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AN APPLICATION OF SCREW THEORY FOR THE IDENTIFICATION OF SINGULARITIES IN A NOVEL RECONFIGURABLE PARALLEL ROBOT

UNA APLICACIÓN DE TEORÍA DE TORNILLOS PARA LA IDENTIFICACIÓN DE SINGULARIDADES EN UN NOVEDOSO ROBOT PARALELO RECONFIGURABLE

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ABSTRACT

This paper reports the application of the screw theory as a tool for the determination of the singular configurations of a reconfigurable parallel robot composed of two parallel sub-manipulators. The Jacobian matrices of the robot, key elements for the identification of singularities, are easily determined when the input-output equation of velocity of the robot is obtained by the application of some screw theory basic operations. Through this application, the inverse, direct and combined singularities are clearly identified, and their graphical representations can be obtained almost intuitively.

Palabras claves: Screw theory, Jacobian matrix, Singularity analyses, Reconfiguration, Parallel robot.

RESUMEN

Este trabajo reporta la aplicación de teoría de tornillos como una herramienta para la determinación de las configuraciones singulares de un robot paralelo reconfigurable conformado por dos sub-manipuladores paralelos. Las matrices Jacobiana del robot, piezas fundamentales para la identificación de las singularidades, se determinan fácilmente cuando la ecuación de entrada y salida de velocidad del robot es obtenida a través de la aplicación de algunas operaciones básicas de teoría de tornillos. Mediante esta aplicación, las singularidades directas, inversas y combinadas son claramente identificadas, y su representación gráfica puede ser obtenida casi intuitivamente.

Keywords: Teoría de tornillos, Matriz jacobiana, Análisis de singularidades, Reconfiguración, Robot paralelo.

1. INTRODUCTION

A reconfiguration strategy is probably the most reasonable approach to enhance the flexibility of a manipulator robot. In the case of the parallel robots, the major progress in this area can be divided into two categories (Zhang and Shi, 2012); those based on a modular design and those based on a variable geometry approach.

The modular design consists of a set of standardized modules which can be connected and disconnected in order to obtain different configurations (Xi *et al.*, 2011; Yu *et al.*, 2012; Plitea *et al.*, 2013; Carbonari *et al.*, 2014). On the other hand, in a variable geometry approach the dimensions of the geometric parameters of the robot are modified to achieve new configurations (Zhang and Shi, 2012; Bande *et al.*, 2005; du Plessis and Snyman, 2006; Kumar *et al.*, 2009; Borrás *et al.*, 2009; Chen, 2012; Ye, 2014). The variable geometry approach is more used than the modular design because it offers the advantage of easy implementation during the operation of the robot. This approach is used for the reconfiguration of the robot under study in this work.

Parallel robots have two major complications associated with the kinematic model: (i) a very difficult forward displacement analysis and (ii) the existence of multiple singular configurations. When a variable geometry reconfiguration is applied, the complexity of the forward displacement analysis is not necessarily increased, however, the singular configurations may be removed, relocated or new singular configurations may appear due to the modification of the geometry of the manipulator. For this reason the identification of these configurations is a very important task in the design stage of a reconfigurable manipulator.

Many works addressed the identification of the singular configurations of parallel robots with multiloop architecture (Mayer and Gosselin, 2000; Huang and Cao, 2005; Bandyopadhyay and Ghosal, 2006; Jiang and Gosselin, 2009), however, given the importance of this issue, more efficient strategies for the identification of these singularities continue being developed. In this sense, the screw theory has proven to be a very useful tool for this purpose (Bonev *et al.*, 2003; Gallardo-Alvarado *et al.*, 2006), even for hexapods (Gallardo-Alvarado *et al.*, 2013).

Taking into account the aforementioned, in this paper the screw theory is used to identify the singular configurations of a novel reconfigurable parallel robot composed of two parallel sub-manipulators.

