Report Number: WUCS-90-32

1990-10-01

Performance Evaluation of a User Network Interface for ATM Networks

Andreas D. Bovopoulos and Einir Valdimarsson

In this paper the functionality requirements of a user-network interface for bandwidth allocation are discussed. Such requirements include the capability to provide end users with a variety of non-deteriorating connection types. In addition effective policy enforcement and traffic shaping mechanisms are required to facilitate network management and the efficient utilization of network resources. Based on these requirements, a user-network interface model is proposed, and its performance is studied. Its intrinsic properties are revealed for two cases, a Poisson source and a bursty source.

PERFORMANCE EVALUATION OF A USER NETWORK INTERFACE FOR ATM NETWORKS

Andreas D. Bovopoulos and Einir Valdimarsson

WUCS-90-32

October 1990

Department of Computer Science
Washington University
Campus Box 1045
One Brookings Drive
Saint Louis, MO 63130-4899

PERFORMANCE EVALUATION OF A USER NETWORK INTERFACE FOR ATM NETWORKS

Andreas D. Bovopoulos and Einir Valdimarsson

Computer and Communications Research Center
Washington University, St. Louis, MO 63130-4899

ABSTRACT

In this paper the functionality requirements of a user-network interface for bandwidth allocation are discussed. Such requirements include the capability to provide end users with a variety of non-deteriorating connection types. In addition effective policy enforcement and traffic shaping mechanisms are required to facilitate network management and the efficient utilization of network resources. Based on these requirements, a user-network interface model is proposed, and its performance is studied. Its intrinsic properties are revealed for two cases, a Poisson source and a bursty source.

1. Introduction

The broadband integrated services digital network (B-ISDN) is revolutionizing the telecommunications industry. B-ISDN is an all purpose digital network that can in theory support any kind of traffic service [MIN89], thus decoupling network evolution from service evolution. Over the past few years, substantial progress has been made concerning many aspects of B-ISDN development. Standards committees have decided on the Asynchronous Transfer Mode (ATM) as the target transport technique of the B-ISDN. ATM is a connection-oriented transport technique that can support both connection-oriented and connectionless services. In an ATM network, information is transmitted in packets of equal size called *cells*. Each cell consists of an information field (48-octets long) and a header field (5-octets long). Equally noticeable progress has been made in the design of broadband switches that can be used in a B-ISDN/ATM environment [HUA84], [HUI87], [DEV88], [TUR88a].

Despite tremendous progress in communications technology, there is not yet a sufficient understanding of the teletraffic and resource allocation problems appearing in a broadband setting [GEC88], [MIN89], [O'RE89], [COU89]. Current research has addressed some of the statistical multiplexing problems that appear in an ATM environment [JEN84], [HEF86], [SRI89],[LI90]. However, network-wide problems have not yet received the required attention [HUI88], [FIL89], [JAC90].

The performance of an ATM statistical multiplexer depends on the traffic mix. As a result, without sufficient control provisions, the statistical multiplexing of different sources fails to provide quality of service (QOS) guarantees for the multiplexed sources. In order to develop ways of guaranteeing QOS, the study of network-wide resource sharing and traffic control techniques is of utmost importance [GEC88]. This area of research still contains many open problems and is one of the least understood. In order for the ATM concept to succeed, it is necessary to introduce network-wide control procedures that guarantee QOS to network users.

In ATM networks the emphasis is on preventive control schemes. As a result source-based control plays a prominent role. Whereas it is not yet clear if source based control suffices, it is necessary for the management and efficient utilization of an ATM network.

In this paper we focus on the functionality requirements of a user-network interface (UNI). In Section 2, we present the functionality requirements of the UNI for bandwidth allocation and present a system that appears to be able to provide effectively these requirements. In Sections 3, 4 and 5, we analyze the UNI performance for a Poisson source. In Sections 6, 7, 8 and 9, we analyze the UNI performance for a bursty source. In Section 10, we discuss the UNI operation using a number of detailed examples and demonstrate its intrinsic desirable properties. We end the paper with conclusions and a description of issues that remain to be addressed.

2. The User Network Interface (UNI)

The UNI is the system that is located at the edge of a network and that interconnects the customer premises with the public network; it is the point at which a user's traffic leaves the user's premises and enters the public network. The UNI should have some of the properties possessed by any system interconnecting two different network domains. It should be able to monitor the traffic that enters the network and to enforce the operational policy which the user negotiates with network management at the time of the connection establishment. These capabilities are necessary in a network that provides services to end users with performance guarantees. At the time at which a particular type of connection is negotiated between a user and the network, it is necessary that the network be able to project the user's requirements into a traffic behavior that improves the utilization of the network resources and facilitates the network management. This capability is sometimes called *traffic shaping.* Traffic shaping does not merely refer to the dropping or marking of a cell but to traffic transformation, at a cost. The cost is related not only to the system cost but also to the possible reduction of the utilization of the system due to the acceptance of a particular connection type. Traffic shaping is an additional functionality that the UNI should provide. For the user it is important that trade-offs between different connection types be made explicitly clear and available at connection time. For the network management the issues of policy enforcement and traffic shaping are of paramount importance. Whereas the user is concerned with end-to-end performance, the network management is concerned with the efficient utilization of network resources and the availability of services to end users in a cost effective way. Last but not least, inter-operability issues between the user, the UNI and the network management along the virtual path of the connection should be considered during the design of a UNI system.

Most of the previously studied UNI models are variations of the "leaky bucket" model [TURN88b], [NIES90]. A leaky bucket model monitors and controls one particular parameter of the user's traffic. In case multiple parameters have to be regulated, the utilization of variations of the leaky bucket model has been suggested [NIE90].

In the rest of this section we present a UNI model which addresses the requirements presented above. The UNI model that we describe in this section can effectively implement a wide variety of alternative acceptance policies at different costs and therefore can satisfy the requirements of different connection types and services. While issues related with its effective use in a network environment will not be addressed in this paper, it should be mentioned that this particular UNI model facilitates network management, as it demonstrates ways in which connections along a virtual path should be controlled.

The UNI has two buffers: the token buffer of size M and the cell buffer of size K. An arriving cell enters without delay into the network, as long as the token pool contains a token. One token is removed from the token pool each time a cell enters the network. If an arriving cell finds no tokens available at the UNI, it is stored in the cell buffer, as long as the cell buffer is not full. As it will appear in the application section, the source buffer introduces a trade-off between cell loss and cell delay.

As we mentioned in the introduction, in an ATM network we favor predictable control policies because we want to minimize the need for feedback control. This is the reason why in the UNI being analyzed, tokens are generated *with a deterministic pattern*. Such a behavior is predictable and yet at the same time rich enough to implement a wide variety of control policies. The number of tokens that are generated is equal to s. The time between two token generations is D seconds. The tokens that are provided to a particular connection may have a maximum lifetime. If such an option is exercised, a token that is not used during a particular time interval is eliminated. Such a policy could be very important for network management. For the rest of this paper we assume that the lifetime of the tokens is infinite.

With the time scale D, the two buffers K and M, and the number of tokens s that are generated at each renewal, a wide variety of options, with respect to performance, capacity requirements and cost can be provided to an end user.

It is clear that such a UNI can perform both traffic monitoring, traffic shaping and policy enforcement functionalities. Therefore it can be described as an effective UNI. The UNI model being analyzed behaves like a leaky bucket model if the source buffer is removed and the token pool is full at the beginning of the connection.

The performance of this model was also addressed in [SID89] and [BERG90a,b]. In [SID89] for a Poisson source, the Laplace transform of the waiting time distribution and the inter-departure distribution were computed for the case in which $s = 1$ and $K = \infty$. The throughput was also computed for the case of a finite K. In [BERG90a,b] it was shown that for a Markovian arrival process source and an independent renewal token source, the throughput is a function of $K + M$.

3. Performance Analysis of the UNI Under a Poisson Traffic Source

In this section we analyze the UNI performance for the case in which the traffic source generates cells according to a Poisson distribution. The inter-arrival time is exponentially distributed with rate c. Every D seconds the token source generates s tokens, where s is a deterministic positive number.

Let q be the number of cells in the input buffer and w be the number of tokens in the token pool. Notice that the probability that both q and w are positive is zero. This is because outstanding cells always utilize any available tokens. The system state is described by the number of tokens that are required for the service of all outstanding cells in the input buffer plus the number of tokens that are needed to fill up the empty positions in the token buffer. Therefore when $(q, w) = (0, w)$, the state of the UNI is $M - w$, for $0 \leq w \leq M$. When the state of the two buffers is $(q, 0)$ for $0 \leq q \leq K$, the state of the UNI is $q + M$. This representation was first utilized in [SID89].

We observe the state of the UNI just prior to the generation of the s tokens. We call these instances in time the renewal epochs. At those instances the behavior of the UNI is described by an embedded Markov chain on the state space $\{0, 1, \cdots, K + M\}$. Let the state before the last renewal epoch be j. In general

$$q = (j - M)^+ \tag{3.1}$$

and

$$w = (M - j)^+ \quad . \tag{3.2}$$

Now let

$$w(j) \stackrel{\text{def}}{=} min\{(w - q + s)^+, M\} \; = \; min\{\left((M - j)^+ - (M - j)^- + s\right)^+, M\} \quad .$$

Then,

$$w(j) \; = \; min\{(M - j + s)^+, M\} \quad . \tag{3.3}$$

Similarly, let

$$q(j) \stackrel{\text{def}}{=} (q - s)^+ \; = \; (j - M - s)^+ \quad . \tag{3.4}$$

Notice that if j is the state of the system before the last renewal, $w(j)$ and $q(j)$ are the total number of available tokens and cells respectively after the renewal. Furthermore notice that it is impossible for both $w(j)$ and $q(j)$ to be positive.

Let $A(u)$ be the total number of arrivals in the u seconds immediately following the last renewal, and let $\alpha_k(u)$ be the probability that $A(u) = k$, for $k \geq 0$. Then $\alpha_k(u) \stackrel{\text{def}}{=} P[A(u) = k] = e^{-cu}\frac{(cu)^k}{k!}$ for $k \geq 0$. For simplicity we write α_k instead of $\alpha_k(D)$. Let

$$b_{j_1,j_2} \stackrel{\text{def}}{=} \begin{cases} \alpha_{j_2-(j_1-s)^+}, & \text{if} j_2 - (j_1 - s)^+ \geq 0 \text{ and } 0 \leq j_2 < K + M \\ 0, & \text{otherwise} \end{cases} \tag{3.5}$$

and

$$\bar{b}_{j_1,j_2} \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if} j_2 < K + M \\ 1 - \sum_{n=0}^{K+M-1} b_{j_1,n}, & \text{if} j_2 = K + M \end{cases} \tag{3.6}$$

for every value of j_1, $0 \leq j_1 \leq K + M$.

