Applying Newton's Law of Cooling When The Target Keeps Changing Temperature, Such As In Stratospheric Ballooning Missions

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Newton's Law of Cooling describes how a "small" system, such as a thermometer, comes to thermal equilibrium with a "large" system, such as its environment, as a function of time. It is typically applied when the environment is in thermal equilibrium and the conditions are such that the thermal decay time for the thermometer is a constant. Neither of these conditions are met when measuring environmental (i.e. atmospheric) temperature using a thermometer mounted in a payload lofted into the stratosphere under weather balloons. In this situation the thermometer is in motion so it encounters layer after layer of atmosphere which differ in temperature, and the changing environmental conditions can influence the thermal decay time "constant" for the thermometer as well. We have used Newton's Law of Cooling in spreadsheet-based computer simulations to explore how thermometer readings react under these conditions, to better-understand how logged temperature records from stratospheric balloon flights during both ascent (relatively slow) and descent (much faster, especially at altitude) are related to actual environmental temperatures at various altitudes.

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I. Introduction

Newton's Law of Cooling, for use in situations where heat is transferred by convection, states that the rate of heat flow dQ/dt is proportional to the difference an object's temperature T and the environmental temperature T_{env}.

$$dQ/dt = k * (T - T_{env})$$

Here the proportionality value k depends on heat transfer area and heat transfer efficiency and is usually assumed to be a constant.

If an object that starts at temperature T_0 in an environment of constant temperature T_{env} this law suggests that the object's temperature T[t] will decay exponentially in time toward T_{env} as

$$T[t] = T_{env} + (T_0 – T_{env}) * \exp[- t / \tau]$$

Here the characteristic decay time constant τ is how long it takes for the temperature it get within 1/e = 0.368 of the final value T_{env} (not to be confused with "half-life" $\tau_{1/2}$ for exponential decay situations which is how long it takes to get within 1/2 of the final value).

II. Excel Simulation Results

An Excel spreadsheet was written to simulate Newton's Law of Cooling using small time steps (much smaller than τ). Figure 1(a) shows T[t] for two objects which start at different temperatures but tend toward the same environmental temperature with the same decay time constant. Figure 1(b), on the other hand, shows T[t] for two objects with the same initial and final temperatures, but which have different decay time constants.

Figure 1(a) and Figure 1(b)

If the object in question is a thermometer and it is being used to measure the environmental temperature, a fair question might be “How long before the thermometer is reporting the environmental temperature?” The answer, as illustrated by the figures above, is “Eventually.” The farther apart the initial thermometer temperature is from the environmental temperature and the longer the decay time constant, the longer this will take. But at least after a few τ’s have elapsed the thermometer temperature will be indistinguishable from the environmental temperature.

However if the environmental temperature is constantly changing, such as is the case during stratospheric balloon flights, the answer becomes “Never!” Now the thermometer will “chase” the environmental temperature but will never come into equilibrium with it. The same Excel simulation was used to explore the changing-environmental-temperature situation. The simplest possible situation to consider is one where the environmental temperature changes linearly in time. Figure 2(a) shows how a specific thermometer (with a specific decay time constant) will eventually parallel the environmental temperature with the same offset temperature when the thermometer starts out warmer than or colder than (or even the same temperature as – not shown), the environment. Figure 2(b) shows how two thermometers with different decay time constants behave similarly, though the “faster” thermometer will reach the parallel-temperature condition more quickly and with a smaller temperature offset. Figure 2(c) illustrates how the temperature offset for a given thermometer is directly proportional to the rate at which the environmental temperature is changing. Simply put, Newton’s Law of Cooling suggests that a thermometer needs a specific temperature offset from the environmental temperature to drive it to change at a specific rate. Once that temperature offset is reached, the thermometer temperature will maintain that offset rather than continuing to get closer and closer to the environmental temperature.

