Exponential Synchronization for Fractional-order Time-delayed Memristive Neural Networks

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***Abstract*—Considering the fact that the exponential synchronization of neural networks has been widely used in theoretical research and practical application of many scientific fields, and there are a few researches about the exponential synchronization of fractional-order memristor-based neural networks (FMNN). This paper concentrates on the FMNN with time-varying delays and investigates its exponential synchronization. A simple linear error feedback controller is applied to compel the response system to synchronize with the drive system. Combining the theories of differential inclusions and set valued maps, a new sufficient condition concerning exponential synchronization is obtained based on comparison principle rather than the traditional Lyapunov theory. The obtained results extend exponential synchronization of integer-order system to fractional-order memristor-based neural networks with time-varying delays. Finally, some numerical examples are used to demonstrate the effectiveness and correctness of the main results.**

Keywords-Exponential Synchronization; Memristor-based Neural Networks; Fractional-order; Linear Error Feedback Control; Time-varying Delays.

I. INTRODUCTION

Chua already supposed the existence of memristor in 1971 [1], however, the practical device of memristor in electronics is obtained in [2] until 2008. In addition to the existing three kinds of circuit elements, memristor is regarded as the fourth basic circuit element and is defined by a nonlinear charge-flux characteristic. As everyone knows, resistors can be used to work as connection weights so that it can emulate the synapses in artificial neural networks. However, in the neural networks of biological individual, long-term memories is essential in the synapses among neurons, but for the general resistors, it is impossible to have the function of memory. Recently, due to the memory characteristics of memristor, memristor can replace the resistor to develop a new neural networks that is memristor-based neural networks (MNN) [3-6].

In recent years, more and more attentions have been put on the dynamical analysis of memristor-based neural networks, such as the investigation of stability [7-10], periodicity [11-13], system synchronization [14-22], passivity analysis [23], dissipativity [24-25] and attractivity [26]. Particularly, the stability and synchronization of MNN has been widely studied in [27-30]. In fact, synchronization means the dynamics of nodes share the common time-spatial property. Therefore we can understand an unknown dynamical system by achieving the synchronization with the well-known dynamical systems [18]. Moreover, in the transmission of digital signals, communication will become security, reliable and secrecy by achieving synchronization between the various systems. Therefore, the synchronization of MNN is still worth further research.

Moreover, the fractional-order models can better describe the memory and genetic properties of various

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materials and process, so the fractional-order models have received a lot of research attentions than integer-order models. In recent years, with the improvement of fractional-order differential calculus and fractional-order differential equations, it is easy to model and analyze practical problems [31, 32]. Therefore, there have been a lot of researches about the dynamical analysis and synchronization of fractional-order memristor-based neural networks (FMNN) [34-39]. Finite-time synchronization, hybrid projective synchronization and adaptive synchronization of FMNN have all been researched [34-36]. However, there are only a very few research results on exponential synchronization of FMNN. In fact, the exponential synchronization of neural networks has been widely used in the theoretical research and practical application of many scientific fields, for example, associative memory, ecological system, combinatorial optimization, military field, artificial intelligence system and so on [40-43]. So the exponential synchronization of FMNN is still worth further studying as it is a significant academic problem.

On the other hand, the stability and synchronization of FMNN without time delay have been deeply studied such as in [33]. However, in hardware implementation of neural networks, time delay is unavoidable owing to the finite switching speeds of the amplifiers. And it will cause instability, oscillation and chaos phenomena of systems. So the investigation for stability and synchronization of FMNN cannot be independent on the time delay.

Motivated by the above discussion, this paper studies the exponential synchronization of FMNN with time-varying delays. The main contributions of this paper can be listed as follow. (1) This is the first attempt to achieve exponential synchronization of FMNN with time-varying delays by employing a simple linear error feedback controller. (2) The sufficient condition for exponential synchronization of FMNN with time delays is obtained based on comparison principle instead of the traditional Lyapunov theory. (3) Some previous research results of exponential synchronization for integer-order memristor-based system are the special cases of our results. Furthermore, some numerical examples are given to demonstrate the effectiveness and correctness of the main results.

The rest of this paper is organized as follows. Preliminaries including the introduction of Caputo fractional-order derivative, model description, assumptions, definitions and lemmas are presented in Section 2. Section 3 introduces the sufficient condition for exponential synchronization of the FMNN. In Section4, the numerical simulations are presented. Section5 gives the conclusion of this paper.

II. PRELIMINARIES

Compared to the integer-order derivatives, we know the distinct advantage of Caputo derivative is that it only requires initial conditions from the Laplace transform of fractional derivative, and it can represent well-understood features of physical situations and making it more applicable to real world problems [36]. So in the rest of this paper, we apply the Caputo fractional-order derivative for the fractional-order memristor-based neural networks (FMNN) and investigate the exponential synchronization of FMNN.

A. *The Caputo fractional-order derivative*

Definition1 [32] The Caputo fractional-order derivative is defined as follows:

$$D_t^q f(t) = \frac{1}{\Gamma(m-q)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau, \tag{1}$$

where q is the order of fractional derivative, m is the first integer larger than q, $m-1 \le q < m, \Gamma(\cdot)$ is the Gamma function,

$$T(x) = \int_0^{\infty} t^{x-1} e^{-t} dt. \tag{2}$$

Particularly, when $0 < q < 1,$

$$D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-t_0)^q} d\tau. \tag{3}$$

B. Model description

In this paper, referring to some relevant works on FMNN [35,36], we consider a class of FMNN with time-varying delays described by the following equation,

$$\begin{aligned} &D^q x_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_j(t))) g_j(x_j(t-\tau_j(t))) + I_i, \\ &t \ge 0, i \in N. \\ &a_{ij}(x_j(t)) = \frac{M_{ij}}{C_i} \times \delta_{ij}, \quad b_{ij}(x_j(t-\tau_j(t))) = \frac{W_{ij}}{C_i} \times \delta_{ij}, \quad \delta_{ij} = \begin{cases} 1, & i \ne j, \\ -1, & i = j. \end{cases} \end{aligned} \tag{4}$$

where $x_i(t)$ is the state variable of the i th neuron (the voltage of capacitor C_i), q is the order of fractional derivative, $c_i > 0$ is the self-regulating parameters of the neurons, $0 \le \tau_j(t) \le \tau$ and (τ is a constant) represents the transmission time-varying delay. $f_j, g_j : R \to R$ are feedback functions without and with time-varying delay. $a_{ij}(x_j(t))$ and $b_{ij}(x_j(t-\tau_j(t)))$ are memristive connective weights, which denote the neuron interconnection matrix and the delayed neuron interconnection matrix, respectively. W_{ij} and M_{ij} denote the memductances of memristors R_{ij} and F_{ij} respectively. And R_{ij} represents the memristor between the feedback function $f_i(x_i(t))$ and $x_i(t)$, F_{ij} represents the memristor between the feedback function $g_i(x_i(t-\tau_i(t)))$ and $x_i(t)$. I_i represents the external input. According to the feature of memristor, we denote

$$a_{ij}(x_j(t)) = \begin{cases} \hat{a}_{ij}, & x_j(t) \le 0, \\ \breve{a}_{ij}, & x_j(t) > 0, \end{cases} \quad b_{ij}(x_j(t-\tau_j(t))) = \begin{cases} \hat{b}_{ij}, & x_j(t-\tau_j(t)) \le 0. \\ \breve{b}_{ij}, & x_j(t-\tau_j(t)) > 0. \end{cases} \tag{5}$$

C. Assumptions, Definitions and Lemmas

In the rest of paper, we first make following assumption for system (4) .

