A Mix Model of Discounted Cash-Flow and OWA Operators for Strategic Valuation

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Abstract **— The stock market volatility and the actual stock Exchange activity have increased the need of counting with effective methods on the part of financial analysts to achieve a division in relation to the investment actions, being also growing the demand of methodological instruments that reduce and minimize the risks and uncertainty when valuating financial actives and companies. These systems not only must use quantitative information but the inclusion of qualitative information must also bear heavily on them, as an improvement element in the adjustment of these valuating methods, with the aim of throwing a more well-conceived or less mistaken decision.**

In this work, the use of Discounted Cash-Flow model is proposed, with quantitative information together with the OWA operators as an inclusion method of qualitative information in the traditional valuating models, with the aim of generating an strategic valuating system which allows to develop more agreed and less mistaken valuations.

Keywords **— Discounted Cash-Flow, OWA operators, Linguistic Information, Strategic Valuation.**

XII. Introduction

Nowadays, the success of the stock exchange activity as well as actives valuations into the business market mostly depend on the capacity of anticipating to the stock market trends and the achievement of a quick reply. Managers must assimilate the information and adopt the decisions in a chaotic environment, provided with risk and uncertainty, most of times without counting with experience and an adequate planning, and even without having enough time to carry out an strict and systematic analysis (Besoun, 2004; Cross & Brodt, 2006). All these circumstances lay the reasons why the knowledge of the recent decision techniques have a special outstanding into the business and stock market environment. Generally it is not feasible to establish in all these processes and standard criterion of decision which could be useful in any case, given that every operation is ´different in itself` from the rest, motivated by the several circumstances and risk elements. That´s the reason why the end in a decision system, within this field, is banded to a negotiation process or consensus between both interested parts with its aim.

In many cases, such mechanisms have being established by the different opinions applied to the experts, who facilitate a series of valuations which allow the obtaining of a final value of satisfactory decision. In these conditions, it is necessary to enable different mechanisms which allow to generate representative results of the group and operate with the risk produced, related to the uncertainty of the opinions expressed by the decision-makers, that most of the time, will be defined in a qualitative ways (Kaufmann & Gil, 1986).

In the strategic valuation it is pretended to determine an interval of reasonable values in which the definitive value of the considered element will be included. For instance, when valuating a company the aim is to obtain an estimation which may never be a unique or exact number due to the difficulties belongings to the decision process. However this will depend on the company situation, the transaction moment and the method we use. To determine the right valuation it is necessary to establish hypothesis and future uncertainty scenes due to the possibility of event in relation to the risk elements inherent to the event scene. These hypotheses are involved in a risk and uncertainty universe, so that the final result will be an interval or series of values, and not only one of them. Finally, the information derived from the valuation report developed by the experts will mean the base in the parties' negotiation, from which the definitive transaction price will arise.

The valuation methods use future estimations which, in many cases, are being giving out by experts according to their experience or reality perception, what means an added risk. In these conditions, it is necessary the disposition of several instruments which allow to operate with the

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uncertainty or risk of the expressed opinions, which normally will be defined in linguistic values in different ways of expression. It is also necessary that these instruments should be able to add the opinions in a representative value of them.

A new strategic valuation model is represented in this work, not only to a business manager level but also to the agents and stock market investors, offering to the eloquent a new instrument based on operators of aggregate OWA, with the aim to provide a better quality decision in a context with lack of information and with the need of taking it with celerity, permitting our decision be as correct as possible.

The article structure is the following: In the next section the valuating model ´Discounted Cash-Flow` will be introduced; in the third section it is shown the information using linguistic labels with the two tuplas model and the proposed aggregate operator OWA; in the fourth section the new model of strategic valuation will be presented, developing a detailed example of application, and finally, conclusions will be shown.

XIII. Strategic Valuation. Discounted Cash-Flow

One of the most important sides of the modern financial theory is the one referred to the strategic valuation of enterprises. The demand for adequated methods when valuating actins and enterprises is increasing. The role which has been played by the fusion processes and the acquisition in the actual strategy of business is requiring adequate financial models which allow inferring the potential synergies of all kinds of operations of combinations and/or societies restoration (Ruiz & Gil, 2004).