The transition matrix of the embedded Markov chain is given by

$$\mathbb{P} \stackrel{\text{def}}{=} \begin{pmatrix} b_{0,0} & b_{0,1} & \cdots & \bar{b}_{0,M+K} \\ b_{1,0} & b_{1,1} & \cdots & \bar{b}_{1,M+K} \\ \vdots & \vdots & \ddots & \vdots \\ b_{M+K,0} & b_{M+K,1} & \cdots & \bar{b}_{M+K,M+K} \end{pmatrix} \tag{3.7}$$

Let π_j be the equilibrium probability that the system is in state j immediately before the generation of tokens, and $\pi \stackrel{\text{def}}{=} [\pi_0, \pi_1, \cdots, \pi_{M+K}]$ be a vector such that $\pi\mathbb{P} = \pi$ and $\pi\mathbf{e} = 1$. π is the equilibrium probabilities vector of the state of the UNI at the instances immediately before the renewal epochs of the token source. $\mathbf{e}$ is a $M + K + 1$ vector with all its elements equal to one.

Let $E\gamma$ be the throughput, *i.e.* the expected number of cells that pass through the UNI system in the time period between two renewal epochs of the token source. Then

$$E\gamma = \sum_{j=0}^{K+M} \pi_j \Big\{ \sum_{n=0}^{K+w(j)-q(j)-1} n \times \alpha_n + \sum_{n=K+w(j)-q(j)}^{\infty} \alpha_n \times \Big(K + w(j) - q(j)\Big) \Big\} .$$

Therefore,

$$E\gamma = \sum_{j=0}^{K+M} \pi_j \Big\{ \big(K + w(j) - q(j)\big) - \sum_{n=0}^{K+w(j)-q(j)-1} \alpha_n \times \Big(K + w(j) - q(j) - n\Big) \Big\} \tag{3.8}$$

4. The Waiting Time Distribution of a Poisson Source

In order to compute the waiting time distribution, we have to take into account the probability that a particular cell enters the system u seconds after the last renewal epoch, for $u \in (0, D]$.

Let us tag a cell that arrives u seconds after the last renewal epoch, and let j be the state of the UNI before the last renewal.

If $q(j) + A(u) + 1 - w(j) \leq 0$, the tagged cell is served instantaneously.

If $1 \leq q(j) + A(u) + 1 - w(j) \leq K$, then the tagged cell has to wait until the token source generates enough tokens to serve all the cells in front of the tagged cell. The time that it takes the token source to generate enough tokens is

$$T \stackrel{\text{def}}{=} \min_{n} \{n | n \times s \geq q(j) + A(u) + 1 - w(j)\} \quad , \tag{4.1}$$

and the waiting time of the tagged cell is

$$W_u \stackrel{\text{def}}{=} D - u + (T - 1)D \quad . \tag{4.2}$$

If $q(j) + A(u) + 1 - w(j) > K$, the tagged cell is lost.

Therefore, the probability that a cell arriving at time u enters the system is given by

$$P_{accept} \stackrel{\text{def}}{=} P[q(j) + A(u) + 1 - w(j) \leq K] \quad . \tag{4.3}$$

We can now compute the waiting time distribution of a cell that arrives at the UNI u seconds after the last renewal, under the conditions that the cell was accepted and that the state of the system before the last renewal was j.

$$P[W_u > x |\; accepted,\; j] = \frac{P[W_u > x,\; accepted,\; j]}{P[accepted,\; j]} =$$

$$\frac{\sum_{k=0}^{K+M} P[W_u > x,\; accepted,\; j, A(u) = k]}{\sum_{n=0}^{K+M} P[accepted,\; j,\; A(u) = n]} = \frac{\sum_{k=0}^{K+M} P[W_u > x,\; accepted,\; j | A(u) = k]\alpha_k(u)}{\sum_{n=0}^{K+M} P[accepted,\; j | A(u) = n]\alpha_n(u)}$$

for all $x \geq 0$. Therefore,

$$P[W_u > x | accepted,\; j] =$$

$$\frac{\sum_{k=\left(w(j)-q(j)\right)^+}^{K+w(j)-q(j)-1} 1\big(D - u + (T-1)D > x | A(u) = k\big) a_k(u)}{\sum_{n=0}^{K+w(j)-q(j)-1} a_n(u)} \quad . \tag{4.4}$$

where 1() is a boolean function which returns one, if true, and zero, if false.

Given that the state of the system before the last renewal was j, the waiting time distribution of a cell that enters the UNI is

$$P[W > x | j] = \frac{\int_0^D P[W_u > x | accepted, j] (\sum_{n=0}^{K+w(j)-q(j)-1} \alpha_n(u)) du}{\sum_{n=0}^{K+w(j)-q(j)-1} \int_0^D \alpha_n(u) du} =$$

$$\frac{\sum_{k=(w(j)-q(j))^+}^{K+w(j)-q(j)-1} \int_0^D 1(D-u+(T-1)D > x|A(u)=k)a_k(u)du}{\frac{K+w(j)-q(j)}{c} - \frac{1}{c}\sum_{n=0}^{K+w(j)-q(j)-1}(K+w(j)-q(j)-n)\alpha_n} \quad .$$

The last equation holds because

$$\sum_{n=0}^{B} \int_0^D \alpha_n(u)du \;=\; \frac{B+1}{c} - \frac{1}{c}\sum_{n=0}^{B}(B-n+1)\alpha_n \quad .$$

Finally, for any $x \geq 0$, the waiting time distribution of any accepted cell is

$$P[W > x] \;=\; \sum_{j=0}^{K+M} P[W > x|j]\pi_j \quad . \tag{4.5}$$

Similarly,

$$P[W_u = 0|accepted,\ j] \;=\; \frac{\sum_{k=0}^{w(j)-q(j)-1} a_k(u)}{\sum_{n=0}^{K+w(j)-q(j)-1} a_n(u)} \quad , \tag{4.6}$$

and

$$P[W = 0|j] \;=\; \frac{\frac{w(j)-q(j)}{c} - \frac{1}{c}\sum_{n=0}^{w(j)-q(j)-1}(w(j)-q(j)-n)\alpha_n}{\frac{K+w(j)-q(j)}{c} - \frac{1}{c}\sum_{n=0}^{K+w(j)-q(j)-1}(K+w(j)-q(j)-n)\alpha_n} \quad . \tag{4.7}$$

The computation of the probability that an accepted cell is served instantaneously readily follows from the equation

$$P[W = 0] \;=\; \sum_{j=0}^{K+M} P[W = 0|j]\pi_j \quad . \tag{4.8}$$

5. The Loss Distribution of a Poisson Source

The probability that L *consecutive cells are lost* in an interval $[0, D)$ is given by

$$P[L = k] \;=\; \sum_{j=0}^{K+M} P[L = k|j]\pi_j \tag{5.1}$$

for $k \geq 0$. Notice that

$$P[L = k|j] \;=\; \begin{cases} \sum_{n=0}^{K+w(j)-q(j)} \alpha_n, & \text{if } k = 0, \\ \alpha_{K+w(j)-q(j)+k}, & \text{if } k \geq 1. \end{cases}$$

Therefore,

$$P[L = k] \;=\; \begin{cases} \sum_{j=0}^{K+M} \sum_{n=0}^{K+w(j)-q(j)} \alpha_n \pi_j, & \text{if } k = 0, \\ \sum_{j=0}^{K+M} \alpha_{K+w(j)-q(j)+k}\pi_j, & \text{if } k \geq 1. \end{cases} \tag{5.2}$$

From the last expression we can compute the expected number of lost cells between two renewal epochs of the token source.

$$E[L] \;=\; \sum_{j=0}^{K+M} \sum_{k=1}^{\infty} k\alpha_{K+w(j)-q(j)+k}\pi_j \;=\; cD \;-\; E\gamma \quad . \tag{5.3}$$

Therefore, the loss probability P_L is given by

$$P_L \;=\; \frac{E[L]}{cD} \;=\; 1 \;-\; \frac{E\gamma}{cD} \quad . \tag{5.4}$$

6. Analysis of the Performance of a Bursty Source

In an ATM environment the majority of traffic sources will have bursty characteristics. In this section we study the behavior of a bursty source. We model a bursty source as a two state Markov process. One of the states is the idle state i, while the other state is the active state a. λ is the transition rate from state i to state a, whereas μ is the transition rate from state a to state i. The source alternates between the idle and active states.

The behavior of a two state Markov process is determined by the generator

$$Q = \begin{pmatrix} -\lambda & \lambda \\ \mu & -\mu \end{pmatrix} \tag{6.1}$$

If $\mathbf{P}(u) = [p_i(u), p_a(u)]$ is the 1×2 vector that describes the probability that the Markov process is in a particular state at time u, then $\mathbf{P}(u) = \mathbf{P}(0)e^{Qu}$,

$$p_i(u) = \frac{\mu}{\mu+\lambda} + \{p_i(0) - \frac{\mu}{\mu+\lambda}\}e^{-(\lambda+\mu)u} \quad , \tag{6.2}$$

and

$$p_a(u) = \frac{\lambda}{\mu+\lambda} + \{p_a(0) - \frac{\lambda}{\mu+\lambda}\}e^{-(\lambda+\mu)u} \quad . \tag{6.3}$$

Let $T_a(u)$ be the total time that the source spends in the active state in $[0, u)$, and let $T_i(u)$ be the total time that the source spends in the idle state in $[0, u)$.

Let $F_a(u)$ be the distribution of the time that the system spends in the active state and $F_i(u)$ be the distribution of the time that the system spends in the idle state. Let $F_i^{(k)}(u)$ (similarly $F_a^{(k)}(u)$) be the k^{th} convolution of $F_i(u)$ (similarly $F_a(u)$) distribution. Furthermore let S_u be the state of the Markov process at time u.

