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Interest on Investments, and Amortization of Premiums Paid and Accumulation of Discounts Allowed Thereon

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INTEREST ON INVESTMENTS, AND AMORTIZATION OF PREMIUMS PAID AND ACCUMULATION OF DISCOUNTS ALLOWED THEREON

(Continued from May Issue)

THE CALCULATION OF INTEREST

As already noted, by custom interest is calculated at an agreed percentage per annum on each dollar, or fraction, of indebtedness.[39] To quote a rate of 6% per annum, means 6c per year on 100c, or $1, of indebtedness. The rate expressed as a fraction would be 6/100 of the principal per annum.

Simple interest may be easily calculated. Multiply the rate by the number of dollars; the product obtained will equal the amount of interest for one year. Knowing the amount for one year, the amount for any fraction of a year, or for any number of years, may easily be found by simple proportion. There are various methods for making this calculation, by dividing or multiplying the rate according to the proportion of the year or the number of years to be considered, or by dividing or multiplying the amount of annual interest, according to such proportion, or by dividing or multiplying the amount of the debt according to such proportion, and proceeding as in the first instance. Simple interest is calculated by arithmetical methods, as explained in any book on business arithmetic taught in primary schools. In addition there are combination methods and short methods, and also interest tables are provided for those who have much interest to calculate and need to be speedy and accurate in their work.

While the usual calendar year contains 365 days and a leap year 366 days, it is the custom of banks and merchants to calculate interest on basis of 360 days to the year, and to consider the interest for one day equal to 1/360 of amount for the year, for all periods of fractional years. Where the rate of interest is less than the highest legal rate allowed by law in a state, no question of usury is encountered by using the 360 days method, as the usury laws will not be infringed upon. Strictly speaking, the 360 days method causes an overcharge of interest equal to 5/365, or 1/73rd, in the usual calendar year (6/366 in a leap year), and to that extent error is made in the calculation of interest on the debt. As the error is slight, it is deliberately overlooked and generally accepted even by the courts.[40] However, when

39. Mo. R. S. 1909, § 7182.
40. Patton v. Bank, 124 Ga., 965.

the highest legal rate of interest is the rate on a loan, then to use the 360 days method will result in the usurious charge of interest, and the loan may be subject to the penalties of usury.[41] In the latter case, it would seem advisable to follow the exact method in the calculation of interest. Where an institution uses the 360 days method for calculating interest in its favor, proper appreciation of proprieties should induce it to use the same method for interest it pays, instead of the exact 365 days method; but this is not always the case.

It is the custom in banking institutions to discount short time obligations; that is, deduct in advance from the proceeds of loans the interest to accrue thereon for the period of each loan. Again, so long as the rate of discount is less than the highest legal rate allowed, no question of usury is encountered; but, if the highest legal rate is used, then the transaction may be tainted with usury, because the deduction in advance of the highest legal rate on the whole amount of the loan would result in a charge for interest in excess of the legal limit on the net amount of the loan.[42] Stated in another way, the present worth of an amount of interest, deducted in advance from a loan, is less than the amount of interest payable at the maturity of the loan, and, to the extent of the excess, would be usurious: $1 to be paid in the present is worth more than $1 to be paid at the end of the loan period, considering the money to have an interest earning value. In the last case, it would seem advisable to follow the exact method in figuring interest at the highest rate, and, for purpose of discount, calculate the present worth thereof, instead of taking the whole amount as interest. The present worth is readily ascertained by dividing the amount of interest by one-plus-the-rate-percent of the interest for the year, or pro rata of the per cent for fractional year, as the case may be.

