A Literature Review: Modelling Dynamic Portfolio Strategy under Defaultable Assets with Stochastic Rate of Return, Rate of Inflation and Credit Spread Rate

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Abstract—**This research aims to find an optimal solution for dynamic portfolio in finite-time horizon under defaultable assets, which means that the assets has a chance to be liquidated in a finite time horizon, e.g corporate bond. Besides investing on those assets, investors will also have benefit in the form of consumption. As a reference in making investment decisions the concept of utility functions and volatility will play a role. Optimal portfolio composition will be obtained by maximizing the total expected discounted utility of consumption in the time span during the investment is executed and also to minimize the risk, the volatility of the investment. Further the reduced form model is applied since the assets prices can be linked with the market risk and the credit risk. The interest rate and the rate of inflation will be allowed as a representation of market risk, while the credit spread will be used as a representation of credit risk. The dynamic of asset prices can be derived analytically by using Ito Calculus in the form of the movement of the three risk factors above. Furthermore, this problem will be solved using the stochastic dynamic programming method by assuming that market is incomplete. Depending on the chosen utility function, the optimal solution of the portfolio composition and the consumption can be found explicitly in the form of feedback control. This is possible since the dynamic of the wealth process of the control variable is linear. To apply dynamic programming as well as to find solutions we use Backward Stochastic Differential Equation (BSDE) where the solution can be solved explicitly, especially where the terminal value of the investment target is chosen random. Further, it will be modeled with Monte Carlo simulation and, calibrated using Indonesia data of stock and corporate bond.**

***Keywords*—Optimal Portofolio, Defaultable Assets, Dynamic Programming, Optimal Stochastic Control.**

I. Introduction

In determining investment decision, an investor must be able to have a dynamic strategy to manage their portfolio optimally. Dynamic strategy means that investors have to rebalance continually their portfolio, which always contain risky assets, in response to the fluctuations in their portfolio's value. It is also a nature that investor wants to maximize the return for a given level of risk.

A static portfolio optimization based on mean-variance from the benchmark risk of an investor in a single period) has been introduced by Markowitz (1952). However, his model is independent of time and was unable to connect the wealth process changes of investor with the investment. Samuelson (1969) has continued the work of Markowitz by entering the dynamic way, as well as assessing the optimal investment by including the consumption function into the discrete time model. Merton (1969, 1971) continued Samuelson model using continuous time and added discounted utility of terminal wealth. Also he was able to separate the optimal value function of the portfolio and made the solution explicitly. Their seminal works have introduced a new term of modelling portfolio, called dynamic portfolio model, where the portfolio model is time-dependent, and further developed by others. In modelling dynamic portfolio strategy, it has to be set first the fundamental model of portfolio, between continuous time model and discrete-time model. Second of all is choosing the type of market, such as complete market and incomplete market. In complete market there is no friction in the market and the trading assets are equal with the amount of traders, while by incomplete market the trading assets are less than the amount of traders. Furthermore, the dynamics of assets prices and budget constraints will be derived, which will be linkage

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with the consumption and portfolio composition. These assets prices will be defined as risk-free asset and risky asset. The risky-asset is usually defined as the defaultable asset, which means that the asset has a chance to be liquidated in finite time horizon. Next step is to define the objective of the investor whether it is to maximize her wealth or to minimize her risk, and to set other underlying parameters such as time-horizon (long life investor or finite-time investor), variables that investor wants to control her portfolio such as portfolio composition and consumption.

In order to model the asset pricing, as already mentioned above, there are some risks that need to be considered. First of all it is credit risk which is connected with the defaultable asset. Default value is considered as the main parameter for credit risk when an obligor fails to pay its debt or shows deteriorating its credit worthiness. The credit risk has become an important point of view since the corporate bond markets are developing very fast nowadays. As we know that the largest global financial crisis in history in 2008 has shaken United States and the world tremendously until today, with a collapse of capital flow and international trading. Lehman was the fourth largest US investment bank at the time is collapsed. They filed for bankruptcy on September 15, 2008 with $639 billion in assets and $619 billion in debt. The default triggering value cause Lehman Brothers had registered for Chapter 11 & US Bankruptcy code. Since then there are many literatures brought the study of credit risk into their portfolio studies.

