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Effective Teaching for Place Value Understanding: A Case Study of a Literacy-Integrated Math Curriculum Module

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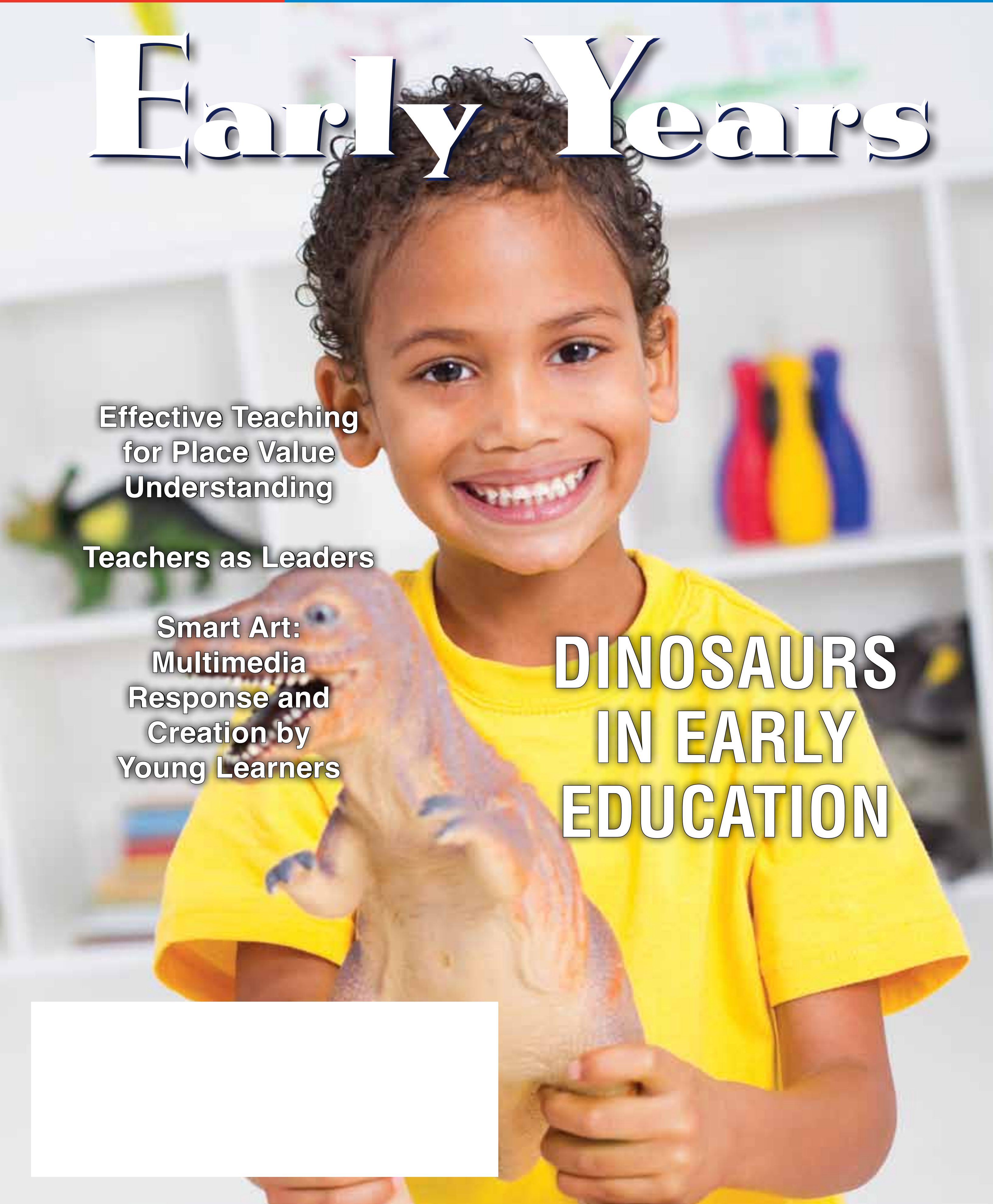
Early Years