Figure 2(a) and Figure 2(b) and Figure 2(c)

Even though the thermometer never reports the true environmental temperature in this situation, except if the two temperature curves happen to cross, the thermometer can still be used to determine the environmental temperature. To do this first document the decay time constant τ for the thermometer by doing exponential time decay fits of T[t] as the thermometer comes into equilibrium with a fixed-temperature environment. Then, when the thermometer is in contact with an environment whose temperature is changing linearly in time, note the drift rate R that the thermometer ultimately reaches which will be the same rate that the environmental temperature is changing with time. The time derivative of T[t] above, in the limit where t approaches zero, is $dT/dt = \Delta T * (- 1 / \tau)$. If this is to equal R, then the temperature offset must be $\Delta T = - R \tau$, a particularly simple result. Note: slight discrepancies from this result arise from the finite step size of the simulation. As anticipated above, the temperature offset grows linearly with the environmental temperature drift rate R. The minus sign indicates that the thermometer temperature always lags the environmental temperature. If R is positive (i.e. the environment is warming), then ΔT will be negative (i.e. the thermometer will always be behind (i.e. cooler than) the environment). Conversely, if the environmental temperature is going down then R will be negative so ΔT will be positive (i.e. the thermometer will again be behind (i.e. now warmer than) the environment).

An extension of the Excel spreadsheet allows us to simulate the response of a thermometer to the 5 phases of a typical stratospheric balloon flight: (1) temperature decreasing slowly during ascent through the troposphere, (2) temperature increasing slowing during ascent into the stratosphere, (3) temperature decreasing quickly during (post-burst) descent to the tropopause, (4) temperature increasing quickly during descent to the ground, and (5) temperature not changing back on the ground. Figure 3 shows how two thermometers with different decay time values, assumed to be constant throughout the flight, would respond to this actual temperature profile. Simply put, the thermometers report a "distorted" version of the profile. As before, the "faster" thermometer (i.e. the one with the shorter decay time constant) is more responsive and follows the environmental temperature changes more exactly, with less temperature offset.

Figure 3

The ultimate goal of this exploration is basically do this backwards – to determine the actual environmental temperature from stratospheric balloon flights using actual thermometer-reported temperature profiles such as those shown in the next section. "Correcting" the temperatures will require experimentally-determined knowledge of the decay time constants for thermometers.

III. Preliminary Experimental Results

The high-altitude ballooning teams at the University of Minnesota – Twin Cities and at St. Catherine University have historically made environmental temperature measurements during stratospheric balloon flights using (a) Onset Computer's Air/Water/Soil 1-foot temperature sensors for HOBO data loggers <http://www.onsetcomp.com/products/sensors/tmc1-hd>, (b) Neulog's wide-range temperature (thermocouple) sensors <https://neulog.com/wide-range-temperature/>, and Maxim's (Arduino-logged) DS18B20 Dallas 1-Wire digital temperature sensors <https://datasheets.maximintegrated.com/en/ds/DS18B20.pdf>.

To characterize the time decay constants for all 3 types of thermometers at the same time, a "triple-temperature" device was built (see Figure 4(a)) which included a HOBO, a Neulog chain, and an Arduino Uno, with the three sensors listed above mounted within 1.5 centimeters of each other (see Figure 4(b)).

Figure 4(a) and Figure 4(b)

This device was moved between a deep freeze and home-temperature air multiple times to characterize the decay time constant for each type of sensor. The time decay constant results in standard atmospheric pressure were as follows:

τ_{HOBO} = 223 seconds; τ_{Neulog} = 23 seconds; τ_{Dallas} = 190 seconds

To determine if τ values change with environmental pressure – we hypothesized that the sensors would be slower (i.e. have larger τ values) at reduced pressure – the device was "slim-mounted" on a sled that could fit into a 3-inch diameter pvc tube which was then evacuated using a vacuum pump. The two ends of the tube were held at different temperatures by covering one end in ice cubes. The sled started at the cold end but then was slid to the warm end without breaking the vacuum seal by tipping the tube – one could hear it slide through the tube easily. Figure 5(a) shows the slim-mounted version and Figure 5(b) shows the pvc experimental set-up for the reduced-pressure test.