Assumption1: For $j \in N, \forall s_1, s_2 \in R$, the neuron activation functions f_j, g_j bounded, $f_j(0) = g_j(0) = 0$ and satisfy

$$0 \le \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \le \sigma_j,$$
$$0 \le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le \rho_j, \tag{6}$$

where $s_1 \neq s_2$ and σ_j, ρ_j are nonnegative constants.

We consider system (4) as drive system and corresponding response system is given as follows:3

$$D^q y_i(t) = -c_i y_i(t) + \sum_{j=1}^{n} a_{ij}\left(y_j(t)\right) f_j\left(y_j(t)\right) + \sum_{j=1}^{n} b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right) g_j\left(y_j\left(t-\tau_j(t)\right)\right) + I_i + u_i,$$
$$t \ge 0,\ i \in N, \tag{7}$$

Where

$$a_{ij}\left(y_j(t)\right) = \begin{cases} \hat{a}_{ij}, & y_j(t) \le 0, \\ \check{a}_{ij}, & y_j(t) > 0, \end{cases} \quad b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right) = \begin{cases} \hat{b}_{ij}, & y_j\left(t-\tau_j(t)\right) \le 0, \\ \check{b}_{ij}, & y_j\left(t-\tau_j(t)\right) > 0, \end{cases} \tag{8}$$

and $u_i(t)$ is a liner error feedback control function which defined by $u_i(t) = \omega_i\left(y_i(t) - x_i(t)\right)$, where $\omega_i,\ i \in N$ are constants, which denotes the control gain. Next, we define the synchronization error $e(t)$ as $e(t) = \left(e_1(t), e_2(t), \ldots, e_n(t)\right)^T$, where $e_i(t) = y_i(t) - x_i(t)$. According to the system (4) and system (7), the synchronization error system can be described as follows:

$$D^q e_i(t) = -c_i e_i(t) + \left[\sum_{j=1}^{n} a_{ij}\left(y_j(t)\right) f_j\left(y_j(t)\right) - \sum_{j=1}^{n} a_{ij}\left(x_j(t)\right) f_j\left(x_j(t)\right)\right]$$
$$+\left[\sum_{j=1}^{n} b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right) g_j\left(y_j\left(t-\tau_j(t)\right)\right) - \sum_{j=1}^{n} b_{ij}\left(x_j\left(t-\tau_j(t)\right)\right) g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right]$$
$$+u_i(t),\ t \ge 0, i \in N \tag{9}$$

where $a_{ij}\left(y_j(t)\right), b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right), a_{ij}\left(x_j(t)\right), b_{ij}\left(x_j\left(t-\tau_j(t)\right)\right)$ are the same as those defined above, $u_i(t) = \omega_i\left(y_i(t) - x_i(t)\right) = \omega_i e_i(t)$, where $\omega_i,\ i \in N$ are constants, which denotes the control gain.

According to the theories of differential inclusions and set valued maps [40], if $x_i(t)$ and $y_i(t)$ are solutions of (4) and (7) respectively, system (4) and system (7) can be written as follow:

$$D^q x_i(t) \in -c_i x_i(t) + \sum_{j=1}^{n} co\left[a_{ij}\left(x_j(t)\right)\right] f_j\left(x_j(t)\right)$$
$$+\sum_{j=1}^{n} co\left[b_{ij}\left(x_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(x_j\left(t-\tau_j(t)\right)\right) + I_i,\ t \geq 0, i \in N \tag{10}$$

And

$$D^q y_i(t) \in -c_i y_i(t) + \sum_{j=1}^{n} co\left[a_{ij}\left(y_j(t)\right)\right] f_j\left(y_j(t)\right)$$
$$+\sum_{j=1}^{n} co\left[b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(y_j\left(t-\tau_j(t)\right)\right) + I_i + u_i,\ t \geq 0, i \in N, \tag{11}$$

Where

$$co\left[a_{ij}\left(x_j(t)\right)\right] = \begin{cases} \hat{a}_{ij}, x_j(t) < 0, \\ co\left\{\hat{a}_{ij}, \breve{a}_{ij}\right\}, x_j(t) = 0, \\ \breve{a}_{ij}, x_j(t) > 0, \end{cases} \quad co\left[a_{ij}\left(y_j(t)\right)\right] = \begin{cases} \hat{a}_{ij}, y_j(t) < 0 \\ co\left\{\hat{a}_{ij}, \breve{a}_{ij}\right\}, y_j(t) = 0 \\ \breve{a}_{ij}, y_j(t) > 0 \end{cases} \tag{12}$$

And

$$co\left[b_{ij}\left(x_j\left(t-\tau(j)\right)\right)\right] = \begin{cases} \hat{b}_{ij}, x_j\left(t-\tau(j)\right) < 0, \\ co\left\{\hat{b}_{ij}, \breve{b}_{ij}\right\}, x_j\left(t-\tau(j)\right) = 0, \\ \breve{b}_{ij}, x_j\left(t-\tau(j)\right) > 0, \end{cases}$$
$$co\left[b_{ij}\left(y_j\left(t-\tau(j)\right)\right)\right] = \begin{cases} \hat{b}_{ij}, y_j\left(t-\tau(j)\right) < 0, \\ co\left\{\hat{b}_{ij}, \breve{b}_{ij}\right\}, y_j\left(t-\tau(j)\right) = 0, \\ \breve{b}_{ij}, y_j\left(t-\tau(j)\right) > 0, \end{cases} \tag{13}$$

where $co\{u, v\}$ denotes the closure of convex hull generated by real numbers u and v or real matrices u and v.