II. Literature Review

A. *Model of Dynamic Portfolio*

Within the dynamic portfolio model, we have to define first the fundamental model of portfolio. The choices are between continuous-time model and discrete-time model. In continuous-time model, according to Merton (1978), the underlying stochastic variables follow diffusion type motion within continuous sample path, and the trading takes place continuously in time. The solutions will be both simplier and richer than that from the usual discrete-time model assumption. Samuelson (1969) was a pioneer in discrete-time model of dynamic portfolio, and the followed by many researchers such as: Grauer & Hakansson (1993) using mean variance approximation, Karatzas & Shreve (1998), Campbell & Viceira (1998), Campbell et.al (2001), Callegaro et.al (2010) and all references in it. In continuous-time model it was pioneered by Merton (1969,1971), then followed by: Cox & Huang (1989), Karatzas et.al (1991), Zariphapolou (2001), Brennan & Xia (2002) , Hou & Jin (2002), Hou (2003), Zhou & Li (2002), Castaneda-Leyva & Hernandez (2005), Stoikov & Zariphopolou (2005), Shouda (2006, Bielecki & Jang (2007), Callegaro et.al (2009), Bo et.al (2010), Jial & Pham (2011), Jiao et.al (2012), Jiao et.al (2013), Bo et.al (2013).

Furthermore we also have to choose the underlying market for our dynamic portfolio model between complete market and incomplete market. The first one means that the market has no friction and the trading assets are equal with the amount of traders, while the second one is assumed that the market has friction where the trading assets are less that the amount of traders. Incomplete market assumption is closed with the actual condition in financial market (Bjork, 2009). In complete market there are literatures such as Merton (1969, 1971), Cox & Huang (1989), Campbell et.al (2001), Dai et.al (2008), Bjork (2009), Jiao & Pham 2011. While study of dynamic portfolio under incomplete market are done among other things by: Karatzas et.al (1989), Zariphapolou (2001),Brennan & Xia (2002), Oksendal & Sulem (2004), Castaneda-Leyva & Hernandez (2005), Stoikov & Zariphopolou (2005), Bielecki & Jang (2007), Lackner & Liang (2008), Callegaro et.al (2009), Bo et.al (2010), Callegaro et.al (2011), Jiao et.al (2013) and Bo et.al (2013).

Another underlying paramater is investor's time horizon. We have to choose the type of investor's time-horizon between infinite and finite time-horizon. The assumption for infinite time-horizon investor, is that the investor invests her assets during her long-life time-span and never redraw her portfolio. In finite-time horizon the investor has an exit time to end her investment. The later parameter is in fact actual with the fact. Investors are usually like to have an exit for their portfolio in order to have benefit of their wealth or because the situation of the financial market push the investors to have an exit time e.g bankruptcy. The studies under infinite time horizon are done by: Samuelson (1969), Merton (1969, 1971), Hakansson (1971), Karatzas et.al (1987), Cox & Huang (1989), Campbell & Viceira(1998), Campbell et.al (2001), Bielecki & Jang (2007), Lackner & Liang (2008), Bo et.al (2010), Bo et.al (2013) and all references in it. While under finite time horizon, the studies are done by: He & Pearson (1991), Karatzas et.al (1991), Zariphapolou (2001), Brennan & Xia (2002), Hou & Jin (2002), Hou (2002), Castaneda-Leyva & Hernandez (2005), Stoikov & Zariphopolou (2005), Lackner & Liang (2008), Oksendal & Sulem (2009), Jiao & Pham (2011), Jiao et.al (2013).

B. The Dynamic Pricing of Assets

There are two assets that are usually used in dynamic portfolio model, defaultable asset and default-free asset. Defaultable asset is also called with risky asset or defaultable asset, which means that the asset can be default at any time. Stock is categorized as the defaultable asset, and denoted with S_t. The price process of defaultable asset is given as follows:

$$dS(t) = \alpha\big(t, S(t)\big).S(t)dt + \sigma\big(t, S(t)\big).S(t)dW \quad [1]$$

where α is known as the local mean rate of return of S, σ is known as the volatility of *S*, and , *W* is a random "white noise" process as a wiener process.

Default free assets is also called with risk-free asset. The price process of risk-free asset, given $B(t)$, according to Merton (1969), is defined as follows:

$$dB(t) = r(t).Bdt \quad [2]$$

where *r(t)* is the rate of return (Bjork, 2009).

