (Sections 3.2 and 4.1) [30].
  - The characterization of these patterns in the basic Bernoulli MDP so that admission decisions can be executed via a simple threshold test (Section 3.2) [32].
  - The characterization of these patterns in the Shannon MDP so that admission and modulation decisions can be made in $O(1)$ time (Section 4.2) [30].

- The cross-validation of several models of RF spectrum performance.

  - The design and implementation of a discrete-event simulation model of message delivery over RF spectrum (Section 5.3) [31].

– The development of an $M/M/c/c$ queueing theoretic model and its comparison with the performance predictions of the discrete-event simulation model (Section 5.3) [31].

– The comparison of the Bernoulli MDP models with the discrete-event simulation model, both in terms of performance predictions and local optimality of the MDP's value-optimal policies (Section 5.4) [32].

Chapter 2 discusses related research on RF spectrum and its management, Markov decision processes and their use, and general model validation approaches. Chapter 3 introduces both our Bernoulli and Shannon MDP models, supporting admission (Bernoulli) and admission and modulation (Shannon) decisions. Chapter 4 performs a parametric empirical evaluation of the Shannon MDP, illustrates the patterns that exist in the evaluation, and exploits those patterns to construct heuristics that mimic the MDP's value-optimal policy. Chapter 5 assesses the viability of using MDP models generally in RF spectrum management, by comparing properties of the Bernoulli MDP with both a discrete-event simulation and an analytic queueing model. The results are summarized in Chapter 6 with a brief discussion of potential future work.

Chapter 2

Background and Related Work

2.1 RF Spectrum

RF communications is a rich field with a long history [37, 51, 62]. This dissertation's focus is on the management of RF spectrum to help ensure its efficient utilization. Specifically, we are interested in maximizing the effective data transmission throughput in a managed region of spectrum.

There are a number of control parameters that have substantial influence on the overall data throughput (i.e., management choices that RF system designers could potentially have at their disposal). These include (but are not limited to) the following:

- **admission decisions** – should candidate transmitters be allowed to transmit,
- **placement decisions** – what frequencies should be occupied by a transmission,
- **modulation decisions** – which modulation technique should be used,
- **power levels** – at what power level should transmissions occur,

- **coding choices** – what channel codes, error codes, etc., should be used,
- **spectrum organization** – how should the spectrum be divided into channels, and
- **message size** – how many channels should an individual message occupy.

In this dissertation, we focus on a subset of the above parameter space, concentrating on admission and modulation decisions, with a limited look at placement decisions.

There are, of course, many other factors that also influence the data throughput, which are not under the control of a system designer. These include environmental noise, fading, multipath interference, offered load, etc. This work assumes that the spectrum manager has no control over these factors, but does have knowledge of their extent, which will be quantified using commonly utilized aggregations (articulated in Section 3.1).

While the admissions decision itself is fairly straightforward to understand in simple terms (i.e., when a transmitter wishes to send a message, the management function is to decide whether or not to allow that message to be sent), there has been substantial prior work in how to make this decision effectively. For example, Fu et al. [12] describe a mechanism for re-using channels in a cellular system that allows for greater capacity (i.e., more admissions).

There are a multitude of modulation options available today, including amplitude modulation (AM), frequency modulation (FM), phase shift keying (PSK), pulse-position modulation (PPM), orthogonal frequency division multiplexing (OFDM), frequency hopping spread spectrum (FH-SS), direct-sequence spread spectrum (DS-SS), and quadrature amplitude modulation (QAM) [62]. We next describe the two modulation options we consider in this work, and defer consideration of others to future work.

Many RF systems have predefined modulation selected based on the type of service (e.g., video, text, voice), desired quality, available bandwidth, and other factors. Two common modulation types used for voice and data are orthogonal frequency division multiplexing (OFDM) and direct-sequence spread spectrum (SS). OFDM and SS offer two distinctly different modulation types that have different system characteristics, which is helpful to show how such difference may impact modeling and policy generation issues explored in this dissertation.

OFDM modulation is characterized by concentrating the RF signal power within a single channel of some fixed bandwidth. The signal power of a real OFDM transmitter does not evenly fill the channel with power (i.e. a peak signal is located in the center of the channel with exponential decay of the power beyond the channel boundary) as shown in Figure 2.1. OFDM modulators are simpler to construct (relative to SS) but the resulting system has less tolerance of interfering signals. OFDM is relatively robust against multipath fading and inter-symbol interference. In this work, we assume there is perfect channel independence (i.e., we do not model the interference due to imperfect channel separation).

SS modulation distributes the signal power across multiple channels. This implies a proportionally lower signal strength in each individual channel. In this work, we will focus exclusively on direct-sequence SS modulation, which uses pseudo-noise codes to phase shift the signal as the spreading mechanism. We make the simplifying assumption that the SS modulation distributes the signal power across the entire region of spectrum being managed, and defer consideration of SS modulation over smaller sub-regions of spectrum to future work. Figure 2.2 illustrates the signal power distribution for a 4 channel example spectrum using SS modulation. Note that the area under the curve (which represents total signal power) is comparable to that of the OFDM example shown in Figure 2.1.

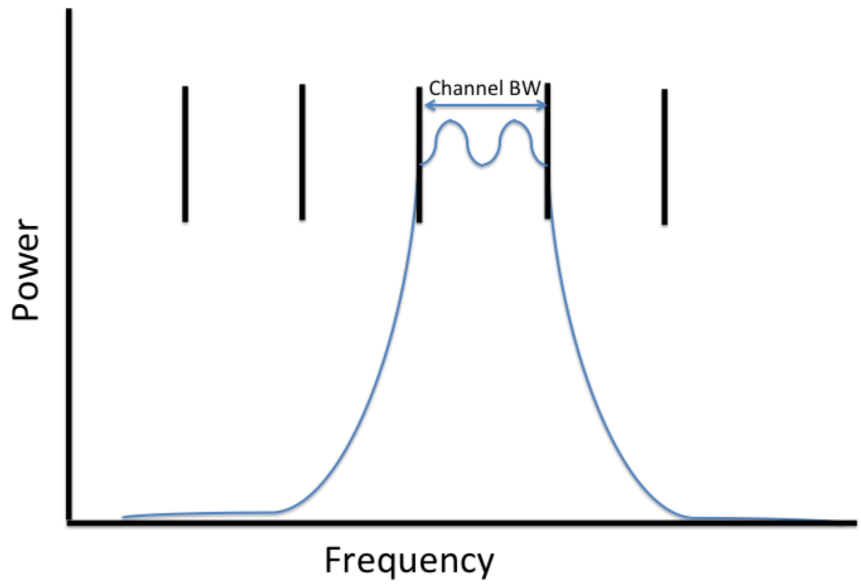


Figure 2.1: OFDM spectrum shape.

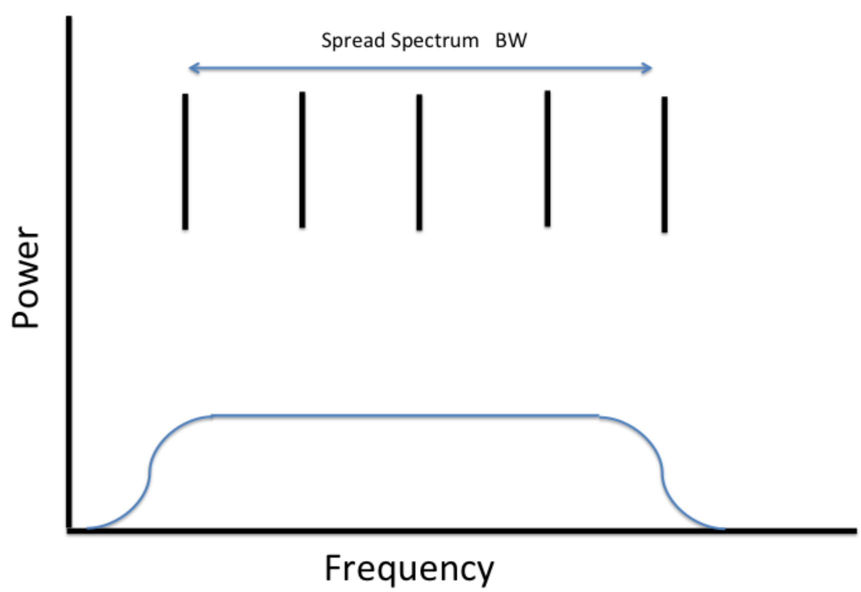


Figure 2.2: SS spectrum shape.

2.2 Markov Decision Process Models

The following exposition is derived from Tidwell [56], which applies a Markov Decision Process (MDP) model to processor allocation problems[1]. The five-tuple $(\mathcal{X}, \mathcal{A}, T, R, \gamma)$ describes a discrete-time MDP. The *states* are designated as $\chi \in \mathcal{X}$, and *actions* are designated as $a \in \mathcal{A}$. The transition system, T, gives the probability $P_T(\chi' \mid \chi, a)$ of transitioning from state χ to state χ' on action a. The reward function $R(\chi, a, \chi') \in \mathbb{R}_{\geq 0}$ describes the reward that accrues when transitioning from state χ to state χ' via action a. The discount factor, γ, provides a means to ensure the convergence of the long term reward, and is a value greater than 0 but less than or equal to 1, denoted as [0, 1). This discount factor defines how potential future rewards are weighed against immediate rewards when evaluating the impact of taking a given action in a given state.

A policy, π, maps states in $\mathcal{X}$ to actions in $\mathcal{A}$. At each discrete decision epoch k the agent observes the state of the MDP χ_k, then selects an action $a_k = \pi(\chi_k)$. The MDP then transitions to state χ_{k+1} with probability $P_T(\chi_{k+1} \mid \chi_k, a_k)$ and yields immediate reward $r_k = R(\chi_k, a_k, \chi_{k+1})$.

Given discount factor γ, the value of a policy, denoted by V^π, is the expected sum of long-term, discounted rewards obtained while following that policy,

$$V^\pi(\chi) = E\left\{\sum_{k=0}^{\infty} \gamma^k r_k \mid \chi_0 = \chi, a_k = \pi(\chi_k)\right\}. \tag{2.1}$$

[1] Despite the differing resources (RF spectrum vs. processor cycles) and semantic models (allocation vs. time-utility scheduling) involved, our formulation of the MDPs in this work is similarly motivated by the challenge of optimal resource use considered in that work.

R^π denotes the expected reward obtained when executing action $a = \pi(\chi)$ in state χ.

$$R^\pi(\chi) = \sum_{x \in \mathcal{X}} P_T\left(x \mid \chi, \pi(\chi)\right) V^\pi(x) \tag{2.2}$$

Then we may equivalently define V^π as the solution to the linear system

$$V^\pi(\chi) = R^\pi(\chi) + \gamma \sum_{x \in \mathcal{X}} P_T\left(x \mid \chi, \pi(\chi)\right) V^\pi(x) \tag{2.3}$$

for each state χ. When $|R^\pi(\chi)|$ is bounded for all states, the discount factor γ prevents V^π from diverging for any choice of policy, and can be interpreted as the prior probability that the system persists from one decision epoch to the next [22]. In practice this value is almost always set very close to 1 and in this work we set γ to 0.99 (following established convention [39, 57]).

There are several techniques for computing the value-optimal policy for an MDP with finite state and action spaces [39]. These techniques calculate the optimal action, $\pi^*(\chi)$, for every state $\chi \in \mathcal{X}$:

$$\pi^*(\chi) = \operatorname*{arg\,max}_{a \in \mathcal{A}} \left\{ R(\chi, a) + \gamma \sum_{x \in \mathcal{X}} P_T\left(x \mid \chi, a\right) V^*(x) \right\} \tag{2.4}$$

where the optimal value, $V^*(\chi)$, is given by:

$$V^*(\chi) = \max_{a \in \mathcal{A}} \left\{ R(\chi, a) + \gamma \sum_{x \in \mathcal{X}} P_T\left(x \mid \chi, a\right) V^*(x) \right\}. \tag{2.5}$$

The value-optimal policy is the policy that optimizes long term value within the MDP, in contrast to immediate reward.

2.3 Uses of Markov Decision Process Models

Markov decision processes have been used extensively to optimize control of systems [1, 47], particularly those that include stochastic elements [7]. Here, we investigate the viability of using an MDP to optimize decisions in the context of managing RF spectrum. These decisions might be admission decisions, channel allocation decisions, modulation decisions, transmitter power level decisions, or any number of other choices that are germane to managing the shared use of the spectrum.

Our work closely follows the approaches of Glaubius et al. [14, 15, 16] and Tidwell et al. [56, 57], which used MDP theory to inform resource scheduling decisions in a real time embedded systems context, where the duration of resource allocations is stochastic. The work of Glaubius focused on proportional sharing of a single discretely time-sliced resource. The work of Tidwell considered arbitrary utility functions in the valuation of individual scheduling decisions. In these works, the authors identified boundaries in the MDP state space that separate the value-optimal actions, which then led to efficient heuristics that closely approximate the value-optimal policies.

Our work follows a similar path, with some important distinctions. First, both Glaubius and Tidwell were working in the domain of real-time scheduling. Our domain is RF spectrum management. By describing a family of MDP models, we demonstrate the applicability of MDP theory over a range of applications in spectrum management. This is further supported by the use of more than one reward function within the family of models we consider.

Second, their initially specified state space was infinite, and (using various techniques that they introduced) they formulated bounded versions that were demonstrably equivalent to the infinite spaces. In our case, the initial state space is constructed in such a way that it naturally has bounded extent. This implies that there are edge case considerations in our MDPs that must be explicitly handled, which was not the case in the previous work.

Third, the applicability of the underlying MDP models to the problem domain was not a point of investigation by Glaubius or Tidwell. In our domain, we use a discrete-event simulation model to assess the appropriateness of the family of MDP models to spectrum management.

This dissertation is not the first to suggest using MDPs to guide decision making in RF systems. Zhao et al. [61] propose the use of MDPs for guiding what they call opportunistic spectrum access (the ability of secondary users to identify and exploit instantaneous spectrum use opportunities that arise because of the bursty traffic patterns of primary users). Tradeoffs between optimality and complexity in such cases are examined by Djonin et al. [11]. Akbar and Tranter [2] use hidden Markov models (HMMs) to model and predict the spectrum occupancy of licensed radio bands for this same purpose.

Markovian models have been used to characterize other properties of RF systems as well. Wang et al. [59] use a Markov transition system to characterize different handoff delays associated with connections in cognitive radio networks. Geirhofer et al. [13] propose a continuous-time semi-Markov model of a WLAN's behavior, towards a better understanding of primary users' activities.

2.4 Validation of Models

In Chapter 5 we assess the applicability of Markovian models to managing wireless radio spectrum. Perhaps the most relevant results in model applicability come from the domain of finite-element modeling [54, 55], where explicit error estimation is used to select between p-methods and h-methods for analysis. In that work, a quantitative estimate of model error is used to assess whether or not a model is appropriate for a given task.

When to use (or not use) a performance model is a subject that is covered in many performance modeling texts (see [21, 26, 27] for but a few examples). Unfortunately, the methods described in these texts are often quite labor intensive, typically requiring measurement of the system being modeled for empirical validation. Sargent has written extensively on the subject, with a focus on simulation models rather than analytic models, from origins in the 1970s [42] to a recent comprehensive review [43].

The areas of model selection and validation also have been extensively studied. Model selection literature is rooted in multiple fields from operations research [60] where the focus is typically on relating a model to a particular physical process, to machine learning literature [10] where the focus is often on selecting the best predictive model.

Krishnamurthy and Chamberlain [25] directly addressed the question of when their proposed models for bounded queueing systems were applicable, by proposing a pair of explicit tests. The first was derived from a slightly relaxed set of assumptions and the second was empirically based. If either test failed, the model was considered to be unreliable. Beard and Chamberlain [6] have investigated the use of flow models combined with queueing models to show that while such models can be quite effective at throughput prediction, they are prone to significant error in predicting queue occupancy.

More commonly, assessment is an empirical exercise, in which the model in question (or more precisely, a set of predictions made by the model) is compared either with measurements of the physical system being modeled (e.g., see [6]) or with a (presumably) more robust model. For example, a frequent practice is to utilize simulation models to assess analytic models [28], as we do in this work. Such an approach makes the most sense when: (1) the simulation model explicitly incorporates aspects of the physical system being modeled that are either simplified or completely ignored in the analytic model, and (2) when the simulation model (or other reference model) has been independently evaluated, as are both the case in this work. To assess the applicability of Markovian models to the problem of managing RF spectrum, we compare model predictions to a discrete-event simulation.