Proposition 1: *The distribution of the total time that the system spends in the active state in the time interval [0,u), given that the initial state at time 0 was the idle state, is given by*

$$P[T_a(u) \leq x | S_0 = i] = \sum_{k=0}^{\infty} F_a^{(k)}(x) \times \{F_i^{(k)}(u-x) - F_i^{(k+1)}(u-x)\} \quad , \tag{6.4}$$

for every x, $0 \leq x \leq u$.
The distribution of the total time that the system spends in the active state in the time interval [0,u), given that the initial state at time 0 was the active state, is given by

$$P[T_a(u) < x | S_0 = a] = 1 - \sum_{k=0}^{\infty} F_i^{(k)}(u - x) \times \{F_a^{(k)}(x) - F_a^{(k+1)}(x)\} \tag{6.5}$$

for every x, $0 \leq x \leq u$.

Proof:

We first prove (6.4). Let τ_a^k (respectively τ_i^k) be the total time that the source spends in the active (respectively idle) state when it visits the active (respectively idle) state for the k^{th} time, for all k, $k \geq 1$. Let

$$N_i(u) \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } \tau_i^1 > u \\ sup\{n : \sum_{n=1}^{k} \tau_a^n \leq u\}, & \text{otherwise} \end{cases} .$$

Then

$$P[T_a(u) \leq x | S_0 = i] \ = \ 1 \times P[N_i(u-x) = 0] \ + \ \sum_{k=1}^{\infty} P[\sum_{n=1}^{k} \tau_a^n \leq x] \times P[N_i(u-x) = k] \quad .$$

Therefore,

$$P[T_a(u) \leq x | S_0 = i] \ = \ \sum_{k=0}^{\infty} F_a^{(k)}(x) \times \{F_i^{(k)}(u-x) \ - \ F_i^{(k+1)}(u-x)\} \quad ,$$

and the proof of (6.4) is complete. Notice that

$$P[T_a(u) \leq x | S_0 = i] = P[u - T_i(u) \leq x | S_0 = i] = P[T_i(u) \geq u - x | S_0 = i].$$

Therefore,

$$P[T_i(u) < u - x | S_0 = i] = 1 \ - \ P[T_a(u) \leq x | S_0 = i] \quad . \tag{6.6}$$

Equations (6.6) and (6.4) prove (6.5).

Equation (6.5) can be also proven as follows:

$$P[T_a(u) \leq x | S_0 = a] \ = \ \int_{y=0}^{u} \{\sum_{k=0}^{\infty} F_a^{(k)}(x-y) \times \{F_i^{(k)}(u-x) \ - \ F_i^{(k+1)}(u-x)\}\} dF_a(y) \ =$$

$$\sum_{k=0}^{\infty} F_a^{(k+1)}(x) \times \{F_i^{(k)}(u-x) \ - \ F_i^{(k+1)}(u-x)\} \ =$$

$$\sum_{k=0}^{\infty} F_a^{(k+1)}(x) \times F_i^{(k)}(u-x) \ - \ \sum_{k=0}^{\infty} F_a^{(k+1)}(x) \times F_i^{(k+1)}(u-x) \ =$$

$$\sum_{k=0}^{\infty} F_a^{(k+1)}(x) \times F_i^{(k)}(u-x) \ - \ \sum_{k=0}^{\infty} F_a^{(k)}(x) \times F_i^{(k)}(u-x) \ + \ 1 \quad .$$

Therefore,

$$P[T_a(u) \leq x | S_0 = a] \ = \ 1 \ - \ \sum_{k=0}^{\infty} F_i^{(k)}(u \ - \ x) \times \{F_a^{(k)}(x) \ - \ F_a^{(k+1)}(x)\} \quad .$$

■

Given the bursty source model is a Markov process, $F_i(u) \ = \ 1 \ - \ e^{\lambda u}$, and $F_a(u) \ = \ 1 - e^{\mu u}$ for $u \geq 0$. Equation (6.4) thus becomes:

$$P[T_a(u) \leq x | S_0 = i] \ = \ \sum_{k=0}^{\infty} e^{-\lambda(u-x)} \frac{(\lambda(u-x))^k}{k!} \sum_{n=k}^{\infty} e^{-\mu x} \frac{(\mu x)^n}{n!} \ =$$

$$\sum_{n=0}^{\infty} e^{-\mu x}\frac{(\mu x)^n}{n!}\sum_{k=0}^{n} e^{-\lambda(u-x)}\frac{(\lambda(u-x))^k}{k!} =$$

$$\sum_{n=0}^{\infty} e^{-\mu x}\frac{(\mu x)^n}{n!}\{1 - \sum_{k=n+1}^{\infty} e^{-\lambda(u-x)}\frac{(\lambda(u-x))^k}{k!}\} =$$

$$1 - \sum_{n=0}^{\infty} e^{-\mu x}\frac{(\mu x)^n}{n!}\int_0^{u-x} \lambda e^{-\lambda y}\frac{(\lambda y))^n}{n!}dy =$$

$$1 - \lambda e^{-\mu x}\int_0^{u-x} e^{-\lambda y} I_0(2\sqrt{\lambda\mu x y})dy$$

where $I_0(y) \stackrel{\mathrm{def}}{=} \sum_{n=0}^{\infty} \frac{(\frac{y}{2})^{2n}}{(n!)^2}$ is the modified Bessel function of zero order. Therefore,

$$P[T_a(u) \leq x|S_0 = i] = 1 - \lambda e^{-\mu x}\int_0^{u-x} e^{-\lambda y} I_0(2\sqrt{\lambda\mu x y})dy \quad , \tag{6.7}$$

for every $x \in [0, u)$. Similarly (6.5) becomes

$$P[T_a(u) < x|S_0 = a] = 1 - \sum_{k=0}^{\infty} e^{-\mu x}\frac{(\mu x)^k}{k!}\sum_{n=k}^{\infty} e^{-\lambda(u-x)}\frac{(\lambda(u-x))^n}{n!} \quad . \tag{6.8}$$

After some algebraic manipulation the previous relation becomes

$$P[T_a(u) < x|S_0 = a] = \mu e^{-\lambda(u-x)}\int_{[0,x)} e^{-\mu y} I_0(2\sqrt{\lambda\mu(u-x)y})dy \tag{6.9}$$

for every x, $x \in [0, u)$.

Proposition 2: *The distribution of the total time that the system spends in the active state in the time interval [0,u], given that the initial state at time 0 was the idle state, is given by*

$$P[T_a(u) \leq x|S_0 = i] = 1 - \lambda e^{-\mu x}\int_0^{u-x} e^{-\lambda y} I_0(2\sqrt{\lambda\mu x y})dy \quad , \tag{6.10}$$

for every x, $0 \leq x \leq u$.

The distribution of the total time that the system spends in the active state in the time interval [0,u], given that the initial state at time 0 was the active state, is given by

$$P[T_a(u) \leq x|S_0 = a] = \begin{cases} 0, & \text{if } x = 0; \\ \mu e^{-\lambda(u-x)}\int_0^x e^{-\mu y} I_0(2\sqrt{\lambda\mu(u-x)y})dy, & \text{if } 0 < x < u; \\ 1, & \text{if } x = u. \end{cases} \tag{6.11}$$

Let ${}^a\alpha_k(u) \stackrel{\mathrm{def}}{=} P[A(u) = k|S_0 = a]$, and ${}^i\alpha_k(u) \stackrel{\mathrm{def}}{=} P[A(u) = k|S_0 = i]$. For simplicity we write ${}^i\alpha_k$ and ${}^a\alpha_k$ instead of ${}^i\alpha_k(D)$ and ${}^a\alpha_k(D)$, respectively.

The Deterministic Case

When a source switches to the active state, one cell is generated every $1/c$ seconds. In addition the packetizer flushes its content to the UNI before a renewal. The following equations are representations of this packetization process:

$$ {}^{i}\alpha_0(u) = P[T_a(u) = 0 | S_0 = i] \quad , $$

$$ {}^{a}\alpha_0(u) = P[T_a(u) = 0 | S_0 = a] \quad , $$

$$ {}^{i}\alpha_k(u) = P[\frac{k-1}{c} < T_a(u) \leq \frac{k}{c} | S_0 = i] \quad , \quad for\ k \geq 1, $$

and

$$ {}^{a}\alpha_k(u) = P[\frac{k-1}{c} < T_a(u) \leq \frac{k}{c} | S_0 = a] \quad , \quad for\ k \geq 1. $$

These expressions can be computed using (6.10) and (6.11). Notice that in every interval between two successive renewal epochs, at most $\lceil cD \rceil$ cells can be generated.

The Interrupted Poisson Case

If instead we assume that the source behaves like an interrupted Poisson source, then while in the active state, the source will generate Poisson traffic with rate c. In this case

$$ {}^{i}\alpha_k(u) = \int_{\tau \in (0,u]} e^{-c\tau} \frac{(c\tau)^k}{k!} dP[T_a(u) \leq \tau | S_0 = i] \quad , $$

and

$$ {}^{a}\alpha_k(u) = \int_{\tau \in (0,u]} e^{-c\tau} \frac{(c\tau)^k}{k!} dP[T_a(u) \leq \tau | S_0 = a] \quad , $$

for all k, $k \geq 0$.

7. Performance Analysis of the UNI Under a Bursty Traffic Source

In this section we analyze the UNI system when the state of the source is observed every D seconds, immediately before the renewal of the token source. At those instances the behavior of the UNI is described by an embedded Markov chain. Let $p_{ii}(D)$ be the probability that the source is in idle state immediately before a renewal epoch, given that its state immediately before the previous renewal epoch was idle. Then $p_{ii}(D) \stackrel{\text{def}}{=} Prob(S_D = i|S_0 = i)$. Similarly, $p_{ia}(D) \stackrel{\text{def}}{=} Prob(S_D = a|S_0 = i)$, $p_{ai}(D) \stackrel{\text{def}}{=} Prob(S_D = i|S_0 = a)$, and $p_{aa}(D) \stackrel{\text{def}}{=} Prob(S_D = a|S_0 = a)$. From Equations (6.2) and (6.3) we have

$$p_{ii}(D) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}e^{-(\lambda+\mu)D} \quad ,$$

$$p_{ia}(D) = \frac{\lambda}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda}e^{-(\lambda+\mu)D} \quad ,$$

$$p_{ai}(D) = \frac{\mu}{\mu+\lambda} - \frac{\mu}{\mu+\lambda}e^{-(\lambda+\mu)D} \quad ,$$

and

$$p_{aa}(D) = \frac{\lambda}{\mu+\lambda} + \frac{\mu}{\mu+\lambda}e^{-(\lambda+\mu)D} \quad .$$

The state of the UNI system is described by the state of the UNI's buffers and the state of the source.