The value is searching its support in a logical or mathematical basis being as rigorous as possible. It looks for the objectivity, neutrality and independence opposite to the parts, strong relations in the stock market and even the market situation itself. However the need of predicting future scenes in which the own activity is developing, could create the impossibility to determine a specific and certain value, this may origin an interval of possible values within which the most certain and possible value of the enterprise will be found. The definitive value will come by consent and negotiation between the interested parts. As a result, the extent of possible values interval will distinguish the valuation report before the decision.

It is precisely in this point where we want to improve the quality of the available information to the investor, if it is possible to decrease interval extent of the possible values with the methodology proposed, the position of the interested parts will be closer to each other. Being like this, the possibility of agreement to finalize the operation will have increased in a well-balanced consensual price and even minimally negotiated. In this way, a rise in the stock market efficiency and fluidity is produced.

Within the last few years, with the stock markets worldwide extension, the technological development of these ones and the appearance of new financial instruments, have promoted new valuation techniques improving the ones already existing.

This fact has meant a growth not only in the valuating methods and its possible action setting but also in the need to discriminate against which methods are applicable in certain circumstances and the veracity or credibility of the results.

In this report it is used one of the methods which is actually the most accepted in the professional and scientific community, the ´Discounted Cash-Flow` model. The following expression distinguishes it.

$$V_E = \sum_{t=1}^{n} \frac{CFL_t}{\prod_{j=1}^{t} (1+K_j)^t}$$

Where V_E represents the enterprise actual value; *CFL* is the ´Cash Flow` free from the enterprise for the period – *t* (including the residual value); K_j is the adequate updating valuation and agreed for risk (WACC) to the period – *j* and *n* is the valuation horizon.

XIV. Linguistic Model and OWA Operator

Actually the concept of linguistic variable is widely used in those decision making problems with imprecise assessments given in a linguistic way for some of its elements. Usually, many aspects of different activities cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables.

This approach is adequate in some situations, for example, when attempting to qualify phenomena related to human perception, we are often led to use words in natural language. This may arise for different reasons. There are some situations where the information may not be quantified due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating financial situations terms like “bad”, “poor”, “tolerable”, “average”, “good” can be used). In other cases, precise quantitative information may not be stated because either it is not available or the cost of its computation is too high, then an “approximate value” may be tolerated (e.g., when evaluating the cost of a infrastructure, terms like “expensive”, “very expensive”, “cheap” are used instead of numerical values). The fuzzy linguistic approach has been applied with very good results in different problems, such as, information retrieval, decision-making, etc.

This linguistic information model used to define the proposed valuation system is designated 2 - tuple fuzzy linguistic defined in ´Herrera & Martinez`, 2000. This model presents the advantage of permitting to equalize the information expressed by the experts in different properties without lost of information.

From this concept, in "Herrera & Martinez" (2000) is developed a linguistic representation model which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in$ [-0.5, 0.5). r_i represents the linguistics label center of the information and α_i is a numerical value that represents the translation from the original result β to the closest index label in the linguistic term set (r_i), i.e., the Symbolic Translation.

This linguistic representation model defines a set of functions to make transformations among linguistic terms, 2-tuples and numerical values.

Definition. Let $s_i \in S$ be a linguistic term, them its equivalent 2-tuple representation is obtained by means of the function θ as:

$$\theta : S \rightarrow \left(S\ x\left[-0.5, 0.5\right)\right), \qquad \theta(s_i) = (s_i, 0) / s_i \in S$$

Definition . Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \rightarrow Sx\left(\left[-0.5, 0.5\right)\right),$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in \left[-0.5, 0.5\right) \end{cases}$$

where round is the usual operation, s_i has the closest index label to β and $\propto$ is the value of the symbolic translation.

Definition. Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set and (s_i,$\propto$) be a linguistic 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0,g]$.

$$\Delta^{-1} : Sx\left[-0.5, 0.5\right) \rightarrow \left[0, g\right],\ \Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

To carry out the aggregate stage of the linguistic information produced in the valuation process, the use of operators OWA is proposed, mainly because since its definitions they have been shown as one of the most effective option to choose when taking a decision in group (Herrera et al. ,1996; Pasi & Yager, 2006; Peláez & Doña, 2006; Llamazares, 2007), not only for the satisfied specific properties (Yager, 1988; Liu, 2006; Amin 2007) but also for the possibility of representing blurred concepts as the majority through the aggregate semantic of operators and its combination with linguistic quantifiers (Pasi & Yager 2006; Peláez & Doña 2006).

