Critical Crossover Behavior of Itinerant Weak Antiferromagnets around Quantum Critical Point

Rikio Konno, Nobukuni Hatayama, Toyoharu Itoh, and Yoshinori Takahashi

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***Abstract*—Magnetic and thermal critical behaviors of itinerant electron magnets around the quantum critical point (QCP) have recently attracted much interest. On the other hand, crossover phenomena between classical and quantum critical behaviors have not drawn much attention. The purpose of this study is, therefore, clarifies how the predominance of thermal and quantum critical behaviors will change around the QCP based on the spin fluctuation theory. We show that in the region very close to the Neel temperature T_N in the paramagnetic phase, the temperature dependence of the inverse of staggered susceptibility obeys the classical $(T\text{-}T_N)^2$-linear behavior. With increasing temperature T, it changes into the quantum critical $(T^{3/2}\text{-}T_N^{3/2})$-linear dependence above the crossover temperature T^*. Since T^* tends to vanish in the quantum critical limit $T_N \rightarrow 0$, the classical critical region disappears and predominant $T^{3/2}$-linear behavior prevails.**

***Index Terms*—magnetic susceptibility, the Neel temperature, spin fluctuations, itinerant weak antiferromagnets, quantum critical point**

I. INTRODUCTION

The effects of spin fluctuations on the magnetism of itinerant electron magnets have long been studied and various interesting properties have been derived [1]. Recently, crossover between classical- and quantum-critical behaviors of itinerant weak ferromagnets have been studied by Takahashi paying particular attention on the effects of spin fluctuations around QCP [1]. The aim of our present study is to show that the similar crossover behavior is observed for itinerant electron weak antiferromagnets around the QCP. We also show how the predominance of thermal and quantum spin fluctuations of itinerant weak antiferromagnets will change as systems approach the QCP by paying particular attention on the relative dominance of the thermal and the zero-point spin fluctuation amplitude.

We organize the subsequent section as follows. In section II, we will formulate the antiferromagnetic spin fluctuations by following Takahashi's theory. In section III, the results will be provided. In section IV, conclusions will be given.

II. FORMULATION

Let us begin with the assumption that the following local spin amplitude squared in the paramagnetic phase, for instance, is conserved:

$$< S_{loc}^2 >=< \delta S_{loc}^2 >_T + < \delta S_{loc}^2 >_Z \quad (1)$$

where the first and the second terms represent the thermal and the zero-point spin fluctuation

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amplitudes, respectively. In terms of reduced parameters, they are written in the form,

$$< \delta S_{loc}^2 >_T = \frac{9T_0}{T_A} A(y,t)\,, \quad (2)$$

$$< \delta S_{loc}^2 >_Z (y) = < \delta S_{loc}^2 >_Z (0) - \frac{9T_0}{T_A} cy\,, \quad (3)$$

where $t=T/T_0$ and c are the reduced temperature and a numerical constant, respectively. Two temperature scales, T_0 and T_A, are introduced as measures of spectral widths of spin fluctuations, i.e. the imaginary part of the dynamical susceptibility, in frequency and wave-vector spaces. In place of the magnetic susceptibility $\chi(Q)$ in units of $(2\mu_B)^2$ for antiferromagnetic wave-vector Q, its reduced inverse parameter defined by $y = 1/2T_A\chi(Q)$ is used in the following discussion. The temperature dependence of y is then derived from the following equation,

$$A(y,t) - cy = A(0,t_N)\,,\ t_N = T_N/T_0\,, \quad (4)$$

which is derived from (1).

According to [1] the thermal amplitude $A(y,t)$, for $t<<1$, is represented in the form of the scaling function:

$$A(y,t) \cong t^{3/2} F(z),\ z = y/t \quad (5)$$

$$F(z) = \lim_{t\to 0} \int_0^{1/t^2} ds s^2 \left[\ln u - \frac{1}{2u} - \psi(u) \right]$$

where $u = z + s^2$ and $\psi(u)$ is the digamma function. Depending on the magnitude of z, $F(z)$ is approximated by

$$F(z) = \begin{cases} F(0) - \frac{\pi}{4}\sqrt{z} & (z<<1) \\ \frac{\pi}{48\sqrt{z}} & (z>>1) \end{cases}. \quad (6)$$

Just at the QCP, (4) is written in the form:

$$F(0) - (\frac{\pi}{4}\sqrt{z} + c\frac{z}{t^{1/2}}) = 0, \quad (7)$$

the solution of which is given by $z = t^{1/2}F(0)/c$, i.e. $y = t^{3/2}F(0)/c$ is obtained. In the critical region where z<<0.1 is satisfied, it can be written in the form:

$$F(0)(t^{3/2} - t_N^{3/2}) - t^{3/2}(\frac{\pi}{4}\sqrt{z} + c\frac{z}{t^{1/2}}) = 0\,. \quad (5)$$

To find the solution of (5), it is convenient to introduce the variable $\zeta = y/t^2$ in place of y and the following function $g(t)$ by

$$g(t) = \frac{1}{t^2}(t^{3/2} - t_N^{3/2}) \quad (8).$$

Equation (5) is then written in the form:

$$\frac{F(0)}{c} g(t) = \varsigma + \frac{\pi}{4c}\sqrt{\varsigma}\ . \quad (7)$$

III. RESULTS

The solution of ς in (7) in the critical region is simply obtained as

$$\sqrt{\varsigma} = \frac{F(0)g(t)/c}{\sqrt{\frac{F(0)}{c} g(t) + \left(\frac{\pi}{8c}\right)^2} + \frac{\pi}{8c}} \quad (8)$$

in terms of the function $g(t)$. It is easy to see that it has the following two limiting solutions depending on the magnitude of $g(t)$.