Let us order the states as follows: $\{(0,i),(1,i),\cdots(M+K,i);(0,a),(1,a),\cdots,(M+K,a)\}$. Let

$${}^{i}b_{j_1,j_2} \stackrel{\text{def}}{=} \begin{cases} {}^{i}\alpha_{j_2-(j_1-s)^+}, & \text{if} j_2-(j_1-s)^+ \geq 0 \text{ and } 0 \leq j_2 < K+M \\ 0, & \text{otherwise} \end{cases} \tag{7.1}$$

and

$${}^{i}\bar{b}_{j_1,j_2} \stackrel{\text{def}}{=} \begin{cases} 0, & \text{if } j_2 < K+M, \\ 1-\sum_{n=0}^{K+M-1} {}^{i}b_{j_1,n}, & \text{if } j_2 = K+M, \end{cases} \tag{7.2}$$

for $0 \leq j_1 \leq K+M$. Similarly we define the functions ${}^{a}b_{j_1,j_2}$ and ${}^{a}\bar{b}_{j_1,j_2}$.

Let

$${}^{i}\mathbf{P} \stackrel{\text{def}}{=} \begin{pmatrix} {}^{i}b_{0,0} & {}^{i}b_{0,1} & \cdots & {}^{i}\bar{b}_{0,M+K} \\ {}^{i}b_{1,0} & {}^{i}b_{1,1} & \cdots & {}^{i}\bar{b}_{1,M+K} \\ \vdots & \vdots & \ddots & \vdots \\ {}^{i}b_{M+K,0} & {}^{i}b_{M+K,1} & \cdots & {}^{i}\bar{b}_{M+K,M+K} \end{pmatrix} \tag{7.3}$$

In a similar way we can define the matrix ${}^{a}\mathbf{P}$. The transition matrix is given by

$$\mathbb{P} = \begin{pmatrix} {}^{i}\mathbf{P} \times p_{ii}(D) & {}^{i}\mathbf{P} \times p_{ia}(D) \\ {}^{a}\mathbf{P} \times p_{ai}(D) & {}^{a}\mathbf{P} \times p_{aa}(D) \end{pmatrix} \tag{7.4}$$

Let ${}^{i}\pi_j$ be the equilibrium probability that the UNI is in state (j,i) immediately before a renewal epoch, and let ${}^{i}\pi \stackrel{\text{def}}{=} [{}^{i}\pi_0, {}^{i}\pi_1, \cdots, {}^{i}\pi_{M+K}]$. Similarly we define the vector ${}^{a}\pi$. Finally we define the vector of equilibrium probabilities as the vector $\pi \stackrel{\text{def}}{=} [{}^{i}\pi, {}^{a}\pi]$, such that $\pi\mathbb{P} = \pi$ and $\pi\mathbf{e} = \mathbf{1}$.

The expected number of cells that pass through the UNI system per D seconds is

$$E\gamma = \sum_{j=0}^{K+M} {}^{i}\pi_j \Big\{ (K + w(j) - q(j)) - \sum_{n=0}^{K+w(j)-q(j)-1} {}^{i}\alpha_n \times \Big(K + w(j) - q(j) - n\Big) \Big\} +$$

$$\sum_{j=0}^{K+M} {}^{a}\pi_j \Big\{ (K + w(j) - q(j)) - \sum_{n=0}^{K+w(j)-q(j)-1} {}^{a}\alpha_n \times \Big(K + w(j) - q(j) - n\Big) \Big\} \quad . \tag{7.5}$$

Notice that the previous approach results in a state space with twice as many states as in the Poisson source case. In a second approach now introduced, we once again have a Markov process as a source. We do not observe its state at the time of the renewal. In this case the probability that k cells arrive at the UNI in u seconds after the last renewal of the token source, is given by $P[A(u) = k] = P[A(u) = k|S_0 = i]\frac{\mu}{\lambda + \mu} + P[A(u) = k|S_0 = a]\frac{\lambda}{\lambda + \mu}$. With $\alpha_k(u) \stackrel{\text{def}}{=} P[A(u) = k]$,

$$\alpha_k(u) = {}^{i}\alpha_k(u)\frac{\mu}{\lambda + \mu} + {}^{a}\alpha_k(u)\frac{\lambda}{\lambda + \mu} \quad .$$

After the definition of the probability that k packets arrive in the interval $[0, u)$, the analysis and the equations for the throughput, delay and loss distributions derived for the Poisson source case hold.

8. The Waiting Time Distribution of a Bursty Source

As in the Poisson source case, let us tag a cell that arrives u seconds after the last renewal epoch, and let (j, i) or (j,a) be the state of the UNI before the last renewal.

If $q(j) + A(u) + 1 - w(j) \leq 0$, the tagged cell is served instantaneously.

If $1 \leq q(j) + A(u) + 1 - w(j) \leq K$, then the tagged cell has to wait until the source generates enough tokens to serve all the cells in front of the tagged cell. The time that it takes the token source to generate enough tokens is

$$T = \min_{n} \{n | n \times s \geq q(j) + A(u) + 1 - w(j)\} \quad ,$$

and the waiting time of the tagged cell is

$$W_u = D - u + (T - 1)D \quad .$$

If $q(j) + A(u) + 1 - w(j) > K$, the tagged cell is lost.

The probability that a cell arriving at time u enters the system is given by

$$P_{accept} = P[q(j) + A(u) + 1 - w(j) \leq K] \quad . \tag{8.1}$$

As in the Poisson case, let $P[W_u > x|\ accepted,\ (j,i)]$ be the waiting time distribution of a cell arriving at time u, given that it was accepted and the state of the UNI immediately before the last renewal epoch was (j,i).

$$P[W_u > x|\ accepted,\ (j,i)] \;=\; \frac{P[W_u > x,\ accepted,\ (j,i)]}{P[accepted,\ (j,i)]} \;=$$

$$\frac{\sum_{k=0}^{K+M} P[W_u > x,\ accepted,\ (j,i), A(u) = k]}{\sum_{n=0}^{K+M} P[accepted,\ (j,i),\ A(u) = n]} \quad ,$$

for all $x \geq 0$. Therefore,

$$P[W_u > x|accepted,\ (j,i)] \;=\; \frac{\sum_{k=\left(w(j)-q(j)\right)^+}^{K+w(j)-q(j)-1} 1\Big(D-u+(T-1)D > x|A(u)=k\Big)^i\alpha_k(u)}{\sum_{k=0}^{K-q(j)+w(j)-1} {}^i\alpha_k(u)} \quad , \tag{8.2}$$

Similarly,

$$P[W_u > x|accepted,\ (j,a)] \;=\; \frac{\sum_{k=\left(w(j)-q(j)\right)^+}^{K-q(j)+w(j)-1} 1\Big(D-u+(T-1)D > x|A(u)=k\Big)^a\alpha_k(u)}{\sum_{k=0}^{K-q(j)+w(j)-1} {}^a\alpha_k(u)} \quad . \tag{8.3}$$

Therefore, given that the state of the UNI before the last renewal was (j,i), the waiting time distribution of a cell that enters the UNI is

$$P[W > x|(j,i)] = \frac{\sum_{k=\left(w(j)-q(j)\right)^+}^{K+w(j)-q(j)-1} \int_0^D 1\Big(D-u+(T-1)D > x|A(u)=k\Big)^i\alpha_k(u)du}{\sum_{n=0}^{K+w(j)-q(j)-1} \int_0^D {}^i\alpha_n(u)du} \quad . \tag{8.4}$$

Similarly,

$$P[W > x|(j,a)] = \frac{\sum_{k=\left(w(j)-q(j)\right)^+}^{K-q(j)+w(j)-1} \int_0^D 1\Big(D-u+(T-1)D > x|A(u)=k\Big)^a\alpha_k(u)du}{\sum_{n=0}^{K+w(j)-q(j)-1} \int_0^D {}^a\alpha_n(u)du} \quad . \tag{8.5}$$

Finally, for any $x \geq 0$ the waiting time distribution of any accepted cell is

$$P[W > x] \;=\; \sum_{j=0}^{K+M} \left\{P[W > x|(j,i)]^i\pi_j + P[W > x|(j,a)]^a\pi_j\right\} \quad . \tag{8.6}$$

For $x = 0$,

$$P[W_u = 0|accepted,\ (j,i)] \;=\; \frac{\sum_{k=0}^{w(j)-q(j)-1} {}^i\alpha_k(u)}{\sum_{k=0}^{K+w(j)-q(j)-1} {}^i\alpha_k(u)} \quad , \tag{8.7}$$

$$P[W_u = 0|accepted,\ (j,a)] \;=\; \frac{\sum_{k=0}^{w(j)-q(j)-1} {}^a\alpha_k(u)}{\sum_{k=0}^{K+w(j)-q(j)-1} {}^a\alpha_k(u)} \quad , \tag{8.8}$$

$$P[W = 0|(j,i)] = \frac{\sum_{k=0}^{w(j)-q(j)-1} \int_0^D {}^i\alpha_k(u)du}{\sum_{n=0}^{K+w(j)-q(j)-1} \int_0^D {}^i\alpha_n(u)du} \quad , \tag{8.9}$$

and

$$P[W = 0|(j,a)] = \frac{\sum_{k=0}^{w(j)-q(j)-1} \int_0^D {}^a\alpha_k(u)du}{\sum_{n=0}^{K+w(j)-q(j)-1} \int_0^D {}^a\alpha_n(u)du} \quad . \tag{8.10}$$

Therefore, the probability that an accepted cell is served instantaneously follows from the equation

$$P[W = 0] \;=\; \sum_{j=0}^{K+M} \left\{P[W = 0|(j,i)]^i\pi_j + P[W = 0|(j,a)]^a\pi_j\right\} \quad . \tag{8.11}$$

9. The Loss Distribution of a Bursty Source

The probability that L consecutive cells are lost in an interval $[0, D)$ is given by

$$P[L=0] \;=\; \sum_{j=0}^{K+M} \sum_{n=0}^{K+w(j)-q(j)} \{{}^{i}\alpha_n \times {}^{i}\pi_j + {}^{a}\alpha_n \times {}^{a}\pi_j\} \quad , \tag{9.1}$$

and

$$P[L=k] \;=\; \sum_{j=0}^{K+M} \{{}^{i}\alpha_{K+w(j)-q(j)+k} \times {}^{i}\pi_j \;+\; {}^{a}\alpha_{K+w(j)-q(j)+k} \times {}^{a}\pi_j\}, \tag{9.2}$$

for $k \geq 1$. From the last expression we can compute the expected number of lost cells between two renewal epochs.

$$E[L] \;=\; \sum_{j=0}^{K+M} \sum_{n=K+w(j)-q(j)+1}^{\infty} \Big(n - \big(K + w(j) - q(j)\big)\Big) \times \{{}^{i}\alpha_n \times {}^{i}\pi_j \;+\; {}^{a}\alpha_n \times {}^{a}\pi_j\} \quad . \tag{9.3}$$

From (7.5) and (9.3), we find that the total number of cells arriving at the UNI is

$$E[L] + E\gamma \;=\; \sum_{j=0}^{K+M} \sum_{n=0}^{\infty} n \times \{{}^{i}\alpha_n \times {}^{i}\pi_j \;+\; {}^{a}\alpha_n \times {}^{a}\pi_j\} \quad . \tag{9.4}$$

This expression represents the "load" of the UNI. It corresponds to the value cD which is the expected number of arriving cells in $[0, D)$ for the Poisson source case.

