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A band formula for a Toeplitz commutant lifting problem

Abstract

The band method plays a fundamental role in solving a Toeplitz and Nehari interpolation problem; see [2]. The solution to the Nehari problem involves the inverses of $I - HH^*$ and $I - H^*H$ where H is the corresponding Hankel matrix. Here we will derive a similar result for a certain commutant lifting problem.

Let Θ be an inner function in $H^\infty(\mathcal{E}, \mathcal{Y})$ and $\mathcal{H}(\Theta)$ the subspace of $\ell_+^2(\mathcal{Y})$ defined by

$$\mathcal{H}(\Theta) = \ell_+^2(\mathcal{Y}) \ominus T_\Theta \ell_+^2(\mathcal{E})$$

where T_Θ is the Toeplitz operator determined by Θ . Clearly, $\mathcal{H}(\Theta)$ is an invariant subspace for the backward shift $S_{\mathcal{Y}}^*$. Consider the data set $\{A, T', S_{\mathcal{Y}}\}$ where A is a strict contraction mapping $\ell_+^2(\mathcal{U})$ into $\mathcal{H}(\Theta)$, the operator T' on $\mathcal{H}(\Theta)$ is the compression of $S_{\mathcal{Y}}$ to $\mathcal{H}(\Theta)$, that is,

$$T' = \Pi_{\mathcal{H}(\Theta)} S_{\mathcal{Y}}|_{\mathcal{H}(\Theta)}.$$

Here $\Pi_{\mathcal{H}(\Theta)}$ is the orthogonal projection from $\ell_+^2(\mathcal{Y})$ onto $\mathcal{H}(\Theta)$. Moreover, A intertwines $S_{\mathcal{U}}$ with T' , that is, $T'A = AS_{\mathcal{U}}$. Given this data set the commutant lifting problem is to find all contractive Toeplitz operators T_Ψ such that

$$\Pi_{\mathcal{H}(\Theta)} T_\Psi = A. \tag{1}$$

This lifting problem includes the Nevanlinna-Pick and Leech interpolation problems. Using two different methods we will show that the set of all solutions are given by

$$\Psi = (\Upsilon_{12} + \Upsilon_{11}g)(\Upsilon_{22} + \Upsilon_{21}g)^{-1}.$$

Here g is a contractive analytic function acting between the appropriate spaces. Analogous to the band formulas in the Nehari interpolation problem, Υ_{jk} are determined by the inverses of $I - AA^*$ and $I - A^*A$. The proofs rely on different techniques. Finally, this is joint work with S. ter Horst and M.A. Kaashoek.

References

- [1] C. Foias, A.E. Frazho, I. Gohberg, and M. A. Kaashoek, *Metric Constrained Interpolation, Commutant Lifting and Systems*, Operator Theory: Advances and Applications, **100**, Birkhäuser-Verlag, 1998.
- [2] I. Gohberg, S. Goldberg, and M.A. Kaashoek, *Classes of Linear Operators*, Vol. II, Operator Theory: Advances and Applications, **63**, Birkhäuser-Verlag, Basel, 1993.

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