1. Classical limit for $g(t)$<<1
   In this case, ς is given by

$$\varsigma(t) \cong [\frac{F(0)}{\pi} g(t)]^2 \cong (\frac{6F(0)}{\pi t_N^2})^2 (t - t_N)^2\,, \quad (9)$$

From the definition of ς , the $(t-t_N)^2$ -linear dependence of $y(t)$ is thus derived, that is characteristic to the classical critical behavior.

2. Quantum critical limit for $g(t)$>>1

The function ς, in this case, is given by

$$\varsigma(t) \cong \frac{F(0)}{c} g(t) = \frac{F(0)}{ct^2} (t^{3/2} - t_N^{3/2}). \quad (10)$$

The $(t^{3/2} - t_N^{3/2})$ -linear dependence of $y(t)$ is derived in this case, which corresponds to the quantum critical behavior.

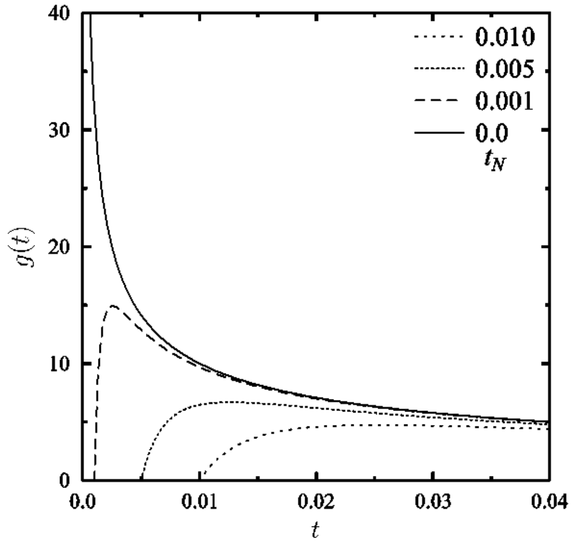

Fig. 1. The temperature dependence of $g(t)$ for various values of t_N

Fig. 1 shows the temperature dependence of the function $g(t)$ for various values of t_N . Let us now define the crossover temperature t^* from the condition, $g(t^*) \cong 1$, for each curve in Fig.1. If we define $\delta = t^* - t_N$, its value is estimated by

$$g(t^*) = \frac{(t_N + \delta)^{3/2} - t_N^{3/2}}{(t_N + \delta)^{3/2}} \cong \frac{3\delta}{2t_N^{3/2}} = 1, \quad (11)$$

Fig. 2 shows t_N dependence of t*. The black dotted line and the red dotted line represent the numerical result by the condition $g(t^*) \cong 1$ and We then expect that the classical critical behavior is observed in the range, $t_N \le t < t^*$. For higher temperatures where the condition, $t^* < t$, is satisfied, the quantum critical behavior is observed. The smaller the value of t_N, the classical critical region becomes narrower and vanishes at t_N=0.

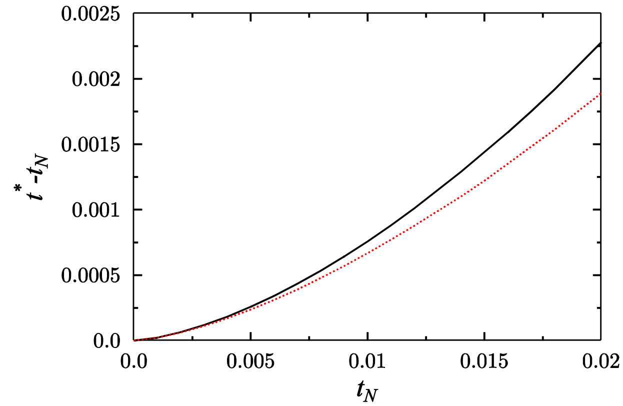

Fig. 2. t_N dependence of t^*.

VI. Conclusions

We have discussed the crossover of temperature dependence of magnetic susceptibility of itinerant antiferromagnets between the classical and quantum critical behaviors. We have found that the quantum critical $(t^{3/2} - t_N^{3/2})$ -linear dependence of the inverse of magnetic susceptibility prevails at low temperatures for magnets with very low t_N. The classical critical $(t - t_N)^2$ -linear dependence is observed only in the restricted region very close to t_N.

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