Chapter 3

Markov Decision Process Models for RF Spectrum Management

The central goal of this dissertation is to assess the viability of using Markov decision theory for context aware configuration of parameters affecting radio frequency (RF) spectrum allocation, and in doing so to improve throughput across a range of relevant operating conditions. We investigate that goal in this chapter by addressing the following hypothesis.

> Markov Decision Process (MDP) models can be developed for the purpose of selecting and evaluating different combinations of factors affecting RF spectrum management, with specific management decisions guided by the value-optimal policies determined from the specific MDP in use. Specifically, we hypothesize that MDP models, whose parameters encode the different factors, can be developed to guide both admission and modulation decisions effectively with respect to relevant throughput measures.

We develop models of radio frequency (RF) spectrum semantics that are intended to capture throughput, environmental interference, channel interference, message duration, and other relevant factors. We start with a description of the RF spectrum system model that we will use throughout this work. We refer to this as the *physical model*; it is intended to capture the essential aspects of the RF spectrum that we will consider for the management techniques developed in the dissertation. We then present three distinct *MDP models* that capture different characteristics of the physical model, handle different sets of input parameters, and have different spectrum management goals.

The MDPs we develop in this dissertation are intended to represent faithfully the semantics of the physical model described next. We investigate this relationship further in Chapter 5, where the basic Bernoulli MDP developed here is cross-validated using a separately developed discrete-event simulation model.

3.1 RF Spectrum Physical Model

In our physical model, the RF medium is a range of radio wave frequencies (e.g., from F1 to Fn), divided into some number of channels (illustrated in Figure 3.1), and a centralized manager that makes decisions about the use of those channels. With C channels, each channel has bandwidth (Fn−F1)/C. The channels are rigid, non-overlapping regions of the RF spectrum for which a centralized manager makes usage decisions.

Messages arrive from transmitters via a Poisson process (utilizing a separate *control channel*), are allocated to a channel if admitted, and depart the system if not admitted (i.e., we do not model retries). The manager is responsible for making these admission decisions, which are delivered to the appropriate transmitters (again via a separate control channel that is not explicitly modeled).

We assume the existence of a centralized spectrum resource manager to make decisions that effectively control the use of the media. For example, when the resource manager assigns a

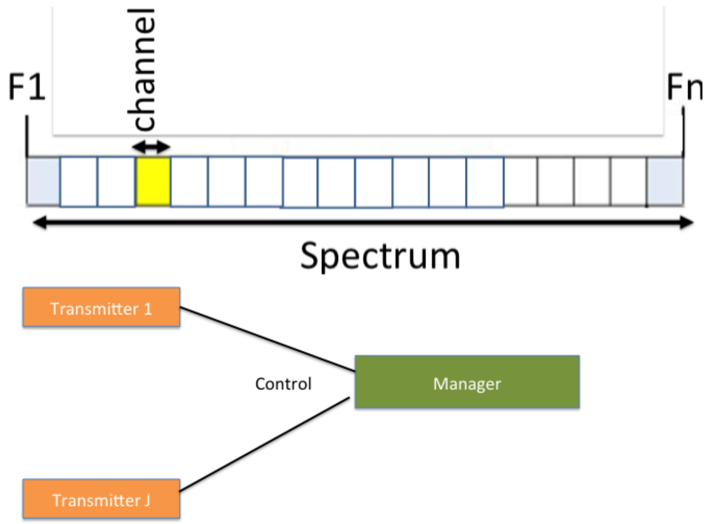


Figure 3.1: RF spectrum physical model, divided into fixed width channels. Individual transmitters communicate with the centralized spectrum manager for permission to transmit (admission) as well as channel assignment (allocation).

message to a particular channel, the transmitter is then required to transmit exclusively using that single channel. For the purposes of this dissertation, we limit such decision-making to admission, allocation (i.e., selection of a particular channel, including whether or not messages are allowed to overlap), and modulation type. Although the approach developed here could be extended to a de-centralized control scheme, the design issues associated with constructing such distributed management approaches are beyond the scope of this dissertation. Furthermore, having a centralized resource manager allows it to utilize global information during decision making.

We denote the mean message arrival rate by λ, and message durations are assumed to be uniformly distributed with mean $1/\mu$ (following the convention in queuing theory that μ is a *service rate*). The total rate of departure at any specific time, therefore, is proportional to the number of messages in the system. Multiple messages may be allocated into one channel (i.e., they can overlap); in that case, the probability of successful message delivery is a function of the RF channel model, which we describe next. In what follows, we will use a pair of different channel models, the first referred to as the *Bermoulli* model and the second referred to as the *Shannon* model. Neither is a new proposition, and each has its foundations in the RF communication literature [37, 51, 62].

The Bernoulli channel model characterizes the probability of success (or failure) of message delivery in terms of environmental factors and conflicts due to common channel occupancy [19]. Unlike more sophisticated models that attempt to account individually for distinct interference mechanisms [12, 29, 41], the environmental factors (e.g., background noise, reflections, etc.) are aggregated into a single term, denoted P_{env}, representing the probability of a message delivery failure due to these factors. Message conflict is similarly characterized by a single term, denoted P_{conf}, which parametrizes a Bernoulli model of message failure. The probability that an individual message is successfully delivered, denoted P_{succ}, is therefore

$$P_{succ} = (1 - P_{env})(1 - P_{conf})^{(N_m - 1)} \tag{3.1}$$

where N_m is the number of messages sharing the channel.

The Shannon channel model characterizes the throughput achievable on an individual channel via the classic Shannon capacity [48]. Shannon's Theorem provides a measure of the channel capacity as a function of the available bandwidth and the signal-to-noise ratio

$$C_s = \gamma_e B_C \log_2(1 + S/N), \tag{3.2}$$

where C_s is the achievable channel capacity (in bits/s), γ_e is the modulation efficiency, B_C is the channel bandwidth, S is the average signal power (at the receiver), and N is the average noise power. When multiple modulation types are used, we assume that the "signal" power for an alternate message overlapping in the same spectrum is perceived by the receiver as noise when processing the message of interest.

In both channel models, we assume that transmitters have been sufficiently power-controlled such that the receiver signal strength is common.[2] Note that the basic measures provided by the Bernoulli model and the Shannon model are different from one another, and thus the reward function of any MDP model based on either a Bernoulli or Shannon channel model will need to account for this distinction.

3.2 Basic MDP Model with a Bernoulli Reward Function

We illustrate the development of MDP models by starting with an initial basic MDP that is designed to make admission decisions based on the Bernoulli channel model. We will refer to this as the *basic Bernoulli* MDP model. The goal of the basic Bernoulli MDP model is to set *admission* policy, essentially deciding whether an individual channel is to be allocated (or not allocated) to a newly arriving message (i.e., allowing the transmitter to send the message or not).

This MDP does not make decisions about where to allocate each message. Instead, it assumes the existence of an omniscient (i.e., best possible) allocator that achieves either one channel per message or minimizes message overlap. This is trivially realizable in the circumstances where the number of admitted messages is less than the number of channels available in the spectrum. When the number of messages is greater than the number of channels, we assume that the quantity of messages overlapping one another (i.e., allocated to the same channel)

[2] Our model does not account for message transmission overhead (routing information, and error coding).

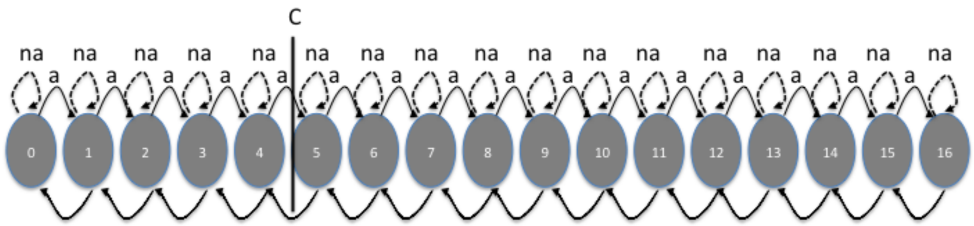


Figure 3.2: State transition diagram for basic Bernoulli MDP.

can be minimized. This latter assumption is tested (by comparing with a greedy algorithm) in Chapter 5.

The success or failure of a message delivery is characterized by the Bernoulli channel model, which incorporates environmental effects (via P_{env}) and message conflicts (i.e., message overlap, via P_{conf}). In terms of the effects of such an admission policy, admitting more messages has the potential to yield higher throughput. However, if the policy admits sufficient messages such that message overlap will occur (i.e., more messages are admitted than there are channels available), the channel model reflects this by a diminished probability that either of the messages occupying the same channel will succeed: this in turn diminishes throughput. By designing a reward function that reflects this tradeoff, we can ask the MDP to provide a value-optimal policy that can then be used to make on-line admissions decisions.

3.2.1 States, Actions, and Transitions for the Basic Bernoulli MDP

The basic Bernoulli MDP's state transition system is illustrated in Figure 3.2. Each state $\chi = i$ encodes the number of messages occupying a channel of spectrum (i.e., currently being transmitted). This MDP model assumes a perfect allocation of messages to channels, so that if there are C channels and $\chi \leq C$, each message is assumed to be allocated to a distinct channel. If $\chi > C$, we assume the number of conflicts (i.e., with other messages transmitting on the same channel) experienced by any one message is the minimum possible.

For admission control, the two possible actions are to accept or not accept. One of these actions is taken whenever a message arrives in the system. In Figure 3.2, actions to accept are indicated by edges labeled 'a' in the transition from a state i to the neighboring state $i+1$. Similarly, actions to not accept are indicated by edges labeled 'na' and are self-loops (i.e., they transition back into the same state rather than to a new state).

Since the Poisson arrival process has mean rate λ the transition rates for the edges labeled 'a' and 'na' are both λ. The basic Bernoulli MDP treats the set of messages as a traditional birth-death process (of the type described by Kleinrock [23]), so the departure rate is determined by the mean service rate to be $\chi\mu$.

While this state space could, in principle, be infinite in extent, we artificially bound it to some maximum size under the expectation that above some number of messages it is unreasonable for further admissions to be beneficial in terms of increased throughput.

The above described (continuous-time) model is converted into a discrete-time MDP by adding self-loops and converting the transition rates to transition probabilities using the uniformization technique described by Grassmann [18] with uniform rate parameter δ which is set to be greater than the largest rate in the continuous-time model.[3] As a result, the probability of an arrival is λ/δ (for an action to accept) and the probability of a departure is $\chi\mu/\delta$. The probability of a self-loop is $1-(\lambda+\chi\mu)/\delta$ (for an action to accept) and $1-\chi\mu/\delta$ (for an action to not accept). Value (determined according to the reward function described in the next section) is accrued on departure transitions.

After uniformization, the resulting discrete-time MDP is illustrated in Figures 3.3 and 3.4. The basic Bernoulli MDP is then a four-tuple $(\mathcal{X}, \mathcal{A}, T, R, \gamma)$ that consists of a collection of

[3]In the literature the uniform rate parameter is frequently represented by γ, but we reserve that symbol for use as part of the MDP definition.

states $\mathcal{X}$, actions $\mathcal{A} = \{\mathrm{a}, \mathrm{na}\}$, a transition system T, a reward function R that specifies the expected benefit of each action in each state, and a discount factor γ. Figure 3.3 shows the transition table, where $P_\lambda = \lambda/\delta$ and $P_\mu = \chi\mu/\delta$. The leftmost column indicates the starting state and action (one per row), while the remaining columns are marked on the top row by the destination state. The entries in the table represent the probability of transitioning from the starting state to the destination state for each action.

	0	1	2	i-1	i	i+1	n-2	n-1	n
0 a ηa	$(1-P_\lambda)$ 1.0	P_λ 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
1 a ηa	P_μ P_μ	$1-P_\mu-P_\lambda$ $1-P_\mu$	P_λ 0	0 0	0 0	0 0	0 0	0 0	0 0
2 a ηa	0 0	P_μ P_μ	$1-P_\mu-P_\lambda$ $1-P_\mu$	0 0	0 0	0 0	0 0	0 0	0 0
i a ηa	0 0	0 0	0 0	P_μ P_μ	P_μ P_μ	$1-P_\mu-P_\lambda$ $1-P_\mu$	P_λ 0	0 0	0 0
n a ηa	0 0	0 0	0 0	0 0	0 0	0 0	0 0	P_μ P_μ	0 $1-P_\mu$

Figure 3.3: Basic Bernoulli MDP state transition table.

Figure 3.4 represents the same information in graphical form, with nodes representing states and action-labeled edges representing transitions. The state diagram provides a visual indication of the transition probability structure of the MDP. In both the state transition table and diagram, we assume that there is an upper bound of n concurrent messages possible, and indicate the general interior state with the letter i. Putting these figures into the MDP notation presented in Chapter 2, for state χ, action $a_\chi = \pi(\chi)$. The MDP then transitions to state χ' with probability $P_T(\chi' \mid \chi, a_\chi)$.

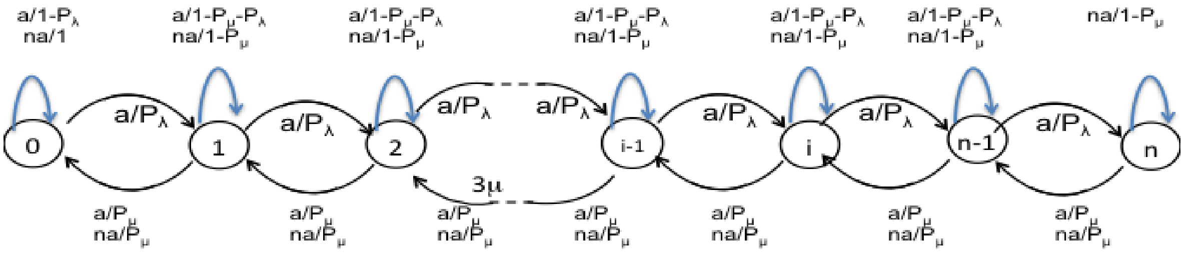


Figure 3.4: Basic Bernoulli MDP state transition diagram.

3.2.2 The Basic Bernoulli Model Reward Function

A policy will be generated by an MDP that maps states in $\mathcal{X}$ to actions in $\mathcal{A}$. At each discrete decision epoch, the policy observes the state of the MDP $\chi \in \mathcal{X}$, then selects an action $a_\chi \in \mathcal{A}$. The MDP then transitions to state χ' with probability $P_T(\chi'|\chi, a_\chi)$ and the controller receives reward $r = R(\chi, a_\chi, \chi')$.

The reward function R is defined over the domain of state-action-state tuples, such that $R(\chi, a, \chi')$ is the immediate reward for taking action a_χ in state χ and ending up in state χ'. For the basic Bernoulli MDP, reward is accrued when messages depart the system (i.e., transition edges moving from state χ to $\chi - 1$). The amount of reward is equal to the the expected duration of the message time multiplied by the probability that the message is successfully delivered over the channel, P_{succ} (see eqn. (3.1)). For all other transitions (arrivals or self-loops), the reward is zero.

Our approach is based on solving for a policy that is optimal in expectation of accrued long-term reward according to the specified reward function. We used software originally developed by Glaubius et al. [14] to calculate a value-optimal policy for each of the MDPs investigated in the dissertation, by simply via encoding the specifics of the new MDPs in that framework.

Table 3.1: Value-optimal policy for $P_{conf} = 0.97$.

State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Policy	a	a	a	a	na	na	na	na	na	na	na	na	na	na	na	na	na

Informally, we expect value-optimal admission decisions made by this MDP to result in an admission policy of accepting incoming messages up to some system occupancy (i.e., number of messages in the system) and then not accepting new messages at any higher occupancy, with the acceptance threshold being influenced in part by the value of P_{conf}, which parametrizes the penalty of sharing individual channels. This policy is frequently used in practice for existing real-world spectrum allocation, e.g., when using FM modulation.

This intuition is confirmed by ramping the value of P_{conf} down from 1 and looking for the above pattern when solving for the value-optimal policy using the MDP. In a 4-channel system ($C = 4$, $n = 16$, $P_{env} = 1$) the pattern is evident at a P_{conf} of 0.97. This value-optimal policy is shown in Table 3.1. At this high value of P_{conf} (i.e., high probability of message delivery failure due to conflict), sharing of channels is unlikely to benefit throughput, and the policy that is chosen via the MDP is to accept up to 4 messages, but no more. This result is resilient to variation in offered load.

We continue this investigation, ramping down to $P_{conf} = 0.7$ in the same 4-channel system and again solve for the value-optimal policy (shown in Table 3.2). In this case, the resulting policy is to accept up to 8 messages, but no more. This time each channel is potentially shared by up to 2 messages. This pair of experiments helps give us confidence that the MDP is making reasonable admission decisions.

Table 3.2: Value-optimal policy for $P_{conf} = 0.7$.