Effective Teaching for Place Value Understanding

Teachers as Leaders

Smart Art: Multimedia Response and Creation by Young Learners

DINOSAURS IN EARLY EDUCATION

PROMOTING EQUITY IN THE EARLY LEARNING ENVIRONMENT

UNDERSTANDING
PLACE VALUE

Effective Teaching for Place Value Understanding

A case study of a literacy-integrated math curriculum module

By Young Rae Kim & Mi Sun Park

Abstract

A lesson sequence for place value was developed as an early intervention for kindergarteners. The sequence begins with a children's picture book involving language familiar to children and continues with hands-on activities for children to make connections between and within multiple representations of place value. Through engaging in the literacy-integrated math curriculum module, kindergartners deepened their understanding of place value and the base-ten number system, as they were consistently engaged in problem solving and mathematical discourse triggered by their own mathematical thinking, as well as purposeful questions prompted by the teacher.

Keywords: early childhood mathematics education, storytelling, problem solving, multiple representations, place value, base-ten

Developing a conceptual understanding in regard to the principles underlying place value is challenging for primary students. However, early exposure (much earlier than first or second grade) to the foundational knowledge for place value understanding is necessary to prevent student misunderstanding, which may persist until the fourth and fifth grades (Baroody, 1990; Baroody, Lai, & Mix, 2006; Fraivillig, 2017; McGuire & Kinzie, 2013).

A fundamental goal for students in early primary grades is to develop an initial understanding of place value and the base-ten number system (Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000, 2006). The Common Core State Standards for Mathematics (CCSSM) maintain that kindergarten students should be able to at least "compose and decompose numbers from 11 to 19 into ten ones and some further ones" (CCSSI, 2010, p. 12). For example, they should understand that the number 1 in 17 means ten ones, and thus, that the number 17 is composed of ten ones (one ten) and seven further ones.

The current article presents a literacy-integrated math curriculum module—a lesson sequence of a children's picture book and two hands-on activities—to support young children in developing place value understanding. A case study was conducted to explore how kindergarten students developed place value understanding by engaging in the curriculum module.

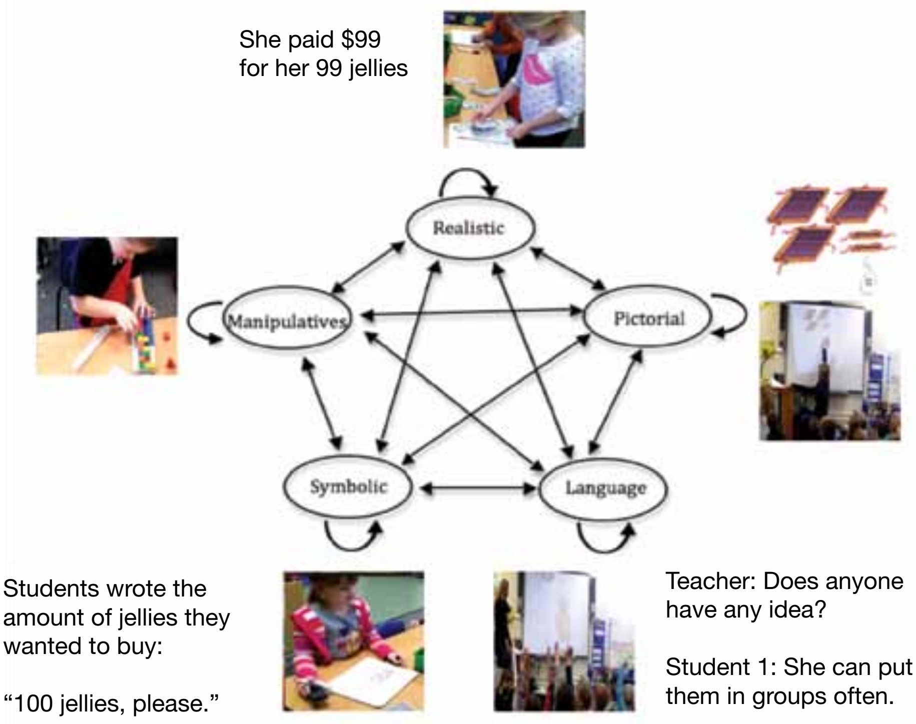


Figure 1. A literacy-integrated math curriculum module designed, based on the Lesh Translation Model.

