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Witten Index and spectral shift function

Abstract

Let D be a selfadjoint unbounded operator on a Hilbert space and let $\{B(t)\}$ be a one parameter norm continuous family of self-adjoint bounded operators that converges in norm to asymptotes $B_\pm$. Then setting $A(t) = D + B(t)$ one can consider the operator $\mathbf{D_A} = d/dt + A(t)$ on the Hilbert space $L_2(\mathbb{R}, H)$. We present a connection between the theory of spectral shift function for the pair of the asymptotes (A_+, A_-) and index theory for the operator $\mathbf{D_A}$.

Under the condition that the operator B_+ is a p-relative trace-class perturbation of A_- and some additional smoothness assumption we prove a heat kernel formula for all $t > 0$,

$$\operatorname{tr}\Big(e^{-t\mathbf{D_A}\mathbf{D_A}^*} - e^{-t\mathbf{D_A}^*\mathbf{D_A}}\Big) = -\Big(\frac{t}{\pi}\Big)^{1/2} \int_0^1 \operatorname{tr}\Big(e^{-tA_s^2}(A_+ - A_-)\Big)ds,$$

where $A_s, s \in [0, 1]$ is a straight path joining A_- and A_+.

Using this heat kernel formula we obtain the description of the Witten index of the operator $\mathbf{D_A}$ in terms of the spectral shift function for the pair (A_+, A_-).

Theorem. *If 0 is a right and a left Lebesgue point of the spectral shift function $\xi(\cdot; A_+, A_-)$ for the pair (A_+, A_-) (denoted by $\xi_L(0_+; A_+, A_-)$ and $\xi_L(0_-; A_+, A_-)$, respectively), then the Witten index $W_s(\mathbf{D_A})$ of the operator $\mathbf{D_A}$ exists and equals*

$$W_s(\mathbf{D_A}) = \frac{1}{2}\big(\xi(0+; A_+, A_-) + \xi(0-; A_+, A_-)\big).$$

We note that our assumptions include the cases studied earlier. In particular, we impose no assumption on the spectra of $A_\pm$ and we can treat differential operators in any dimension.

As a corollary of this theorem we have the following result.

Corollary. *Assume that the asymptotes $A_\pm$ are boundedly invertible. Then the operator $\mathbf{D_A}$ is Fredholm and for the Fredholm index* $\operatorname{index}(\mathbf{D_A})$ *of the operator $\mathbf{D_A}$ we have*

$$\operatorname{index}(\mathbf{D_A}) = \xi(0; A_+, A_-) = \operatorname{sf}\{A(t)\}_{t=-\infty}^{+\infty},$$

where $\operatorname{sf}\{A(t)\}_{t=-\infty}^{+\infty}$ *denotes the spectral flow along the path* $\{A(t)\}_{t=-\infty}^{+\infty}$.

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