2. DESCRIPTION OF THE RECONFIGURABLE PARALLEL ROBOT

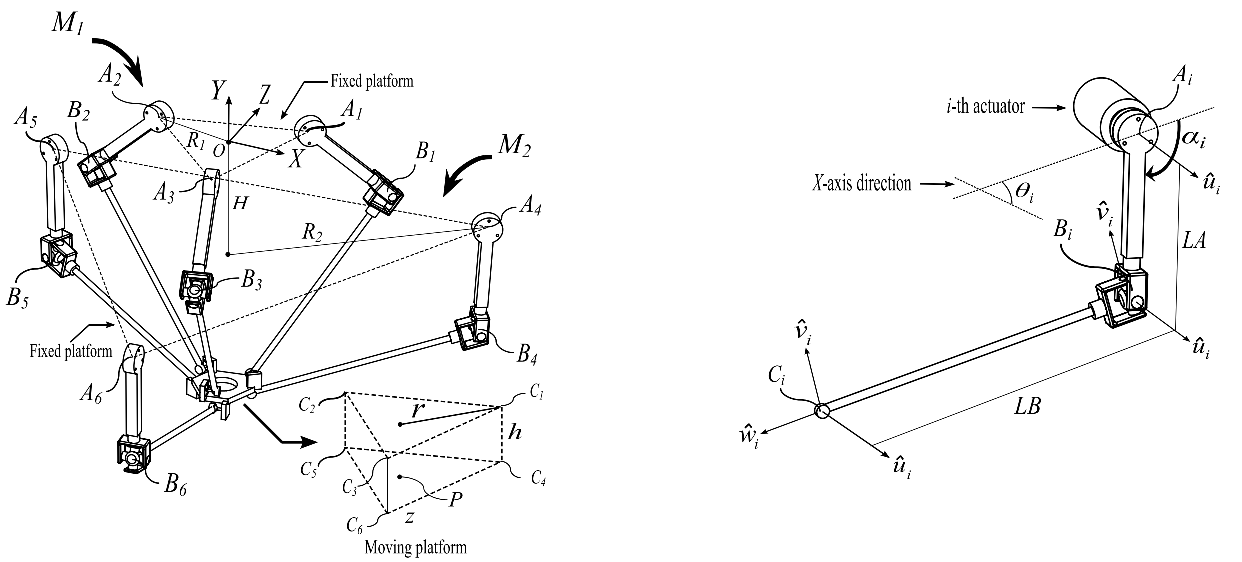
The robot under study is based on a 6-DOF parallel robot, which hereafter will be called initial configuration model. The description of the initial configuration model and the reconfiguration strategy implemented are shown below.

2.1. Initial Configuration Model

The robot (patenting process: MX/a/2013/011009) consists of two 3-$\underline{R}$US parallel sub-manipulators, which share a common three-dimensional moving platform (Figure 1a), where R, U, and S denote the revolute, universal and spherical joints, respectively, and the underline represents the active joint. Unless otherwise specified, hereafter, the subscripts $i = 1, 2, 3$ refer to elements of the sub-manipulator M_1, whereas the subscripts $i = 4, 5, 6$ refer to elements of the sub-manipulator M_2.

The fixed platform of the manipulator is represented by two parallel equilateral triangles $A_1A_2A_3$ and $A_4A_5A_6$, which are separated by a distance H. The fixed coordinate system is attached to the center O of the triangle $A_1A_2A_3$, its X- and Z-axes lie on the plane defined by this triangle, and the Y-axis points upward.

The active revolute joints, whose nominal positions $A_i = (A_{Xi}, A_{Yi}, A_{Zi})$ are located by vectors $\boldsymbol{A_i}$, determine the vertices of the equilateral triangles $A_1A_2A_3$ and $A_4A_5A_6$, where R_1 and R_2 represent the circumradii of these triangles, and θ_i represents the orientation of the i-th kinematic chain (Figure 1b), which is measured from the X-axis direction to vector $\boldsymbol{A_i}$ for $i = 1, 2, 3$ and to the projection of vector $\boldsymbol{A_i}$ on the XZ plane for $i = 4, 5, 6$. On the other hand, $Bi = (B_{Xi}, B_{Yi}, B_{Zi})$ denotes the nominal position, which is located by vectors $\boldsymbol{B_i}$, of the universal joint that connects the link of length LA to the link of length LB in the same kinematic chain. Similarly, $C_i = (C_{Xi}, C_{Yi}, C_{Zi})$ denotes the nominal position, which is located by vectors $\boldsymbol{Ci}$, of the spherical joint that connects the moving platform to the link of length LB. Points C_i form a triangular prism of height h defined by the equilateral triangles $C_1C_2C_3$ and $C_4C_5C_6$, where r represents the circumradius of these triangles. The rotation axes of the active revolute joints, which are denoted by $\hat{u}_i$, are tangential to the circumscribed circle of the triangles $A_1A_2A_3$ and $A_4A_5A_6$. Moreover, the rotation axes of the universal joints are $\hat{u}_i$ and $\hat{v}_i$, where $\hat{v}_i$ is orthogonal to $\hat{u}_i$ and to the direction $\hat{w}_i$ of the link of length LB. For the spherical joints, the rotation axes are $\hat{u}_i$, $\hat{v}_i$ and $\hat{w}_i$. Finally, point $P = (P_X, P_Y, P_Z)$, which is located by vector $\boldsymbol{P}$, is the interest point in the moving platform (end effector) and is conveniently located at the center of the triangle $C_4C_5C_6$.



