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1989-06-01

Decentralized Network Flow Control

Andreas D. Bovopoulos and Aurel A. Lazar

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Bovopoulos, Andreas D. and Lazar, Aurel A., "Decentralized Network Flow Control" Report Number: WUCS-89-29 (1989). *All Computer Science and Engineering Research*. https://openscholarship.wustl.edu/cse_research/742

Department of Computer Science & Engineering - Washington University in St. Louis
Campus Box 1045 - St. Louis, MO - 63130 - ph: (314) 935-6160.

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Complete Abstract:

In this paper, the problem of finding the decentralized flow control of a BCMP network is investigated. The packets of each of the users correspond to different classes of customers. The servers in the network are exponential and serve packets with FIFO policy. Each network user operates with either a state-dependent arrival rate (i.e. an arrival rate which depends upon the number of the user's packets that have not yet been acknowledged) or a state-dependent arrival rate (which the user chooses). The decentralized flow control problem is formulated under two optimization criteria. Under the first optimization criterion, the decentralized flow control corresponding to each of the network users maximizes the throughput of the network, under the constraint that the expected time delay of the packets in the network does not exceed a preassigned upper bound. Under the second optimization criterion, the decentralized flow control corresponding to each of the network users maximizes the throughput of the network, under the constraint that the expected time delay of each particular class of packets does not exceed a preassigned (user dependent) upper bound. In this paper all the previous classes of problems are handled uniformly, using efficient nonlinear optimization techniques.

DECENTRALIZED NETWORK
FLOW CONTROL

Andreas D. Bovopoulos
Aurel A. Lazar

WUCS-89-29

June 1989

Department of Computer Science
Washington University
Campus Box 1045
One Brookings Drive
Saint Louis, MO 63130-4899

Decentralized Network Flow Control*

Andreas D. Bovopoulos

Department of Computer Science
Washington University, St. Louis, MO 63130-4899
e-mail: andreas@patti.wustl.edu

and

Aurel A. Lazar

Department of Electrical Engineering
and
Center for Telecommunications Research
Columbia University, New York, NY 10027-6699
e-mail: aurel@ctr.columbia.edu

ABSTRACT

In this paper, the problem of finding the decentralized flow control of a BCMP network is investigated. The packets of each of the users correspond to different classes of customers. The servers in the network are exponential and serve packets with FIFO policy. Each network user operates with either a state-dependent arrival rate (i.e., an arrival rate which depends upon the number of the user's packets that have not yet been acknowledged) or a state-independent arrival rate (which the user chooses). The decentralized flow control problem is formulated under two optimization criteria. Under the first optimization criterion, the decentralized flow control corresponding to each of the network users maximizes the throughput of the network, under the constraint that the expected time delay of the packets in the network does not exceed a preassigned upper bound. Under the second optimization criterion, the decentralized flow control corresponding to each of the network users maximizes the throughput of the network, under the constraint that the expected time delay of each particular class of packets does not exceed a preassigned (user

* Proceedings of the 9th International Conference on Computer Communication, Tel Aviv, Israel, October 30-November 3, 1988.

dependent) upper bound. In this paper all the previous classes of problems are handled uniformly, using efficient nonlinear optimization techniques.

Index Terms: decentralized flow control, linear programming, network optimization, nonlinear optimization, resource allocation,

The research reported here was supported in part by the National Science Foundation under Grant # CDR-84-21402.

Decentralized Network Flow Control

1. Introduction

This paper presents new results in the area of single -objective flow control problems. Previous work in the area is reported in [5], [6] and [3]. The methodology introduced in this paper is a uniform one. It is shown that *the network flow control problem can be handled in the same way regardless of whether each class of packets is subject to state dependent or state independent flow control. Furthermore prime optimization techniques are utilized for the solution of the decentralized flow control problem.* This approach is first introduced in [3], where feasible direction techniques are applied for the solution of a number of resource allocation problems appearing in computer communication networks.

The analysis presented in this paper exemplifies the applicability of efficient nonlinear optimization techniques (such as the prime optimization techniques). One such technique, the reduced gradient projection method seems very promising in network optimization.

Organization of the Paper

In order to accomplish the above goals, the problem being analyzed and the criteria being used are specified in section 2.

In Section 3, a BCMP network shared by a number of customer classes, each operating under state independent flow control, is analyzed. In Sections 4 and 5, the decentralized flow control problem corresponding to a simple multiclass network shared by two classes of customers is analyzed. In Section 4, both classes of packets are subject to state dependent flow control. In Section 5, only one class of packets is subject to state dependent flow control, where the other class is subject to state independent flow control.

