Report Number: WUCS-92-08

1992

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January 1992

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COMMUN. STATIST.— STOCHASTIC MODELS, 8(2), 289-305 (1992)

OPTIMAL BURST LEVEL ADMISSION CONTROL IN A BROADBAND NETWORK

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Key Words: Admission control, ATM network, linear programming, Norton equivalent, statistical multiplexing.

ABSTRACT

In a broadband ATM network the traffic of a virtual circuit is defined at the cell, burst and call levels. All virtual circuits sharing the resources of a switch are statistically multiplexed at the cell level. In this paper the issue of how to control the admission of bursts of a particular virtual circuit is analyzed. It is demonstrated that under two optimization criteria, the optimal burst level admission control of a virtual circuit is a window control. This result suggests that while the cells of all virtual circuits sharing the resources of a switch should be serviced using the statistical multiplexing technique, at the burst level, the total number of bursts of a particular virtual circuit admitted inside the network should be monitored and controlled in such a way that the number of bursts does not exceed a given upper bound, which is the quota for that particular virtual circuit.

1. Introduction

Research concerning ATM networks is currently focused in the study of teletraffic problems appearing in an ATM environment. Each virtual circuit in the ATM layer results in a cell sequence (CS) that can be both analyzed and controlled at one or more of the following time scales: call, burst, and cell [1,10]. At each of the time scales for which a CS control is provided, a resource allocation scheme can be introduced and classified as corresponding to the call, burst, or cell time scale.

In order to design an integrated traffic characterization and control infrastructure capable of guaranteeing a grade of service (GOS) and providing that GOS at the minimum cost, the following are required: (i) the design of a traffic control and resource allocation infrastructure capable of efficiently handling a wide variety of traffic behaviors, (ii) given a particular traffic control and resource allocation scheme, the determination of the GOS that can be provided to an incoming CS, (iii) given a particular GOS requirement, the determination of the traffic control and resource allocation scheme required to provide to an incoming CS the desired GOS at the minimum cost. The work presented in this paper is part of an ongoing effort directed towards satisfying these requirements [2].

In a broadband network cells are statistically multiplexed and are served on a first-come first-served (FCFS) basis. For many of the services that will be supported by a future broadband network, cell level control of an incoming CS will not be sufficient. This is particularly true for data services, such as file or image transfer, in which information is transferred in bursts which are potentially several Mbits long. These services could be better supported by a network with switches capable of reserving resources for the transmission of the whole burst.

Previous work on burst level resource allocation has been reported in [1, 10]. In both of these references, alternative ways in which a switching system might support burst level resource allocation have been investigated. No study has been made, however, of the network-wide implications of burst level control. This paper attempts to address this issue by focusing on the burst level admission control for a network virtual circuit.

A burst is defined by its cell rate and its size, which is expressed in terms of number of cells. For the purposes of this paper, it is assumed that the burst size of a particular virtual circuit is exponentially distributed. In addition it is assumed that, at any given moment, the rate at which bursts are admitted into the network is a function of only the total number of bursts of that particular virtual circuit that are still in the network. In practice this information can be easily obtained by acknowledging successfully delivered bursts.

This paper is organized as follows. In Section 2, the problem formulation is introduced. In Section 3, some properties concerning the stochastic monotonicity of a finite birth-death process are presented and are utilized in subsequent sections. In Sections 4 and 5, the optimal burst admission control problem is analyzed under two distinct optimization criteria. In Section 6, a number of properties relevant to the admission control policy are proven.

2. The Statement of the Problem

It is assumed that each of the I switches of the broadband network can serve any number of bursts by appropriately dividing the switch's bandwidth among the number of contending bursts. It is also assumed that the length of each burst is exponentially distributed, and therefore a switch can be modelled as an exponential server with infinite buffer size and with exponential service rate. At the cell level, bursts are multiplexed using statistical multiplexing, which results in the cells being served in the order in which they are accepted. The behavior of a switch serving a number of bursts can be modelled as an exponential processor that completely shares its resources among the queued bursts. A network which operates under these conditions is a product-form network.

Let μ_i be the service rate of the i^{th} switch, $i \in I$. The routing parameter of a particular connection can be defined by the $(I+1) \times (I+1)$ routing matrix $(0 \leq i \leq I,\ 0 \leq j \leq I)$. In this notation, bursts join the network at switch i with probability r_{0i}. Upon completion of service at switch i, bursts leave the network with probability r_{i0} or are routed from switch i to switch j with probability r_{ij}.