Background and Framework

Research argues that there has been a lack of intentional instruction for early childhood mathematics education in the United States (Ginsburg, 2009; Hachey, 2013; Li, Chi, DeBey, & Baroody, 2015). In particular, there is insufficient place-value specific instruction and research in the early primary grades (McGuire & Kinzie, 2013), despite the call from national mathematics education standards for young children (grades Pre-K-2) to develop an understanding of place value and the base-ten number system (CCSSI, 2010; NCTM, 2000, 2006). The current article addresses the need for intentional instruction and research on the development of place value understanding in Pre-K.

We chose to develop activities by integrating storytelling and problem solving with multiple representations. We hypothesized that the processes of teaching and learning place value would benefit from integrating storytelling and problem solving with multiple representations. These methods are widely accepted in mathematics education today (Amies, 2002; CCSSI, 2010; NCTM, 2000, 2006, 2014).

The Lesh Translation Model

Specifically, we used the Lesh Translation Model, which is a multiple representations model, to guide our development of a literacy-integrated math curriculum module consisting of a math storybook and two hands-on activities (*see Figure 1*).

The Lesh Translation Model suggests that when students can translate within and between five modes of representation: 1) realistic, 2) pictorial, 3) language (verbal and written), 4) symbolic, and 5) manipulative (through concrete models), the processes deepen their understanding of mathematics concepts and procedures (Cramer, 2003; Lesh & Doerr, 2003).

We incorporated opportunities for children to make connections among the five representations for place value into a literacy-integrated math curriculum module (see Figure 1). Thus, we anticipated that the curriculum module would support children's conceptual understanding of place value. This anticipation was also supported by a recognizable body of research that takes into account the importance of multiple representations to promote the development of place value understanding (Fraivillig, 2017; Fuson & Briars, 1990; Hiebert & Wearne, 1992; Kamii & Joseph, 1988).

Storytelling and Problem Solving

Based on the Lesh Translation Model, our first step in developing an early intervention was to choose effective activities/strategies to help kindergarten students (ages 5-6) develop place value understanding. Storytelling was the first choice. It is an understandable and memorable way to convey mathematical concepts, which also increases children's interest and engagement in mathematics (Butterworth & Lo Cicero, 2001; Egan, 2001; Goral & Gnadinger, 2006; Green, 2004; Zazkis & Liljedahl, 2009). Storytelling can also easily incorporate multiple representations and problem solving, which are other important national recommendations in mathematics education (Amies, 2002; CCSSI, 2010; NCTM, 2000, 2006, 2014).

We wrote the children's picture book, Ten jellies in a long box and one hundred jellies in a flat box (Kim & Park, 2013), whose storyline will be summarized in the case study below. We incorporated multiple representations into the picture book based on the Lesh Translation Model. For example, pictures of jelly boxes visualize the concepts of place value (*see Figure 2*).

We intentionally use this pictorial representation to translate between the pictorial and base-ten blocks—manipulatives commonly used in instructional activities for place value—and to help children move from the concrete to the linguistic (ones, tens, hundreds) and then to the symbolic (1s, 10s, 100s).

A Preset Groupable Model

We next developed physical manipulatives: a long box of ten "jellies"—2 centimeter plain wooden cubes—and a flat box of 10 long boxes (*see Figure 3*).