The OWA operator used in this work is the LAMA (Peláez & Doña, 2003b), due to this operator is adequate to synthesize linguistic information in decision making environments producing aggregated results with a majority semantic (Peláez et al., 2007).

The LAMA operator is based in most of the process (Peláez & Doña 2003a) and is a mapping function $F: R^n \rightarrow R$ that has associated a weighting vector $W = \left[w_1, w_2, \dots, w_n\right]^T$ where $w_i \in \left[0,1\right]$ y $\sum_{i=1}^{n} w_i = 1$.

$$LAMA(a_1, a_2, \dots, a_n) = b_1 \otimes w_1 \oplus b_2 \otimes w_2 \oplus \dots \oplus b_n \otimes w_n$$

with b_j being the j^{th} largest element of the a_i, and $\oplus$ is the sum of labels and $\otimes$ is the product of a label by a positive real defined in (Herrera & Martinez, 2000).

The weights used in the LAMA operator are usually calculated from majority process (Peláez & Doña 2003a) as follow:

Let δ_i the cardinality for the element i with $\delta_i > 0$, then.

$$w_i = f_i(b_1, \dots, b_n) = \frac{\gamma_i^{\delta_{\min}}}{\theta_{\delta_{\max}} \cdot \theta_{\delta_{\max}-1} \cdot \dots \cdot \theta_{\delta_{\min}+1} \cdot \theta_{\delta_{\min}}} + \frac{\gamma_i^{\delta_{\min}+1}}{\theta_{\delta_{\max}} \cdot \theta_{\delta_{\max}-1} \cdot \dots \cdot \theta_{\delta_{\min}+1}} + \dots + \frac{\gamma_i^{\delta_{\max}}}{\theta_{\delta_{\max}}}$$

where

$$\gamma_i^k = \begin{cases} 1 & if\ \ \delta_i \geq k \\ 0 & otherwise \end{cases}$$

and

$$\theta_i = \begin{cases} (number\ of\ item\ with\ cardinality \geq i) + 1 & if\ \ i \neq \delta_{\min} \\ number\ of\ item\ with\ cardinality \geq i & otherwise \end{cases}$$

The majority operators aggregate in function of δ_i that generally represents the importance of the element i using its cardinality. Most of the processes are considered the formation of discussion of majority groups depending on similarities or distances among the experts' opinions. All values with a minimum of separation are considered inside the same group. The calculation method for the value δ_i is independent from the definition of most of the operators.

XV. Strategic Valuation. Proposed Model

Firstly, we take the estimation of the updating valuation appropriate and agreed to the risk, which is usually the balance of average cost of capital (WACC). We should start with an analysis which considers every possible section among those we expected a valuated fluctuation to the periods which are considered in the research, in order to be a start point in the decision process between the parts which are taking over such process. In the following example, it has been established an analysis period of three years, and it has been considered the following intervals for the interest rate:

Table IV.1. Intervals for the Updating valuations

Updating valuations

Year 1	Year 2	Year 3
[0,04; 0,05]	[0,045; 0,06]	[0,05; 0,06]

Next we ask, for instance, ten experts who express their valuations about the intervals, making use of the following linguistic group: S = { S_8 (practically sure), S_7 (very high), S_6 (high), S_5 (little high), S_4 (medium), S_3 (little low), S_2 (low), S_1 (very low), S_0 (practically low)}.

Table IV.2. Linguistic values. Valuate fluctuation. Expressed values by the experts

	[0,04 ; 0,05]	[0,045 ; 0,06]	[0,05 ; 0,06]
e_1	$(S_6,0)$ - $(S_8,0)$	$(S_8,0)$	$(S_2,0)$ - $(S_4,0)$
e_2	$(S_5,0.33)$ - $(S_7,-0.37)$	$(S_5,0.33)$ - $(S_8,0)$	$(S_0,0)$ - $(S_4,0)$
e_3	$(S_6,0)$	$(S_8,0)$	$(S_0,0)$ - $(S_3,0)$
e_4	$(S_5,0)$	$(S_7,0)$ - $(S_8,0)$	$(S_2,0)$ - $(S_4,0)$
e_5	$(S_6,0)$ - $(S_8,0)$	$(S_6,0)$ - $(S_8,0)$	$(S_2,0)$
e_6	$(S_8,0)$	$(S_8,0)$	$(S_0,0)$ - $(S_2,0)$
e_7	$(S_8,0)$	$(S_5,0)$ - $(S_7,0)$	$(S_2,0)$ - $(S_4,0)$
e_8	$(S_5,0)$ - $(S_7,0)$	(S5,0) - $(S_6,0)$	$(S_5,0)$ - $(S_7,0)$
e_9	$(S_8,0)$	$(S_1,0.33)$ - $(S_3,-0.33)$	$(S_7,-0.37)$ - $(S_8,0)$
e_{10}	$(S_0,0)$ - $(S_1,0)$	$(S_5,0)$ - $(S_6,0)$	$(S_6,0)$ - $(S_8,0)$