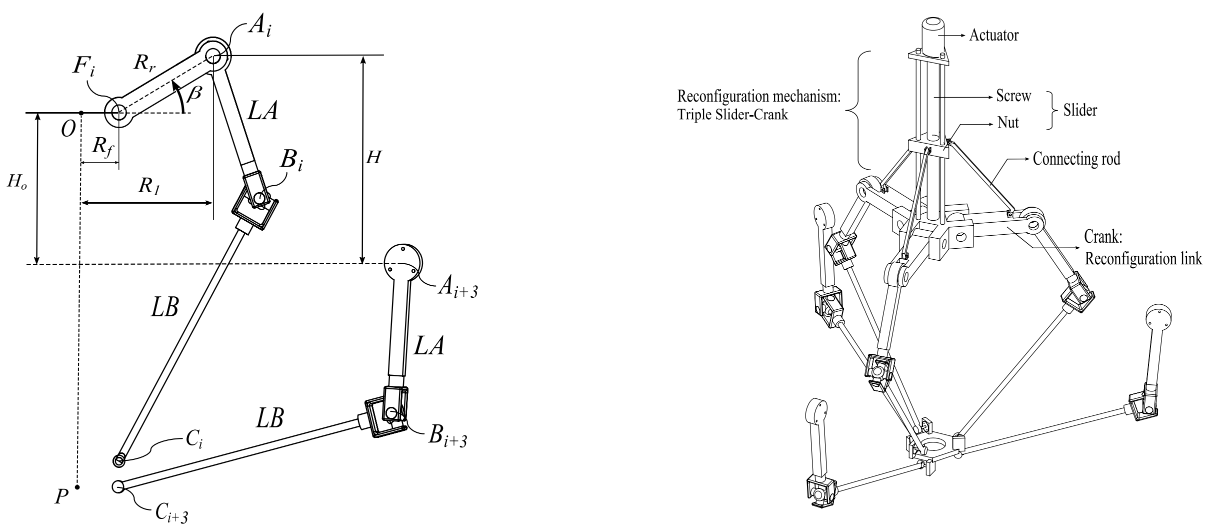
(a) (b)

Figure 1. Initial configuration model. (a) General view. (b) *i*-th kinematic chain.

2.2 Reconfiguration Strategy

The reconfiguration strategy involves adding to the $i = 1, 2, 3$ kinematic chains of the sub-manipulator M_1 a link of length R_r, which will be called reconfiguration link. The angular positioning β of the reconfiguration link result in a relocation of the joints defined in points A_i, which can be seen as a simultaneous modification of the parameters R_1 and H (Figure 2a). The nominal position $F_i = (F_{Xi}, F_{Yi}, F_{Zi})$ of the active revolute joint that allows the mobility of the reconfiguration link is defined by vector $\boldsymbol{F_i}$, whose magnitude and orientation are R_f and θ_i, respectively. The aforementioned indicates that the fixed platform of the reconfigurable robot is represented by the parallel equilateral triangles $F_1F_2F_3$ and $A_4A_5A_6$.

Figure 2b shows a conceptual design for the reconfigurable robot. This design is based on incorporating a mechanism which is composed of three slider-crank sub-mechanisms. When the actuator located on the top of the reconfiguration mechanism is driven, a simultaneous angular positioning β of the reconfiguration links is performed. This strategy allows to modify the geometry of the robot by including only one degree of freedom to the mechanism. Additional information about this reconfiguration concept can be found in (Balmaceda-Santamaría *et al.*, 2014)



(a) (b)

Figure 2. Reconfiguration strategy. (a) Structure of the *i*-th and the *i+3*-th kinematic chain. (b) General view of the reconfigurable robot.

3. SINGULARITY ANALYSIS

To identify the singularities it is necessary to develop the velocity model of the robot. Because of the reconfiguration mechanism is based on slider-crank sub-mechanisms, whose analysis does not imply any contribution, the velocity model will be performed only considering the angular positioning β of the reconfiguration links (Figure 2a), which, together with the angular positioning α_i of the link length LA (Figure 1b), for $i = 1, 2 \ldots 6$, are the input parameters of the robot. Output parameters are defined by the pose (three translations and three orientations) of the moving platform.

3.1 Velocity model

The velocity model is addressed using screw theory. For a detailed explanation of the method of infinitesimal kinematics that we used here, please consult (Rico y Duffy, 2000).