In Section 6, the previous problems are unified. It is shown that the decentralized flow control problem of a multi-class network subject to state independent or state dependent flow control can be handled uniformly. Specifically, a feasible direction approach results naturally in good decentralizable flow control policies.

2. The Statement of the Problem

A BCMP network with M exponential servers is shared by K different classes of packets that are processed according to the FIFO policy.

Let μ^i be the service rate of the i^{th} server, and \mathbf{M} be the $1 \times M$ matrix $[\mu^1 \ \dots \ \mu^M]$. Let $\mathbf{R}^k = [r^{kij}]$ be the $M \times M$ routing matrix of the k^{th} class of packets ($1 \leq k \leq K$, $1 \leq i \leq M$, $1 \leq j \leq M$). In this notation, class k packets are routed from node i to node j with probability r^{kij} and join node j with probability $r^{k \cdot j}$. $\Lambda = [\Lambda^1 \ \dots \ \Lambda^K]$, where $\Lambda^k = \{\lambda_l^k, \text{ for } l \geq 0\}$, represents the set of arrival rates of the K classes of packets; it also denotes the set of controls. λ_l^k is the arrival rate of the k class of packets when there are l of its packets in the network. Furthermore, $0 \leq \lambda_l^k \leq c^k$, for $1 \leq k \leq K$ and $l \geq 0$.

$E\gamma^k$ and $E\tau^k$ are the expected throughput and the expected time delay of the packets of the k^{th} user in the network. T^k is the upper bound of the expected time delay of the packets of the k^{th} user. Let $E\gamma \stackrel{\text{def}}{=} \sum_{k=1}^K E\gamma^k$, and $EQ \stackrel{\text{def}}{=} \sum_{k=1}^K EQ^k$.

The network optimization problem may be formulated in two different ways.

Network Optimization: First Formulation

The performance of the network is optimized if the total throughput of the packets via the network is maximized so that the expected time delay of the packets of the k^{th} user will not exceed an upper bound T^k for $k = 1, \dots, K$;

$$\max_{EQ^k - T^k E\gamma^k \leq 0; 1 \leq k \leq K} \left\{ \sum_{k=1}^K E\gamma^k \right\} .$$

If instead of requiring that each of the classes of packets obeys its own time delay constraint, it is required that the expected time delay of all the packets be smaller than or equal to a given upper bound, the network optimization problem may be formulated as follows:

Network Optimization: Second Formulation

The performance of the network is optimized if the total throughput of the packets via the network is maximized so that the expected time delay of the packets will not exceed an upper bound T ;

$$\max_{E\mathcal{Q}-TE\gamma \leq 0} \left\{ \sum_{k=1}^K E\gamma^k \right\} .$$

3. State Independent Flow Control of a BCMP Network

Let λ^k be the state independent arrival rate of the k class of packets.

Let the $1 \times M$ matrix $\Theta^k = [\theta^{k1} \dots \theta^{kM}]$ be the solution of the traffic flow equations for the k^{th} class of packets of the BCMP network defined in the previous section, i.e.,

$$\Theta^k = \Lambda^k + (\Theta^k \wedge M)\mathbf{R}^k .$$

Here Λ^k denotes the load vector of the input traffic flows of the k^{th} traffic class:

$$\Lambda^k = \lambda^k [r^{k \cdot 1} \dots r^{k \cdot M}] ,$$

for all k , $k = 1, 2, \dots, K$. If the BCMP network is stable:

$$\Theta^k = \Lambda^k + \Theta^k \mathbf{R}^k ,$$

or

$$\Theta^k = \Lambda^k (\mathbf{I} - \mathbf{R}^k)^{-1} .$$

With

$$\alpha^k = [\alpha^{k1} \dots \alpha^{kM}] \stackrel{\text{def}}{=} [r^{k \cdot 1} \dots r^{k \cdot M}] (\mathbf{I} - \mathbf{R}^k)^{-1} ,$$

we obtain

$$\theta^{kj} = \lambda^k \alpha^{kj} .$$

Thus the expected time delay of user k packets in the network is given by

$$E\tau^k = \sum_{j=1}^M \frac{\alpha^{kj}}{\mu^j - \sum_{l=1}^K \alpha^{lj} \lambda^l} .$$

Network Optimization

In this section the network optimization problem will be formulated with each of the different classes of packets subject to state independent flow control. As will be proven, the network optimization problem can be formulated as a convex optimization problem. Thus, feasible directions algorithms that take into account the fact that the problem is a convex nonlinear optimization problem will be developed for the computation of the decentralized flow control of the network.