Then the synchronization error system can be described as follows:

$$D^q e_i(t) \in -c_i e_i(t) + \left\{ \sum_{j=1}^{n} co\left[a_{ij}\left(y_j(t)\right)\right] f_j\left(y_j(t)\right) - \sum_{j=1}^{n} co\left[a_{ij}\left(x_j(t)\right)\right] f_j\left(x_j(t)\right) \right\}$$
$$+ \left\{ \sum_{j=1}^{n} co\left[b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(y_j\left(t-\tau_j(t)\right)\right) \right.$$
$$\left. - \sum_{j=1}^{n} co\left[b_{ij}\left(x_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(x_j\left(t-\tau_j(t)\right)\right) \right\} + \omega_i e_i,\ t \geq 0, i \in N. \tag{14}$$

Definition2 [8] For $\forall\, t \geq 0,$ the exponential synchronization of system (4) and system (7) can be transformed to the exponential stability of the error system (9) (error approaches to zero). The error system (9) is said to be exponentially stable, if there exist constant $Q_i > 0,\ P_i > 0,$ such that the solution $e(t) = \left(e_1(t), e_2(t), \ldots, e_n(t)\right)^T$ of error system (9) with initial condition $e(s) = \phi(s) \in \left(\left[t_0 - \tau, t_0\right], R^n\right)$ satisfies

$$\left|e_i(t)\right| \leq Q_i \max_{1 \leq i \leq n} \left\{ \sup_{t_0 - \tau \leq s \leq t_0} \left|\phi_i(s)\right| \right\} \exp\left\{-P_i\left(t - t_0\right)\right\},\ t \geq t_0 > 0,$$

$i = 1, 2, \ldots, n$, where P_i is called the estimated rate of exponential convergence.

Lemma1 [14] Under the assumption1, the following estimation can be obtained:

(i) $co\left[a_{ij}\left(y_j(t)\right)\right] f_j\left(y_j(t)\right) - co\left[a_{ij}\left(x_j(t)\right)\right] f_j\left(x_j(t)\right) \leq A_{ij} F_j\left(e_j(t)\right)$,

(ii) $co\left[b_{ij}\left(y_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(y_j\left(t-\tau_j(t)\right)\right) - co\left[b_{ij}\left(x_j\left(t-\tau_j(t)\right)\right)\right] g_j\left(x_j\left(t-\tau_j(t)\right)\right)$

$\leq B_{ij} G_j\left(e_j\left(t-\tau_j(t)\right)\right)$,

where $A_{ij} = \max\left\{\left|\hat{a}_{ij}\right|, \left|\breve{a}_{ij}\right|\right\}, B_{ij} = \max\left\{\left|\hat{b}_{ij}\right|, \left|\breve{b}_{ij}\right|\right\}, i, j \in N,$

$$F_j\left(e_j(t)\right) = f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right), G_j\left(e_j\left(t-\tau_j(t)\right)\right) = g_j\left(y_j\left(t-\tau_j(t)\right)\right) - g_j\left(x_j\left(t-\tau_j(t)\right)\right), j \in N.$$

Proof: If $y_i(t) = 0, x_i(t) = 0, i \in N$ we can easily have part(i) hold. From (9) and(10), we can get

(1) For $y_i(t) < 0, x_i(t) < 0$, then

$$co\left[a_{ij}\left(y_j(t)\right)\right] f_j\left(y_j(t)\right) - co\left[a_{ij}\left(x_j(t)\right)\right] f_j\left(x_j(t)\right) = \hat{a}_{ij} f_j\left(y_j(t)\right) - \hat{a}_{ij} f_j\left(x_j(t)\right)$$

$$= \hat{a}_{ij} F_j\left(e_j(t)\right) \leq A_{ij} F_j\left(e_j(t)\right).$$

(2) For $y_i(t) > 0, x_i(t) > 0$, then

$$co\left[a_{ij}\left(y_j(t)\right)\right] f_j\left(y_j(t)\right) - co\left[a_{ij}\left(x_j(t)\right)\right] f_j\left(x_j(t)\right) = \breve{a}_{ij} f_j\left(y_j(t)\right) - \breve{a}_{ij} f_j\left(x_j(t)\right)$$

$$=\breve{a}_{ij}F_j\left(e_j\left(t\right)\right)\le A_{ij}F_j\left(e_j\left(t\right)\right).$$

(3) For $x_i\left(t\right)<0<y_i\left(t\right)$ or $y_i\left(t\right)<0<x_i\left(t\right)$, then

$$\begin{aligned}&co\left[a_{ij}\left(y_j\left(t\right)\right)\right]f_j\left(y_j\left(t\right)\right)-co\left[a_{ij}\left(x_j\left(t\right)\right)\right]f_j\left(x_j\left(t\right)\right)\\&=\breve{a}_{ij}(f_j\left(y_j\left(t\right)\right)-f(0))+\hat{a}_{ij}(f(0)-f_j\left(x_j\left(t\right)\right))\\&\le A_{ij}\left(f_j\left(y_j\left(t\right)\right)-f(0)\right)+A_{ij}(f(0)-f_j\left(x_j\left(t\right)\right))=A_{ij}(f_j\left(y_j\left(t\right)\right)-f_j\left(x_j\left(t\right)\right))=A_{ij}F_j\left(e_j\left(t\right)\right).\end{aligned}$$

Then complete the proof of part (i). In the similar way, part(ii) can be easily hold.

III. MAIN RESULTS

We present the exponential stability results for the synchronization error system of FMNN，when the error system (9) is exponentially stable, the system (4) and system (7) will achieve the exponential synchronization.

Theorem1 If there exist positive constant $\varepsilon,\eta_1,\eta_2,...,\eta_n$ such that for any $t\ge t_0>0,i\in\{1,2,...,n\}$

$$(-c_i+\omega_i+\varepsilon)\eta_i+\sum_{j=1}^{n}A_{ij}\sigma_j\eta_j+\sum_{j=1}^{n}B_{ij}\rho_j\eta_j\exp\left\{\varepsilon\tau_j\left(t\right)\right\}<0, \tag{15}$$

then the error system (9) is globally exponentially stable.