For the deterministic case, the last equation becomes

$$E[L] + E\gamma \;=\; \sum_{j=0}^{K+M} \sum_{n=0}^{\lceil cD \rceil} n \times \{{}^{i}\alpha_n \times {}^{i}\pi_j \;+\; {}^{a}\alpha_n \times {}^{a}\pi_j\} \quad . \tag{9.5}$$

For the interrupted Poisson case (9.4) becomes

$$E[L] + E\gamma \;=\; \sum_{j=0}^{K+M} \{cE[T_a(D)|S_0 = i] \times {}^{i}\pi_j \;+\; cE[T_a(D)|S_0 = a] \times {}^{a}\pi_j\} \quad , \tag{9.6}$$

where

$$E[T_a(D)|S_0] \;=\; \int_0^D p_a(u)du \;=\; \frac{\lambda D}{\mu+\lambda} \;+\; \{p_a(0) \;-\; \frac{\lambda}{\mu+\lambda}\}\frac{1-e^{-(\lambda+\mu)D}}{\mu+\lambda} \quad .$$

From the expressions (9.5), (9.6), and (7.5), we can compute $E[L]$ for the deterministic and the interrupted Poisson cases. The loss probability is given by the equation

$$P_L \;=\; \frac{E[L]}{E[L] + E\gamma} \quad . \tag{9.7}$$

10. Examples

The Case of a Poisson Source

In Figures 2 and 3, we present the throughput and the cell loss probability of the UNI for different values of the offered load and for different values of the buffer sizes. The dotted points are points derived with simulation. We observe that the theoretical results match the simulation results. In addition we observe that the throughput and the cell loss are a function of $K + M$. The throughput is an increasing function of the size of the source buffer, and an increasing function of the size of the token buffer.

In Figure 4, we observe that the waiting time decreases with the size of the token buffer M. In addition the waiting time increases with the size of the source buffer K. In addition, notice that if $K + M$ is constant and we increase the size of the token buffer at the expense of the size of the source buffer, the waiting time decreases. Recall that from the Figures 2 and 3, such a change does not affect the throughput and the loss probability. In the previous figures, $s = 1$, and $D = 1.0$.

The Case of a Bursty Source

In Figures 5 and 6, the distribution of the time that the bursty source is in active state, given its state at the last renewal epoch, is shown. In this paper we provide results for a bursty source that is subject to packetization. Similar results can be derived for the interrupted Poisson case. In Figure 7, we present the throughput and the cell loss for the case in which $c = 5$ $s = 3$, $D = 1.0$, and $\mu = 0.5$. Notice that the throughput is an increasing function of λ, and of the offered load. In addition notice that the throughput increases with $K + M$. In Figure 8, the throughput and the cell loss are shown as functions of c and the offered load for different values of $K + M$, for the case in which $\lambda = .5$, D=1.0, $\mu = .5$, and s=3. The packetization effect on the throughput and cell loss is clearly demonstrated. When the value of c is up to a few times the value of D, the packetization effect cannot be ignored. As c increases, with respect to a given D, the packetization effect becomes less and less noticeable. In Figure 9, we notice that for a given value of $K + M$, if we increase the token buffer at the expense of the source buffer, the waiting time decreases. In this case D=1, λ= 2.0, $\mu = 0.5$, $c = 5$, s =3. Obviously the throughput and the cell loss are the same for both cases.

11. Conclusions

In the present paper the functionality requirements of a UNI were studied. Based on these requirements a UNI model was suggested and studied for two cases, a Poisson source and a bursty source. Some of the extensive studies that we conducted using the derived theoretical results were also presented. In the future we plan to study the behavior of this UNI model for different sources, and we plan to address issues related to the inter-operability between the user, the UNI, the network systems and the network management.

12. References

[BER90a] Berger, A. W.,"Performance Analysis of a Rate Control Throttle Where Tokens and Jobs Queue," *Proceedings of INFOCOM'90,* San Francisco, California, June 5-7, 1990, pp. 30-38.

[BER90b] Berger, A. W.,"Overload Control Using a Rate Control Throttle: Selecting Token Bank Capacity for Robustness to Arrival Rates, to appear in *IEEE Transactions on Automatic Control,* Fall, 1990.

[COU89] Coudreuse, J. P., "ATM: Status of Definition and Discussion of Some Open Issues," *Proceedings of the MULTUMEDIA'89: 2nd IEEE Communications Society International Workshop on Multimedia Communications,* April 20-23, 1989.

[Chu90] Chuah, M. C., and Cruz, R. L., "Approximate Analysis of Average Performance of (σ, ρ) Regulators," *Proceedings of INFOCOM'90,* San Francisco, California, June 5-7, 1990, pp. 874-880.

[DEV88] Devault M., Cochennec, J. Y., and Servel, M., "The Prelude ATD Experiment: Assessments and Future Prospects," *IEEE Journal on Selected Areas in Communications,* Vol. SAC-6, No. 9, December 1988, pp. 1528-1537.

[FIL89] Filipiak, J., "M-Architecture: A Structural Model of Traffic Management and Control in Broadband ISDNs," *IEEE Communications Magazine,* Vol. 27, No. 4, May 1989, pp. 25-31.

[GEC88] Gechter, J., and O'Reilly, P., "Standardization of ATM," *Proceedings of IEEE GLOBECOM'88,* November 28- December 1, 1988 pp. 107-111.

[HEF86] Heffes, H., and Lucantoni, D. M., "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Performance," *IEEE Journal on Selected Areas in Communications,* Vol. SAC-4, No. 6, September 1986, pp. 856-868.

[HUA84] Huang, A., and Knauer, S., "Starlite: a Wideband Digital Switch," *Proceedings of IEEE GLOBECOM'84,* December 1984, pp. 121-125.

[HUI87] Hui, J., and Arthurs, E., "A Broadband Packet Switch for Integrated Transport," *IEEE Journal on Selected Areas in Communications,* Vol. SAC-5, No. 8, October 1987, pp. 1264-1273.

[HUI88] Hui, J., "Resource Allocation for Broadband Networks," *IEEE Journal on Selected Areas in Communications,* Vol. SAC-6, No. 9, December 1988, pp. 1264-1273.

[JAC90] Jacobsen, S.B., Moth, K., and Dittmann, L., "Load Control in ATM Networks," *Proceedings of XIII International Switching Symposium,* Stockholkm-Sweden, May 27-June 1, 1990.

[JEN84] Jenq, V. C., "Approximations for Packetized Voice Traffic in Statistical Multiplexer," *Proceedings of IEEE INFOCOM'84,* April 1884, pp. 256-259.

[LI90] Li, S.Q., "A General Solution Technique for Discrete Queueing Analysis of Multi-Media Traffic on ATM," *Proceedings of IEEE INFOCOM'90,* pp. 1144-1155.

[MIN89] Minzer, S. E., "Broadband ISDN and Asynchronous Transfer Mode (ATM)," *IEEE, Communications magazine,* Vol. 27, No.9, September 1989, pp. 17-24.

[NIE90] Niestegge, G., "The "Leaky Bucket" Policing Method in the ATM Network," *International Journal of Digital and Analog Cabled Systems,* June 1990.

[O'RE89] O'Reilly, P., "ATM: Ready for the Marketplace?" *Proceedings of IEEE INFOCOM'89,* April 23-27, 1989, pp. 495-496.

[SID89] Sidi, M., Liu, W. Z., Cidon, I., and Gopal, I., "Congestion Control Through Input Rate Regulation," *Proceedings of GLOBECOM'89,* Dallas, Texas, November 27-30, 1989, pp. 1764-1768.

[SRI89] Sriram, K., and Lucantoni, D. M., "Traffic Smoothing Effects of Bit Dropping in a Packet Voice Multiplexer," *IEEE Transactions on Communications,* Vol. 37, No. 7, July 1989, pp. 703-712.

[TUR88a] Turner, J. S., "Design of a Broadcast Packet Switching Network," *IEEE Transactions on Communications,* Vol. 36, No. 6, June 1988, pp. 734-743.

[TUR88b] Turner, J. S.,"The Challenge of Multipoint Communication," in *Traffic Engineering for ISDN Design and Planning,* M. Bonnatti and M. Decina (Editors), North-Holland, 1988.