Figure 5(a) and Figure 5(b)

Plots of the reduced-pressure test results are shown in Figure 6(a) – Pressure (all versus time), Figure 6(b) – Arduino (Dallas) temperature, and Figure 6(c) – HOBO temperature. The Neulog sensors turned off mid-test so no useful data was forthcoming. The time decay constant results in reduced pressure are listed below. As anticipated, they were larger than the values at atmospheric pressure (for the two sensors tested).

τ_{HOBO} = 370 sec; τ_{Neulog} = TBA (sensor failed); τ_{Dallas} = 236 sec

Figure 6(a) and Figure 6(b) and Figure 6(c)

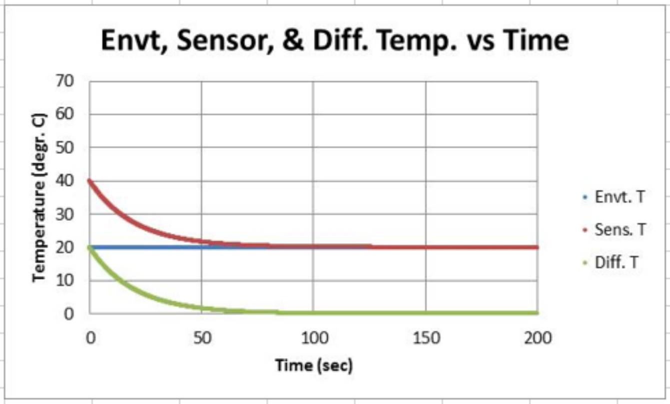
The triple-temperature device has been flown on two stratospheric balloon missions so far. Figure 7(a-d) shows temperature versus time graphs for the 3 types of temperature sensors (plus one pressure versus time graph) from one actual flight. Future work includes trying to apply simulation capabilities and experimental knowledge of decay time constants and standard pressure and at one reduced pressure to determine what single/actual atmospheric temperature profile is simultaneously consistent with all the plots below.

Figure 7(a) and Figure 7(b) and Figure 7(c) and Figure 7(d)

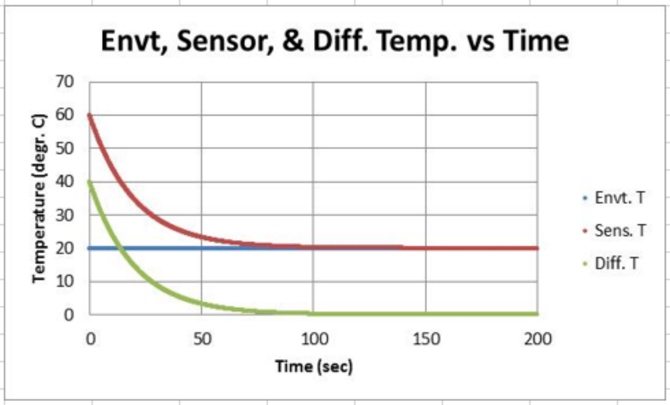
IV. Conclusions

Implementing Newton's Law of Cooling using an Excel spreadsheet has allowed us to apply it to situations where the environmental temperature is not constant. This has given us insight into differences between thermometer readings and various time-dependent profiles of actual environmental conditions, with an eye toward ultimately reaching conclusions about atmospheric temperatures during high-altitude balloon flights using "distorted" thermometer records due to decay time constants. Comparison of thermometer decay times at atmospheric pressure and at reduced pressure suggest that these values are not in fact constant over the wide range of pressures encountered during stratospheric ballooning missions, further complicating analysis of (and correction of) logged temperature data.