Proof: Consider $W_i\left(t\right)=\left|e_i\left(t\right)\right|/\eta_i\,,i=1,2,...,n$, according to the error system (9) or (14) and lemma1, we can get the following inequality

$$\begin{aligned}D^qe_i\left(t\right)&\le-c_ie_i\left(t\right)+\sum_{j=1}^{n}A_{ij}F_j\left(e_j\left(t\right)\right)+\sum_{j=1}^{n}B_{ij}G_j\left(e_j\left(t-\tau_j\left(t\right)\right)\right)+\omega_ie_i\left(t\right)\\&=(-c_i+\omega_i)e_i\left(t\right)+\sum_{j=1}^{n}A_{ij}F_j\left(e_j\left(t\right)\right)+\sum_{j=1}^{n}B_{ij}G_j\left(e_j\left(t-\tau_j\left(t\right)\right)\right).\end{aligned} \tag{16}$$

Evaluating the fractional order derivative of $W_i\left(t\right)$ along the trajectory of error system, then

$$\begin{aligned}D^qW_i\left(t\right)&\le\left(-c_i+\omega_i\right)\left|e_i\left(t\right)\right|/\eta_i+1/\eta_i\left[\sum_{j=1}^{n}A_{ij}\sigma_j\left|e_j\left(t\right)\right|+\sum_{j=1}^{n}B_{ij}\rho_j\left|e_j\left(t-\tau_j\left(t\right)\right)\right|\right]\\&=\left(-c_i+\omega_i\right)W_i\left(t\right)+1/\eta_i\left[\sum_{j=1}^{n}A_{ij}\sigma_j\eta_jW_j\left(t\right)+\sum_{j=1}^{n}B_{ij}\rho_j\eta_jW_j\left(t-\tau_j\left(t\right)\right)\right].\end{aligned} \tag{17}$$

Define $\tilde{W}_i\left(t\right)=W_i\left(t\right)-\bar{W}\left(t_0\right)\exp\left\{-\varepsilon\left(t-t_0\right)\right\},t\ge t_0>0,i=1,2,...,n$, where

$$\bar{W}(t_0) = \max_{1\le i\le n}\left\{\sup_{t_0-\tau\le s\le t_0} |e_i(s)|/\eta_i\right\}.$$

We will prove that $\tilde{W}_i(t) \le 0, i = 1,2,...,n$, for any $t \ge t_0 > 0$. Otherwise, since $\tilde{W}_i(t) \le 0,\ i = 1,2,...,n$ for $t \in [t_0-\tau,\ t_0]$, there must exist $t_1 \ge t_0$ and some ς such that $D^q\tilde{W}_\varsigma(t_1) \ge 0$ and $\tilde{W}_\varsigma(t_1) = 0$. Then

$$\begin{aligned}
&D^q\tilde{W}_\varsigma(t_1) \le (-c_\varsigma + \omega_\varsigma)W_\varsigma(t_1) + 1/\eta_\varsigma\left[\sum_{j=1}^n A_{\varsigma j}\sigma_j\eta_j W_j(t_1) + \sum_{j=1}^n B_{\varsigma j}\rho_j\eta_j W_j(t-\tau_j(t_1))\right]\\
&+\varepsilon\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\\
&= (-c_\varsigma + \omega_\varsigma)\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\} + 1/\eta_\varsigma\left[\sum_{j=1}^n A_{\varsigma j}\,\sigma_j\eta_j\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\right.\\
&\left.+\sum_{j=1}^n B_{\varsigma j}\rho_j\eta_j\bar{W}(t_0)\exp\{-\varepsilon(t_1-\tau_j(t_1)-t_0)\}\right] + \varepsilon\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\\
&= (-c_\varsigma + \omega_\varsigma + \varepsilon)\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\} + 1/\eta_\varsigma\left[\sum_{j=1}^n A_{\varsigma j}\,\sigma_j\eta_j\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\right.\\
&\left.+\sum_{j=1}^n B_{\varsigma j}\rho_j\eta_j\bar{W}(t_0)\exp\{-\varepsilon(t_1-\tau_j(t_1)-t_0)\}\right]\\
&= (-c_\varsigma + \omega_\varsigma + \varepsilon)\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\\
&+1/\eta_\varsigma\left[\sum_{j=1}^n A_{\varsigma j}\,\sigma_j\eta_j\bar{W}(t_0) + \sum_{j=1}^n B_{\varsigma j}\rho_j\eta_j\bar{W}(t_0)\exp\{\varepsilon\tau_j(t_1)\}\right]\exp\{-\varepsilon(t_1-t_0)\}.
\end{aligned} \tag{18}$$

Moreover, from inequality(15), we have

$$(-c_i + \omega_i + \varepsilon) + 1/\eta_i\left[\sum_{j=1}^n A_{ij}\sigma_j\eta_j + \sum_{j=1}^n B_{ij}\rho_j\eta_j\exp\{\varepsilon\tau_j(t)\}\right] < 0, t \ge t_0 > 0, i = 1,2,...,n,$$

Therefore

$$\begin{aligned}
&(-c_i + \omega_i + \varepsilon)\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\}\\
&+1/\eta_i\left[\sum_{j=1}^n A_{ij}\sigma_j\eta_j + \sum_{j=1}^n B_{ij}\rho_j\eta_j\exp\{\varepsilon\tau_j(t)\}\right]\bar{W}(t_0)\exp\{-\varepsilon(t_1-t_0)\} < 0,\\
&t \ge t_0 > 0, i = 1,2,\dots,n,
\end{aligned} \tag{19}$$

so it is easy to find that $D^q\tilde{W}_\varsigma(t_1) < 0$, which contradicts $D^q\tilde{W}_\varsigma(t_1) \ge 0$. That shows $\tilde{W}_i(t) = W_i(t) - \bar{W}(t_0)\exp\{-\varepsilon(t-t_0)\} \le 0,\ t \ge t_0 > 0, i = 1,2,...,n$. Thus

$$|e_i(t)|/\eta_i \le \max_{1\le i\le n}\left\{\sup_{t_0-\tau\le s\le t_0}|e_i(s)|/\eta_i\right\}\exp\{-\varepsilon(t-t_0)\},\ t \ge t_0 > 0, i = 1,2,...,n.$$

It shows

$$\left|e_i(t)\right| \le \eta_i \max_{1\le i\le n}\left\{\sup_{t_0-\tau\le s\le t_0}\left|e_i(s)\right|/\eta_i\right\}\exp\left\{-\varepsilon(t-t_0)\right\},\ t\ge t_0>0, i=1,2,...,n. \tag{20}$$

This completes the proof.

IV. NUMERICAL RESULTS

In this section, we will give two numerical examples to demonstrate our analysis on exponential synchronization of FMNN.