The screws that are associated with each joint of the $i = 1, 2, 3$ kinematic chains of the sub-manipulator M_1 (Figure 3a) are: $\$_i^r$, which is associated with the active revolute joint defined in F_i; $\$_i^1$, which is associated with the active revolute joint defined in A_i; $\$_i^2$ and $\$_i^3$, which are associated with the universal joint defined in B_i, and finally $\$_i^4$, $\$_i^5$ and $\$_i^6$ are the screws associated with the spherical joint defined in C_i. These screws are also associated with the joints defined in A_i, B_i and C_i for the $i = 4, 5, 6$ kinematic chains of the sub-manipulator M_2 (Figure 3b), however the screw $\$_i^r$ is not taken into account, and a screw

$\$_i^f$, collinear to $\$_i^6$, is conveniently added as a screw associated with a virtual prismatic joint. As will be shown below, the inclusion of these virtual screws facilitates the systematic obtainment of an input-output equation of velocity in a useful form for the singularity analysis.

A screw associated with a revolute joint is defined in Plücker coordinates as: $\$ = \left[\hat{S} \quad \vec{S}_O\right]^T$, where the primal part, $\hat{S}$, is a unit vector along the rotation axis of the joint associated with the screw, whereas the dual part, $\vec{S}_O$, is the moment produced by $\hat{S}$ about the reference pole *O*. On the other hand, a screw associated with a prismatic joint is defined in Plücker coordinates as: $\$ = \left[0 \quad \hat{S}\right]^T$, where $\hat{S}$ is a unit vector along the translation axis of the prismatic joint. Taking into account the above: $\$_i^r = \left[\hat{u}_i \quad \boldsymbol{F_i}\times\hat{u}_i\right]^T$, $\$_i^1 = \left[\hat{u}_i \quad \boldsymbol{A_i}\times\hat{u}_i\right]^T$, $\$_i^2 = \left[\hat{u}_i \quad \boldsymbol{B_i}\times\hat{u}_i\right]^T$, $\$_i^3 = \left[\hat{v}_i \quad \boldsymbol{B_i}\times\hat{v}_i\right]^T$, $\$_i^4 = \left[\hat{u}_i \quad \boldsymbol{C_i}\times\hat{u}_i\right]^T$, $\$_i^5 = \left[\hat{v}_i \quad \boldsymbol{C_i}\times\hat{v}_i\right]^T$, $\$_i^6 = \left[\hat{w}_i \quad \boldsymbol{C_i}\times\hat{w}_i\right]^T$, $\$_i^f = \left[0 \quad \hat{w}_i\right]^T$.

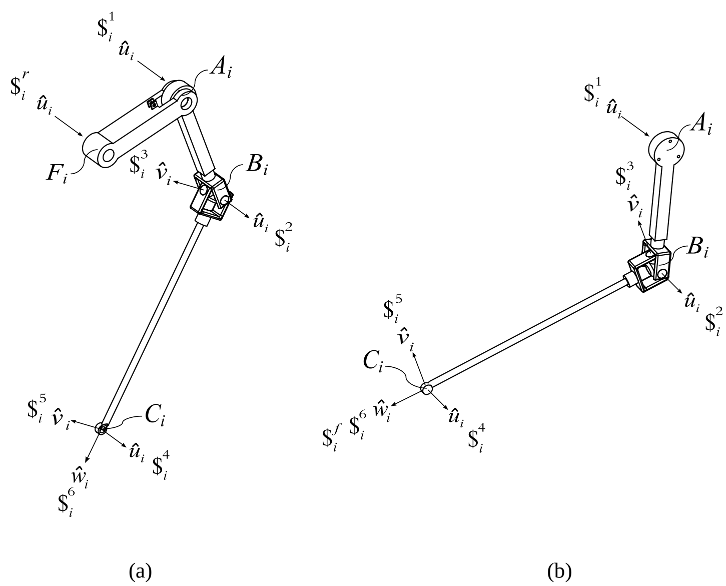


Figure 3. Infinitesimal screws. (a) Kinematic chain of M_1. (b) Kinematic chain of M_2.

The velocity state $\boldsymbol{V}$ of the moving platform may be expressed in screw form as:

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{v} \end{bmatrix} = \omega_r^i \$_i^r + \omega_1^i \$_i^1 + \omega_2^i \$_i^2 + \omega_3^i \$_i^3 + \omega_4^i \$_i^4 + \omega_5^i \$_i^5 + \omega_6^i \$_i^6, \qquad \text{for } i = 1, 2, 3 \qquad (1)$$

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{v} \end{bmatrix} = \omega_1^i \$_i^1 + \omega_2^i \$_i^2 + \omega_3^i \$_i^3 + \omega_4^i \$_i^4 + \omega_5^i \$_i^5 + \omega_6^i \$_i^6 + \omega_f^i \$_i^f, \qquad \text{for } i = 4, 5, 6 \qquad (2)$$

where $\boldsymbol{\omega}$ and $\boldsymbol{v}$ are the angular and linear velocity vectors of the moving platform, respectively, with respect to the fixed platform, whereas ω_r^i, ω_n^i (*n = 1, 2… 6*) and $\omega_f^i = 0$ are the joint velocity rates associated with the screws, with $\omega_r^i = \dot{\beta}$ and $\omega_1^i = \dot{\alpha}_i$ as the generalized speeds of the robot. Note that the joint velocity rate associated with the virtual screws is zero, which prevents any effect in the velocity state due to the inclusion of these screws.