Thus, we directly connected the story to the real world (e.g., classroom). The physical model for base-ten concepts is different from the "pregrouped" models commonly used in instructional activities, such as base-ten blocks. It is also different from the "groupable" models that are "put-together-take-apart" models, but not inherently proportional (not ready-made to show the ten-to-one relationships), such as connecting cubes (Van de Walle, Karp, & Bay-Williams, 2015). In other words, the physical model allows children to put together single pieces/units (e.g., wooden cubes) to make a proportionally (ten times) preset unit (e.g., a full long box of ten wooden cubes). Children can also take single pieces/units (e.g., full long boxes of ten wooden

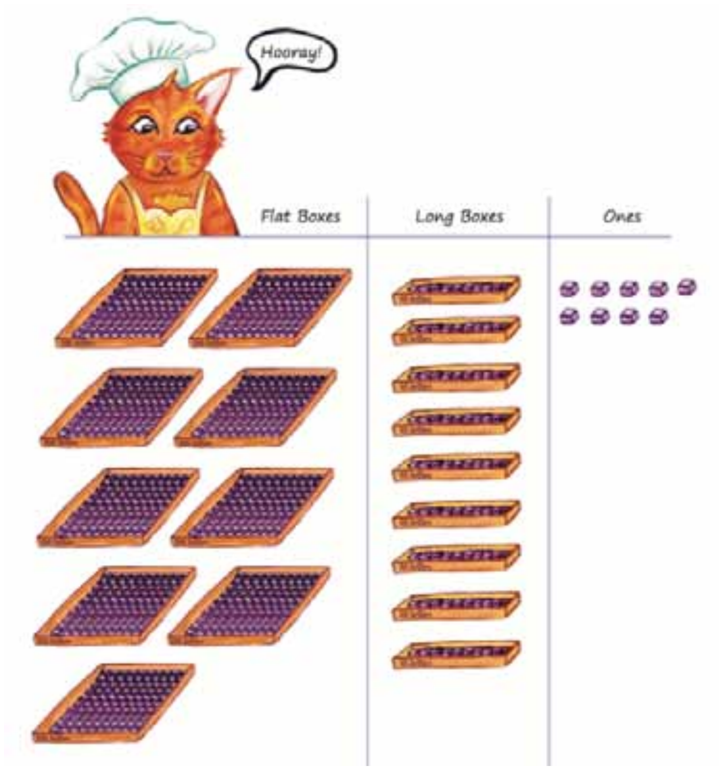


Figure 2. Ten Jellies in a Long Box and One Hundred Jellies in a Flat Box.

Figure 3. I Need, Who Has? and Place Value Shopping activities.

cubes) from a preset unit (e.g., a full flat box of 10 long boxes).

We call this physical model for base-ten concepts "a preset groupable model" or "a semi-pregrouped model." We believe that this model for base-ten concepts is easy to use and effective in supporting a conceptual understanding of the principles underlying place value. It makes ten-to-one relationships more clearly visible by allowing children to put together/take apart ones to/from preset tens. For example, children can create the number 132 (an amount of 132 jellies) using a full flat box of 10 full long boxes, 3 full long boxes, and 2 wooden cubes. Children can recognize that they have 13 full long boxes and 2 wooden cubes, or 123 individual cubes if the pieces all come apart. Children can see tens or hundreds as composite units of ones or tens, respectively.

Hands-on Activities

Finally, we developed two hands-on activities, "I Need, Who Has?" and "Place Value Shopping," utilizing physical manipulatives (see Figure 3). We anticipated that these activities would aid in the development of place value understanding by providing children with opportunities to make connections to multiple representations. For example, when children represent the number 23 (called out by a teacher or shown by peers) using jelly boxes, they are translating *from* verbal language/symbols *to* manipulatives. When children explain sixteen with base-ten language—that is, "one ten and six ones,"—they are translating *within* the language mode. The activities will be described in the following section with excerpts from student work.

A Case Study

A kindergarten class from the US Midwest participated in a curriculum module with a teacher and a researcher/observer during two consecutive classes meeting for approximately 60 minutes total over two days. The teacher started the class with a children's picture book, *Ten jellies in a long box and one hundred jellies in a flat box* (Kim & Park, 2013). Next, the children engaged in two hands-on activities, "I Need, Who Has?" and "Place Value Shopping."