The network throughput is a linear function. Thus the network optimization problem may have multiply solutions. In order to get a unique solution the objective function is made to be concave by raising each user's throughput to the power of β , where $0 < \beta < 1$, but β is arbitrarily close to one.

Nonlinear Problem 1 (Under the First Optimization Criterion):

$$\max\{(\lambda^1)^\beta + (\lambda^2)^\beta + \dots + (\lambda^K)^\beta\}$$

under the following constraints:

$$0 \leq \lambda^l \leq c^l ,$$

for $l = 1, 2, \dots, K$,

$$\lambda^l \sum_{j=1}^M \frac{\alpha^{lj}}{\mu^j - \sum_{k=1}^K \lambda^k \alpha^{kj}} - T^l \lambda^l \leq 0 , \quad (1)$$

for $l = 1, 2, \dots, K$

$$\sum_{k=1}^K \lambda^k \alpha^{kj} \leq \mu^j \quad , \quad (2)$$

for every node $j = 1, 2, \dots, M$.

The Constraint (2) is never active as an equality, because if $\sum_{k=1}^K \lambda^k \alpha^{kj} = \mu^j$, Equation (1) would be violated. ■

Observe that Equation (1) is the sum of a convex equation and a linear equation with respect to the arrival rates. So, *Nonlinear Problem 1 is a convex nonlinear optimization problem with respect to the arrival rates.*

Constraint (2) is never active. Therefore, a feasible direction technique may ignore its presence.

Similarly, the network optimization problem under the second optimization criterion can be formulated as follows:

Nonlinear Problem 1 (Under the Second Optimization Criterion):

$$\max\{(\lambda^1)^\beta + (\lambda^2)^\beta + \dots + (\lambda^K)^\beta\}$$

under the following constraints:

$$0 \leq \lambda^l \leq c^l \quad ,$$

for $l = 1, 2, \dots, K$,

$$\sum_{l=1}^K \lambda^l \sum_{j=1}^M \frac{\alpha^{lj}}{\mu^j - \sum_{k=1}^K \lambda^k \alpha^{kj}} - T \sum_{l=1}^K \lambda^l \leq 0 \quad ,$$

$$\sum_{k=1}^K \lambda^k \alpha^{kj} \leq \mu^j \quad , \quad (3)$$

for every node $j = 1, 2, \dots, M$.

Constraint (3) is never active. Therefore, a feasible direction technique may ignore its presence.

We now present the Kuhn and Tucker conditions for an optimal solution in order to fully understand the dependence of the behavior of the network to both the upper bound T of the time delay and the upper bound c of the arrival rates. Let w_0 be the Lagrange coefficient corresponding to the time delay constraint. Let w_k be the Lagrange coefficient corresponding to the upper bound of the arrival rate of the k^{th} user, for $k = 1, 2, \dots, K$. Then, the Kuhn and Tucker condition can be written as follows:

$$-\nabla_{\lambda^k} E\gamma + w_0(\nabla_{\lambda^k} EQ - T\nabla_{\lambda^k} E\gamma) + w_k = 0 \quad ,$$

for $k = 1, \dots, K$. Furthermore, an optimal point should validate the following equations:

$$w_k(\lambda^k - c^k) = 0 \quad ,$$

for $k = 1, \dots, K$,

$$w_0(EQ - TE\gamma) = 0 \quad ,$$

and

$$w_i \geq 0 \quad ,$$

for $i = 0, 1, \dots, K$.

From the previous equations we find that

$$\nabla_{\lambda^k} E\gamma(1 + w_0T) = w_k + w_0\nabla_{\lambda^k} EQ \quad ,$$

for $k = 1, \dots, K$.

If $EQ - TE\gamma < 0$, then $w_0 = 0$ and $\nabla_{\lambda^k} E\gamma = w_k > 0$ for $k = 1, \dots, K$. The fact that $w^k > 0$ for all k means that $\lambda^k = c^k$ for all k as well.

$$\text{If } \lambda^k < c^k \text{ then, } \frac{\nabla_{\lambda^k} E\gamma}{\nabla_{\lambda^k} EQ} = \frac{w_0}{1 + w_0T} > 0.$$

If $\lambda^k = c^k$ then, $\frac{\nabla_{\lambda^k} EQ}{\nabla_{\lambda^k} E\gamma} \leq \frac{1+w_0T}{w_0} > 0$.