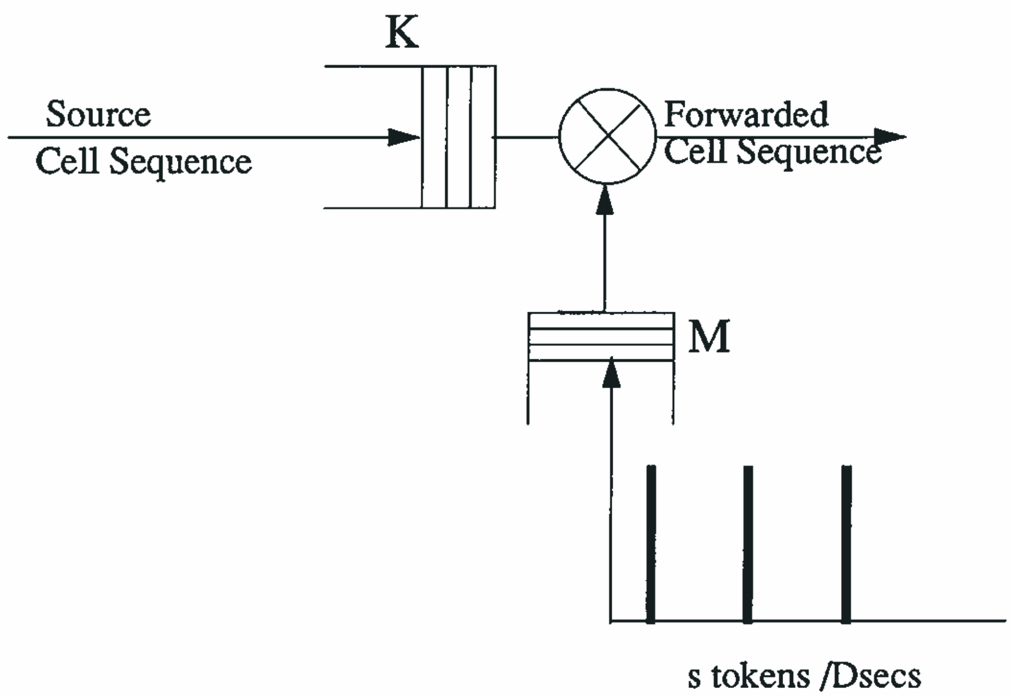

K
Source
Cell Sequence
Forwarded
Cell Sequence
M
s tokens /Dsecs

Figure 1

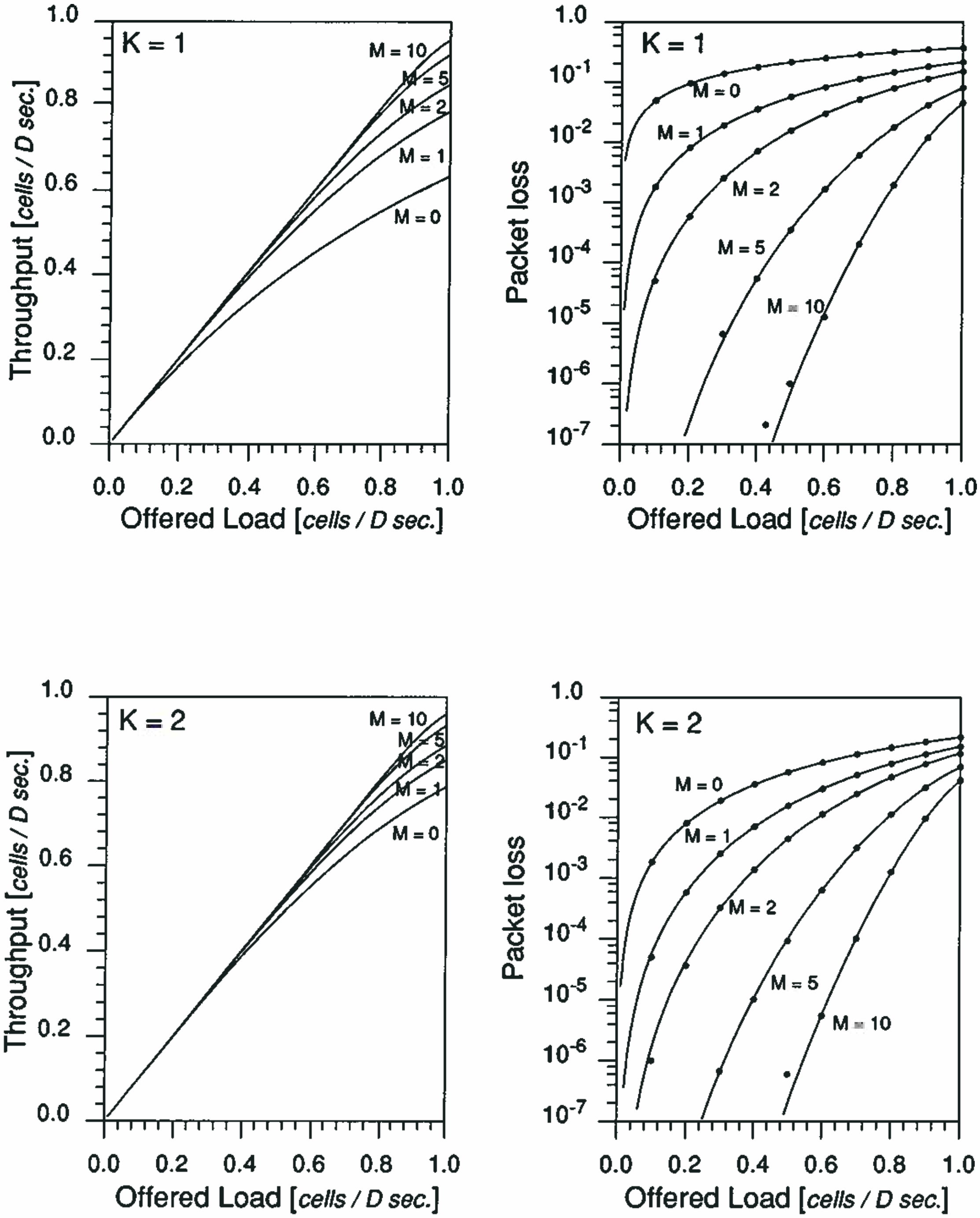

K = 1
K = 2
M = 0
M = 1
M = 2
M = 5
M = 10
Throughput [cells / D sec.]
Packet loss
Offered Load [cells / D sec.]

Figure 2

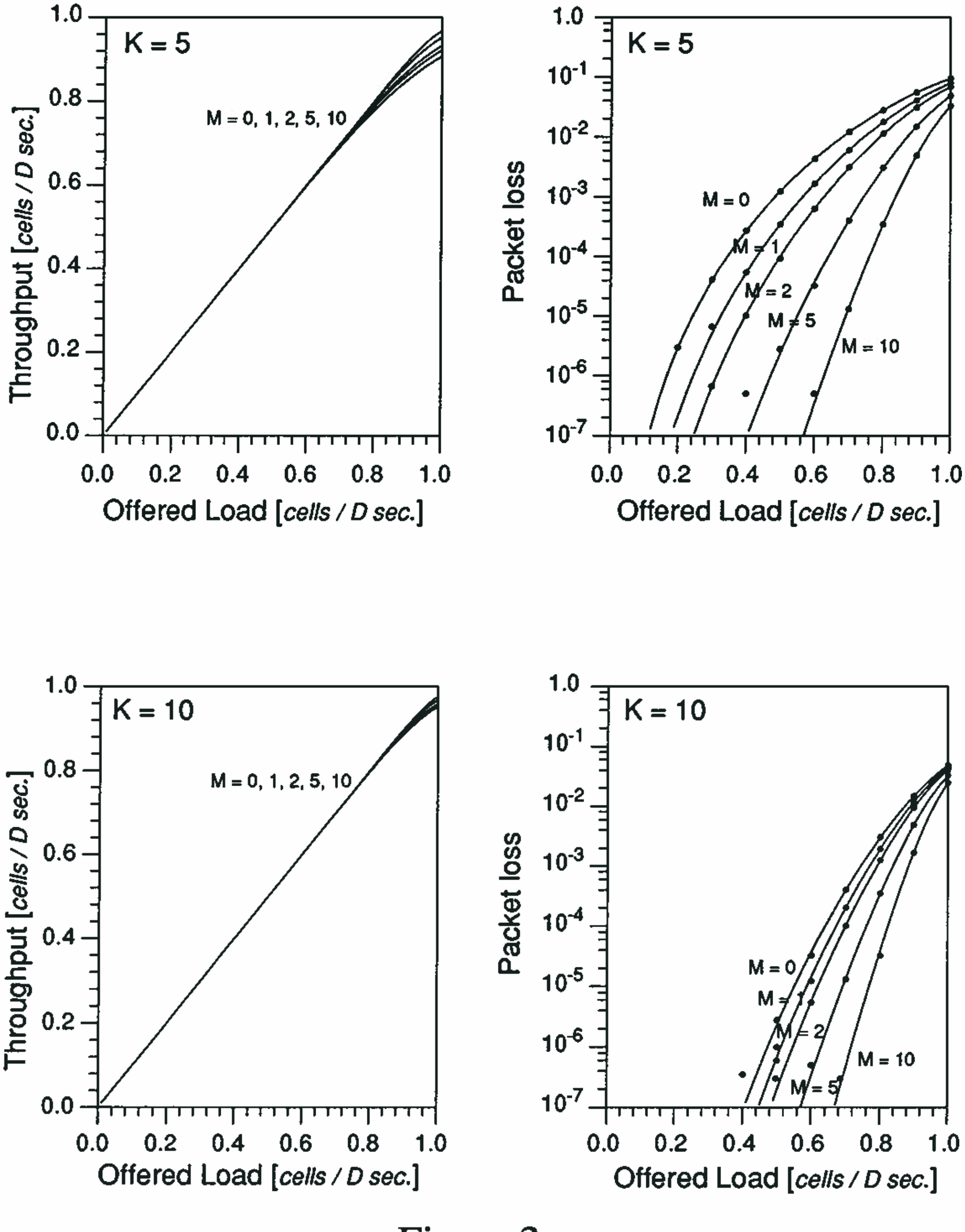
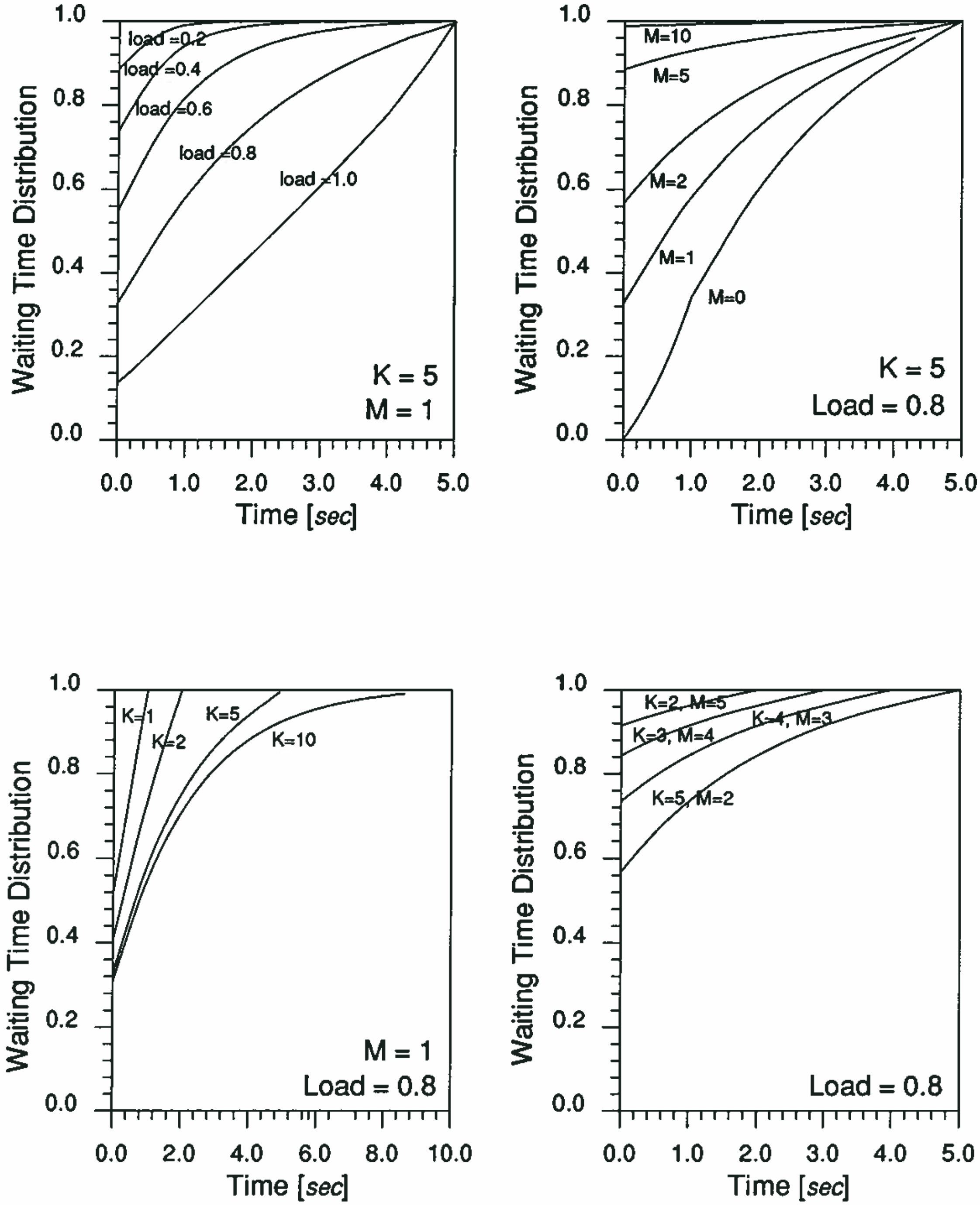