Moreover, we can see that the screw $\$_i^6$ is reciprocal to all screws in the same kinematic chain, except the screws $\$_i^r$ and $\$_i^1$ for *i = 1, 2, 3*, and the screws $\$_i^1$ and $\$_i^f$ for *i = 4, 5, 6*. Taking into account the above, the application of the Klein form $\{*;*\}$ of the screw $\$_i^6$ to both sides of Eqs. (1) and (2) with the reduction of terms yields the following

$$\{\boldsymbol{V};\$_i^6\} = \dot{\beta}\{\$_i^r;\$_i^6\} + \dot{\alpha}_i\{\$_i^1;\$_i^6\}, \qquad \text{for } i = 1, 2, 3 \qquad (3)$$

$$\{\boldsymbol{V};\$_i^6\} = \dot{\alpha}_i\{\$_i^1;\$_i^6\} + \omega_f^i\{\$_i^f;\$_i^6\}, \qquad \text{for } i = 4, 5, 6 \qquad (4)$$

For a quick review of the screw theory basic operations, including the Klein form, please consult (Gallardo-Alvarado *et al.*, 2006).

Taking into account that $\omega_f^i = 0$ and $\{\$_i^f;\$_i^6\} = 1$ (collinear screws), Eqs. (3) and (4) can be presented in a matrix-vector form to represent the input-output equation of velocity of the robot as:

$$\boldsymbol{J}_x^T \Delta \boldsymbol{V} = \boldsymbol{J'}_q \dot{\boldsymbol{q}}_\beta + \boldsymbol{J}_q \dot{\boldsymbol{q}}_\alpha \qquad (5)$$

where the inverse Jacobian matrices are: $\boldsymbol{J'_q} = \text{diag}\left[\{\$_1^r;\$_1^6\} \; \{\$_2^r;\$_2^6\} \; \{\$_3^r;\$_3^6\} \; 1 \; 1 \; 1\right]$ and $\boldsymbol{J_q} = \text{diag}\left[\{\$_1^1;\$_1^6\} \; \{\$_2^1;\$_2^6\} \; \{\$_3^1;\$_3^6\} \; \{\$_4^1;\$_4^6\} \; \{\$_5^1;\$_5^6\} \; \{\$_6^1;\$_6^6\}\right]$, whereas $\boldsymbol{J_x} = \left[\$_1^6 \; \$_2^6 \; \$_3^6 \; \$_4^6 \; \$_5^6 \; \$_6^6\right]$ is the direct Jacobian matrix. Furthermore, $\Delta = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}$ is an operator of polarity in terms of the 3×3 identity matrix ***I*** and the 3×3 zero matrix **0**. Finally, $\dot{\boldsymbol{q}}_\beta = \left[\dot{\beta} \; \dot{\beta} \; \dot{\beta} \; 0 \; 0 \; 0\right]^T$ and $\dot{\boldsymbol{q}}_\alpha = \left[\dot{\alpha}_1 \; \dot{\alpha}_2 \; \dot{\alpha}_3 \; \dot{\alpha}_4 \; \dot{\alpha}_5 \; \dot{\alpha}_6\right]^T$ are the first-order driver matrices of the robot.

The Eq. (5) has a suitable matrix-vector form for the singularity analysis due in large part to the inclusion of the virtual screws $\$_i^f$. The above does not mean that the velocity model of the robot cannot be performed via screw theory with-out the inclusion of the virtual screws. The sole aim in the use of these screws is to allow the matrices $\boldsymbol{J'_q}$ and $\boldsymbol{J_q}$ to have the same size.

3.2 Singularity analysis

For parallel robots, singularities are configurations where the moving platform gains or loses degrees of freedom. Singular configurations may be classified into three types, each of which has a specific physical interpretation (Gosselin y Angeles, 1990).