Figure 1(a)

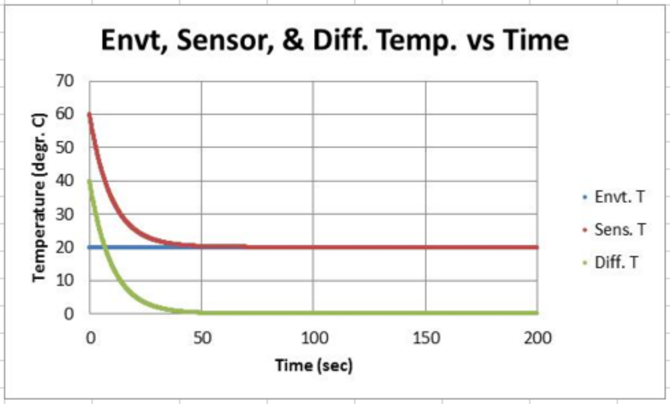


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	40
time step size (s)	0.1
envt temp drive (degr/s)	0
asymptotic diff (degr)	0.00

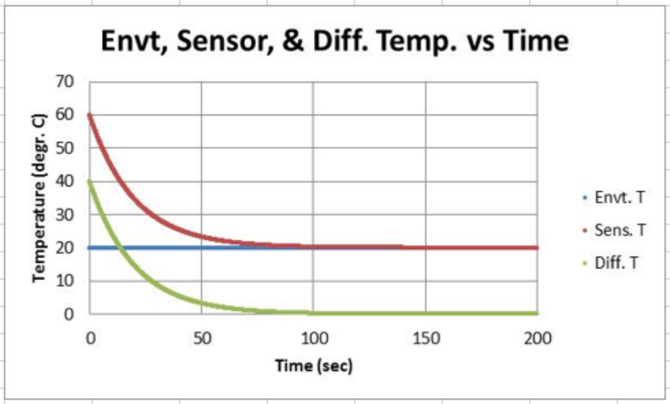


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	0
asymptotic diff (degr)	0.00

Figure 1(b)

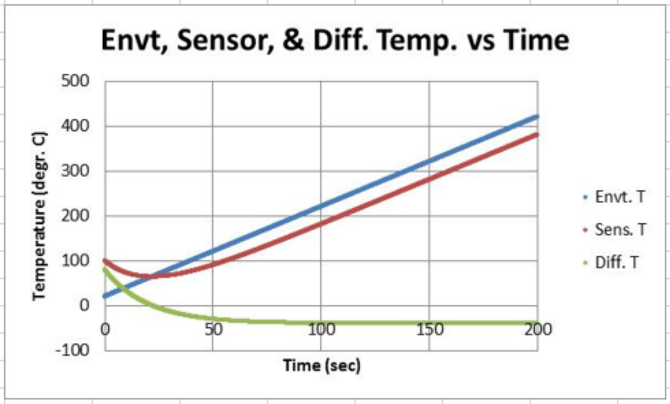


sensor decay time (s)	10
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	0
asymptotic diff (degr)	0.00

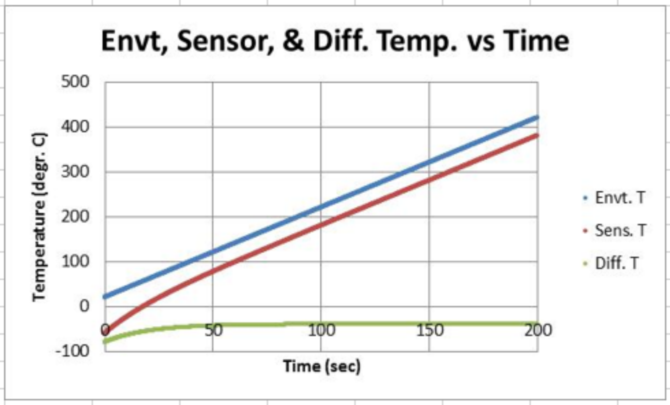


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	0
asymptotic diff (degr)	0.00