State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Policy	a	a	a	a	a	a	a	a	na	na	na	na	na	na	na	na	na

Given the value-optimal policies generated by the MDP, it is reasonable to consider the development of an on-line run-time admission heuristic that mimics the actions of the value-optimal policy. Such a heuristic can be expressed as follows:

$$a = \begin{cases} \text{`a'} & \text{if } \chi < Th \\ \text{`na'} & \text{otherwise} \end{cases} \tag{3.3}$$

where $a \in \mathcal{A}$ is the action chosen, $\chi \in \mathcal{X}$ is the number of messages currently in the system, and Th is a fixed threshold. Effectively, the value of the threshold, Th, identifies a decision boundary, in which the value-optimal action is different above and below the boundary. We can therefore use the MDP off-line to choose the threshold value, Th, and then use the heuristic on-line to make the individual admission decisions for each message. Although the threshold differs between the two experiments (i.e., it is dependent upon the value of P_{conf}), its presence in both illustrates the potential for exploiting such decision boundaries, which we develop further in Chapter 4.

In this case, with an appropriate choice of threshold, the on-line heuristic actually mimics the value-optimal policy indicated by the MDP precisely. In Chapter 4, we will return to this approach where an on-line heuristic might not provide a perfect match to the value-optimal policy, but does closely mimic it. In both cases, the on-line decisions take polynomial time, and are therefore considerably more computationally efficient than repeated solving of the MDP for value-optimality.

3.3 Extending the Basic Bernoulli MDP to Support Multiple Message Sizes

The basic Bernoulli MDP introduced above is but one example of a whole family of potential MDP models. Here, we extend the basic MDP to support messages that require more than one channel. This is a common technique in RF systems to enable faster delivery of a message, taking advantage of the resulting higher bit rate available when more channels are used [62].

In this extended version of our basic Bernoulli MDP model, we will assume the size of the message (i.e., the number of channels that it occupies) is requested by the transmitter, rather than being decided by the centralized manager. In effect, the manager is still only performing admission decisions. The distinction is that the incoming messages might require more than one channel. Without loss of generality (i.e., by rounding the number of channels needed to the next binary exponent), in the extended MDP model described below the number of channels supported is restricted to be a power of 2.

Figure 3.5 illustrates the state space of an extended Bernoulli MDP model that supports messages of size 1 (i.e., 1 channel) and 2 (i.e., 2 channels). The model encodes the message size in the dimensionality of the state space. Specifically, each state is labeled with an ordered pair, (x_2, x_1), where the value of x_1 increases horizontally and x_2 increases vertically. The value of x_1 represents the number of messages of size 1 (requiring one channel) and the value of x_2 represents the number of messages of size 2 (requiring 2 channels). For example, the state $(1, 1)$ corresponds to the circumstance where there are 2 messages currently occupying RF spectrum, with one of the messages consuming one channel and the other message consuming two channels.

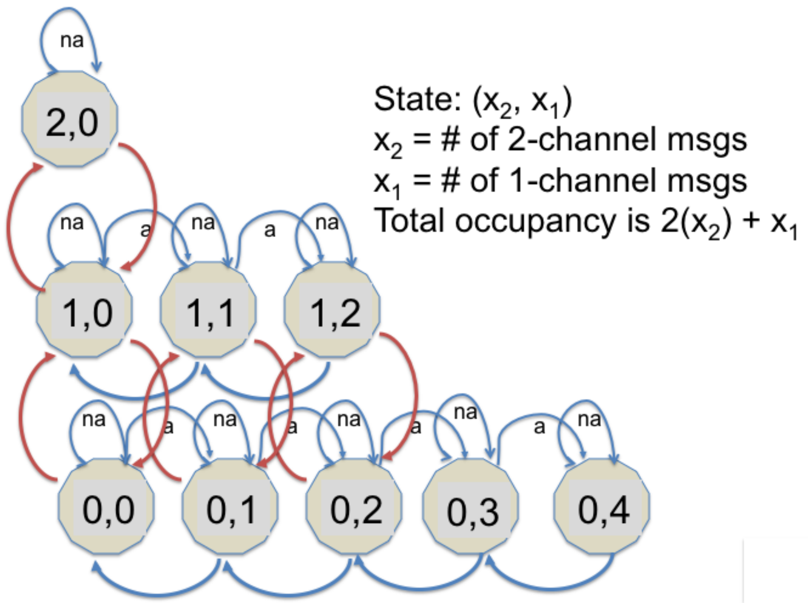


Figure 3.5: Extended Bernoulli MDP state space diagram, supporting message sizes of 1 or 2 channels.

As with the basic Bernoulli MDP, actions are still to accept ('a') or to not accept ('na'), which are shown on the figure only in the horizontal direction. The maximum number of simultaneous messages is bounded by the extent of the MDP state space in each dimension. For the figure, up to 4 single channel messages can be considered and up to 2 dual channel messages can be considered. Given a total channel count of C, the reward function reflects the same Bernoulli channel model as above: i.e., allocation is assumed to be ideal (i.e., there are no conflicts) when the number of occupied channels is less than or equal to the number of channels ($2x_2 + x_1 \leq C$), and the number of conflicts experienced by any one message is minimized when $2x_2 + x_1 > C$.

This MDP can be extended to additional dimensions (e.g., 3 dimensions to consider message sizes of 1, 2, and 4) as illustrated in Figure 3.6. In this case, states are labeled via the 3-tuple (x_3, x_2, x_1), where x_1 encodes the number of messages using 1 channel, x_2 encodes the number of messages using 2 channels, and x_3 encodes the number of messages using 4 channels. Actions remain accept ('a') and no accept ('na'). The total number of messages in the system is indicated by $4x_3 + 2x_2 + x_1$. The number of channels is still encoded by C, and the same Bernoulli reward function is simply extended to the new state encoding (i.e., reflecting whether or not there is overlap). In this example, the number of channels, C, is 4 since the state space supports a message that consumes 4 channels, yet the maximum occupancy for single channel messages is also 4 (i.e., $x_1 \leq 4$).

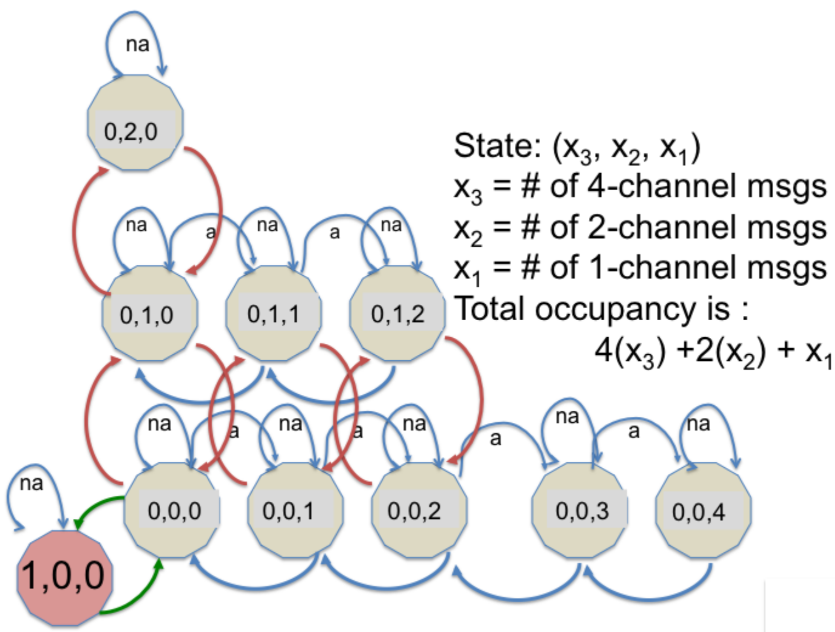


Figure 3.6: Extended Bernoulli MDP state space diagram, supporting message sizes of of 1, 2, and 4 channel widths.

3.4 Basic MDP Model with a Shannon Reward Function

In this second family of MDP models, we will revert to messages of a common size (one channel), but expand the function of the MDP to include new responsibilities. Here, we will ask the MDP to guide the management of both admission decisions and also modulation decisions. The two types of modulation that we will consider are Orthogonal Frequency Division Multiplexing (OFDM), in which each message consumes only a single channel, and Spread Spectrum (SS), in which each message's transmission is spread across the entire RF spectrum under consideration.

We will also alter the reward function to represent more faithfully the presence of more than one modulation type in common channels of spectrum. This new reward function is based on classic Shannon capacity theory.

3.4.1 States, Actions, and Transitions for the Basic Shannon MDP

The state space diagram for this new MDP is shown in Figure 3.7. States $(y_2, y_1) \in \mathcal{X}$ represent the number of transmitters using each modulation type. Here, y_2 is the number of SS transmitters and y_1 is the number of OFDM transmitters. In this case, the actions include accept SS, '`as`', accept OFDM, '`af`', and no accept, '`na`'; and therefore $\mathcal{A} = \{\mathtt{as}, \mathtt{af}, \mathtt{na}\}$. As in the earlier MDP, this continuous-time model is converted to a discrete-time MDP using the uniformization technique described by Grassmann [18]. In the Bernoulli MDPs, value was accrued only on message departure. Here, value is accrued within each state as specified by the Shannon reward function described below.

3.4.2 Shannon Reward Function Design

The reward function for this family of MDPs is based on the Shannon channel model described in Section 3.1. Given that our high-level goal is to maximize data throughput, we will

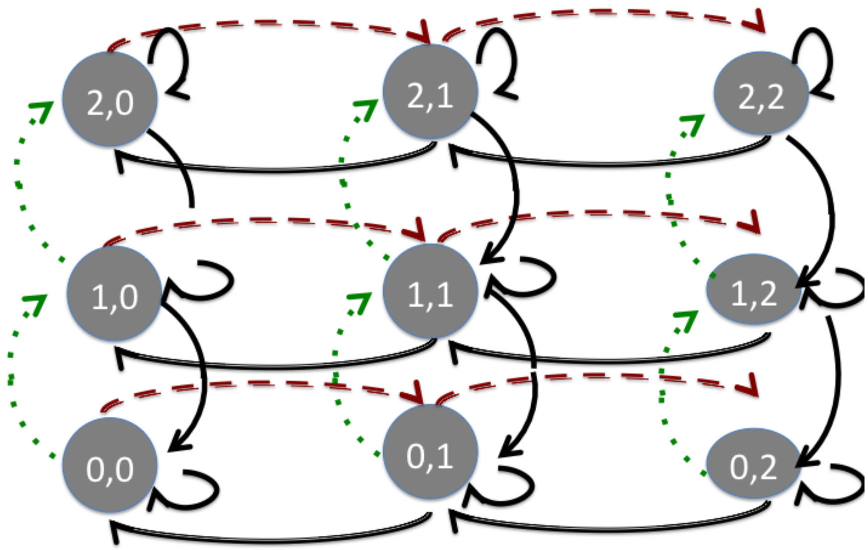


Figure 3.7: State space diagram for multiple modulation types. Number of SS transmitters is shown vertically and number of OFDM transmitters is shown horizontally. Self loops represent `na` actions, vertical rising edges represent `as` actions, horizontal edges to the right represent `af` actions, vertical falling edges and horizontal edges to the left represent message departures (all actions have the same effect).

ask the MDP to optimize a reward function that reflects data throughput. Equation (3.4) below is a restatement of equation (3.2) in Section 3.1.

$$C_s = \gamma_e B_C \log_2(1 + S/N), \tag{3.4}$$

Recall that C_s is the achievable channel capacity (in bits/s), γ_e is the modulation efficiency, B_C is the channel bandwidth, S is the average signal power (at the receiver), and N is the average noise power.

In what follows, we will explore distinct values of modulation efficiency, γ_e, for each modulation type: $\gamma_{e.SS}$ and $\gamma_{e.OFDM}$. Channel bandwidth, B_C, is assumed to be fixed (and given). Signal power, S, is for a single transmitter that has been admitted to the spectrum by the central manager. If using OFDM modulation, this signal is limited to a single channel. When using SS modulation, this signal is spread across all channels. We assume that sufficient power control management is in place so that the signal power for each transmitter (for either modulation technique) is the same at the receiver (i.e., transmitters at a farther distance have a greater transmit power). The noise power is a combination of background (environmental) noise and the interference noise generated by other transmitters that are using common spectrum. This implicitly makes the assumptions that (1) OFDM transmitters do not share channels, (2) SS transmitters all use distinct codes so that they appear as pseudo-random noise to each other and to OFDM transmitters, (3) the interference noise power per transmitter is effectively power controlled in the same way as the intended signal, and (4) noise power is uniformly distributed across each channel.

For each admitted transmitter, equation (3.4) provides the capacity for that individual transmitter. The reward is computed as the number of transmitters accepted to transmit using OFDM modulation multiplied by the associated capacity within each channel and the number of spread spectrum transmissions multiplied by their individual capacity. This can be expressed as follows,

$$R = y_2 \cdot C_{SS} + y_1 \cdot C_{OFDM} \tag{3.5}$$

where $C_{SS} = C_s\delta$ for spread spectrum transmitters, $C_{OFDM} = C_s\delta$ for OFDM transmitters, and δ is the uniform rate factor, which changes the units of channel capacity, C_s, from bits/s to simply bits (i.e., δ is expressed as time).

We illustrate this reward function via an example, a 2 channel spectrum allocation/modulation problem with a maximum of four transmitters capable of either OFDM or SS modulation. Here, the signal is assumed to perfectly fill the spectrum (i.e. modulation efficiency, γ_e, is assumed to be unity). There are nine total states. We examine each state in turn and formulate an expression for the reward. We do this by first articulating values for C_{SS} and C_{OFDM} in terms of the received signal from the transmitter of interest, S, and the environmental noise, N. When interfering transmitters are included in the expression, the "noise" due to these transmitters is accounted for by additional instances of S.

The states are configured into a matrix, as shown in Figure 3.8, with the horizontal dimension, y_1, representing the OFDM modulated signals and the vertical dimension, y_2, representing the SS modulated signals. Starting with state $(0, 0)$, we progress horizontally across the bottom row examining first the OFDM modulated channels (no SS modulation), then vertically up the left-most column allocating SS modulated channels only (no OFDM modulation), and finally examining the remaining states.

State $(0, 0)$ indicates all transmitters are off and therefore the reward is zero. Here, both $C_{SS} = 0$ and $C_{OFDM} = 0$.

State $(0, 1)$ represents the circumstance where one OFDM transmitter is on. The reward is given by $C_{SS} = 0$ and $C_{OFDM} = B_C \log_2((S/N) + 1)$ understanding that there is only one transmitter enabled using OFDM modulation (i.e., one transmitter's worth of signal, S, only environmental noise, N, and no interfering transmitters). This is illustrated in Figure 3.8 by the red rectangle in the left channel. The orange rectangle that covers both channels represents the environmental noise. The overall reward is therefore $R = 1 \cdot C_{OFDM} = B_C \log_2((S/N) + 1)$.

State $(0,2)$ represents the case where two OFDM transmitters are on. The reward is given by $C_{SS} = 0$ and $C_{OFDM} = B_C \log_2((S/N)+1)$, illustrated in the figure by the red and black rectangles in the two channels. The capacity of each channel is the same as above since they are independent. Since there are two channels in use, the overall reward is therefore $R = 2 \cdot C_{OFDM} = 2B_C \log_2((S/N)+1)$. This completes the first row of the state space.

Moving vertically up the initial column, state $(1,0)$ indicates one SS transmitter is sending signal power S spread over two channels ($2B_C$). The capacity of the spectrum to transmit the information is given by $C_{SS} = 2B_C \log_2((S/2N)+1)$, with the environmental noise of 2 channels represented by $2N$. This is shown in the figure by the blue rectangle which overs both channels. Since there are no OFDM transmitters, $C_{OFDM} = 0$. The overall reward is therefore $R = 1 \cdot C_{SS} = 2B_C \log_2((S/2N)+1)$.

State $(2,0)$ represents a pair of SS transmitters, each spread over the 2 channels of spectrum. In this case, each of the transmitters will appear as noise to the other transmitter, contributing an S in the denominator of the Shannon capacity expression. This gives $C_{SS} = 2B_C \log_2\left(\frac{S}{2N+S}+1\right)$. Since there are no OFDM transmitters, $C_{OFDM} = 0$, and the overall reward is therefore $R = 2 \cdot C_{SS} = 4B_C \log_2\left(\frac{S}{2N+S}+1\right)$. This completes the first column of the state space.