From the previous equations we find that for $k = 1, \dots, K$

$$\frac{\nabla_{\lambda^k} E\gamma}{\nabla_{\lambda^k} EQ} \begin{cases} = \frac{w_0}{1+w_0T} & \text{if } \lambda^k < c^k \\ \geq \frac{w_0}{1+w_0T} & \text{if } \lambda^k = c^k \end{cases}$$

The previous expressions can be used for the creation of an effective feasible direction optimization technique.

Observe that an arrival rate λ^k is feasible if it does not violate the constraint $0 \leq \lambda^k \leq c^k$, for $k = 1, \dots, K$, if it does not saturate any of the processors in the network, and if the expected time delays are acceptable.

Algorithm:

Initially, all the arrival rates are equal to zero.

Step 1: *If possible, increase the arrival rates to values that are feasible; if not, stop.*

Step 2: *Adjust the arrival rates so that the Kuhn and Tucker conditions are validated; repeat Step 1.*

By expressing the arrival rates with respect to the path flows and by considering the path flows of each of the classes of packets as unknowns, *the optimal load balancing* can be computed as well.

4. Decentralized State Dependent Flow Control of a BCMP Network

Using a linear programming formulation ([6]) and the feasible direction methodology introduced in [3], it can be proven that under the second optimization criterion each user operates with an arrival rate with at most one random point. No algorithm was, however, suggested in [6] for the derivation of the decentralized flow control policy.

In a similar fashion, using the linear programming formulation and Proposition 2 of [2], it can be proven that under the first optimization criterion, a BCMP network which is shared by K users each of the users' policy has at most K random points.

Using the optimization methodology introduced in [2], it can be shown that under a state dependent flow control policy, a centralized solution can be derived which has at most as many random points as the number of time delay constraints. However, not every centralized solution can be implemented in a decentralized fashion. Thus nonlinear optimization techniques that directly lead to decentralizable policies must be used. For this it is required that the controller of each of the classes that share the resources of the network be only a function of the number of packets for which the controller has not yet received acknowledgement.

In order to make the presentation transparent, the simplest possible case of an exponential server with rate μ shared by two different classes of customers will be analyzed. The presentation of the more general case consisting of a BCMP network shared by a number of different users will be presented in Section 6.

The equilibrium probability that there are j packets from the first class and k packets from the second class in the network is given by

$$p_{jk} = \binom{j+k}{j} \prod_{i_1=1}^j \left(\frac{\lambda_{i_1}^1 - 1}{\mu} \right) \prod_{i_2=1}^k \left(\frac{\lambda_{i_2}^2 - 1}{\mu} \right) p_{00} ,$$

where $k_i \geq 0$, for $i = 1, 2$.

The optimal arrival rates can be derived from the solution of the following nonlinear program:

Nonlinear Problem 2 (Under the First Optimization Criterion):

$$\max \left\{ \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \binom{j+k}{j} \prod_{i_1=1}^j \left(\frac{\lambda_{i_1}^1 - 1}{\mu} \right) \prod_{i_2=1}^k \left(\frac{\lambda_{i_2}^2 - 1}{\mu} \right) \right\}$$

under the constraints

$$0 \leq \lambda_{k_i}^i \leq c^i ,$$

for $i = 1, 2$ and $0 \leq k_i$,

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} k_i p_{k_1 k_2} - T^l(\mu - \mu p_{00}) \leq 0 \quad .$$

for $l = 1, 2$.

Observe that the arrival rates $\lambda_{k_l}^l = 0$ for $k_l \geq 0$ and $l = 1, 2$ represent a feasible solution. Furthermore, in order to move to a better operating point, what we must determine is whether it is possible to improve the values of the parameters λ_0^l for $l = 1, 2$. For a problem with K users, the optimization problem that must be solved involves $2K$ equations.

Nonlinear Problem 2 (Under the Second Optimization Criterion):

$$\max\left\{\sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \binom{j+k}{j} \prod_{i_1=1}^j \left(\frac{\lambda_{i_1-1}^1}{\mu}\right) \prod_{i_2=1}^k \left(\frac{\lambda_{i_2-1}^2}{\mu}\right)\right\}$$

under the constraints

$$0 \leq \lambda_{k_i}^i \leq c^i \quad ,$$

for $i = 1, 2$ and $0 \leq k_i$.