Another practice in the calculation of interest generally indulged in by banking institutions throughout the United States is to calculate interest from the date of a note to the maturity of the note, both

41. Talbot v. Bank, 185 U. S., 172.

42. Loganville B. Co. v. Forrester, 143 Ga., 302; Contra, Bank of Newport v. Cook, 60 Ark., 288. Since the foregoing was written, note decision by United States Supreme Court, December, 1919, in case of Evans v. Bank, from Georgia. By divided opinion of four to three, the court holds that a National Bank did not subject itself to the penalties prescribed for taking usury, by discounting short time notes and charging interest in advance at the highest contract rate allowed in the state. The majority of the court reached this conclusion because of the special provisions of the National Bank Act, authorizing discounting by National Banks and referring to the state law only to determine the maximum rate permitted. The state courts in Georgia hold to the contrary in similar matters in their jurisdiction. U. S. Sup. Ct., Cause No. 67, Oct. Term, 1919.

days inclusive. This method of calculation gives the institution interest for one extra day on the amount of each note and is an illegal method for figuring an agreed rate; for the rule of law, governing matter of time of performance of contracts, is to exclude either the first day or the last day, in finding the number of days the obligation is to run.[43] If, instead of actually paying his note on the day of maturity, the maker gives a renewal note on such day of maturity, for the same debt, the result is that the institution for such day receives two days' interest, and the day has already been taken as an extra day. So long as the rate of interest taken is not the highest legal rate allowed in the state, the question of usury will not enter. If the highest rate is taken, usury may again taint the contract, unless exact interest is calculated for the legal number of days the contract is to run.[44]

The general rule in the United States for the calculation of interest on a debt, where partial payments are made from time to time, is the rule known as the United States Supreme Court Rule, because approved by that court. Under the rule, interest is to be calculated on the debt up to time of first payment; then add interest to principal and deduct payment; then cast interest on remainder, to the second payment; add the new interest to the remainder and deduct therefrom the second payment; and so on, until the last partial payment; except in any case where the interest up to any partial payment shall exceed the partial payment, in such case the partial payment is to be deducted from the interest, and the excess of interest is to be carried forward without casting interest thereon to the next payment that will discharge the excess.[45] While courts are opposed to the allowance of compound interest, yet it will be noted that, in the case of partial payments, compounding against such payments, where more than the accrued interest for the period covered, is allowed without an express contract for such compounding of interest. This rule is general and must, therefore, be taken as an exception to the rule of the courts never to allow compound interest without an express permissible contract. On the other hand, the equitable rule, known as the Merchants Rule, for calculating interest on partial payment contracts, is to calculate the interest on the debt without regard to partial payments, and ascertain the amount of such interest; then separately to calculate interest on each partial payment, from the time of payment to maturity of the debt, and credit such interest on partial payments against the interest first calculated on the debt; and the

43. Bank v. Durkee, 1 Vt., 399.
44. Bank v. Durkee, supra.
45. Story v. Livingston (U. S.), 13 Pet., 359.

reduced amount of interest is then taken as the true simple interest to be paid on the debt.

When money is loaned at interest compounded once a year, or, as it is said, at annual rests, it means that the interest accrued to the end of the year is to be added to the principal, to form a new increased principal, upon which interest is to be calculated for the second year, and so from year to year while the debt remains unpaid.[46] If the interest is to be compounded semi-annually, it means that the interest accrued at the end of each half year is to be added to the principal and the increased amount each half year shall form a new principal for the calculation of interest. Compound interest may be calculated arithmetically, by repeated calculations for each period, on the principal and interest of the preceding period, for the number of years or periods the loan is to run. Where the periods are few, the amount may be so calculated without very much work, beyond an ordinary interest calculation. Where the periods are many, then the arithmetical work becomes very laborious, and, as it also is liable to error at each step of the process, the tedious calculation becomes uncertain. Rules for making computations of compound interest cannot be conveniently expressed in arithmetical form; they can better be expressed algebraically. To make compound interest calculations resort is usually had to a table of compound interest for the desired number of years or periods, at the required rate of compound interest.