Immediately after, we proceed to obtain an agent for each interval. In this way it is applied most of the linguistic operator afore defined, with the aim of obtaining a value which represents the whole collection of opinions made by the different experts in a majority way.

Extreme [0,04]

$$\phi = (S_8,0)\otimes 0.433 \oplus (S_6,0)\otimes 0.433 \oplus (S_5,0)\otimes 0.1 \oplus (S_5,0.33)\otimes 0.017 \oplus$$

$$\oplus (S_0,0)\otimes 0.017 = (S_7,-0.35)$$

Extreme [0,05]

$$\phi = (S_8,0)\otimes 0.947 \oplus (S_7,-0.33)\otimes 0.0106 \oplus (S_7,0)\otimes 0.0106 \oplus$$

$$\oplus (S_6,0)\otimes 0.0106 \oplus$$

$$\oplus (S_5,0)\otimes 0.0106 \oplus (S_1,0)\otimes 0.0106 = (S_8,-0.15)$$

Future valuation for year 1:

$$i_1 = [0{,}04] + (0{,}01)(\cdot)[0{,}738;0{,}872] = [0.04738;0{,}04872]$$

Future valuation for year 2:

$$i_2 = [0{,}045] + (0{,}015)(\cdot)[0{,}6172;0{,}8577] = [0{,}05426;0{,}05787]$$

Future valuation for year 3:

$$i_3 = [0{,}05] + (0{,}01)(\cdot)[0{,}1831;0{,}4362] = [0{,}05183;0{,}05436]$$

The following step needs to establish some values which customers and sellers are agree with according to the possible Cash Flows free to obtain in the considered periods. In order to get it, firstly we start with intervals to qualify the CFL which will be useful as a reference to apply for the opinion of the experts at such content. These must be established not only for the customers' part, but also for the seller´s one. To operative effects of the practical decision it has been established the following intervals indicating the possible CFL in financial units for the three analysis periods: year 1 [4.000; 6.000]; year 2 [3.000; 6.000]; year 3 [2.000; 5.000].

From the previous valuations, it is possible to apply for the cooperation of experts when expressing their opinions through linguistic valuations taking customers and seller positions.

Table IV.3. Linguistic Valuations; Cash Flow Free

	[4.000 ; 6000]	[3.000 ; 6000]	[2.000 ; 5.000]
Customer			
e_1	$(S_4,0)$ - $(S_6,0)$	$(S_2,0)$ - $(S_4,0)$	$(S_2,0)$ - $(S_4,0)$
e_2	$(S_3,-0.33)$ - $(S_4,0)$	$(S_4,0)$ - $(S_7,-0.33)$	$(S_0,0)$ - $(S_4,0)$
e_3	$(S_5,0)$	$(S_1,0)$ - $(S_2,0)$	$(S_0,0)$ - $(S_3,0)$
e_4	$(S_5,0)$	$(S_5,0)$ - $(S_6,0)$	$(S_2,0)$ - $(S_4,0)$
e_5	$(S_2,0)$ - $(S_4,0)$	$(S_4,0)$ - $(S_6,0)$	$(S_2,0)$
Seller			
e_1	$(S_7,-0.33)$ - $(S_8,0)$	$(S_4,0)$ - $(S_5,0.33)$	$(S_5,0.33)$ - $(S_7,-0.33)$
e_2	$(S_5,0)$ - $(S_6,0)$	$(S_7,0)$ - $(S_8,0)$	$(S_3,0)$ - $(S_4,0)$
e_3	$(S_4,0)$ - $(S_6,0)$	$(S_6,0)$	$(S_2,0)$ - $(S_4,0)$
e_4	$(S_2,0)$ - $(S_6,0)$	$(S_2,0)$ - $(S_4,0)$	$(S_2,0)$ - $(S_6,0)$
e_5	$(S_5,0)$ - $(S_6,0)$	$(S_7,0)$ - $(S_8,0)$	$(S_6,0)$ - $(S_7,0)$

Then, the unified information will be aggregated being used again the last operator OWA. In order not to reaffirming the calculus, we only develop the operations bellowing to the first period for the customers.