Moving to the interior states, state $(1,1)$ indicates two transmitters are on, one using SS modulation and the other using OFDM modulation. Here, the OFDM transmission experiences both the environmental noise in its assigned channel, N, and one half of the signal power of the spread spectrum transmitter, $0.5S$. This yields $C_{OFDM} = B_C \log_2\left(\frac{S}{N+0.5S}+1\right)$. Similarly, the SS transmission experiences both environmental noise (across both channels in this case, $2N$), and all of the signal power of the OFDM transmitter, S. This yields

$C_{SS} = 2B_C \log_2 \left(\frac{S}{2N+S} + 1\right)$. The overall reward function is therefore

$$R = 1 \cdot C_{SS} + 1 \cdot C_{OFDM} = 2B_C \log_2 \left(\frac{S}{2N+S} + 1\right) + B_C \log_2 \left(\frac{S}{N+0.5S} + 1\right). \quad (3.6)$$

Clearly, what is distinct in each state is the expression for the denominator in the Shannon capacity (i.e., the noise experienced by each transmission). We can summarize the last three states in the following table showing the expression for C_{SS} and C_{OFDM} for each case.

State	C_{SS}	C_{OFDM}
$(2,1)$	$2B_C \log_2 \left(\frac{S}{2N+2S} + 1\right)$	$B_C \log_2 \left(\frac{S}{N+S} + 1\right)$
$(1,2)$	$2B_C \log_2 \left(\frac{S}{2N+2S} + 1\right)$	$B_C \log_2 \left(\frac{S}{N+S} + 1\right)$
$(2,2)$	$2B_C \log_2 \left(\frac{S}{2N+3S} + 1\right)$	$B_C \log_2 \left(\frac{S}{N+S} + 1\right)$

The reward for each of the 9 possible states is visually depicted in Figure 3.8.

Given the above reward function, we calculated a a value-optimal policy for an illustrative example application. Table 3.3 depicts the policy for a 4-channel spectrum, with other parameters as specified. In the table, the green entries labeled `as` indicate an 'accept SS' action, the blue entries labeled `af` indicate an 'accept OFDM' action, and the red entries labeled `na` indicate a 'no accept' action.

Table 3.3: Value optimal policy that results given the following parameters: 4 channels of spectrum, offered load = $OL = 0.4$ E (erlangs), signal-to-noise ratio = $S/N = 1$, and SS and OFDM efficiency = $\gamma_e = 1$.

4	af	af	af	af	na
3	as	as	af	af	as
2	as	as	af	af	as
1	as	as	af	af	as
0	as	as	af	af	as
SS/OFDM	0	1	2	3	4

We make several observations about this value-optimal policy.

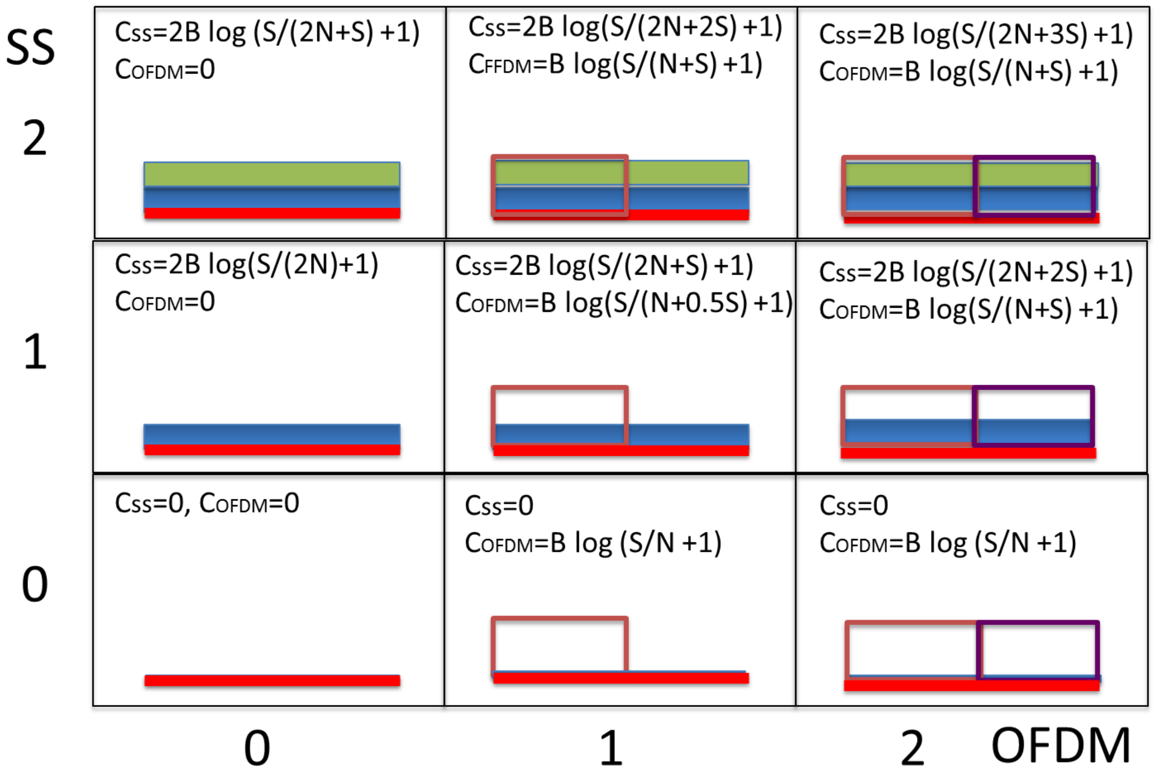


Figure 3.8: Shannon reward function visual representation example.

1. The 'no accept' action indicated in the top-right corner is the only option available to the MDP in that state, since the state space does not allow increased number of transmitters of either type beyond this state. I.e., the spectrum is at full capacity for both modulation types.

2. Across the top row, the only modulation type that is available is OFDM (since we are at capacity for SS modulation), and any accept action must be an 'accept OFDM' action. Here, the MDP is indicating that these transmitters should be admitted (using the available OFDM modulation).

3. Similarly, in the rightmost column, the only modulation type that is available is SS (since we are at capacity for OFDM modulation), and any accept action must be an

'accept SS' action. Again, the MDP is indicating that these transmitters should be admitted.

4. We next turn our attention to the remaining entries. Here, the MDP has the ability to recommend any modulation type (in what follows we refer to this as the *unrestricted region*), and there is a regular pattern that emerges from the value-optimal policy. If there are fewer than 2 OFDM transmitters, the policy is exclusively to choose SS modulation. If there are 2 or more OFDM transmitters, the policy is to exclusively choose OFDM modulation.

With the region where the MDP has the full set of options available to it called the unrestricted region, we will refer to the top row and the rightmost column collectively as the *restricted region*, reflecting the notion that the full set of options is not available to the MDP.

Given the fact that the reward function is tracking data throughput, it is not surprising that the value-optimal policy is to admit a transmitter whenever possible. In Chapter 4 we investigate the pattern in item 4 above, and eventually develop heuristic spectrum management algorithms that exploit this pattern. The motivation for the heuristic approximations is the same as described in Section 3.2.

3.5 Generalization of Spectrum Allocation Models

In this chapter, we have presented 3 different MDP models that are applicable to managing RF spectrum. They include two different families of reward functions (Bernoulli and Shannon), they support two different action sets (admission with and without modulation type selection), and one supports multiple message sizes.

Using these MDP models as examples, one could reasonably develop additional MDP models that are tailored to different circumstances. For example, one could merge the basic Shannon MDP with the extended Bernoulli MDP to yield an extended Shannon MDP that supports multiple message sizes.

One management task that the above MDPs do not directly address is allocation (i.e., which channel(s) should be assigned to an incoming message?). While this would clearly require an expansion of the state space, it does not entail any fundamentally new insights.

3.6 Chapter Summary

This chapter has introduced a series of MDP models that reflect a family of options for managing RF spectrum, with the goal of improving throughput. This includes a pair of distinct reward functions (Bernoulli and Shannon) and a pair of distinct management functions (admission and modulation decisions).

For the basic Bernoulli MDP model, the resulting value-optimal policy is synonymous with that currently used in industry practice. Using the Shannon MDP model, the value-optimal policy indicates a richer structure of decisions, reflecting the wider variety of circumstances that the MDP state space represents. In particular there are clearly distinguishable regions of the state space where one modulation type vs. the other is preferable. However, the MDP approach is, in general, exponentially expensive in the number of states present in the model. In the next chapter, we will explore heuristic methods for characterizing the distinct regions more efficiently, so that on-line admission and modulation choices can be realized in practice.

Chapter 4

Heuristic Approximation of Value-Optimal Policies

The discussion in the previous chapter indicates the potential utility of using Markov decision processes to generate policies for managing RF spectrum. However, the exponential cost of computing a value-optimal policy may be impractical in an on-line setting. Figure 4.1 shows the execution time to compute the value-optimal policy of the Shannon MDP as a function of the number of states on a 2.3 GHz Opteron with 16 GB of memory.

Although for small numbers of transmitters value-optimal policies can be generated in a matter of seconds to minutes, for moderate numbers of transmitters (32, 64, or 128) doing so would take hours, days, or even weeks, which is unsuitable for on-line use. For even larger numbers of transmitters (e.g., as is envisioned for so-called "Internet of Things" applications [5]), direct generation of value-optimal policies becomes intractable, and approximation is then necessary.

Recall, however, that the basic Bernoulli model of Section 3.2 resulted in a simple heuristic that faithfully reproduced the value-optimal policy that was derived from the MDP model.

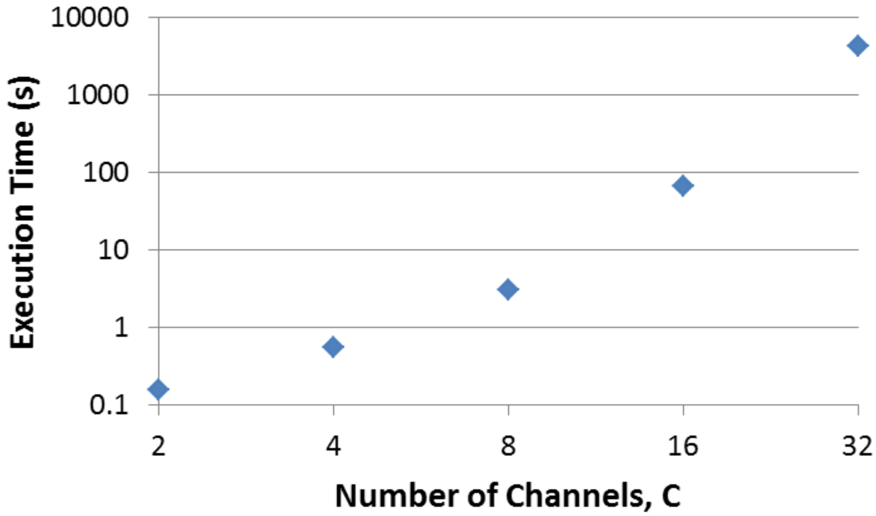


Figure 4.1: Execution time required to compute value-optimal policies vs. state space size.

Here, we hypothesize that this notion generalizes to other models as well. Specifically, we assert the following.

> We can formulate efficient and effective heuristics that mimic the value-optimal policy of the MDP models we consider.

These heuristics are based on the discovery of efficiently computable boundaries between regions that are characterized by a common action.

We evaluate this hypothesis as follows. First, we choose one of the more complex MDP models from Chapter 3 for investigation. Second, we empirically explore the value-optimal policies produced by the chosen MDP model, looking for patterns that can be exploited for use by a heuristic. Third, we formulate a candidate heuristic based on these observations and assess its effectiveness.

4.1 Empirical Parametric Study

Since the Shannon MDP model both supports a larger action space and has a distinct reward function (relative to the Bernoulli MDP models) we will focus our investigation of the heuristic approach using that MDP. For the empirical exploration of the value-optimal policies produced by the Shannon MDP model, we will first consider each input parameter individually. The parameters we consider include the size of the spectrum (in channels), the offered load of messages into the system, the modulation efficiency of each modulation type, and the signal-to-noise ratio experienced within the RF spectrum (which we express as r_{SN}, i.e., $S = r_{SN} \cdot N$).

For each of the sections below, we vary one of the above parameters, keeping the other parameters fixed. In each case, we illustrate the observed trends with a small set of figures. Additional supporting evidence for the trends that we identify can be found in the appendices of this dissertation.

4.1.1 Impact of Spectrum Size

To investigate the effect that spectrum size has on the value-optimal policy, we show the value-optimal policy for 2-, 4-, 8-, and 16-channel spectra in Tables 4.1 to 4.4, respectively. The other input parameters for each of these examples are fixed as follows: offered load $= OL = 0.4$ E (erlangs), SS and OFDM efficiency $= \gamma_e = 1$, and signal-to-noise ratio $= r_{SN} = 1$). Note that Table 4.2 is the same policy shown earlier in Table 3.3.

The pattern of actions for these policies follows that described in Chapter 3. Independent of the spectrum size, at the top right, the only available action is a 'no accept'; across the

Table 4.1: Value-optimal policy for a 2-channel spectrum.

2	af	af	na
1	as	af	as
0	as	af	as
SS/OFDM	0	1	2

Table 4.2: Value-optimal policy for a 4-channel spectrum.

4	af	af	af	af	na
3	as	as	af	af	as
2	as	as	af	af	as
1	as	as	af	af	as
0	as	as	af	af	as
SS/OFDM	0	1	2	3	4

Table 4.3: Value-optimal policy for an 8-channel spectrum.

8	af	af	af	af	af	af	af	af	na
7	as	as	as	as	af	af	af	af	as
6	as	as	as	as	af	af	af	af	as
5	as	as	as	as	af	af	af	af	as
4	as	as	as	as	af	af	af	af	as
3	as	as	as	as	af	af	af	af	as
2	as	as	as	as	af	af	af	af	as
1	as	as	as	af	af	af	af	af	as
0	as	as	as	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8

Table 4.4: Value-optimal policy for a 16-channel spectrum.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

top, the action is consistently 'accept OFDM'; on the right, the action is consistently 'accept SS' (these are the restricted regions of the state space); and in the remaining states (the unrestricted region) there is a line that demarcates the 'accept SS' actions from the 'accept OFDM' actions. As was the case for the basic Bernoulli MDP, there is an easily identifiable boundary that partitions the actions chosen by the value-optimal policy. Here, however, rather than the line being strictly vertical, it has a finite, positive slope.

4.1.2 Impact of Offered Load

To investigate the effect that offered load has on the value-optimal policy, we vary the input rate while keeping the message duration (which determines the effective service rate) constant. Defining the offered load as the ratio between input rate and service rate, its units are therefore erlangs (E). Tables 4.5 to 4.8 show the value-optimal policy for offered loads ranging from 0.2 to 2.4 E. Tables showing the value-optimal policies for several additional values of offered load, ranging from 0.2 to 2.4 E, are presented in Appendix B. The other input parameters for each of these examples are fixed as follows: spectrum size = C = 16 channels, SS and OFDM efficiency = $\gamma_e = 1$, and signal-to-noise ratio = $r_{SN} = 2$).

Table 4.5: Offered Load of 0.2 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

As was the case for spectrum size, we observe that the boundary partitioning the actions chosen by the value-optimal policy is not significantly effected by changes in the offered load (i.e., the pattern is relatively insensitive to offered load). In the unrestricted region, the boundary line is near the center of the region, and as offered load increases, the slope of the line gets steeper, but only slowly.

Table 4.6: Offered Load of 0.8 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.7: Offered Load of 1.6 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.8: Offered Load of 2.4 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

4.1.3 Impact of Modulation Efficiency

To investigate the effect that modulation efficiency has on the value-optimal policy, we vary the spread spectrum modulation efficiency, $\gamma_{e.SS}$, and separately the OFDM modulation efficiency, $\gamma_{e.OFDM}$, while keeping the other parameters constant.

Tables 4.9 to 4.16 show the value-optimal policy for modulation efficiencies ranging from 0.7 to 0.999. Tables showing the value-optimal policies for several additional values of modulation efficiency ranging from 0.7 to 0.999 are presented in Appendices C and D. The other input parameters for each of these examples are fixed as follows: offered load $= OL = 0.6$ E, spectrum size $= C = 16$ channels, and signal-to-noise ratio $= r_{SN} = 2$).

The first observation we make from these results is that the modulation efficiency has a much stronger impact than either spectrum size or offered load on the value-optimal policies that

Table 4.9: Spread Spectrum Efficiency of 70 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.10: Spread Spectrum Efficiency of 80 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.11: Spread Spectrum Efficiency of 90 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.12: Spread Spectrum Efficiency of 99.9 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.13: Orthogonal Frequency Division Multiplexing Efficiency of 70 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.14: Orthogonal Frequency Division Multiplexing Efficiency of 80 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.15: Orthogonal Frequency Division Multiplexing Efficiency of 90 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.16: Orthogonal Frequency Division Multiplexing Efficiency of 99.9 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

are determined by the MDP. This is not surprising, given that at a lower efficiency for one modulation type, the MDP is choosing to use the other modulation type more frequently.