As already described in equation [2], the default-free asset is considered as the bond pricing when *r(t)* is a deterministic function. In fact for the corporate bond, it is affected not only by market risk as rate of return but also by rate of inflation, and another thing that is more direct to link with the corporate default is the credit risk. To describe the credit risk into the asset pricing model and to determine the asset pricing, there are two methods used namely, the reduced form method and the structural method. The first method is more applicable because the assets price can be linkaged with the credit risk, while the second method the asset pricing is linkaged only with the movements of the firm's value (Bielecki & Kurtowski, (2007)).

1) Dynamic Asset Pricing Under Market Risk

The market risk is usually related with the rate of return and rate of inflation. There are not many studies allowing rate of inflation on to the portfolio model. Usually they allow only interest rate, and rarely put inflation on asset pricing model. The studies that only use deterministic interest rate are: Samuelson (1969), Merton (1969,1971), Karatzas et.al (1987), Grauer & Hakansson (1999), Campbell et.al (2001), Zariphopolou (2001), Brennan & Xia (2002), Hou & Zin (2002), Hou (2003), Zhou & Li (2002), Castaneda-Leyva & Hernandez (2005), Stoikov & Zariphopolou (2005), Bielecki & Jang (2007), Dai et.al (2008), Ankirchner & Blanchet-Scalliet (2010), Callegaro et.al (2011), Jiao et.al (2012) and Bo, et.al (2013).

Nonetheless, there are some studies using the uncertain inflation on portfolio strategy such as Campbell and Viceira (2001), Brennan and Xia (2002), Munk et.al (2004). Campbell and Viceira (2001) allow consumption, but in infinite life-span setting and using Vasicek model as one factor model of interest rate and rate of inflation. Brennan and Xia (2002) used also vasicek model for both under interest rate and rate of inflation, but in finite time horizon. Munk et.al (2004) used the same constructions as Brennan and Xia (2002) did, except that they made differences between nominal interest rate and real interest rate. They used one-factor vasicek model for the first and two-factor model for the latter.

2) Dynamic Asset Pricing Under Credit Risk

The credit risk is concerned with the modeling of the random time when the default event occurs, for example the default time. According to Bielecki & Kurtowski (2002), "a default risk is a possibility that a counterparty in a financial contract will not fulfill a contractual commitment to meet her/his obligations stated in the contract". The financial instruments that credit risk sensitive are corporate bonds, credit derivatives, claims etc. Corporate bonds are the bond which bear the biggest credit risk, when the obligors fail to repay its debt and/or coupon. The credit risk is usually described with the credit spread. By definition credit spread is the difference in yield between two types of bond, corporate bond and risk-free bond (usually they are treasury and the government bond) at the same time-maturity Bielecki & KurtowskI (2002). Credit risk consists of default risk, recovery risk, correlation risk and migration risk, the most fundamental is default risk (Hou, 2003). It is called default risk when the yield of corporate bond is lower than the risk-free bond. Bielecki & KurtowskI (2002) have explained that there are two methods to value and hedge credit risk; they are structural-form- and reduced-form model.

Structural-form is also called as the firm value approach. In this method the modeling and pricing the credit risk is specific to a corporate obligor. Bielecki & Kurtowski (2002) also mentioned that the movements of the firm's value which relative to some barriers, will trigger the credit events. One type of credit events in most structural model is the firm's default and it will be defined endogenously in the model. By modeling the credit events in terms of the firm's value, this method can link the credit events to the firm's economic fundamental. The framework of this approach is to make a model of the firm's value and firm's capital structure.

In contrast with structural form, the firm's value and firm's capital structure are not modeled at all in the reduced form model. The credit events are defined in terms of some exogeneously process. Bielecki & Kurtowski (2002) also said that the reduced form model can be distinguished between the modeling of the default time, called as the intensity based models, and the modeling of the migration between credit rating classes, called as the credit migration models.

Studies in structural form are done among other things by: Zariphapolou (2001), Hou & Jin (2002), Hou (2002), Korn & Kraft (2003), Shouda (2006), Bielecki & Jang (2007) and Bo et.al (2010). Most of them are using constant rate of return. In reduced-form approach are done among other things by: Oksendal & Sulem (2009), Jiao & Pham (2011), Jiao et.al (2012), Bo et.al (2013).