AMORTIZATION OF PREMIUMS AND ACCUMULATION OF DISCOUNTS ON INVESTMENTS

We shall now consider questions touching the amortization of premiums paid and accumulation of discounts allowed on bonds and similar obligations, purchased at a price greater than par (at a premium) or at a price less than par (at a discount). It requires no demonstration to establish the fact as to a bond purchased at par, bearing interest at 6% per annum, payable annually, upon which interest was paid regularly until maturity, and upon which the principal was promptly paid at maturity at par, that the bond was a net 6% investment. It is usual for interest on bonds to be paid semi-annually. Mathematically the payment of annual 6% interest, in semi-annual installments of equal amount, is payment at a slightly higher rate than 6% interest annually, since the present worth of half of the annual interest, payable at the end of six months, is greater in amount than the present worth of such payment if due at the end of the year.

46. Camp v. Bates, 11 Conn., 487.

Purchases of bonds at a premium or discount are made according to prices or bases shown in tables known as bond tables. Bond tables are a practical necessity in the purchase and sale of bonds at a price other than par. Such tables take into consideration the present worth, at the time of purchase, at compound discount, of the future payments of interest and principal to be made on the bond. Bond tables indicate what price above or below par must be paid for a security, having a designated number of years to run, bearing a designated rate of interest, payable periodically, to net the investor a desired rate of income at compound interest; or what rate of income, at compound interest, a security will net the purchaser, purchased at a stated price above or below par, the security bearing a certain rate of interest, payable periodically, the security having stated number of years to run. Where the interest on a security is payable annually, tables based on annual interest payments should be used. Where the interest is payable semi-annually, tables based on semi-annual interest payments should be used. On bonds with interest payable annually at 6%, the tables are based on the ratio of 1.06 once a year, and on semi-annual interest payment 6% bonds the tables are based on the ratio of 1.03 twice a year. Because of these differences in ratios, between annual and semi-annual interest payment bonds, the results of present worth calculations are not the same, and differences in values are bound to appear.

Where a bond is purchased at a premium or discount, the income earning of the bond will be affected by the amount of premium paid or discount allowed. If such investment is made by the trustee of an estate and under the terms of the trust the income is to be paid to one beneficiary for life (designated life tenant), and, on the death of the life tenant, the principal of the estate is to be transferred to another (designated remainderman), it becomes legally important that proper accounts be kept of the investment and that the accounts be fundamentally and legally correct, so that the true legal net income on the investment is paid to the one, and the true legal principal of the estate is kept for the other.[47]

The correct method of writing off premiums or writing up discounts is gradually, by charging or crediting to premiums or discounts each year a proper amount, so that at maturity the security will stand at par, the amount at which it will be paid off. This process is designated as amortization of premiums or accumulation of discounts. The depreciation by loss of the premium, and the appreciation by gain of the discount, are not considered depreciation or appreciation

47. 17 R. C. L., § 18.

of property, but as affecting the income of a bond only, because purchased on a basis that such depreciation or appreciation will certainly take place during the life of the bond, and, therefore, to be charged or credited to the income of the trust account;[48] whereas technical depreciation or appreciation would fall on, or inure to the benefit of, the remaindermen, because theoretically beyond the control of the trustee.[49] If, during the life of any bond purchased, the trustee should sell the same at a price other than the amortized premium value or the accumulated discount value, then the difference between such value of the bond and the sale price would, as a loss or gain, affect the remaindermen, as technical depreciation or appreciation of property.[50]

In the purchase of securities, it is important to know whether the securities selected may be called before maturity, as call before maturity will affect the income rate, based on the expectation that the bond will run to maturity. If a high rate security, having twenty years to run, be purchased at a premium of 20%, then the premium is required to be amortized, so that approximately one-twentieth is amortized each year; whereas, if the bond be subject to call in five years, and be called at the end of five years for payment, then the amortization of the premium would have to be at approximately the rate of one-fifth each year, greatly depreciating the income basis of the bond. On the other hand, if a low rate security, having twenty years to run, be purchased at a discount of 20%, then the discount is required to be accumulated at approximately one-twentieth each year; whereas, if the bond be subject to call in five years, and be called at the end of five years for payment, then the accumulation would end in five years and be at the rate of one-fifth each year, greatly increasing the income basis of the bond.