The second observation we make is that the unrestricted region of the state space exhibits the same pattern as we have previously observed: there exists a linear boundary that separates the region into two, with the value-optimal policy being all actions to one side of the boundary are `'as'` and all actions to the other side of the boundary are `'af'`. As the input efficiencies vary, both the area on each side of the boundary change and the slope of the boundary line may change.

4.1.4 Impact of Signal-to-Noise Ratio

To investigate the effect that signal-to-noise ratio has on the value-optimal policy, we vary the signal-to-noise ratio, r_{SN}, while keeping other parameters constant. Tables 4.17 to 4.20 show the value-optimal policy for signal-to-noise ratios ranging from 1 to 12. Tables showing the value-optimal policies for several additional values of SNR ranging from 1 to 12 are presented in Appendix E. The other input parameters for each of these experiments are fixed as follows: spectrum size = $C = 16$ channels, offered load = $OL = 0.6$ E, and SS and OFDM efficiency = $\gamma_e = 1$.

The first observation here is the restricted regions behave differently than what we have previously observed. At sufficiently high SNR, the value-optimal policy includes some "no accept" (`'na'`) actions, where for the previously investigated parameter settings (each of which had low SNR) this was not the case. We speculate that this is due in part to the fact that the value-optimal policies are generally observed to be keeping the modulation types consistent and do not mix modulation types except when forced to do so in these restricted

Table 4.17: Signal-To-Noise Ratio of 1.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.18: Signal-To-Noise Ratio of 4.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.19: Signal-To-Noise Ratio of 8.

16	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table 4.20: Signal-To-Noise Ratio of 12.

16	na	na	na	na	na	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

regions of the state space. At high values of SNR, the MDP appears to prefer awaiting a future new transmission over the required mixing of modulation types that would be implied by an accept action in the restricted region. These value-optimal actions are distinct from the greedy actions for the same states, illustrating the differences between value-optimality in expectation as determined by Markov decision theory vs. simple greedy techniques.

The second observation is that the boundary line between distinct value-optimal actions still exists in the unrestricted region of the state space. In this case, the slope of the boundary line is strongly determined by the signal-to-noise ratio.

4.1.5 Observations from Empirical Results

Here we summarize the observations made from the empirical study. First, the value-optimal policy has distinct patterns in the restricted region of the state space vs. the unrestricted regions. In the unrestricted region, the value-optimal action are effectively divided by a linear boundary line and all actions to the left of the line are "accept SS" (`'as'`) and all action to the right of the line are "accept OFDM" (`'af'`). This line is effectively characterized by the set of parameters explored above. Generally, the spectrum size and offered load have a limited impact. Modulation efficiency has a much more substantial impact, varying the number of states which each action, and the SNR impacts the slope of the boundary line.

In the restricted region, SNR has a significant effect, establishing a boundary between "accept" action or "no accept." Which "accept" action is restricted by the region.

4.2 Development of Heuristics

Following the example of Glaubius et al. [16] and Tidwell et al. [57], it is often possible to exploit regular structure of the value-optimal policies to differentiate between regions of the state space that have common action. The results of our empirical evaluations suggest that a similar exploitation of such structure is viable here. In our empirical investigation of the parameter space, each and every value-optimal policy had the following properties.

1. The action in the upper-right corner is to not accept (the only action available).

2. The action in the top-most row is to either not accept or accept OFDM (the only two options available). States to the left of a boundary are "no accept" actions and states to the right of that boundary are "accept OFDM" actions.

3. The action in the right-most column is similar in form to that of the top-most row. States below a boundary are "no accept" actions and states above that boundary are "accept SS" actions.

4. The remaining actions (in the unrestricted region) are divided by a line that separates the state space between SS and OFDM allocation.

The various parameters impact the position and orientation of the boundaries, but do not alter their form.

We define a heuristic approach to approximating the true value-optimal policies for the parameter space as follows. First, using off-line analysis, we describe the boundaries as a function of the parameters. For the unrestricted region, given the offered load, OL, modulation efficiency, $\gamma_{e.SS}$ and $\gamma_{e.OFDM}$, and signal-to-noise ratio, r_{SN},

$$B_u(y_2, y_1 \mid OL, \gamma_{e.SS}, \gamma_{e.OFDM}, r_{SN}) = 0 \tag{4.1}$$

represents the boundary between where the value-optimal decision is "accept SS" vs. "accept OFDM." For the restricted region, given the signal-to-noise ratio, r_{SN}, $B_{rs}(r_{SN})$ and $B_{rf}(r_{SN})$ are boundaries between where where the value-optimal decision is "accept SS" vs. "no accept" for B_{rs} and "accept OFDM" vs. "no accept" for B_{rf}.

Second, the on-line algorithm follows a similar structure as the heuristic of Chapter 3, expressed there as Equation (3.3), with the threshold test replaced with a decision boundary described by the boundary line $B(y_2, y_1) = 0$ and the boundaries B_{rs} and B_{rf}.

4.2.1 Run-time Algorithm

We first describe the run-time algorithm, given the parametrized boundaries that separate the state space into regions of distinct action. Next we will describe the approach we use to identify the boundaries.

We can approximate the value-optimal policy via the run-time algorithm of Figure 4.2. Here, the boundary line $B_u(y_2, y_1) = 0$ is expressed in the form

$$y_1 = m \cdot y_2 + b \tag{4.2}$$

where m and b are coefficients that are determined using the techniques below.

(1) `if` $y_1 = C$ `and` $y_2 = C$ `then`
(2) $a \leftarrow$ 'na'
(3) `else if` $y_2 = C$ `then`
(4) if $y_1 < B_{rf}$ `then`
(5) $a \leftarrow$ 'na'
(6) `else`
(7) $a \leftarrow$ 'af'
(8) `endif`
(9) `else if` $y_1 = C$ `then`
(10) if $y_2 < B_{rs}$ `then`
(11) $a \leftarrow$ 'na'
(12) `else`
(13) $a \leftarrow$ 'as'
(14) `endif`
(15) `else if` $y_1 \leq m \cdot y_2 + b$ `then`
(16) $a \leftarrow$ 'as'
(17) `else`
(18) $a \leftarrow$ 'af'
(19) `endif`

Figure 4.2: Run-time algorithm to approximate value-optimal policies. Lines (1) to (14) correspond to the restricted region of the state space, and lines (15) to (19) correspond to the unrestricted region.

Note that this formulation expresses the horizontal component of the state as a function of the vertical component (i.e., the opposite of what is traditional). This helps us with vertical lines (which we have) and would be problematic with horizontal lines (which we don't have). This is equivalent to transposing the earlier tables and showing the number of spread spectrum transmitters, y_2, horizontally and the number of OFDM transmitters, y_1, vertically. This is illustrated in Table 4.21, which is the transposed version of Table 4.4 shown earlier.

Table 4.21: Transposed representation of value-optimal policy from Table 4.4.

16	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	na
15	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
14	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
13	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
12	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
11	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
10	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
9	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af
8	af	af	af	af	af	af	af	af	af	as	as	as	as	as	as	as	af
7	af	af	af	as	as	as	as	as	as	as	as	as	as	as	as	as	af
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
5	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
4	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
3	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
2	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
1	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
0	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af
OFDM/SS	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

4.2.2 Determining the Boundary Line for the Unrestricted Region

Our working hypothesis is that one can express the coefficients of the line B_u in terms of the parameters explored in Section 4.1, specifically: offered load, modulation efficiency, and signal-to-noise ratio. We capture this relationship via multi-variable linear regression.

Using 10 variations of each input parameter within the operational space, we use the MDP solver to determine the true value-optimal policy for each parametrization. This is used as input into MATLAB's multi-variable regression function `mvregress` (version R2013a), which generates a multivariate general linear model, to determine the coefficients of the line B_u. The range of input parameter values are the same as those shown in Appendix B for the offered load, $0.2 \leq OL \leq 2.4$, and Appendix E for the signal-to-noise ratio, $1 \leq r_{SN} \leq 12$. For the modulation efficiencies, we limited the parameters to the range $0.91 \leq \gamma_e \leq 1$, since modern modulation techniques typically have efficiencies in that range [37], and we limited the spectrum size to $C = 16$. This results in a 4-predictor regression (offered load, SS efficiency, OFDM efficiency, and signal-to-noise ratio) that yields the coefficients of the line B_u expressed in Equation (4.2).

4.2.3 Determining Boundaries in the Restricted Region

In the empirical study presented above (Section 4.1), we observe that the boundary between accept and no accept decisions in the restricted region appears to be only a function of the signal-to-noise ratio and not influenced by the other parameters. This hypothesis was examined further by varying more than one parameter at once (the previous empirical studies altered only one parameter at a time). Value-optimal policies for varied SNR and modulation efficiencies are presented in Appendix E. We observe that only SNR influenced the boundaries between actions in the restricted regions. For the restricted regions, we therefore can omit multivariate regression for determination of the boundaries, and instead do single variable regression based on the signal-to-noise ratio, r_{SN}.

4.3 Assessment of the Heuristic

4.3.1 Restricted Region

To assess the heuristic in the restricted region, we are interested in comparing two thresholds. First, the threshold that represents the boundary between actions in the value-optimal policy, and second, the threshold determined by the heuristic using the run-time algorithm of Figure 4.2. Figure 4.3 shows a plot that compares these two thresholds for the portion of the restricted region in which only OFDM modulation is allowed. The points indicate the value-optimal threshold and the line indicates the heuristic threshold. Clearly, the alignment is quite good ($r^2 = 0.97$). For the restricted region in which only SS modulation is allowed, the heuristic is equally effective ($r^2 = 0.93$).

4.3.2 Unrestricted Region

To assess the effectiveness of the heuristic approach in the unrestricted region, we applied the multi-variable linear regression with a test set of 48 configurations (varying each parameter independently). For each of these configurations, we asked the MDP solver for the true value-optimal policy and provided that as the desired result for the regression training process.

For testing, 48 additional randomly generated configurations (constrained to be in the ranges used for training) were used. The randomly generated configurations were additionally constrained not to be in the test set. In each case, we executed the run-time algorithm of Figure 4.2 and compared the result of that execution to the the true value-optimal policy generated by the MDP.

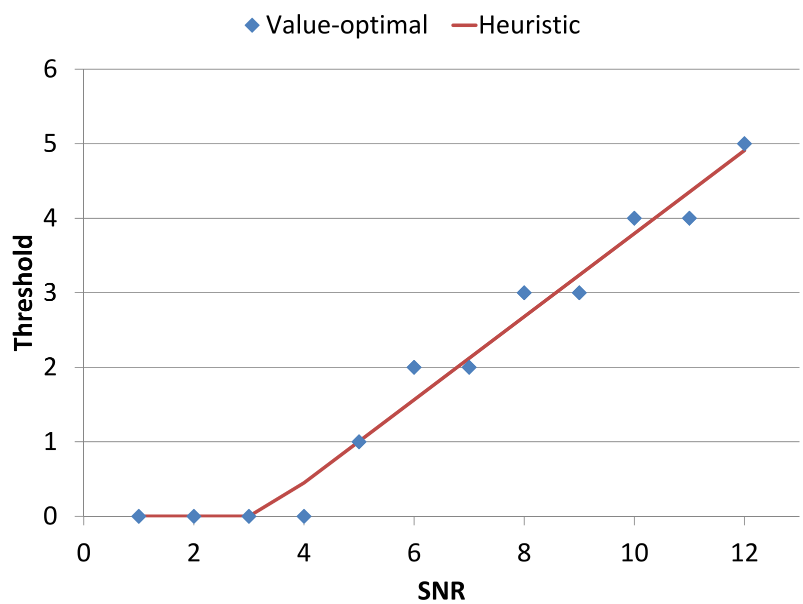


Figure 4.3: Value-optimal and heuristic thresholds separating actions in the restricted region in which OFDM modulation is allowed.

Two specific comparisons were made between the heuristic run-time results and the true value-optimal policies. First, we informally assessed the slope and intercept of each policy (both true and heuristic) to ensure that they were similar. Second, we use the r-squared test to measure quantitatively the differences between the heuristic run-time algorithm policy and the true value-optimal policy. For the testing set, $r^2 = 0.95$. This tells us that there is close agreement between the true value-optimal policy as specified by the MDP and the heuristic run-time approximation.

4.4 Chapter Summary

In this chapter we performed an empirical study of the Shannon MDP over a wide range of parameters. Each parameter was varied independently, and the implications of that variation where assessed. In both the restricted and unrestricted regions, we observed consistent patterns that separate the actions in the value-optimal policies. The boundaries that separate the actions are described by linear function, which is then used to guide a heuristic approximation to value-optimal policies that are expensive to compute. In the restricted region, the boundary is a function of SNR only, while in the unrestricted region the boundary is a function of all the parameters.

The patterns in the value-optimal policies lend themselves to heuristic approximation. Using multivariate linear regression in the unrestricted region, and single variable linear regression in the restricted region, we calibrate the boundaries between actions to develop a run-time decision algorithm that exploits those boundaries. The heuristic approximation was evaluated over a set of randomly generated parameterizations and shown to accurately track the value-optimal policies from the MDP.

Chapter 5

Cross-Validation of MDP Models via Discrete-Event Simulation

The use of Markov decision theory for decision making in RF systems is predicated in significant part on the underlying Markovian models being reasonable representations of the physical RF system we are trying to manage. Critical to the viability of MDP approaches is the question of whether or not assumptions inherent in an MDP model over-simplify the underlying reality and, as a result, even an optimal choice in the space of the MDP model might or might not be an optimal choice in the real world. In this chapter we evaluate the use of MDPs for RF spectrum allocation by comparing (empirically) decisions and performance predictions made by a representative MDP model with performance predictions made by a discrete-event simulation (DES) model. While some modeling assumptions are common to both the MDP model and the DES model (and these assumptions therefore will not be evaluated), points of difference between the models are investigated here.

One of the properties of discrete-event simulation models, in general, is that they have very few restrictions on the underlying physical model that they represent. As a result, issues such as distributional assumptions, etc., are fairly straightforward to assess. Here, we pose a set of hypotheses about the correspondence between the MDP models described earlier in the dissertation and the DES model described in this chapter. Explicit experiments are designed for the basic Bernoulli MDP model. As is discussed later in this chapter, results of those experiments were suitably indicative of the reasonableness of the MDP modeling approach. Further comparative experiments with other MDP models developed in this dissertation are deferred as future work.

We start by returning to the MDP that models a straightforward admission control problem, the basic Bernoulli MDP. This is followed by a set of hypotheses that examine several aspects of the assumptions made in the MDP modeling process. For each of these hypotheses, a specific experiment is designed and executed, and the validity of the hypothesis is assessed. The three hypothesis are:

1. performance can be reasonably characterized by the mean of message durations and is relatively insensitive to their distribution;

2. even though imperfect channel allocations will occur in any real system, they are infrequent enough that ignoring them does not have a significant impact on an MDP model's ability to predict throughput accurately; and

3. value-optimal policy decisions made by the MDP are at least locally optimal as determined by the DES model.

The first two hypotheses are assessed by experiments utilizing the discrete-event simulator. The third hypothesis is assessed by comparing the performance predictions of the MDP model to those of the DES model. In all three cases, we find strong evidence to support the hypotheses, providing an indication that Markov decision processes offer a suitable mechanism for managing shared RF spectrum.

5.1 Basic Bernoulli MDP

Figure 5.1 repeats the structure of the basic Bernoulli MDP we are using for admission control (it was previously presented in Chapter 3). Each state $\chi = i$ encodes the number of messages occupying a channel (i.e., currently being transmitted). This MDP model assumes a perfect allocation of messages to channels, so if there are C channels and $\chi \leq C$, each message is assumed to be allocated to a distinct channel. If $\chi > C$, we assume the number of conflicts (i.e., messages transmitting on a common channel) is the minimum possible.

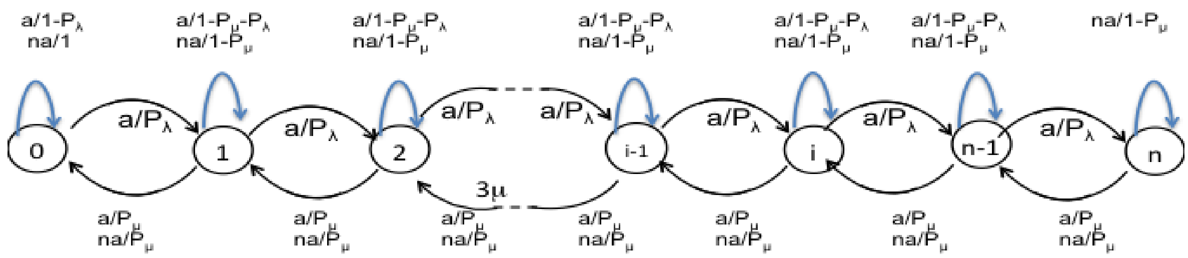


Figure 5.1: Basic Bernoulli MDP state transition diagram.