Example1 Consider two-dimension fractional-order memristor-based neural networks

$$\begin{cases} D^q x_1(t) = -c_1 x_1(t) + a_{11}(x_1(t)) f_1(x_1(t)) + a_{12}(x_2(t)) f_2(x_2(t)) \\ +b_{11}(x_1(t-\tau_1(t))) g_1(x_1(t-\tau_1(t))) + b_{12}(x_2(t-\tau_2(t))) g_2(x_2(t-\tau_2(t))) + I_1 \\ D^q x_2(t) = -c_2 x_2(t) + a_{21}(x_1(t)) f_1(x_1(t)) + a_{22}(x_2(t)) f_2(x_2(t)) \\ +b_{21}(x_1(t-\tau_1(t))) g_1(x_1(t-\tau_1(t))) + b_{22}(x_2(t-\tau_2(t))) g_2(x_2(t-\tau_2(t))) + I_2 \end{cases} \tag{21}$$

where $c_1 = c_2 = 1,\ a_{11}(x_1(t)) = 1,\ a_{22}(x_2(t)) = 1.8,$

$$a_{12}(x_2(t)) = \begin{cases} 12, & x_2(t)\le 0, \\ 14, & x_2(t)>0, \end{cases} \quad a_{21}(x_1(t)) = \begin{cases} 0.1, & x_1(t)\le 0, \\ 0.05, & x_1(t)>0, \end{cases}$$

$$b_{11}(x_1(t-\tau_1(t))) = \begin{cases} -1.2, & x_1(t-\tau_1(t))\le 0, \\ -1.5, & x_1(t-\tau_1(t))>0, \end{cases} \quad b_{12}(x_2(t-\tau_2(t))) = \begin{cases} 0.8, & x_2(t-\tau_2(t))\le 0, \\ 1.0, & x_2(t-\tau_2(t))>0, \end{cases}$$

$$b_{21}(x_1(t-\tau_1(t))) = \begin{cases} 0.05, & x_1(t-\tau_1(t))\le 0, \\ 0.1, & x_1(t-\tau_1(t))>0, \end{cases} \quad b_{22}(x_2(t-\tau_2(t))) = \begin{cases} -1.6, & x_2(t-\tau_2(t))\le 0, \\ -1.4, & x_2(t-\tau_2(t))>0, \end{cases}$$

where $\tau_j(t) = e^t/1+e^t$, $I = (I_1, I_2)^T = (0,0)^T$, $q = 0.92$ and take the activation function as $f_i(x_i) = \sin(x_i)$, $g_i(x_i) = 0.5(|x_i+1| - |x_i-1|),\ i,\ j = 1,2.$ The model (21) has chaotic attractors with initial values $x(0) = (0.45, 0.65)^T$ which can be seen in Figure1.

We consider system (21) as the drive system and corresponding response system is defined as Eq.(7). And for the controller $u_i(t) = \omega_i(y_i(t) - x_i(t))$, the parameter ω_i is chosen as $\omega_1 = -9.5,\ \omega_2 = -10.5.$ From Theorem1, when we take $\varepsilon = 0.7, \tau_j(t) = 1,$ $\eta_1 = \eta_2 = \rho_1 = \rho_2 = \sigma_1 = \sigma_2 = 0.1,$ we can easily know

$(-c_i+\omega_i+\varepsilon)\eta_i+\sum_{j=1}^{n}A_{ij}\sigma_j\eta_j+\sum_{j=1}^{n}B_{ij}\rho_j\eta_j\exp\{\varepsilon\tau_j(t)\}<0$ $\omega_1=-9.5,\omega_2=-10.5,$ we can get is true when $\omega_1<-1.703,\omega_2<-0.232$. So when

$$(-c_1+\omega_1+\varepsilon)\eta_1+A_{11}\sigma_1\eta_1+A_{12}\sigma_2\eta_2+B_{11}\rho_1\eta_1\exp\{\varepsilon\tau_1(t)\}+B_{12}\rho_2\eta_2\exp\{\varepsilon\tau_2(t)\}=-0.798<0,$$
$$(-c_2+\omega_2+\varepsilon)\eta_2+A_{21}\sigma_1\eta_1+A_{22}\sigma_2\eta_2+B_{21}\rho_1\eta_1\exp\{\varepsilon\tau_1(t)\}+B_{22}\rho_2\eta_2\exp\{\varepsilon\tau_2(t)\}=-1.027<0.$$

It satisfies the condition of Theorem 1, then the exponential synchronization of drive-response system is achieved.

When the response system with this controller, we get state trajectories of variable $x_1(t),y_1(t)$ and $x_2(t),y_2(t)$ are depicted in Figure2a and 2b. Moreover, Figure3a and 3b depict the synchronization error curves $e_1(t),e_2(t)$ between the drive system and response system. These numerical simulations show the state trajectories of variable $x_1(t),y_1(t)$ and $x_2(t),y_2(t)$ are synchronous and synchronization error $e_1(t),e_2(t)$ are converge to zero. These prove the correctness of the Theorem1.

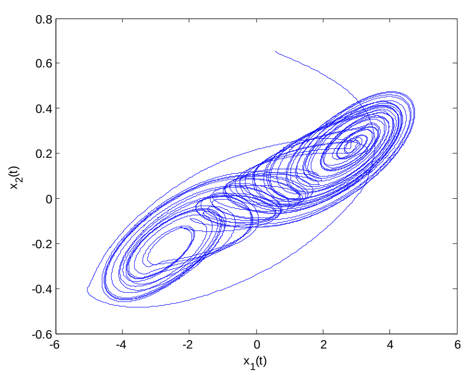

Figure 1. The chaotic attractors of fractional-order memristor-based neural networks(18)

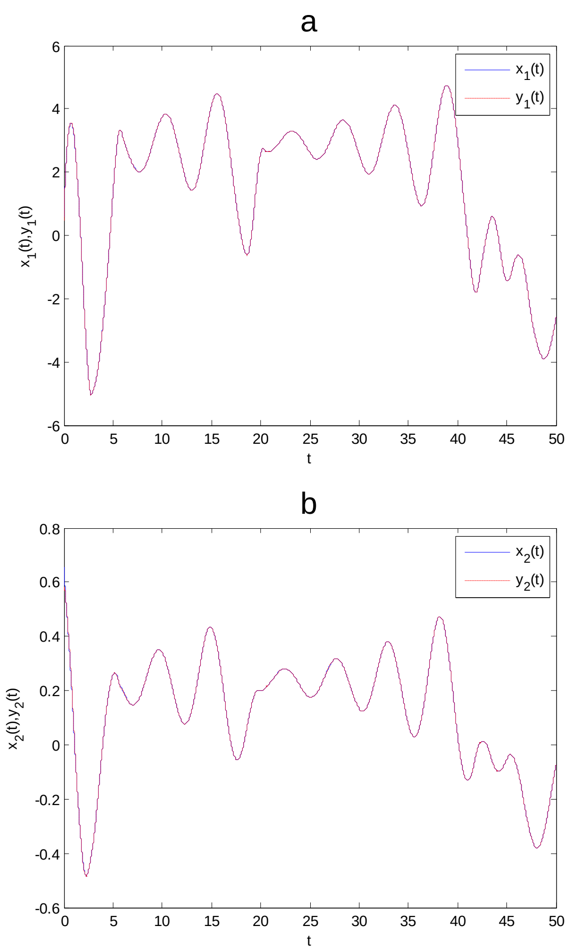

Figure 2. Exponential synchronization of state variable with cntroller $(a:x_1(t),y_1(t),b:x_2(t),y_2(t))$