K = 5
M = 0, 1, 2, 5, 10
Throughput [cells / D sec.]
Offered Load [cells / D sec.]
K = 5
M = 0
M = 1
M = 2
M = 5
M = 10
Packet loss
Offered Load [cells / D sec.]
K = 10
M = 0, 1, 2, 5, 10
Throughput [cells / D sec.]
Offered Load [cells / D sec.]
K = 10
M = 0
M = 1
M = 2
M = 5
M = 10
Packet loss
Offered Load [cells / D sec.]

Figure 3



Waiting Time Distribution
Time [sec]
load =0.2
load =0.4
load =0.6
load =0.8
load =1.0
K = 5
M = 1
M=10
M=5
M=2
M=1
M=0
K = 5
Load = 0.8
K=1
K=2
K=5
K=10
M = 1
Load = 0.8
K=2, M=5
K=3, M=4
K=4, M=3
K=5, M=2
Load = 0.8

Figure 4

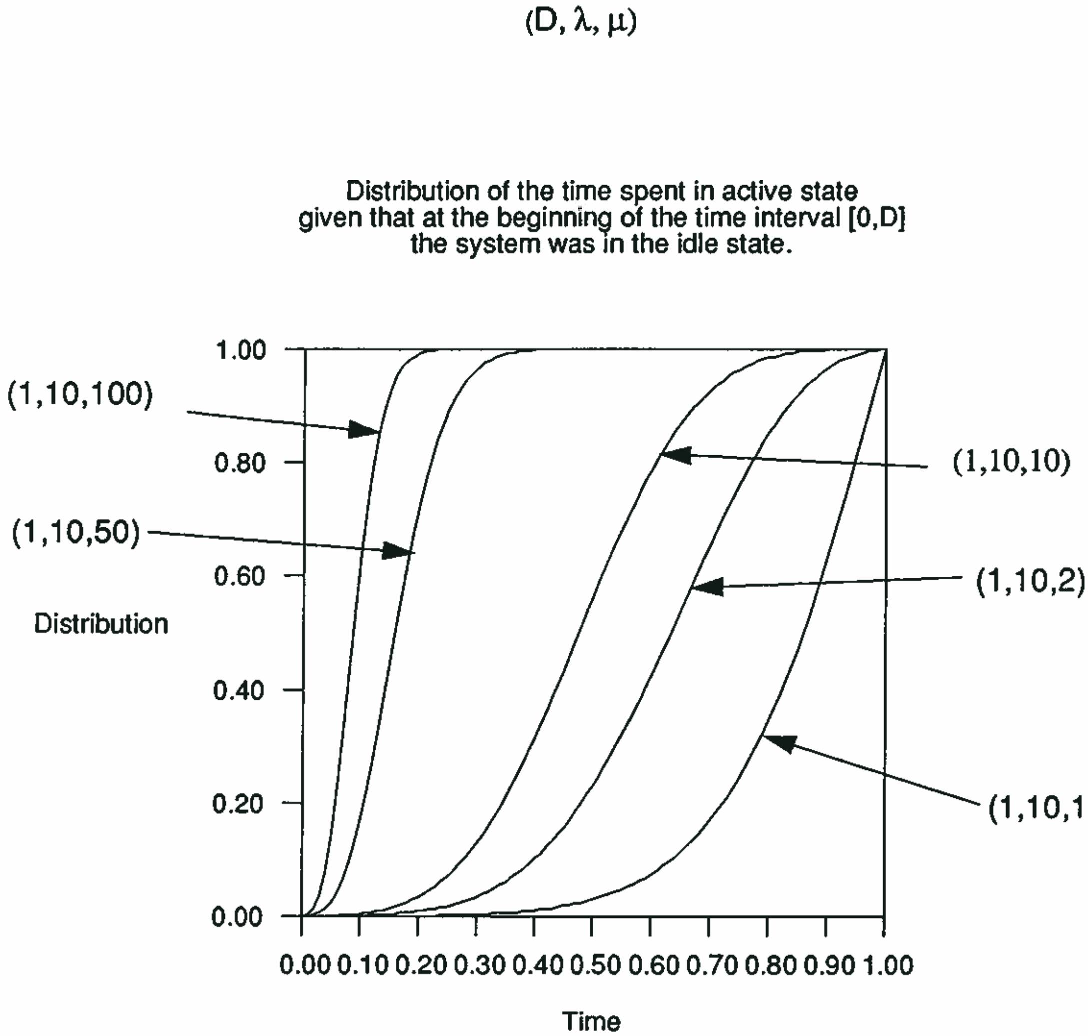

(D, λ, μ)
Distribution of the time spent in active state
given that at the beginning of the time interval [0,D]
the system was in the idle state.
Distribution
1.00
0.80
0.60
0.40
0.20
0.00
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
Time
(1,10,100)
(1,10,50)
(1,10,10)
(1,10,2)
(1,10,1)