For admission control, the two possible actions are to allocate or not allocate. In Figure 5.1, actions to allocate are indicated by edges labeled 'a' in the transition from a state χ to the neighboring state $\chi + 1$ (this edge will be traversed with probability P_λ, reflecting the probability of an arrival, given arrival rate λ). Similarly, actions to not allocate are indicated by edges labeled 'na' and are self-loops. Transitions from state χ to state $\chi - 1$ represent message completions (messages departing from the system), and are therefore independent of action. Additional details of this MDP are available in Chapter 3, which describes its development.

5.2 Evaluation Approach

In this section we evaluate the use of a Markov decision processes for RF spectrum management by comparing empirically decisions and performance predictions made by the above MDP model with performance predictions by a discrete-event simulation (DES) model.

A key premise of this dissertation is that there is inherent value in using MDP models to manage and control RF spectrum resources. Our approach to help evaluate that premise is to compare an MDP model with a different (i.e., DES) model. To appreciate both the benefits and limitations of this approach, it is important to distinguish what this empirical comparison does and does not validate. Specifically, any modeling assumption that is made in common by both the MDP model and the DES model will not be tested by this approach. We first articulate some of these common assumptions:

1. The arrival process of messages is assumed to be Poisson with a given mean arrival rate.
2. Messages each consume one channel of RF spectrum (i.e., the spectrum is decomposed into discrete, equal-sized channels) and collectively have a given mean duration.
3. The underlying channel model that predicts the success or failure of an individual message delivery, based on environmental factors and/or conflict with other messages, is common across the MDP and DES models.
4. Messages that fail depart the system. We do not model retries.

Although one may question these assumptions, it is impractical to test them given the available experimental infrastructure, since the DES model (which also makes those same assumptions) is our comparison vehicle.

While the above list describes what we will not test, the purpose of our evaluation is to assess the three primary hypotheses (stated above), which we can test:

1. the independence of message throughput on the distribution of message durations,
2. the insignificance of imperfect allocations, and
3. that value-optimal policies chosen by the MDP are at least locally optimal according to the DES model.

5.3 Discrete-event Simulation Model

The DES model uses traditional event-driven simulation techniques, in which state changes to the modeled system are represented by time-stamped events that are maintained in a time-ordered priority queue. The event with the smallest time-stamp is removed from the queue, the state change represented by that event is executed, and any subsequent future state changes implied by the event's execution are scheduled in the priority queue.

5.3.1 DES Model

The discrete-event simulator maintains an explicit representation of the set of channels within the RF spectrum range being modeled, including occupancy of each channel over time by

```
sample random variable X ∈ U[0,1)
if X < P_env
  message m fails
else
  sample random variable Y ∈ U[0,1)
  if m overlaps another message in any channel and Y < P_conf
    message m fails
  else
    message m succeeds
```

Figure 5.2: Pseudo-code of message success or failure.

specific sets of messages. The events supported by the simulator include message arrival and message departure. As part of the execution of a message arrival event, the simulator performs a greedy allocation algorithm (i.e., it allocates the newly arrived message to a channel with the fewest conflicts with other, pre-existing messages currently using the spectrum). During the execution of a message departure event, the Bernoulli model of Section 3.2 is used to determine whether or not the message was successfully delivered. The resulting simulation model considers two types of effects: conflict failure (denoted by P_{conf}) and environmental transmission failure (denoted by P_{env}). The pseudo-code of Figure 5.2 shows the algorithm used to account for success or failure of a message.

To support the evaluations we wish to perform in this dissertation, the simulator was extended in two specific ways. First, the uniform distribution assumption for message duration was expanded to also include the option for an exponential distribution. Second, the effectiveness of the greedy allocation algorithm was measured by counting the number of messages that were delivered under imperfect allocation decisions (i.e., the message was delivered using a shared channel when a free channel was available but, at the time, unknown to the simulator).

Many features of the discrete-event simulator are unnecessary for its use in this dissertation (e.g., it supports a binary buddy algorithm for spectrum allocation). Since the present investigation is constrained relative to the simulator's capabilities, we will only describe features that are directly relevant to our evaluations. Each rectangle in Figure 5.3 corresponds to a fixed size unit of allocation (which we call a *channel*). Each message is allocated a fixed number of contiguous channels, and occupies those channels until a given time relative to the time of allocation (called EOL for message end-of-life). The messages shown in Figure 5.3 have been allocated to adjacent channel ranges, leaving a single unallocated gap in the region shown. Different models for allocation would naturally give other states; for example in a buddy allocation scheme (the subject of [31]) messages would be aligned on boundaries that are binary exponentials, while in an allocation scheme that allows overlaps, messages could be allocated atop regions already occupied by other messages.

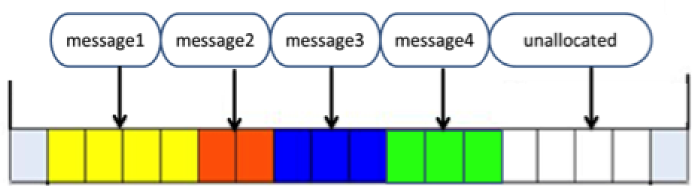


Figure 5.3: Example simulator state during a run.

5.3.2 DES Evaluation

To evaluate the simulator, performance predictions made by the simulator are compared with an M/U/c/c queueing model (i.e., Markovian arrival process, Uniformly distributed service process, c servers, and c total jobs allowed in the system). For all of these experiments, we fixed the number of channels, C, maximum message size, m_{MAX}, and maximum end-of-life for a message, EOL_{MAX}. Parameters that are varied include the number levels, L (encoding the degree of spectrum overlap allowed), failure probabilities P_{env} and P_{conf}, and the message arrival rate, λ. The ranges of values used in the evaluation are listed in Table 5.1.

Table 5.1: Evaluation Parameters.

Symbol	Parameter	Value(s)
C	number of channels	1024
m_{MAX}	maximum message size	256
EOL_{MAX}	maximum end of life	64 ms
L	number of levels	{1,3}
P_{env}	environmental failure prob.	{0.0, 0.1, 0.2}
P_{conf}	conflict prob.	{0.0, 0.2}
λ	mean message arrival rate	(0 to 1] msgs/ms

For all of the simulation results, the plotted mean delivered throughput represents the mean value across five distinct simulation executions (e.g., 5 distinct pseudo-random number generator seeds). Error bars represent 99% confidence intervals based on Student's t-distribution.

Queueing Theoretic Approximate Model

We use queuing theory to provide an approximate model of the resource (spectrum) usage. This helps us both validate the simulation models and better understand the underlying causes for the effects that are observed. Additional symbols used by the analytic model are shown in Table 5.2.

Table 5.2: Symbol Definitions.

Symbol	Description	Units
r	delivered message throughput	msgs/ms
μ	mean message service rate	msgs/ms
c	number of servers	
$B(c, \lambda/\mu)$	Erlang's loss formula	

Using Kendall's notation, we model the spectrum allocation as an M/U/c/c queuing system, i.e., Markovian (Poisson) arrival process with mean arrival rate λ msgs/ms; uniformly distributed service process with mean service rate μ msgs/ms; c servers; and c total jobs (msgs) allowed in the system. Given a uniformly distributed service time with a maximum of EOL_{MAX} ms,

$$\mu = \frac{2}{EOL_{MAX} + 1} \text{ msgs/ms.} \tag{5.1}$$

Also given a uniformly distributed message size with a maximum of m_{MAX} channels,

$$c = L \times \frac{1}{m_{MAX}} \sum_{i=1}^{m_{MAX}} 2^{\lceil \log_2 i \rceil} \tag{5.2}$$

where the ceiling function accounts for internal fragmentation (e.g., due to the buddy algorithm).

The actual spectrum use differs from the queuing model primarily in the fact that the queueing theoretic results make the assumption that the number of servers is fixed, whereas the simulation model accounts for the varying instantaneous number of concurrent messages that can be supported by the spectrum. This discrepancy will result in the queuing theory model not necessary precisely matching the simulation results, but we expect both throughput predictions to be reasonably close to one another. Where they diverge, the simulator is likely to be more accurate, since its model more closely matches reality.

Using the above queuing model, the delivered throughput in the absence of failures is predicted as

$$r = \lambda(1 - B(c, \lambda/\mu)), \tag{5.3}$$

where B is Erlang's loss formula [3]. The modeled capacity of the spectrum is μc. The degradation in throughput due to environmental causes is modeled as being linear, e.g.,

$$r = (1 - P_{env})\lambda(1 - B(c, \lambda/\mu)), \tag{5.4}$$

and the degradation in throughput due to conflicts is linear when the expected number of messages in the system,

$$N_M = \lambda/\mu(1 - B(c, \lambda/\mu)), \tag{5.5}$$

is greater than c/L.

$$r = \begin{cases} (1 - P_{env})\lambda(1 - B(c, \lambda/\mu)), & \text{if } N_M < c/L \\ (1 - P_{conf})(1 - P_{env})\lambda(1 - B(c, \lambda/\mu)), & \text{if } N_M > c/L \end{cases} \tag{5.6}$$

Simulation Results

We present the results of our evaluations through a series of graphs containing the following elements: (1) each plot has fixed values for the number of levels and failure probabilities, and these quantities vary across plots; (2) the horizontal axis shows the mean message arrival rate, λ, in messages per ms; (3) the vertical axis shows the delivered throughput, r, in messages per ms; (4) the dotted line (labeled q.t. capacity) represents the constant-valued queuing theoretic capacity approximation, μc; (5) the dashed line (labeled q.t. tput) shows the queuing theoretic model of delivered throughput, r, in messages per ms; and (6) the points (labeled mean sim tput) show the simulation model predictions for delivered throughput.

Figure 5.4 shows the baseline performance predictions: 1 level and failure probabilities all 0. As can be seen, there is reasonable alignment between the simulation model predictions and the approximate analytic model derived from queuing theory. At low arrival rates (i.e., $\lambda \leq 0.1$ msgs/ms) the delivered throughput is essentially equal to the arrival rate for both the simulation model and the analytic model. The transition region is entered at only slightly higher arrival rates, and while the alignment is not perfect, the simulation results and analytic approximations are reasonably close across the rest of the graph.

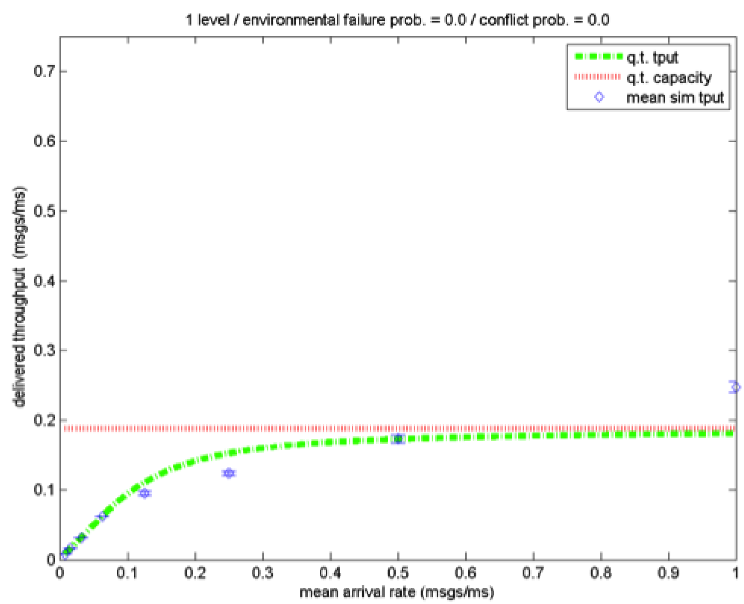


Figure 5.4: Baseline performance, no overlap ($L = 1$), no failures (P_{env}=0.0).

This graph gives us assurance that the simulation model is reasonable (i.e., it tracks the general trends of the analytic approximation). We anticipate that the discrepancies that exist are due primarily to the fact that the analytic model makes the simplifying assumption that the number of queuing theoretic servers (c in the M/U/c/c queuing system) is constant, while the underlying truth is that the number of messages that can be delivered simultaneously by the spectrum depends upon the instantaneous message size. As is true for all of the simulation results, the error bars are very tight, indicating relatively small uncertainty in predictions due to statistical variability.

Figure 5.5 explores how this baseline performance is altered when multiple simultaneous messages are allowed to overlap in each channel. Here, the number of levels of allocation has been increased to 3, yet the failure probabilities are still all 0. Two features of this graph reflect the trebling of effective capacity. First, the analytic model shows a three-fold increase in delivery rate at or near saturation. Second, both analytic and simulation models show the transition region extending approximately three times as wide as the previous case (i.e., to $\lambda \leq 0.3$ msgs/ms).

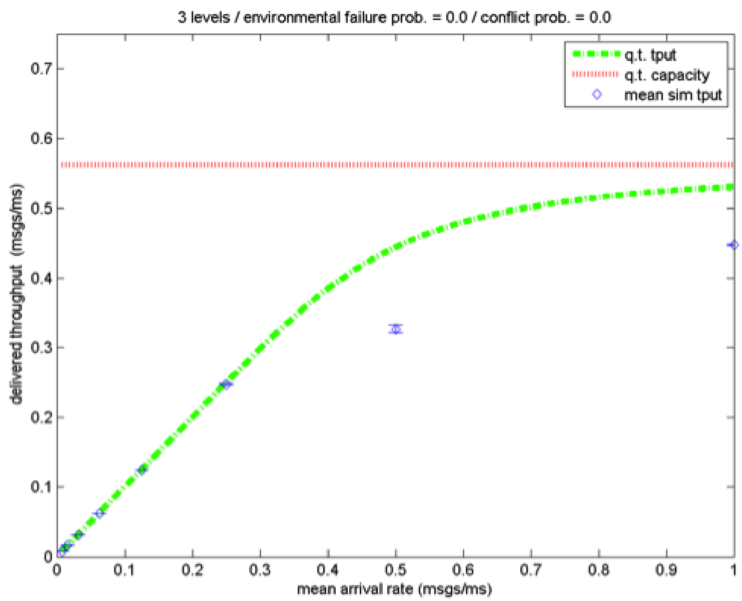


Figure 5.5: Overlap allowed ($L = 3$), no failures (P_{env}=0.0).

Figures 5.6 and 5.7 return to the no overlap case (1 level) and explore the impact on delivered throughput of environmental failures. The plots for $P_{env} = 0.1$ and $P_{env} = 0.2$ can be compared to Figure 5.4 (with $P_{env} = 0.0$). Across the board, the analytic and simulation models show a linear decrease in throughput as the environmental failure probability increases. Note that the queuing theoretic capacity approximation does not change: only the predicted throughput is impacted.

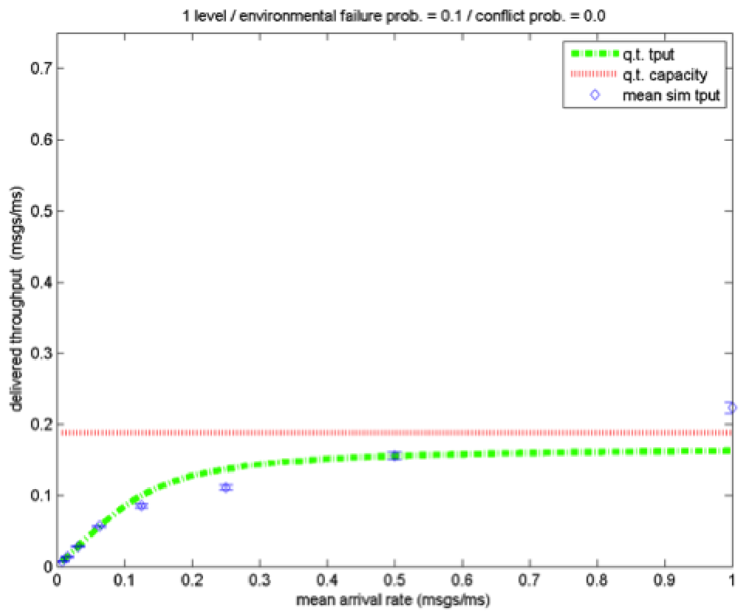


Figure 5.6: No overlap allowed ($L = 1$), with failures (P_{env}=0.1).