Singularity type 1. This inverse singularity occurs when det ($\boldsymbol{J_q}$) = 0, which arises when any diagonal element of $\boldsymbol{J_q}$ vanishes. Physically, this condition implies that in at least one kinematic chain the links of length *LA* and *LB* are in a full extended or contracted configuration, which makes $\$_i^1$ and $\$_i^6$ reciprocal and $\{\$_i^1;\$_i^6\} = 0$. An example of this type of singularity is shown in Figure 4a, where the links of length *LA* and *LB* of the sub-manipulator M_1 are in a full extended configuration.

An inverse singularity can also occur when det (J'_q) = 0, which arises when any diagonal element of J'_q vanishes. This condition implies that in at least one kinematic chain of the sub-manipulator M_1 the links of length R_r and LB are collinear. This situation can occur in three cases: (i) when the links of length R_r, LA and LB are in a full extended configuration (Figure 4b), (ii) when the links of length R_r and LA are in a full extended configuration forming a segment in a full contracted configuration with respect to the link of length LB, (iii) when the links of length LA y LB are in a full extended configuration forming a segment in a full contracted configuration with respect to the link of length R_r. Note that in all three cases the links of length LA and LB are in a full extended or contracted configuration, therefore a singularity associated with J'_q can occur only if a singularity associated with J_q also occurs. This is: det (J'_q) = 0 $\Rightarrow$ det (J_q) = 0.

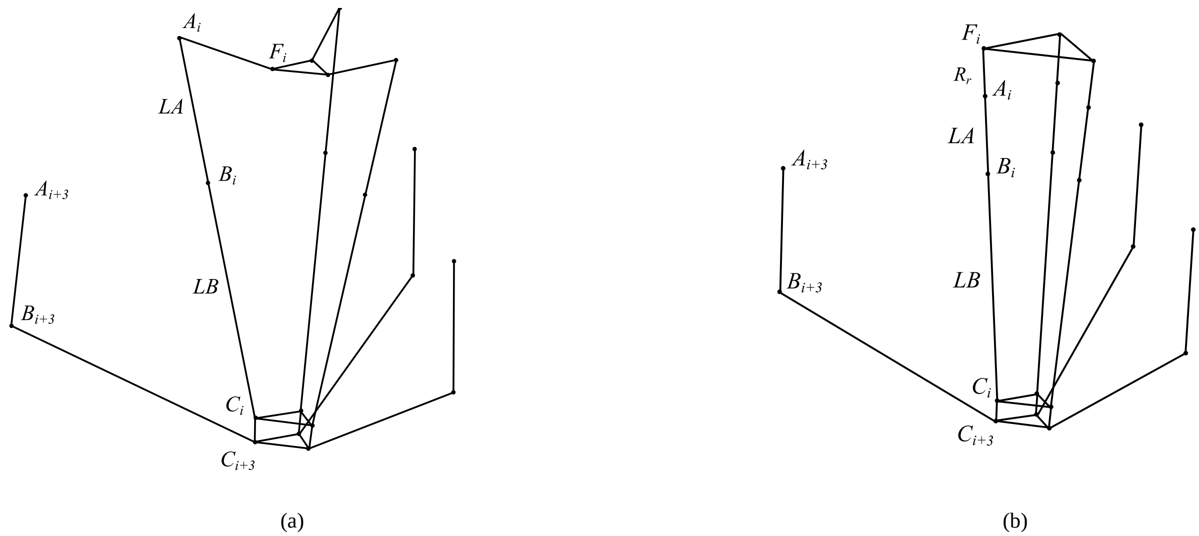


Figure 4. Some inverse singularities. (a) J_q is singular. (b) J'_q is singular.

Singularity type 2. This direct singularity occurs when det (J_x) = 0, which arises when the reciprocal screws $\$_i^6$ are linearly dependent. Because of the arrangement of the spherical joints in two parallel planes, the probability of occurrence of this condition is notably low. However, the manipulator can be in a singular con-figuration if the screws $\left[\$_1^6 \quad \$_2^6 \quad \$_3^6\right]$ or $\left[\$_4^6 \quad \$_5^6 \quad \$_6^6\right]$ are linearly dependent; the former is because the robot may be seen as two parallel sub-manipulators. This situation may occur, e.g., when for any sub-manipulator, the links of length LB are located in the same plane and their reciprocal screws converge to a common point or are parallel, as happens in a plane mechanism (Bonev *et al.*, 2003). An example of this situation is shown in Figure 5. To prevent this situation, the following relation must be satisfied: $LB + r > LA + R$, where $R = R_f + R_r cos\beta$ for the sub-manipulator M_1 and $R = R_2$ for the sub-manipulator M_2. If this condition is only satisfied for one sub-manipulator, this sub-manipulator can pull or push the other to avoid a singularity in the robot or release it if it already exists.