The evolution of the queueing network is described by the stochastic process

$$\mathbf{Q}_t^* \stackrel{def}{=} (Q_t^1, \cdots, Q_t^I) \quad ,$$

where Q_t^i refers to the number of bursts at switch i, $1 \leq i \leq I$. The state space of the system is given by

$$E^* \stackrel{def}{=} \left\{\mathbf{k} \;=\; (k_1, \cdots, k_I) \,\middle|\, 0 \leq k_i, \quad i = 1, 2, \cdots, I\right\} \quad .$$

In what follows, the states of $\mathbf{Q}_t^*$ will be aggregated to form a new state space. A new process Q_t is defined by

$$Q_t \stackrel{def}{=} Q_t^1 + \cdots + Q_t^I \quad .$$

The state space of Q_t is given by

$$E \stackrel{def}{=} \left\{k_1 + \cdots + k_I \,\middle|\, 0 \leq k_i, \quad i = 1, 2, \cdots, I\right\} \quad .$$

Let λ_k refer to the burst arrival rate when there are k bursts in the network. Let EQ be the expected number of bursts in the network, $E\gamma$ the throughput, and $E\tau$ the expected delay of a burst in the network. Let c_k represent the maximum rate by which the virtual circuit's controller can send bursts into the network when the state of the network is k, for every k, $k \geq 0$, where c_k is a function of the internals of virtual circuit's source and the virtual circuit's network interface and the state of the network. Thus $0 \leq \lambda_k \leq c_k$, for every k, $k \geq 0$.

Let the $1 \times (I+1)$ matrix $\Theta \stackrel{def}{=} [\theta_0\ \theta_1\ \cdots\ \theta_I]$ be the solution of the traffic flow equations

$$\Theta \;=\; \Theta \mathbf{R} \quad ,$$

where $\theta_0 = 1$.

The service rate of the process Q_t is the conditional service rate of the network given that there are Q_t bursts in the network. For product form networks this is given by the well known Norton equivalent [4, 9, 11]. Let

$$g_k \stackrel{def}{=} \sum_{k_1+k_2+\cdots+k_I=k} \prod_{j=1}^{I} \left(\frac{\theta_j}{\mu_j}\right)^{k_j} \quad , \tag{2.1}$$

where $0 \le k_i$, for $i = 1, \cdots, I$, and for all k, $k \ge 1$. If k is the total number of bursts in switches $1, 2, \cdots, I$, then the Norton equivalent, symbolized by ν_k, is given by

$$\nu_k \stackrel{def}{=} \frac{g_{k-1}}{g_k} \quad , \tag{2.2}$$

and is a concave increasing function with respect to k, for all $k \ge 1$ [9].

The decision concerning which virtual circuit controller policy to enforce requires the introduction of an optimization criterion that is based on the information available to the controller, namely the total number of bursts currently in the network. In this paper two different criteria are utilized.

First Optimization Criterion: Maximize the throughput of the network, under the constraint that the expected time delay of a burst in the network does not exceed an upper bound:

$$\max_{EQ-TE\gamma\le 0} E\gamma \quad . \tag{2.3}$$

Observe that the time delay constraint is not written in the form $\frac{EQ}{E\gamma} \le T$, since $\frac{EQ}{E\gamma}$ is not defined if $\lambda_k = 0$, for all $k \in E$. Instead, using Little's formula, the criterion is written in the form $EQ - TE\gamma \le 0$, where $EQ - TE\gamma$ is a continuous differentiable function with respect to the arrival rates.

Second Optimization Criterion: Minimize the expected time delay of a burst in the network, under the constraint that the expected throughput does not fall bellow a lower bound Γ:

$$\min_{E\gamma\ge\Gamma} E\tau \quad . \tag{2.4}$$

3. Stochastic Monotonicity of a Finite Birth-Death Process

From the point of view of a particular virtual circuit, the network behaves as a birth-death process. If the state of the network (i.e. the total number of bursts in the network) is k, then the service rate of the birth-death process (*i.e.* the rate at which bursts corresponding to the virtual circuit are serviced) is ν_k, and the arrival rate is λ_k. In order to further an understanding of the network performance under different state-dependent admission control policies, a number of properties concerning the stochastic monotonicity of a finite birth-death process are presented.