Figure 5

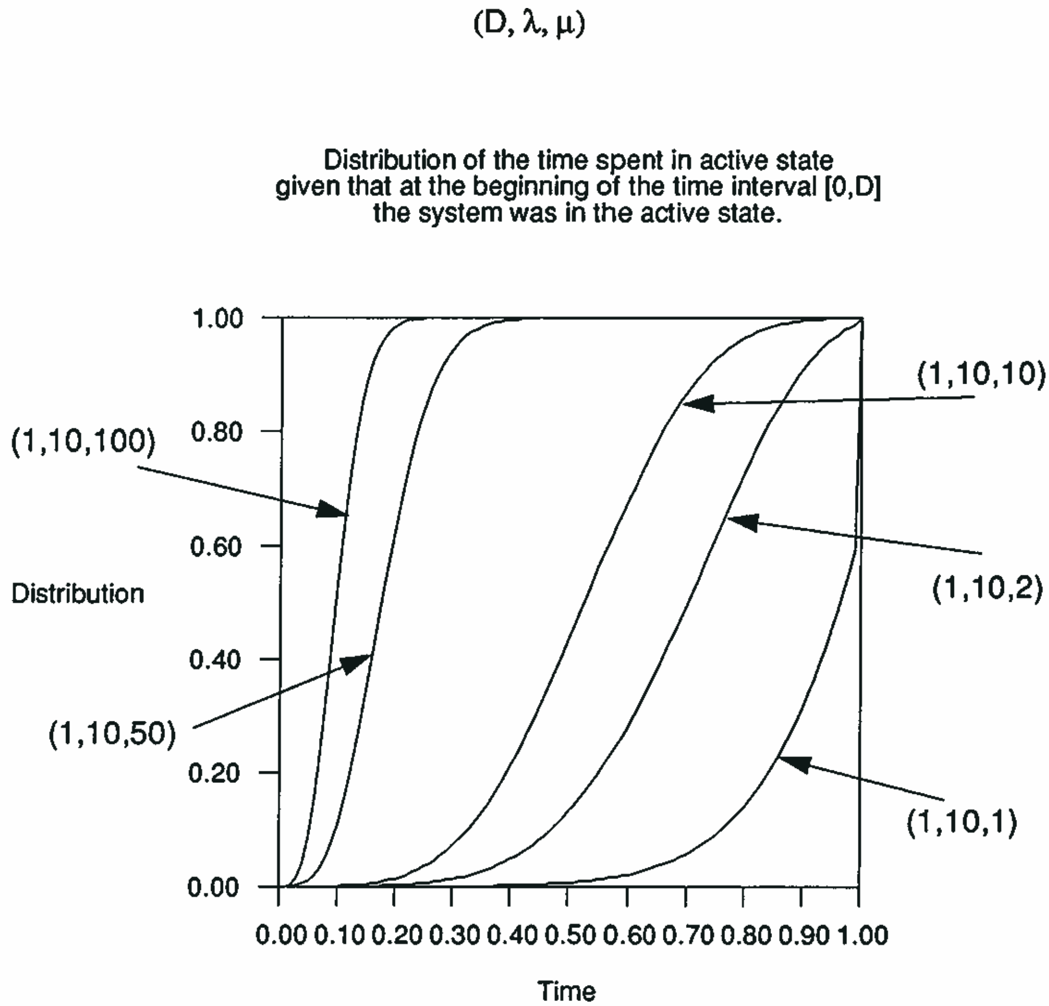
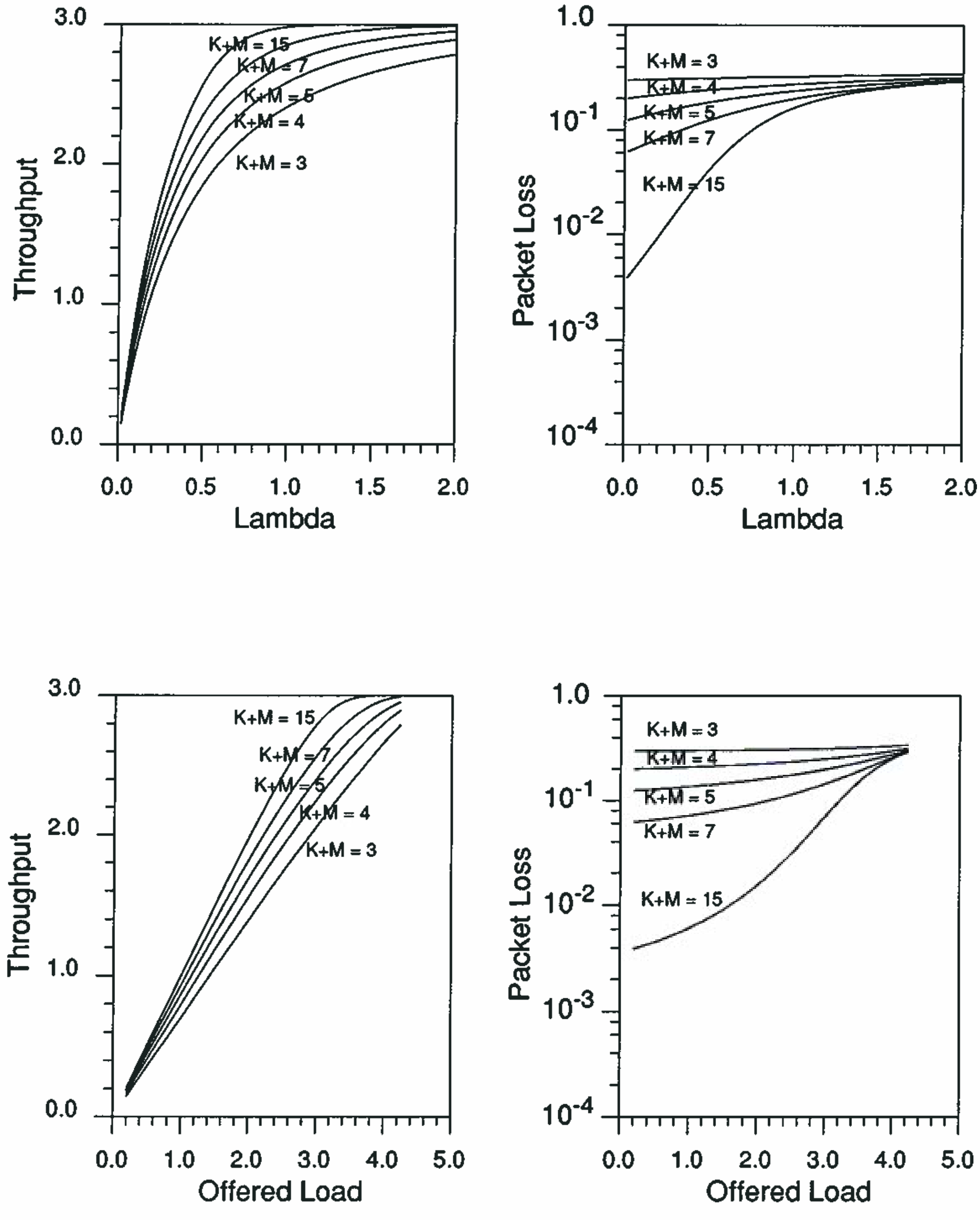

(D, λ, μ)
Distribution of the time spent in active state
given that at the beginning of the time interval [0,D]
the system was in the active state.
1.00
0.80
0.60
0.40
0.20
0.00
Distribution
(1,10,100)
(1,10,50)
(1,10,10)
(1,10,2)
(1,10,1)
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00
Time

Figure 6



Throughput
Lambda
Packet Loss
Offered Load
K+M = 15
K+M = 7
K+M = 5
K+M = 4
K+M = 3

Figure 7

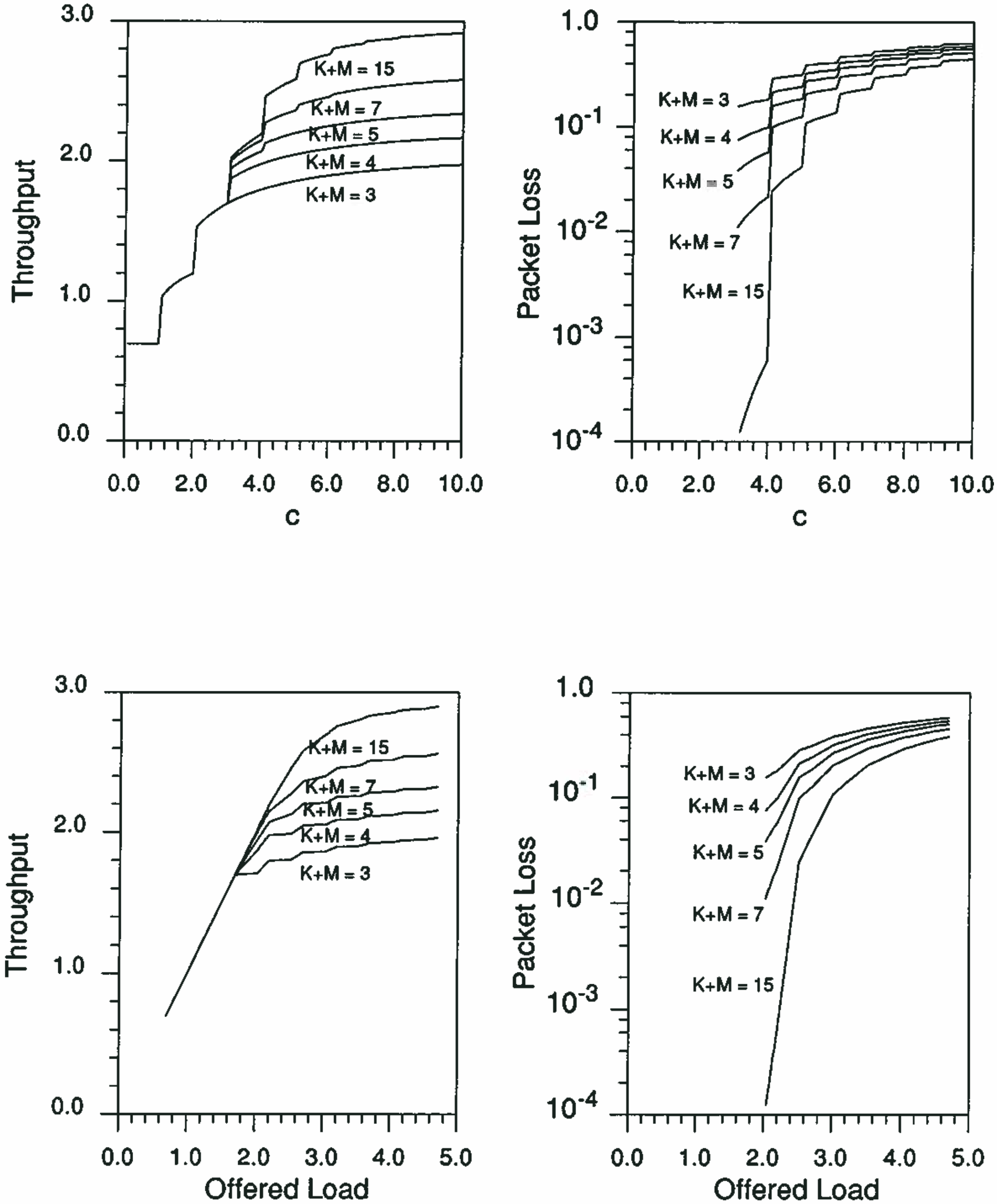


Figure 8

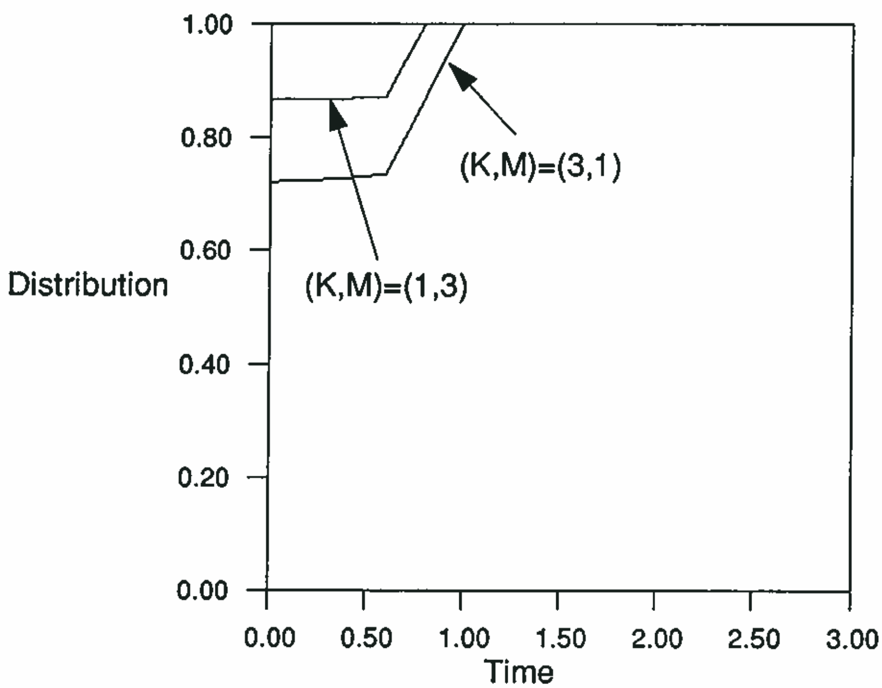

1.00
0.80
0.60
0.40
0.20
0.00
Distribution
(K,M)=(3,1)
(K,M)=(1,3)
0.00
0.50
1.00
1.50
2.00
2.50
3.00
Time

Figure 9