If the trustee mentioned purchase at 110 a bond for $1,000 par value, bearing 6% interest, having ten years to run, the bond would cost the estate $1,100. Each year the trustee would collect $60 as interest on the bond, and during the ten years would collect total of $600 as interest. At maturity he would receive the $1,000 principal of the bond, that cost him $1,100, and thus apparently lose $100 premium paid. Deducting the premium loss of $100 from the $600 interest received, would leave $500 as the apparent earning of the bond, or 5%. The net income on a semi-annual interest payment bond on the basis stated would approximate 4.70%. If at the same time the trustee

48. No case reported on this point.
49. 17 R. C. L., § 22.
50. No case reported on this point.

should puprchase at 90 a bond for $1,000 par value, bearing 4% interest, having ten years to run, the bond would cost the estate $900. Each year the trustee would collect $40 as interest on the bond, and during the ten years would collect total of $400 as interest. At maturity he would receive the $1,000 principal of the bond that cost him $900, and thus apparently gain the $100 discount allowed. Adding the discount gain of $100 to the $400 interest received would make $500 as the apparent earning of the bond, or 5% also in the case of the second bond. The net income on such a bond, with interest payable semi-annually, would approximate 5.30%.

Premiums or discounts may be written off (a) at once at time of purchase of the security, (b) at one time on collection of security at its maturity, or (c) gradually from time to time during the life of the security.[51]

If the premium paid on a bond be written off in full at the time of purchase by the trustee, the effect will be to require the life tenant to pay in advance the amount of the premium and before such payment is legally due.[52] This would be unjust to the life tenant, and, where the amount invested is considerable, may cause hardship on the life tenant, by suspending for some time the payment of other income, because of the sudden large charge to income. Should the life tenant die before the maturity of the bond, then the life tenant would have been charged part of the premium for which the life tenant should not be held liable. If the discount allowed on a bond be written up in full at the time of purchase by the trustee, the life tenant will receive credit in advance for the amount of the discount and before such discount has accrued.[53] This would be unjust also, as the life tenant would then receive credit for income before it was earned. Should the life tenant die before the maturity of the bond, then the life tenant would have received credit as income of that part of the discount that had not accrued before death of the life tenant, and the remaindermen would have cause to complain.

If the premium paid on a bond purchased by the trustee be not written off until maturity of the bond, and the life tenant survive until such time, then there would be a charge of the amount of the premium against income that would affect the life tenant suddenly, and, if the amount be considerable, cause embarrassment to the life tenant until the amount be made up from other income. As the life tenant would

51. 1909 amendment to N. Y. Ins. Cos. Act., requires gradual amortization.
52. No case reported on this point.
53. No. case reported on this point.

have received an over-payment of income in the meantime, the life tenant could not legally complain at the adjustment, although the income would be unexpectedly affected. Should the life tenant die before the maturity of the bond, then the over-payment of income to the life tenant would become an item for which the remaindermen could hold the trustee.[54] If the discount allowed on a bond be not written up until maturity and collection of the bond, if the life tenant survive until that time, then the life tenant would suddenly receive a delayed increase of income to which the life tenant was theretofore entitled from year to year, but the final payment would adjust the matter, although the life tenant would have had cause of complaint against previous withholding of the accruing discount from year to year. Should the life tenant die before maturity of the bond, then the life tenant will not have received credit for the discount accrued and earned to death of the life tenant, and the executor or administrator of the life tenant could claim the amount from the trustee, as against the remaindermen, and the trustee will be liable therefor. If none was claimed by the legal representative of the deceased life tenant, and if the amount of the accrued discount shall have been paid to the remaindermen, then the remaindermen will have received over-payment by the amount involved, and that would not be legal.[55]

The premiums paid on a bond purchased for investment, or the discount allowed thereon, may be amortized or accumulated on what is known as the level basis plan; that is, a proportionate part may be written off or on each year during the life of the security. In the case of a bond having ten years to run, on the level basis plan, one-tenth would be written off or on each year, so that at the end of the ten years the bond would stand at par value. Such plan is practical and readily understood, but is not accurate, especially in case of long time bonds: as during the first few years the amount reserved to cover the premium paid on a bond would be too great and during the last few years the amounts would be too small; while, on bonds purchased at a discount, the reverse would be the case. Except in case of bonds having a long time to run, and except in case of large amounts of bonds in an estate, the level basis plan of amortization of premiums or accumulation of discounts gives substantially correct results, although not exact. The preferred method of amortization and accumulation is the scientific method. By the latter method the income basis of the bond is determined from the bond table at the time of purchase, and is the basis on which to carry the bond to maturity.