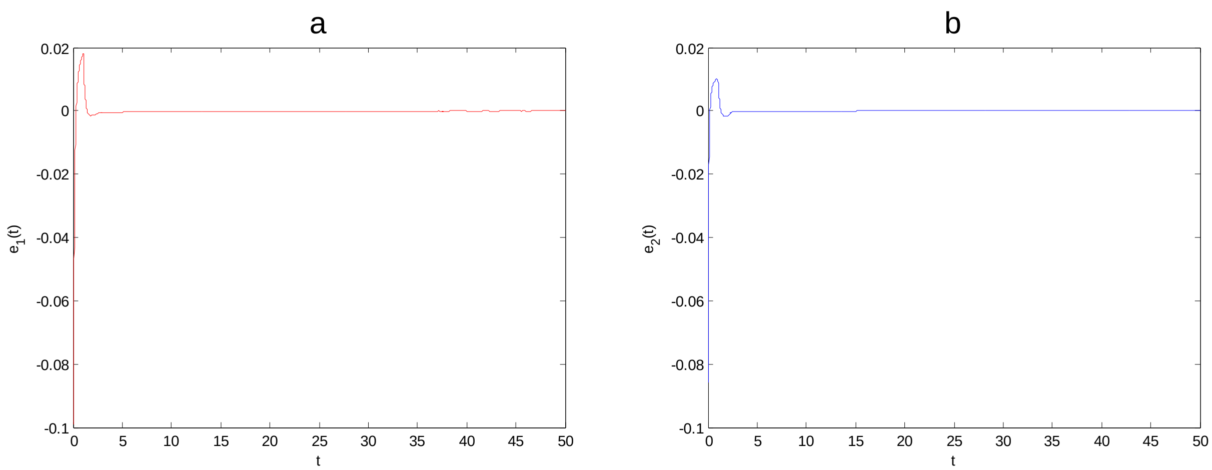

Figure 3. Synchronization error between the drive and response system $(a:e_1(t),b:e_2(t))$

Example2 Consider three-dimension fractional-order memristor-based neural networks

$$\begin{cases} D^q x_1(t) = -c_1 x_1(t) + a_{11}(x_1(t)) f_1(x_1(t)) + a_{12}(x_2(t)) f_2(x_2(t)) + a_{13}(x_3(t)) f_3(x_3(t)) \\ +b_{11}(x_1(t-\tau_1(t))) g_1(x_1(t-\tau_1(t))) + b_{12}(x_2(t-\tau_2(t))) g_2(x_2(t-\tau_2(t))) \\ +b_{13}(x_3(t-\tau_3(t))) g_3(x_3(t-\tau_3(t))) + I_1 \\ D^q x_2(t) = -c_2 x_2(t) + a_{21}(x_1(t)) f_1(x_1(t)) + a_{22}(x_2(t)) f_2(x_2(t)) + a_{23}(x_3(t)) f_3(x_3(t)) \\ +b_{21}(x_1(t-\tau_1(t))) g_1(x_1(t-\tau_1(t))) + b_{22}(x_2(t-\tau_2(t))) g_2(x_2(t-\tau_2(t))) \\ +b_{23}(x_3(t-\tau_3(t))) g_3(x_3(t-\tau_3(t))) + I_2 \\ D^q x_3(t) = -c_3 x_3(t) + a_{31}(x_1(t)) f_1(x_1(t)) + a_{32}(x_2(t)) f_2(x_2(t)) + a_{33}(x_3(t)) f_3(x_3(t)) \\ +b_{31}(x_1(t-\tau_1(t))) g_1(x_1(t-\tau_1(t))) + b_{32}(x_2(t-\tau_2(t))) g_2(x_2(t-\tau_2(t))) \\ +b_{33}(x_3(t-\tau_3(t))) g_3(x_3(t-\tau_3(t))) + I_3 \end{cases} \tag{22}$$

where $c_1 = c_2 = c_3 = 1,$

$$a_{11}(x_1(t)) = \begin{cases} 1, & x_1(t) \le 0, \\ -1, & x_1(t) > 0, \end{cases} \quad a_{21}(x_1(t)) = \begin{cases} 1, & x_1(t) \le 0, \\ -1, & x_1(t) > 0, \end{cases} \quad a_{31}(x_1(t)) = \begin{cases} 1, & x_1(t) \le 0, \\ -1, & x_1(t) > 0, \end{cases}$$

$$a_{12}(x_2(t)) = \begin{cases} -1, & x_2(t) \le 0, \\ 1, & x_2(t) > 0, \end{cases} \quad a_{22}(x_2(t)) = \begin{cases} 1, & x_2(t) \le 0, \\ -1, & x_2(t) > 0, \end{cases} \quad a_{32}(x_2(t)) = \begin{cases} -1, & x_2(t) \le 0, \\ 1, & x_2(t) > 0, \end{cases}$$

$$a_{13}(x_3(t)) = \begin{cases} -1, & x_3(t) \le 0, \\ 1, & x_3(t) > 0, \end{cases} \quad a_{23}(x_3(t)) = \begin{cases} 1, & x_3(t) \le 0, \\ -1, & x_3(t) > 0, \end{cases} \quad a_{33}(x_3(t)) = \begin{cases} 1, & x_3(t) \le 0, \\ -1, & x_3(t) > 0, \end{cases}$$

$$b_{11}\left(x_1\left(t-\tau_1\left(t\right)\right)\right)=\begin{cases}1, & x_1\left(t-\tau_1\left(t\right)\right)\le 0,\\ -1, & x_1\left(t-\tau_1\left(t\right)\right)>0,\end{cases}\qquad b_{12}\left(x_2\left(t-\tau_2\left(t\right)\right)\right)=\begin{cases}-1, & x_2\left(t-\tau_2\left(t\right)\right)\le 0,\\ 1, & x_2\left(t-\tau_2\left(t\right)\right)>0,\end{cases}$$

$$b_{21}\left(x_1\left(t-\tau_1\left(t\right)\right)\right)=\begin{cases}1, & x_1\left(t-\tau_1\left(t\right)\right)\le 0,\\ -1, & x_1\left(t-\tau_1\left(t\right)\right)>0,\end{cases}\qquad b_{22}\left(x_2\left(t-\tau_2\left(t\right)\right)\right)=\begin{cases}1, & x_2\left(t-\tau_2\left(t\right)\right)\le 0,\\ -1, & x_2\left(t-\tau_2\left(t\right)\right)>0,\end{cases}$$