Additional figures are presented in [31]. Across the board, our simulation model closely approximates the analytic queueing model, giving us confidence that the discrete-event simulation model is a reasonable approximation of the RF spectrum resource.

5.4 Experimental Predictions

We now describe experimental predictions related to each hypothesis provided at the beginning of the chapter. Each experiment is designed to assess the distinctions between the MDP model and the DES model. The first two hypotheses are assessed by experiments utilizing the discrete-event simulator. The third hypothesis is assessed initially by comparing the throughput performance predictions of the MDP model to those of the DES model, followed by using the simulator to examine local optimality of the decisions made by the MDP.

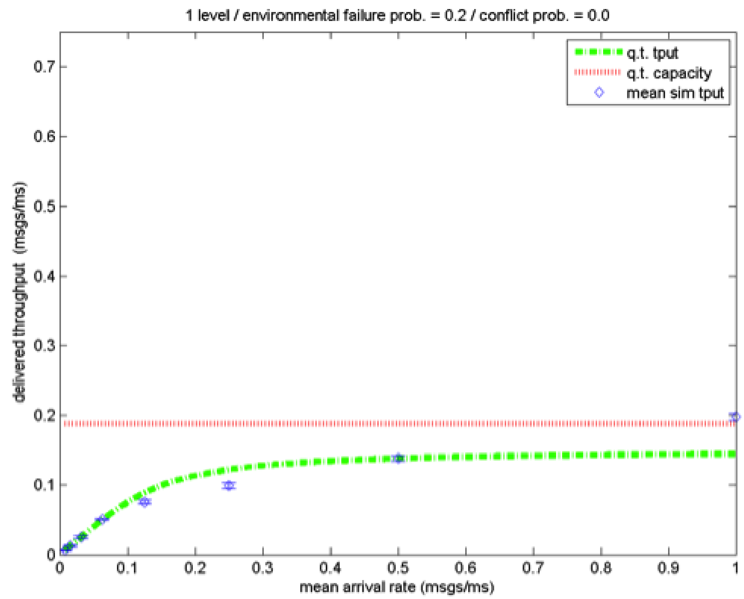


Figure 5.7: No overlap allowed ($L = 1$), with failures (P_{env}=0.2).

The first experiment evaluates the impact of the distribution of message durations on the predicted throughput of the system. The MDP model makes the standard memory-less assumption, modeling message durations via an exponential distribution. The DES model follows a common convention in RF systems and models message durations via a uniform distribution. To test our first hypothesis we configure the simulator to use both exponential and uniform distributions for message duration, and compare the throughput predictions for uniformly distributed message durations with that for exponentially distributed message durations. We intentionally perform this experiment exclusively in the simulation model. Our experimental prediction is that there will not be a significant difference between the throughput for the two (different) distributions of message duration. In effect, we are testing whether or not the insensitivity property that is well established for Erlang-loss systems [45] holds here.

The second experiment evaluates the impact of imperfect allocations on the throughput of the system. The MDP model implicitly assumes that allocation decisions are perfect (i.e., if a free channel exists in the spectrum, some other channel isn't shared). The DES model makes no such assumption, but rather implements the specific actions of a greedy allocator. By measuring the frequency of imperfect allocations (which do not meet the ideal assumptions made by the MDP model) in the simulation, we can assess the impact of this assumption on the performance predictions made by the MDP model. We predict that the frequency of these imperfect allocations is sufficiently small that its effect on throughput is within the statistical variation of the throughput predictions made by the simulator. As in the previous experiment, we use the discrete-event simulator to assess the validity of the perfect channel allocation assumption.

The third experiment is designed to assess the appropriateness of the MDP's value-optimal policy decisions. We predict that a reasonable correspondence (based on local optimality) exists between the performance predictions of the proposed MDP model and the DES model, such that value-optimal policy decisions made by the MDP correspond to optimal throughput predictions by the DES model. As true optimality is computationally impractical to test, we explicitly check for local optimality (i.e., the policy chosen by the MDP is at least a locally optimal choice as predicted by the DES model). Starting from the value-optimal policy chosen by the MDP, we ask the DES to predict performance given that policy and several "nearest neighbor" policies, with the neighborhood chosen to represent what we mean by locality.

We now describe the experiments we conducted to evaluate the hypotheses discussed above. First we describe the design details and infrastructure used to conduct the experiments, and then we present and discuss their results.

5.4.1 Experimental Setup

The MDP and DES models are configured to evaluate the models' behaviors with either a single channel or four channels of spectrum available for allocation. Each of these channels can have up to four levels of redundant use (or reuse). We describe the set up for each of these experiments, and how the evaluations take place. The MDP model and DES model are set up using a Bernoulli reward function (described in Chapter 3). All throughput predictions are made over a range of input rates that provide normalized offered load, ρ, between 0.5 and 2.5 (i.e., $0.5 \leq \rho = \lambda/\mu \leq 2.5$), P_{conf} ranging from 0.1 to 0.9, and $P_{env} = 0$ (the latter since P_{env} was shown in Section 5.3.2 to have a simple linear impact on throughput). All DES model throughput results are analyzed using the method of batch means, with 100 independent runs decomposed into 10 batches each of batch size 10.

DES Experiments to Assess Significance of Message Duration Distribution

Our MDP model assumes exponentially distributed message durations. The DES model assumes uniformly distributed message durations by default, but we configure the DES to test both uniform and exponential message durations and evaluate the impact. We plot the predicted throughput for both uniform and exponential message durations (using single standard deviation whiskers) for a variety of channel counts and system utilization. We call this experiment a *distribution assessment*.

DES Experiments to Count and Assess Significance of Imperfect Allocations

We also modified the DES to count imperfect allocations. This number was compared to the number of normal allocations, constructing a ratio for comparison. We modified the simulator to record the number of imperfect spectrum allocations by counting whether or not a free channel is available each time a message completes using a shared channel.

The resulting throughput ratio, R_T, provides a normalized measure of variability in throughput that can be used to judge the impact of imperfect allocations. Defined as twice the coefficient of variation, its intent is to provide a comparison point for imperfect allocations (i.e., if the ratio of imperfect allocations to total allocations is lower than R_T, they can be considered to be infrequent enough to be within the normal stochastic variation inherent in the simulation model).

$$R_T = \frac{2 \times \textit{throughput std. dev.}}{\textit{mean throughput}} \tag{5.7}$$

The ratio of total messages transmitted relative to the number of imperfect allocations provides a measure of their significance.

$$R_I = \frac{\textit{imperfect allocations}}{\textit{total allocations}} \tag{5.8}$$

We contrast the two ratios, R_T and R_I, to evaluate the effects of imperfect allocations on the two models' throughput predictions. We call this experiment an assessment of *imperfect allocations.*

Comparison of MDP to DES

We plot the throughput predictions made by the MDP and DES models for different experimental configurations and compare trends between the two models' results. We then configure the MDP to generate a value-optimal policy for spectrum allocation. We test the local optimality of the allocations dictated by that policy by evaluating DES-predicted throughput in configurations in the neighborhood of those that use the value-optimal policy. In these experiments, the neighborhood is defined by varying the allowed message overlap. We call these experiments a *throughput evaluation*.

5.4.2 Experimental Results

Here, we are interested in exploring the quantitative results of the set of experiments described above. We organize the results according to the three hypotheses that we wish to evaluate:

Distribution Assessment Experiment

The intent of the distribution assessment experiment is to determine whether or not the mean of the message size distribution is sufficient to characterize the achievable throughput (i.e., how important is the shape of the distribution).

Figure 5.8 plots message throughput predicted by the DES model as a function of offered load for a pair of different circumstances:

1. $P_{conf} = 0.1$, single channel ($C = 1$), no overlap of messages ($L = 1$), and offered load ranging from 0.5 to 2.5.

2. $P_{conf} = 0.1$, four channels ($C = 4$), overlap of up to 4 messages ($L = 4$), and offered load ranging from 0.5 to 2.5.

For each of these circumstances, the simulator is configured to use both the uniformly distributed message duration and the exponentially distributed message duration. Points represent the mean over 100 simulation executions and the whiskers represent standard deviation of 10 batches (each of size 10).

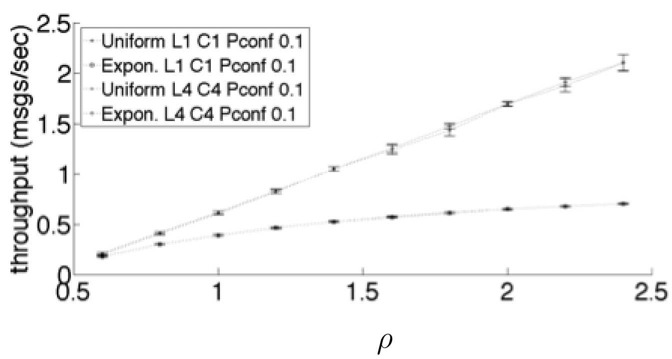


Figure 5.8: Throughput vs. offered load for both uniform and exponential distribution of message durations, ρ is offered load (λ/μ). Whiskers represent standard deviation.

We observe that the throughput predictions are reasonable, with greater throughput achievable with greater capacity. As is readily apparent in the plots, the distinction in throughput between these two distribution assumptions is quite small and is well within the expected deviations due to statistical variation. This evidence thus strongly supports hypothesis 1, that "performance can be reasonably characterized by the mean of message durations and is relatively insensitive to their distribution."

To be clear, we have only truly verified the correspondence between the uniform distribution (commonly used in the RF literature and the original assumption present in the discrete-event simulation model) and the exponential distribution (used in the Markov decision process model). Where this hypothesis might yet not hold is for extreme values of the true distribution's variance. E.g., deterministic (fixed duration) messages with zero variance or more heavy-tailed distributions with large variance. We leave this investigation for future work.

Imperfect Allocations Experiment

Hypothesis 2 posits insignificant performance variation due to imperfect allocations that are not faithfully reproduced in the MDP model. The imperfect allocations are recorded using the simulator to assess the significance of ignoring these occurrences. We record the occurrences when a channel is empty but allocations are yet made on already allocated channels. We examine:

1. The four channel system ($C = 4$), overlap of up to 4 messages ($L = 4$), and offered load ranging from 0.5 to 2.5, with

2. $P_{conf} = 0.1$, 0.5, and 0.9.

We constrain the experiment to the four channel system with overlap since the other configurations cannot exhibit imperfect allocation. The 1000 independent DES simulations executed for each value of P_{conf} constitute over 500,000 allocations. The results are presented in Table 5.3. Recall that R_I is the ratio of imperfect allocations to total allocations and that R_T is twice the coefficient of variability in the resultant throughput predicted by the simulator.

These results indicate that only a limited number of messages are affected by the imperfect allocations not accounted for with the MDP model (approximately equal to 2 standard

Table 5.3: Frequency of Imperfect Allocations.

P_{conf}	R_I	R_T
0.1	0.0160	0.0156
0.5	0.0165	0.0156
0.9	0.0162	0.0156

deviations of the simulation's statistical variability). As a result, the maximum impact on throughput is well within the statistical variation illustrated in Figure 5.8, and this evidence supports hypothesis 2. Also, the low impact of imperfect allocations implies that the greedy allocation algorithm is working quite well, i.e., essentially indistinguishable from a perfect allocator. Note that this conclusion is only valid for the MDP being considered, and is not a more general result.

Throughput Evaluation Experiments

Hypothesis 3 examines the use of the MDP model for evaluating optimal throughput characterizations. We assess this hypothesis by examining two things: (1) throughput predictions made by both the DES and MDP models; and (2) the local optimality (as confirmed by the DES model) of value-optimal policies chosen by the MDP. The throughput predictions are from the following set of experiments:

1. $P_{conf} = 0.9$, using single channel ($C = 1$), no overlap of messages ($L = 1$), and offered load ranging from 0.5 to 2.5.

2. $P_{conf} = 0.5$, using four channels ($C = 4$), no overlap of messages ($L = 1$), and offered load ranging from 0.5 to 2.5.

3. $P_{conf} = 0.1$, using four channels ($C = 4$), overlap of up to 4 messages ($L = 4$), and offered load ranging from 0.5 to 2.5.

The first experiment is limited to no overlap of messages primarily because with such a large value of P_{conf}, overlapping messages do not effectively improve throughput in any event. With only a single channel and no message overlap allowed, we expect a maximum throughput bounded above by an individual channel's capacity. The latter two experiments explore the use of additional channels and message overlap for two different values of P_{conf}. Here, we would expect to see some variation in achievable throughput between the two experiments. In each of the above experiments, we are comparing the throughput predictions of the MDP to that of the DES. To accomplish this while manually controlling the MDP policy (i.e., number of overlap messages allowed), we disable the value-optimal policy evaluation within the MDP and manually set the policy we wish to explore. This manual policy setting action transforms the Markov decision process into a traditional Markov process, for which we can determine the throughput by solving for the steady-state occupancy probabilities for each state χ in the original MDP.

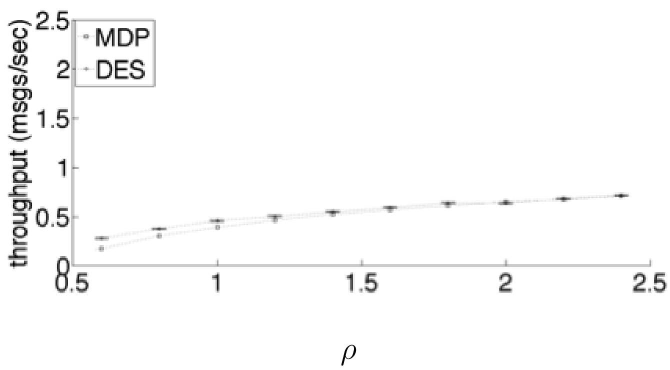
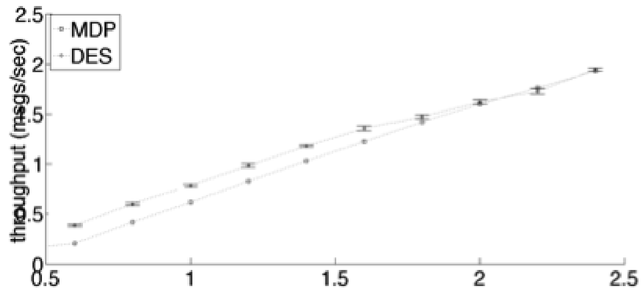
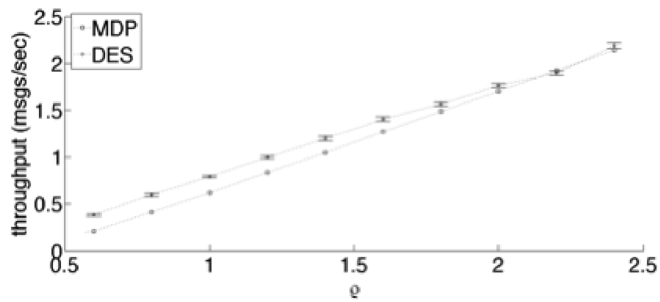


Figure 5.9: MDP vs. DES model throughput predictions for $P_{conf} = 0.9$, $C = 1$, $L = 1$.



ρ

Figure 5.10: MDP vs. DES model throughput predictions for $P_{conf} = 0.5$, $C = 4$, $L = 1$.



ρ

Figure 5.11: MDP vs. DES model throughput predictions for $P_{conf} = 0.9$, $C = 4$, $L = 4$.

As was expected, the maximum throughput achievable with only a single channel (Figure 5.9) is quite limited. Achievable throughputs, however, increase as additional resources are made available (Figures 5.10 and 5.11).

All three plots show reasonable agreement between the throughput predictions made by the MDP model and the DES model. While the MDP model's throughput predictions are not always within one standard deviation of the DES model's mean throughput, even when they separate it is not by far. Additionally, as the offered load increases, the separation between the two models actually diminishes. Given that imperfect information is less important to an admission algorithm at low load, we are much more interested in the correspondence between the two models under high-load conditions.

After concluding that we are able to model the throughput using the MDP model, we re-enable the ability of the MDP to choose a value-optimal policy. This allows us to test the (local) optimality of the policy chosen by the MDP by using the simulator. We confirm local optimality by doing a local neighborhood search with the discrete-event simulator and assessing whether or not the policy chosen by the MDP corresponds to the local throughput maximum.

The experiment is run with $P_{conf} = 0.7$, which gives a value-optimal admission policy (provided by the MDP) of using up to 2 levels of overlap (i.e., up to 2 messages per channel, but no more). Using the simulator, we assess the throughput predictions for $L = 1$ (no overlap), $L = 2$ (value-optimal according to the MDP), and $L = 3$. The mean throughput predictions from the DES model are shown in Table 5.4.

Table 5.4: Local Optimality.