54. Curtis v. Osborn, 79 Conn., 555.
55. No case reported on this point.

From year to year the amount of interest received on coupons over the income basis of the bond is set apart and taken as a partial payment of premium on the bond and as ceasing to bear interest, and the exact unpaid principal and unpaid premium only is considered as bearing interest at the income rate. Also, from year to year the accruing discount is taken as increasing the investment in a bond which is at a discount, the income basis of the bond being maintained, but the amount of income increasing with the increase in the amount taken as invested in the bond.[56]

It is the custom, when a security bearing periodical interest runs for a fractional period over a regular period, to take from the proper bond table the value just short of the last period and then average the fractional period according to the time over, using for the purpose of average the next value beyond the fractional period. While the custom is universal, it is not accurate, and the fractional value shown cannot be checked arithmetically. A better method is to use an annual or semi-annual bond table, for the exact number of annual or semi-annual periods the security has to run, and take the value shown; then ascertain the interest to accrue for the fractional period on the value shown, at the income basis of the bond, and add to the value taken from the table. But even the last method is not exact. The most exact method is to take from the bond table the value shown for the full periods the bond has to run; next, calculate interest to accrue on bond for fractional period at contract rate and add to value taken from bond table; next, from a supplementary true discount table ascertain present worth of $1 for the number of days in the fractional period; next, multiply value of bond shown in bond table with aforesaid interest added by present worth of $1 in true discount table, and product will represent true value of bond with interest for whole time to run; except interest accrued from last interest payment date to date of purchase, not taken into consideration. The error in the universal custom may be pointed out by illustration, to-wit : With a security, netting 6% interest, payable annually, having three years and two months to run, the ratio of the interest rate for the three years is 1.06, to which the table is planned; but for the two months period (1/6 of a year), the ratio of the interest rate is 1.01, and not according to the plan of the 1.06 table.[57]

While altogether unusual, yet it may be in the contemplation of the seller and buyer of a bond that the purchaser shall receive interest, on the income basis of the bond, on the amount of accrued interest

56. 1909 amendment to Ins. Cos. Law of N. Y., permits amortization by level basis plan or by scientific basis plan.
57. No case reported on this point.

paid at the time of purchase to the date of payment of the first maturing interest coupon. If so, then an additional calculation is necessary, namely: To reduce the amount of accrued interest due to its present worth, considering that the accrued interest paid will be refunded with the first maturing interest coupon.

The question of the amortization of premiums paid on bonds purchased by trustees has been before the courts of various states numbers of times and the conclusions reached do not coincide. All courts primarily endeavor to give effect to the clear intent of the person creating a trust, as to amortization, as expressed in the instrument or will creating the trust.[58] (A) In a Connecticut case the trustee bought securities at a premium and it was held that the trustee ought to withhold from the life tenant annually such part of the income as by proper investment would create a fund equal to the premium paid.[59] (B) In an early Massachusetts case the loss of premium value of bonds bequeathed in trust was placed on the remainderman; and loss of premiums paid by the trustees on bonds purchased by him after the death of the testator was also placed on the remainderman.[60] In a later case in the same state the court held that on collection of interest on bonds purchased at a premium the trustee should deduct a sufficient sum from the income of the life tenant to make good to the remainderman the amount of premium paid.[61] And in a case in the same state, subsequent to the two cases mentioned, the court placed the loss of premium on the remainderman; apparently receding from the second case and going back to the ruling of the first case.[62] In the last case the court did not attempt to reconcile the two prior cases. (C) In a New York case,[63] overruling older cases in point, the court held the loss of premiums paid on bonds falls on the life tenant, basing the decision on the ground that premiums on bonds are paid to purchase an increased rate of interest. In the same state it is held, if bonds are bequeathed in trust, then the loss of the premium value falls on the remainderman.[64] (D) The courts of New Jersey require trustees to amortize premiums paid on bonds purchased by them.[65] (E) In a Wisconsin case, where a trustee invested trust funds in bonds at a