Promoting Mathematical Reasoning and Problem Solving

Using a classroom document camera, the teacher first read the picture book to the kindergartners. The story's main character, Chef Chloe, starts her own jelly store in the Kitty Cat Village. Soon her jellies become famous all over the village. Many animals visit her store to buy yummy jellies. At first, she counts the jellies one by one whenever a customer wants to buy them. This process takes a long time and creates long lines. Many customers return home without any jellies after waiting in long lines. Chef Chloe wants to solve this problem.

During the storytelling, the teacher encouraged the children to engage in the story by posing questions: "*Why do you think the line was taking so long?*"; "Do you remember when you had to count things out?"; "*Can you turn and tell a neighbor what you think she [Chef Chloe] will do?*"; "Does anyone have an idea?" and so on. We could see that the kindergartners were actively engaged in the processes of reasoning and responding to the questions arising from the story. For example, almost all the children raised their hands to answer the questions (see the picture bottom right in *Figure 1*).

The kindergartners were naturally engaged in mathematical reasoning and problem solv-

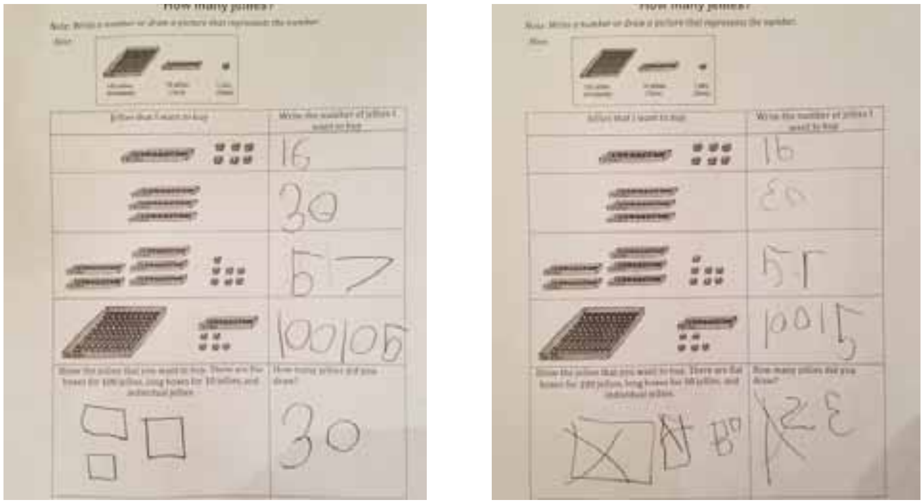


Figure 4. Samples of students' post-test answers.

ing: "*She [Chef Chloe] had to count every jelly out,*"; "*She can make a big box,*"; "*She can put them in groups of ten,*"; "[She would count them] by tens" and so on. The kindergartners also compared their solution strategies to the way that Chef Chloe made her customers happy with her jellies (see Figure 2): "*Oh, she [Chef Chloe] made long boxes of ten*"; "*Count boxes*"; "*By tens*"; "*Bigger trays*"; "*They are boxes of 100,*" and so on. Several children voluntarily translated from the pictures to verbal language using base-ten terms: "Oh, seven hundreds, nine tens, and nine ones." Finally, the teacher showed a picture (see the picture middle right in *Figure 1*) and asked "how many jellies can you see?" Almost all the kindergartners loudly answered together: "Three hundreds, two tens, and four ones," thereby connecting the picture to base-ten language.

Conceptual Understanding and Procedural Fluency with Multiple Representations

After the storytelling, the teacher started the activity "I Need, Who Has?" by asking the kindergartners to choose an amount of jellies within a range of numbers, and to represent that number by using wooden cubes (jellies), long boxes, and flat boxes: "*It is going to be a new number less than 20.*" This activity connected the story to the real world (the children's classroom experience). Then the teacher called out a number and asked a child who had represented the number to explain how she/he knew that her/his representation was that number: "*I need 17 jellies. Who has that?*"; "*Mason, how did you know so fast that this was 17 jellies?*"; "Anna, come on up and tell the class…" and so on. The children thus translated the teacher's verbal language to manipulatives. Then the children justified how they knew that their representations were correct by translating the manipulatives into base-ten language: "*This is ten [pointing out a full long box of ten wooden cubes]*"; "*11, 12, 13, 14, 15, 16, 17 [counting the seven wooden cubes together], seven more ones,*" and so on.