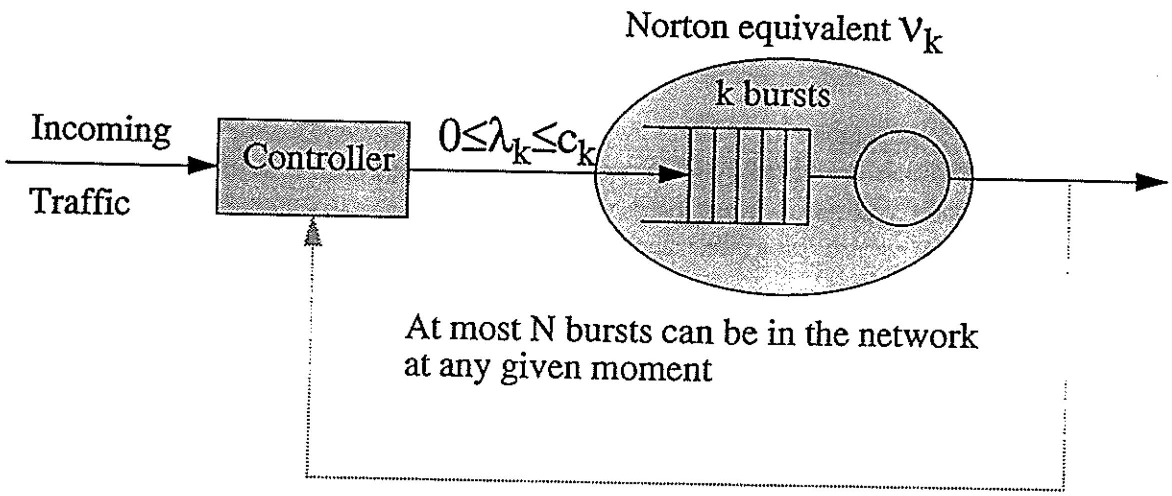

Figure 3.1

Assume that at most N bursts can be in the network at any given moment. The dynamic behavior of such a virtual circuit is described by the network depicted in Fig. 3.1.

Let $E\gamma_N$ be the throughput of this network. Let EQ_N^c and $E\tau_N^c$ be the expected number of bursts and the expected time delay of the bursts, respectively, in the virtual circuit's controller. Similarly, let EQ_N and $E\tau_N$ be the expected number of bursts and the expected time delay of the bursts, respectively, in the network. Observe that

$$E\gamma_N = \frac{EQ_N}{E\tau_N} = \frac{EQ_N^c}{E\tau_N^c} = \frac{N}{E\tau_N + E\tau_N^c} \quad .$$

Statements (i), (ii), and (v) of Proposition 3.2 are from [12] and are reported here in the context of the theory of stochastic monotonicity for finite state Markov chains.

Definition [6, pp. 164] *Let* $\mathbf{P}_\mathrm{N}$ *be the set of stochastic vectors in N-space, i.e.* $\mathbf{P}_\mathrm{N} = \{\mathrm{p} \overset{\mathrm{def}}{=} (p_0, p_1, \cdots, p_{N-1}) \mid \sum_{i=0}^{N-1} p_i = 1\}$ *and let* $\mathrm{p}^\#$ *and* p^* *be elements in* $\mathbf{P}_\mathrm{N}$. $\mathrm{p}^\#$ *is said to be larger stochastically than* p^* *or to dominate* p^*, *if* $\sum_{i=n}^{N-1} p_i^\# \geq \sum_{i=n}^{N-1} p_i^*$, *for all* n, $n = 0, 1, \cdots, N-1$. *One then writes* $\mathrm{p}^\# \succ \mathrm{p}^*$.

Proposition 3.1[6, pp. 170]: *If* $\mathrm{p}^\# \succ \mathrm{p}^*$, *and* ν_k *is an increasing function of* k, *then* $\sum_{k=0}^{N-1} \nu_k p_k^\# \geq \sum_{k=0}^{N-1} \nu_k p_k^*$.

Proposition 3.2

(i) If ν_k *is an increasing function of* k, *the expected throughput* $E\gamma_N$ *is increasing in* λ_k *for* $k = 0, 1, \cdots, N-1$.

(ii) The expected number of bursts EQ_N is increasing in λ_k for $k = 0, 1, \cdots, N-1$.

(iii) The expected number of bursts EQ_N^c is decreasing in λ_k for $k = 0, 1, \cdots, N-1$.

(iv) If ν_k is an increasing function of k, the expected time delay of the bursts in the controller $E\tau_N^c$ is decreasing in λ_k for $k = 0, 1, \cdots, N-1$.

(v) If $\frac{k}{\nu_k}$ is an increasing function of k, the expected time delay $E\tau_N$ is increasing in λ_k for $k = 0, 1, \cdots, N-1$.