Extreme [4.000]

$$\phi = (S_5,0)\otimes 0.625 \oplus (S_4,0)\otimes 0.125 \oplus (S_2,0)\otimes 0.125 \oplus$$

$$\oplus(S_1, 0.33)\otimes 0.125 = (S_4, 0.04)$$

Extreme [6.000]

$$\phi = (S_5, 0)\otimes 0.625 \oplus (S_4, 0)\otimes 0.125 \oplus (S_2, 0)\otimes 0.125 \oplus$$

$$\oplus(S_1, 0.33)\otimes 0.125 = (S_4, 0.04)$$

CFL Seller

$$CFL_1^C = [4.000] + (2.000)(\cdot)[0{,}4490; 0{,}5185] = [4.898; 5.037]$$

To the customers we obtain:

CFL Seller

$$CFL_1^V = [4.000] + (2.000)(\cdot)[0{,}4768; 0{,}6805] = [4.953; 5.361]$$

And for the remaining intervals:

Interval CFL Customer - Seller [3000, 6000]

$$CFL_2^C = [3.000] + (3.000)(\cdot)[0{,}388; 0{,}6018] = [4.164; 4.805]$$

$$CFL_2^V = [3.000] + (3.000)(\cdot)[0{,}6527; 0{,}7688] = [4.958; 5.306]$$

Interval CFL Customer - Seller [2000, 5000]

$$CFL_3^C = [2.000] + (3.000)(\cdot)[0{,}1666; 0{,}4166] = [2.500; 3.249]$$

$$CFL_3^V = [2.000] + (3.000)(\cdot)[0{,}3379; 0{,}5601] = [3.013; 3.680]$$

In the table IV is presented a calculus summary.

Table IV. 4. Calculus Summary

Updating valuations	Year-1	
Interval-K	0,040	0,050
Interval-ϕ	0,738	0,872
Interval-$K_{adjusted}$	0,04738	0,04872
Cash-Flow Free	Year-1	
Interval-CFL	4.000	6.000
Interval-ϕ	0,4490	0,6805
Interval-$CFL_{adjusted}$	4.898,00	5.361,00
Updating valuations	Year-2	
Interval-K	0,045	0,060
Interval-ϕ	0,617	0,858
Interval-$K_{adjusted}$	0,05426	0,05787
Cash-Flow Free	Year-2	
Interval-CFL	3.000	6.000
Interval-ϕ	0,3880	0,7688
Interval-$CFL_{adjusted}$	4.164,00	5.306,40
Updating valuations	Year-3	
Interval-K	0,050	0,060
Interval-ϕ	0,183	0,436
Interval-$K_{adjusted}$	0,05183	0,05436
Cash-Flow Free	Year-3	
Interval-CFL	2.000	5.000
Interval-ϕ	0,1666	0,5601
Interval-$CFL_{adjusted}$	2.499,80	3.680,30

We notice how using the majority operator we get to reduce the interval of variable values considered in the valuation (table 5), which leads us to consider that the enterprise value derived from them will equally present a more reduced interval than if we do not use such operators.

Table IV.5. Range of the interval

Updating valuation	Year-1	Year-2	Year-3
Range of Interval-K	0,010	0,015	0,010
Range of Interval -$K_{adjusted}$	0,00134	0,00361	0,00253
Cash-Flow Free	Year-1	Year-2	Year-3
Interval-CFL	2.000,0	3.000,0	3.000,0
Range of Interval –$CFL_{adjusted}$	463,00	1.142,4	1.180,5

In fact, it is shown in the table 6 the comparative calculus of the two related versions, proving that the use of the majority operators OWA reduces the interval of the positive estimated values in a considerable form, reaching like this our targets. In our example, if we apply directly to the first information the classic expression of ´Discounted Cash-Flow` we will obtain the following interval [8.971; 17.122] with a breadth of 8.151, whereas if we consider the information in the form proposed by the majority operators OWA we will thus obtain a more appreciably narrow interval, that is [11.514; 14.379] of breadth 2.864. The reduction of the range interval is due to the increase of the inferior extreme and the decrease of the higher one.