Figure 2(a)

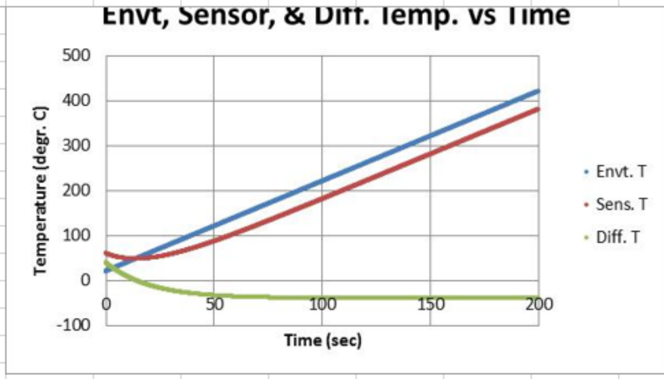


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	100
time step size (s)	0.1
envt temp drive (degr/s)	2
asymptotic diff (degr)	-40.09

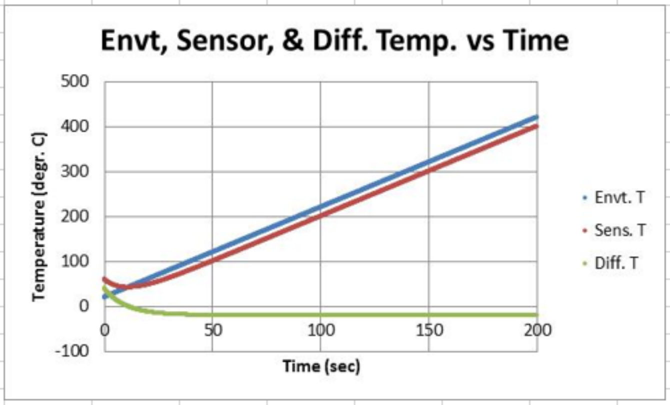


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	-60
time step size (s)	0.1
envt temp drive (degr/s)	2
asymptotic diff (degr)	-40.10

Figure 2(b)

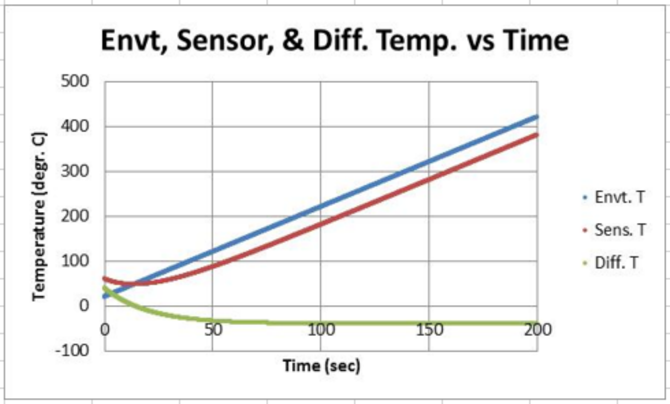


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	2
asymptotic diff (degr)	-40.10

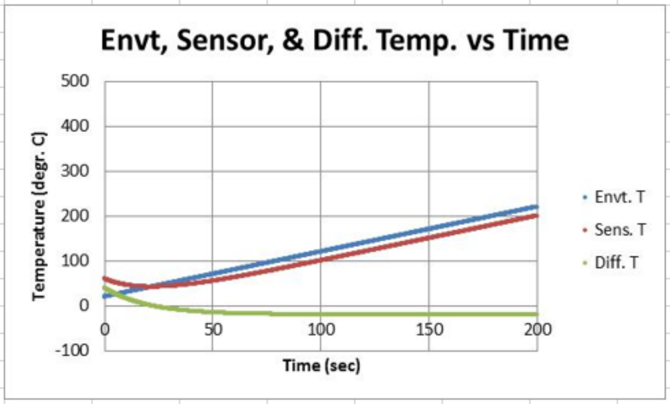


sensor decay time (s)	10
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	2
asymptotic diff (degr)	-20.1