C. *Discussion Among Literature Reviews In Dynamic Asset Pricing*

In most of literature studies, that we have discussed above, they use deterministic function of interest rate *r*, except for Brennan and Xia (2002), they use a stochastic rate of return function, in the form of Vasicek function, which is given below:

$$dr = \gamma(\bar{r} - r)dt + \sigma_r \, dz_r \qquad [3]$$

where γ is the coefficient to represent that the Ornstein Uhlenbeck process is an elastic random walk process (which possess a stationary distribution, only for $\gamma > 0$), $\bar{r}$ is the long term mean of return, σ_r is the variance of rate of return

Vasicek model has a mean reverting behavior (Luenberger, 1998), where it tends to a constant mean in the long term period. This behavior is very reasonable for interest rate, rather than using random walks, because it is not possible for economically reason that interest rate become arbitrary large.

Beside using interest rate as for the market risk, Brennan & Xia (2002) also use the rate of interest of inflation which also following the Vasicek model:

$$dI = \gamma_I(\bar{I} - I)dt + \sigma_I \, dz_I \qquad [4]$$

where $\gamma_I > 0$, $\bar{I}$ is the long term mean of rate of inflation, and σ_I is the variance of rate of inflation.

Further, Hou and Jin (2002) and Hou (2003) were integrating the rate of interest as market risk and credit risk using Vasicek process and martingale approach (this will be described in the next subsection) .

For studies under credit risk with deterministic interest rate, it has been done by Zariphapolou (2001), Hou & Jin (2002), Hou (2002), Korn & Kraft (2003), Shouda (2006), Bielecki & Jang (2007), Bo et.al (2010) under structural-form method and Oksendal & Sulem (2009), Jiao & Pham (2011), Jiao et.al (2012), Bo et.al (2013) under reduced form model.

III. RESEARCH POSITION

There are five benchmarks studies that will be the starting point of this research, Hou (2003), Oksendal & Sulem (2004), Zhou & Li (2000), Lim & Zhou (2002), Brennan & Xia (2002), Ankirchner et.al (2010), Bo, Wang, Yang (2010), Jiao & Pham (2011) and Bo et.al (2013). The position of their studies will be given as follows:

1. The study under Hou (2003) was only used the rate of interest as the representation of market risk, and the credit spread as the representation of credit risk, no rate of inflation was considered in it. Furthermore, the consumption is also not included into the wealth process and the objective was only to find the optimal return.The result of the weight portfolio as the optimal portfolio strategy for stock is not depend on wealth process and time, where in fact the solution should be depend on the wealth process and time.

2. The study under Brenan & Xia (2002) was only considered the rate of return and the rate of inflation as the representation of market risk and no credit risk was taken. and the objective of their study was only to find the optimal return. The result given is in the form of optimal wealth, not the weight composition.

3. The study under Ankirchner et.al (2010) used the credit risk in the form of jump diffusion, but they used structural form method to find the solution. By using this method, the credit risk cannot be linkaged directly from the macro condition, but only from the internal firm itself.

4. The study under Bo, Wang & Yang (2010) and Bo et.al (2013) used the constant rate of return as the representation of the market risk, and using the credit risk in the form of jump process. The objective of this research is to find the optimal return only.The solution was given in the form of control strategy for portfolio composition and consumption. As in their result, the solution was found constant (see chapter II for further discussion).

5. The study under Jiao & Pham (2013) used jump process as the representation of default time and used the structural form method to find the solution. Again, this is the same as Ankirchner et.al (2010) had

done, that the credit risk cannot be linkaged directly from the macro condition.

6. Oksendal & Sulem (2004) are using BSDE to find the solution the portfolio problem under jump diffusion process. The objective of their study is to find the maximum return and minimum risk, taking no consumption and the rate of return is deterministic. There are no rate of inflation and credit spread.

7. Zhou & Li (2000) and Lim & Zhou (2002) studied about mean variance portfolio problem in complete market with random interest rate, no consumption, and volatility coefficients. To find the solution they used Linear-quadratic control and Lim & Zhou added BSDE as the method to find the solution The rate of rate inflation and credit spread are not involved here,

From those aforementioned studies none of them combined the market risk which contain of both rate of return and rate of inflation with the the credit spread as the credit risk and none of them are taking the consumption. Those three risks will be in the form of vasicek model which are done in Brennan & Xia (2002) and Hou (2003). It is possible because it exhibits mean reverting behavior (Luenberger 1998) and can be written in affine structure (Bjork, 2009). To describe the risks into the asset pricing the reduced form method will be used in this study.