Proof :

Let $\mathbf{p}^* \stackrel{\text{def}}{=} (p_0^*, \cdots, p_N^*)$ correspond to the control $\lambda^* \stackrel{\text{def}}{=} (\lambda_0^*, \cdots, \lambda_{N-1}^*)$, and let $\mathbf{p}^\# \stackrel{\text{def}}{=} (p_0^\#, \cdots, p_N^\#)$ correspond to the control $\lambda^\# \stackrel{\text{def}}{=} (\lambda_0^\#, \cdots, \lambda_{N-1}^\#)$. Assume that $\lambda_k^\# \geq \lambda_k^*$ for $k = 0, 1, \cdots, N-1$. Then

$$\frac{p_k^*}{p_{k-1}^*} = \frac{\lambda_{k-1}^*}{\nu_k} \leq \frac{\lambda_{k-1}^\#}{\nu_k} = \frac{p_k^\#}{p_{k-1}^\#} \quad ,$$

for $k = 1, \cdots, N$, from which it follows that $\mathbf{p}^* \prec \mathbf{p}^\#$. Using Proposition 3.1, $\sum_{k=1}^N \nu_k p_k^* \leq \sum_{k=1}^N \nu_k p_k^\#$.

(ii) $EQ_N = \sum_{k=1}^N k p_k$. The arguments of (i) hold here if k is substituted for ν_k.

(iii) $EQ_N^c = N - EQ_N$. The statement is then true because of *(ii)*.

(iv) This statement holds because $E\tau_N^c = \frac{EQ_N^c}{E\gamma_N}$.

(v) $E\tau_N$ is a weighted average of $\frac{k}{\nu_k}$, for $k = 1, \cdots, N$, with weights $q_k^* = \frac{\nu_k p_k^*}{\sum_{i=1}^N \nu_i p_i^*}$. The arguments of *(i)* hold if q_k^* is substituted for p_k^*, and the statement follows.

∎

In a similar way, the following proposition can be proven.

Proposition 3.3

(i) If λ_k is a decreasing function of k, the expected throughput $E\gamma_N$ is increasing in ν_k for $k = 0, 1, \cdots, N$.

(ii) The expected number of bursts EQ_N^c is increasing in ν_k for $k = 0, 1, \cdots, N$.

(iii) The expected number of bursts EQ_N is decreasing in ν_k for $k = 0, 1, \cdots, N$.

(iv) If λ_k is a decreasing function of k, the expected time delay of the bursts in the network $E\tau_N$ is decreasing in ν_k for $k = 0, 1, \cdots, N$.

(v) If $\frac{N-k}{\lambda_k}$ is a decreasing function of k, the expected time delay $E\tau_N^c$ is increasing in ν_k for $k = 0, 1, \cdots, N$.

4. Optimal Burst Level Admission Control under the First Optimization Criterion

Let the probability that there are k bursts in the network (*i.e.* the state of the network is k) be given by p_k. Let r_k^1 refer to the probability that an incoming burst is allowed to enter the network, when the state of the network is k. Then $r_k^0 \stackrel{\text{def}}{=} 1 - r_k^1$ is the probability that the burst is not allowed to enter the network. Let $x_k \stackrel{\text{def}}{=} p_k r_k^1$ and $y_k \stackrel{\text{def}}{=} p_k r_k^0$. Then $\lambda_k = c_k r_k^1$, for every k.

EQ_N and $E\gamma_N$ represent the expected number of bursts in the network and the throughput of the network, respectively, given that at most N bursts can be in the network at any given moment. Similarly $E\tau_N$ denotes the expected delay of a burst in the network, given that at most N bursts can be in the network at any given moment. Thus

$$EQ_N = \sum_{k=1}^{N} k p_k = \sum_{k=1}^{N} k\,(x_k + y_k) \quad , \tag{4.1}$$

$$E\gamma_N = \sum_{k=1}^{N} p_k \nu_k = \sum_{k=1}^{N} (x_k + y_k)\,\nu_k \quad , \tag{4.2}$$

and

$$E\tau_N = \frac{EQ_N}{E\gamma_N} = \frac{\sum_{k=1}^{N} p_k k}{\sum_{k=1}^{N} p_k \nu_k} = \frac{\sum_{k=1}^{N} (x_k + y_k)\,k}{\sum_{k=1}^{N} (x_k + y_k)\,\nu_k} \quad . \tag{4.3}$$

The global balance equations (GBEs) are given by the following equations:

$$p_k r_k^1 c_k = p_{k+1} \nu_{k+1} \quad ,$$

or, equivalently,

$$x_k c_k = (x_{k+1} + y_{k+1}) \nu_{k+1} \quad , \tag{4.4}$$

for all k, $0 \le k \le N-1$.