58. Kemp v. Macready, 150 N. Y. Supp., 618; Am. S. & T. Co. v. Payne (D. C.), 33 App., 178; Shaw v. Cordis, 143 Mass., 443; Pell v. Mercer, 14 R. I., 412.
59. Curtis v. Osborn, 79 Conn., 555.
60. Hemenway v. H., 134 Mass., 446.
61. New Eng. Trust Co. v. Eaton, 140 Mass., 532.
62. Shaw v. Cordis, 143 Mass., 443.
63. In re Stevens, 187 N. Y., 471; also see later case In re Guaranty Trust Co., 195 N. Y., 611.
64. In re Fanoni, 216 N. Y., 640.
65. Ballantine v. Young, 76 N. J. Eq., 613.

premium, the court held he should restore to the corpus of the estate, from the interest on the investment, the amount of premiums paid.[66]

However, there are cases in Pennsylvania, holding that loss of premiums paid by a trustee on investments in bonds should be charged to principal of the estate, and not amortized against the income. See Penn-Gaskell's Est., 208 Pa., 346. While, in Kentucky is a case holding that a trustee need not concern himself as to premiums paid or discounts allowed on investments by the trustee; that such matters balance themselves. See Hite v. Hite, 93 Ky., 257.

The writer has not found any case where the courts have passed on the question of the accumulation of discounts on bonds purchased by a trustee.[67] That question is less apt to arise, as trustees should prefer premium investments, on account of the fact that the true income can readily be paid out of coupons on such bonds (the collections on such coupons being in excess of income payments) while on discount bonds the collections on coupons are not equal to the true income on discount bonds, and the excess must be made up out of other funds in the estate.

While there is no legal requirement on individuals or corporations generally to amortize premiums or accumulate discounts on investments (except insurance companies and banks in several states), yet it seems advisable that individuals and corporations do so, to ascertain real income earnings on their bonds.

It is the rule that profits other than income earning on investments should go to the credit of principal of a trust estate, being considered as in the nature of a profit by appreciation.[68] This certainly is true in the case of a bond not redeemable before maturity and purchased by a trustee at a price and subsequently sold at a higher price. In the event a bond is issued to mature at a fixed period, with a provision that the obligor may call such bond before maturity at some fixed bonus above par, on purchase by a trustee of such a bond the trustee cannot presume that the bond will be called before it is payable and he must make the investment on the basis that the bond will remain outstanding until it matures. If the bond is purchased at a premium, the principles for amortization of the premium should be applied, and from interest period to interest period the cost of the bond in the account of the trustee should be adjusted accordingly, until redemption of the bond according to provisions for its call before maturity. If the call is upon an interest payment date, the proceeds of the last

66. In re Allis, 123 Wisc., 223.

67. See passing remark by court in Hite v. Hite, 3 Ky., 257, indicating no attention need be given by trustees to either premiums or discounts.

68. Scovel v. Roosevelt (N. Y.), 5 Redfield Surr., 121.

coupon collected are adjusted on the usual amortization basis of the bond, and the proceeds of collection of the principal of the bond with call bonus thereon are credited to principal of the trust estate. If such proceeds with bonus are more than the then amortized cost of the bond, as carried in the account of the trustee, a profit by appreciation to the principal of the trust estate is the result, or, if less than such cost, a loss by depreciation is the result.[69] If the call is made between interest periods, the proceeds of the interest accrued from the last matured interest coupon should necessarily be adjusted on the amortization basis for the fractional period.