Figure 2(c)

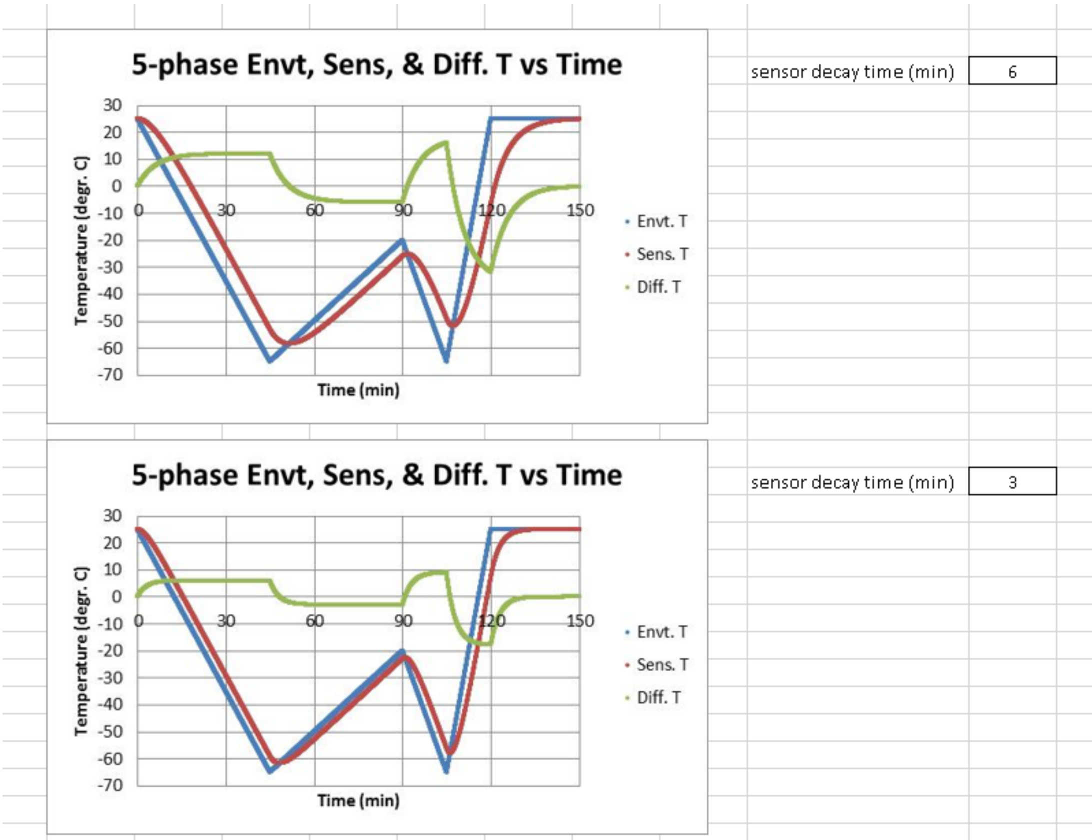


sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	2
asymptotic diff (degr)	-40.10



sensor decay time (s)	20
init env temp (degr)	20
init sensor temp (degr)	60
time step size (s)	0.1
envt temp drive (degr/s)	1
asymptotic diff (degr)	-20.05

Figure 3



5-phase Envt, Sens, & Diff. T vs Time
Temperature (degr. C)
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
0
30
60
90
120
150
Time (min)
Envt. T
Sens. T
Diff. T
sensor decay time (min)
6
5-phase Envt, Sens, & Diff. T vs Time
Temperature (degr. C)
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
0
30
60
90
120
150
Time (min)
Envt. T
Sens. T
Diff. T
sensor decay time (min)
3