Finally, if the total number of bursts in the network cannot exceed N, $\sum_{k=0}^{N} p_k = 1$. Equivalently,

$$\sum_{k=0}^{N} (x_k + y_k) = 1. \tag{4.5}$$

Proposition 4.1 *The optimal admission control parameters λ_k, $k \in E$, are given by the equations:*

$$\lambda_k = \begin{cases} c_k \left(\frac{x_k}{x_k + y_k} \right) & \text{if } x_k + y_k > 0 \\ 0 & \text{if } x_k + y_k = 0 \end{cases} \quad . \tag{4.6}$$

where (x_k, y_k), $k \in E$, *is the solution of the following iterative algorithm:*

Step 0: *L=1.*

Iteration:

Step 1: *For the current value of L, solve the following linear optimization problem:*

$$max \sum_{k=1}^{L} (x_k + y_k)\, \nu_k \tag{4.7}$$

under the following constraints:

$$\sum_{k=1}^{L} (x_k + y_k)\, k \le T \sum_{k=1}^{L} (x_k + y_k)\, \nu_k \quad , \tag{4.8}$$

$$x_k c_k = (x_{k+1} + y_{k+1})\, \nu_{k+1} \;\; for\; 0 \le k \le L-1 \quad , \tag{4.9}$$

$$\sum_{k=0}^{k=L} (x_k + y_k) = 1 \quad , \tag{4.10}$$

where

$$x_k \ge 0 \; and \; y_k \ge 0 \; for \; 0 \le k \le L \quad . \tag{4.11}$$

Step 2: *If* $E\gamma_L = E\gamma_{L-1}$, *stop; the derived admission control is optimal and is given by the Equation 3.12. Else,* $L := L + 1$, *and repeat all the steps of the iteration, using the optimal solution of the linear program as the initial feasible point of the next iteration.*

From the solution of the above iterative algorithm one can easily recover the parameters of interest. In particular $p_k = x_k + y_k$, for all k, $k \ge 0$. Finally for all the accessible states (*i.e.* for all k, such that $p_k > 0$,) $r_k^1 = \frac{x_k}{x_k + y_k}$ and $\lambda_k = c_k \frac{x_k}{x_k + y_k}$. Furthermore notice that if $p_L = 0$, then $p_k = 0$, for all $k \ge L$.

5. Optimal Burst Level Admission Control under the Second Optimization Criterion

As mentioned above, there are cases which require the minimization of the time delay, such that the throughput is greater than or equal to a given lower bound referred to as Γ. In the sequel, a methodology that reduces the derivation of the optimal burst admission control policy to a linear optimization problem is presented.

Lemma 5.1: *The optimization problem which requires the minimization of the expected time delay $E\tau_N$ under the contraint that the throughput $E\gamma_N \geq \Gamma$ can be formulated as the following optimization problem:*

$$\min \frac{\sum_{k=1}^{N}(x_k+y_k)\,k}{\sum_{k=1}^{N}(x_k+y_k)\,\nu_k} \quad , \tag{5.1}$$

under the throughput constraint

$$\sum_{k=1}^{N}(x_k+y_k)\,\nu_k \;\geq\; \Gamma \quad , \tag{5.2}$$

the GBE constraints

$$x_k c_k \;= (x_{k+1} \;+\; y_{k+1})\,\nu_{k+1} \quad for\ 0 \leq k \leq N-1 \quad , \tag{5.3}$$

the constraint that the sum of the probabilities equal one

$$\sum_{k=0}^{N}(x_k+y_k) \;=\; 1 \quad , \tag{5.4}$$

and the constraint that the probabilities be non-negative, i.e. $x_k \;\geq\; 0$, $y_k \;\geq\; 0$, for all k, $0 \;\leq\; k \;\leq N$.

Following the methodology introduced in [3, pp. 128], let

$$g \stackrel{\text{def}}{=} \frac{1}{E\gamma_N} = \frac{1}{\sum_{k=1}^{N}\nu_k(x_k+y_k)} \quad , \tag{5.5}$$

$$x_k^* \stackrel{\text{def}}{=} g x_k \quad , \tag{5.6}$$

and

$$y_k^* \stackrel{\text{def}}{=} g y_k \quad , \tag{5.7}$$

for all k, $\;0 \;\leq\; k \;\leq\; N$.