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} (k_1 + k_2) p_{k_1 k_2} - T^l(\mu - \mu p_{00}) \leq 0 \quad .$$

for $l = 1, 2$.

5. Decentralized Flow Control with Classes of Packets Subject to State-Dependent and State-Independent Flow Control

In this section a BCMP network consisting of an exponential processor with rate μ and shared by two classes of customers will be analyzed. The packets of the first class of customers are subject to state-independent flow control, and the packets of the second class are subject to state dependent flow control.

The equilibrium probability that there are j packets from the first class and k packets from the second class in the network is given by

$$p_{jk} = \binom{j+k}{j} \left(\frac{\delta}{\mu}\right)^j \prod_{l=0}^{k-1} \left(\frac{\lambda_l}{\mu}\right) p_{00} \quad ,$$

where δ is the arrival rate of the first class of packets. The conditional service rate of the network for packets of the first class given that there are j packets of the first class in the network, is given by ([4])

$$\nu^1(j) = \frac{\sum_{k=0}^{\infty} \mu \frac{j}{j+k} p_{jk}}{\sum_{k=0}^{\infty} p_{jk}} \quad .$$

The conditional service rate of the network for packets of the second class corresponds to that of an M/M/1 exponential server with rate ([4])

$$\nu^2(k) = \frac{\sum_{j=0}^{\infty} (\mu \frac{k}{j+k}) p_{jk}}{\sum_{j=0}^{\infty} p_{jk}} = \mu - \delta \quad .$$

The network optimization problem can be formulated as a nonlinear optimization problem similarly to the one presented in the previous section.

6. Decentralized Flow Control in a General BCMP Network

In Sections 5 and 6, the network which was analyzed consisted of an exponential processor of rate μ , was a product form network. A general BCMP network has a product form solution as well. Therefore, the expressions concerning the network throughput as well as the expressions regarding the time delay of the packets of each class are continuous twice differentiable functions.

(Even more generally, a Markovian queueing network has equilibrium probabilities which are continuous functions with respect to the arrival rates [3]. This statement is a consequence of the Implicit Theorem [7].)

The first and second derivatives of the throughput and time delay constraints can be computed algorithmically using finite differences. Having established the fact that all the expressions introduced in the nonlinear optimization problems studied in this paper, are continuous twice differentiable we would like to develop a general methodology for deriving the optimal network flow control of a general BCMP network shared by a number of classes, some of which are subject to state dependent and some of them being subject to state independent flow control.

ν_{k_1, \dots, k_K}^j represents the Norton equivalent expression of the network with respect to the j class of packets, given that there are k_i packets of the i^{th} class of packets in it for $i = 1, 2, \dots, K$. In a network with K classes of packet under state dependent flow control and with at most N_k packets in the network from class k , the behavior of the network is described by the rates ν_{k_1, \dots, k_K}^j , $j = 1, 2, \dots, K$, for $0 \leq k_i \leq N_i$ where $i = 1, 2, \dots, K$, and the arrival rates of the classes of packets which are subject to state independent flow control through the network's processors. Given the expressions concerning the Norton equivalent, the network optimization problem can be solved using the following iterative algorithm. For simplicity, the algorithm will be presented for a network with one class of packets (the first) being under state dependent flow control and with one class of packets (the second) been under state independent flow control.

Let $N_1 = 0$ and $\lambda_{k_1}^1 = 0$, for $k_1 \geq 0$.

Iteration

Step 1:

If k_1 is the minimum nonnegative integer for which $\lambda_{k_1} \neq 0$, then $N_1 := k_1 + 1$. Find the Norton equivalent of the service rates of the network given that the arrival rate for the class of packets subject to state independent flow control is δ . Obviously the expressions for the throughput and time delays are continuous functions with respect to the arrival rates δ and $(\lambda_0^1, \dots, \lambda_{N_1-1}^1)$.

Step 2: Find the optimal values of the arrival rates δ and $(\lambda_0^1, \dots, \lambda_{N_1-1}^1)$. If there is improvement in network performance, go to Step 1; else stop. The decentralized flow control is derived when two consecutive repetitions of the iteration result in the same control policy.

For the solution of each optimization problem appearing in Step 2, an efficient nonlinear optimization technique (one of which is the reduced gradient projection method) can be applied. The complexity of the problem is greatly simplified if some of the variables $(\lambda_0^1, \dots, \lambda_{N_1-1}^1)$ are forced to remain invariant during an iteration of this algorithm.

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