Figure 4(a) – Triple-temperature device, basic mounting – Neulog modules on underside

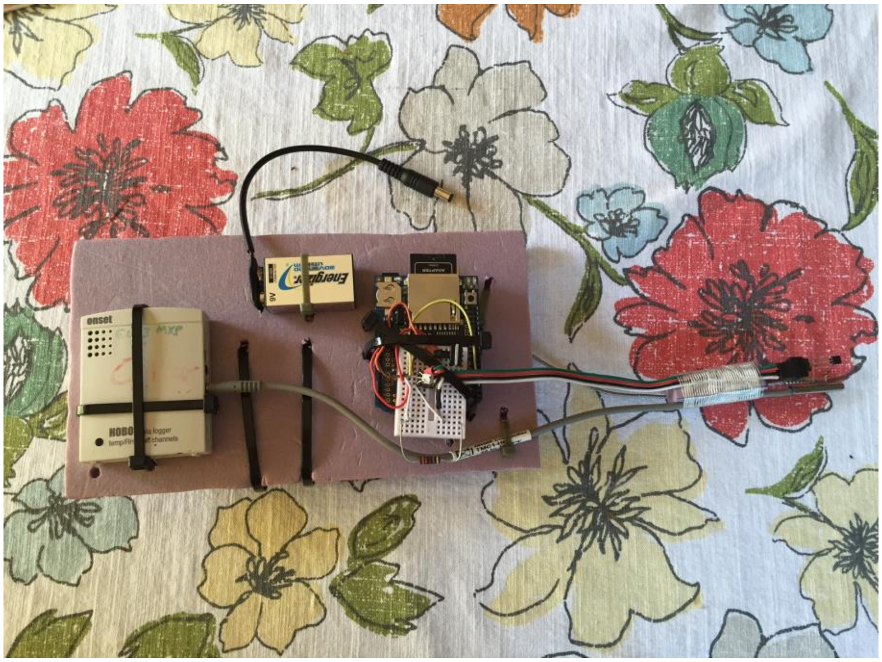


Figure 4(b) – close-up of the triple-temperature tip

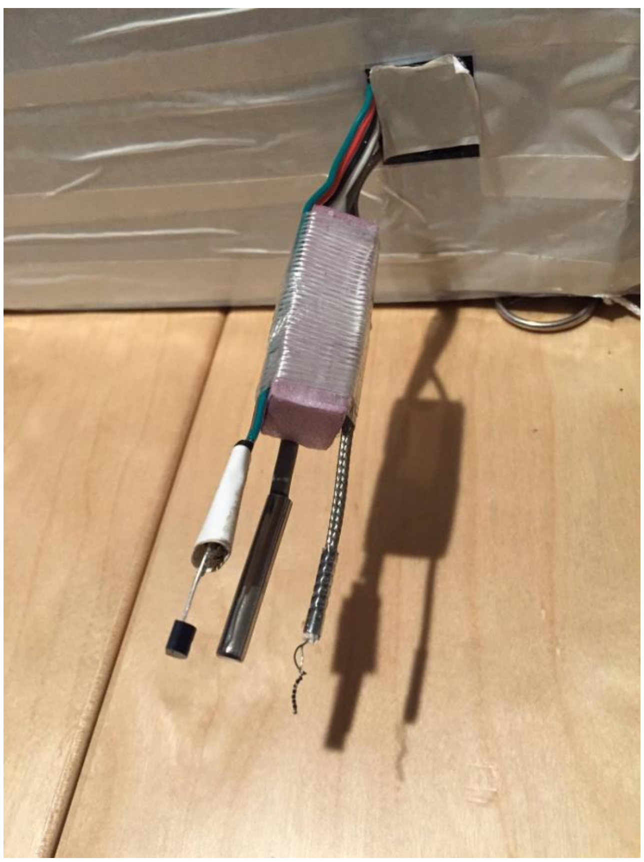


Figure 5(a) – Slim mount for reduced-pressure test.