A special case of this type of singularity occurs when a row or column of J_x vanishes, e.g., when the equilateral triangles $F_1F_2F_3$ and $A_4A_5A_6$ (fixed platform) and $C_1C_2C_3$ and $C_4C_5C_6$ (moving platform) coincide, i.e., they are concentric and have identical orientations. Naturally, this condition can only arise along the negative Y direction. If this condition is satisfied, the second element in the dual part of the screws $\$_i^6$ of J_x is zero, which makes det (J_x) = 0 because the direction of $\$_i^6$ (primal part) and the location vector $\boldsymbol{C_i}$ of each spherical joint form an orthogonal plane to the XZ plane, which makes the dual part of $\$_i^6$ a vector of the plane *XZ* without the *Y* component. This singular configuration may be easily avoided if the orientations of $F_1F_2F_3$ and $A_4A_5A_6$ are properly selected to prevent all triangles from coinciding, considering the following condition: $\theta_i \neq \theta_i+3$.

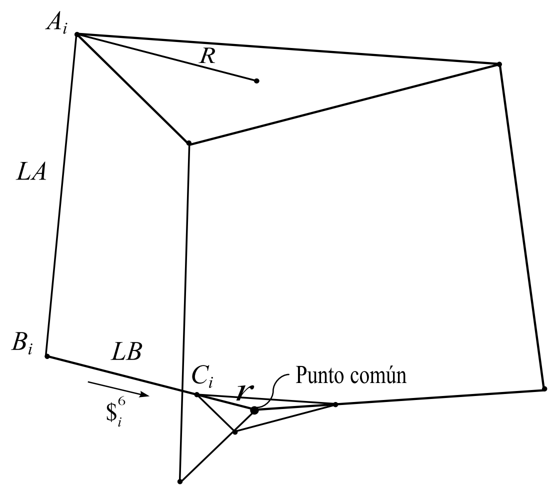


Figure 5. A direct singularity.

Singularity type 3. This combined singularity occurs when exist simultaneously inverse and direct singularities. This situation can occur in two cases: (i) det (J_q) = det (J_x) = 0, (ii) det (J'_q) = det (J_q) = det (J_x) = 0. The option where only det (J'_q) = det (J_x) = 0 cannot exist, remember that: det (J'_q) = 0 $\Rightarrow$ det (J_q) = 0. An example of the first case can arise if in any kinematic chain of a sub-manipulator the links of length *LA* and *LB* are in a full extended or contracted configuration, whereas in the other sub-manipulator, the reciprocal screws are coplanar and converge to a common point. The second case can arise if the fixed and the moving platforms are identical in size, i.e., $F_1F_2F_3 = A_4A_5A_6 = C_1C_2C_3 = C_4C_5C_6$, and the kinematic chains are all vertical (naturally parallel), which occurs only if the moving platform is located between triangles $F_1F_2F_3$ and $A_4A_5A_6$, which is a meaningless configuration for the robot (Figure 6).

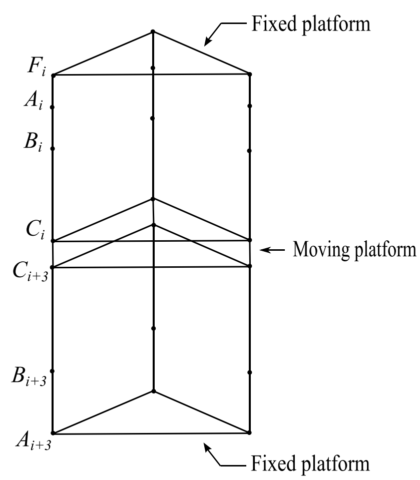


Figure 6. A combined singularity.

4. CONCLUSIONS

Screws theory was applied as a tool to identify the singular configurations of a novel reconfigurable parallel robot. The process involves solving the velocity model of the robot to obtain the Jacobian matrices. Subsequently, the study of these matrices allows to identify the singular configurations.

With classical approaches, Jacobian matrices are functions of the sums and products of sines and cosines of the angles formed by active and passive joints, where finding the singular configurations is somewhat complicated. However, using screw theory, this task becomes notably easy because these matrices are functions of infinitesimal screws which are intuitively associated with a vector direction.

The implemented method is systematic and the singularity analysis was successfully performed for the robot under study. This work allows to highlight some advantageous features of the robot associated with the reduced singular regions. In the case of the inverse singularities, which are associated with the full extended or contracted configuration of the kinematic chains, it was found that the inclusion of the reconfiguration links in the robot does not generate independent singularities. On the other hand, for the direct singularities, which are associated with the linear dependence of the screws related to the direction of the links of length *LB*, all these singularities can be avoided by properly selecting the dimensions of the robot. Finally, the combined singularities can be avoided if the direct or in-verse singularities are also avoided.