Under a bequest of specific bonds the face value of the bonds is determined by the testator as the principal of the estate to be held in trust for the remainderman, notwithstanding the market value is above par or below par, and the life tenant is generally entitled to the income on the face value of the specific bonds, at the contract rate of the interest thereon.[70] In case of a residuary bequest, if the residue should consist entirely or in part of bonds held by the testator and having a market value above or below par, on receipt of such bonds by the trustee for a life tenant and remainderman, the question at once arises whether the life tenant should receive interest at the contract rate on such bonds, or should the trustee in his accounts enter bonds included in the residuary bequest at their market value and consider such value as the cost to the trust estate of such bonds, and on the basis of such value amortize part of the proceeds of the interest coupons thereon as collected from time to time or accumulate the discount. In the opinion of the writer such amortization or accumulation should be made. In the distribution of the bonds to the trustee they are necessarily distributed by the executor at their market value and received by the trustee at such value, instead of their par value. In case of the specific bequest of bonds, the testator is conclusively presumed to have fixed the amount and kind of securities upon which the life tenant is to receive the interest as income.[71] In case of a residuary bequest, the rule is that the life tenant, from date of death of testator, is entitled to the net income on the clear residue when ascertained.[72] To determine the value of the clear residue of the estate, any bonds and shares must be taken at their market value at the date of death of the testator. A share of stock having a market value of $200 per share would not be taken at its par value of $100, unless specifically so directed. No more should a bond be taken at

69. No case reported on this point.
70. Robertson v. De Brulator, 188 N. Y., 301.
71. Robertson v. De Brulator, supra.
72. Lawrence v. S. Co., 56 Conn., 423.

par, if its market value is above or below par, unless specifically so directed. If the residue of an estate consists entirely of cash and is bequeathed to a trustee, and if soon after the date of death the cash is paid over to the trustee for investment and is invested in bonds at a premium or discount, it is clear that the principles of amortization or accumulation should be applied for the protection of the remainderman. It seems to the writer to follow, if the residue consists of bonds invested in by the testator, that the bonds should be received at their cash value, unless there is an express direction of the testator to the contrary. Suppose, pursuant to authority, a trustee one day after the receipt of bonds under a residuary bequest sell such bonds at their market value, the sale being presumed to be on an interest payment date of the bonds, so no question of accrued interest is to be considered, would not the trustee credit the proceeds of sale to the principal of the trust estate and hold the amount in principal for the remainderman? There would be no question as to his duty in so doing.[73] Any investment of the trust funds thereafter would be upon the basis of the principal so fixed. Should not the principal of the trust estate be fixed at the same amount, in a case where there is no sale of the bonds, as well as in a case where a sale is made? The writer answers, yes: that as a matter of principle the law should not leave the fixing of the amount of the principal of an estate to the chance of a sale of bonds by the trustee; that impartiality between the remainderman and life tenant requires the bonds to be taken at their cash or market value at the death of the testator, in the absence of an express direction of the testator to the contrary; since the bonds are to be taken at such value, that impartiality between the remainderman and life tenant requires amortization of the premium or accumulation of the discount, so that the life tenant will receive all of the net income on the value of the bonds at the taking effect of the bequest, but no more or less, and the remainderman, on the termination of the life estate, receive the full value of the estate bequeathed, but no more or less. In such case, any profit by appreciation or loss by depreciation, in the event of a sale of the bonds by the trustee at a price other than their amortized or accumulated value, should be credited to or charged against the remainderman, and by the trustee entered in the principal of the trust estate, as is required in case of bonds purchased by the trustee.

The following reported cases are opposed to the view of the writer, above mentioned; they are based on the presumption, if bonds pass to a trustee as part of the residue of an estate, that it must have been

73. Scovel v. Roosevelt, supra; In re Cutler, 52 N. Y. Supp., 842.

the unexpressed intention of the testator for the bonds to be received by the trustee at their par value, independent of their actual value at death of the testator, and whether above or below par, to-wit: Conn. Trust Co. Appeal, 80 Conn., 540; Higgins v. Beck, 116 Me., 127; Whitridge v. Williams, 71 Md., 105; Shaw v. Cordis, 143 Mass., 443; Ballantine v. Young, 74 N. J. Eq., 572. Why should bonds be taken at their par value, and not their market value, when other securities, passing under a residuary bequest, are taken at their market value?