$$b_{31}\left(x_2\left(t-\tau_2\left(t\right)\right)\right)=\begin{cases}1, & x_2\left(t-\tau_2\left(t\right)\right)\le 0,\\ -1, & x_2\left(t-\tau_2\left(t\right)\right)>0,\end{cases}\qquad b_{32}\left(x_2\left(t-\tau_2\left(t\right)\right)\right)=\begin{cases}-1, & x_2\left(t-\tau_2\left(t\right)\right)\le 0,\\ 1, & x_2\left(t-\tau_2\left(t\right)\right)>0,\end{cases}$$

$$b_{13}\left(x_3\left(t-\tau_3\left(t\right)\right)\right)=\begin{cases}-1, & x_3\left(t-\tau_3\left(t\right)\right)\le 0,\\ 1, & x_3\left(t-\tau_3\left(t\right)\right)>0,\end{cases}$$

$$b_{23}\left(x_3\left(t-\tau_3\left(t\right)\right)\right)=\begin{cases}1, & x_3\left(t-\tau_3\left(t\right)\right)\le 0,\\ -1, & x_3\left(t-\tau_3\left(t\right)\right)>0,\end{cases}$$

$$b_{33}\left(x_3\left(t-\tau_3\left(t\right)\right)\right)=\begin{cases}1, & x_3\left(t-\tau_3\left(t\right)\right)\le 0,\\ -1, & x_3\left(t-\tau_3\left(t\right)\right)>0.\end{cases}$$

And $\tau_j\left(t\right)=e^t/1+e^t$, $I=(I_1,I_2,I_3)^T=(0,0,0)^T$ $q=0.92$ and take the activation function as $f_i\left(x_i\right)=g_i\left(x_i\right)=\tanh(x_i), i=1,2,3$. We consider system(22) as the drive system and the corresponding response system is defined in Eq.(7). And for the controller $u_i\left(t\right)=\omega_i\left(y_i\left(t\right)-x_i\left(t\right)\right)$, ω_i is chosen as $\omega_1=-9.5, \omega_2=-10.5, \omega_3=-11$. From Theorem1, we take $\varepsilon=0.7, \tau_j\left(t\right)=1$ and choose $\eta_1=\eta_2=0.1$ $\rho_1=\rho_2=\sigma_1=\sigma_2=0.1$. According to $A_{ij}=\max\left\{\left|\hat{a}_{ij}\right|,\left|\breve{a}_{ij}\right|\right\}, B_{ij}=\max\left\{\left|\hat{b}_{ij}\right|,\left|\breve{b}_{ij}\right|\right\}$ $i,j=1,2,3$ $A_{ij}=B_{ij}=1,$ we can easily know

$$(-c_i+\omega_i+\varepsilon)\eta_i+\sum_{j=1}^{n}A_{ij}\sigma_j\eta_j+\sum_{j=1}^{n}B_{ij}\rho_j\eta_j\exp\left\{\varepsilon\tau_j\left(t\right)\right\}<0$$

is true when $\omega_i<-0.604$. So when $\omega_1=-9.5,\ \omega_2=-10.5, \omega_3=-11$ we can get

$$(-c_1+\omega_1+\varepsilon)\eta_1+A_{11}\sigma_1\eta_1+A_{12}\sigma_2\eta_2+A_{13}\sigma_3\eta_3+\left(B_{11}\rho_1\eta_1+B_{12}\rho_2\eta_2+B_{13}\rho_3\eta_3\right)\exp\left\{\varepsilon\tau_j\left(t\right)\right\}=-0.89<0,$$

$$(-c_2+\omega_2+\varepsilon)\eta_1+A_{21}\sigma_1\eta_1+A_{22}\sigma_2\eta_2+A_{23}\sigma_3\eta_3+\left(B_{21}\rho_1\eta_1+B_{22}\rho_2\eta_2+B_{23}\rho_3\eta_3\right)\exp\left\{\varepsilon\tau_j\left(t\right)\right\}=-0.99<0,$$

$$(-c_3+\omega_3+\varepsilon)\eta_1+A_{31}\sigma_1\eta_1+A_{32}\sigma_2\eta_2+A_{33}\sigma_3\eta_3+\left(B_{31}\rho_1\eta_1+B_{32}\rho_2\eta_2+B_{33}\rho_3\eta_3\right)\exp\left\{\varepsilon\tau_j\left(t\right)\right\}=-1.04<0.$$

It suggests the condition of Theorem 1 is satisfied, then drive-response system achieves the synchronization.

When the response system with this controller, we get state trajectories of variable $x_1\left(t\right), y_1\left(t\right)$ and $x_2\left(t\right), y_2\left(t\right)$

and $x_3(t), y_3(t)$ are depicted in Figure 4a,4b,4c. Moreover, Figure 5a,5b,5c depict the synchronization error curves $e_1(t), e_2(t), e_3(t)$ between the drive system and response system. It's easy to see that the state trajectories of variable $x_1(t), y_1(t)$, $x_2(t), y_2(t)$ and $x_3(t), y_3(t)$ are synchronous and synchronization error $e_1(t), e_2(t), e_3(t)$ are converge to zero. So the Theorem1 is proved to be correct.

In addition, we choose $\omega_1 = -9.5$, $\omega_2 = -10.5, \omega_3 = -11$, according to the Theorem1, it needs the following inequalities to hold:

$$\begin{cases} \tau < \dfrac{1}{\varepsilon} \ln\left(\dfrac{117}{3} - \dfrac{10}{3} \varepsilon \right) \\ \tau < \dfrac{1}{\varepsilon} \ln\left(\dfrac{112}{3} - \dfrac{10}{3} \varepsilon \right) \\ \tau < \dfrac{1}{\varepsilon} \ln\left(\dfrac{102}{3} - \dfrac{10}{3} \varepsilon \right) \end{cases}$$

So, we just need $\tau < \frac{1}{\varepsilon} \ln\left(\frac{102}{3} - \frac{10}{3} \varepsilon \right)$ holds. We have the exponential convergence rate $0 < \varepsilon < 1$, figure 6 depicts the relation of time-varying delay τ and exponential convergence rate ε.