From (5.5) it follows that

$$\sum_{k=1}^{N}(x_k^*+y_k^*)\,\nu_k \;=\; 1 \quad . \tag{5.8}$$

Equation (5.1) for the expected time delay becomes

$$E\tau_N = \frac{\sum_{k=1}^{N}(x_k + y_k)\,k}{\sum_{k=1}^{N}(x_k + y_k)\,\nu_k} = \sum_{k=1}^{N}(x_k^* + y_k^*)\,k \quad . \tag{5.9}$$

Because $\sum_{k=0}^{N}(x_k + y_k) = 1$,

$$g = \sum_{k=0}^{N}(x_k^* + y_k^*) \quad .$$

By multiplying both sides of the throughput inequality constraint (5.2) by g, it follows that

$$gE\gamma_N \geq g\Gamma \quad ,$$

which in turn (becauuse $gE\gamma_N = 1$) can be re-written as

$$\Gamma \sum_{k=0}^{N}(x_k^* + y_k^*) \leq 1 \quad . \tag{5.10}$$

The GBEs (5.3) become

$$x_k^* c_k = \left(x_{k+1}^* + y_{k+1}^*\right)\nu_{k+1} \quad for\ 0 \leq k \leq N-1 \quad . \tag{5.11}$$

Lemma 5.2: *The optimization problem that requires the minimization of the expected time delay $E\tau_N$ under the contraint that the throughput $E\gamma_N \geq \Gamma$ can be formulated as the following optimization problem:*

$$\min \sum_{k=1}^{N}(x_k^* + y_k^*)\,k \quad ,$$

under the throughput constraint

$$\Gamma \sum_{k=0}^{N}(x_k^* + y_k^*) \leq 1 \quad . \tag{5.12}$$

the GBE constraints

$$x_k^* c_k = \left(x_{k+1}^* + y_{k+1}^*\right)\nu_{k+1} \quad for\ 0 \leq k \leq N-1 \quad , \tag{5.13}$$

the constraint that the sum of the probabilities equal one

$$\sum_{k=1}^{N}(x_k^* + y_k^*)\,\nu_k = 1 \quad , \tag{5.14}$$

and the constraint that $x_i^* \geq 0$, $y_i^* \geq 0$, for all k, $0 \leq k \leq N$.

From the solution of the linear optimization problem, it follows that $g = \sum_{k=0}^{N}(x_k^* + y_k^*)$, $E\gamma_N = \frac{1}{g}$, $x_k = \frac{x_k^*}{g}$, $y_k = \frac{y_k^*}{g}$, and $p_k = x_k + y_k$, for all k, $0 \leq k \leq N$. Finally for all accessible states (*i.e.* for all k such that $p_k > 0$,) $r_k^1 = \frac{x_k}{x_k + y_k} = \frac{x_k^*}{x_k^* + y_k^*}$ and $\lambda_k = c_k \frac{x_k^*}{x_k^* + y_k^*}$. Furthermore notice that if $p_L = 0$, then $p_k = 0$, for all $k \geq L$.

Proposition 5.3 *The optimal burst admission control parameters $\lambda_k, k \in E$, are given by the equations:*

$$\lambda_k = \begin{cases} c_k \left(\frac{x_k^*}{x_k^* + y_k^*} \right) & \text{if } x_k^* + y_k^* > 0 \\ 0 & \text{if } x_k^* + y_k^* = 0 \end{cases} , \tag{5.15}$$

where (x_k^*, y_k^*), $k \in E$, is the solution of the following iterative algorithm:

Step 0: *L=1.*

Iteration:

Step 1:

$$min \sum_{k=1}^{L} (x_k^* + y_k^*)\, k \tag{5.16}$$

under the following constraints:

$$\sum_{k=1}^{L} (x_k^* + y_k^*)\, \nu_k = 1 , \tag{5.17}$$

$$\Gamma \sum_{k=0}^{L} (x_k^* + y_k^*) \leq 1 , \tag{5.18}$$

$$x_k^* c = \left(x_{k+1}^* + y_{k+1}^* \right) \nu_{k+1} \quad for \quad 0 \leq k \leq L-1 , \tag{5.19}$$

where

$$x_k^* \geq 0 \text{ and } y_k^* \geq 0 \text{ for } 0 \leq k \leq L . \tag{5.20}$$

Step 2: *If the linear program has a feasible solution; stop. The derived admission control is optimal. Else, L := L+1, and repeat all the steps of the iteration.*

In the above iterative algorithm, each time a linear program needs to be solved, the solution of the previous step is used as the initial feasible point. This systematic approach to optimal admission control requires less computation than any other approach. Since the dimensionality of the problem is kept at a minimum, this iterative algorithm is optimal for the solution of the above class of problems.

6. Properties of the Optimal Burst Level Admission Control Policy under the Two Optimization Criteria

In this section structural properties of the optimal solution of each of the two linear programs presented in Sections 4 and 5 are presented.