LOGARITHMS

It may be interesting to inquire how compound interest tables and bond tables above necessarily referred to are calculated. They are calculated by the use of logarithms.

A preliminary explanation of terms may be helpful. When we multiply a number by itself, the number is said to be "raised to the second power;" when we multiply the result so obtained by the number, the number is said to be "raised to the third power," and so on. In Algebra this process is for brevity indicated thus, for example, 10^3, which means that the number 10 is raised to the third power; in other words, 10 times 10 times 10 is 1000; that is, $10^3=1000$. Now, in the example stated the number 3 is called the exponent (or the exponent of the power), and the number 10 is called the base, and the number 3 is said to be the logarithm of 1000 to the base 10. Hence, the logarithm of any given number is the exponent which indicates the power to which the base number must be raised in order to produce that given number.

We learn in algebra (1) that by adding exponents, we multiply numbers; (2) by subtracting exponents, we divide numbers; (3) by actually multiplying exponents, we raise numbers to powers; and (4) by actually dividing exponents, we extract the roots of numbers. The third process is the process used in computing compound interest. The process is briefly, as follows: In a table of logarithms find the logarithm corresponding to the ratio of one plus the interest rate i. e., for 5% annual rate, 1.05; multiply that logarithm by the number of full periods the loan is to run; to the power so obtained, add the logarithm of the amount of the loan; the total will be the logarithm of the amount required; then from the table find the corresponding amount.

Bond tables are prepared by the use of logarithms, as follows: First calculate the present worth at compound discount of the principal of the bond for the number of periods it has to run at stated rate of compound discount; next calculate the present worth of an annuity payable annually or semi-annually equal to the amount of annual or semi-annual interest coupons on the bond; next find the numerical value of the two present worths by adding them together; their sum will be the value of the bond. (1) To determine the present worth of the principal, find in a table of logarithms the logarithm corresponding to the ratio of one plus the interest rate—i. e., for 6% annual rate, payable semi-annually, 1.03; multiply that logarithm by the number of full interest periods the bond has to run, thus giving the power to which the ratio must be raised; next find the logarithm of the amount of principal due at maturity; next from the logarithm of the principal deduct the logarithm of the power, and find the difference between the two logarithms; next find the numerical value of the difference. The numerical value of such difference will be the desired present worth of the principal at compound discount. (2) The present worth of an annuity payable at the end of each year, may be found by calculating by logarithms the present worth of each annual payment at compound discount for the number of years the respective payments have to run, as explained with respect to the present worth of the principal. This calculation becomes tedious where a number of interest periods are involved. Such present worth is more readily ascertained by use of algebraic formula for calculation of problems in geometrical progression.

Logarithms were devised by the famous mathematician Napier (1550-1617) and published about 1614. They marked a great advance in mathematics. His table of logarithms is based on the constant known to mathematicians as "e," the numerical value of which, to five decimal places, is 2.71828. The constant "e" may be found by calculation of the following series, to-wit:

$$1 + \frac{1}{1} + \frac{1}{1x2} + \frac{1}{1x2x3} + \frac{1}{1x2x3x4} + \text{——}$$

Each process may be continued to as many decimal places as desired. The next step is to determine to what power the base must be raised to equal each number in the table. This is usually done systematically, from unity up to 100. The table of logarithms of these numbers may then be extended to length desired. To transform the logarithms of one base to another base, the logarithms are multiplied by a modulus determined from the value of one base as compared with the other. Briggs (1561-1631), also a mathematician of note, a contemporary of Napier, realizing the value of the invention of logarithms, journeyed to confer with Napier, and suggested the plan of preparing a table of logarithms to the base ten, as one that would be of easier use than the one based on the constant "e." Briggs and associates thereafter prepared a table to the base ten, and the latter is the foundation of the various tables of logarithms to that base now in general use; they are known as common or decimal logarithms. Any number other than unity (one) may be used as a base for logarithms.

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