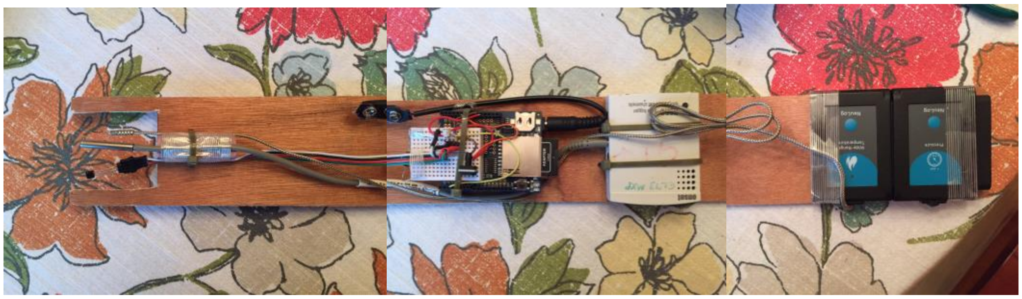


Figure 5(b) – PCV pipe set-up for reduced-pressure test

Figure 6(a) and Figure 6(b) and Figure 6(c)

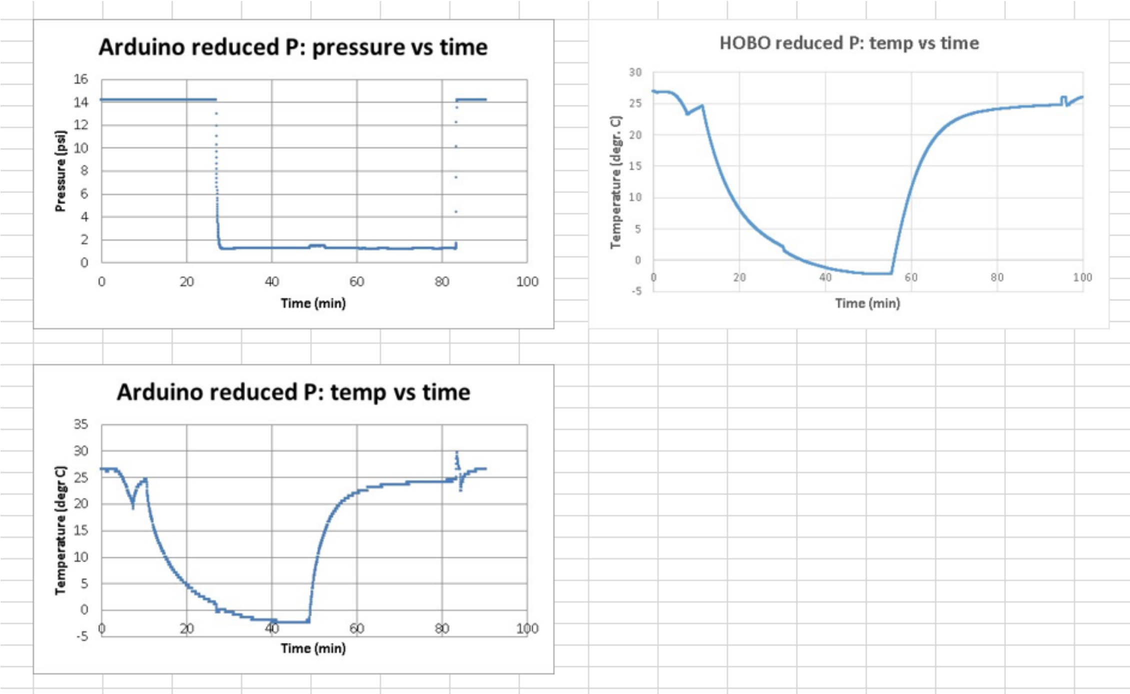


Figure 7(a) and Figure 7(b) and Figure 7(c) and Figure 7(d)