Proposition 6.1 *The optimal solution of each of the two linear programs contains at most one random point. Hence the optimal solution of each linear program is of the form:*

$$\lambda_k = \begin{cases} c_k & \text{if } 0 \le k \le L-1 \text{ and } k \ne m \\ 0 < \lambda_m \le c_m & \text{if } k = m \\ 0 & \text{if } L \le k \end{cases}$$

Proof : First, this proposition is proved for the first optimization criterion. Let S be the set of states which are accessible under the solution of the linear program in Step 1 of the iterative algorithm in Proposition 4.1 at a particular iteration. That is

$$S = \{k \mid p_k > 0\} \quad .$$

Associated with every state k, $k \epsilon S$, is the global balance Equation 4.9. In addition the linear program must satisfy the independent inequality constraint 4.8. Thus if the total number of accessible states is L, the total number of independent constraints is $L+1$, the total number of variables is $2L$ and thus, the optimal basic solution should have $L-1$ variables which are non-basic and equal to zero. From the global balance equations 4.9, it is clear that if state k is accessible under the optimal basic solution, $x_l > 0$ for all l, $0 \le l \le k-1$. These variables represent L of the basic variables under the optimal solution. The optimal basic feasible solution has one additional basic variable, which must be chosen from the set of variables $\{y_k\}$ for $0 \le k \le L$. If that particular variable is y_m, then the solution of the iteration will be of the form

$$\lambda_k = \begin{cases} c_k & \text{if } 0 \le k \le L-1 \text{ and } k \ne m \\ 0 < \lambda_m \le c_m & \text{if } k = m \\ 0 & \text{if } L \le k \end{cases}$$

Therefore the optimal solution of the iterative algorithm in Proposition 4.1 must satisfy the same structural property. Thus under the first optimization criterion, there exists an optimal policy with no more than one random point is the whole state space.

Under the second optimization criterion, the proof is identical with the proof under the first optimization criterion, with the exception of the following substitutions. The variables $\{x_k^*, y_k^*\}$ are substituted for the variables $\{x_k, y_k\}$. The GBEs Equation 5.18 is used in place of the GBE Equation 4.9, and the inequality constraint Equation 5.17 is used in place of Equation 4.8. ∎

The problem under the first optimization criterion has been studied in [7] in the context of the optimal flow control of a Jackson network. In that paper it was proven that the optimal flow control of a Jackson network whose Norton equivalent is a concave increasing function with respect to the number of bursts in it [9], is a window flow control with the random point, if it exists, at the end of the window. In the proof presented in [7], it was assumed that at most N bursts can be in the network at any given moment. This assumption in turn has led many researchers to interpret the previous cited result as meaning that the optimal flow control of a *closed network* is a window flow control [11, pp. 20]. The problem analyzed in [7] was reformulated as a linear program in [12], where the window flow control result was proven using an important proposition appearing in this paper as Proposition 6.2. This proposition is utilized here because it gives a straightforward proof of the form of the optimal flow control, even under the second optimization criterion.

Proposition 6.2.([12]) *If ν_k is a concave increasing function with respect to k, the optimal flow control under the first and second optimization criteria is of a window type with the random point, if it exists, corresponding to the last burst of the window. Thus, the optimal flow control is of the form*

$$\lambda_k = \begin{cases} c_k & \text{if } 0 \le k \le L-2 \\ 0 < \lambda_{L-1} \le c_{L-1} & \text{if } k = L-1 \\ 0 & \text{if } L \le k \quad . \end{cases}$$

Proof: Let $(\lambda_0, \lambda_1 \cdots)$ and $(\lambda_0^*, \lambda_1, \cdots)$ correspond to two different control policies. It is now shown how to choose $c_m \ge \lambda_m^* > \lambda_m$, $\lambda_{m+1}^* < \lambda_{m+1}$, and $\lambda_k^* = \lambda_k$ for all k, $k \ne m$ and $k \ne m+1$, such that $E\gamma^* \ge E\gamma$ and $E\tau^* \le E\tau$. Observe that

$$(p_0^*, p_1^*, \cdots) = ((1-\delta)p_0, (1-\delta)p_1, \cdots, (1-\delta)p_{m-1}, (1+\epsilon)p_m, (1-n)p_{m+1}, \cdots)$$

δ, ϵ, and n are chosen such that

$$\sum_{k=0}^{\infty} p_k \ = \ 1$$

and

$$EQ^* \ = \ EQ \quad .$$

In other words,

$$\delta \sum_{k=0}^{m-1} p_k \ + \ n \sum_{k=m+1}^{\infty} p_k = \epsilon p_m$$

and

$$\delta \sum_{k=0}^{m-1} kp_k \ + \ n \sum_{k=m+1}^{\infty} kp_k \ = \ \epsilon m p_m \quad .$$