This problem will be solved using the stochastic dynamic programming method which also differ from Ankirchner et.al (2010); Bo, Wang & Yang (2010); Jiao & Pham (2012) and Bo, Li, Wang & Yang (2013). We take the underlying parameters as close as the real fact in financial market, such as incomplete market, and under finite time horizon since we use the defaultable assets. The objective of this research is beside maximizing return, but also to minimize the risk as Zhou & Li (2000), and Lim & Zhou (2002) and Oksendal & Sulem (2004) had done, the optimal solution of the portfolio composition and the consumption can be found explicitly in the form of feedback control.

To enrich this case, beside of finding the optimal solution, we also want to minimize the volatility of the investment. The volatility of investment according to Oksendal & Sulem (2004), is linear with the volatility of stock price. This can be define by applying BSDE where the solution can be solved especially where the terminal value of the investment target is chosen random, Oksendal & Sulem (2004). For other strategy to find the solution, we can use Linear-Quadratic Control (LQ Control Method) together with BSDE as in Lim & Zhou (2002), they said that LQ control can be used both under deterministic and stochastic process, and when the target is random.

In case of Indonesia, the financial investment in financial market is not yet attractive widely as in other countries. But it develops well year by year, especially since the Indonesian people gaining their knowledge about the benefit for having an investment in the form of portfolio. It is well seen that in every year there are many companies launch IPO in the Indonesian financial market lately, the Indonesia Corporate start to launch their corporate bonds. According to the report of Asian Development Bank (ADB) in June 2013 (http://investasi.kontan.co.id, retrieved February 5th, 2014), Indonesia Corporate Bond market has reached U.S$ 20 billion at the end of March 2013. The growth is more than 26%. It is very prospecting said the Head of Fixed-Income Security of Bank Central Asia (BCA), Herdi Ranu Wibowo, because of the macroeconomic condition in Indonesia is quite good, it makes corporate bond issuance will be much more interesting rather than to seek a loan from the bank as a source of funding. This will bring to a very good prospective for the corporate bond growth in Indonesia.

Therefore the study in dynamic portfolio is very interesting to do, especially in Indonesia. Under these reasons the solution will be tested with Monte Carlo simulation and, calibrated using Indonesia data of stock and corporate bond.

IV. RESEARCH QUESTIONS

The research questions then lead to:

1. How to define the assets pricing model under default risky assets?

Research Question 1 will lead to model the assets pricing. We will define those assets as the defaultable assets. The defaultable assets will be under risky asset as stock and corporate bond. The problem is to define the bond pricing, since we add the market risk and credit risk, as for rate of return, rate of inflation and credit spread in form of vasicek model. This question will be splitted up into these questions below:

a. How the closed form of the dynamic for corporate bond pricing, under the rate of return and the credit spread will be defined?

b. How the closed form of the dynamic for corporate bond pricing, under the rate of return, the rate of inflation and the credit spread will be defined?

2. How the portfolio and the consumption optimization strategy will be obtained using default risky assets that build up from Research Question 1, in finite time horizon?

Applying stochastic dynamic programming, the explicit solution of the optimal portfolio composition and consumption can be determined. The optimal solution can be solved explicitly in the form of feedback control by finding the maximal utility. For cases where the target investment of an investor is to minimize the volatility of the investment, it is necessary to apply dynamic programming as well as to find solutions of Backward Stochastic Differential Equation (BSDE). The solution can be solved especially where the random terminal value of the investment target is chosen random. By this, the following research questions will be added with:

a. How is the optimal portfolio composition will be obtained if the target of investor is also to minimize the volatility of investment in the time span during the investment ?

b. Will this model have a nice and smooth calibration when it is implemented into the real data of Indonesian stock price and corporate bond?

V. CONCLUSION

From literature review that has been explained above the study in dynamic portfolio by linking the market risk and credit risk in vasicek model into the asset, is still not done and interesting to do, especially when the subject is not only to maximize return but also to minimize risk volatility. For next stage of this study the closed form solution for dynamic portfolio with those constrains will be analyzed. The solution will be tested and calibrated using Indonesia data of stock and corporate bond.

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