Table IV.6. Comparative results

Valuation	Interval of Values		Range
$V_{E(CFL)}$	8.971	17.122	8.151
$V_{E(adjusted)}$	11.514	14.379	2.864

XVI. Conclusions

In this report it has been presented a new strategic valuation system based on the ´Discounted Cash-Flow´ model, aggregate operators OWA and linguistic information. Due to the importance that valuation process is representative of the main part of the estimation done by the experts, it has been used the majority operator LAMA extended to the linguistic representation of 2-tuple, which allows to work with a manifold information in the attaching process.

The use of majority operators OWA in uncertainty contexts, risk and where consent estimations are based on subjective opinions (training and experience) of the stock market agents may produce a greatest ease to come to a value of consent or balance in a quick and objective way. These linguistic operators summarize the first information, allowing an attitude approach which provides the achievement of a consent value or the fast attainment of a balance price in the stock market.

Furthermore, at the same time they generate possibilities of arbitrage and a volatility reduction. In other words, they generate quality information providing efficient decisions, which produces a greatest market efficiency and fluidity.

Finally, we would like to indicate that the proposed methodology is absolutely flexible and adaptable to whichever decision stage both on business and stock market, then allowing having the valuation weights by means of a previous calculus, being used again in other valuation processes, making the method application almost immediate. This is not possible with the traditional methods.

REFERENCES

[1] G. R. Amin. Notes on properties of the OWA weights determination mode. *Computers & Industrial Engineering* 52. 533–538. 2007.

[2] J.-J. Besoun. El nuevo reto del director financiero: riesgo y beneficio. *Revista Estrategia Financiera*. Anuario 2004. 2004.

[3] R. L. Cross, S. E. Brodt. El valor del juicio intuitivo en la toma de decisiones. Expansión/Harvard Deusto. *Dirigir en la Incertidumbre*, pp. 153-176. 2006.

[4] F. Herrera, E. Herrera-Viedma, J. I. Verdegay. Direct approach processes in group decision making using linguistic OWA operators. *Fuzzy Set and Systems 79*, 175-190. 1996.

[5] F. Herrera, L. Martínez. A 2-tuple fuzzy linguistic representation on model for computing with words. *IEEE Transactions on Fuzzy Systems 8:6*, 746-752. 2000.

[6] A. Kaufmann, J. Gil Aluja. *Introducción de la teoría de los subconjuntos borrosos en la gestión de empresas*. Ed. Milladoiro, Santiago de Compostela. 1986.

[7] B. Llamazares. Choosing OWA operator weights in the field of Social Choice. *Information Sciences: an International Journal*. 177, 21, 4745-4756. 2007.

[8] X. W. Liu. Some properties of the weighted OWA operator. *IEEE Transactions on Systems, Man and Cybernetics*, 36, 1, 118-127. 2006.

[9] G. Pasi & R. Yager. Modeling the Concept of Majority Opinion in Group Decision Making. Recent Advancements of Fuzzy Sets: Theory and Practice. *Information Sciences*, 176, 4, 390-414. 2006.

[10] J. I. Peláez, J. M. Doña. LAMA: A Linguistic Aggregation of Majority Additive Operator. *International Journal of Intelligent Systems*. 2003.

[11] J. I. Peláez, J. M. Doña. Majority Additive-Ordered Weighting Averaging: A New Neat Ordered Weighting Averaging Operators Based on the Majority Process. *International Journal of Intelligent Systems*, 18, 4, 469-481. 2003.

[12] J. I. Peláez, J. M. Doña. A Majority Model in Group Decision Making Using QMA-OWA Operators. *International Journal of Intelligent Systems*, 193-208, 2006.

[13] J. I. Peláez, J. M. Doña, J. A. Gómez-Ruiz. Analysis of OWA Operators in Decision Making for Modelling the Majority Concept. *Applied Mathematics and Computation*. 2007.

[14] R. J. Ruiz, A. M. Gil. *El valor de la empresa*. Instituto superior de técnicas y prácticas bancarias. Madrid. 2004.

[15] R. Yager. On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Trans. On Systems, Man and Cybernetics 18*. 183-190. 1988.