L	1	2	3
throughput	1.93	2.11	2.07

Although the separation between these throughput predictions is not very large, the throughput for $L = 2$ is clearly above that of $L = 1$ and $L = 3$. The value-optimal policy chosen by the MDP is confirmed to be locally optimal as assessed by the DES model. This provides evidence for confirming hypothesis 3.

5.5 Chapter Summary

This chapter has assessed the use of MDP models for making management decisions for RF spectrum. We have formulated 3 distinct hypotheses, developed and conducted experiments to assess each of these hypotheses, and the empirical results all support the confirmation of the hypotheses. We therefore conclude that the use of MDPs for RF spectrum management is a reasonable (and potentially fruitful) path to explore. The example MDP we used in this chapter is suitable for admission decisions, and we have provided evidence that (for this simple case) a greedy placement algorithm is effective. Further validation studies, for the other MDP models developed in this dissertation in particular, are warranted prior to applying any of them in practice. However, as those models explore a larger space of modeling design issues and assumptions, doing so is deferred to future work.

Chapter 6

Conclusions and Future Work

In this chapter we first summarize the contributions of this dissertation. We then describe future directions for research to extend this work, and connect those new directions to specific advances in this work.

6.1 Conclusions

This dissertation focuses on the efficient use of RF spectrum, with a particular emphasis on the design of management techniques for controlling the spectrum use. To this end, we have developed Markov Decision Process (MDP) models for both admission and modulation decisions within RF spectrum. An initial MDP model (the Bernoulli MDP) confirms the previously known result that overlays of narrowband transmitters using a common modulation type are ineffective at increasing throughput. Another MDP model (the Shannon MDP) provides guidance for the combination of both admission and modulation decisions (either direct sequence spread spectrum or orthogonal frequency division multiplexing).

These MDPs are but two examples in a potentially large family of Markovian models that explore a wide range of parameterizations and management decisions. The examples described in the dissertation include multiple transmitters, modulation types, message sizes, as well as variable modulation efficiency, offered load, and signal-to-noise ratio. The MDPs support two distinct action sets and two fundamentally different reward functions. Potential expansions of this set of examples are described in the next section below.

Upon examination of the value-optimal policies, we observed repeatable patterns in the action choices. These patterns identified portions of the state space for which the action is common, separated by readily quantifiable boundaries. These patterns were retained across multiple parameterizations.

In an effort to develop more computationally efficient management decisions, we forgo the explicit solution of value optimal policies. These require time exponential in the size of the state space. Instead, we exploit the identified patterns to develop heuristics that effectively mimic the value optimal policies produced by the MDPs.

Using multivariate regression, we characterize the boundaries between regions of common action as functions of the input parametrization (e.g., offered load, modulation efficiencies, and signal-to-noise ratio). These boundaries are then utilized within an on-line decision algorithm that makes both admission and modulation decisions (i.e., chooses the appropriate action). When compared with the true value optimal action decisions, the heuristic consistently does a good job mimicking the MDP (r^2 values universally over 0.9).

To assess the appropriateness of using MDP models in RF spectrum management, we compared a number of properties of the MDP to a distinct discrete-event simulation model. This include the insensitivity of throughput to the distribution of message durations, the appropriateness of using a greedy allocation algorithm (and assuming it results in near perfect allocations), and the local-optimality (verified by the discrete-event simulation) of value optimal policies produced by the MDP.

6.2 Future Work

In Chapter 2, we identified a number of control parameters that can have substantial influence on the overall data throughput (i.e., management choices that RF system designers could potentially have at their disposal). In this dissertation, we focused on a subset of this parameter space, concentrating on admission and modulation decisions. Opportunities for future work include the addition of the following into the model(s):

- **placement decisions** – what frequencies should be occupied by a transmission,
- **power levels** – at what power level should transmissions occur,
- **coding choices** – what channel codes, error codes, etc., should be used,
- **spectrum organization** – how should the spectrum be divided into channels, and
- **antenna control** – how do we manage a multi-element antenna, and
- **message size** – how many channels should an individual message occupy.

In addition, the input parameters we consider could be expanded beyond the current set of spectrum size, offered load, modulation efficiencies, and signal-to-noise ratio. Options here include: additional modulation types, loss parameters (e.g., bit-error rates), antenna properties, etc. Furthermore, the ranges of input parameter values could be expanded, both in terms of the specific ranges and also the heterogeneity of their combinations. As an example, the empirical results presented here (in Chapter 4) are limited to a square MDP state space, while other rectangular state spaces clearly are relevant in the real world (e.g., with C OFDM channels and K SS codes, the state space would be of dimension $C \times K$).

Beyond control or input parameters, this work exploited both Shannon capacity and Bernoulli models of wireless channels. Additional physical phenomena that could be considered with more comprehensive channel models include distance, power, fading, multi-path effects, terrain, etc.

As noted in Chapter 3, another important extension of this work would be to replace the assumed centralized control manager with a distributed control protocol. We expect that the results obtained here would apply locally within distinct geographic regions of a distributed system (e.g., cells) with federated coordination among those regions being an open area of investigation.

While we dedicated a reasonable effort to validating both the models and their applicability to RF spectrum management, any empirical validation is subject to additional expansion. Specifically, we focused our validation efforts on the Bernoulli MDPs, and therefore the calibration and evaluation of the Shannon MDP warrants additional effort.

In all of these potential research directions, the work presented in this dissertation serves as a precursor to further investigation. In addition, the research presented here is applicable to a meaningful class of wireless communication scenarios (i.e., those with fixed channel boundaries and a centralized spectrum manager).

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Appendix A

Glossary

RF spectrum physical model			
Term	**Definition**	**Units**	**Comments**
C	number of channels		size of the RF spectrum
$F1$	spectrum boundary	Hz	low end of RF spectrum
Fn	spectrum boundary	Hz	high end of RF spectrum
B_C	channel bandwidth	Hz	$(Fn - F1)/C$
EOL	message end of life	s	individual message duration
μ	mean message service rate	msgs/s	inverse of mean EOL
λ	mean message arrival rate	msgs/s	

Bernoulli MDP models			
Term	**Definition**	**Units**	**Comments**
P_{env}	prob. of environmental failure		
P_{conf}	prob. of conflict failure		
N_m	num. of messages sharing a channel		
P_{succ}	prob. of successful message delivery		
$\mathcal{X}$	set of states		
χ	num. of current transmitters	$\chi \in \mathcal{X}$	basic model
(x_2, x_1)	num. of current transmitters of size i	$(x_2, x_1) \in \mathcal{X}$	extended model
$\mathcal{A}$	set of actions		$\mathcal{A} = \{\mathtt{a}, \mathtt{na}\}$
δ	uniformization rate parameter	s	
P_λ	prob. of message arrival		λ/δ
P_μ	prob. of message departure		$\chi\mu/\delta$

Shannon MDP models			
Term	**Definition**	**Units**	**Comments**
S	signal power		at receiver
N	noise power		at receiver
r_{SN}	signal-to-noise ratio		S/N
γ_e	modulation efficiency	bits/s/Hz	
$\gamma_{e.SS}$	SS modulation efficiency	bits/s/Hz	
$\gamma_{e.OFDM}$	OFDM modulation efficiency	bits/s/Hz	
C_s	channel capacity	bits/s	
(y_2, y_1)	num. of transmitters of modulation type i	$(y_2, y_1) \in \mathcal{X}$	
$\mathcal{A}$	set of actions		$\mathcal{A} = \{\texttt{as}, \texttt{af}, \texttt{na}\}$
OL	offered load		utilization λ/μ

Appendix B

Empirical Results Varying Offered Load

The tables below (Table B.1 to B.12) show the effect that offered load has on the value-optimal policy, we vary the input rate while keeping the message duration (which determines the effective service rate) constant. Defining the offered load as the ratio between input rate and service rate, its units are therefore in erlangs (E). The other input parameters for each of these examples are as follows: spectrum size = $C = 16$ channels, SS and OFDM efficiency = $\gamma_e = 1$, and signal-to-noise ratio = $r_{SN} = 2$).

Here, we observe that the pattern of an easy to identify boundary segregating actions chosen by the value-optimal policy is not significantly effected by changes in the offered load (i.e., the pattern is relatively insensitive to offered load). In the unrestricted region, the boundary line is near the center of the region, and as offered load increases, the slope of the line slowly gets steeper.

Table B.1: Offered Load of 0.2 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.2: Offered Load of 0.4 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.3: Offered Load of 0.6 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.4: Offered Load of 0.8 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.5: Offered Load of 1 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.6: Offered Load of 1.2 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.7: Offered Load of 1.4 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.8: Offered Load of 1.6 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.9: Offered Load of 1.8 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.10: Offered Load of 2 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.11: Offered Load of 2.2 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table B.12: Offered Load of 2.4 Erlangs.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Appendix C

Empirical Results Varying SS Modulation Efficiency

To investigate the effect that modulation efficiency has on the value-optimal policy, we vary the spread spectrum modulation efficiency, $\gamma_{e.SS}$, while keeping other parameters constant. Tables C.1 to C.24 show the value-optimal policy for spread spectrum modulation efficiencies ranging from 0.7 to 0.999. The other input parameters for each of these examples are as follows: offered load = $OL = 0.6$ E, spectrum size = $C = 16$ channels, OFDM modulation efficiency = $\gamma_{e.OFDM} = 1$, and signal-to-noise ratio = $r_{SN} = 2$).

Here, we observe that the unrestricted region of the state space exhibits the same pattern as we have previously seen. There exists a linear boundary that separates the region into two, with the value-optimal policy being all actions to one side of the boundary are `‘as’` and all actions to the other side of the boundary are `‘af’`. As the input efficiencies vary, not only does the area on each side of the boundary change, but the slope of the boundary line changes as well.

Table C.1: Spread Spectrum Efficiency of 70 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.2: Spread Spectrum Efficiency of 72 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.3: Spread Spectrum Efficiency of 74 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.4: Spread Spectrum Efficiency of 76 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.5: Spread Spectrum Efficiency of 78 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.6: Spread Spectrum Efficiency of 80 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.7: Spread Spectrum Efficiency of 82 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.8: Spread Spectrum Efficiency of 84 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.9: Spread Spectrum Efficiency of 86 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.10: Spread Spectrum Efficiency of 88 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.11: Spread Spectrum Efficiency of 90 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.12: Spread Spectrum Efficiency of 94 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.13: Spread Spectrum Efficiency of 97 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.14: Spread Spectrum Efficiency of 98 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.15: Spread Spectrum Efficiency of 99 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.16: Spread Spectrum Efficiency of 99.1 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.17: Spread Spectrum Efficiency of 99.2 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.18: Spread Spectrum Efficiency of 99.3 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.19: Spread Spectrum Efficiency of 99.4 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.20: Spread Spectrum Efficiency of 99.5 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.21: Spread Spectrum Efficiency of 99.6 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.22: Spread Spectrum Efficiency of 99.7 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.23: Spread Spectrum Efficiency of 99.8 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table C.24: Spread Spectrum Efficiency of 99.9 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Appendix D

Empirical Results Varying OFDM Modulation Efficiency

To investigate the effect that modulation efficiency has on the value-optimal policy, we vary the OFDM modulation efficiency, $\gamma_{e.OFDM}$, while keeping other parameters constant. Tables D.1 to D.24 show the value-optimal policy for OFDM modulation efficiencies ranging from 0.7 to 0.999. The other input parameters for each of these examples are as follows: offered load = OL = 0.6 E, spectrum size = C = 16 channels, SS modulation efficiency = $\gamma_{e.SS} = 1$, and signal-to-noise ratio = $r_{SN} = 2$).

Here, we observe that the unrestricted region of the state space exhibits the same pattern as we have previously seen. There exists a linear boundary that separates the region into two, with the value-optimal policy being all actions to one side of the boundary are `'as'` and all actions to the other side of the boundary are `'af'`. As the input efficiencies vary, not only does the area on each side of the boundary change, but the slope of the boundary line changes as well.

Table D.1: Orthogonal Frequency Division Multiplexing Efficiency of 70 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.2: Orthogonal Frequency Division Multiplexing Efficiency of 72 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.3: Orthogonal Frequency Division Multiplexing Efficiency of 74 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.4: Orthogonal Frequency Division Multiplexing Efficiency of 76 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.5: Orthogonal Frequency Division Multiplexing Efficiency of 78 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.6: Orthogonal Frequency Division Multiplexing Efficiency of 80 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.7: Orthogonal Frequency Division Multiplexing Efficiency of 82 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.8: Orthogonal Frequency Division Multiplexing Efficiency of 84 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.9: Orthogonal Frequency Division Multiplexing Efficiency of 86 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.10: Orthogonal Frequency Division Multiplexing Efficiency of 88 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.11: Orthogonal Frequency Division Multiplexing Efficiency of 90 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.12: Orthogonal Frequency Division Multiplexing Efficiency of 94 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.13: Orthogonal Frequency Division Multiplexing Efficiency of 97 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.14: Orthogonal Frequency Division Multiplexing Efficiency of 98 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.15: Orthogonal Frequency Division Multiplexing Efficiency of 99 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.16: Orthogonal Frequency Division Multiplexing Efficiency of 99.1 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.17: Orthogonal Frequency Division Multiplexing Efficiency of 99.2 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.18: Orthogonal Frequency Division Multiplexing Efficiency of 99.3 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.19: Orthogonal Frequency Division Multiplexing Efficiency of 99.4 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.20: Orthogonal Frequency Division Multiplexing Efficiency of 99.5 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.21: Orthogonal Frequency Division Multiplexing Efficiency of 99.6 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.22: Orthogonal Frequency Division Multiplexing Efficiency of 99.7 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.23: Orthogonal Frequency Division Multiplexing Efficiency of 99.8 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table D.24: Orthogonal Frequency Division Multiplexing Efficiency of 99.9 Percent.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Appendix E

Empirical Results Varying Signal-to-Noise Ratio

To investigate the effect that signal-to-noise ratio has on the value-optimal policy, we vary the signal-to-noise ratio, r_{SN}, while keeping other parameters constant. Tables E.1 to E.12 show the value-optimal policy for signal-to-noise ratios ranging from 1 to 12. The other input parameters for each of these experiments are as follows: spectrum size = $C = 16$ channels, offered load = $OL = 0.6$ E, and SS and OFDM efficiency = $\gamma_e = 1$.

The first observation here is the restricted regions behave differently than what we have previously observed. At sufficiently high SNR, the value-optimal policy includes some "no accept" (`'na'`) actions, where for the previously investigated parameter settings (each of which had low SNR) this was not the case.

To investigate whether or not this boundary is soley a function of SNR, two additional tables show the value-optimal policy for a signal-to-noise ratio of 8, varying first SS (Figure E.13) and then OFDM efficiency (Figure E.14) from 1 to 0.97. This provides evidence that the "no accept" decisions in the restricted regions of the state space are a function of SNR alone.

The second observation is that the boundary line between distinct value-optimal actions still exists in the unrestricted region of the state space. In this case, the slope of the boundary line is a strong function of the signal-to-noise ratio.

Table E.1: Value-optimal policy, Signal-to-Noise Ratio of 1.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.2: Value-optimal policy, Signal-to-Noise Ratio of 2.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.3: Value-optimal policy, Signal-to-Noise Ratio of 3.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.4: Value-optimal policy, Signal-to-Noise Ratio of 4.

16	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	as
0	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.5: Value-optimal policy, Signal-to-Noise Ratio of 5.

16	na	af	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.6: Value-optimal policy, Signal-to-Noise Ratio of 6.

16	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.7: Value-optimal policy, Signal-to-Noise Ratio of 7.

16	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.8: Value-optimal policy, Signal-to-Noise Ratio of 8.

16	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.9: Value-optimal policy, Signal-to-Noise Ratio of 9.

16	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.10: Value-optimal policy, Signal-to-Noise Ratio of 10.

16	na	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.11: Value-optimal policy, Signal-to-Noise Ratio of 11.

16	na	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.12: Value-optimal policy, Signal-to-Noise Ratio of 12.

16	na	na	na	na	na	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.13: Value-optimal policy, Signal-to-Noise Ratio of 8, SS modulation efficieny of 0.97.

16	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Table E.14: Value-optimal policy, Signal-to-Noise Ratio of 8, OFDM modulation efficieny of 0.97.

16	na	na	na	af	af	af	af	af	af	af	af	af	af	af	af	af	na
15	as	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	as
14	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
13	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
12	as	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	as
11	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
10	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
9	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
8	as	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	as
7	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
6	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
5	as	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	as
4	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
3	as	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	na
2	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
1	as	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	na
0	as	as	as	as	as	as	af	af	af	af	af	af	af	af	af	af	na
SS/OFDM	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

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August 2015