REFERENCES

Balmaceda-Santamaría, A., Castillo-Castañeda, E., Gallardo-Alvarado, J., Sánchez-Alonso, R. (2014). Una familia de manipuladores paralelos reconfigurables tipo Delta. In: XVI Congreso Mexicano de Robótica, Mazatlán, México.

Bande, P., Seibt, M., Uhlmann, E., Saha, S. and Rao, P. (2005). Kinematics Analyses of Dodekapod. Mech. Mach. Theory, 40 (6), 740-756.

Bandyopadhyay, S. and Ghosal, A. (2006). Geometric characterization and parametric representation of the singularity manifold of a 6-6 Stewart platform manipulator. Mech. Mach. Theory, 41 (11), 1377-1400.

Bonev, I., Zlatanov, D. and Gosselin C. (2003). Singularity analysis of 3-DOF planar parallel mechanisms via screw theory. J. Mech. Design, 125 (3), 573-581.

Borrás, J., Thomas, F., Ottaviano, E. and Ceccarelli, M. (2009). A reconfigurable 5-DoF 5-SPU parallel platform. In: ASME/IFToMM International Conference on Reconfigurable Mechanisms and Robots, 617-623, IEEE.

Carbonari, L., Callegari, M., Palmieri, G. and Palpacelli, M.-C. (2014). A new class of reconfigurable parallel kinematic machines. Mech. Mach. Theory, 79, 173-183.

Chen, Ch-T. (2012). Reconfiguration of a parallel kinematic manipulator for the maximum dynamic load-carrying capacity. Mech. Mach. Theory, 54, 62-75.

du Plessis, L.J. and Snyman, J.A. (2006). An Optimally Re-Configurable Planar Gough-Stewart Machining Platform. Mech. Mach. Theory, 41 (3), 334-357.

Gallardo-Alvarado, J., Rico, J-M. and Alici, G. (2006). Kinematics and singularity analyses of a 4-dof parallel manipulator using screw theory. Mech. Mach. Theory, 41 (9), 1048-1061.

Gallardo-Alvarado, J., García-Murillo, M. and Castillo-Castaneda, E. (2013). A 2(3-RRPS) parallel manipulator inspired by Gough-Stewart platform. Robotica, 31 (3), 381-388.

Gosselin, C., Angeles, J. (1990). Singularity analysis of closed-loop kinematic chains. IEEE T Robotic. Autom., 6 (3), 281-290.

Huang, Z. and Cao, Y. (2005). Property identification of the singularity loci of a class of Gough-Stewart manipulators. Int. J. Robot. Res., 24 (8), 675-685.

Jiang, Q. and Gosselin, C. (2009). Determination of the maximal singularity-free orientation workspace for the Gough-Stewart platform. Mech. Mach. Theory, 44 (6), 1281-1293.

Kumar, S., Nagarajan, T. and Srinivasa, Y.G. (2009). Characterization of reconfigurable Stewart platform for contour generation. Robot Com.-Int. Manuf., 25 (4-5), 721-731.

Mayer, B. and Gosselin, C. (2000). Singularity analysis and representation of the general Gough-Stewart platform. Int. J. Robot. Res., 19 (3), 271-288.

Plitea, N., Lese, D., Pisla, D. and Vaida, C. (2013). Structural design and kinematics of a new parallel reconfigurable robot. Robot Com.-Int. Manuf., 29 (1), 219-235.

Rico, J-M. and Duffy, J. (2000). Forward and inverse acceleration analyses of in-parallel manipulators. J. Mech. Design, 122 (3), 299-303.

Xi, F., Li, Y. and Wang, H. (2011). Module-Based Method for Design and Analysis of Reconfigurable Parallel Robots. Front. Mech. Eng., 6 (2), 151-159.

Ye, W., Fang, Y., Zhang, K. and Guo, S. (2014). A new family of reconfigurable parallel mechanisms with diamond kinematotropic chain. Mech. Mach. Theory, 74, 1-9.

Yu, H., Li, B., Wang, Y. and Hu, Y. (2012). Conceptual design and workspace analysis of reconfigurable fixturing robots for sheet metal assembly. Assembly Autom., 32 (3), 293-299.

Zhang, D. and Shi, Q. (2012). Novel Design and Analysis of a Reconfigurable Parallel Manipulator Using Variable Geometry Approach. In: Practical Applications of Intelligent Systems, edited by Yingling Wang and Tianrui Li, 124, 447-457. Advances in Intelligent and Soft Computing. Shanghai, China: Springer International Publishing.

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