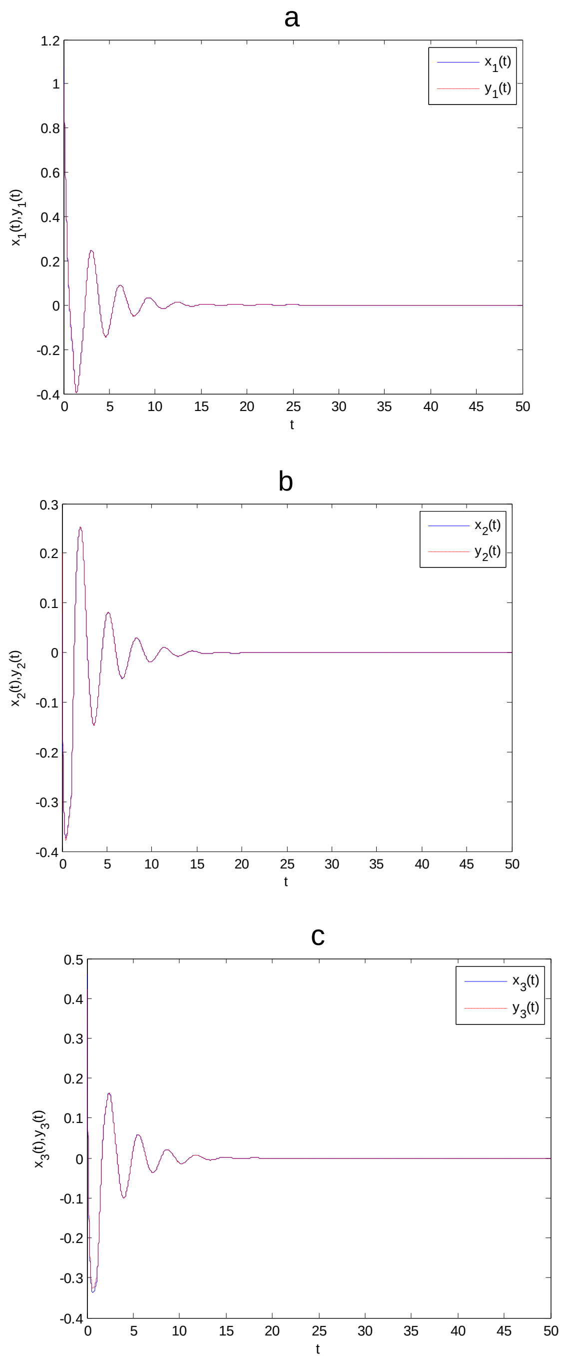

Figure 4. Synchronization of state variable with controller $\left(a: x_1(t), y_1(t), b: x_2(t), y_2(t), c: x_3(t), y_3(t)\right)$

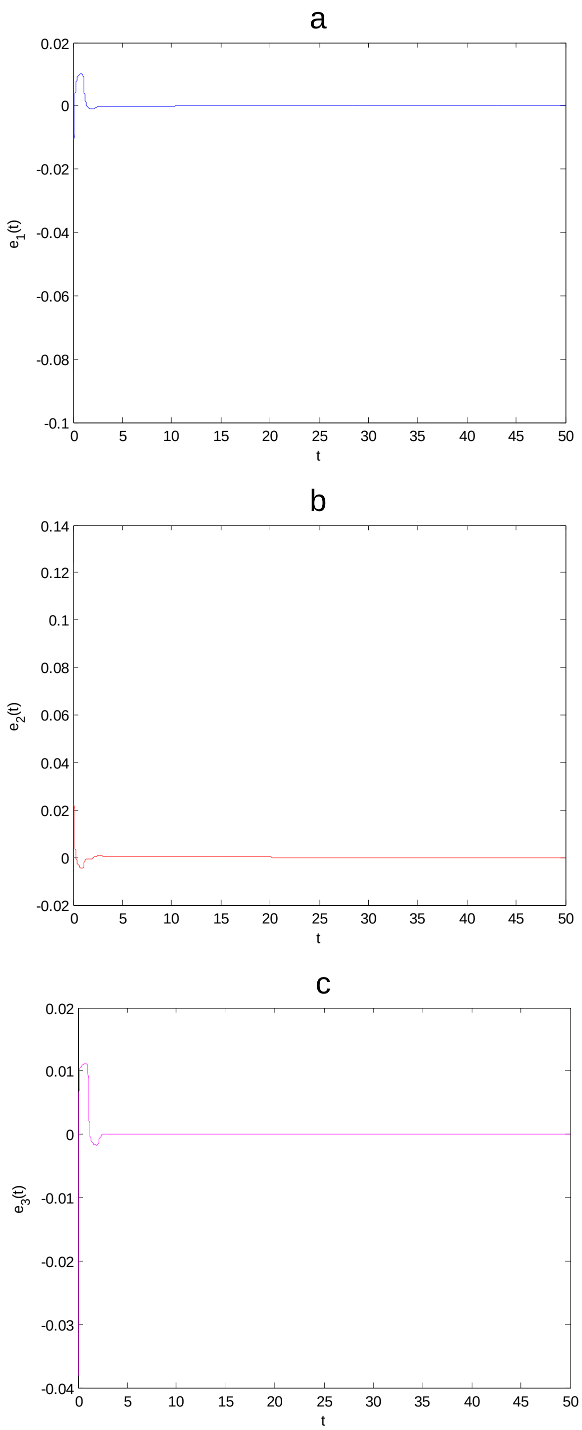

Figure 5. Synchronization error between the drive and response system $\left(a:e_1(t),b:e_2(t),c:e_3(t)\right)$

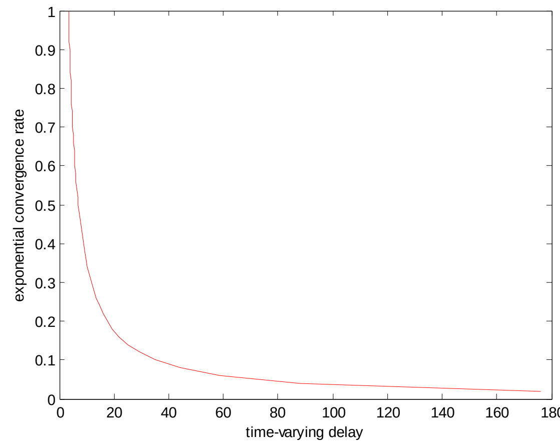

Figure 6. The relation of time-varying delay τ and exponential convergence rate ε .

V. CONCLUSION

This paper achieves the exponential synchronization of a class of FMNN with time-varying delays by using linear error feedback controller. Based on comparison principle, the new theorem is derived to guarantee the exponential synchronization between the drive system and response system. The methods proposed for synchronization is effective and it is easy to achieve than other complex control methods. Moreover, it can be extended to investigate other dynamical behaviors of fractional-order memristive neural networks, such as realizing the lag synchronization or anti-synchronizaton of this system based on the suitable controller. These issues will be the topic of future research. Finally, numerical examples are given to illustrate the effectiveness of the proposed theory.

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