We also observed that the kindergartners were involved in meaningful mathematical discourse, prompted by purposeful questions arising from the teacher and the children themselves. The teacher engaged the kindergartners in math discussions, clarifying the mathematical ideas elicited by them to support and extend their learning. For example, the teacher often seemed to intentionally repeat the children's verbal thinking (e.g., "*So, I just added 3 left over*"; "*I took one away [pointing out a long box, which he took one away from, filled with nine cubes]…there are 9 left over.*") in a loud, slow voice: "*Added 3 left over…*"; "*TAKE one AWAY…there are 9 left over.*" The teacher emphasized these phrases to introduce the preliminary concepts of addition and subtraction.

After repeated activities with increasing numbers, such as less than 40 and less than 50, the kindergartners played "Place Value Shopping," a role-play game in groups of three. The teacher (customer) demonstrated the activity with two volunteers (a chef and a cashier) setting up their jelly store. To reinforce the children's conceptual understanding and procedural fluency on place value concepts, she used fake play money as a realistic representation with symbols (e.g., 1, 5, 10, etc.). Since money is a nonproportional model, we used only $1, $10, and $100 to reflect place-value relationships. Thus, each jelly cost $1. The children were challenged to use their understanding of place value, the manipulatives (wooden cubes, long boxes, and flat boxes), and base-ten language to represent numbers with a realistic model: the fake play money.

In "Place Value Shopping," a customer budgets to buy jellies (wooden cubes) for her/his family. The customer writes the amount of jellies on a mini dry-erase board (making connections/translating between realistic and symbolic representations). The chef prepares the jellies for the customer (translating from symbols to manipulatives), and the cashier receives and changes the money (translating from symbols to a realistic representation: fake paper money). Children create math problems by choosing their own numbers, representing these numbers with written symbols, manipulatives, and fake paper money. They then make self-evaluations by double-checking their solutions and strategies with one another (*see Figure 3*).

Effective Teaching of Place Value

Through participation in the literacy-integrated math curriculum module, the kindergartners had multiple opportunities to build their conceptual understanding of place value as they translated from one representation of numbers to another. This lesson sequence allowed the children to explore place value concepts by making connections and translating among the story, the real world, manipulatives, written symbols, and language. The children were encouraged throughout the curriculum module to use base-ten language. After the lesson sequence, more than 90 percent of the students were able to translate from the pictures and manipulatives to verbal language that included *ones*, *tens*, and *hundreds*.

Not surprisingly, the symbolic representation of place value was more difficult for the kindergartners to use, even though their understanding of place value concepts had improved. After the lesson sequence, more than 80 percent of the kindergarteners correctly wrote the symbolic representations of two-digit numbers with minor errors, but only 30 percent correctly wrote the symbolic representations of three-digit numbers (see *Figure 4*). By contrast, the pretest result showed about 40 percent of correct answers for two-digit numbers and about 10 percent for three-digit numbers.

This article also shares insights as to which pictorial and concrete (manipulatives) models are effective in developing place value understanding. The children engaged in the two hands-on activities by using our preset groupable model (or semi-pregrouped model) for the base-ten concepts (the physical model: wooden cubes, the long boxes, and the flat boxes). Yet they developed an understanding of the preliminary concepts of addition and subtraction themselves. It is likely that the children developed the ability to use mathematics with different representations.

Finally, this article illustrates how the Lesh Translational Model can provide early childhood teachers with guidelines while planning intentional mathematics instruction with explicit teaching goals and developing purposefully planned mathematics activities. Although this article focuses on an early intervention for kindergartners, the activities in the lesson sequence could also be modified for various students: as a supplement to the curriculum for first- and second-grade students, or as a remedial program for third-grade students who are struggling with place value and the base-ten number system.

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