It is easy to see that if $\epsilon > 0$, then $\delta > 0$ and $n > 0$. Letting

$$\alpha_k = \begin{cases} \frac{\delta p_k}{\epsilon p_m} & \text{if } 0 \leq k < m \\ \frac{n p_k}{\epsilon p_m} & \text{if } m < k \end{cases} ,$$

we have $\alpha_k > 0$, for $k \neq m$ with

$$\sum_{k \neq m} \alpha_k = 1$$

and

$$\sum_{k \neq m} k\alpha_k = m .$$

By the concavity of ν_k with respect to k,

$$\sum_{k \neq m} \nu_k \alpha_k \leq \nu_m .$$

Thus,

$$E\gamma^* \geq E\gamma ,$$

and

$$E\tau^* \leq E\tau .$$

The last two equations prove that transformation of the arrival rate control policy in a way that makes it of a window type with the random point, it it exists, at the end of the window will simultaneously increase the throughput and decrease the expected time delay. From this it is straightforward to see that the optimal flow control under both optimization criteria is a window flow control with a random point, it it exists, at the end of the window.

Because $n > 0$,

$$\sum_{i=k}^{\infty} p_i^* < \sum_{i=k}^{\infty} p_i , \tag{6.1}$$

for all k, $k > m$. Similarly, because $\delta > 0$,

$$\sum_{i=k}^{\infty} p_i^* > \sum_{i=k}^{\infty} p_i , \tag{6.2}$$

for all k, $k < m$.

∎

It is straightforward to see how the above results simultaneously prove that the optimal admission control of burst is of a window type under both optimization criteria. Therefore, if ν_k is a concave increasing function of k, the optimal feasible point which permits L bursts to enter the network is of the form

$$\lambda_k = \begin{cases} c_k & \text{for } 0 \leq k \leq L-2 \\ 0 < \lambda_{L-1} \leq c_{L-1} & \text{for } k = L-1 \end{cases}$$

under the first and second optimization criteria.

■

For the case in which the Norton equivalent of the network is a concave increasing function with respect to the number of bursts in the network the following simplified iterative algorithms can be introduced.

Iterative Algorithm for the First Optimization Criterion

Step 0: *$L := 1$. Set $\lambda_0 := c_0$ and $\lambda_k := 0$ for all k, $k \geq 1$.If $E\tau_1 \leq T$, continue to Step 1. Otherwise stop; no bursts can enter into the network.*

Step 1: *$L := L+1$. Set $\lambda_k := c_k$ for all k, $0 \leq k \leq L-1$. If $E\tau_L \leq T$, repeat Step 1. Else, find the exact rate (which is between 0 and c_L) with which the last burst should be accepted and which results in $EQ - TE\gamma = 0$; the resulting admission control is the optimal admission control; thus stop.*

Iterative Algorithm for the Second Optimization Criterion

Step 0: *$L := 0$. Set $\lambda_k := 0$ for all k, $k \geq 0$.*

Step 1: *$L := L+1$. Set $\lambda_k := c_k$, for all k, $0 \leq k \leq L-1$. If $E\gamma_L \geq \Gamma$, go to Step 2; else, repeat Step 1.*

Step 2: *If $E\gamma = \Gamma$, the resulting admission control is the optimal admission control; thus stop. Else, find the value of λ_{L-1} (which is between 0 and c_{L-1}) which results in $E\gamma = \Gamma$; the resulting admission control is the optimal admission control; stop.*

For the case in which the Norton equivalent of the network is a concave increasing function with respect to the number of bursts in the network, *there is a one-to-one mapping between the admission control solution given under the first and the second optimization criteria.* The optimization problems

$$\max_{E\tau \leq T} E\gamma$$

and

$$\min_{E\gamma \geq \Gamma} E\tau$$

have the same class of window admission control solutions. If the window size is the same, it is easy to see that

$$\Gamma T = EQ \quad .$$

7. Conclusions

In this paper the optimal admission control of bursty traffic in a broadband network has been studied. It has been demonstrated that under two distinct optimization criteria, the optimal admission control at the burst level is a window flow control, which implies that a virtual circuit should not be allowed to have more than a given number of bursts in the network at any time. While, at the cell level, each network switching system services cells on a FCFS basis using statistical multiplexing, the results of this paper suggest that at the end-to-end level, the network should monitor and control each connection with respect to the number of bursts that the connection has in the network at any given moment.

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Received: 4/20/1990

Revised: 2/20/1992

Accepted: 3/24/1992

Recommended by V. Kulkarni, Editor